trouve un grand nombre d'observations qui confirment nos constatations [1]).

Mais le charbon en site n'est pas précisement de la matière incohérente. Aussi quand on l'abat, le front n'est pas muni d'un boisage. Le fait qu'il ne s'écoule pas de lui même, qu'il faut l'abattre au marteau-pic pneumatique qu'on introduit dans les plans de clivage, démontre que nous n'avons pas encore la loi exacte qui commande les déplacements à l'intérieur de la couche de charbon exprimé vers le vide. En effet toute la masse n'est pas pulvérulente. Il y a de gros morceaux qui se réfractent. Pour les matières plastiques la résistance à la traction et à la compression sont égales, mais le charbon ne supporte que très peu de traction et la résistance à la compression est grande quand on ajoute de la pression dans tous les sens. Dans l'annexe 4 nous appliquons à la fin la loi, la condition de rupture pour cette espèce de matière et trouvons de nouveau la distribution, l'accroissement rapide de la pression, selon la formule exponentielle. Ce que nous avons dit de l'influence du frottement qui fait croître la pression près du front outre mesure, s'applique à plus forte raison pour le charbon encore cohérent, sauf que le front n'a même pas besoin de soutènement.

Par ce long chemin nous sommes arrivés à la conclusion qu'aux profondeurs où nous exploitons le charbon l'accroissement des pressions près du front est tel que le toit et le mur ne peuvent pas rester indemnes. Il s'y produit au moins des crevasses, mais si l'on examine le toit avec attention on trouve des dérangements de blocs qui certainement sont dus à l'abatage du charbon.

Jusqu'à présent nous avons toujours supposé que le rocher demeurait intact, mais de cette manière nous ne pouvions pas expliquer le comportement du chantier pendant l'exploitation. Nous espérons traiter dans un troisième chapitre la distribution des tensions dans le rocher et dans la veine autour de la taille quand on aura dépassé la résistance des deux matières à la rupture.

[1]) WEISSNER, Gebirgsbewegungen beim Abbau flachgelagerter Steinkohlenflösse, Glückauf 22 Okt. 1932, p. 945.
LÖFFLER, Zur Abbaudynamik bei streichendem Blindortbetrieb. Der Bergbau 9 Juni 1938.

Hydrodynamics. — *On the application of viscosity data to the determination of the shape of protein molecules in solution.* By J. M. BURGERS. (Mededeeling N°. 38 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft.)

(Communicated at the meeting of February 24, 1940.)

1. In Ch. III of the "Second Report on Viscosity and Plasticity" formulae have been given for the resistance of small particles of elongated form, and for their influence upon the effective viscosity of the liquid in which they are suspended; and the application has been discussed of these formulae to the results obtained with suspensions of polystyrenes by STAUDINGER and SIGNER [1]). A discussion of their application to suspensions of methyl cellulose has been given by SIGNER and V. TAVEL [2]). POLSON has applied the formula for the influence of such particles upon the effective viscosity to the analysis of data obtained with suspensions of proteins [3]), and a report of this work recently has been given by PEDERSEN in Part I, Ch. B, of SVEDBERG and PEDERSEN's new book "The Ultracentrifuge" [4]).

The way in which the formulae for the viscosity and for the frictional coefficient have been applied by POLSON and the discrepancy which has been found between certain calculated and observed values, calls for some remarks which will be collected in the following pages. In connection with these remarks a few data also will be supplied for some cases not treated in the "Second Report", *viz.* for disk-shaped particles (oblate rotational ellipsoids), and for certain systems consisting of rigidly connected spheres.

[1]) J. M. BURGERS, Ch. III of the "Second Report on Viscosity and Plasticity", Verhand. Kon. Nederl. Akad. v. Wetenschappen (1e sectie) Vol. **16**, No. 4 (Amsterdam 1938), pp. 122—126 (resistance formulae), 145—153 (influence upon the effective viscosity), 168—181 (application to polystyrenes).

[2]) R. SIGNER und P. V. TAVEL, Die Form und Grösse von Methylcellulose-Molekeln in Lösung, Helv. Chim. Acta **21**, 535 (1938). The subject also has been treated in the chapter contributed by R. SIGNER to TH. SVEDBERG and KAI O. PEDERSEN's book "The Ultracentrifuge" (Oxford 1940), pp. 431—442, whereas cellulose acetates and some other linear high polymers are considered by E. O. KRAEMER and J. B. NICHOLS, *ibid.* pp. 416—431. — See also footnote 20) below.

