Acoustics. — *The residue, a new component in subjective sound analysis.*

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§ 1. *Introduction.*

In a former paper [1] we described some experiments performed with an optic siren which permits to realize periodic sounds of any prescribed wave form. It was found that the non-linear distortion in the human ear is by far less pronounced than is stated by some authors and in particular that if a complex sound of moderate loudness objectively misses the fundamental tone, this tone is *not* generated in any appreciable amount in the ear. It is a fact that a pitch equal to that of the fundamental tone is ascribed even to those sounds in which the fundamental tone is not present. The almost generally accepted hypothesis to account for that behaviour consists in the assumption that this fundamental tone is generated within the ear by means of non-linear distortion. Our experiments thus proved this hypothesis to be invalid.

We were finally led to speculations as to how the ear might ascribe a pitch to a complex sound and suggested that the periodicity of the wave form rather than the distance of the harmonics in the Fourier spectrum might be the physical property determining this pitch. No mechanism presenting itself to enable the ear to perceive that periodicity, the question was left there unsolved.

Further investigations and in particular the studying and repeating of experiments almost a century old [2] led us to the conclusion that the fundamental problem underlying these and similar paradoxical phenomena is not a question of perception of pitch, but rather a question of subjective sound analysis.

Once a radical change is made in OHM's seemingly trivial acoustical law of sound analysis [3] the explanation of the "case of the missing fundamental" [4] and similar problems follows quite naturally.

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[1] J. F. SCHOUTEN, The perception of subjective tones. Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **41**, 1086 (1938).

[2] A. SEEBECK, Beobachtungen über einige Bedingungen der Entstehung von Töne. Pogg. Ann. **53**, 417 (1841). A. SEEBECK, Ueber die Sirene. Pogg. Ann. **60**, 451 (1843).

[3] G. S. OHM, Ueber die Definition des Tones, usw. Pogg. Ann. **59**, 513 (1843). G. S. OHM, Noch ein Paar Worte über die Definition des Tones. Pogg. Ann. **62**, 1 (1844).

[4] S. S. STEVENS and H. DAVIS, Hearing, New York 1938, p. 99.

This extension of OHM's law involves an important consequence both as regards our conceptions of subjective sound analysis, as well as regards those of the mechanism of sound perception.

We shall, therefore, although the essential clues for solving the problem can be found on page 1092 of our former paper, reintroduce the matter from the very beginning.

§ 2. *Subjective analysis of a periodic impulse.*

Periodic sounds containing a great number of higher harmonics present themselves to the untrained ear as one sound of a certain sharp tone quality with a pitch equal to that of the fundamental tone. Since HELMHOLTZ' careful investigations [5] we know that the suitably trained ear is able to perceive the lowest dozen of harmonics with ease separately in the sound. This confirms OHM's acoustical law which states that sinusoidal vibrations only are perceived as a pure tone, that a complex sound is analysed by the ear into its different sinusoidal components and that these components will be perceived as pure tones having a pitch determined by the respective frequencies.

As a first experiment we listen to a periodic impulse of width $\frac{2\pi}{20}$ [6] which contains the lowest score of harmonics in slowly decreasing amplitude (Fig. 1*a*). The fundamental frequency used was 200 cycles/sec. Our conclusions to be made are naturally restricted in that respect.

In this periodic impulse a strong and sharp note of pitch 200 is imme-

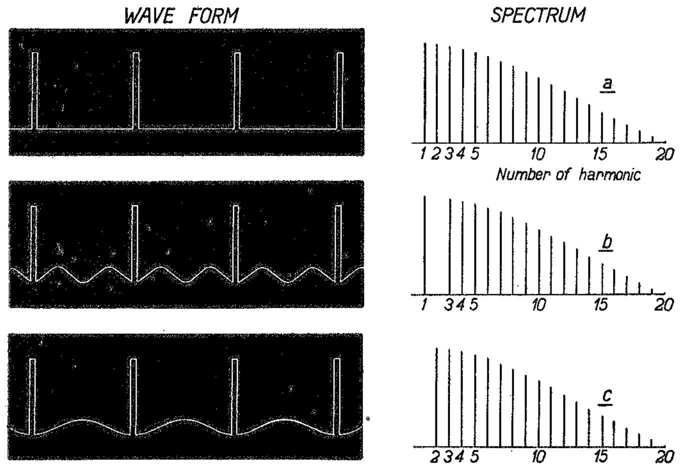

Fig. 1. *Elimination of components. a.* Periodic impulse of width $\frac{2\pi}{20}$. *b.* Periodic impulse without second harmonic. *c.* Periodic impulse without fundamental tone.

