Comparative Pathology. A second group of observations regarding Dimension and Form with the growth of the seeds of Phaseolus vulgatis ${ }^{1}$ ). By G. P. Frets. (Communicated by Prof. J. Boeke.)

## (Communicated at the meeting of January 28, 1939.)

This summer I have measured beans still smaller than those of last year ${ }^{2}$ ). For that purpose I used a binocular lens with object micrometer: these measurements are very reliable. For somewhat greater dimensions I used sliding callipers with vernier scale and the larger beans were measured in the usual manner with the measuring instrument of Johannsen ${ }^{3}$ ). All beans were measured and weighed and the indices calculated. From such data various tables have been compiled.
Table 1 contains the result of the calculations for $L$ and $L B$ for the I-line. A correlation table has been drawn up for the length and the length-breadth index. The smallest length measured is 0.5 mm . With the smallest dimensions measured, small differences of dimensions give great differences of indices. Thus we find e.g. among the observations of the 4th bean of a pod $L=0.7, B=0.5 \mathrm{~mm}$, index $=71.5$ and of the 5 th bean $L=0.7, B=0.6 \mathrm{~mm}$, thus index $=85.7$. For the last bean of a pod I measured $L=0.6, B=0.55 \mathrm{~mm}$, thus index $=91.7$ and for the last bean of another $\operatorname{pod} L=0.6, B=0.4 \mathrm{~mm}$, thus $l=66.8$. For slight differences of the dimensions there is thus a great divergence of the indices.
Let us now see what indices are found for different lengths of beans. Of the smallest bean measured, $L=0.5 \mathrm{~mm}$, there are 3 beans. For 2 beans of the same pod I find $L=0.5, B=0.5-0.4$ and $D=0.2 \mathrm{~mm}$, thus index $L B=90$ and for one more bean, the last of a pod, of which 3 other beans are somewhat larger, I find $L=0.5, B=0.4$ and $D=0.3 \mathrm{~mm}$, thus index $L B=80$.
For the dimensions $L=0.6 \mathrm{~mm}$, there are 9 observations; 5 of these beans have a low index $L=67-75$ and 4 a high index $=83.5$ (3) and $I=91.5$. A few of these smallest beans are thus round, others are more oval. They are also seen thus under the lens.
In the case of the smallest dimensions, I have therefore calculated 2 averages; this only holds good for the dimensions $0.5-0.9 \mathrm{~mm}$. Commencing with the dimension $L=1.0 \mathrm{~mm}$, the very high indices do not again appear.

With increasing length we find first a sharp fall of the average

[^0]TABLE 1. Beans in growth. I-line, 1938. The average $L B$-index with increasing lengthclasses

| Lengthclass. <br> in mm | Number | $M \pm m$ |  | $\sigma \pm m$ | Gr. Var. | Sm. Var. |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | Var. br. 9.

$L B$ indices (Table 1). This fall is clear from $L=1.01-3.0 \mathrm{~mm}$, if the lists for the sizes and indices of the individual beans are examined, and is also well expressed in the average indices. It is true that the standard deviation and the probable error are great.

Commencing with $L=3.1 \mathrm{~mm}$, the average $L B$-index begins to rise (Table 1) and increases until $L=12-12.5 \mathrm{~mm}$; for $L=12.1-12.5 \mathrm{~mm}$, we find ind. $L B=68.3$. The numbers in some groups are small. I have at my disposal a group of observations which were made by an assistant where also up to $L=12.5 \mathrm{~mm}$ there is a rise of the average $L B$-index. Moreover we also find in my earlier observations ${ }^{4}$ ) this increase of the average indices up to 12.0 mm . For the classes $4.1-7.0 \mathrm{~mm}$, I have for some classes included in the table 1 averages, (not published here), because the observations of my assistant and myself are perhaps not entirely comparable.
From $L=12.5 \mathrm{~mm}$ we find a very regular fall of the average $L B$-index (Table 1); finally for $L=19.6-20.0 \mathrm{~mm}$ the index $L B=58.3$.

[^1]From this result, that with increasing length the $L B$-index first falls, then rises, and finally again falls, we deduce that with the growth, the breadth with regard to the length is in a varying relation. (CF. p. 229.)


Fig. 1a. Average $L B$-index with increasing length. Fig. 1b. Average $L D$-index with increasing length.