[3]) A. POLSON, Ueber die Berechnung der Gestalt von Proteinmolekülen, Kolloid-Zeitschr. **88**, 51 (1939). — See also: Nature **137**, 740 (1936).

[4]) TH. SVEDBERG and KAI O. PEDERSEN, The Ultracentrifuge (Oxford 1940), pp. 38—44. The results of the calculations also have been given by TH. SVEDBERG in the "Opening Address to a discussion on the protein molecule", Proc. Roy. Soc. (London) B **127**, 9—10 (1939). — It may be mentioned that the problem of the determination of the shape of tobacco mosaic virus particles in solution has been discussed by J. R. ROBINSON, Nature **143**, 923 (1939).

The procedure followed by POLSON in order to obtain the molecular weight of a protein exclusively from viscosity and diffusion data, without having recourse to sedimentation measurements, was as follows: The length-diameter ratio L/d of the molecules, which were assumed to have the form of elongated rotational ellipsoids, was deduced from the specific increase of the viscosity of a solution, $\eta_{sp} = \eta/\eta_0 - 1$, making use of the known value of the partial specific volume V of the dissolved protein. The result was applied to derive the absolute dimensions of the molecule from its experimentally determined frictional constant f_{exp}, which can be obtained from the diffusion constant D (measured by means of an optical method [5]) through the equation $f_{exp} = RT/D$. From the dimensions and V the molecular weight then can be calculated.

However, on comparing the molecular weight found in this way with that deduced from observations on the sedimentation equilibrium, as is done *e.g.* in table 5 of "The Ultracentrifuge" [6]), it appears that the calculated values are much too low: by about 30 % when KUHN's formula for η_{sp} [7]) is used, and by about 50 % when the more exact formula, derived from JEFFERY's calculations by BURGERS [8]), is applied. A correct result could be obtained only when instead of these theoretical formulae, an empirical expression, given by POLSON [9]), is taken.

2. It would appear to the present writer that a more convenient basis for a discussion of the cause of the discrepancy can be obtained by arranging the calculations in a different way.

As explained in the "Second Report on Viscosity and Plasticity" [10]), molecular weights always should be deduced from direct measurements, in the present case preferably from the sedimentation equilibrium (M_e), or else from the sedimentation velocity in combination with the value of f_{exp} as obtained from diffusion measurements (M_s) [11]). These values of

[5]) See A. POLSON, Nature **137**, 740 (1936), where it is stated that the diffusion constants were measured by the refractometric method of O. LAMM, Zeitschr. f. physik. Chem. A **138**, 313 (1928) and B **143**, 177 (1929).

[6]) "The Ultracentrifuge", Table 5, p. 44. — The same table occurs in TH. SVEDBERG, Proc. Roy. Soc. (London) B **127**, 10 (1939) and in A. POLSON, Kolloid-Zeitschr. **88**, 59 (1939).

[7]) W. KUHN, Zeitschr. f. physik. Chemie A **161**, 24 (1932).

[8]) Second Report, pp. 152—153.

[9]) This formula can be written in our notation as follows:

$$\eta_{sp}/c = V[4{,}0 + 0{,}098\,(L/d)^2],$$

as the quantity G used by POLSON is equal to cV. See A. POLSON, Kolloid-Zeitschr. **88**, 56 (1939); K. O. PEDERSEN, The Ultracentrifuge, p. 43.

[10]) Second Report, p. 184.

[11]) TH. SVEDBERG, The Ultracentrifuge, p. 9. — The fact that the two methods, in those cases where both can be applied, practically lead to the same values for the molecular weight, proves that diffusion and sedimentation velocity are both governed by the same mean frictional coefficient.

the molecular weights will be assumed as trustworthy here. When now the following assumptions are made: (1) that the volume taken in by the molecule in the solution can be deduced from M by means of the value measured for V; (2) that the molecule has the form of an elongated rotational ellipsoid, the value of η_{sp} can be used to find the value of L/d (under the circumstances of the viscosity experiment). The formula to be applied is:

$$\eta_{sp}/c = V\,\Lambda_{II} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where η_{sp}/c is obtained from POLSON's data for $d(\eta/\eta_0)/dn$ [12]); V from data given in table 48, p. 406, of "The Ultracentrifuge"; while Λ_{II} is a function of L/d, tabulated in the "Second Report" [13]).