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[5] H. HELMHOLTZ, Die Lehre von den Tonempfindungen, 1862, Kapitel IV.

[6] J. F. SCHOUTEN, l.c. p. 1090.

diately perceived, whereas some higher harmonics, although by far weaker than this gravest note, can be heard separately one after another if the attention is fixed upon them.

A very critical method of drawing the attention even of an untrained listener to a particular harmonic consists in adding first (by means of the second holder of the optic siren) that harmonic in the same amplitude but in opposite phase to the impulse. The harmonic, e.g. the second, (Fig. 1*b*) is thus cancelled from the sound. By then covering the second holder the harmonic is made to reappear and can be heard in often surprising clearness. The loudness of the harmonics decreases rapidly with increasing order. The twelfth harmonic is about the last one which can be heard separately, the higher harmonics are *not* separately perceptible.

So far the experiments, but for the great loudness of the gravest note (generally identified with the fundamental tone), present nothing essentially new. If, however, we now cancel the fundamental tone (Fig. 1*c*) we find that the sharp note of pitch 200 remains unchanged present in the perceived sound. Moreover, if thereupon the fundamental tone is again added to the sound, *it is heard separately as a pure tone of pitch 200* of low loudness comparable to that of the second and third harmonic. After some training this fundamental tone may even be heard without any help, although with more difficulty than the next harmonics.

The crucial point is thus that, as to subjective analysis, the sound contains *two* components of pitch 200, one of which, having a pure tone-quality is identical with the fundamental tone, whereas the other, having a sharp tone quality and great loudness, is of different origin.

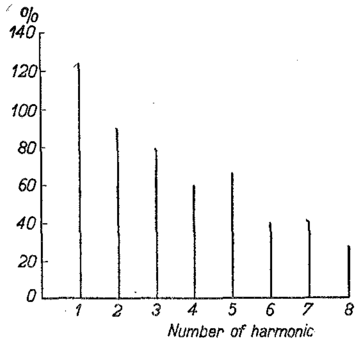

Fig. 2.

Loudness of subjective pure components (compare Fig. 1*a*).

Measurements of the loudness of the various harmonics were taken by comparing in successive contrast a particular harmonic in the periodic impulse and a pure tone of the same frequency. In Fig. 2 the relative amplitudes of the test frequencies, necessary for the match, are given. It will be seen that the fundamental tone in the periodic impulse is heard slightly too strong, the second harmonic in almost its true loudness and the higher harmonics gradually weaker and weaker.

If the harmonics are eliminated one by one, starting from the lowest, the sharp note does not, at first, materially change either in character or in loudness. If, however, the same is done starting from the highest the sharp note gradually loses in sharpness as well as in loudness. This behaviour suggests that the sharp note is connected with the presence of *high* harmonics in the complex sound.

If the intensity is raised, the pitch of the sharp note remains practically unchanged. The fundamental tone, however, exhibits the well-known fall up to about half a tone.

The totally different nature of the fundamental tone and the sharp note of pitch 200 is best brought out by adding the fundamental tone in increasing amplitude to the impulse. The increasing loudness of the fundamental tone is easily perceived, whereas the sharp note does not materially change in character and merely seems slightly to *decrease* in loudness.

Thus the fact that the lowest note in a complex sound is more easily perceptible than e.g. the second, third and fourth harmonic is not due to a particular enhancement of the loudness of the fundamental tone or to a general impairment of the harmonics in question, but to the presence of a hitherto unknown additional subjective component 7) of almost the same pitch as the fundamental tone.