Table 2 shows how with the beans of the $I$-line, the average $L D$-index varies with increasing length. We see that the movement of the $L D$-index (Table 2) does not entirely correspond to that of the $L B$-index (Table 1 and Fig. 1).
With increasing length the average $L D$-index decreases till $L=3.0 \mathrm{~mm}$. This is a regular decrease: the indices are also regularly distributed over the individual beans. Amongst the beans with the smallest lengths we do not meet with examples with a very high $L D$-index (p. 230).
The beans with $L=3.1-5.0 \mathrm{~mm}$ have on an average higher indices than those of the previous group $L=2.1-3.0 \mathrm{~mm}$; the indices, thus also the thicknesses, are somewhat more divergent here in the case of the individual beans.
There is a fairly great difference between the average $L D$-index of beans of $4.6-5.0 \mathrm{~mm}$ and of $5.1-6.0 \mathrm{~mm}$. The group $5.1-6.0$ contains a
large number of beans because a portion of them were measured by my assistant. It is possible that with these fairly small beans the thickness was measured with the sliding callipers by my assistant somewhat less

TABLE 2. Beans in growth. I-line, 1938. The average $L D$-index with increasing lengthclasses.

| increasing lengthclasses. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lengthclass <br> in mm | Number | $M_{L D}$ | Gr. Var. | Sm. Var. | Var. br. |
|  | 33 | 45.2 | 60 | 30 | 30 |
| $1.1-1.0$ | 39 | 41.1 | 50 | 33 | 17 |
| $2.1-3.0$ | 37 | 37.2 | 41 | 30 | 21 |
| $3.1-4.0$ | 23 | 39.1 | 48.4 | 32.3 | 16 |
| $4.1-5.0$ | 25 | 38.5 | 46.3 | 32.3 | 14 |
| $5.1-6.0$ | 41 | 34.2 | 40 | 30.5 | 9.5 |
| $6.1-7.0$ | 32 | 34 | 42.8 | 29 | 14 |
| $7.1-8.0$ | 24 | 36.2 | 43.8 | 29.1 | 15 |
| $8.1-9.0$ | 17 | 37 | 43.6 | 33.4 | 10 |
| $9.1-10.0$ | 12 | 39.1 | 41.2 | 32.3 | 9 |
| $10.1-11.0$ | 15 | 39.1 | 44.2 | 36.7 | 7.5 |
| $11.1-12.0$ | 7 | 40.2 | 43.7 | 35.1 | 9 |
| $12.1-13.0$ | 23 | 42.6 | 46.8 | 37.7 | 9 |
| $13.1-14.0$ | 32 | 43.5 | 56 | 3.2 | 19 |
| $14.1-150$ | 39 | 43 | 50 | 35.4 | 15 |
| $15.1-16.0$ | 44 | 42.5 | 48.7 | 38.6 | 10 |
| $16.1-17.0$ | 45 | 42.2 | 45.5 | 38.1 | 7 |
| $17.1-18.0$ | 40 | 42.3 | 47.4 | 38.1 | 9 |
| $18.1-19.0$ | 31 | 42.5 | 47.3 | 39.4 | 8 |
| $19.1-20.0$ | 19 | 42.6 | 48.1 | 39 | 9 |
| $20.1-$ | $(1)$ | $(42.2)$ |  |  |  |
|  |  |  |  |  |  |

than by me. The small beans up to $L=5 \mathrm{~mm}$ were exclusively measured by me. 15 beans of the groups $L=5.1-6.0 \mathrm{~mm}$ were measured by me; of these beans the index $L D=36.3$; also of the beans measured by me


Fig. 2. Beans. 1938. I-line. Average $B D$-index with increasing breadth.
the average index- $L D$ for this group is thus lower than for the previous group.

