about only 10 % of the total vitamin A activity of the provitamins). The average real vitamin A level of prisoners and Europeans, etc. is practically equal, notwithstanding the fact that the former in contrast with the latter have to draw all their vitamin A from he provitamin mentioned. The low α- and β-carotene level may be explained by the fact that much more than in the case of Europeans, etc. this is (and has to be) converted into vitamin A. The conversion of α- and β-carotene into vitamin A is apparently easier than that of cryptoxanthene. It may be questionable whether cryptoxanthene (which in the vegetable foodstuffs of tropical countries occurs much more frequently than in those of colder regions) is indeed a provitamin A for man as it is for the rat. To these and similar questions we shall revert more explicitly in future publications.

Finally we wish to express our thanks to the Institute for Nutrition Research for the financial aid received.

Batavia C., September 1937.

Chemical Department of the Central Medical Laboratory.

Hydrodynamics. — *Preliminary records of the velocity fluctuations in a boundary layer before and after the transition to turbulent motion.* (Mededeeling N°. 33 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft.) By G. BROERSMA. (Communicated by Prof. J. M. BURGERS.)

(Communicated at the meeting of October 30, 1937.)

1. *Introduction.*

In an earlier communication from the Laboratory for Aero- and Hydrodynamics of the Technical University at Delft [1]) some experiments have been described on the simultaneous recording of the fluctuations of the air velocity in a windtunnel by means of two hot wire anemometers, which were placed at a small distance from each other in the same plane perpendicular to the direction of the motion of the air. At distances of the order of magnitude of 1 cm or less a distinct correlation can be remarked in the records given by the two wires, which correlation gradually disappears when the distance becomes larger. It was then planned to perform similar experiments in the boundary layer along a flat plate, and extensive series of measurements have been performed already in 1924, on which, however, only a short communication has been published [2]). Afterwards, in 1930

1) J. M. BURGERS, Experiments on the fluctuations of the velocity in a current of air, Proc. Royal Acad. Amsterdam, **29**, 547 (1926).

2) J. M. BURGERS, Experimental investigation of the motion of the air in the boundary layer along a smooth surface (in russian), Journ. of Applied Physics (Moscow), **4**, 7—9 (1927).

and 1931, experiments were made by ZIEGLER with two hot wires of very small dimensions, connected with the amplifying sets described in "Mededeelingen N^0. 20, 21" [3]), and it had been observed by ZIEGLER that when two hot wires were placed one at a distance of *e.g.* 40 cm downstream of the other, in the boundary layer developing along a flat plate with a velocity in the windtunnel of the order of 5 m/sec (so that the state of motion in the layer for the main part was laminar), fluctuations recorded by the foremost wire sometimes were found back, smeared out over a longer interval of time and often with more rapid fluctuations developed, in the records obtained with the second wire.

Plans were made for investigating more systematically the development of disturbances arising in a laminar boundary layer, but a delay arose in consequence of other work, and it was only recently that some preliminary experiments on the simultaneous recording of fluctuations in the laminar and in the turbulent part of a boundary could be taken in hand.

2. *Installation of the glass plate for the boundary layer measurements.*

Before beginning with the measurements proper the velocity distribution was investigated in a cross section of the windtunnel which was to be used for the experiments. This tunnel is of the closed type; the measuring space has a square cross section of 50 × 50 cm^2, and a length of 270 cm. The results of the exploration at a distance of 25 cm behind the entrance of the measuring space proved to be sufficiently favourable both with a mean velocity of 6 m/sec and of 16 m/sec (deviations from the mean value of less than 2 %, with a rather regular distribution, over an area of 40 cm width and 30 cm height); it was thought unnecessary to explore also sections further downstream.

A difficulty which is encountered with experiments on the flow along glassplates in windchannels of restricted dimensions is to give such a position to the plate that the velocity of the flow outside of the boundary layer shall have a constant velocity over the whole length of the plate. In the present case the leading edge of the glassplate was sharpened unsymmetrically; moreover it was considered desirable to bring the plate rather near to the lower wall of the channel. The plate was first laid parallel to the axis of the tunnel; investigation, however, of the velocity distribution in a vertical section of the boundary layer at a distance of 6,0 cm from the leading edge of the plate, gave a curve with a distinct *S*-shape, thus revealing that separation of the flow from the plate had occurred. After several adjustments finally a position was found in which this shape had disappeared, and for which at the same time in points at 70 and 111 cm respectively from the leading edge (the distance at which

