be a consequence from that. This cranial vault-height appears to have, anyhow, as little phylogenetic significance as it has in the Neanderthal Man skull.

The cephalization has nothing in common with this cranial vault-height, for the real cephalization-factor of the palm squirrel which died in the Bandoeng zoological garden appears to be somewhat below the normal one, on account of the body weight having a little increased in confinement. Comparing indeed, proportionally, the cube of the head-and-body length of *Sciurus palmarum* with the cubes of those lengths and the body weights of *Sciurus indicus* (*Ratufa indica*) and *Sciurus locria*, all of these lengths and weights given by BLANFORD [4]), we find for the body weight of *Sciurus palmarum* about 69 gr. and 64.5 gr. resp.

Assuming, further, equality of the normal cephalization-factor of *Sciurus palmarum* with that of *Sciurus vulgaris*, we find that the normal body weight of the Oriental squirrel should be 56.5 gr. The specimen from the Bandoeng zoological garden weighed only a little more. Generally, indeed the body weight of squirrels alters only slightly in such a confinement. *E.g.* the average cephalization-factor (in terms of brain weight) of 15 *Sciurus carolinensis* and of 5 *Sciurus hudzonicus* from the National Zoological Park in Washington City, as quoted by ALEŠ HRDLIČKA [5]), was 0.248 and 0.245 resp. This is different with some species becoming fat in confinement. In consequence the average cephalization-factor of 6 *Sciuropterus volans*, also quoted by HRDLIČKA, was only 0.202. Of 3 *Sciuropterus sagitta*, the data concerning which other species of Flying Squirrel, as concerning many other species of Mammals from Java, I am much indebted for to Prof. DAMMERMAN, the cephalization-factor (in terms of brain volume) was averagely 0.214. Undoubtedly, however, in the state of nature the cephalization-factor of the Flying Squirrels equals that of the other arboreal Squirrels.

[4]) O.c., pp. 372, 376 and 383.

[5]) Brain Weight in Vertebrates. Smithsonian Miscellaneous Collections, Vol. **48**, p. 97.

Geophysics. — *Moon and earth*. By B. G. ESCHER. (Communicated by Prof. J. H. OORT.)

(Communicated at the meeting of January 28, 1939.)

I. *Continents, Oceans and the Granite-problem.*

It is our custom to regard the fold-tectogens (= orogens) and the volcanoes as the most important forms of the face of the earth. Much more important in volume, however, are the continents which not only have an average height of 735 m above sea-level, but which rise about 5 km above the average depth of the bottoms of the oceans.

The most important characteristic in the face of the earth is the occurrence of two favourite levels; the one between 0 and 1000 metres, the other between —4000 and —5000 metres. G. K. GILBERT (1892) emphatically pointed this out in his "Presidential address before the Geological Society of America". He called the two levels "Continental Plateau" and "Oceanic Plateau" respectively, and gave (1892, p. 164, fig. 1) a graphic representation of the distribution of the areas of the earth at different heights and depths. Later, A. PENCK (1894, p. 136, fig. 9) and later still KRÜMMEL (1907, p. 87, fig. 15) represented such a graph, and they called it "hypsographische Kurve der Erdkruste" and "der Erdoberfläche" respectively.

The first question we want to put here is: *How did continents and oceans originate*? This problem is just as important to geology as another problem, viz. *How did granite originate?* is to petrology.

In one of his theoretical treatises DARWIN (1879, p. 535 and 537) mentioned in passing that the moon may have originated from the earth at the time when the earth had a time of rotation of 4 or 5 hours. This theory has later been called the "resonance theory" because it implied that half the time of rotation of the earth was equal to the period of distortion called forth by the solar semi-diurnal tide.

