Physics. — *Ionization and conductivity in gases at high pressures.* By J. CLAY.

(Communicated at the meeting of November 27, 1937.)

For the determination of the intensity of electromagnetic radiation of a small wavelength (roentgen and gamma radiation) and of corpuscular radiation with great energy (betha radiation, cosmic radiation) in most cases the ionization in gases is employed.

Recently ionization chambers filled with gas under a high pressure have been used for the measuring of very small intensities. However, some phenomena were observed which gave rise to differences of opinion. The increase of ionization with the pressure, which at low pressures is proportional to them, soon at increasing pressure appears to be smaller than the proportional value and repeatedly the further course of the pressure curve turned out to be different with different investigators. Here two factors play a part.

In the first place, at a high pressure it is not possible to catch all the ions formed by the rays in the gas. At a high pressure, namely, such a considerable recombination of the ions takes place as to make it impossible to expel all ions from the gas even by means of high electric fields, and yet this number has to be known in order to enable us to determine the required intensity of radiation.

There are two ways in which the ions might be recombined. On the whole it may be said that practically in all cases the ions are formed along the paths of the charged corpuscles moving through the gas. They will escape recombination in the column more easily according as the field is higher and the pressure lower, so dependent on $\left(\frac{X}{p}\right)$. However, whether the number of paths is larger or smaller, i.e. whether the intensity of radiation is more or less considerable, this will neither diminish nor increase the chance of escape from the column. It will be clear that the higher is the pressure, the stronger must be the field in order to liberate an equal number of ions from the column.

Once having escaped from the column, the ions still have a chance of recombination on their way to the electrodes, the so-called volume-recombination. This chance is proportional to the square of the number of ions present per unit of volume (if the positive and the negative number are the same). Consequently in this case the chance of recombination will very rapidly increase with the intensity of the radiation.

The column ionization theory of JAFFÉ (1) now enables us to calculate from the current i, occurring in a certain field X, the saturation current which might be found if the field was infinitely large and all formed ions were collected.

The form given to this formula by ZANSTRA (2) is more useful, if we introduce the time t, which is required to collect a certain ionization charge, and the time T which would be required for the saturation current, so if the field is infinitely large. Consequently we have

$$t = T + qTf(x) \qquad \text{in which } q = \frac{\alpha N_0}{8\pi D}$$

α is the recombination coefficient
N_0 is the specific ionization
D is the diffusion coefficient

$f(x)$ is the cylinder function $f(x) = e^x \frac{i\pi}{2} H_0^{(1)}(ix)$

$$x = \left(\frac{b u \sin \varphi X}{2D}\right)^2.$$

In most cases this can be simplified to

$$x = c\left(\frac{X}{p}\right)^2.$$

X is the force of the field in Volts per cm
p is the pressure in atmospheres
b is a column parameter
u is the mobility of the ions
φ is the angle between the path and the direction of the field.

The value of c is determinated from our experiments.

For $X = \infty$ is $f(x) = 0$ and consequently the linear relation between the measured time t and the value of $f(x)$, calculated from the force of the field and the constant c, immediately yields the value of T by extrapolation to $f(x) = 0$ and from this we can find the saturation current and the number of pairs of ions formed per second and per unit of volume.

That there is indeed a linear relation between t and $f(x)$ has been proved in a number of cases in the Amsterdam Laboratory, by ionization in air, nitrogen, argon, carbon dioxide, xenon, krypton, neon, helium, with roentgen and gamma rays (3), and further also for cosmic rays in air, argon and carbon dioxide (4) (figs. 1, 2, 3).

To facilitate the use of the formula ZANSTRA has made a graph (4*a*, 4*b*) of the HANKEL cylinderfunction, so that we find directly $f(x)$ for each value of x from X, p and the constant c.

The form found just now shows two things. Expressing the time for the transition of a certain charge in the time which would be required if the field were infinitely large, we find (5)

$$\vartheta = \frac{t}{T} = 1 + \frac{a N_0}{8 \pi d} f(x).$$

That means that in two cases with rays of different intensity, as long as

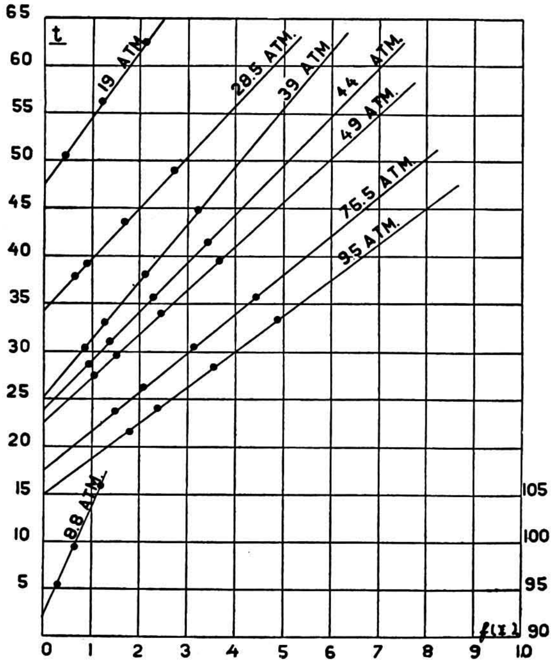

Fig. 1. CLAY and v. TIJN.
Measurements of ionization by gamma rays in air between two flat plates at 0,6 cm distance at different pressures and different fields. According to formula we may expect that the relation between I and $f(x)$ is linear for every pressure.