[3]) M. ZIEGLER, The application of the hot-wire anemometer for the investigation of the turbulence of an airstream, Proc. Royal Acad. Amsterdam, **33**, 723 (1930); A complete arrangement for the investigation, the measurement and the recording of rapid airspeed fluctuations with very thin and short hot wires, *ibid.* **34**, 663 (1931).

the records have been taken to be considered below) the velocities at 7 cm from the plate proved to be equal, so long as they did not surpass *ca.* 11 m/sec. In this case the plate from leading edge to trailing edge inclined upwards at an angle of 0,89° with respect to the bottom of the windtunnel. The leading edge was 3 cm behind the entrance plane of the measuring space.

In order to keep the tunnel free from unessential apparatus which might influence the airflow, the windspeed at the points mentioned was compared with the pressure drop between two points of the wall of the tunnel, one well upwards from the measuring space, where the cross sectional area of the tunnel was much larger, and one near to the measuring space. The windspeed then afterwards was found from this pressure drop.

3. *The transition point between the laminar and the turbulent parts of the boundary layer.*

The object of the experiments was to produce a situation in which the wire at 70 cm from the leading edge still should be in the laminar part of the boundary layer, whereas the wire at 111 cm should be in the turbulent part, in order to obtain a comparison between the simultaneous fluctuations at those two places. It was necessary therefore to ascertain the position of the transition point for various velocities, so that a velocity might be chosen at which the transition point would be situated between those two places.

This was done by measuring the velocity distribution at the two places chosen by means of hot wires, for velocities V outside the boundary layer increasing with steps of about 0,5 m/sec. With increase of the tunnel velocity the transition point moves towards the leading edge; when it passes the position of a measuring wire, a characteristic change occurs in the shape of the curve representing the distribution of the velocity as a function of the distance from the plate. So long as the motion is either laminar or turbulent, there is a gradual increase of the velocity gradient at the surface of the plate, and simultaneously a gradual decrease of the boundary layer thickness; when the transition from the laminar motion occurs to the turbulent motion, however, the velocity gradient at the surface of the plate takes a greater value while at the same time the thickness of the boundary layer *increases*: hence the curve for the case of turbulent motion crosses the curve for the case of laminar motion in a way which has been described already earlier [4]), and which is not difficult to recognize.

The measurements were repeated a number of times on various days; they gave the following results:

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[4]) J. M. BURGERS and B. G. VAN DER HEGGE ZIJNEN, Preliminary measurements of the distribution of the velocity of a fluid in the immediate neighbourhood of a plane, smooth surface, Verhandel. Kon. Akad. v. Wetensch. Amsterdam (1e sectie) XIII, N^0. 3 (1924); J. M. BURGERS, Versl. Kon. Akad. v. Wetensch. Amsterdam, **32**, p. 856 (1923).

a. mean value of V when the transition point was at $x = 70$ cm from the leading edge: 7,75 m/sec;

corresponding mean value of REYNOLDS' number ($R = \varrho\ V\ x/\mu$): 362.000;

b. mean value of V when the transition point was at $x = 111$ cm from the leading edge: 5,55 m/sec;

corresponding mean value of REYNOLDS' number: 412.000.

The difference between the two values found for the REYNOLDS number must be ascribed to the fact that the fluctuations of the velocity in the flow outside of the boundary layer are more damped at the larger distance from the leading edge.

The experiments further were made with a velocity $V = 6{,}4$—$6{,}5$ m/sec, for which the transition point could be assumed to be at 90,5 cm.

4. *Arrangement and calibration of the hot wires.*

The hot wires used had a length between 2 and 3 mm, and a thickness of 0,015 mm. They could be moved upwards and downwards from the glassplate; measurements were performed at distances of 1, 2, 3, ... 10, 15 and 50 mm from the latter. The electrical connection for every wire was the ordinary Wheatstone bridge arrangement, two independent circuits being used; no amplifiers were inserted, as torsion string galvanometers were used, which had sufficient sensitivity for the currents applied here, and which responded to fluctuations with a maximum frequency of about 50 per second. The deviations of the galvanometers were recorded photographically on a film of sensitive paper, together with a time signal.

From the curves for the velocity distribution in function of the distance from the plate the value 7 mm was estimated for the thickness of the boundary layer at $x = 70$ cm, and the value 9 mm at $x = 111$ cm.