For the group $6.0-7.0 \mathrm{~mm}$ we find (Table 2) index- $L D=34$; of 15 beans measured by me the index $L D=35$. Here, I assume, any difference in the manner of measuring of myself and of my assistant is thus almost obviated. For the larger beans, such is certainly the case.
In the case of beans commencing with $L=7.1 \mathrm{~mm}$, we find with increasing length a regularly increasing average index up to $L=14.0 \mathrm{~mm}$ or 15.0 mm . Commencing with 15.0 mm the average index with further increasing length remains equal or becomes somewhat less (Table 2 and Fig. 1). In a small material, measured entirely by me, the increase is still somewhat greater; for 10 beans of $-L \models 14.1-15.0 \mathrm{~mm}$ the average $L D$-index $=46.4$.
With increasing length the average $L D$-index thus runs in a downward direction (Fig. 1), afterwards there is a small rise, again a fall and then follows a rise which corresponds to the rise in the $L B$ curve (Fig. 1), then comes a horizontal or very slightly falling portion of the curve (p. 229).

We have compared the $B D$-index of the beans of the I-line with increasing breadth (Table 3, Fig. 2). The smallest breadth measured is 0.4 mm . Very strongly diverging indices, very high, are not found with the very smallest breadths.
With increasing breadth the average $B D$-index for the smallest breadths increases until $B=2.1-2.5 \mathrm{~mm}$ (Table 3 and Fig. 2). We find three times a very high index; for $B=1.3 \mathrm{~mm}$ we find once $B D$-index $=84.5$, for $B=1.4 \mathrm{~mm}$ once $B D$-index $=85.2$ and for $B=1.8 \mathrm{~mm}$ once $B D$-index $=83.3$. In these exceptional individual cases the thickness is thus in advance of the breadth.

TABLE 3. Beans in growth. I-line, 1938. The average $B D$-index with increasing breadthclasses.

| Breadthclass <br> in mm | Number | $M B D$ | Gr. Var. | Sm, Var. | Var br. |
| :---: | :---: | :---: | :---: | :---: | :--- |
| $0.6-1.0^{1}$ ) | 42 | 64.1 | 77 | 45.4 | 31.5 |
| $1.6-2.0$ | 24 | 65.3 | 74 | 53 | 21 |
| $2.6-3.0$ | 18 | 62.4 | 75 | 55 | 20 |
| $3.6-4.0$ | 48 | 52 | 51.5 | 42 | 19.5 |
| $4.6-5.0$ | 33 | 53.8 | 70 | 40 | 30 |
| $5.1-6.0$ | 38 | 56.1 | 68.5 | 45.5 | 23 |
| $6.1-7.0$ | 31 | 60 | 72 | 50 | 22 |
| $7.1-8.0$ | 17 | 63.9 | 72 | 62.5 | 20 |
| $8.1-9.0$ | 55 | 66.9 | 85.5 | 54.5 | 31 |
| $9.1-10.0$ | 92 | 69.5 | 82.5 | 56 | 26.5 |
| $10.1-11.0$ | 95 | 69.8 | 82 | 62 | 20 |
| 11.12 .0 | 38 | 70.4 | 81 | 65.5 | 15.5 |
| $12.1-$ | 4 | $(68.6)$ | 71.5 | 65.5 | 6 |
|  |  |  |  |  |  |

[^2]From $B=2.5-4.5 \mathrm{~mm}$ the average $B D$-index decreases with increasing breadth. Afterwards commencing with $B=5.0 \mathrm{~mm}$ follows with increasing breadth a regular increase of the average $B D$-index until the end of the observations, i.e. to $B=12.0 \mathrm{~mm}$.
The 3 curves for $L$ and $L B, L$ and $L D$ and $B$ and $B D$ for beans of the I-line can be summarized as follows:
In the first portion of the curves the direction of the curve for $L$ and $L B\left(=\frac{100 B}{L}\right)$ and for $L$ and $L D\left(=\frac{100 D}{L}\right)$ is falling (Fig. 1), the average indices become smaller, i.e. $B$ and $D$ increase proportionately less than $L$. The fall of the curve for $L$ and $L B$ is somewhat steeper than of the curves for $L$ and $L D$.
Of the curve for $B$ and $B D\left(=\frac{100 D}{B}\right)$ the direction in the first portion of the curve is rising, the average $B D$-index becomes greater, i.e. $D$ increases proportionately more than $B$. This result is the confirmation of the result of the comparison of the curves for $L$ and $L B$ and $L$ and $L D$; $B$ increases less than $D$.
After the first falling portion of the curve for $L$ and $L D$ there follows a second larger rising portion of $L=3$ up to 13 mm (Fig. 1). Within this range the curve for $L$ and $L D$ consists of a very small rising portion, then a falling portion and finally a large rising portion. The breadth and thickness in this second portion of the 2 curves do not bear the same proportion to the length; this is also expressed in the curve for $B$ and $B D$ (Fig. 2).
The rising portion of the curve for $L$ and $L B$ is less steep than of the curve for $L$ and $L D, D$ thus increases more than $B$. The curves for $B$ and $B D$ have a very steep portion, which also indicates that $D$ increases more strongly than $B$.

