Kristallisation schnell vor sich geht und einen ausgeprägten Temperaturgradienten aufweist, wachsen die primären Kristalle fortwährend, so dass das Auftreten von isolierten Körnern von vornherein unwahrscheinlich ist. In dieser Weise kann die Umwandlung durch die ganze Masse fortschreiten.

12. Nach dem von MASON und FORGENG beim Bi Beobachteten liegt es auf der Hand die anderen von uns studierten Metalle, welche in Spuren eine Verzögerung des Fortschreitens der Zinnpest herbeizuführen im stande sind, von dem neu gewonnenen Standpunkt eingehend zu studieren, da derselbe nicht allein für die Kenntnis der U.G. polymorpher Metalle, sondern ebenfalls für die der Korrosionserscheinungen im Allgemeinen von grösster Bedeutung sein dürfte.

Wir hoffen darüber demnächst zu berichten.

ZUSAMMENFASSUNG.

Die vorliegende Untersuchung ergab, dass auch der Zusatz von äusserst geringen Mengen Magnesium zum weissen Zinn die akute Zinnpest hervorzurufen im stande ist; auch hier lässt sich das Eintreten des HEYN–WETZELeffekts als Erklärung heranziehen.

Die enorm verzögernde Wirkung von äusserst geringen Spuren Wismut auf das Eintreten der Zinnpest findet ihre Erklärung in den von MASON und FORGENG beobachteten Erscheinungen, welche die Kristallisation der Zinn-Wismutlegierung begleiten.

Utrecht, August 1938. VAN 'T HOFF-*Laboratorium.*

Mathematics. — *Rectilinear congruences in the three-dimensional projective space built up of quadratic reguli.* By W. VAN DER WOUDE and J. J. DRONKERS.

(Communicated at the meeting of September 24, 1938.)

Of the above-mentioned congruences we wish to prove a few properties and to mention special cases which to us seem to be interesting.

§ 1. We consider a congruence $\mathfrak{K}$, built up of quadratic reguli λ. Simultaneously with this one a second congruence $\mathfrak{K}^+$ is produced, constructed of quadratic reguli λ^+, both λ and λ^+ always lying on the same quadratic surface L; λ and λ^+ are called complementary reguli, $\mathfrak{K}$ and $\mathfrak{K}^+$ complementary congruences.

Theorem. *On the surface* $\mathbf{L}$, *the enveloping surface of system* L, *the focal curves of the two congruences* $\mathfrak{K}$ *and* $\mathfrak{K}^+$ *are lying.*

Proof. $\mathbf{L}$ is produced as the locus of the curve (C_4), the characteristic of a surface L. Let us assume for the present that (C_4) has not degenerated. It is then intersected in two points by each straight line lying on L, whether the latter belongs to λ or λ^+. In each point of (C_4) L is touched by $\mathbf{L}$; if b is a straight line on L, then b touches $\mathbf{L}$ in the above-mentioned points of intersection and consequently b is a double tangent to $\mathbf{L}$. The two focal curves of $\mathfrak{K}$ and likewise those of $\mathfrak{K}^+$, therefore, lie on $\mathbf{L}$.

§ 2. Before proceeding we shall establish the results analytically. The two congruences are at the same time represented by

$$\varrho x_i = a_i(w) + b_i(w)\,v + c_i(w)\,u + d_i(w)\,u\,v \qquad (i = 1, \ldots, 4),$$

or still more briefly by

$$\varrho x = a(w) + b(w)\,v + c(w)\,u + d(w)\,uv \quad . \quad . \quad . \quad . \quad (1)$$

The quadratic surfaces L are indicated by $w = w_0$ (constant), the straight lines of congruence $\mathfrak{K}$ by $w = w_0$, $v = v_0$, those of $\mathfrak{K}^+$ by $w = w_0$, $u = u_0$. The straight lines of $\mathfrak{K}$ are called u-lines, those of $\mathfrak{K}^+$ v-lines.

