Summary.

With the interferometric method of quartz LUMMER plate crossed with a quartz spectrograph the ZEEMAN-effect of the thorium lines has been investigated. A number of 50 energy levels of the doubly ionized atom Th. III have been detected. A list of classified Th. III lines is given. The structure of the Th. III spectrum is not analogous to Ra. I but to Ce. III. The g-values have been compared.

Laboratory "Physica" of the University of Amsterdam.

April, 1940.

Mathematics. — *Self-projective point-sets.* By Dr. O. BOTTEMA. (Communicated by Prof. W. VAN DER WOUDE).

(Communicated at the meeting of April 27, 1940.)

1. If we consider in n-dimensional space S_n a set of $(n+2)$ points (no $n+1$ of which belong to a S_{n-1}) taken in a given order, there always exists a non-singular collineation, which interchanges the points of the set in a given way. This is a consequence of the well-known fact that a collineation is determined by giving $(n+2)$ pairs of conjugated points. The theorem does not hold for a set of $(n+3)$ points (no $n+1$ of which belong to a S_{n-1}) taken in a given order and which may be called a *throw* (dutch: *worp*). If we exclude the case $n=1$ (the four points then being permutable according to the *Vierergruppe*) there does not generally exist a collineation, differing from identity, so that the set, taken as a whole, is not altered. The question arises to construct-analogous with harmonic and equi-anharmonic sets in the case n=1-throws which are invariant for certain finite collineation groups, thus showing „projective symmetry" which the general throw lacks.

For $n=2$ and $n=3$ the question was completely solved by BARRAU [1]). Answering a prize-question of the Wiskundig Genootschap for the year 1938 following BARRAU's line of thought I gave additional remarks to the general theory.

In the following by a new method a complete solution is given.

2. As invariants for the throw $A_1, A_2, \dots A_{n+3}$, BARRAU takes the set of homogeneous coordinates $(a_1 ; a_2 ; \dots a_{n+1})$ of the point A_{n+3} with regard to a system where the first $(n+1)$ points are fundamental points and A_{n+2} is the unit-point. The classes of projective throws are thus represented by the points of an S_n. By the $(n+3)!$ permutations of the $(n+3)$ points of the throw, the invariants a_i are transformed in a well-defined way and take, for $n>1$, in general $(n+3)!$ values. In the S_n an involution of degree $(n+3!$ is created. The coinciding points of the involution represent the self-projective throws.

The method here given is based on the well-known fact, that the $(n+3)$ points of a throw in S_n always lie on a rational normal curve C_n of degree n; the curve is uniquely determined by the points. If t is the rational parameter on C_n, the points of the throw can be given by

1) BARRAU, Proc. Kon. Akad. v. Wetensch., Amsterdam **39**, 955—961 (1936); **40**, 150—155 (1937).

a set of $(n+3)$ values of t. We suppose that $t=t_i$ corresponds with the point A_i. If $(p\,q\,r\,s)$ stands for the anharmonic ratio

$$\frac{(s-q)(r-p)}{(r-q)(s-p)},$$

we consider the set of n non-homogeneous values

$$p_i=(t_{n+3}\,t_{n+2}\,t_{n+1}\,t_i). \qquad (i=1,2,\ldots n)$$

According to their geometrical meaning the p_i are invariants for the projective group in S_n. They are a complete set of invariants for the throw: two throws which have the same set of p_i are projective. This follows immediately from the following theorems:

1°. All the C_n in S_n are projective.

2°. A C_n is invariant for a group of ∞^3 collineations in S_n, which corresponds with the group of linear transformations of the rational parameter.

It can easily be shown, that the p_i notwithstanding their different origin are not essentially unlike BARRAU's invariants. If $A_1, A_2, \ldots A_{n+1}$ are taken as fundamental points for the system of coordinates, a C_n which passes through these points has the equations

$$x_i=B_i\,\frac{P(t)}{(t-t_i)} \qquad (i=1,2,\ldots n+1),$$

where B_i are constants and $P(t)$ stands for

$$(t-t_1)(t-t_2)\;\ldots\;(t-t_{n+1}).$$

If the curve moreover passes through the points A_{n+2} and A_{n+3}, the corresponding parameter-values being t_{n+2} and t_{n+3}, the coordinates of these points are respectively

$$x_i'=B_i\,\frac{P(t_{n+2})}{(t_{n+2}-t_i)} \qquad (i=1,2,\ldots n+1)$$

and

$$x_i''=B_i\,\frac{P(t_{n+3})}{(t_{n+3}-t_i)} \qquad (i=1,2,\ldots n+1)$$

If we take A_{n+2} as unit-point the homogeneous coordinates of A_{n+3}, being the BARRAU-invariants a_i of the set, are obviously

$$a_i=\frac{x_i''}{x_i'}=\frac{P(t_{n+3})}{P(t_{n+2})}\cdot\frac{t_{n+2}-t_i}{t_{n+3}-t_i},$$

or

$$a_i=\varrho\,\frac{t_i-t_{n+2}}{t_i-t_{n+3}}$$

where ϱ is a constant.