Making use of the equation:

$$\pi L\, d^2/6 = M\, V/N_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(where N_A is AVOGADRO's number $= 6{,}06 \,.\, 10^{23}$), it then becomes possible to calculate the dimensions L and d of the molecule. These now can be used to find the frictional constants, f_1 for the motion in the direction of the axis of the molecule, and f_2 for the motion in the direction perpendicular to the axis; from these the mean frictional constant is derived by the equation: $1/f_m = 1/(3f_1) + 2/(3f_2)$. When we write:

$$f_m = 3\,\pi\,\eta\,L\,N_A/\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

then for elongated rotational ellipsoids [14]):

$$\lambda = \frac{log\,(L/d + \sqrt{L^2/d^2 - 1})}{\sqrt{1 - d^2/L^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(the *log* being the Napierian logarithm). Finally the sedimentation constant [15]) $S = (dx/dt)/\omega^2 x$ is given by:

$$S = \frac{1 - \varrho\,V}{3\,\pi\,\eta}\,\frac{M}{N_A\,L}\,\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

When the calculations are executed in this way for the proteins mentioned by POLSON and PEDERSEN, and the results are compared with the experimental values of the sedimentation constant, it appears that the

[12]) A. POLSON, Kolloid-Zeitschr. **88**, 58, Tab. VI (1939).

[13]) Second Report, p. 153.

[14]) The formulae for the resistance of an ellipsoid have been derived by OBERBECK, and are given *e.g.* in C. W. OSEEN, Hydrodynamik (Leipzig 1927), pp. 186—189. — See also: J. PERRIN, Journ. de Physique et le Radium (VII) **5**, 409 (1934) and **7**, 10—11 (1936).

[15]) See "The Ultracentrifuge", p. 5.

correct order of magnitude is obtained, but that on the average: $S_{obs} \simeq = 1{,}3\ S_{calc}$, as will be seen from the following table: [16])

TABLE I.

(Elongated ellipsoids, unhydrated).

Name of the protein	M	$\frac{10^{24}\ MV}{N_A}$	$\frac{\eta_{sp}}{cV}$	$\frac{L}{d}$	$10^8 L$	$10^8 d$	$\frac{1-\varrho V}{3\pi\eta}$	10^{13}. S_{calc}	10^{13}. S_{obs}
Gliadin	27000	32200	14.55	20.9	300	14.3	2.94	1.64	2.1
Lactoglobulin	38000	47200	5.98	10.1	210	20.8	2.64	2.38	3.12
Ovalbumin	40500	50100	5.70	9.6	207	21.5	2.66	2.54	3.55
Haemoglobin	68000	84100	5.38	9.1	237	26.1	2.66	3.67	4.41
Serum albumin	68000	84000	6.52	11.0	269	24.5	2.67	3.44	4.46
Serum globulin	150000	184400	9.0	14.5	420	29.0	2.70	5.46	7.1
Amandin	330000	407000	7.04	11.8	476	40.4	2.69	9.8	12.5
Thyroglobulin	650000	770000	9.87	15.6	710	45.6	2.96	15.5	19.2
Homarus haemocyanin	800000	977000	6.39	10.8	600	55.7	2.75	18.7	22.6
Octopus „	2800000	3420000	9.03	14.5	1110	76.7	2.75	38.6	49.3
Helix pomatia „	6700000	8160000	6.36	10.7	1210	113	2.77	78	98.9

3. The discrepancy between calculated and observed results thus again turns up before us, but now in a form in which a better judgment can be made.