§ 3. *The hypothesis of the residue.*

Hitherto, as expressed in OHM's law, it was generally accepted that the ear analyses a complex sound into components of a pure tone-quality each of which corresponds with one frequency of the inner-ear sound field 8). The difficulty remained that the highest harmonics, although not separately present in subjective analysis, add materially to the loudness as well as to the tone-quality of the complex sound.

We now find that, apart from those pure components, an additional component of sharp tone-quality may exist which cannot be correlated with any single frequency of the sound field.

We propose to call such an additional subjective component a "*residue*". It is very well possible that in a complex sound several residues are present.

OHM's law of subjective sound-analysis may now be extended as follows:

1. *The ear analyses a complex sound into a number of components each of which is separately perceptible.*
2. *A number of these components corresponds with the sinusoidal oscillations present in the inner-ear sound field. These components have a pure tone-quality.*
3. *Moreover, one or more components may be perceived which do not correspond with any individual sinusoidal oscillation, but which are a collective manifestation of some of those oscillations which are not or scarcely individually perceptible. These components (residues) have an impure, sharp tone-quality.*

§ 4. *The pitch of the residue.*

The loudness of the residue in a periodic impulse is greatly diminished

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7) The alternative hypothesis, consisting in the supposition that the observed sharp note is not a *component* of the sound, but the *total sound itself* leads to a great many difficulties and can, we believe, be discarded.

8) Thus including the components generated within the ear by non-linear distortion.

if the higher harmonics are cut off, e.g. by means of a low-pass filter. The residue is thus a collective manifestation of those higher harmonics.

Which physical property of these harmonics might determine the pitch of the residue? Two possibilities present themselves: either the distance between the harmonics, or the periodicity of the total wave form of the harmonics in question. In the case of the periodic impulse the two properties lead to the same pitch, the distance between the harmonics and the periodicity both being 200 cycles/sec.

Comparison of the two wave forms represented in Fig. 3 enables us to answer this question. The first wave form contains the even harmonics only and is thus the octave of the periodic impulse used hitherto. The distance between the harmonics is 400 cycles/sec., the periodicity of the wave form, as well as of any group of adjoining harmonics is also 400 cycles/sec. The pitch of the residue is found to be 400. The second wave form contains the odd harmonics only. In this case the distance between the harmonics is again 400 cycles/sec., the periodicity, however, be it of the total wave form or of any particular group of adjoining harmonics is 200 cycles/sec. We found that a residue is present, although less pronounced than for the first wave form, and that the pitch of this residue is 200. No indication of any subjective component of pitch 400 was found.

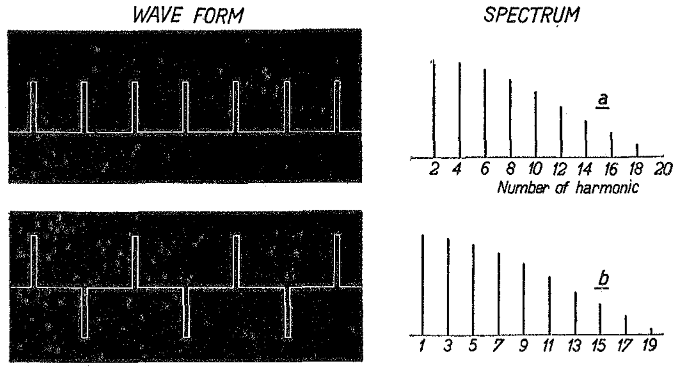

Fig. 3. *Two wave forms having the same distance between the harmonics but a different periodicity.*

The ear thus ascribes a pitch to a residue by virtue of the periodicity of the total wave form of the harmonics which are responsible for this residue.

Summarizing we are thus led to the following view upon subjective analysis of a periodic sound containing a great number of harmonics. The lower harmonics can be perceived individually and have almost the same pitch as when sounded separately. The higher harmonics, however, cannot be perceived separately but are perceived collectively as one component (the residue) with a pitch determined by the periodicity of the collective wave form, which is equal to that of the fundamental tone. We are thus confronted with the surprising result that the harmonics *highest* in frequency, are perceived as a subjective component almost *lowest* in pitch. In a periodic impulse the residue must be the most prominent component due to its richness in harmonics and to the comparatively high sensitivity of the ear to these harmonics.

