Physics. — *Second order disturbance terms in pendulum observations at sea.* By F. A. VENING MEINESZ.

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An investigation of Mr. B. C. BROWNE, Cambridge, about the measurement of gravity at sea, which by kind courtesy of Mr. B. the writer has been able to study before it appeared in the Geophysical Supplement of the Monthly Notices of the R.A.S., has revealed that there are some second order terms of the disturbances caused by the ship's movements in these measurements, which cannot always be neglected and which, even for measurements in submerged submarines, may attain values of more than 10 milligals. The writer feels deeply indebted for this study. In this note he wishes to give a few general remarks about it as far as it concerns the measurement of gravity at sea by means of pendulum observations in a submerged submarine.

In this case the second order disturbance term is

$$\delta g = -\frac{\overline{\ddot{x}^2}}{4g} + \frac{\overline{\ddot{y}^2}}{2g} + \frac{\overline{\ddot{z}^2}}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$\ddot{x}$ is the vertical acceleration of the apparatus and $\ddot{y}$ and $\ddot{z}$ the horizontal accelerations perpendicular to the ship's axis and in the direction of the ship's axis resp. The dashes above the terms indicate mean values over the time of observation. The first term originates from the fact that the pendulum observations provide us with the mean value of $\sqrt{g}$; in general this does not equal the root of the mean value of g — which last quantity is undisturbed by the vertical accelerations of the apparatus — but differs from it by terms of second and higher order. The two last terms result from the vectorial addition to g of the horizontal accelerations in y and z sense; the resultant differs likewise from g by second and higher order terms of these accelerations. The formula for this effect, however, is only true for accelerations whose periods are long compared with the periods of the pendulums and of the damped pendulums of the pendulum apparatus; for smaller periods the formulas change. In the case of quickly varying accelerations of a period small compared to that of the pendulums and of the damped pendulums, the effect becomes negligible. This explains that there have never been found any traces of second order terms in harbours where the gravity value in the harbour could be compared to that on land. The horizontal accelerations in the harbours, although usually especially large, appear to be exclusively of the short periodic type and so the second order terms seem to be negligible notwithstanding the great size of the accelerations.

Two points have now to be considered. In the first place whether the accelerations can be determined; this would enable us to apply the corrections and thus to maintain the old standard of accuracy for these observations even for strong ship's movements, which otherwise would be correspondingly reduced. In the second place whether the past gravity material at sea can be corrected for them and whether the figures given for the mean errors of these results have to be increased. In connection with this point an investigation of the ship's movements and of their effect on these corrections is important.

The determination of the vertical accelerations does not present a serious problem. Their effect appears in the records of the gravity apparatus as fluctuations of the time-marks in the pendulum records. The vertical accelerations can thus be computed for the ascending and descending parts of the time-mark curve, i.e. for about one third part of each coincidence interval and this is sufficient for determining the correction for gravity for the whole observation with satisfactory accuracy. The formula for computing the correction is

$$\delta g = \frac{g}{2}\left(\frac{T}{\tau}\frac{\alpha}{a}\right)^2$$

in which

T = pendulum period,
a = pendulum amplitude on the record,
τ = half period of fluctuations of time-marks on the record,
α = amplitude of fluctuations of time-marks at the centre of the record.

The determination of the horizontal accelerations is a more difficult problem. They bring about fluctuations of the direction of the apparent gravity and the problem is to measure these fluctuations. The possibility mentioned by BROWNE of making a record of the position of the horizon, is excluded in a submerged submarine. Experiments are under way to do it by installing on the gravity apparatus a slow pendulum, of which the period is long compared with the period of the horizontal accelerations. As the period of the long ocean-swell, which is practically the only one that is perceptible at the depths where the observations are made, may be as long as 12 seconds for a complete revolution, we need a pendulum whose period is at least 30 or 40 seconds for a double swing.