OSMOND FISHER (1882 and 1889), a man with a clear vision, in whom the thought of convection currents in the substratum had already risen, used this theory to explain the origin of the oceans, and at the same time that of the continents. His chapter XXV (1889, p. 336—341) is entitled: A speculation on the origin of Ocean Basins. He raises the question whether the Pacific Ocean could not be the scar of the separation of the moon. Presupposing a specific gravity of granite of 2.68, if the shell-surface corresponding with the area of the oceans had been removed from the earth, this shell would have to be 41 miles thick to furnish the mass of the moon. Then he fancies that the bottom of the wound thus formed, consisted of "heavy substratum" risen from the depths and solidified, and that the flakes left of the original granitic crust of the earth, caused the continents to originate. "We have suggested" (this refers to 1882, p. 224) "that the "original crust would break up into larger and smaller fragments, and "float towards the cavity. This would explain a certain rude parallelism "which exists between opposite coastlines. Such is traceable between the "western coast of America and the eastern coast of the old World." (1889, p. 339.)

OSMOND FISHER expressed this idea in "Nature" as early as 1882.

Thus it appears that neither WEGENER (1912 and 1929) nor TAYLOR (1910) nor PICKERING (1907) was the first to think of Continental Drift. And it is of equal importance that OSMOND FISHER pointed out already in 1882, how the two most important levels on earth originated and that the higher of the two must be granitic. It seems necessary to me to re-introduce these ideas into the world of geological thought.

To JEFFREYS we owe a modern, critical treatise on G. H. DARWIN's "resonance theory" (1924 and 1929). He reaches the result, that the moon may have been separated from the earth, provided the earth was not homogeneous (1929, 3.2). So a differentiation in shells and central core must have taken place, before the separation of the moon from the earth occurred. Besides, the moon must consist of the materials of these shells (1929, 3.5). According to GOODACRE (1931, p. 353) the specific weight of the moon amounts to 3.34, which agrees with this. The birth of the moon from the earth occurred when the latter was practically entirely fluid, i.e. within 10.000 years after the formation of the earth. The oceans originated on earth after the separation of the moon (1929, 5.8). The moon was formed "when solidification was starting" (1929, 5.9).

Originally (1924, 11.4, p. 150) JEFFREYS thought that, in connection with a treatise by JEANS, he could not accept OSMOND FISHER's theory on the origin of Oceans and Continents. However, he comes back on this in the second edition of "The Earth" (1929, 15.7), and declares, that the origin of continents, as imagined by FISHER, is certainly possible. So this is the criticism of the theory from a geophysical point of view.

Whereas WEGENER (1912 and 1929) pointed out the great importance of the two privileged levels on the earth, and gave the explanation: the high level owes its origin to the floating of sial-flakes in sima, the surface of which caused the low level, it must be stated that OSMOND FISHER pointed this out already thirty years before.

A considerable difference of opinion prevails at present about the *granite-problem*. All petrologists try to derive granite from the magmas known to us through their rocks. Also in connection with the oldest

quartziferous sediments, the discussion gives again and again rise to the thought that granite must, already very early, have formed part of the earth's crust. It is not my intention to trace this problem in its historical development. N. L. BOWEN regards the granitic magma as a rest-magma after fractional crystallisation of a magma with the composition of the plateau-basalts. Against this supposition R. A. DALY (1933, p. 425) raises the objection that, in that case, an enormous quantity of basic material, which could not possibly have kept the composition of plateau-basalts, would have had to be present under the original sial crust.

KENNEDY and ANDERSON (1938) point out two important basalt-groups: the alkali basalts with olivine and the tholeiites without olivine; the latter form the plateau-basalts. Like BOWEN they think that a granitic liquid may, by fractional crystallisation, originate from the tholeiitic magma type. On the other hand A. RITTMANN (1938, p. 53) joined JAGGAR (1931, p. 57) recently, and he thinks that the original parental magma from which sial would have been formed, is alkali basalt. The way he follows in explaining the origin of the sial flakes, is, both petrologically and geologically, very intricate. His hypothesis was published in its provisional form to provoke a discussion which, undoubtedly, promises to become interesting.

