

too the same result was obtained; a shortening of the induction period and no influence on the reaction velocity. It appeared that tin is less active than copper. So for the original oil, without any supply of metal, the induction period was found to be 60 hours. By supplying a certain surface of copper filings this time was reduced to 8 hours. Using the same surface of tin however we got an induction period of 36 hours. As most of the anti-oxidants contain polarisable nitrogen containing groups<sup>10)</sup> this experiment affirms the well known fact of a higher relative affinity of adsorption for these compounds in the case of copper as in that of tin.

In this investigation not only the alteration of the dipole moment but also that of peroxides, acid value, saponification number, density and viscosity was measured. All these quantities are absolutely independent from the presence of copper or tin, provided one takes the induction period into account. It may be possible that an influence on the reaction velocity exists if one adds such compounds as f.i. copper stearate getting thus a homogeneous system. However such an influence, if it exists, must be regarded as a phenomenon of induced oxydation. It is therefore that we consider the addition of soluble organic metal compounds for the study of specific metal actions as incorrect.

Finally we also tried to get an idea about the mechanism of the oxidation of our oil. Therefore the number of particles which were estimated respectively as peroxides, acids and saponifiable compounds, were plotted on a logarithmic scale as a function of the time of oxidation. For small values of the time straight lines were obtained which show the tangent 1.0 for peroxides (indicating a first oxidation stage) and 2.9 for acids (third oxidation stage).

For the increase of the saponification number however we found in this way the tangent 1.4; a value which is difficult to explain.

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Rotterdam, }  
Utrecht, } December 1939.

<sup>10)</sup> E. VELLINGER and N. SAITO, 2e Congrès Mondial du Pétrole. Paris, juin, 1937.

**Physics.** — *The decay of the penetrating cosmic rays.* III. By E. M. BRUINS. (Communicated by Prof. J. CLAY.)

(Communicated at the meeting of December 30, 1939.)

§ 1. *The GROSZ-reduction.*

The relation between the unidirectional intensity  $\psi$  and the total intensity  $I$  of cosmic rays has been discussed by B. GROSZ<sup>1)</sup>, the fundamental assumptions being:

1. the primary rays are isotropic
2. the absorptionfunction is the same in any direction
3. the intensity is a unique function of the mass per cm<sup>2</sup> traversed.

This well-known GROSZ-reduction, normalising  $\psi$  in such a way that at the top of the atmosphere  $\psi = I$ , is given by

$$\psi = I - x \frac{dI}{dx} \dots \dots \dots (1)$$

$x$  being the depth below the top of the atmosphere in gr per cm<sup>2</sup>.

The conditions 2 and 3 do not hold for the penetrating cosmic rays. Therefore the GROSZ-reduction cannot be correct; neither in the atmosphere nor in a layer of absorbing material. It is easy to prove, that for non-isotropic rays, obeying the conditions 2 and 3 the relation (1) has to be replaced by (see § 3 and § 4)

$$\psi = I - \frac{1}{p+1} x \frac{dI}{dx} \dots \dots \dots (2)$$

the intensitydistribution of the primary rays with the zenith angle  $\vartheta$  being:

$$F_{\vartheta} = F_0 \cos \vartheta^p.$$

§ 2. We wish to stress an important relation between the correction-factors  $K_{\vartheta}$  for the decay in different directions. Suppose the differential energyspectrum in the direction  $\vartheta$  at the height  $H$ , where the penetrating rays are created, is  $F_{\vartheta}(E)$ . Then the energy spectrum at a depth  $h$  below this layer will be<sup>2)</sup>:

$$N(E - \Delta E, h) = F_{\vartheta}(E) \cdot K_{\vartheta}(E)$$

<sup>1)</sup> B. GROSZ, Zs. f. Physik 83, 214 (1933).

<sup>2)</sup> These relations have been deduced and used in these Proceedings 42, 54 and 740 (1939).

where

$$\Delta E = A \sec \vartheta (e^{\beta h} - 1) \quad A = \frac{m_0 a}{\beta} e^{-\beta H}$$

$$K_\vartheta(E, h) = \left[ \frac{E - A \sec \vartheta (e^{\beta h} - 1)}{E e^{\beta h}} \right]^{\frac{\alpha \sec \vartheta}{\beta(E + A \sec \vartheta)}} \dots (3)$$

From (3) we obtain the important relation used in the previous papers.

$$K_\vartheta(E \sec \vartheta, h) = K_0(E, h) \dots (4)$$

§ 3. Reduction in a strongly absorbing layer.