We have therefore

$$p_i=\frac{a_i}{a_{n+1}},$$

the new invariants thus being shown to be the ratios of BARRAU's.

3. If we have a self-projective throw in S_n, there exists a group of collineations which interchanges the separate points, leaving the throw as a whole invariant. The collineations then leave invariant also the rational normal curve C_n which passes through the $(n+3)$ points and therefore belong to the group of ∞^3 collineations having that property. The latter group is isomorphic with the group of linear transformations in one variable. Therefore the finite group of collineations which leaves the throw invariant is isomorphic with a group of linear transformations for the rational parameter t and can be produced by the latter.

Thus in order to find a self-projective throw, we must consider a set of parameter-values $t_i\;(i=1,2,\ldots n+3)$, which has the property that there exists a group of linear transformations of t by which the set as a whole is not altered. Now our investigation is highly facilitated by the fact, that the theory of linear groups in one variable was developed a long time ago. As KLEIN has pointed out, there are no other groups but the cyclic groups (of order k), the dihedron groups (of order $2\,k$), the tetrahedron group (of order 12), the octahedron group (of order 24) and the icosahedron group (of order 60). In consequence of this we shall not proceed by summing up the possible self-projective throws for a given value of n, but we start from a given group and construct the point-sets which are invariant for this group. For this construction we can make use of the idea of *Diskontinuitätsbereich* as defined by KLEIN. The points of the complex t-plane by means of a stereographic projection can be represented by the points of a sphere the rotations of which correspond with the linear transformations of t. Finite groups of these transformations correspond with groups of rotations belonging to regular polyhedra inscribed in the sphere. The symmetry-planes of these polyhedra divide the surface of the sphere in a number of regions. A point of the sphere, submitted to the rotations of a group of order r, generates a set of points the number of which is r (respectively a factor of r) if the original point belongs to one region (respectively to two or more regions).

4. First considering the cyclic groups and choosing $t=0$ and $t=\infty$ as fixed points, the group is given by

$$t'=\varepsilon_k^{\nu}\,t \qquad (\nu=0,1,\ldots k-1),$$

where ε_k stands for $e^{\frac{2\pi i}{k}}$. From an arbitrary point (not coinciding with a fixed point) arises a set of k points, which is invariant for the cyclic group. Thus a point-set which remains unaltered by the group consists of mk, $mk+1$ or $mk+2$ points, where $m \geqslant 0$ stands for an integer. As t-values of the points of the set we can take $a_\mu \overset{\nu}{\varepsilon}_k$ ($\mu = 1, 2, \ldots m$; $\nu = 0, 1, \ldots k-1$) to which are added 0, 1 or 2 of the points $t=0$ and $t=\infty$. The numbers a_μ can be chosen arbitrarily with the exception of the points of the set having to be different.

It is possible that the sets thus obtained are invariant for a wider group of transformations than the cyclic group. If $k=2$, $m=1$ or $m=2$, we have sets of four points, which are invariant for the Vierergruppe mentioned above; if k is arbitrary, $m=1$, the sets containing k or $k+2$ points are invariant for a dihedron group of order $2k$.

5. The dihedron group can be generated by adding to the cyclic group $t' = \overset{\nu}{\varepsilon}_k t$ the transformation $t' = \frac{1}{t}$. It contains besides the cyclic group the transformations $t' = \frac{\overset{\nu}{\varepsilon}_k}{t}$, whose second powers are the unity-transformation. They interchange the points $t=0$ and $t=\infty$. If an arbitrary point is submitted to the transformations of the group, it generates a set of $2k$ points, which is invariant for the group. This number is reduced to k if the original point is chosen in one of the points $t = \overset{\nu}{\varepsilon}_k$, and to 2 if the point is chosen in $t=0$ or $t=\infty$.

