

Physics. — *The decay of the penetrating cosmic rays. II.* By E. M. BRUINS. (Communicated by Prof. J. CLAY).
(Communicated at the meeting of September 30, 1939.)

§ 1. *The absorption of the penetrating cosmic rays.*

We assume¹⁾ that the penetrating cosmic rays consist of decaying heavy electrons which are created at a height H in the atmosphere by the primary cosmic rays, the differential energy spectrum being

$$N(E) = N_0 E^{-s}.$$

As the lifetime is proportional to the energy E , we have for the number N of the rays incident at an angle ϑ with the vertical

$$\frac{dN}{N} = -\frac{a \sec \vartheta dh}{E - \Delta E}$$

where h is the depth below the layer where the penetrating rays are created. The mass per cm^2 per cm at a depth h in the atmosphere at angle ϑ may be written as

$$m(h) = m_0 e^{-\beta(H-h)}$$

and therefore the energyloss ΔE becomes

$$\Delta E = \int_0^h m_0 e^{-\beta(H-h)} a \sec \vartheta dh = A \sec \vartheta (e^{\beta h} - 1)$$

$A = 1.64 \times 10^8 \text{ eV}$, with $H = 20 \text{ km}$ and $a = 2 \times 10^6 \text{ eV}$ in air per gram/cm^2 . The solution of the equation (1) is

$$N(E) = N_0 E^{-s} \left[\frac{E - A \sec \vartheta (e^{\beta h} - 1)}{E e^{\beta h}} \right]^{\frac{a \sec \vartheta}{\beta(E + A \sec \vartheta)}}.$$

If we assume here that

$$E = \frac{A \sec \vartheta q}{y}$$

we obtain for the integral energyspectrum

$$N(q) = \frac{N_0 \cos \vartheta^{s-1}}{(s-1)(Aq)^{s-1}} C(q)$$

where

$$C(q) = (s-1) \int_0^1 y^{s-2} \left[\frac{q + y(1-e^{\beta h})}{q e^{\beta h}} \right]^{\frac{a y}{A \beta(y+q)}} dy \quad \left. \right\} . . . (I)$$

is the factor by which the intensity is diminished according to the decay. With $a = 0$, we obtain $C(q) = 1$.

The factor $C(q)$ is completely determined by q , s , h and does not depend on the angle ϑ . By numerical integration we obtained with $q = (e^{\beta h} - 1)t$ the following values of $C(t)$ for different t and s .

TABLE I. $C(t)$.

t	$\tau = 10^{-6} \text{ sec.}$	$\tau = 2 \times 10^{-6} \text{ sec.}$	$\tau = 4 \times 10^{-6} \text{ sec.}$	$\tau = 8 \times 10^{-6} \text{ sec.}$	$\tau = 10^{-5} \text{ sec.}$
1	0.125	0.131	0.286	0.293	0.492
1.154	0.165	0.171	0.358	0.365	0.579
1.414	0.231	0.237	0.451	0.456	0.660
2	0.360	0.366	0.585	0.589	0.760
4	0.608	0.612	0.776	0.778	0.880
8	0.782	0.785	0.884	0.885	0.941
16	0.885	0.887	0.941	0.941	0.970
32	0.941	0.941	0.970	0.970	0.985
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s	3	2.93	3	2.93	3
					2.93
					3
					2.93

From these results we see:

1. The angular distribution is not altered by the decay of the heavy electrons and remains at every depth $\cos \vartheta^{s-1}$.

2. If E_{min} represents the minimum energy of the particle which penetrates to a shielded instrument

$$E_{min} = E_{atm} + E_{sh}$$

we have, when q_v represents the limiting value for vertical incidence with an unshielded instrument:

$$q = \frac{E_{min} \cos \vartheta}{A} = q_v + \frac{E_{sh} \cos \vartheta}{A}$$

or:

¹⁾ comp. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam **42**, 54, (1939).

The decrease of intensity of the penetrating rays at an angle ϑ by a shield which is $\sec \vartheta$ times that used in the vertical direction is the same in any direction.

3. The absorption in 0—150 cm Pb for different values of the life-time and for $s = 3$ and $s = 2.93$ are shown in fig. 1, giving the relative intensity under various layers of lead. The curves are constructed in order to determine by extrapolation the intensity of the penetrating rays at sealevel.