In a layer of strongly absorbing material we can neglect the decay as compared to the absorption. The intensity in the direction  $\vartheta$  under a total absorbing layer of  $x$  gr per  $\text{cm}^2$  being  $\psi_\vartheta(x)$  we have, when  $t$  represents the energyloss per gram per  $\text{cm}^2$ , in the absorbing layer:

$$\psi_\vartheta(x \sec \vartheta) = \int_{tx \sec \vartheta}^{\infty} F_\vartheta(E) K_\vartheta(E) dE$$

from which it follows

$$\frac{d\psi_0}{dx} = -t F_0(tx) K_0(tx) \dots (5)$$

The total intensity  $I(x)$  will be

$$I(x) = \int_0^{\pi/2} \sin \vartheta d\vartheta \int_{tx \sec \vartheta}^{\infty} F_\vartheta(E) K_\vartheta(E) dE.$$

In the absorbing layer the correctionfactor  $K_\vartheta(E, h)$  must be calculated with  $h = \text{constant} = \text{equal to the depth of the top of this layer. Using (4) and } F_\vartheta(E) = F_0(E) \cos \vartheta^p$ , we find

$$\frac{dI}{dx} = -t K_0(tx) \int_0^{\pi/2} \sin \vartheta F_0(tx \sec \vartheta) \cos \vartheta^{p-1} d\vartheta$$

or with  $tx \sec \vartheta = \xi$

$$\frac{dI}{dx} = -t^{p+1} K_0(tx) x^p \int_{tx}^{\infty} \frac{F_0(\xi)}{\xi^{p+1}} d\xi$$

$$\frac{d^2 I}{dx^2} = \left( \frac{d \ln K_0}{dx} + \frac{p}{x} \right) \frac{dI}{dx} + \frac{t K_0(tx) F_0(tx)}{x}$$

or, using (5)

$$\frac{d\psi_0}{dx} = \left( p + x \frac{d \ln K_0}{dx} \right) \frac{dI}{dx} - x \frac{d^2 I}{dx^2} \dots (6)$$

Normalising  $\psi_0$  in such a way, that at the top of the absorbing layer ( $x = x_a$ ),  $\psi = I$  the integration of (6) leads to

$$\psi = I - \frac{1}{p+1} x \frac{dI}{dx} + \frac{1}{p+1} \int_{x_a}^{x_1} x \frac{dI}{dx} \cdot \frac{d \ln K_0}{dx} dx \dots (7)$$

§ 4. Special remarks.

a. For non decaying particles we have  $K \equiv 1$ , and (7) reduces to (2). With  $p = 0$  we obtain (1).

b. As generally  $\frac{dI}{dx} < 0$  the second term in (7) is positive, whereas the third term is negative. The ordinary or the corrected GROSZ-reduction (2) will be too great for decaying particles.

c. From (6) it follows<sup>3)</sup> that  $I$  and  $\psi$  are proportional only when the primary energyspectrum  $F(E)$  is proportional to  $E^{-s}$ . If  $I = c\psi$  we find  $cs = (p+1)(1-c)$ .

d. From (3) we obtain  $G$  being the wateraequivalent of the total atmosphere when

$$x = G\zeta \quad x_a = G\eta$$

$$\ln K_0 = \frac{\alpha}{\beta t G} \frac{\ln \left( 1 - \frac{1}{\eta} \right) (1 - \eta/\zeta)}{(\zeta + 1 - \eta)}$$

<sup>3)</sup> From (7) we have, using  $\frac{d\psi}{dx} = u$  and  $I = c\psi$

$$(p+1-cp)u = -cx \frac{du}{dx} + cu x \frac{d \ln K}{dx}$$

or

$$\ln u = \ln K + \frac{cp-p-1}{c} \ln x + \text{constans}$$

$$\frac{d\psi}{dx} = \text{constans} \cdot K x^{\frac{cp-p-1}{c}}$$

Comparing with (5) we see that the differential energyspectrum

$$F = \text{constans} x^{\frac{cp-p-1}{c}}$$

The integral spectrum is therefor proportional to  $x^{-s}$  if  $cs = (p+1)(1-c)$ .

The constant  $\frac{\alpha}{\beta t G} \simeq \frac{1}{\tau}$  ( $\tau$  being the lifetime of the particles in  $10^{-6}$  sec. The third term in the relation (7) is inversely proportional to the lifetime.

§ 5. *On the reduction in the atmosphere.*

In the atmosphere the correction factor is a function of  $x$ . The reduction can therefore, generally speaking, become very complicated.