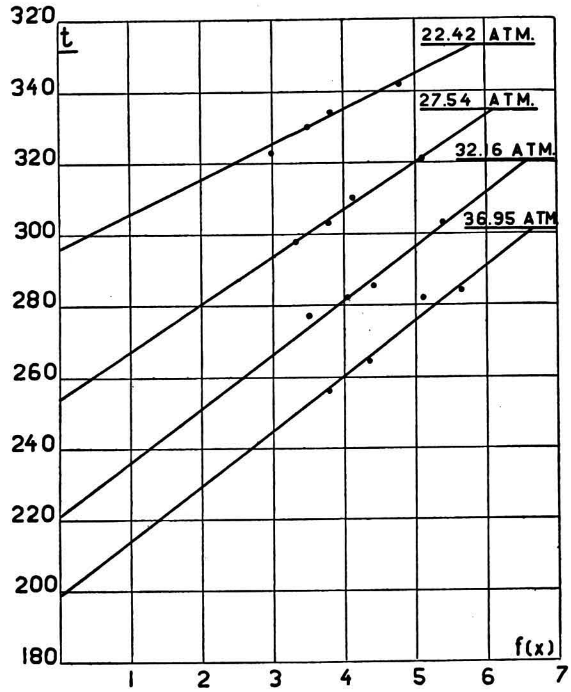

Fig. 2. CLAY and JONGEN.
Measurements of ionization in air by cosmic rays between two coaxial cylinders at 3 cm distance at different pressures at different fields.

there is no volume ionization, ϑ will always be the same, owing to the fact that t and T change in the same proportion. If, therefore, with two different degrees of ionization we find the same linear relation between ϑ and $f(x)$, we may conclude that no volume recombination has taken place. In two cases this test was made by VAN KLEEF and myself (6), one in air and one in argon (figs. 5 and 6). For small intensities of the radiation we see that the volume-recombination occurs only for weak fields.

At the same time we see that, in case of rays of a different kind and with a different specific ionization, the gradients of the lines observed for ϑ in $f(x)$ in the same gas at the same pressure and the same field are a direct measure for the proportion of the specific ionization of these radiations. In fig. 6 we find the proportion of the specific ionization in argon: 1st by roentgen rays of a wavelength between 0,3 and 0,4 Å, 2nd by gamma rays of *Ra* and 3rd by cosmic rays.