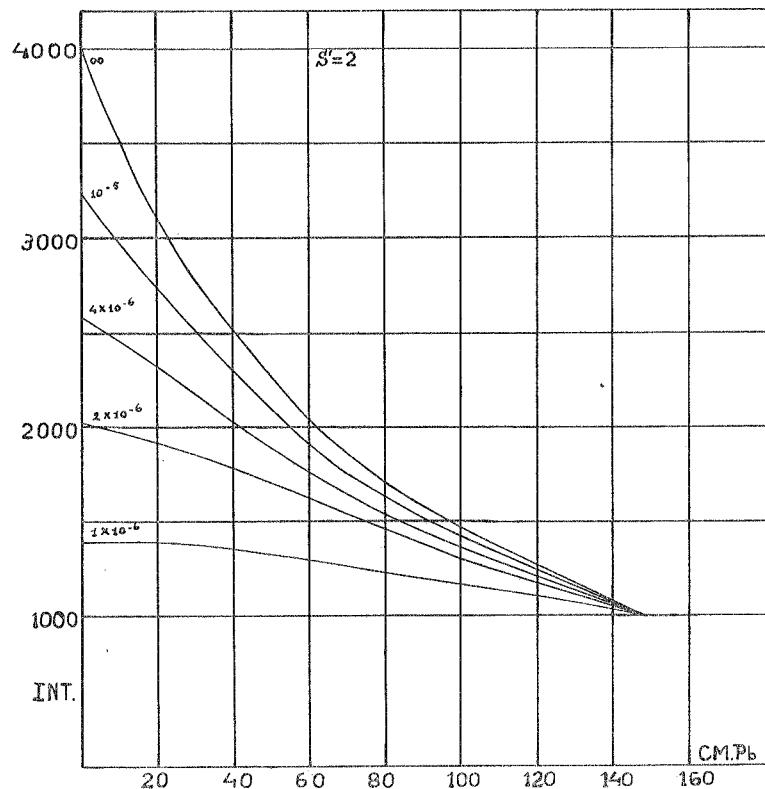


Fig. 1a. Absorption in lead for vertical incidence. $s' = 2$.

§ 2. The influence of the barometric pressure.

The energyloss of a particle can be written as

$$\Delta E = \int_H^h m_0 e^{-\beta h} a dh = \frac{m_0 a}{\beta} (e^{-\beta h} - e^{-\beta H}).$$

From this we obtain, at sealevel $h = 0$, the change of intensity with

barometric pressure, the integral energyspectrum of non-decaying particles being

$$N = N_0 E^{-s'}$$

$$(B \cdot E)_0 = -100 \frac{d \ln N}{dB} = \frac{100 s'}{B_0 - B}$$

where B represents the pressure of the layer where the penetrating particles are created²⁾.

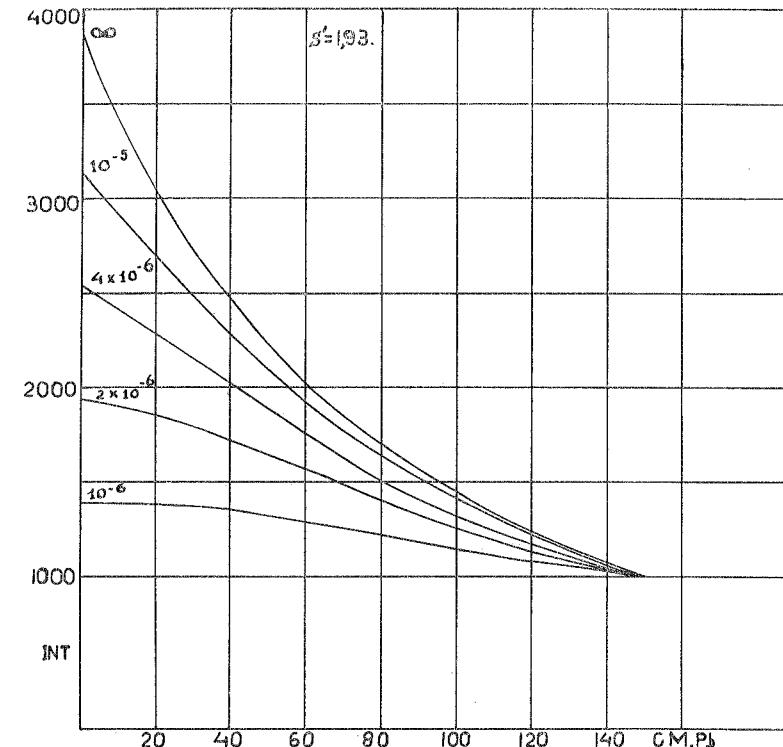


Fig. 1b. Absorption in lead for vertical incidence. $s' = 1.93$.