The determination of the scale of the records was made in the following way:

The calibration curve for a hot wire anemometer can be represented by a formula:

$$I^2 = f(V, W) \quad \ldots \ldots \ldots \quad (1)$$

where V is the velocity of the wind, I the electric current through the wire and W the electrical resistance of the wire, the equation referring to the state in which the production of heat by the electric current is equal to the loss of heat due to the wind. From this equation one deduces by differentiation:

$$2I\left(\frac{\partial I}{\partial W}\right)_{V=const.} = -\frac{\partial f}{\partial V}\left(\frac{\partial V}{\partial W}\right)_{I=const.} \quad \ldots \quad (2)$$

As the function f generally has the form: $f = a + b\sqrt{V}$, (a and b

being functions of W), we have $\partial f/\partial V = b/2\sqrt{V}$, and hence it is found:

$$\left(\frac{\partial V}{\partial W}\right)_{I=const.} = -\left[\frac{4I\sqrt{V}}{b}\right]\left(\frac{\partial I}{\partial W}\right)_{V=const.} \quad . \quad . \quad . \quad . \quad (3)$$

The expression [ ] can be derived from the calibration curve corresponding to the state in which the hot wire was used for the recording of the velocity fluctuations. The differential quotient $\left(\frac{\partial I}{\partial W}\right)_V$ can be obtained by taking calibration curves (I *vers.* V) for different values of the resistance of the wire. These different resistances are obtained by balancing the bridge for zero velocity at different values of the electric current through the wire.

A variation of the resistance of the wire with the amount $\triangle W$ now will cause a shift $\triangle s$ on the record. Hence eq. (3) leads to the following one:

$$\frac{dV}{ds} = \frac{\partial V}{\partial W}\frac{dW}{ds} = -\left[\frac{4I\sqrt{V}}{b}\right]\left(\frac{\partial I}{\partial W}\right)_V \frac{dW}{ds}.$$

The value of the quotient $\frac{dW}{ds}$ we find as follows: substitute for the hot wire successively two resistances of known values, slightly differing from the resistance of the hot wire. Let the electric current through the substituting resistance have the same value as the one used when the record was taken (the distance of the galvanometer from the film being, of course, unchanged). Then the shift $\triangle s$ corresponding to a known value of $\triangle W$ can be determined.

As an instance we give the values obtained for dV/ds for the records with the two wires at a distance of 1 mm from the plate:

	Forward wire	Backward wire	
Series I	0,051	0,098	m/sec windvelocity per mm
Series II	0,043	0,082	on the record.

5. *Results of the measurements.*

The obtained records revealed a marked difference in the character of the fluctuations recorded respectively in the laminar and in the turbulent part of the boundary layer. (See f.i. fig. 1 in which are reproduced some of the analysed parts of records taken on 10-IX-1937).

An extensive search was made in order to find whether a disturbance could be pointed out, which had been recorded by both wires. Owing to the fact that the records were too full of detail no sharp evidence could be found.

It was then tried to arrive at a numerical characterisation of the records by determining the mean square of the deviations of the velocity from its mean value, and the mean frequency. In order to find the mean square