After the rising portion there follows a falling portion for the curves $L$ and $L B$ and a very slightly falling portion for the curves $L$ and $L D$; also in this final portion of the curves $D$ thus increases more strongly than $B$, whilst the fall of the curves shows that $L$ increases more strongly than $B$ and $D$.
The curve for $B$ and $B D$ continues to rise, but in the final portion only slightly, this will thus again show that $D$ increases proportionately more than $B$ (p.232).

## II-line.

For the beans of the li-line, corresponding Tables and curves have been prepared as for those of the I-line. Of the smallest beans measured the outline is sometimes round to wide oval; we then find high $L B$-indices. Of the smallest bean measured, e.g. $L=0.5 \mathrm{~mm}$ and the $L B$-index $=90$. Of 7 beans of which $L=0.6 \mathrm{~mm}$ the Index $L B=83.5$ of 3 specimens; of the remainder the index is resp. $=66.7,74$ and 77 ( 2 specimen); these
are thus oval. Small variations of the breadth give large variations of indices. With very small beans the breadth often seems to be composed of 2 portions; the outer portion of the bean is oval, to this is attached an inner portion almost as wide, the portion connecting it with the pod.

For lengths rising by 0.1 mm , as for the I-line, we have calculated the average $L B$-indices and afterwards combined them into classes with respectively increases of 0.5 and with 1 mm (Table 4). The class $L=1.1$ 1.5 mm has a very low average $L B$-index. Afterwards, in the succeeding classes the average $L B$-index begins to rise fairly regularly till the group $L=5.6-6.0 \mathrm{~mm}$. Most of these groups contain a few very high $L B$-indices (Table 4).

From the group $L=6.1-6.5 \mathrm{~mm}$ onwards the average $L B$-index regularly falls with increasing length. (Fig. 3.) The curves for the average $L B$-index with increasing length for the I and II-line show a very great resemblance (Fig. 1 and Fig. 2, p. 226 and 227). The IL-line contains a much greater number of individual cases with a very high $L B$-index ${ }^{1}$ ). The fact

TABLE 4. Beans in growth. II-line, 1938. The average $L B$-index with
increasing lengthclasses.

| Lengthelass in mm | Number | $M \pm m$ | $\sigma \pm m$ | Gr. Var. | Sm. <br> Var. | Var. br. | Spec. with very high ind. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $-1.0$ | 27 | $71.1 \pm 1.6$ | $8.1 \pm 1.1$ | 84 | 50 | 34 | 3 |
| 1.1-2 | 34 | $61.3 \pm 0.8$ | $4.9 \pm 6$ | 70 | 50 | 20 | 0 |
| 2.1-3.0 | 108 | $64.6 \pm 0.6$ | $6.6 \pm 0.5$ | 82 | 50 | 32 | 1 |
| 3.1-4.0 | 94 | $67.3 \pm 0.7$ | $6.8 \pm 0.5$ | 86 | 55 | 31 | 2 |
| 4.1-5.0 | 96 | $72.5 \pm 0.8$ | $7.7 \pm 0.6$ | 89 | 54 | 35 | 5 |
| $51-6.0$ | 74 | $76.9 \pm 0.6$ | $4.8 \pm 0.4$ | 87 | 61 | 26 | s. text |
| 6.1-7.0 | 67 | $75 \pm 0.4$ | $3.6 \pm 0.3$ | 86 | 65 | 21 | 3 |
| $7.1-8.0$ | 57 | $72.9 \pm 0.5$ | $4.1 \pm 0.4$ | 81 | 63 | 18 | 0 |
| 8.1-9.0 | 49 | $73.7 \pm 0.4$ | $3.1 \pm 0.3$ | 80 | 68 | 12 | 0 |
| 9.1-10.0 | 43 | $72.7 \pm 0.4$ | $2.6 \pm 0.3$ | 79 | 68 | 11 | 0 |
| 10 1--11.0 | 71 | $71.4 \pm 0.3$ | $2.7 \pm 0.2$ | 79 | 65 | 14 | 0 |
| 11.1-12.0 | 156 | $71 \pm 0.2$ | $2.6 \pm 0.2$ | 76 | 64 | 12 | 1 |
| 12.1-13.0 | 290 | $69.5 \pm 0.1$ | $2.4 \pm 0.1$ | 77 | 64 | 13 | 0 |
| 13.1-14.0 | 181 | $68.8 \pm 0.16$ | $2.1 \pm 0.1$ | 77.5 | 63 | 15 | 0 |
| 14.1-15.0 | 48 | $68.0 \pm 0.5$ | $3.6 \pm 0.4$ | 75 | 522) | 232) | 0 |
| 15.1- | 2 | (67.3) |  | 69.5 | 65 | 5 | 2 |