In order to explain this discrepancy SVEDBERG, PEDERSEN and POLSON have advanced the hypothesis that the protein molecules in the solution might be hydrated to such an extent, that their actual volume is equal to about 1,59 times the value MV/N_A [17]). It is logical to assume that the consequent increase in dimensions then also must be taken into account in calculating the value of the frictional constant. It has been pointed out by KRAEMER that the hydration practically does not affect the driving force acting on a protein molecule in an aqueous solution during the

[16]) The data used in the calculations mostly have been taken from table 48, p. 406, of "The Ultracentrifuge". For the molecular weight the value of M_e has been taken, with the exception of that of *Octopus* haemocyanin, where M_s is used. The values of η_{sp}/c have been derived from POLSON's data, Kolloid-Zeitschr. **88**, 58, Tab. VI (1939); in the cases of serum globulin and *Helix pomatia* haemocyanin, however, the values of η_{sp}/cV have been derived from POLSON's Tab. III, *l.c.* p. 56, as it was not evident how the most suitable mean value should be obtained from the numbers given in Tab. VI.

[17]) See A. POLSON, Kolloid-Zeitschr. **88**, 56 (1939); K. O. PEDERSEN, The Ultracentrifuge, p. 43. — According to POLSON the so-called electroviscous effect can be neglected under suitably chosen conditions (see also PEDERSEN, The Ultracentrifuge, p. 26).

sedimentation process [18]); nor does it affect the number of molecules present in a solution of a given concentration of c grams of dry weight per unit of volume. Hence we can use the equation:

$$\frac{\eta_{sp}}{c} = \frac{N_A}{M}\frac{\pi L d^2}{6} A_{II} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

together with equation (5) also in the present case. Instead of eq. (2), we now, however, take:

$$\pi L d^2/6 = 1{,}59 \,.\, MV/N_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

When the calculations are repeated in this way, a smaller value is obtained for L/d; L decreases slightly, whereas d increases, but the value of the sedimentation constant, as calculated from (5) with the new values of L and d, practically remains unchanged, so that the discrepancy is not removed in this way. The results have been given in Table II.

TABLE II.

(Elongated ellipsoids; volume increased by 59 % in consequence of hydration).

Name of the protein	$\frac{L}{d}$	$10^8\ L$	$10^8\ d$	$10^{13}\ S_{calc}$
Gliadin	14.7	271	18.8	1.63
Lactoglobulin	5.75	168	29.2	2.43
Ovalbumin	5.35	163	30.5	2.62
Haemoglobin	4.80	181	37.6	3.79
Serum albumin	6.55	222	33.9	3.51
Serum globulin	9.55	371	38.9	5.33
Amandin	7.2	400	55.5	9.9
Thyroglobulin	10.5	636	60.7	15.3
Homarus haemocyanin	6.4	495	77.4	18.9
Octopus „	9.6	985	103	38.3
Helix pomatia „	6.3	995	158	79

It is possible to arrange the calculations differently, making use neither of eq. (2), nor of eq. (7), but solving L and d directly from eqs. (6) and (5). The results arrived at in this way, however, appear to be extremely unprobable, as they lead to excessive values of L/d and to very low values of d, whereas the volume turns out even smaller than MV/N_A. As examples may be mentioned:

gliadin: $L/d = 360$; $L = 410 \,.\, 10^{-8}$; $d = 1{,}14 \,.\, 10^{-8}$
ovalbumin: $L/d = 710$; $L = 360 \,.\, 10^{-8}$; $d = 0{,}51 \,.\, 10^{-8}$
amandin: $L/d = 265$; $L = 730 \,.\, 10^{-8}$; $d = 2{,}76 \,.\, 10^{-8}$

[18]) E. O. KRAEMER, The Ultracentrifuge, p. 63.

It must be concluded, therefore, that the hypothesis of a large increase of volume in consequence of hydration does not help to remove the discrepancy between the results of the sedimentation and the viscosity measurements, so long as the assumption of an elongated ellipsoidal form is retained.

4. When the values of the ratio L/d for the proteins are calculated from the observed values of the sedimentation constant S, as has been done by POLSON [19]), results are obtained which are more than 50 % smaller than those found from η_{sp}, as given in table I above. It might be suggested, therefore, as a possibility for an explanation of the discrepancy, that the molecules should have different shapes in the two types of experiments to which they are subjected.

This possibility has been considered in the "Second Report" in connection with the application of the formulae to the results obtained with suspensions of polystyrenes, where a similar discrepancy between the calculated and the observed values of the sedimentation constant was found [20]). It was shown there that the forces exerted by the liquid upon a

[19]) A. POLSON, Kolloid-Zeitschr. **88**, 56, Tab. III (1939).