The pitch of a complex sound now follows quite naturally: *The pitch ascribed to a complex sound is the pitch of that component to which the attention, either by virtue of its loudness, or of its contrast with former sounds is strongest drawn.*

Therefore the pitch of a complex sound may be different depending upon the circumstances under which it is heard. An example of this behaviour was given on page 1092 of our former paper.

§ 5. SEEBECK's *experiments.*

It should be definitely stated that the views proposed here, although rather radical in comparison with the modern acoustical point of view, are no more than an extension of the work of the admirable and ununderstood SEEBECK. In 1841 SEEBECK (l.c.) described a great number of experiments performed with his acoustic siren. This lead OHM (l.c.) in 1843 to the formulation of his famous law. Far from being satisfied SEEBECK in the same year published a second paper in which he proved that OHM's law, although qualitatively accounting for the observed phenomena, could by far not describe these quantitatively. Therefore SEEBECK proposed an extension of OHM's law. The discussion is closed by OHM in 1844 with a second paper, which, though humorous, added nothing essential to clear the difficulties.

HELMHOLTZ (l.c.) too, while discussing the controversy between SEEBECK and OHM impresses the opinion upon the reader that SEEBECK, though otherwise a very keen observer, must have been mistaken here.

We shall describe three of SEEBECK's experiments. In the first experiment air was blown from a pipe against a revolving disk containing a number of concentrically arranged equidistant holes. SEEBECK thus produced a "periodic impulse". If a second pipe was placed against the same side of the disk at half-interval distance from the first the pitch was heard to jump an octave upwards (Figs. 5*a* and 5*b*). The number of impulses per second is thus doubled, the spectrum will contain the even harmonics in double amplitude and no odd harmonics. SEEBECK's objection to this formulation as a means of quantitatively describing the observed phenomena is the following: in the first sound the loudness of the gravest note is very great compared to that of the second and third harmonic. If the odd harmonics are compensated and the even ones doubled, the second harmonic (now the fundamental tone of the new sound) should be a little stronger than in the first sound but not so strikingly strong as is actually heard.

HELMHOLTZ suggests that SEEBECK, while not using the proper means of drawing his attention to the second harmonic of the first sound, may have underrated its loudness. The experiment, however, is easily repeated with our optic siren and completely confirms SEEBECK's description. In view of the hypothesis of the residue the explanation is simple. The strong gravest note heard by SEEBECK is not the fundamental tone (which must have been of almost the same loudness as the second harmonic) but the residue. Doubling the frequency of the impulse not only doubles the amplitude of the second harmonic but *also* shifts the residue an octave upwards, thereby seemingly increasing the loudness of the second harmonic.

SEEBECK's second experiment is more immediately convincing. He compares three wave forms (Fig. 4). The first (Fig. 4*a*) is again obtained by one pipe. The second (Fig. 4*b*) by two pipes at a distance equal to the interval of the holes. This merely results in doubling the amplitude of the periodic impulse. The third wave form (Fig. 4*c*) is obtained by placing the second pipe against the opposite side of the disk at half-interval distance from the first pipe.

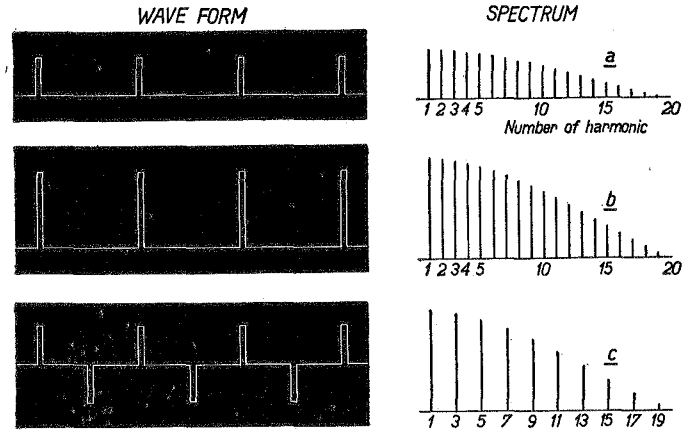

Fig. 4. SEEBECK's *experiment 2.* The amplitude of the fundamental tone in *b* and *c* is twice that in *a*. Yet as to subjective analysis the gravest note in *c* is barely stronger than in *a* and strikingly weaker than in *b*.