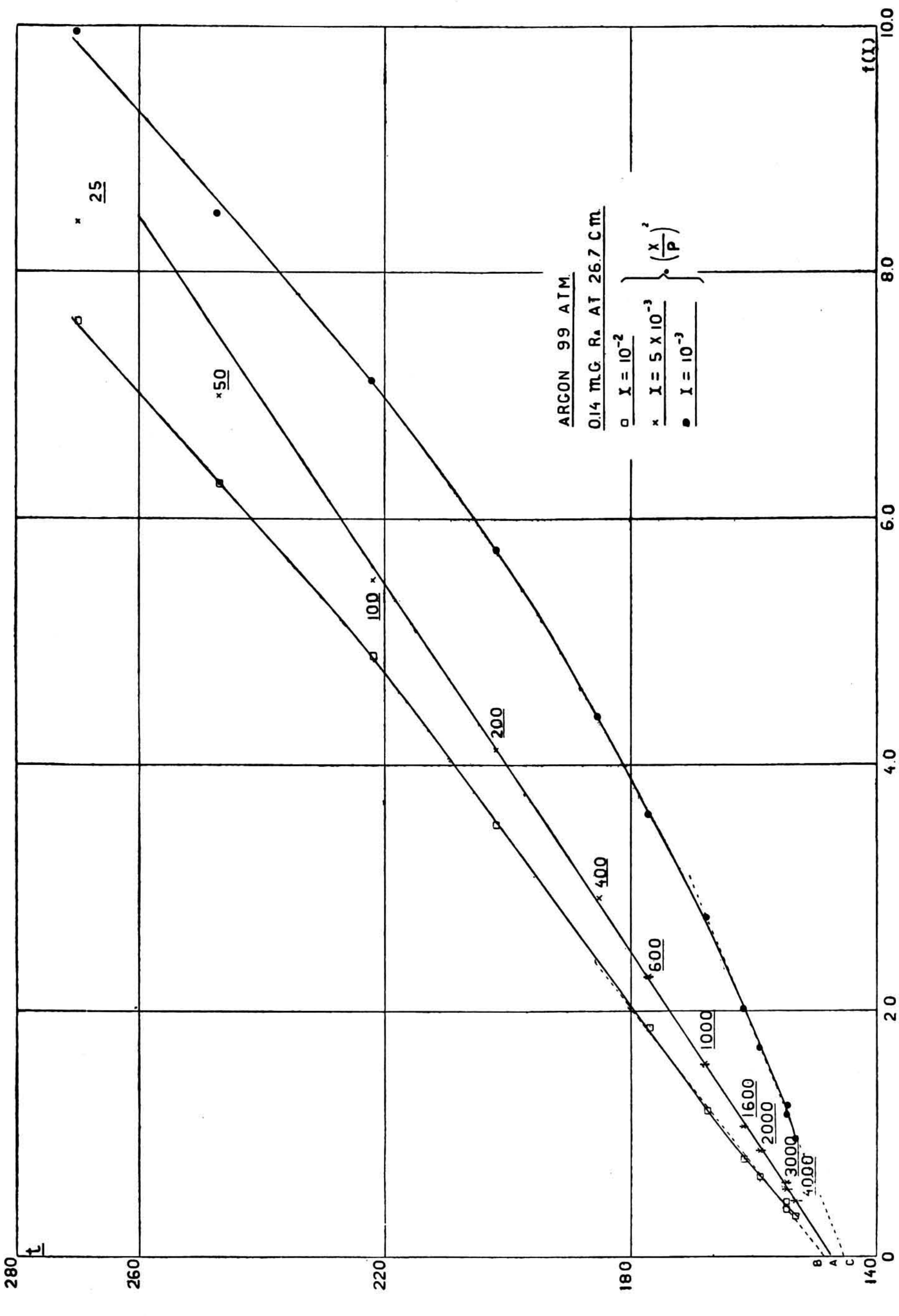

Fig. 3. CLAY and VAN KLEEF. Measurements of ionization in argon at 99 atm. by gamma rays between two plates at a distance of 5 mm.

When by the above-mentioned method the total number of produced electrons in the vessel has been determined, yet another difficulty has to be overcome in order to find out how many would have been formed *in the* *gas alone*, since the number of ions that is found is the sum of what really was formed by electrons in the gas and the number produced by electrons from the wall. The latter number may be eliminated in two ways.

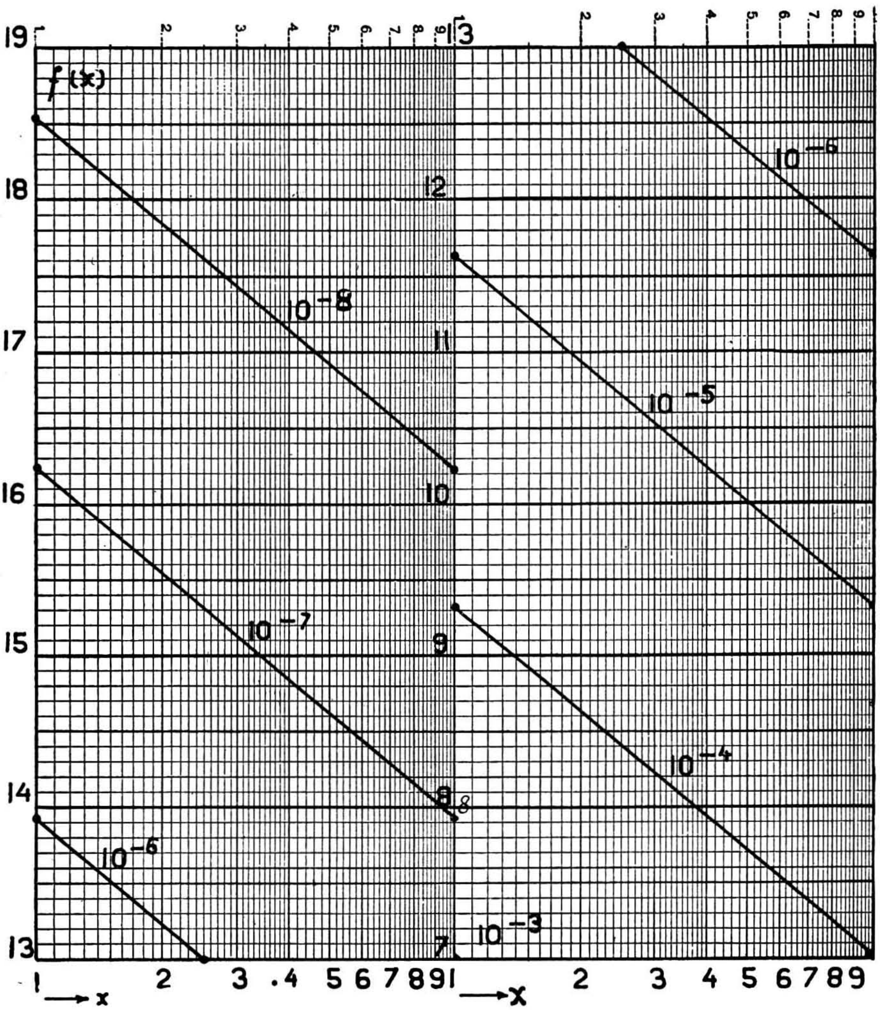

Fig. 4a. ZANSTRA. Values of $f(x) = e^{x} \frac{i\pi}{2} H_0^{(1)}(ix)$ for x between 10^{-8} till 10^{-4}. Example: for $x = 3{,}82 \, . \, 10^{-5}$ is $f(x) = 10{,}28$.