Thus a point-set which is invariant for a dihedron group of order $2k$ consists of

$$m \,.\, 2k + m_1 \,.\, k + m_2 \,.\, 2$$

points; here m is an integer $\geqslant 0$, m_1 is 0 or 1, m_2 is 0 or 1.

It is possible that the point-sets so obtained are invariant for a wider group than the dihedron one. So for $k=4$, $m=0$, $m_1=1$, $m_2=1$, we have a set of six points ($t = 1, i, -1, -i, 0, \infty$) which is the stereographic projection of an octahedron and accordingly is not altered by the octahedron group.

6. The t-values of a point-set which is the stereographic projection of a tetrahedron, an octahedron or an icosahedron and the formulae for the linear transformation groups which belong to them, are not given here. They can o.g. be found in KLEIN's classical monography [1]).

As for the tetrahedron group, it is of order 12. An arbitrary point, submitted to the group, generates a set of 12 points, the t-values of which can be written down by means of KLEIN's formulae. The number decreases to 6, if the original point is taken on the boundary of two Diskontinuitätsbereiche; the six points correspond to the middles of the edges of the tetrahedron and have the octahedron symmetry. The

[1]) KLEIN, Vorlesungen über das Ikosaeder. Leipzig (1884).

number decreases to 4, if the original point is taken on the boundaries of three regions, the points being the vertices of a tetrahedron (which can be chosen in exactly two ways); they correspond with four values of t which have an equianharmonic ratio.

A point-set which is invariant for the tetrahedron group thus consists of

$$12\,m + 6\,m_1 + 4\,m_2$$

points, where m is an integer $\geqslant 0$, $m_1 = 0$ or 1, $m_2 = 0$, 1 or 2. In the case $m = m_2 = 0$, $m_1 = 1$, the set has the symmetry of the octahedron group.

The octahedron group is of order 24. An arbitrary point induces a set of 24 points, invariant for the group. Choosing the original point in a particular way, there arise sets of respectively 12, 8 and 6 points, each having one and but one representative.

A point-set which is invariant for the octahedron group therefore consists of

$$24\,m + 12\,m_1 + 8\,m_2 + 6\,m_3$$

points, where m is an integer $\geqslant 0$, m_1 is 0 or 1, m_2 is 0 or 1, m_3 is 0 or 1.

The icosahedron group has the order 60. An arbitrary point generates an invariant set of 60 points. There are three particular sets, which contain respectively 30, 20 and 12 points. Accordingly a point-set, invariant for the group consists of

$$60\,m + 30\,m_1 + 20\,m_2 + 12\,m_3$$

points; m is an integer $\geqslant 0$, m_1, m_2 and m_3 are each seperately 0 or 1.

7. Making use of these results, we are able to construct in S_n, where n has an arbitrary vaule, all the sets (containing a finite number of points $r \geqslant n+3$), which are invariant for a group of projective transformations. For the present we leave the value of n out of account and investigate the sets of r values of t, which are as a whole invariant for the linear groups of t-transformations mentioned above. This being done and t_ν ($\nu = 1, 2, \ldots r$) being such a set, we distribute these values over a rational normal curve C_n in S_n, e.g. the curve with the parameter-equations

$$x_\mu = t^\mu \qquad (\mu = 0, 1, \ldots n).$$

Thus we obtain the throws in S_n, which are self-projective. In what follows we confine ourselves to $r = n+3$, this case being the starting-point of our investigation.

8. If $n = 2$, we must have throws of 5 points. Considering the linear groups, we have the following possibilities:

Cyclic groups.

$m = 1$, $k = 4$; $t_1 = 1$, $t_2 = i$, $t_3 = -i$, $t_4 = -1$, $t_5 = 0$. The points of the throw are: $A_1 \equiv (1, 1, 1, 1)$, $A_2 \equiv (1, i, -1, -i)$, $A_3 \equiv (1, -i, -1, i)$, $A_4 \equiv (1, -1, 1, -1)$, $A_5 \equiv (1, 0, 0, 0)$. The throw is invariant for a cyclic group of 4 collineations, which permutates the points according to the cycle $(A_1\, A_2\, A_3\, A_4)$. $m = 1$, $k = 2$; $t_1 = a_1$, $t_2 = -a_1$, $t_3 = a_2$, $t_4 = -a_2$, $t_5 = 0$ $(a_1 \neq 0,\ a_2 \neq 0,\ a_1^2 \neq a_2^2)$; $A_1 \equiv (1, a_1, a_1^2, a_1^3)$, $A_2 \equiv (1, -a_1, a_1^2, -a_1^3)$, $A_3 \equiv (1, a_2, a_2^2, a_2^3)$, $A_4 \equiv (1, -a_2, a_2^2, -a_2^3)$, $A_5 \equiv (1, 0, 0, 0)$. The throw is invariant for a group of order 2, the points being permutated according $(A_1\, A_2)\,(A_3\, A_4)\, A_5$.