It is easily verified that the amplitude of the fundamental tone (which has the same pitch for the three wave forms) in the second and third wave form is twice that in the first one.

Contrary to OHM's expectation SEEBECK finds that in the third wave form the gravest note is barely stronger than in the first and strikingly weaker than in the second wave form.

Again the explanation is simple once it is realized that SEEBECK did not observe the fundamental tone, which must have been comparatively weak, but the residue. This residue must be much weaker in the third than in the second wave form since the former contains only half the number of frequencies.

Lastly we describe a third very elegant experiment. If, in the case of two pipes placed against the same side at half-interval distance, this distance is slightly changed, the pitch of the sound immediately jumps an octave downwards. For demonstration of this effect SEEBECK constructed a disk containing four concentric rows of holes. In the first row the distance between the holes was 20°, in the second 10°, in the third alternately 9½° and 10½° and in the fourth 9° and 11°.

The second row, of course, gave a pitch an octave above that of the first row, in the third the octave was still the most prominent although the lower note was distinctly audible, in the fourth this lower note was more prominent than the octave. It is seen from Fig. 5 that even for the last wave form the fundamental tone is still very weak.

We repeated this experiment by putting one impulse in the first, another in the second (turnable) holder of the optic siren. If the two impulses were exactly half a period apart the pitch was 400. A slight turning of the second holder, however, was sufficient to make the pitch drop to 200. The effect is very striking indeed. Using again the impulse of width $\frac{2\pi}{20}$ we observed the same behaviour as that described by SEEBECK.

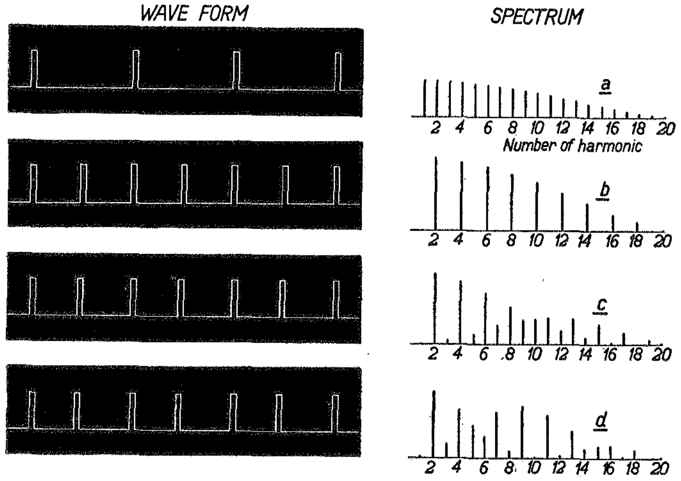

Fig. 5. SEEBECK's *experiment 3.* The pitch of wave form *b* (400) is an octave above that of wave form *a* (200). In wave form *c* a component of pitch 200 is distinctly audible. In wave form *d* it is even more prominent than the component of pitch 400. The fundamental tone, however, is very weak in wave form *c* as well as in wave form *d*.

In terms of the residue the explanation of this effect is that this slight change causes especially the higher odd harmonics to reappear which results in the change of pitch of the residue from 400 to 200. The fundamental tone itself does not play any part in the phenomenon and, in fact, can be heard very weakly after a slight change and more strongly after a great change in the distance between the two impulses, quite in accordance with its objective intensity.

To account for these phenomena, all of which show a discrepancy between the theoretical and experimental loudness of the gravest note in subjective analysis, SEEBECK suggested that *those higher harmonics, which cannot be perceived separately and which have a common period equal to that of the fundamental tone, in some way or other enhance the loudness of this fundamental tone. He even went so far as to suggest that this enhancement is not merely due to non-linear distortion in the ear* (l.c. 1843, p. 480).