The experiments for making such a pendulum give hopeful results. A horizontal beam of about 30 cm length with weights of about 1 kg at each end is suspended by two tiny springs. It is provided with two movable weights for the adjustment of the position of the centre of gravity in a vertical and horizontal sense. It is mounted in an air-tight case from which the air can be removed. Slight air-currents can be let in for bringing the pendulum back to its normal position when desired. The position of the pendulum will be recorded on the same record that serves for the recording of the main pendulum apparatus. The pendulum will only follow the fluctuations of the vertical for a slight part, dependent on the difference in periods, and as the apparatus, hanging in gimbals, follows it for a large part, the record of the angle between the pendulum and the apparatus will show a fluctuation from which the fluctuation of the vertical can be deduced. An accuracy of about $^1/_{25}$ of the fluctuation is enough for allowing the necessary accuracy in the resulting correction for gravity. Fluctuations exceeding half a degree are expected to be exceptional. The reading accuracy on the record can easily be brought to a quarter of a minute of arc.

For the past gravity observations, it is easy to determine the vertical accelerations; they can be deduced from the records by measuring the fluctuations of the time-mark curves. So the first term of formula (1) can be computed. This work is now under way for all the gravity observations at sea made by the Netherlands Geodetic Commission and it is hoped that early in the next year the results will be ready for publication. At this moment only a short provisional statement can be made, resulting from a superficial examination of the records.

For a great number of observations the correction of the gravity result appears to be less than one milligal and the number of observations for which it exceeds a few milligal is relatively small. Referring to the list of stations on page 89 at seq. of "Gravity Expeditions at Sea", vol. II, the corrections seem to be small for all the stations Nos 196 to 486. For those stations in the East Indies enclosed within the Archipelago — i.e. not outside in the Indian Ocean or in the Pacific — they are probably all below one milligal. The same may be said for the stations 426 to 486 in the Atlantic Ocean. For the stations of the last expedition, the voyage of Hr.Ms. O 16 from Holland to Washington and back, which was accomplished under unusually difficult conditions of heavy seas, great values for this correction, up to 30 milligals, do occur.

The last term of formula (1) will probably be below one milligal for practically all the gravity observations which have so far been made; the writer does not think it likely that for work on board submerged submarines it can ever attain values that need be taken into account. This opinion is founded on the supposition that the horizontal accelerations in the sense of the ship's axis are less than one fifth of the vertical accelerations and in that case, as the correction depends on the square of the accelerations, though the denominator is half as great, the corrections are less than 8 % of those for the vertical accelerations. This supposition will, however, have to be proved experimentally and it is hoped that the apparatus mentioned above will provide the means of doing so.

The most difficult point to settle is the value of the second term of the formula (1), i.e. that which depends on the horizontal accelerations perpendicular to the ship's axis. Here future experiments will certainly be necessary for getting further insight. As a first supposition it may perhaps be assumed that the horizontal acceleration in this sense is equal to the vertical acceleration multiplied by the sine of the angle between the direction of the swell and the ship's axis, but it is felt that this supposition, starting from the assumption that the ocean-waves at the depth of the observation are caused by circular movements of the water particles, cannot be considered as more than a tentative idea.

For part of the past gravity observations data are available about the direction of the swell and this will be a valuable help for the deduction of this term. For those observations about which all data are lacking, there is doubtless no other way than to accept the direction of the swell as unknown and to compute the increase of the mean error of the gravity result which corresponds to this uncertainty.

According to formula (1) the correction for the effect of the vertical accelerations makes the gravity anomalies more positive while that for the horizontal accelerations has contrary sign and double its value for the same value of the accelerations. Taking the mean value of the square of the sine of the angle between direction of the swell and the ship's axis for a great many observations at 0.5, the two terms would cancel each other and so the writer believes that the mean total effect for a great many observations will be small, although he is aware of the fact that the mean value of the effect of the horizontal accelerations has probably thus been sowewhat underestimated — for small angles between swell and ship's axis the acceleration is probably not quite proportional to the sine of this angle but greater — and so it is likely that this effect will more or less predominate above that of the vertical accelerations. It is clear, however, that all these considerations are highly speculative and so an experimental investigation is needed before any more exact treatment of the problem can be made.

In a few months the writer will make another expedition on the Atlantic for putting the described method for measuring the horizontal accelerations to a trial and, if it is successful, for making an investigation about the ship's movements at different depths and for different angles between the ship's course and the direction of the main swell. He hopes soon, therefore, to be able to publish further information about these questions.

This paper may give an idea of the importance for gravity work at sea of the effect of the second order disturbance terms, dealt with by Mr. BROWNE. The writer wishes to express his sincere gratitude to him for his valuable investigations.