Now let us determine the focal curves of $\mathfrak{K}$. If in (1) we consider

w and v as functions of u, then a point is determined on each u-line; we need merely express that

$$x^{(0)} \left(= a(w) + b(w)\, v\right),\ x \text{ and } \frac{dx}{du}\left(= \frac{\partial x}{\partial u} + \frac{\partial x}{\partial v}\cdot\frac{dv}{du} + \frac{\partial x}{\partial w}\frac{dw}{du}\right)$$

lie in a straight line. Consequently:

$$\lambda x^{(0)} + \mu x + \nu\left(\frac{\partial x}{\partial u} + \frac{\partial x}{\partial v}\cdot\frac{dv}{du} + \frac{\partial x}{\partial w}\cdot\frac{dw}{du}\right) = 0;$$

The three terms $\lambda x^{(0)}$, μx and $\nu \frac{\partial x}{\partial u}$ may be replaced by $\varrho x + \sigma \frac{\partial x}{\partial u}$. Then we find from these four equations:

$$\left| x,\ \frac{\partial x}{\partial u},\ \frac{\partial x}{\partial v},\ \frac{\partial x}{\partial w} \right| = 0. \quad \ldots \ldots \quad (2)$$

This equation both in u and v is of the second (only apparently of the third) degree. It shows that the focal curves of $\mathfrak{K}$ and $\mathfrak{K}^+$ lie on the same surface, determined by (1) and (2).

It is equally simple to prove that (1) and (2) determine the enveloping surface **L** of the system L, or — if w is regarded as a constant (w_0) — represent the characteristic of the surface $w = w_0$.

§ 3. In the above it has been assumed that (C_4) had not degenerated. However, that is not essential in the proof of the theorem in § 1. In any case, i. e. also if (C_4) has degenerated, each point of (C_4) is a tangent point of L and **L** and consequently the two straight lines of L through that point touch **L**. This supposition, however, made it clear that the locus of the focal curves of $\mathfrak{K}$ and that of the focal curves of $\mathfrak{K}^+$ is the same. Expressed in the usual terminology of differential geometry: the two congruences have the same focal surfaces (the two branches of **L** being regarded as different surfaces).

Now it is remarkable that it would be incorrect to say that the two complementary congruences *always* have the same focal surfaces. That is, at any rate in a literal sense, not always the case.

Let us suppose that each characteristic (C_4) has degenerated into a (C_3) and a straight line b which belongs to $\mathfrak{K}$. Each straight line of a regulus λ will then in two points intersect the curve (C_3) which lies on the same surface L; each straight line of λ^+ intersects (C_3) once and has a point in common with b.

The locus of (C_3) is a surface S, that of b a ruled surface R; together they form **L**.

Consequently R and S are the focal surfaces of $\mathfrak{K}^+$, both focal curves of $\mathfrak{K}$ lie on S; the ruled surface R belongs to $\mathfrak{K}$.

We may, for example, obtain this case by starting from a ruled surface R:

$$\varrho x^{(0)} = a(w) + c(w)\, u. \quad \ldots \ldots \quad (3)$$

It is obvious that the u-curves are the generators of the surface, while of the w-curves three may be taken arbitrarily and the others are determined by the necessity to intersect the generators in projective ranges of points. Congruence $\mathfrak{K}^+$ is produced by the tangents to the w-curves and both congruences are represented by:

$$\varrho x = a(w) + c(w)\, u + a'(w)\, v + c'(w)\, uv. \quad \ldots \quad (4)$$

As before, the v-lines form congruence $\mathfrak{K}^+$, the u-lines congruence $\mathfrak{K}$.

The enveloping surface **L** of the quadratic surfaces $L(w = w_0)$ is found from (4) and

$$v \,|\, a + cu,\quad c + c' v,\quad a' + c' u,\quad a'' + c'' u \,| = 0$$

i. e. **L** has fallen apart into the ruled surface R and another surface S, defined by:

$$|\, a,\ \ c,\ \ a' + c' u,\ \ a'' + c'' u \,| + |\, a + cu,\ \ c',\ \ a',\ \ a'' + c'' u \,|\, v = 0. \quad (5)$$

If, however, in (4) and (5) we consider w as a constant, then (4) and (5) represent a curve of the third degree,

$$\varrho x = a(w) + c(w)\, u + a'(w)\, v + c'(w)\, uv$$

as becomes apparent by resolving v from (5) and substituting it in (4); with the straight line $v = 0$ this (C_3) forms the characteristic of the surface L under discussion.