that we find the beans with very high indices with the II-line also especially in the smallest length classes indicates that they are brought about by an unequal growth of length and breadth in such first period of growth (and thus with further growth do not continue to exist).

For beans in growth of the II-line, the next table (not published) contains the average $L D$-indices with increasing length.

From the table and from the curve (Fig. 3b), it appears that the average

[^3]LD-index of the beans of the II-line, with increasing length, falls in the lowest length classes. The fall is less steep than for the $L B$-index (Fig. 3b) and corresponds to that of the $L D$-index of the I-line.


Fig. 3a. Average $L B$-index with increasing length.
Fig. 3b. Average $L B$-index with increasing length.
Thereafter the course of the curve of the $L D$-index of the II-line (Fig. 3b) differs from that of the l-line (Fig. 1). The curve of the Il-line begins to rise, while that for the I-line continues its fall (page 226). Only


Fig. 3c. Average $B D$-index with increasing length.
later (with $L=7.0 \mathrm{~mm}$ ) does the rise begin. This difference is connected with the fact that the thickness of the beans of the II-line is much greater than that of the I-line.
The rising portions of the curves for $L$ and $L B$ and for $L$ and $L D$ have not quite the same path, that of the former is somewhat steeper. In this portion of the curves the breadth and the thickness thus increase more than the length and the breadth somewhat more than the thickness. This difference does not exist for the breadth and the thickness of the beans of the I-line (p. 228, Fig. 1).
After the rising portions of the curves, there follows for the curves of $L$ and $L B$ a third, falling portion; for the curve of $L$ and $L D$ this third portion runs almost horizontally and in its final portion falls somewhat. This corresponds here with the curves of the I-line. In the greatest length classes the length of the beans thus with increasing length increases relatively more than the breadth.
For the Il-line a table (not published) and a curve (Fig. 3c) have been drawn up for increasing length and average $B D$-index; for the I-line the same was made by us for $B$ and $B D$ (Table 3 and Fig. 2).
For the $L D$-index $M \pm m$ and $\sigma \pm m$ were calculated for length classes increasing by 1 mm . Also for the $L B$-index, both for the II- and the $I$-line, we made these calculations (Tables 4 and 1). They yielded corresponding results; the standard deviation and the probable error is fairly great for most averages.
We thus find with the curve for $L$ and $B D$ (Fig. 3c) first a rise of the average indices until 4 mm . Afterwards a fall to 5 mm and then, from the length class $5.1-6.0 \mathrm{~mm}$, a rise, which with $L=11.0 \mathrm{~mm}$ becomes a horizontal or very slightly rising final portion. The curve for $L$ and $B D$ is to be explained by that for $L$ and $L B$ and $L$ and $L D$. The sharp fall from 4 to 5 mm is a result of the sharp rise of $L$ and $L B$ and the weak fall of $L$ and $L D$ in these length classes. The strong rise in the following portion is to be explained by the fall of $L B$ (Fig. 3a) and the weak rise of $L D$ (Fig. 3b). Finally the slightly rising final portion is to be explained by the weakly falling final portion of $L D$. Also the curve for $L$ and $B D$ of the II-line (Fig. 2) appears to be comparable to that for $B$ and $B D$ of the I-line; there is a great resemblance.
We have made still another curve (Fig. 4) for the average $B D$-index with increasing breadth for the portion of the material with the small dimensions. This curve 4 corresponds very much to the first portion of the curve 3 c .