If, however,  $F_\vartheta(E)$  is a homogeneous function of  $E$  of degree  $-s$ , then we have:

$$\psi_\vartheta(x \sec \vartheta) = \int_{tx \sec \vartheta}^{\infty} F_\vartheta(E) K_\vartheta(E) dE$$

With  $E = ty \sec \vartheta$  we find

$$\psi_\vartheta(x \sec \vartheta) = t \cos \vartheta^{p+s-1} \int_x^{\infty} F(ty) K(ty, x) dy = \cos \vartheta^{p+s-1} \psi_0(x)$$

whereas

$$I(x) = \int_0^{\pi/2} \sin \vartheta d\vartheta \psi_\vartheta(x \sec \vartheta) = \frac{1}{p+s} \psi_0(x)$$

In the atmosphere we find for every function  $F$ , homogeneous in  $E$ , the reduction

$$\underline{I(x) = \frac{1}{p+s} \psi_0(x)}$$

§ 6. *Comparison with experimental data.*

We apply the results of § 3 to the measurements with counters and the ionisation chamber performed by P. H. CLAY and A. V. GEMERT and J. CLAY<sup>5)</sup>, in 0—50 m of water. The ionisation measurements are given in Fig. 1, curve *I*.

4) Using

$$\beta = \frac{1}{8} KM^{-1}, \quad \alpha = \frac{\mu c^2}{\tau_0 c}, \quad tG = 2 \times 10^9 eV, \quad \mu c^2 = 8 \times 10^7 eV$$

we have

$$\frac{\beta t G}{\alpha} = \frac{60 \tau_0 \times 10^{-6}}{64} \simeq \tau.$$

5) These Proceedings 41, 694 (1938) and Physica 6, 184 (1939).

The GROSZ-reduction for isotropic rays created at a water equivalent of 9 and 5 meters above sealevel ( $\eta=0.9$  and  $\eta=0.5$ ) gives us the curves

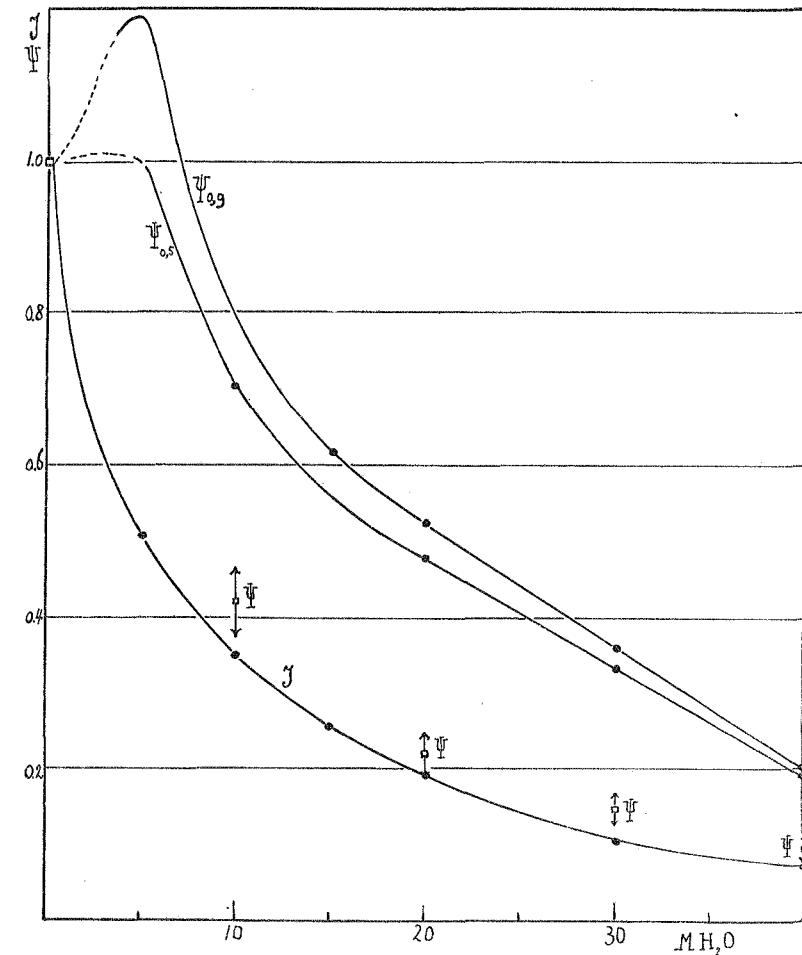


Fig. 1.

$\psi_{0.9}$  and  $\psi_{0.5}$ . The experimental data obtained with a system of counters (opening of the cone  $30^\circ$ ) are only a little above the ionisation curve, but far below those obtained by the ordinary GROSZ-reduction.