We suppose that the number of electrons produced by radiation in the gas is proportional to the density of the gas. Each electron on its way liberates the whole of its energy, if the gas is sufficiently dense, so that the path is confined to the space where the measurements are made. The number of pairs of ions is then also proportional to the density of the gas.

What part now does the wall play here? From the wall a number of electrons is liberated which is proportional to the size of the surface of the wall and each electron will produce a number of ions in accordance with its energy. If the pressure is so high that the whole path of the electrons lies in the vessel, a rising pressure will not produce an increase of the number of ions, but on lowering of the pressure only a number proportional

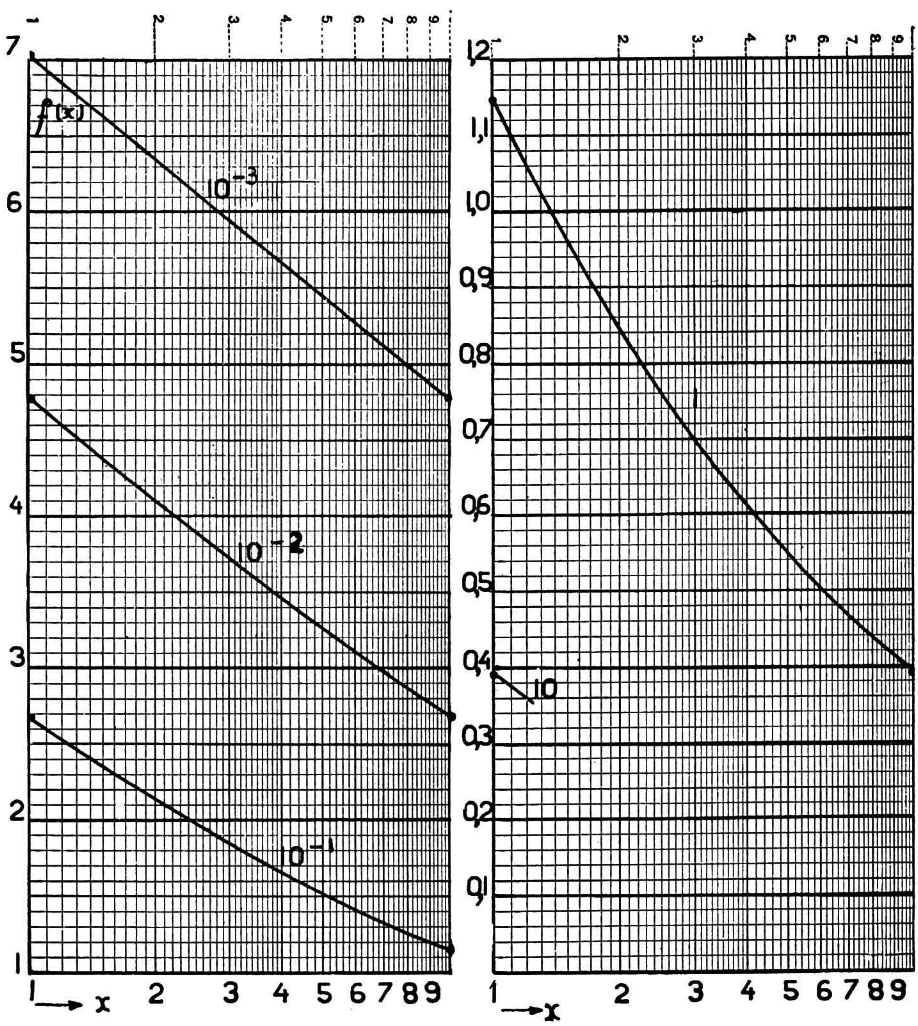

Fig. 4*b*. ZANSTRA. Values of $f(x) = e^x \frac{i\pi}{2} H_0^{(1)}(ix)$ for x between 10^{-3} and 12. Example: for $x = 2{,}40$ is $f(x) = 0{,}774$.

to the pressure will remain in the vessel and then the ionization by the electrons of the wall will likewise be proportional to the pressure. Consequently we find for high pressures

$$I = Ap + B \qquad p > p_0$$

and for lower pressures

$$I = (A + B)p \qquad p < p_0$$

If there are corpuscular rays, passing through the vessel, as is the case with cosmic rays, another factor has to be taken into account, likewise proportional to p.

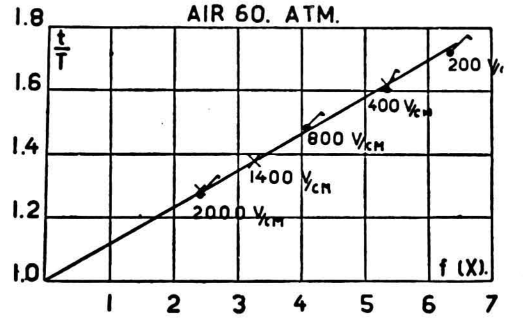

Fig. 5. CLAY and VAN KLEEF.
Measurements in air at 60 atm. by two different gamma radiations at different fields.
($I_1 = 7{,}9 \cdot 10^4$ E.S.U. $\quad I_2 = 5{,}7 \cdot 10^5$ E.S.U.)