Dihedron groups.

$k = 3$, $m = 0$, $m_1 = 1$, $m_2 = 1$; $t_1 = \varepsilon_3$, $t_2 = \varepsilon_3^2$, $t_3 = 1$, $t_4 = 0$, $t_5 = \infty$; $A_1 \equiv (1, \varepsilon, \varepsilon^2, 1)$, $A_2 \equiv (1, \varepsilon^2, \varepsilon, 1)$, $A_3 \equiv (1, 1, 1, 1)$, $A_4 \equiv (1, 0, 0, 0)$. $A_5 \equiv (0, 0, 0, 1)$ $(\varepsilon = \varepsilon_3)$. The throw is invariant for a dihedron group of order 6, the generating permutations being $(A_1\ A_2\ A_3)\ A_4\ A_5$ and $A_1\ (A_2\ A_3)\ (A_4\ A_5)$. $k = 5$, $m = 0$, $m_1 = 1$, $m_2 = 0$; $t_1 = \varepsilon_5$, $t_2 = \varepsilon_5^2$, $t_3 = \varepsilon_5^3$, $t_4 = \varepsilon_5^4$, $t_5 = 1$; $A_1 \equiv (1, \varepsilon, \varepsilon^2, \varepsilon^3)$, $A_2 \equiv (1, \varepsilon^2, \varepsilon^4, \varepsilon)$, $A_3 \equiv (1, \varepsilon^3, \varepsilon, \varepsilon^4)$, $A_4 \equiv (1, \varepsilon^4, \varepsilon^3, \varepsilon^2)$, $A_5 \equiv (1, 1, 1, 1)$, $(\varepsilon = \varepsilon_5)$. The throw is invariant for a dihedron group of order 10; two generating permutations are $(A_1 A_2 A_3 A_4 A_5)$ and $A_1\, (A_2\, A_5)\, (A_3\, A_4)$.

For $n = 2$ the other groups are obviously not possible. The results obtained here agree with those of BARRAU.

9. In the following we omit the statement of the coordinates, these being easily calculated by a substitution of the t-values in the equations of the normal curve.

If $n = 3$, the throw containing 6 points, we have the following cases.

Cyclic groups.

$m = 1$, $k = 5$; $t_1 = \varepsilon_5$, $t_2 = \varepsilon_5^2$, $t_3 = \varepsilon_5^3$, $t_4 = \varepsilon_5^4$, $t_5 = 1$, $t_6 = 0$. The throw is invariant for a cyclic group of order 5, a generating permutation being $(A_1\, A_2\, A_3\, A_4\, A_5)\, A_6$.

If we consider the case $m = 2$, $k = 3$, we find $t_1 = a_1\, \varepsilon_3$, $t_2 = a_1\, \varepsilon_3^2$, $t_3 = a_1$, $t_4 = a_2\, \varepsilon_3$, $t_5 = a_2\, \varepsilon_3^2$, $t_6 = a_2$. But we can always give a linear transformation of t, viz. $t^* = \left(\dfrac{1}{a_1 a_2}\right)^{1/2} t$, so that $t_1 = p\, \varepsilon_3$, $t_2 = p\, \varepsilon_3^2$, $t_3 = p$, $t_4 = \dfrac{\varepsilon_3}{p}$, $t_5 = \dfrac{\varepsilon_3^2}{p}$ $t_6 = \dfrac{1}{p}$, which shows that the set is invariant for $t' = \dfrac{1}{t}$ and thus has dihedron symmetry.

$m = 3$, $k = 2$; $t_1 = a_1$, $t_2 = -a_1$, $t_3 = a_2$, $t_4 = -a_2$, $t_5 = a_3$, $t_6 = -a_3$. The throw is invariant for a cyclic group of order 2, consisting of unity and the permutation $(A_1\, A_2)\,(A_3\, A_4)\,(A_5\, A_6)$.

Dihedron groups.