But why should the parental magma, from which the first granitic crust of the earth was formed through magmatic differentiation, have the exact composition of tholeiite or another rock known to us, that now lies on the surface in large quantities? Is it not more logical to suppose that primary granite as well as basalt are the products of differentiation of a parental magma which, immediately after the liquidification of the earth, formed its outer shell? *This original parental magma would then have had to be of a composition corresponding to x km of sial and y km of sima, to which should be added the quantities that have been deposited on the moon.*

Once the original sial occurs in the continents in the way described above, the oldest sediments may be rich in quartz, and all sorts of other magmas may originate by further differentiation and assimilation of igneous rocks and sediments. Only at that time the far reaching differentiation into the many magmas begins, from which originated the extensive series of igneous rocks, facing petrology with a number of problems.

It may be that not one batholite contains a granite of exactly the same composition as the original granitic shell of the earth, which was, for the greater part, removed to the moon. But it seems to me, that petrology as well as geology would get out of an *impasse*, if some more attention were paid to the first 10.000 years of the earth's existence, and especially to the possibility of the formation of the moon from the earth in that geologically short period of time, which formed the beginning of an evolution that lasted at least 1600 million years.

We must not, of course, forget that we have no certainty that the moon originated from the earth, but at the same time we must remember that this theory on the origin of the moon is the most probable. And if, as we have seen, it is possible to solve two burning problems with the aid of this theory, viz. that of the oceans and continents, and that of the limitation of sial to the continents, then this theory gains in probability.

This, too, OSMOND FISHER (1889, p. 341) realized when he wrote:

"Still it must be admitted that, if this theory of the genesis of the moon "seems to give a fairly rational explanation of the origin of ocean basins, "it will itself receive support from that circumstance."

II. *On the Composition of the Moon.*

In the preceding chapter it has been argued that the continents are the remains of the original outer shell of the earth, after the greater part of it had disappeared through the genesis of the moon.

It is my intention to draw some conclusions from this hypothesis.

The continents rise, on an average, 5 km above the bottom of the oceans, and, for the time being, we ascribe this to an isostatic equilibrium of the continental flakes, just after the separation of the moon. After the flooding with seawater a somewhat different hydrostatic equilibrium occurred. We shall fix the specific gravity of granite at 2.67 (R. A. DALY, 1933, p. 47) that of gabbro, respectively basalt, at 2.97, and that of sea-water at 1.03; the average difference in height between continents and oceans at 5 km and the average height of the continents above the level of the sea at 735 m. The sediments are left out of consideration, because originally they

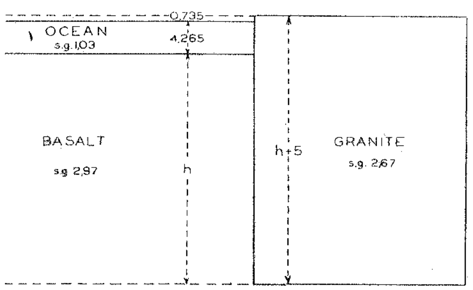

FIG. 1

developed from the igneous rocks and would only cause a slight difference in the result of the following calculation. Then it follows from fig. 1:

$$2.67\ (h + 5) = 2.97\ h + 1.03 \times 4.265$$
$$h = 29.9, \text{ or round } 30 \text{ km},$$

so that the thickness of the granitic flakes amounts to $30 + 5 = 35$ km.

2. We suppose the volume of the moon to be 22.000 million km^3 (21.990 km^3, calculated from $R_m = 1738$ km). The shelves are included in the continents.

The surface of the earth amounts to 510 mil. km^2.

According to the figures given by LITTLEHALES (1932) the surface of the oceans minus the shelves amounts to 335 mill. km^2 or 66 %, that of the continents plus the shelves to 175 mill. km^2 or 34 %.

The average thickness of the portion of the earth's shell that was removed to the moon thus amounts to $\frac{22,000}{353} = 65.7$ km.

But in connection with the way the moon originated from the earth, it is probable that whereas 66 % of the granitic shell contributed to the building up of the moon, the deeper shells contributed a continually decreasing percentage.