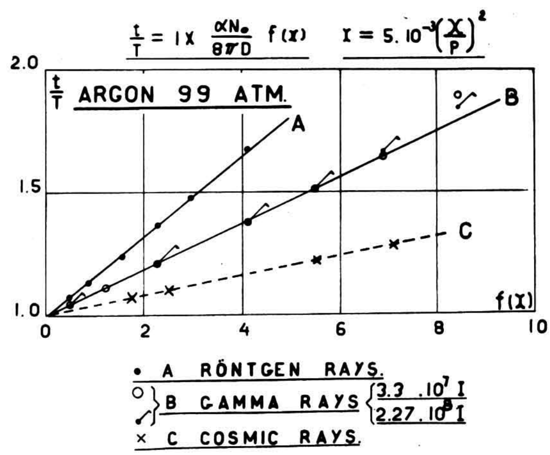

Fig. 6. CLAY and VAN KLEEF.
Ionization in argon by different radiations.

On measuring the ionization between two parallel walls, from the course of the line indicating the saturation current with the pressure the part of the ionization formed in the gas may be separated from the part originating from the wall. This may be seen in the figures 9, 11 and 12.

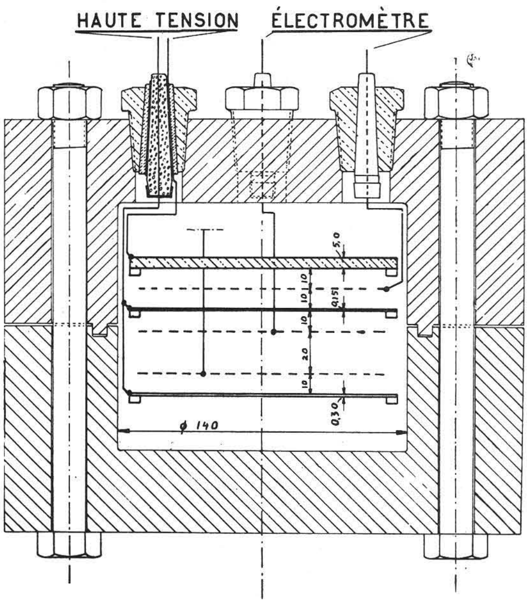

Fig. 7. CLAY.
Ionization chamber for high pressures with different grids.

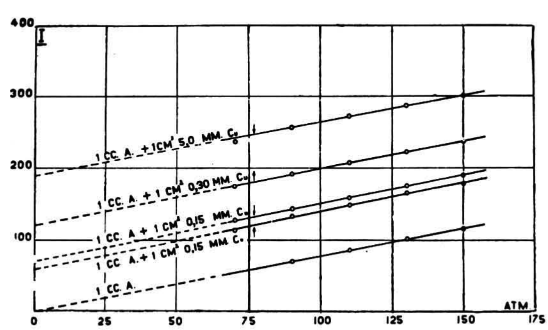

Fig. 8. CLAY and DE BOCK.
Ionization in argon at high pressures by cosmic rays between grids and between copper plates of different thickness.

It is also possible to take walls of gauze, so that the wall has only a small share in the process. Then a pure volume ionization is left. This case is realized in the experiment with a steel vessel filled with argon under a high pressure, up to 150 atm. (fig. 7). The ionization, e.g. by cosmic rays and by gamma rays, may be measured between two grids and, if the pressure is sufficiently high, so that the electrons which by the radiation are ejected from the wall have paths not reaching to the network, the ionization between the grids will be purely proportional to the pressure. The experiment carried out first by myself and afterwards together with DE BOCK proves that the obtained current indeed fulfils this supposition (fig. 8). However, between the solid wall and the network the ionization is larger, viz. so much as corresponds to the ionization by the electrons from the wall. It becomes even apparent that for very thin walls this value is proportional to the thickness of the wall, and that the number of electrons in the direction of radiation and opposite to this direction will not be exactly the same. At the same time we see that this number of electrons from the wall and the ionization caused by them, as might be expected, is absolutely independent of the pressure of the gas, as long as the latter is sufficiently high (fig. 9). If it is no longer sufficiently high, the ionization by the

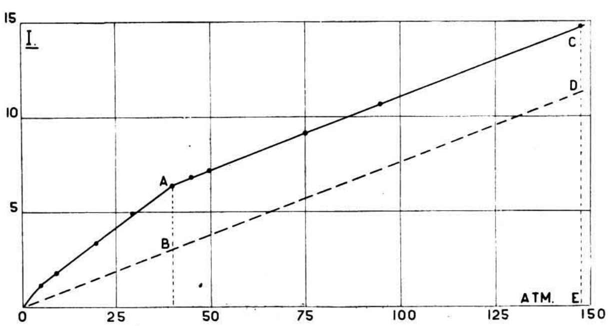

Fig. 9. CLAY, STAMMER and V. TIJN. Ionization in air till 150 arm. by gamma rays.