$k = 2$, $m = 1$, $m_1 = 0$, $m_2 = 1$; $t_1 = a$, $t_2 = -a$, $t_3 = \dfrac{1}{a}$, $t_4 = \dfrac{-1}{a}$, $t_5 = 0$, $t_6 = \infty$. The throw is invariant for a dihedron group of order 4, the permutations being: $(A_1\, A_2)\,(A_3\, A_4)\,(A_5\, A_6)$, $(A_1\, A_3)\,(A_2\, A_4)\,(A_5\, A_6)$, $(A_1\, A_4)\,(A_2\, A_3)\,(A_5\, A_6)$ and unity.

$k = 3$, $m = 1$, $m_1 = 0$, $m_2 = 0$; $t_1 = a$, $t_2 = a\, \varepsilon_3$, $t_3 = a\, \varepsilon_3^2$, $t_4 = \dfrac{1}{a}$, $t_5 = \dfrac{\varepsilon_3}{a}$, $t_6 = \dfrac{\varepsilon_3^2}{a}$. The throw is invariant for a dihedron group of order 6, the permutations being $(A_1\, A_2\, A_3)\,(A_4\, A_5\, A_6)$, $(A_1\, A_3\, A_2)\,(A_4\, A_6\, A_5)$, $(A_1\, A_4)\,(A_2\, A_6)\,(A_3\, A_5)$, $(A_1\, A_5)\,(A_2\, A_4)\,(A_3\, A_6)$, $(A_1\, A_6)\,(A_2\, A_5)\,(A_3\, A_4)$ and unity.

$k = 6$, $m = 0$, $m_1 = 1$, $m_2 = 0$; $t_1 = \varepsilon_6$, $t_2 = \varepsilon_6^2$, $t_3 = \varepsilon_6^3$, $t_4 = \varepsilon_6^4$, $t_5 = \varepsilon_6^5$, $t_6 = 1$. The throw is invariant for a dihedron group of order 12; generating permutations being $(A_1\, A_2\, A_3\, A_4\, A_5\, A_6)$ and $A_1\, A_4\, (A_2\, A_6)\, (A_3\, A_5)$.

In this space we have a throw belonging to the octahedron group, viz. $m = m_1 = m_2 = 0$, $m_3 = 1$. We have: $t_1 = 1$, $t_2 = i$, $t_3 = -1$, $t_4 = -i$, $t_5 = 0$, $t_6 = \infty$. The order of the substitution group is 24. The results obtained here for $n = 3$ agree with those of BARRAU.

10. We proceed by giving the self-projective throws in S_4. A throw now must contain seven points.

Cyclic groups.

$m = 1$, $k = 6$, $t_1 = \varepsilon_6$, $t_2 = \varepsilon_6^2$, $t_3 = \varepsilon_6^3$, $t_4 = \varepsilon_6^4$, $t_5 = \varepsilon_6^5$, $t_6 = 1$, $t_7 = 0$. The throw is invariant for a cyclic group of order 6, generated by $(A_1\ A_2\ A_3\ A_4\ A_5\ A_6)\ A_7$.

$m = 2$, $k = 3$; $t_1 = a_1\, \varepsilon_3$, $t_2 = a_1\, \varepsilon_3^2$, $t_3 = a_1$, $t_4 = a_2\, \varepsilon_3$, $t_5 = a_2\, \varepsilon_3^2$, $t_6 = a_3$, $t_7 = 0$. The throw is invariant for a cyclic group of order 3, generated by $(A_1\, A_2\, A_3)\,(A_4\, A_5\, A_6)\, A_7$

$m = 3$, $k = 2$; $t_1 = a_1$, $t_2 = -a_1$, $t_3 = a_2$, $t_4 = -a_2$, $t_5 = a_3$, $t_6 = -a_3$, $t_7 = 0$. The throw is invariant for a group of order 2, consisting of $(A_1\, A_2)\,(A_3\, A_4)\,(A_5\, A_6)\, A_7$ and unity.

Dihedron groups.

$k = 5$, $m = 0$, $m_1 = 1$, $m_2 = 1$; $t_1 = \varepsilon_5$, $t_2 = \varepsilon_5^2$, $t_3 = \varepsilon_5^3$, $t_4 = \varepsilon_5^4$, $t_5 = 1$, $t_6 = 0$, $t_7 = \infty$. The throw is invariant for a dihedron group of order 10, generated by $(A_1\, A_2\, A_3\, A_4\, A_5)\, A_6\, A_7$ and $A_1\ (A_2\, A_4)\ (A_3\, A_5)\ (A_6\, A_7)$.