The essential feature of our investigations consists in stating that it is not the fundamental tone itself which is so much stronger than should be expected, but that an additional subjective component of almost identical pitch and of often great loudness is present in the sound.

It is OHM's merit to have indicated the general principle underlying the phenomena of subjective tone analysis and to have formulated a law which up till now was considered to be so trivial that a renewed critical testing seemed superfluous.

We cannot but immensely admire SEEBECK to have realized, by means so simple as his acoustic siren the short-comings of OHM's law and even to have suggested an extension of this law which was scarcely considered seriously until now, almost a century later.

In a following paper we hope to investigate the theoretical consequences of the existence of the residue and to show that, as regards the physical part of the analysing mechanism in the inner-ear, the existence of this new component is by no means so improbable as one might expect beforehand.

§ 6. *Additional remarks.*

It is of interest to reconsider now some other older and newer experiments on subjective sound analysis.

KÖNIG 9), experimenting with SAVART's siren, found that while holding a stiff wooden peg against the cogwheel (128 teeth, time of revolution 1 sec.) *a rattling sound as well as the tone c* (128 cycles/sec.) *are heard.* If however, the edge of a paper card is taken, the rattling sound is scarcely perceptible whereas the tone *c* can be heard very clearly.

Our interpretation of this behaviour is that the first sound contains a great many harmonics and thus brings about a residue (the rattling sound) whereas the second sound is comparatively pure.

A highly remarkable consideration can be found in STUMPF's work. STUMPF 10) experimenting on synthetic vowels obtained by sounding together various harmonics of a given frequency finds that the pitch is equal to that of the fundamental tone even if that tone is not objectively present. He ascribes this throughout to a difference tone generated within the ear by non-linear distortion. At one place, however, he wonders: "Yet I have doubted sometimes whether it (the difference tone) is, even subjectively, really present within the ear. One might imagine that in a sound consisting of the objective tones $c^2g^2c^3e^3$, *the pitch* c^1 *only* presents itself to the listener, without the fundamental tone c^1 entering subjectively into the sound".

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9) R. KÖNIG, Ueber den Zusammenklang zweier Töne. Pogg. Ann. **157**, 226 (1876).

10) C. STUMPF, Die Sprachlaute. Berlin 1926, p. 185.

This paradoxical formulation is very similar to our interpretation in our former paper, which we now, however, prefer to abandon in favour of the supposition that an alien component, the residue, is present in the sound. This component may, if loud enough, determine the pitch of the total sound.

The hypothesis of the residue may also be of importance in connection with the *strike note of bells.* We hope to deal more explicitly with that fascinating problem later. It may suffice now to draw attention to its chief characteristics.

In well-designed church bells a prominent note: the strike note, is heard. No partial corresponding to the pitch of that note is present in the unharmonic spectrum. An experimental rule, still adhered to by those workers not sufficiently misled by its theoretical improbability, asserts that the pitch of the strike note is an octave below that of the 5th partial of the bell spectrum. More physical or technical minded people, however, having found that the presence of the strike note is linked up with the presence of at least the 5th and the 7th partial, and having stated that the difference in frequency of those partials is sometimes almost equal to the pitch of the strike note, interpret it as a difference tone (formed within the ear) of those partials. In its essential features we are confronted with the "case of the missing fundamental" all over again.

On account of our hypothesis of the residue one might expect the following behaviour: if, in an unharmonic spectrum, by chance a number of partials are integer multiples of a certain frequency, an additional subjective component of corresponding pitch having a sharp tone quality and eventually great loudness should occur. If no such grouping is possible *no* extra component of definite pitch should be present; if several groupings are possible more than one additional component might occur. All of these expectations correspond to essential features of the strike note of bells.

We are thus led to suppose that the strike note of bells should, indeed, not be interpreted as a difference tone and moreover that it is another example of a residue. Conversely one might call the residue the "strike note" of periodic sounds.

The author wishes to thank Ir. R. VERMEULEN for his valuable criticism.

S. i. H., January 1940.