3. The solid contents of the granitic shell of the earth of a thickness of 35 km amounts to:

$$\tfrac{4}{3}\pi R_A^3 - \tfrac{4}{3}\pi (R_A - 35)^3 = 17700 \text{ million km}^3\ (R_A = 6371 \text{ km}).$$

Of this, 66 % or 11700 million km^3, went to the moon.

Remains: $22000 - 11700 = 10300$ million km^3 as the core of the moon.

So then the moon would consist of 11700 mill. km^3 of shell-material with a specific gravity of 2.67 and 10300 mill. km^3 with a specific gravity of x.

4. According to GOODACRE the specific gravity of the moon is 3.34. JEFFREYS (1929, p. 299) informs us that, in connection with the dimension and gravitation, a compression of rocks occurs on the moon through which the specific gravity of the rocks increases with about $\frac{1}{10}$ of the average increase of the specific gravity of the rocks in the crust of the earth. The specific gravity of the latter is 4.3 and that of identical rocks under a pressure of 1 atmosphere amounts to 3.3. The average specific gravity of the lunar rocks amounts to, not 3.34, but about 3.24.

From the specific gravity of the moon and of granite it follows:

$$11700 \times 2.67 + 10300\,x = 22000 \times 3.24.$$
$$x = 3.89.$$

The specific gravity of the core of the moon would, on the earth's surface, amount to 3.89. This is more than the specific gravity of whatever rock on earth reaches under a pressure of 1 atmosphere (peridotite 3.234, dunite 3.289, lhersolite 3.330. R. A. DALY, 1933, p. 47).

From this it would appear that some nickel-iron had passed from the earth to the moon. This must be understood in such a way, that no material of the core of the earth disappeared to the moon, but material from a ferrosporadic shell. The pallasites among the meteorites render it probable that nickel-iron occurs in silicate rocks in the crust outside the core of the earth. However, in connection with seismic data we must, with JEFFREYS (1929, p. 220) stick to a sudden transition from outer- to core-material at a depth of 2900 km, so that a gradual transition from core to silicate shell, via lithosporadic and ferrosporadic shells, as imagined by ADAMS, WILLIAMSON (1925) and WASHINGTON (1925) cannot occur.

5. For the present we suppose that, in addition to granitic magma, it was principally basaltic magma with a specific gravity of 2.97, that was removed to the moon, as well as some nickel-iron. JEFFREYS (1929, p. 84) informs us that the nickel-iron alloy forming the core of the earth has a specific gravity of about 12, as a consequence of the enormous pressure there, but that this alloy, under a pressure of 1 atmosphere must have a specific gravity of 8. Supposing x million km³ of basaltic material to be present in the moon, there must be $(10300 - x)$ million km³ of nickel-iron. So that the following equation may be drawn up:

$$x \times 2.97 + (10300 - x) \times 8 = 10300 \times 3.89$$

from which follows:

$$x = 8439 \text{ or round } 8400 \text{ million km}^3.$$

So there would have to be round 8400 mill. km³ of basaltic material in the moon, and $10300 - 8400 = 1900$ mill. km³ of nickel-iron; so the core would consist of 82 % basaltic material and 18 % nickel-iron, which would agree with a ferrosporadic composition.

6. We suppose that the moon was torn off the earth at a time when the magmatic differentiation on earth had so far progressed that a shell of viscous granitic magma of a thickness of 35 km formed the outer layer of the earth. 66 % of this went over to the moon and with it a quantity of substratum which was principally basaltic and also contained nickel-iron.

With a radius of 1738 km, the moon consists of a core of basaltic material plus nickel-iron, with a radius of $R_b = \sqrt[3]{\dfrac{10{,}300{,}000{,}000}{\frac{4}{3}\pi}} = 1348$ km, separated from a shell of granitic magma with a thickness of 390 km by a discontinuity (fig. 2).