[20]) Second Report, pp. 176—178. The differences between the calculated and the observed values of the sedimentation constant S in the case of polystyrenes is seen from the table given at p. 176, where it must be kept in mind that the calculated values refer to the motion in the direction of the axis only, and consequently must be divided by about 1,5 in order to give the mean values for all directions of the axis in space. It will be observed that in the case of the polystyrene with $M = 270000$ the discrepancy is much greater than those which are found for the proteins. In the case of the suspensions of methyl cellulose, on the other hand, which were investigated by SIGNER and V. TAVEL (see footnote 2) above), the discrepancy between the calculated and the observed sedimentation velocities is much smaller. In "The Ultracentrifuge", p. 437, Table 54, the values of V and η_{sp}/c have been given; the values of L/d and of L and d calculated from these are collected in Table 55 at the same page. For the calculation of the sedimentation constant SIGNER has used a formula somewhat differing from the one applied in the present text (see Helv. Chim. Acta **21**, 542 (1938) and "The Ultracentrifuge", p. 435); when the values are re-calculated with the aid of eqs. (5) and (4) given above, the following results are obtained:

Fraction	IV	III	II
Molecular weight (M_w)	14100	24300	38100
L/d	77	109	139
$10^{13}\ S_{calc}$	0.70	0.73	0.85
$10^{13}\ S_{obs}$	0.83	0.79	0.89

Here again the calculated values are less than the observed ones, although the difference is becoming smaller for the higher molecular weights. SIGNER remarks that the lengths of the molecules, calculated from η_{sp}, exceed the lengths calculated from the number of

molecule in consequence of the shearing motion which exists in a viscosity experiment, tend to produce an elongation of the molecule; on the other hand, during a sedimentation experiment there is practically no tendency towards a deformation of the molecule. Nevertheless, the fact that the viscosity of a protein solution does not appear to be markedly dependent upon the velocity gradient [21]), makes it unprobable that large deformations are caused by the shearing motion of the liquid, unless it might be supposed that the molecule should possess two relatively stable forms, one of which would appear during the sedimentation experiments, while the other would appear in the viscosity experiments, even if the shearing velocity would be small.

5. There remains, however, an important point which needs consideration, *viz.* that the formulae used in the calculations have been deduced for particles of *ellipsoidal* shape, and that numerically different results must be expected for particles having other shapes. Unhappily there do not exist exact formulae for particles of other shapes; the formulae given in the "Second Report" for cylindrical particles are approximations, which, although useful for great values of L/d, are not sufficiently precise for application in the problem here before us.

It might be supposed that the molecules of the proteins should have the form of *oblate rotational ellipsoids*; for particles of such forms exact expressions can be given. Equations (1), (2) and (5) can be used also in this case; the values of A_{II} and λ then must be calculated anew. When it is assumed that the Brownian movement is sufficiently strong to make all directions of the axis in space equally probable, then for A_{II} we can start from eq. (14. 17) of the "Second Report" (p. 151), provided for C_1, C_2, C_3 we now substitute expressions which are valid for oblate ellipsoids of revolution, and which also have been given by JEFFERY [22]), like those for the elongated ones. A few results are collected in the following table, which is a counterpart to the table given at p. 153 of the "Second Report" for elongated ellipsoids (as before, we have written $d = 2b$ for the equatorial diameter, and $L = 2a$ for the axial diameter of the ellipsoids):

glucose residues present in the molecule. — In the case of the cellulose acetates, considered by KRAEMER and NICHOLS ("The Ultracentrifuge", pp. 426—431), the opposite relation is found: here the lengths calculated from η_{sp}/cV remain below the maximum lengths deduced from the number of structural units, while the calculated sedimentation velocities exceed the observed ones. For other linear high polymers the calculated values of S again are too low; it is mentioned that here the molecules may be coiled, so that similar deformations might be possible as in the case of the polystyrenes.

[21]) A. POLSON, Kolloid-Zeitschr. **88**, 57, (1939).