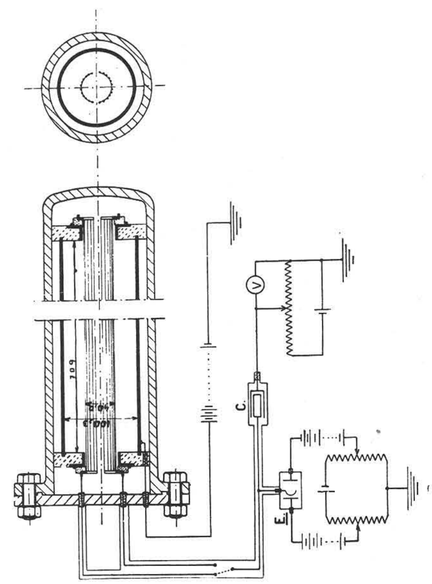

Fig. 10. Ionization chamber for absolute measurements of cosmic rays in different gases at different pressures.

electrons from the wall decreases with the decreasing pressure, as we could demonstrate in the case of the experiment with air in a vessel with two cylindrical walls 3 cm apart (fig. 10) and also for argon (figs. 11 and 12).

Thus the formulas for I are experimentally verified. Consequently it is always possible to eliminate the effect of the walls either by measuring at

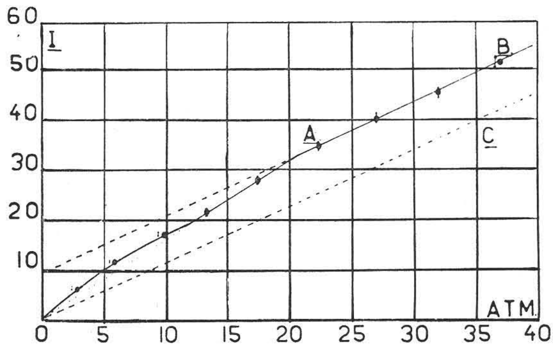

Fig. 11. CLAY and JONGEN.
Ionization in air of different pressures by cosmic rays under 12 cm *Fe*.

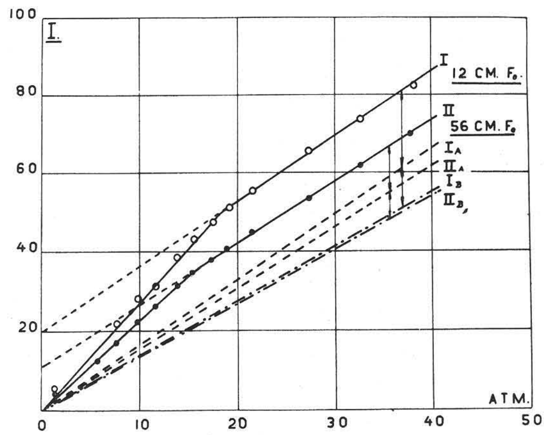

Fig. 12. CLAY and OOSTHUIZEN.
Ionization in argon of different pressures by cosmic gamma rays between two grids (v. fig. 13).

two or more sufficiently high pressure or by making an ionization chamber with electrodes of as little material as possible (figs. 13 and 14) (6).

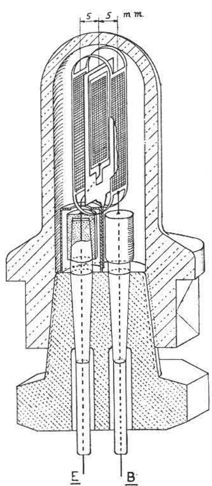

Fig. 13.
Ionization chamber for high pressures and high homogeneous fields.

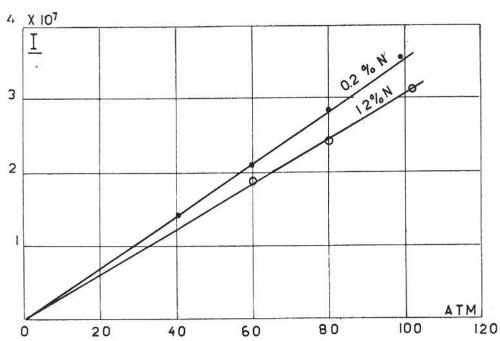

Fig. 14. CLAY and VAN KLEEF.
Ionization in argon at different pressures by gamma rays between two grids (vide fig. 13).

In either case the result is that the total ionization in the gas only is purely proportional to the pressure, also at high pressure, provided the measurements are taken with the saturation current. For this purpose it is necessary in most cases to measure with two or more different electric fields.

We have now composed a program for the examination of the ionization in gases for gamma rays and cosmic rays in connection with density and atomic number, for we know already that the increase of ionization with these quantities is different for the two kinds of rays.

Owing to the assistance of the Managing Board of the Philips Works, for which we are very thankfull, we could include the noble gases in this program. Now the following phenomenon was observed. During an investigation of neon in a suitable vessel, carried out by VAN KLEEF and myself, it was found that at a tension exceeding 1800 Volts in neon there was an increase in the strength of current which could not be explained. Cf. p. 664 (fig. 2) and fig. 15.