The curves (Figs. 1-3) of the beans in growth of the I- and the II-line show the details of the growth more completely than the growth curves (see our previous Communication) because for the investigation of the beans in growth more beans and especially also smaller ones were available. The second investigation confirms that there are three periods during growth. In the first period, both for the I and II-line with
increasing length, the average $L B$ and $L D_{\text {-index }}$ falls (Figs. 1 and 3), the former more than the latter, and the average $B D$-index (consequently) rises (Figs. 2, 3c and 4); the length grows more, thus more rapidly, than the breadth and the thickness, the thickness more quickly than the breadth. This is hardly to be seen in the growth curves (previous Communication, Figs. 1 and 2), because this material contains hardly any beans of the first period of growth (the smallest length measured is here 3 mm ). In the second period of growth there is a strong growth.


Fig. 4. Average $B D$-index with increasing breadth. IIline. Breadthclasses 0.5 5.0 mm With increasing length the average $L B \sim$ and $L D$. index rises (in the II-line the $L B$-index somewhat more, but the $L D$-index continues the rise longer), the $B D$-index first shows a fall and afterwards a rise. By this is expressed that the $L . B$ and the $L D$. index (thus $B$ and $D$ ) do not act in quite the same manner with regard to the length in the 2 nd period of growth. In the growth curves these differences are only to be found to a very slight degree. In the 3rd period of growth there is a decreasing rapidity of growth. With increasing length the average $L B$ index falls; the length increases more than the breadth. The $L D$-index runs horizontal or slightly uness. WD average $B D$-index increases in the 3rd period of
growth, $D$ thus increases more than the breadth. In the growth curves we see the great growth of the length in the 3rd period, that of the breadth is not so great and of the thickness is greater than for the breadth, in the II-line greater than in the I-line.
The research made, as is compiled in the tables and the curves, teaches that in the case of the growth of beans the three dimensions, length. breadth and thickness, possess a certain degree of independence.
Hitherto we have known from the investigations of Johannsen ${ }^{1}$ ) (1907), which were repeated by me in 1934 2), that with an arbitrary bean material, grouped into length classes, with increasing lengths the average $L B$-index falls and this relation was especially regarded as spurious correlation (PEARSON ${ }^{3}$ ) 1897; Frets ${ }^{4}$ ) 1937). Now we find that in the case of beans in their growth, the indices in various periods of growth show a varying relation to the increasing dimension and we also find that there are differences for the three dimensions. The fall of the average $L B$-index, with increasing length, hitherto only known, appears to be only one besides several other possibilities of the conduct of indices

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\({ }^{2}\) ) 1.c.
) Proc. Roy. Soc.. 60, 489 (1897).
4) I.c. p. 457, 1937.
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towards increasing dimensions. In the Figs. 1-4, the latter portion of the curve of $L$ and $L B$, Figs, $1 a$ and $3 a$, represents the experience of JOHANNSEN.

Spurious correlation is, of course, also present. In the manner indicated by Pearson and earlier described by me ${ }^{1}$ ) we have made the calculations for spurious beans for $L$ and $L B$ of the beans in their growth of 1938 for the I-line. Table 5 and curve 5 show the spurious correlation of $L$ and $L B$ as a regular fall from the smallest length class ( 0.6 to 1 mm ) to the greatest length class ( 19.0 to 20 mm ).

TABLE 5. Beans in growth. I line, 1938. Decreasing average $L B^{\prime}$-index with increasing length. Spurious correlation.

|  | increasing length. Spurious correlation. |  |  |
| :---: | :---: | :---: | :---: |
| Lengthclasses <br> in mm | Number | $M \pm m$ | $\sigma \pm m$ |
| $1-1.0$ | 34 | $853 \pm 113$ | $659 \pm 80$ |
| $1.1-2.0$ | 39 | $256 \pm 47$ | $289 \pm 35$ |
| $2.1-3.0$ | 60 | $188 \pm 21$ | $164 \pm 15$ |
| $3.1-4.0$ | 51 | $153 \pm 13$ | $91 \pm 9$ |
| $4.1-5.0$ | 59 | $117 \pm 10$ | $79 \pm 7$ |
| $5.1-6.0^{2}$ | 54 | $95 \pm 8$ | $61 \pm 6$ |
| $7.1-8.0$ | 35 | $80 \pm 8$ | $48.5 \pm 6$ |
| $9.1-10.0$ | 17 | $70 \pm 6.6$ | $27 \pm 4.7$ |
| $11.1-12.0$ | 7 | 55 |  |
| $13.1-14.0$ | 35 | $50 \pm 3.7$ | $22 \pm 2.6$ |
| $15.1-16.0$ | 43 | $30 \pm 4$ | $26 \pm 28$ |
| $17.1-18.0$ | 42 | $38.6 \pm 3.5$ | $23 \pm 2.5$ |
| $19.1-20.0$ | 18 | $48.9 \pm 3.4$ | $14 \pm 2.4$ |