[22]) G. B. JEFFERY, The motion of ellipsoidal particles immersed in a viscous fluid, Proc. Roy. Soc. (London) A **102**, 174—175 (1922—1923). The expressions for C_1, C_2, C_3 can be derived from eq. (61), p. 174, in which the values given at p. 175, eqs. (68), have to be substituted for $\alpha'_0, \beta'_0, \beta''_0$ ($\alpha''_0 = b^2\alpha'_0 - \beta''_0/2$, according to p. 173).

Values of Λ_{II} for oblate rotational ellipsoids.

$b/a = d/L$	Λ_{II}	$b/a = d/L$	Λ_{II}
1	2.50	12	6.39
5	3.56	15	7.64
6	3.95	20	9.74
8	4.74	25	11.85
10	5.56	30	13.96

From $b/a = 35$ onward the approximate formula

$$\Lambda_{II} \simeq 4b/3\pi a + 1{,}19$$

is sufficiently accurate.

When the frictional constant in the present case is written in the form:

$$f_m = 3\pi\eta\, d\, N_A/\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the formula for the sedimentation constant as:

$$S = \frac{1-\varrho V}{3\pi\eta}\frac{M}{N_A d}\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

then the expression for λ becomes, for oblate ellipsoids of revolution

$$\lambda = \frac{\text{arc tg}\sqrt{d^2/L^2 - 1}}{\sqrt{1 - L^2/d^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When the calculations are performed in the same way as was indicated in section 2, again assuming that the volume of a molecule is unaffected by hydration, the curious result is obtained that practically the same values of the sedimentation constant are found as had been derived upon the assumption of an elongated ellipsoidal shape. This will be seen from Table III.

TABLE III.
(Oblate ellipsoids, unhydrated).

Name of the protein	$\frac{d}{L}$	$10^8\ d$	$10^8\ L$	$10^{13}\ S_{calc.}$
Gliadin	31.4	125	3.97	1.62
Lactoglobulin	11.0	100	9.1	2.47
Ovalbumin	10.3	100	9.6	2.65
Haemoglobin	9.6	116	12.1	3.73
Serum albumin	12.3	125	10.2	3.57
Serum globulin	18.3	186	10.2	5.43
Amandin	13.55	219	16.2	10.0
Thyroglobulin	20.3	310	15.3	15.5
Homarus haemocyanin	12.0	282	23.5	19.2
Octopus „	18.3	493	27.0	38.9
Helix pomatia „	11.9	570	48.0	79

One still might wish to repeat the calculations for a different value of the volume of a molecule, in order to see whether the assumption of hydration might help us now. It could be attempted again to calculate L and d directly from eqs. (6) and (5). It is found, however, that upon the supposition of an oblate ellipsoidal form no solution of these equations can be obtained. By way of example in the following table the cases of gliadin and amandin have been considered. Starting with the value of $L\,d^2\,\Lambda_{II} = (6/\pi)\,.\,(\eta_{sp}/c)\,.\,(M/N_A)$ various assumptions are tried for the ratio d/L; for each of these values the corresponding values of L and d can be found, and the calculated value of L/λ can be compared with the experimental value of this quantity, which is equal to $(1-\varrho V)/(3\pi\eta)\,.\,(M/N_A)\,.\,(1/S_{obs})$.

	gliadin	*amandin*
experimental value of $L\ d^2\ \Lambda_{II}$:	891000.10^{-24}	5450000.10^{-24}
„ „ „ L/λ :	$62{,}4.10^{-8}$	117.10^{-8}
oblate ellipsoids		
value assumed for d/L	calculated value of L/λ	
400	81.10^{-8}	149.10^{-8}
100	81.10^{-8}	148.10^{-8}
25	81.10^{-8}	147.10^{-8}
5	77.10^{-8}	141.10^{-8}
sphere 1	71.10^{-8}	130.10^{-8}
elongated ellipsoids		
value assumed for L/d		
2	73.10^{-8}	134.10^{-8}
10	82.10^{-8}	150.10^{-8}
100	70.10^{-8}	128.10^{-8}
200	$65{,}6.10^{-8}$	$119{,}5.10^{-8}$
400	$61{,}5.10^{-8}$	$112{,}4.10^{-8}$

Hence it is found that the only simultaneous solutions of the equations are obtained for elongated ellipsoids with L/d between 200 and 400, which solutions already have been given at the end of section 3.

(To be continued.)