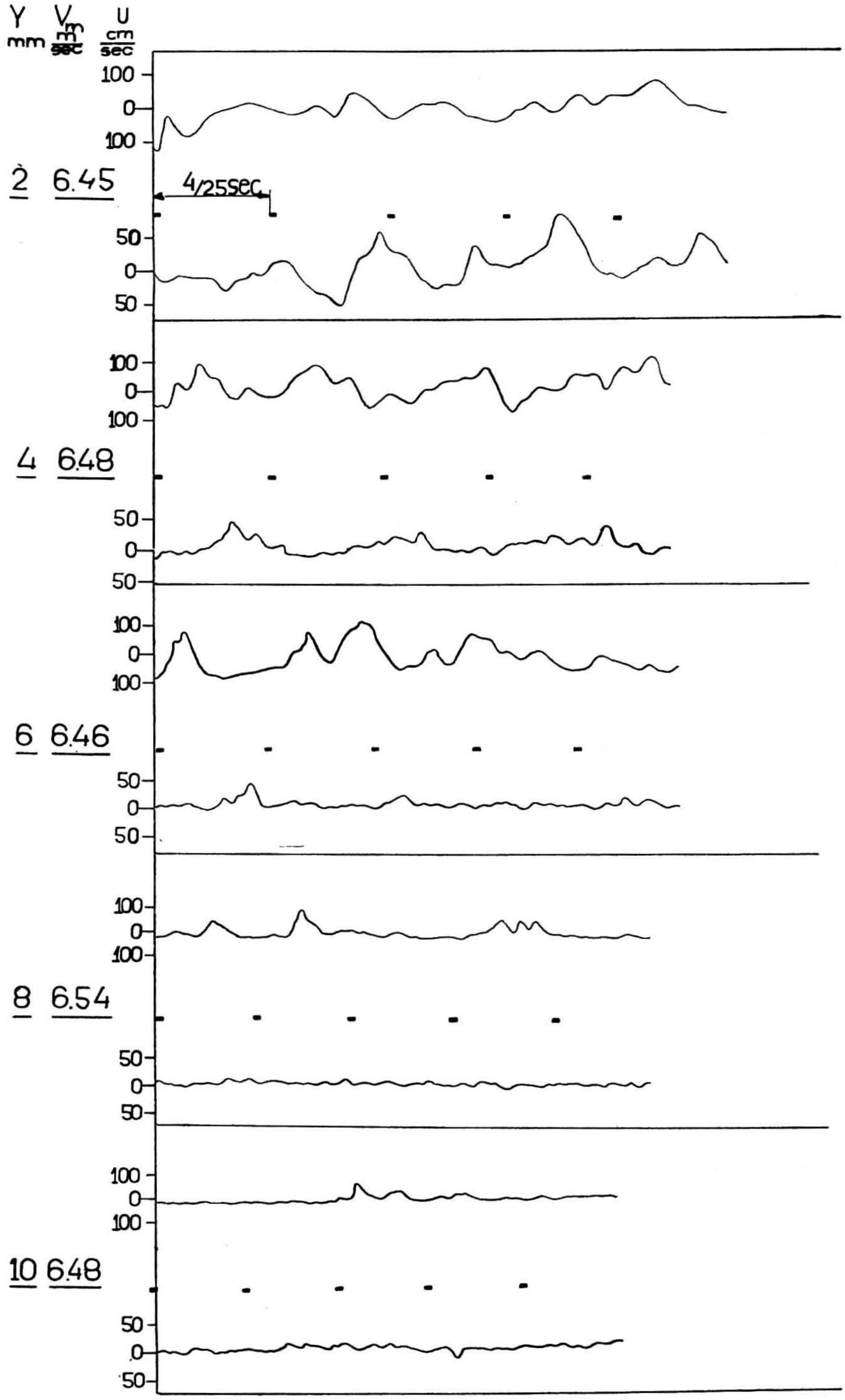

Fig. 1. Sections from records of velocity fluctuations in the boundary layer, obtained at various distances y from the wall. The upper curve of every record refers to the turbulent part of the boundary layer; the lower curve to the laminar part. The mean air velocity in the windtunnel is given to the left of each record; practically: $V = 6{,}5$ m/sec.

of the deviations, a transparent scale of horizontal lines was laid over the record, and for a definite period it was counted how many times every line was crossed by the record. When the lines are numbered 1, 2, 3, ... and when the number of times that the record crosses the line p is indicated by n , then the mean position of the line recorded is given by:

$$a \frac{\Sigma\, p\, n_p}{\Sigma\, n_p},$$

and the mean square of the deviation from this position by:

$$a^2 \left\{ \frac{\Sigma\, p^2\, n_p}{\Sigma\, n_p} - \left(\frac{\Sigma\, p\, n_p}{\Sigma\, n_p} \right)^2 \right\},$$

a in these equations being the change in velocity corresponding to a displacement on the record, equal to the distance between two consecutive lines of the scale. — The frequency was found by counting the number of extremes over a definite period. — Both methods of evaluation are rather rough, but in consequence of the absolutely irregular character of the fluctuations, the application of any other method would lead to an enormous amount of work. In view of the circumstance that records taken over consecutive periods of a second may give quite different results, so that a large number of cases ought to be considered in order to get reliable statistical results, such methods would be out of place here.