The spurious correlation is expressed by this Table and this curve; we have not now calculated the correlation coefficient according to Bravais, It is great, as we found earlier (1937, Table 3, p. 458) and explained. Spurious correlation will be the greatest and approximate - 1 if we do not compare $L$ with $L B^{\prime}$ indices, of which $B$ with regard to $L$ is taken arbi~ trarily (1937, p. 457), but when we calculate the indices of $L$ and arbitrarily selected $L$ 's. For a small material ( 500 examples) of heads of the material of ToCHER, I found for the correlation (thus spurious correlation) of $L$ and $L L^{\prime}$-index $t=0.69 \pm 0.023$.

In our Tables 1-6 (4 tables have been published) and the curves 1-4 there is expressed in addition to the spurious correlation also a very distinct organic correlation.

According to the definition of PEARSON, spurious correlation is always brought about if magnitudes, e.g. indices, are compared which are compound and have a common factor; also e.g. in the case of $L$ and $L B$. I have now also investigated in the ordinary manner (this page) the spurious

[^4]correlation for $L$ and $B D$ in the case of this bean material. From table 6 and curve 6 it appears that in this case there is also spurious correlation. Because there is a strong positive correlation between $L$ and $B$ (this appears from Table 4, Col. 3), there is spurious correlation between $L$ and $B D$. Pearson 1), in his latest publication about this subject (1924) spoke of "covered" correlation.

The rise in the curves for the average indices in curves 5 and 6 for the highest length classes is incidental. The probable error (Table 7 and 8)

TABLE 6. Beans in growth. I-line, 1938. Decreasing average $B D^{\prime}$-index with
increasing length. Spurious correlation. The same classes as in table 5 are omitted.

| Lengthclasses <br> in mm | Number | Mean breadth | $m \pm m$ | $\sigma \pm m$ |
| :---: | :---: | :---: | :---: | :---: |
| $1-1$ | 31 | 0.55 | $779 \pm 108$ | $616 \pm 77$ |
| $1.1-2.0$ | 46 | 0.96 | $254 \pm 47$ | $330 \pm 34$ |
| $2.1-3.0$ | 80 | 1.66 | $175 \pm 18$ | $162 \pm 12.5$ |
| $3.1-4.0$ | 75 | 2.17 | $98.5 \pm 11$ | $97 \pm 8$ |
| $4.1-5.0$ | 64 | 2.94 | $116.6 \pm 11.4$ | $92 \pm 8$ |
| $5.1-6.0$ | 51 | 3.66 | $88.1 \pm 9.5$ | $69 \pm 7$ |
| $7.1-8.0$ | 35 | 5.06 | $82 \pm 9$ | $53 \pm 6.5$ |
| $9.1-10.0$ | 19 | 6.25 | $71 \pm 8.4$ | $33.5 \pm 6$ |
| $11.1-12.0$ | 7 | 7.44 | 55.7 |  |
| $13.1-14.0$ | 27 | 8.89 | $47 \pm 4.1$ | $24.4 \pm 3$ |
| $15.1-16.0$ | 44 | 9.76 | $29.1 \pm 3.9$ | $25.5 \pm 2.7$ |
| $17.1-18.0$ | 43 | 10.65 | $42 \pm 4.4$ | $28.8 \pm 3.1$ |
| $19.1-20.0$ | 19 | 11.4 | $58 \pm 4.4$ | $18 \pm 3$ |

indicates it. That this rise is present in both curves is because the material is transmuted in the same manner.
The investigation regarding the dimensions and the indices of beans in their growth was carried out by me in view of the question as to what extent the dimensions are independent during their growth, the question thus whether the form is primary or whether the dimensions are primary ${ }^{2}$ ), This question is of practical significance in the field of heredity. There we ask: In the case of heredity of beans must we accept genes for the dimensions or for the form, thus e.g. for the indices? The same question is applicable in the case of man with regard to the heredity of the shape of the head.