$k = 7$, $m = 0$, $m_1 = 1$, $m_2 = 0$; $t_1 = \varepsilon_7$, $t_2 = \varepsilon_7^2, \ldots\, t_6 = \varepsilon_7^6$, $t_7 = 1$. The throw is invariant for a dihedron group of order 14, generated by $(A_1\, A_2\, A_3\, A_4\, A_5\, A_6\, A_7)$ and $A_1\, (A_2\, A_7)\, (A_3\, A_6)\, (A_4\, A_5)$.

In this space obviously no other self-projective throws are possible.

11. As for the self-projective throws in S_n for general value of n, each case must be considered for it self. Indeed the solution depends on the arithmetic properties of the number n. Meanwhile some general remarks can be made. In S_n a throw consists of $n+3$ points. If r_1 is a factor of $n+2$ (which may be the number $n+2$ itself) there clearly always exists a throw, which is invariant for a cyclic group of order r_1. If r_2 is a factor of $n+3$ or of $n+1$, there always exists a throw, invariant for a dihedron group of order $2r_2$. In S_5 (more generally: for $n \equiv 1, 5, 7, 9, 11$ (mod. 12)) we have throws which are invariant for a tetrahedron group. In S_5 (more generally: for $n \equiv 3$, 5, 9, 11, 15, 17, 21, 23 (mod. 24)), we have throws with the symmetry of the octahedron group. The case of a throw, which is invariant for the icosahedron group first occurs in S_9 (and generally for $n \equiv 9$, 17, 27, 29, 39, 47, 57, 59 (mod. 60)).

Mathematics. — *Ueber eine Erweiterung der* LAPLACE-*Transformation.* (Erste Mitteilung). Von C. S. MEIJER. (Communicated by Prof. J. G. VAN DER CORPUT).

(Communicated at the meeting of April 27, 1940.)

LITERATURVERZEICHNIS.

S. BOCHNER.

1. Vorlesungen über FOURIERsche Integrale (1932).

F. W. BRADLEY.

2. An extension of the FOURIER–BESSEL integral. Proc. London Math. Soc., (2) **41**, 209—214 (1936).

R. V. CHURCHILL.

3. The inversion of the LAPLACE Transformation by a direct expansion in series and its application to boundary-value problems. Math. Zeitschrift, **42**, 567—579 (1937).

G. DOETSCH.

4. Theorie und Anwendung der LAPLACE-Transformation (1937).

C. FOX.

5. A note on HANKEL's theorem. Proc. London Math. Soc., (2) **30**, 18—22 (1930).

S. GOLDSTEIN.

6. Operational representations of WHITTAKER's confluent hypergeometric function and WEBER's parabolic cylinder function. Proc. London Math. Soc., (2) **34**, 103—125 (1932).

G. H. HARDY.

7. Some further applications of MELLIN's inversion formula. Messenger of Math., **56**, 186—192 (1927).

C. S. MEIJER.

8. Asymptotische Entwicklungen von BESSELschen, HANKELschen und verwandten Funktionen. Proc. Kon. Akad. v. Wetensch., Amsterdam, **35**, 656—667, 852—866, 948—958 und 1079—1090 (1932).
9. Noch einige Integraldarstellungen für Produkte von WHITTAKERschen Funktionen. Ibidem, **40**, 871—879 (1937).
10. Ueber BESSELsche, LOMMELsche und WHITTAKERsche Funktionen. Ibidem, **42**, 872—879 und 938—947 (1939).
11. Ueber Produkte von LEGENDREschen Funktionen. Ibidem, **42**, 930—937 (1939).
12. Ueber WHITTAKERsche bezw. BESSELsche Funktionen und deren Produkte. Nieuw Archief voor Wiskunde, (2) **18**, (4tes Heft), 10—39 (1936).
13. Integraldarstellungen für Produkte von LEGENDREschen Funktionen. Ibidem, (2) **19**, 207—234 (1938).
14. Neue Integraldarstellungen aus der Theorie der WHITTAKERschen und HANKELschen Funktionen. Math. Annalen, **112**, 469—489 (1936).
15. Neue Integraldarstellungen für BESSELsche Funktionen. Compositio Mathematica (Diese Arbeit wird demnächst erscheinen).

A. C. OFFORD.

16. On HANKEL transforms. Proc. London Math. Soc., (2) **39**, 49—67 (1935).

P. M. OWEN.

17. The RIEMANNian theory of HANKEL transforms. Proc. London Math. Soc., (2) **39**, 295—320 (1935).