If the moon had concentrated all the nickel-iron into a core, its radius would be $R_i = \sqrt[3]{0.18} \times 1348 = 763$ km, and a second discontinuity with this radius would occur in the moon (fig. 3).

So the moon would consist of:

11700 mill. km³	or	53 vol. %	granitic magma
8400 ,,	,,	,, 38 ,,	basaltic ,,
1900 ,,	,,	,, 9 ,,	nickel-iron.

7. Of course it is conceivable that under the granitic lunar shell, between basaltic magma and the nickel-iron core, lies a peridotitic or dunitic magma. But the supposition from which we started: the origin of continents and oceans by the separation of the moon from the earth, demands that in any case nickel-iron must occur on the moon.

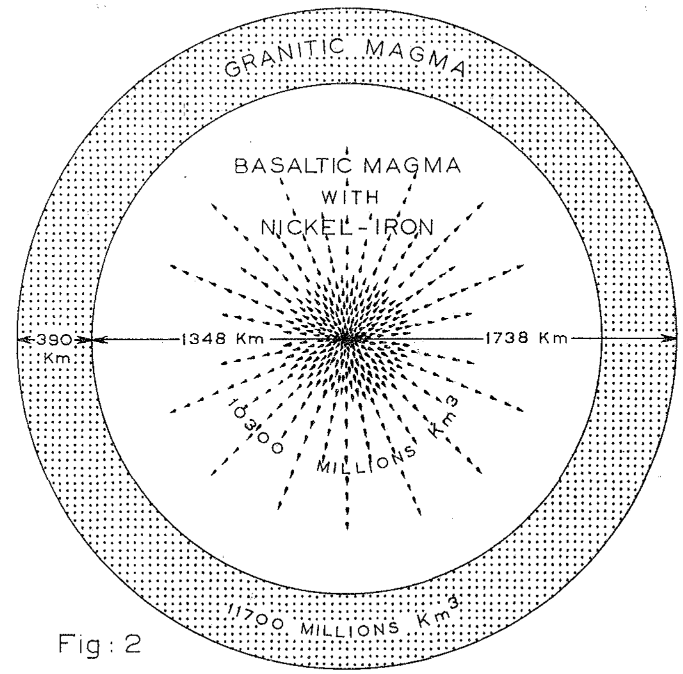

Fig: 2

SECTION THROUGH THE MOON WITH ONE DISCONTINUITY

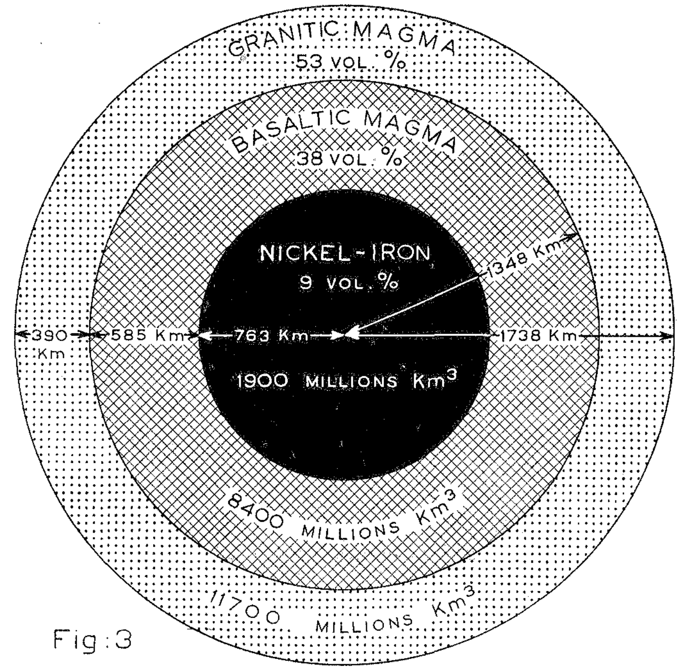

Fig: 3

SECTION THROUGH THE MOON WITH TWO DISCONTINUITIES

III. *On the Parental Magma from which the Granitic Shell originated.*

In the two preceding chapters we discussed the hypothesis wherein the origin of continents and oceans was brought into causal nexus with the separation of the moon from the earth, and a conceivable composition of the moon was calculated from this. We will now discuss another conclusion of this hypothesis.