When, however, the particles are decaying, the intensity at sealevel must be multiplied into a factor $C(H)$. This causes a change in $B \cdot E$ of

$$100 \frac{d \ln C(H)}{d B_0} = \frac{100}{\beta B_0} \frac{d \ln C(H)}{d H}$$

or

$$B \cdot E = (B \cdot E)_0 + \frac{200}{B_0} f.$$

$$f = \frac{\Delta \ln C(H)}{\Delta H} \quad \text{with} \quad \Delta H = 4 \text{ KM.}$$

²⁾ With $s' = 2$ and a B.E. of 6% per cm Hg the pressure in the layer where the penetrating particles are created had to be 43 cm Hg-comp. J. CLAY and E. M. BRUINS, Physica 6, 628, (1939).

The values of $C(H)$ obtained by numerical integration for different values of the lifetime are given in Table II.

TABLE II.

$H \text{ km}$	$\tau \text{ sec.}$	10^{-6}	2×10^{-6}	4×10^{-6}	8×10^{-6}	10^{-5}
8		0.2190	0.4088	0.6032	0.7502	0.7853
16		0.1505	0.3226	0.5271	0.6982	0.7412
20		0.125	0.286	0.492	0.6731	0.719
24		0.1041	0.2539	0.4580	0.6473	0.6968

The corresponding values of f are nearly constant between $H=8$ and $H=24$ km.

τ	1	2	4	8	10	$\times 10^{-6} \text{ sec.}$
f	0.186	0.119	0.072	0.037	0.030	

With $s'=2$, $B \approx 0$ it follows

$$B \cdot E = \frac{200}{B_0} (1 + f) = 2.63 (1 + f) \% \text{ per cm Hg.}$$

The barometer effect decreases for non-decaying particles under a shield which can be penetrated only by particles having an energy $E_{\min} + E_{sh}$ according to the formula

$$(B \cdot E)_0 = \frac{200}{B_0 \left(1 + \frac{E_{sh} \beta}{m_0 a} \right)}.$$

Moreover the influence of the decay will diminish for high energy particles. Therefore we conclude, that the change of $B \cdot E$ caused by the decay must be less than 20% for a lifetime $\tau > 10^{-6} \text{ sec.}$

Mathematics. — *Sur une généralisation du problème de DIRICHLET pour ensembles bornés mesurables (B) quelconques.* Par A. F. MONNA. (Communicated by Prof. W. VAN DER WOUDE).

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§ 1. Construction de la solution.

Dans un article au Bulletin des Sciences Mathématiques¹⁾ récemment M. BRELOT a développé la théorie d'un problème de DIRICHLET pour ensembles bornés fermés. L'idée principale de cette théorie est que tout ensemble borné fermé est limite d'une suite d'ensembles ouverts bornés décroissants. Il est tout naturel de généraliser ses résultats pour les ensembles bornés mesurables (B) quelconques. Les propriétés qu'on obtient dans cette voie sont d'une forme tout à fait analogue à celles que M. BRELOT a donné et le résultat sera une fusion entre la théorie du problème de DIRICHLET pour ensembles ouverts (WIENER) et la théorie de BRELOT pour ensembles fermés. Rappelons que la classe des ensembles mes. (B) est identique à la classe des ensembles O et F de LEBESGUE²⁾, c.à.d. que tout ensemble mes. (B) est la limite finie ou transfinie d'ensembles fermés ou ouverts. Pour arriver à la généralisation mentionnée ci-dessus, nous procédons comme il suit.

1^o. Soit $O^{(1)}$ un ensemble borné O de classe 1. On sait donc que cet ensemble est somme d'ensembles $F^{(0)}$ de classe zéro, donc est limite d'une suite de tels ensembles

$$O^{(1)} = \lim_{n \rightarrow \infty} F_n^{(0)},$$

et l'on a

$$F_1^{(0)} \subseteq \dots \subseteq F_n^{(0)} \subseteq \dots \subseteq O^{(1)}.$$

Soit alors $\phi(P)$ une fonction bornée, continue, définie dans toute l'espace³⁾. D'après BRELOT à chaque $F_n^{(0)}$ et les valeurs de ϕ sur $F_n^{(0)}$

¹⁾ M. BRELOT, Problème de DIRICHLET et majorantes harmoniques. Bull. des Sc. Math. t. LXIII, Mars et Avril 1939.

M. BRELOT, Sur le problème de DIRICHLET et les fonctions sous-harmoniques. C. R. Ac. Sc. t. 206, 1161 (1938).

M. BRELOT, Sur un balayage d'ensembles fermés. C. R. Ac. Sc. t. 207, 1157 (1938).

²⁾ Voir p. ex. CH. DE LA VALLÉE POUSSIN, Intégrales de Lebesgue, Fonctions d'ensembles, classes de Baire. Paris.

³⁾ Puisqu'on n'est pas certain qu'une fonction continue, définie sur un ensemble mes. (B) quelconque, peut être prolongée continûment, on partira dès l'avance d'une fonction continue définie dans toute l'espace.