The results of this investigation, which demonstrate a certain measure of independence of the dimensions, point in the direction of genes or hereditary factors for the dimensions. The index is a deduced notion, the dimensions are primary.

Experience now teaches that e.g. in the case of the shape of the human head, there is heredity of the head form, of the head index, e.g. brachycephaly is often dominant over dolichocephaly and segregation is met with.

[^5]In accepting the hereditary factors for the dimensions, we must thus, in order to be able to explain the heredity of the head-index, further accept that there is a coupling of hereditary factors for the various dimensions, thus that they are situated within the same chromosome.
If we accept hereditary factors for the dimensions, we do not need to


Fig. 5. Spurious correlation. Decreasing $L B^{\prime}$-index with increasing length. Fig. 6. Spurious correlation. Decreasing $B D^{\prime}$-index with increasing length.

Beans. 1938. I-line.
accept separate hereditary factors for the size, and capacity, which is necessary if we accept hereditary factors for the index.
We find the dimensions partly independent of each other. They are probably connected with regard to weight; there is probably a negative correlation of the dimensions for beans of the same weight (compensational growth).
We have found a certain degree of independence of the dimensions and the growth has a definite, varying rapidity. The growth, as we know, is brought about by growth substance (hormone) and the result must be regarded in conjunction with what should be attained. In the case of beans, each dimension has a definite size. In the light of the actual size of each dimension, we must see its growth, also its variability. (See also our previous Communication, p. 232.) Thus we may assume that the growth of the bean is not primarily determined by its form but by the size of its dimensions. The form is not arbitrary, but the result of the growth of dimensions, which are determined each by its own size.

The question whether the form is primary or whether the dimensions are so, and whether we must accept hereditary factors for the form or for the dimensions has not been decided as a result of this investigation. It is also a question of principle of life. My investigation, for me, leads to the solution given: it is the simplest and we choose such in natural science. There is still room for another solution, indicated by another consideration of life. By continual labour it will perhaps become possible to arrive at a certain solution. By additional experience, by the establishment of still more facts, it is possible to solve a problem which hitherto has left room for a different solution, according to the manner in which life is considered (Schopenhauer).

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[^0]:    ${ }^{1}$ ) Cf. also Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, 42, 215-223 (1939).
    ${ }^{2}$ ) Proc. Kon. Ned. Akad. v. Wetensch., 41, 325 (1938).
    ${ }^{3}$ ) Genetica, 16, 46 (1934).

[^1]:    ${ }^{1}$ ) For $l=0.5$ there are 2 beans, of which I $L B=90$. For $l=0.6$ to 1.0 there are 10 beans of which I $L B=83.2$ ( 87.7 to 81.5 ). These beans with very high indices are not included in Table 1 (Cf. p. 224).
    ${ }^{2}$ ) This very low index of 46 and the lowest but one of 51 make a low average index. This bean is recorded as last bean of a pod.
    ${ }^{3}$ ) A bean with the remark "curve" in the case of index 66.5 has not been taken.
    ${ }^{4}$ L.c.p., p. 328, Table 4.

[^2]:    $\left.{ }^{1}\right)$ The classes $-0.5,1.1-1.5,2.1-2.5,3.1-3.5$ and $4.1-4.5$ are omitted.

[^3]:    ${ }^{1}$ ) 1.c. page 438, 1938.
    ${ }^{2}$ ) $I=52$ is followed by $i:=64$; without $i=52, \sigma=2.7$ and $I_{m}=68.3$.

[^4]:    ${ }^{1}$ ) 1.c. p. 457, 1937.
    $\left.{ }^{2}\right)$ The classes $6.1-7,8.1-9,10.1-11,12.1-13,14.1-15,16.1-17,18.1-19$ and 20.1 - are omitted.

[^5]:    $\left.{ }^{1}\right)$ Biom. 16, p. 328.
    ${ }^{2}$ ) Proc. Kon. Ned. Akad. v. Wetensch., 41, 324 (1938).