The values found for $\overline{u^2}$ in the flow outside of the boundary layer were less than 0,01 (m/sec)^2, thus corresponding to a mean amplitude of the fluctuation of about 1 or 2 % of V. (See fig. 2 which gives sets of curves

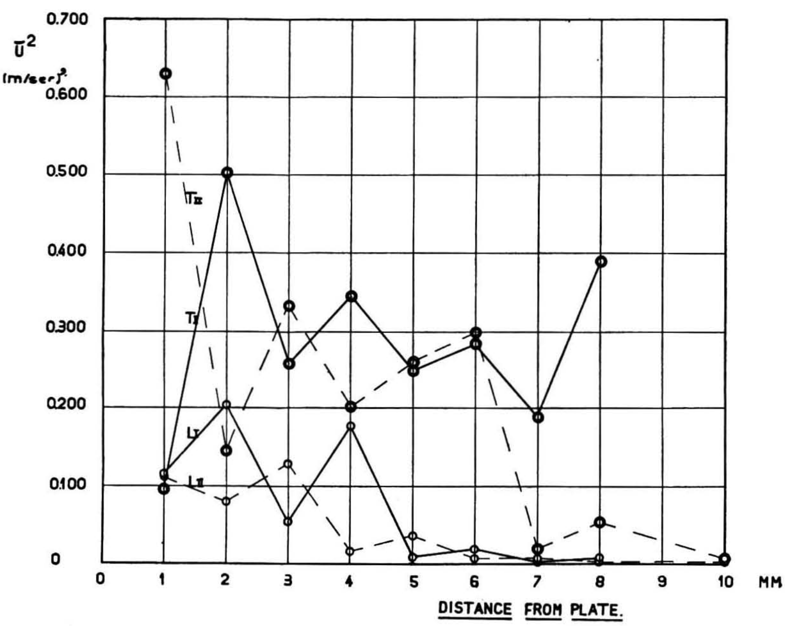

Fig. 2.

calculated from two series of records of the velocity-fluctuations in the laminar and turbulent parts of the boundary layer). Both in the case of the laminar and in that of the turbulent boundary layer the value of $\overline{u^2}$ increases when the distance from the place becomes less; for the laminar layer to values between 0,1 and 0,2 $(m/sec)^2$, and for the turbulent layer to values ranging up to 0,5 or 0,6 $(m/sec)^2$. In the first case we have fluctuations of the order of 5 %; in the second case of more than 10 %. In this respect the difference between the two types of flow seems to be well marked, although it appears that in the laminar layer under the conditions of the experiment (that is, in a section already near to the transition point, which of course must not be considered as a well marked point, but as a more or less broad region), the fluctuations of the velocity are not negligible.

The results for the frequency are less clear, and they are influenced by the fact, that the galvanometer used did not react to frequencies higher than *ca.* 50 per sec. (See fig. 3). In the case of the laminar layer a strong impression is obtained that the frequency decreases when we go to smaller distances from the plate: whereas for the flow outside of the boundary layer a frequency of about 46 per second was noted (which is too near to the

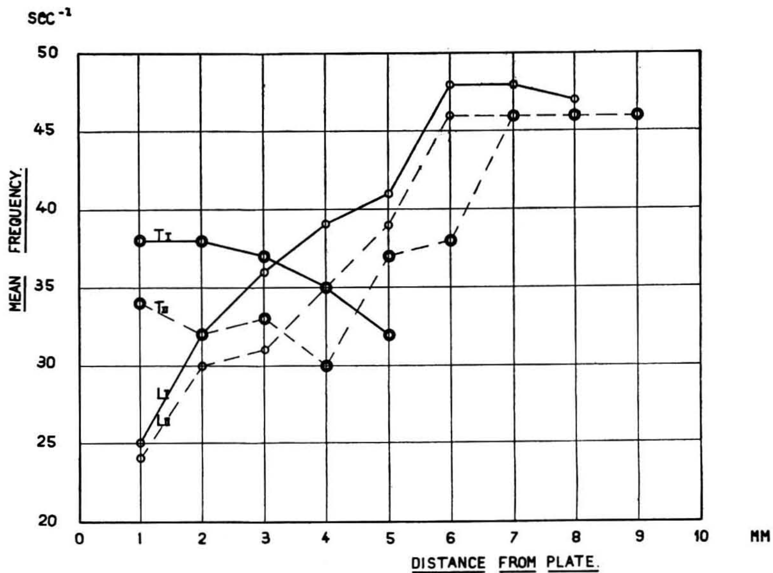

Fig. 3.

limit set by the galvanometers for making further conclusions), at a distance from the plate of 3 mm and less, the frequency had decreased to *ca.* 30 per second. — In the case of the turbulent boundary layer frequencies of a similar order were noted near to the plate, and there seemed to be some indication of a slight increase of the frequency with decreasing distances.