§ 4. We may go yet a step further and cause the enveloping surface of the quadratic surfaces $L(w = w_0)$ to fall apart into surfaces, each of which is a focal surface of $\mathfrak{K}$ or $\mathfrak{K}^+$, none of them being a focal surface of both at the same time. For this purpose we make the characteristic of L degenerate always into 4 straight lines; it is a well known fact that, if two straight lines of λ have part in this characteristic, then two of λ^+ belong to it as well. Each straight line of λ^+ intersects the first pair, each straight line of λ the second. The locus of the first pair consequently consists of the focal surfaces of $\mathfrak{K}^+$, the second pair produces those of $\mathfrak{K}$.

Consequently here belong the focal surfaces of $\mathfrak{K}$ to $\mathfrak{K}^+$ and vice versa.

Again it is easy to give a (well known) example. Let us start once more from

$$\varrho x^{(0)} = a(w) + c(w)\, u \,. \quad \ldots \ldots \quad (6)$$

Here it was required of the w-curves that they intersect the generators in projective ranges. It is well known that the curved asymptotic lines of an arbitrary ruled surface possess this property. Let then the w-curves be curved asymptotic lines of a ruled surface R. Now we consider again the two congruences from

$$\varrho\, x = a\,(w) + c\,(w)\, u + a'\,(w)\, v + c'\,(w)\, uv \quad . \quad . \quad . \quad . \quad (7)$$

The v-lines forming the congruence $\mathfrak{K}^+$ are now the asymptotic tangents of R. It is known that then the two focal surfaces of $\mathfrak{K}^+$ coincide in R. A double counting straight line of R, therefore, is part of the characteristic of each quadratic surface $L\,(w = w_0)$ from (4); the remainder of the characteristic consequently must consist of two asymptotic tangents. That is indeed the case. It is namely well known (see e. g. E. P. LANE, Differential Geometry of Curves and Surfaces, Ch. II, Ruled Surfaces) that such a surface $L\,(w = w_0)$, in other words a quadratic ruled surface described by the asymptotic tangents in the points of a fixed degenerator b, osculates R; i. e. each of these tangents has a contact of the second order with R (usually expressed: has three consecutive points in common with R). However, there are two points on b where the asymptotic tangent has four points in common with R; those points are called the *flecnodal points* and these tangents the *flecnodal tangents*. These two flecnodal tangents together with the double counting straight line b form the characteristic of L. Each of these flecnodal tangents has as locus a ruled surface. These two ruled surfaces consequently are the focal surfaces of $\mathfrak{K}$; they are called the *flecnodal transforms* of R.

It is quite easy to confirm these results analytically. It should merely be borne in mind that the w-curves are asymptotic curves on R, so that in (5):

$$N = |\, a, \quad c, \quad a' + c'\, u, \quad a'' + c''\, u\,| = 0,$$

and proved that then the two values of u from

$$|\, a + c\, u, \quad c', \quad a', \quad a'' + c''\, u\,| = 0$$

indicate the flecnodal points.

§ 5. A second special case — the curve (C_4) need not have degenerated this time — is found by taking linear functions of w in (1) for $a, \ldots d$. Then by

$$\varrho\, x = A + Bu + Cv + Dw + Evw + Fwu + Guv + Huvw, \quad . \quad (8)$$

$A \ldots H$ being constants, three rectilinear congruences are given, of u, v and w-lines respectively, which two by two may be considered as complementary. The three systems of quadratic ruled surfaces

$$u = u_0; \quad v = v_0; \quad w = w_0$$

then have the same enveloping surface $\mathbf{L}$ and on $\mathbf{L}$ the focal curves of all three congruences are lying.

A simple example — the introduced metrical elements may easily be replaced by projective ones — is the following.

We consider the system of normals of the hyperboloid of one sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$$

or, in homogeneous coordinates, of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - t^2 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

A normal is defined by the points (x_0, y_0, z_0, t_0), whose coordinates satisfy (9), and $\left(\frac{x_0}{a^2}, \frac{y_0}{b^2}, \frac{z_0}{c^2}, 0\right)$.