We write  $\varphi(x)$  for  $-x \frac{dI}{dx}$  and obtain,  $\delta$  being the depth expressed in 10 m of water below the sealevel the following values for  $\frac{dI}{dx}$  and, by graphical integration putting  $\bar{a}\bar{k} = \beta t G K$ , the integral of  $\varphi d \ln \bar{k}$

$\delta = 0$	0.5	1	1.5	2	3	4
$-\frac{dI}{dx} = 0$	0.492	0.235	0.151	0.114	0.065	0.026

$$\int_{\delta=0.5}^{\delta=x} \varphi(x) \frac{d \ln \bar{k}}{dx} dx$$

$\eta$	0.95	0.9	0.8	0.7	0.6	0.5
1.0	0.489	0.429	0.324	0.262	0.213	0.167
1.5	0.677	0.580	0.436	0.348	0.281	0.219
2	0.773	0.656	0.492	0.392	0.315	0.244
2.5	0.835	0.704	0.526	0.418	0.336	0.260
3	0.877	0.735	0.547	0.434	0.349	0.269
3.5	0.905	0.753	0.561	0.441	0.357	0.275
4	0.921	0.762	0.563	0.450	0.361	0.278

From (7) we have:

$$\tau = \frac{\int_{x_1}^{x_2} \varphi(x) \frac{d \ln \bar{k}}{dx} dx}{(p+1) \{ \psi(x_2) - I(x_2) - \psi(x_1) + I(x_1) \} + \varphi(x_1) - \varphi(x_2)} \quad (8)$$

For small values of  $p$  the first term in the denominator will be almost negligible as the differences between  $\psi$  and  $I$  are very small as compared to  $\varphi(x_1) - \varphi(x_2)$  for small values of  $\delta$ .

We find for  $\eta > 0.7$  a proportionality between the values of the integral and the corresponding values of  $\varphi(x_1) - \varphi(x_2)$  the resulting lifetime being:

$\eta$	0.95	0.9	0.8	0.7	
$\tau_0$	1.9	1.8	1.5	1.4	$\times 10^{-6} \text{ sec}$

This result is in complete accord with the value found from the deviation of the straight line in the  $\log I - \log d$  diagram. This time  $\tau$  has been deduced without using a special energy distribution. The fact, that the first term in the denominator of (8) is negligible shows that the primary energy-distribution is nearly proportional to  $E^{-s}$  (see § 4, c).

**Mathematics.** — *Sur la capacité des ensembles.* Par A. F. MONNA.  
(Communicated by Prof. W. VAN DER WOUDE.)

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§ 1. La notion de capacité fut introduite par WIENER en 1924. Sa définition n'est valable que pour les ensembles bornés, fermés  $F$ . Il considère dans l'ensemble complémentaire de  $F$  l'ensemble ouvert  $\Omega$  s'étendant vers l'infini. Si  $v(P)$  signifie la solution de WIENER pour  $\Omega$  et valeurs-frontière 1, et si  $S$  est une surface suffisamment régulière entourant  $F$ , on a

$$\text{cap. } F = \frac{1}{4\pi} \iint_S \frac{\partial v}{\partial n} ds$$

( $n =$  normale intérieure).

En 1932 M. DE LA VALLÉE POUSSIN donnait la définition suivante, valable pour ensembles bornés quelconques <sup>1)</sup>:

*La capacité d'un ensemble borné  $E$  est la borne supérieure des charges que l'ensemble peut soutenir sans que son potentiel surpasse l'unité.*

Pour ensembles fermés la valeur qui résulte de cette définition est la même que celle résultant de la définition de WIENER.

Le but principal de cet article est de montrer que la définition de WIENER peut être généralisée pour ensembles bornés quelconques en conséquence d'une généralisation du problème de DIRICHLET.

Nous considérerons le cas d'un espace euclidien à *trois dimensions*; la généralisation aux espaces à plus de dimensions est immédiate.

§ 2. Soit  $E$  un ensemble borné quelconque et  $CE$  son complémentaire. Donnons sur la frontière de  $E$  les valeurs-frontière 1. Soit  $O$  un ensemble ouvert contenant  $E$ . Désignons par  $v_f(P)$  la fonction harmonique dans le complémentaire (fermé) de  $O$ , correspondant aux valeurs-frontière 1 sur la frontière de  $O$ , c.a.d. la fonction appelée par M. BRELOT <sup>2)</sup> „la solution pour  $CO$  et valeurs-frontière 1”.

*Définition.* L'enveloppe inférieure des solutions  $v_f(P)$  pour tous les ensembles ouverts contenant  $E$  (donc borne inférieure en chaque point) sera appelée le potentiel capacitair extérieur  $\bar{v}(P)$  de  $E$ .

<sup>1)</sup> M. DE LA VALLÉE POUSSIN se bornait à des ensembles mesurables  $B$ , mais moyennant une définition plus précise de la notion de répartition de masse, la définition peut être rendue générale.

<sup>2)</sup> M. BRELOT, Problème de Dirichlet et Majorantes harmoniques. Bulletin des Sc. Math. t. LXIII (1939).