Physics. — *On the theory of liquid crystals.* By L. S. ORNSTEIN. (Communication from the Physical Institute of the University of Utrecht).

(Communicated at the meeting of November 26, 1938.)

A. VAN WIJK [1]) and the author have shown that the phenomena in the surface layer of liquid crystals can be described by an equation of the form:

$$A \frac{d^2 \varphi}{d x^2} - a H^2 \sin\varphi \cos\varphi = 0.$$

where x is a direction perpendicular to the surface, φ the angle of the swarms with the x axis, H the strength of the magnetic field, a a constant proportional to the volume of the swarms and the difference in permeability in the directions of their principal axis, and A a constant depending on the interaction of the swarms. The left member of the equation represents the couple on a swarm at the point x of direction φ; this couple is zero for the case of equilibrium.

ZOCHER has deduced an analogous equation on the base of the elastic theory of the liquid crystals.

In the Ann. der Phys. [2]) ZOCHER and FÜRTH have discussed the point: Is it only possible to deduce this equation from the theory of swarms? I will make some remarks on this question which up till now have not been put forward.

If we apply equation (1) to the case that the electric field is zero and that the liquid is contained between two parallel planes, the solution is a value of φ, which depends linearly on x. If φ is zero at both the surfaces, the value is everywhere zero. Now these conclusions are contradictory to observation. It is generally known that for very thin layers a quasi-isotropic state is possible, that however in thick layers, there must exist a random orientation which is the cause of the turbidity of the crystalline liquid.

If — in the second place — we calculate the potential energy from (1) and try to find the probability for the orientation φ, we do not get a distribution consistent with the known facts of the influence of a magnetic field on the dielectric constant.

The cause is that the equation (1) is not complete. We must remember

1) Zeitschrift für Kristallographie, **79**, 112 (1931). „Flüssige Kristalle".
2) Ann. der Phys. **31**, Heft 7 (1938).

that it is obtained from another one as an approximation. The full equation is

$$\int f(x-x')\, g(\varphi-\varphi')\, dx' - a H^2 \sin\varphi \cos\varphi = 0.$$

This equation expresses that the total couple exerted on a swarm at the point x is zero. The integral is composed from contributions of swarms at the points x' with the orientation φ'. The equation (1) is obtained supposing:

a) the function f has only appreciable values for small distances, and that

b) the difference $\varphi-\varphi'$ may be considered as a small quantity.

For the layer at the wall these suppositions are certainly true. However, in the bulk of the liquid the supposition that an equilibrium exists between the interaction and the magnetic field is not true. Here we must introduce the inertial and the BROWNian couple in the equation. If for simplification we assume a rotation in one plane and if the moment of inertia is B, the inertial couple amounts to $B \frac{d^2\varphi}{dt^2}$. The fortuitous forces of the BROWNian movement are represented by the formula

$$-\beta \frac{d\varphi}{dt} + F.$$

where β is the coefficient of internal friction and F a fortuitous force. The full equation of motion for the swarm therefore is

$$B \frac{d^2\varphi}{dt^2} + \beta \frac{d\varphi}{dt} - F = \int f(x-x')\, g(\varphi-\varphi')\, dx' - a H^2 \sin\varphi \cos\varphi \quad . \quad (2)$$

Although this general equation is rather complex, some general conclusions may be deduced without solving it.

Two extreme cases ought to be distinguished:

1. The BROWNian force is small compared to the interaction of the swarms given by the integral.

2. This force is large as compared to the other.

The first case occurs in the surface layer, where the strong forces exerted by the wall orient the first layers of the swarms. Therefore, in good approximation, the BROWNian couple may be omitted — as well as the inertial couple — and the conditions for the development of the integral are fulfilled, so that we get (1).

In the case, however, that the BROWNian couple is strong compared with the individual interaction couples, the neighbouring can thus show fortuitous differences in orientation which are large or small, so that the orientation φ depends no more systemetically on x. The integral can not be developed in the way mentioned, but on the contrary the mean value will be zero, so that in good approximation for the bulk of the liquid its value can be taken zero.