If, therefore, we represent (9) by

$$x = \frac{1}{a}\,(vw + 1), \;\; y = \frac{1}{b}\,(v - w), \;\; z = \frac{1}{c}\,(vw - 1), \;\; t = v + w, \quad . \quad (10)$$

then the congruence of the normals is determined by:

$$\left.\begin{aligned} x &= \left(a + \frac{u}{a}\right)(vw + 1) \\ y &= \left(b + \frac{u}{b}\right)(v - w) \\ z &= \left(c + \frac{u}{c}\right)(vw - 1) \\ t &= \qquad\quad (v + w) \end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

i. e. the congruence of the normals is formed by the u-lines from (11). At the same time, however, (11) represents two other congruences, viz. the following.

The normals, erected in the points of a straight line m of (9) form a quadratic regulus λ, lying on a hyperbolic paraboloid. On that surface a second regulus λ^+ lies, of which e. g. the above-mentioned straight line m of (9) forms part. These reguli λ^+, m taking the place of every straight line of (9), build up the two congruences of v- and w-lines from (11).

These congruences may be characterized in another way. An arbitrary straight line is projected on a surface of the second degree in a (C_4). However, every v- or w-line from (11) is intersected by the normals in the points of a straight line of (9).

The v- and w-lines from (11) consequently are projected on (9) in a straight line and a (C_3).

In connection with § 1 we find from this:

The straight lines, whose projections on a surface of the second degree O_2 (of rank four) have degenerated into a straight line and a (C_3), form two congruences; these lines, like the normals of O_2, are double tangents to the surface of the centres of principal curvature of O_2.

It is easy to be seen that these two congruences have the same degree and class as the congruence of normals, in which case these numbers are 6 and 2. For example, the number of value systems of u, v, w which satisfies (11), the ratios of x, y, z, t being considered as given, indicates the degree of all three congruences.

The two congruences mentioned above are of the sixth degree and the second class.

Botany. — *On the relation between internal and external medium in Artemia salina (L.) var. principalis* SIMON. By HERBERT WARREN, DONALD KUENEN and L. G. M. BAAS BECKING. (From the Botanical Institute, University of Leyden.)

(Communicated at the meeting of September 24, 1938.)

The unique osmotic regulation of Artemia has been commented upon several times, but accurate data on this osmoregulation are still lacking. It seemed, therefore, worth while to investigate the relations between internal- and external milieu in this curious phyllopod. Two of us started to work on this problem at Pacific Grove, California, already in 1929, while the experiments are being continued now at Leyden.

In the earlier experiments living material could be obtained from a near-by salt work, while at Leyden we raise the animals from eggs, collected by one of us in California in 1930. More than 10 % of these eggs are still viable.

The problem was tackled by means of different methods:

1. Direct analysis of the haemocele fluid.
2. Determination of the refractive index of the haemocele fluid.
3. Determination of water endosmosis or exosmosis on transfer from one salt concentration to the other.
4. Volumenometry.

1. *Direct analysis of the haemocele fluid.*

One large female may yield as much as 5.9 mm³ of haemocele fluid, which may be obtained by means of a fine capillary tube. On the average, adult specimens may give 5—6 mm³. We worked chiefly with animals grown in a brine of a s.g. 1.075/15° ($n_D^{25} = 1.3518$). The following table gives the data obtained.

Blood of:	1 ♂	15 ♂	3 ♂	8 ♀	6 ♀	10 ♀	Weighted averages
specific gravity:	—	—	—	1.032	1.044	—	1.037
% dry weight:	4.85	4.83	6.33	—	5.15	6.34	5.36
% Na:	.92	.75	—	—	—	.84	.81⁵

The average weight of a male proved to be 7.5 mg and that of a female 9.2 mg. The average dry weight amounted to 7.48 % of the total body weight. If the Na present in the blood were calculated as NaCl, we would obtain 2.65 % NaCl, while the external milieu contained about 9 % NaCl.

From 9 females grown in a brine of s.g. 1.050 the Na- and chloride contents were determined separately. We found Na .67 % and Cl .87 %.