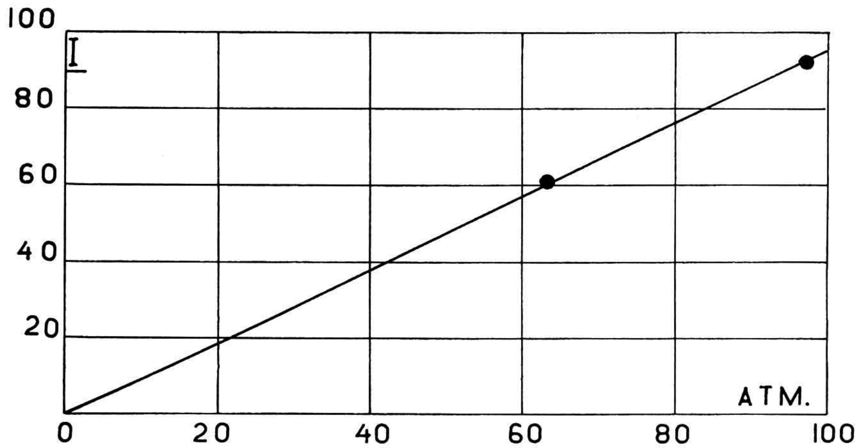

Fig. 15. CLAY and VAN KLEEF.
Ionization in neon by gamma rays between two grids.

This phenomenon was observed to a much larger extent in xenon (fig. 4) p. 665. It may be seen how in xenon at high voltages the current increases rapidly and the linear relation of t with $f(x)$ is broken.

After the external source of ionization had been removed, it appeared that there was still a conductive current and the figure 5, p. 666, shows how strong this is. New measurements are given in fig. 16. We have considered possible causes which probably might give rise to ionization. We think here of other parts between the high tension electrode and the wall, where a stronger field might be formed.

If, however, discharge through the gas might occur there, the ions would come to be situated between the plates, owing to diffusion. In case of discharges at the same field but at a higher pressure, the diffusion of the ions would be much less considerable, so that at a higher pressure the phenomenon would decrease. Beginning at a low pressure, exactly the opposite will be found. At one atmosphere no transfer is observed at all.

Moreover, in the same vessel in which with argon and air not the slightest trace of conduction was observed, conduction by neon and xenon was found, no alterations whatsoever being made to the vessel. Likewise conduction was found in helium (figs. 17 and 18), although previously air

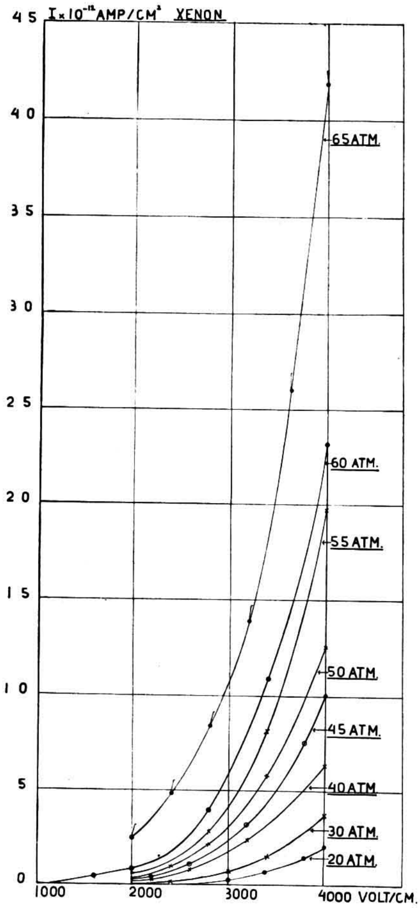

Fig. 16. CLAY and KWIESER. Conductivity of xenon at different pressures in different electric fields.

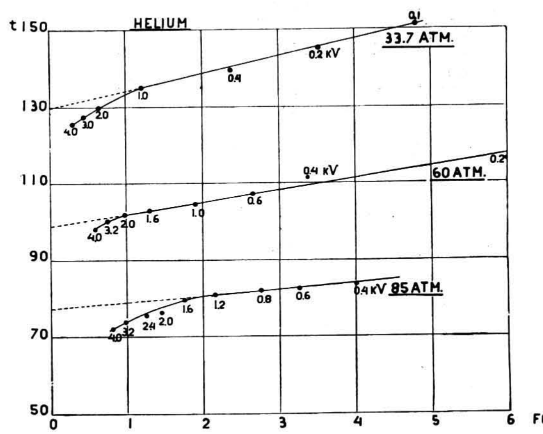

Fig. 17. CLAY and KWIESER. Ionization and conductivity of Helium by gamma rays.

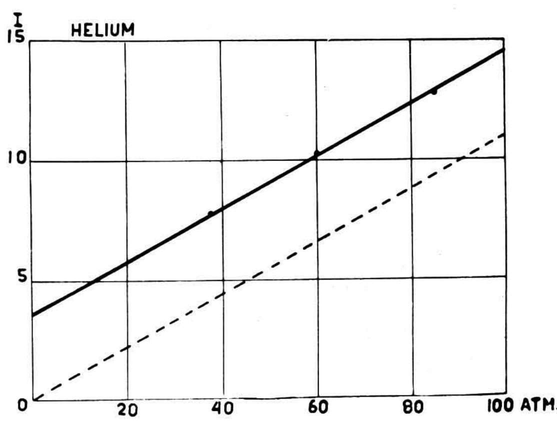

Fig. 18. CLAY and KWIESER. Ionization of helium by gamma rays between two plate electrodes.

in the same vessel did not show conduction. The same holds for another vessel filled with krypton. If it was a conduction kreeping along the surface of the insulator, it might be expected also at a high pressure that the phenomenon would grow less strong, while it would remain inexplicable why it does not occur at all in argon, whereas it is found in neon and xenon.