From the hypothesis it follows that the differentiation in acid and basic magma on earth must have happened in the first 10000 years of its existence (JEFFREYS, 1929, p. 77). It also follows, that there is no sense in looking amongst the rocks known to us, for one out of whose magma granite might have originated by magmatic differentiation. For, if so early in the history of the earth the first quartz-bearing igneous rocks developed through differentiation, the parental magma can no longer be present. The moon can shed some light on the question how that parental magma was composed. For, according to the geophysical laws, the moon must have been formed out of the material of the shells of the earth (JEFFREYS, 1929, chap. III). The moon must be of the same composition as the shells of the earth from which it originated.

We must suppose that nearly all the nickel-iron had already been concentrated within the core of the earth during the birth of the moon. We found (p. 132) that 1900 million km^3 of nickel-iron occurs on the moon. This number is a maximum because we suppose that, under the granitic shell of the moon, only basaltic magma occurs by the side of nickel-iron. If e.g. we had supposed the presence of peridotitic magma on the moon, the calculation would have led to a smaller amount of nickel-iron on the moon.

Supposing that, proportionately, as much nickel-iron occurred at the time in the entire shell of the earth as in the part that was removed to the moon, then the amount would be $\frac{100}{66} \times 1900 = 2878$, round 2900 mill. km^3. The nickel-iron core of the earth has a radius of 3471 km; so it has a volume of 176000 mill. km^3.

The bulk of the nickel-iron that was in the outer crust of the earth would, according to this supposition, be only $\frac{1}{61}$ part of the bulk present in the core of the earth. And the volumes of the nickel-iron in the moon and in the core of the earth would be in a ratio of 1 : 93.

As at the separation of earth and moon especially the light crustal material loosened itself from the earth and 66 % of the outer granitic shell disappeared, less will have disappeared from the basic shell lying under it and even less from the heaviest part of it which was ferrosporadic. So there was probably considerably more than 2900 mill. km^3 nickel-iron in the shell of the earth, consequently more than $\frac{1}{61}$ of the bulk that is now extant in the core of the earth.

So, leaving the nickel-iron of the core of the earth out of consideration, the parental magma must have furnished:

1. a shell, 35 km thick, of granitic magma with a solid contents of about 17700 million km^3.

2. a series of shells, growing more and more basic as they lie deeper, ending in a ferrosporadic rock which contained more than 2900 million km^3 of nickel-iron. This may be a rock such as silicate meteorite with free nickel-iron, like that from Farmington (U.S.A.) or like the basalt from Ovifak in Greenland or, lastly, an olivine rock with nickel-iron (pallasite).

Between the granitic shell and the one containing free iron lie, probably, shells of basaltic (= tachylitic), peridotitic, and dunitic composition.

The average specific gravity of the rocks of the silicate shell of the earth amounts, according to JEFFREYS (1929, p. 299) to 4.3, but is, under a pressure on 1 atmosphere reduced to 3.3. As sial-rocks belong to this too, and the heaviest surface rocks have a specific gravity of 3.33, heavier material must occur in the silicate shell, and that is probably the nickel-iron just mentioned.

IV. *On the Morphology of the Moon.*

Two of the several hypotheses explaining the morphology of the moon are at present being favoured by the students of selenology. By the side of the hypothesis ascribing the surface-formations of the moon to volcanic powers, is the one supposing them to be the consequence of the impact of meteorites. Thus R. A. DALY (1933, pp. 205—206) writes: "Can we "exclude the possibility that the moon's mean density was increased by "the infall of iron-rich bolides during the final organization of the solar "system? It looks as if the impact theory for most of the lunar craters is "going to be ultimately preferred to the theory of gas-controlled eruption "from the interior." and: "On the other hand, one need not doubt the "possibility of some gas-controlled eruptions on the moon and of lava "flooding (areal or fissure eruption) in the broad tracts of the maria."