We thus get

$$B\frac{d^2\varphi}{dt^2} = -\alpha H^2 \cos\varphi \sin\varphi - \beta \frac{d\varphi}{dt} + F \quad . \quad . \quad . \quad . \quad (2)$$

From this equation, by the usual methods of the theory of BROWNian movement, we can deduce the probability for a swarm for its situation between φ and $\varphi + d\varphi$. The result is a BOLTZMANN-function with $-\frac{\alpha}{2} H \cos^2\varphi$ as potential energy of the swarm and containing the temperature of the liquid.

There exists a region of transition between those where the equation (1) and those where (2) holds.

This question is very interesting in connection with the work of the Russian School (FREDERICKZ and others). These physicists have done very fine work on the base of the equation mentioned. Their conclusion, however, on the thickness of the layers gives a proportion of the difference of the dielectric constant to that of the magnetic permeabilities, which ought to be contrary to the facts. They find that this proportion depends on temperature. Now the theory of swarms gives that it must be independent. Experimentally it has been shown by v. WIJK and shortly ago very precisely by P. CHATELAIN [1]) that for the refractive the relation deduced by the theory of swarms holds. The values of the permeability are not sufficiently known but it seems very improbable that the relation deduced from the theory of swarms should not hold. In any case the Russian physicists have neglected the fact that equation (1) holds only for very thin layers and must be replaced by another in the region of transition.

Utrecht, 19 Nov. 1938.

[1]) P. CHATELAIN, Thèse Paris, (1937).

Physics. — *A sensitive method for the measurement of excitationfunction of artificial radioactivity.* By L. S. ORNSTEIN, J. M. W. MILATZ, E. F. M. VAN DER HELD and W. MAAS. (Communication from the Physical Institute of the University of Utrecht.)

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In measurements performed on α-rays with the help of a GEIGER–MÜLLER counter the zero-effect showed to be a function of the time of exposure. This zero-effect is caused by reactions of type:

$$Al_{13}^{27} + He_2^4 \rightarrow P_{15}^{30} + n_0^1$$

$$P_{15}^{30} \rightarrow Si_{14}^{30} + e^+$$

which took place in the wall of the counter. This observation induced us to develop a very sensitive method for the measurement of excitationfunctions of α-rays. The great sensitivity is due to the fact that the full mass of the wall acts on the counter.

The experiment was done with the aid of a *Th*-preparation (10^7 α-particles/sec.) disposed on a *Pt*-wire, which had been exposed during a sufficient time to Thoron. The wire *W* (fig. 1) was put before a slit *S* in such a way, that the counter is striken by the beam of α-rays.

The range of the α-particles could be chosen in varying the pressure of

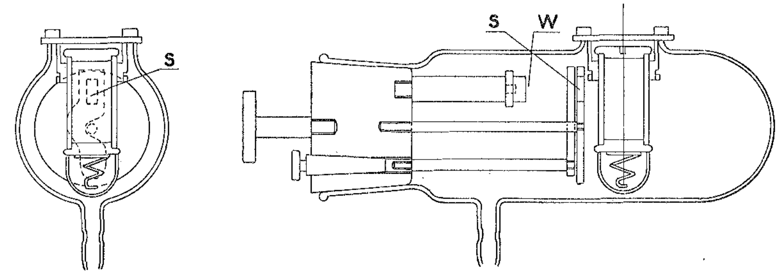

Fig. 1.

the tube, in which the whole was put. In this way it was possible to bombard the wall with α-particles of a given mean energy and known distribution of velocity (straggling). The counter is filled with air of $p = 7.5$ cm Hg, has a length of 4 cm and a diameter of 2 cm, thickness of at the wall 2 mm. The counting was performed with a circuit of NEHER–HARPER, coupled with an amplifier, scale-of-four and a counting-device with a resolving power of $^1/_{100}$ sec.