The phenomenon was observed by VAN TIJN in nitrogen (fig. 19), viz. with two different constructions, different filling and different external ionization and with a corresponding value. Consequently even if a field would be formed between the high tension electrode and the vessel, TOWNSEND's ionization by collision is out of the question, since the force of the field per mm pressure in the gas then has to be more than 1000 times larger than at the values at which we observed conduction.

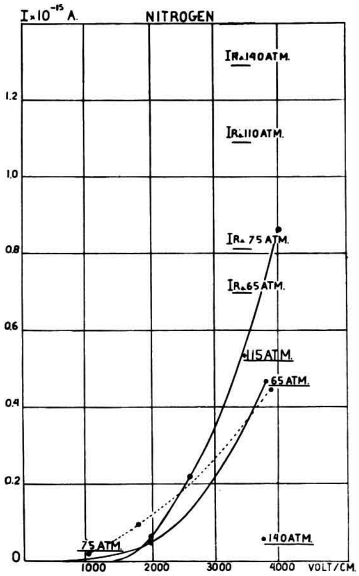

Fig. 19. CLAY and VAN TIJN. Ionization of Nitrogen at different pressures by gamma rays.

Finally it becomes apparent that at high pressures there is influence of the temperature, viz. that the effect becomes smaller at a higher pressure. If the ions by diffusion would laterally penetrate between the electrodes of the measuring space, the reserve might be expected.

However, it will be necessary by all possible means to suppress currents other than between the plate electrodes.

As the result of the measurements, as far as they have been carried out, we may say that the process of ionization with the pressure for gamma rays is totally different from that for cosmic rays. For gamma rays the ionization increases almost in proportion to the square of the atomic number, whereas for cosmic rays the ionization simply increases in proportion to the density

Gas	Z	A	d/d_{Ne}	Z^2/Z^2_{Ne}	Ioni. γ	Ionis Cosm. R.
Helium . . .	2	4	0.2	0.04	(0.13)	
Air	7.5	15	1.5	0.56	0.68	1.50
Neon	10	20	1.0	1.0	1.0	
Argon	18	40	2.0	3.2	2.5	2.03
Krypton . . .	36	83	4.2	12.2	(14)	
Xenon	54	130	6.5	29	2	

of the gas. The latter proportion was also found by JUILFS and MASUCH (8). For gamma rays JUILFS found a considerably smaller increase than we observed. It is very likely that the wall of JUILFS' vessel played an important part, since he worked at atmospheric pressure.

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Mathematics. — *Sur la méthode de* WEYL *dans la théorie des nombres.* Par J. G. VAN DER CORPUT. (Troisième communication).

(Communicated at the meeting of November 27, 1937.)

§ 4. *Sommes de* WEYL *généralisées, de premier degré.*

J'entends par sommes de WEYL généralisées des sommes de la forme

$$S = \sum_{x=P+1}^{P+X} e^{2\pi i f(x)}\,;\; S' = \sum_{x=P+1}^{P+X} \cos 2\pi f(x)\,;\; S'' = \sum_{x=P+1}^{P+X} \sin 2\pi f(x),$$

où P est un nombre entier, X un nombre naturel, $f(x)$ une fonction réelle telle qu'il existe un nombre entier non négatif $k \leqq X-2$, pour lequel

$$\triangle^{k+1} f(x) = f(x+k+1) - \binom{k+1}{1} f(x+k) + \binom{k+1}{2} f(x+k-1) - \ldots \pm f(x)$$

a une valeur proche de zéro lorsque x prend les valeurs $P+1, P+2, \ldots, P+X-k-1$; j'appelle le nombre k le degré de la somme de WEYL généralisée considérée. La notion „proche de zéro" est vague, ce qui rend vague aussi la notion de somme de WEYL généralisée. Une somme de la forme S, ou S', ou S'', pourra être appelée somme de WEYL généralisée, ou non, selon que l'on sera plus ou moins strict en ce qui concerne le sens attaché à la notion „proche de zéro". Le degré d'une somme généralisée n'est pas, non plus, défini avec beaucoup de précision. La question qui se pose est, ici encore, de chercher une borne supérieure pour la valeur absolue des sommes de WEYL généralisées. Si l'on divise l'intervalle $(P+1, P+X)$ en intervalles partiels assez petits, alors, dans chacun de ceux-ci, $\triangle^{k+1} f(x)$ est proche de zéro, ce qui revient à dire que la fonction $f(x)$ est par approximation égale à un polynôme du k^e degré en x. Une somme de WEYL généralisée peut donc, si $k \geqq 1$,