The supporters of this hypothesis have been furnished with new arguments since the Cañon Diablo Crater in Arizona U.S.A. is no longer the only known instance of a scar made in the lithosphere by a meteorite. FR. HEIDE (1934) lately described 4 certain cases and 6 uncertain ones. As may be expected with an impact of meteorites, it appears that now already they confirm: elliptic wounds occur along with circular ones, a.o. near Henbury (Australia). The thousands of craters on the moon are circular, and when they are very large practically circular. We would, however, need an exact stereographic map of the moon to be able to assert categorically that few elliptic craters occur that have not originated from the combination of circular ones. R. A. PROCTOR constructed a selenographic map in stereographic projection (pl. XVIII) which, though antiquated, renders good service in selenologic studies.

As regards this point we shall have to wait till the new map of the moon from the Mt. Wilson Observatory will have been published (F. E. WRIGHT, 1935). But, besides, no lunar scars are known that might be ascribed to the tangential impact of meteorites. This hypothesis seems improbable to me.

On the other hand the volcanic hypothesis gains in probability if we imagine the moon as we did in chapter II. Voices have been raised, arguing that the dimensions of the craters on the earth and those on the moon differ to such a degree — and they indeed do — that they cannot have been caused by the same category of forces.

But if our conception of the composition of the moon is right, there must exist a great difference between the volcanic effects on the earth and those on the moon, because the moon possesses a granitic shell of 390 km, the earth on the other hand a basaltic shell, with here and there a flake of sial, 35 km thick. On the moon volcanism was sialic, on earth principally basaltic.

P. NIGGLI (e.g. 1920) explained in numerous essays that the magmatic gases play an important part in the differentiation of magma and are for the greater part deposited in acid magmas. And BOWEN (1929, pp. 297—298) writes: "The hypothesis of fractional crystallization as here "conceived leads to belief in the derivation of granitic magma as a late "crystallization from more basic, probably basaltic magma, when that "crystallization has been adequately fractional. Granitic magma so derived "would be the natural home of a relatively high concentration of volatiles, "their proportion in the liquid being ordinarily increased continually as "crystallization proceeds."

So when the process of differentiation began on the fluid earth, the viscous, granitic, 35 km thick outer shell became the richest in gas. With the removal of 66 % of this sialic mass to the moon, the earth's discharge of gases had been, for a considerable part indirectly performed. On the other hand a satellite was born which consisted for 53 vol. % of a magma abounding in gases.

If we further keep in mind the small lunar gravity, which is only $\frac{1}{6}$ of that of the earth, it becomes evident that, immediately after the origin of the moon a discharge of gases on a large scale must have taken place. Considering these fundamental differences between the earth and the moon, the morphology of the two celestial bodies must be entirely different too. The moon entirely surrounded by a granitic shell 390 km thick, the earth by a basaltic shell wherein flakes of granitic magma only 35 km thick floated here and there, with a total surface of no more than 34 % of the entire surface of the earth. The moon therefore surrounded by a relatively homogeneous, the earth by a heterogeneous outer shell. The moon with an enormous discharge of gases in a short time, the earth with a slower discharge of gases which now, after 1600 million years is not yet terminated. The earth with its heterogeneous surface, fundamentally disturbed

in its internal equilibrium with, consequently, pressure and tension in tangential direction, causing fold- and fault-tectogens. The moon with principally radially acting powers, characterized by tremendous gas-eruptions that formed craters and outflows of lava, which in the maria hide the original bottom from view, here and there faintly indicating an older crater, and wherein maybe the most recent gas-eruptions left comparatively small, but inviolate craters.

Only some parts of the $\frac{6}{10}$ of the moon-surface visible to us, are nearly free from traces of gas-eruptions. They are the lunar mountains like Caucasus, Apennines and Alps, which, however, show nothing that might point to folding. Straight faults and rifts occur, but they are of minor importance for the morphology of the moon, like the ridges on the maria.

In the preceding pages a hypothesis, that was first indicated by OSMOND FISHER in 1882, has again been taken up, after D. H. DARWIN gave the impetus to it in 1879, and JEFFREYS discussed it critically from a geophysical point of view. Some of its consequences were traced, and some numbers were used, that may be amenable to improvement. We were not interested in details, but in the order of magnitude and in the proportions.

It would seem that this hypothesis gives an acceptable explanation for:

1. The development of continents and oceans.
2. The limitation of granitic magma to the continents.
3. The composition of the moon.
4. The genesis of the first acid rocks, the primary granites.
5. The morphology of the moon, which is so different from that of the earth.

Oegstgeest, January 11, 1939.

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Mathematics. — *Neue Erweiterungs- und Ueberführungssätze.* By HANS FREUDENTHAL. (Communicated by Prof. L. E. J. BROUWER.)

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Die im Folgenden mitgeteilten Begriffsbildungen und Ergebnisse ermöglichen eine sehr weitgehende Verallgemeinerung der Hopfschen Erweiterungs- und Ueberführungssätze und gewisser von L. PONTRJAGIN [1]) und H. WHITNEY [2]) angekündigten Sätze.

1. Sei f eine Abbildung der orientierten e-dimensionalen Sphäre S^e in die orientierte d-dimensionale Sphäre S^d. *Hypergrad* (genauer (d^e)-Hypergrad) von f heiszt das durch f bestimmte Element der Homotopiegruppe (d^e) [3]). Ist t^e ein e-dimensionales Simplex, dessen Rand in den Nordpol von S^d abgebildet ist, so definiert man in naheliegender Weise für diese Abbildung einen Hypergrad.

2. Ist P ein Polytop, so sei P^m das Polytop, das aus den höchstens m-dimensionalen Simplexen von P besteht.

3. Auf dem Teilpolytop Q von P sei eine Abbildung $f(Q) \supset S^d$ gegeben; f sei fortsetzbar auf $Q \vee P^{r-1}$ zu einer Abbildung F. Der Komplex, der angibt [4]), mit welchem Hypergrad die Randsphäre eines beliebigen r-dimensionalen Simplexe t^r durch F in die S^d abgebildet wird, heiszt das r-dimensionale *Hindernis* von F. Verschwinden des r-dimensionalen Hindernisses bedeutet Fortsetzbarkeit von F auf $Q \vee P^r$.

4. Das r-dimensionale Hindernis ist ein oberer Zyklus. Man kann es durch jeden in $P \setminus Q$ homologen Zyklus ersetzen, wenn man F unter Aufrechterhaltung von f geeignet abändert. Umgekehrt entspricht, falls $Q \supset P^{r-2}$, einer Abänderung von F unter Aufrechterhaltung von f eine Abänderung des r-dimensionalen Hindernisses in ein (in $P \setminus Q$) homologes. — Diese Tatsachen hat unabhängig auch Herr S. EILENBERG entdeckt, wie er mir schriftlich mitteilt [5]).

5. Sei $Q \supset P^q$ und $f(P^{q-1}) =$ Nordpol (das letzte bedeutet für $q \leq d$ keine wesentliche Einschränkung). Früher von uns gebrauchte Bezeichnungen verallgemeinernd nennen wir $S^d f^q$ den Komplex, der angibt, mit welchem Hypergrad jedes t^q in die S^d abgebildet ist. Notwendig und hin-

[1]) C. R. Paris, **206**, 1436—1438 (1938).
[2]) Proc. Nat. Acad., **23**, 285—291, speziell 285 (1937).
[3]) Wegen aller Bezeichnungen vergl. man: Verf., Compositio Math., **5**, 299—314 (1937).
[4]) Ein Komplex läszt sich bekanntlich als eine Funktion auffassen.
[5]) Erscheint demnächst in den C. R. Paris.