Acoustics. — *The residue and the mechanism of hearing.* By J. F. SCHOUTEN. (Natuurkundig Laboratorium der N.V. Philips' Gloeilampenfabrieken, Eindhoven–Holland.) (Communicated by Prof. G. HOLST.)

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§ 1. *Introduction.*

Experiments on subjective sound analysis revealed [1]) that the ear but partly analyses a complex sound into pure tones corresponding to the individual frequencies in the Fourier spectrum of that sound. OHM's law of subjective sound analysis applies to the lower harmonics only. In a periodic sound the higher harmonics cannot be perceived separately as pure tones but are perceived collectively [2]) as *one* component of sharp tone quality: the *residue*. The pitch of the residue is equal to that of the fundamental tone.

In terms of this residue some paradoxical phenomena, in particular those related with the problem of the missing fundamental, found a ready explanation.

We shall now turn to the theoretical aspects of the radical change in OHM's law necessitated by the existence of the residue. The validity both of OHM's law and of HELMHOLTZ' rule [3]) always has been held as a supreme test of the *place theory* of hearing. This theory is based upon the assumption that different frequencies excite different restricted areas of the basilar membrane. Such mechanism, indeed, might enable the ear to perform a Fourier analysis of the sound (OHM's law), without its being able to take note of the phase relations between the various sinusoidal components (HELMHOLTZ' rule).

Now what are the consequences of the existence of the residue as to our conceptions of the mechanism of the inner ear? For many sounds, especially those rich in harmonics, one is almost nearer the truth in saying that the ear performs no Fourier analysis at all, then in saying that it does, the residue being by far the most prominent component and only a small portion of the harmonics being separately perceptible.

Is the existence of the residue in conflict with the principles of a *place theory* at all, or with its customary interpretation only? We hope to show that, as to the *analysing* mechanism, there is no conflict, once the limited

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[1]) J. F. SCHOUTEN, The perception of subjective tones, Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **41**, 1086—1093 (1938).

[2]) J. F. SCHOUTEN, The residue, a new component in subjective sound analysis, Proc. Kon. Ned. Akad. v. Wetensch., Amsterdam, **43**, 356—365 (1940).

[3]) H. V. HELMHOLTZ, Lehre von den Tonempfindungen.

resolving power of that mechanism is taken rigorously into account. The actual existence of the residue may then be explained by a particular assumption as regards the functioning of the *transmitting* mechanism.

§ 2. *The Perception of Pitch.*

The place theory proper is a theory regarding the analysing mechanism only. An additional assumption regarding the transmitting mechanism is necessary to account for the capacity of the ear to distinguish different frequencies as tones different in pitch. There are two possibilities. Either it is performed by obtaining knowledge of the *place* of the stimulated receptors on the basilar membrane or by the different *response* of those receptors. In the first case the nerve fibers need only transmit a quantitative message determining the measure of excitation. Whatsoever the stimulus by which a receptor is excited, the pitch ascribed to it will always be that of that particular receptor and fiber. In the second case the nerve fibers also have to transmit a qualitative message, depending upon the quality of excitation of the particular receptor, which will determine the ultimate pitch perception. In that case the perception of a particular pitch is not restricted to excitation of a particular receptor and nerve fiber.

The first case may well be compared with a totally colourblind eye looking into a spectroscope. Different frequencies will be seen as lines different in place but alike in colour. The frequencies can be told from the *place* of each line. The second case corresponds to a normal eye looking into a spectroscope. This eye is able to tell the frequency of the lines, irrespective of their particular place, by virtue of their *colour*.

The first theory (the place-place theory) is the one most generally held, it is often tacitly considered as a necessary extension of the place theory proper. A difficulty arose from the fact that the resolving power of the inner ear is comparatively poor. The excitation curves are of the order of a full tone wide, while for very low frequencies probably even no local excitation exists at all. This led GRAY [4]) to pronounce the "principle of maximal stimulation". He assumed that, although one frequency excites a certain region of the basilar membrane, the point of maximal stimulation will be the point determining the pitch. There are serious objections to be made against that principle, but it affords a very elegant explanation of the phenomena of pitch-shift of weak tones under influence of strong tones of neighbouring frequency.

Returning now to our problem of the residue we find ourselves confronted with the extremely important experimental fact that in a complex sound *two* components may be present which have almost the same pitch, although the excited regions on the basilar membrane are far apart. All harmonics, from the tenth upwards, thus ranging in our case from about 2000 to 4000 cycles/sec. are heard together as one component of pitch 200.

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[4]) A. A. GRAY, Journ. of Anat. and Physiol., **34**, 324 (1900).

This proves, assuming the place theory proper to be correct, that excitation of a given area of the basilar membrane may give rise to perceptions of widely different pitch.

Thus, returning to the analogy with the eye, in which the basilar membrane is compared with the retina, we find that the ear should be considered as an eye equipped with colour sense looking into a spectroscope and deriving the wavelength of the lines *not* from their place in the spectrum but from their colour. Just the same as the eye, in the case of a grating spectroscope, will recognize a red line in a region of otherwise blue lines, so the ear will recognize a component of low pitch in a region which, for pure tones, would give rise to perceptions of very much higher pitch.

The distinctness of pitch, one of the outstanding problems in the first extension of the place theory proper, is, in the second extension, not determined by the narrowness of the stimulated area or by the distinctness of a maximum of stimulation but by the *equality of "colour" of the whole stimulated area*, be it wide or narrow.

Hence even those low tones to which the whole basilar membrane responds uniformly may have a definite pitch by virtue of the "colour" of excitation. The function of the analysing mechanism thus consists merely in dissecting a sound into different components on the basilar membrane which, then, may be separately perceived. Pitch, however, is determined not by the place but by the quality ("colour") of excitation. It is not excluded, and even probable, that the place of excitation (number of receptor or nerve fiber) has some other effect on the ultimate tone quality e.g. a variable amount of sharpness. Thus the pitch of the residue would be determined by its colour, its sharpness by the region of excitation.

§ 3. *The Mechanism of the Residue.*

We now turn to the question whether, on the basis of the place theory proper, the analysing mechanism reveals any property which might give rise to a collective perception of higher harmonics with a pitch equal to that of the fundamental tone.

This property is present indeed. Any pure tone will excite a certain area of the basilar membrane expressed by its excitation curve or, to put it the other way round, any particular receptor will respond to a degree expressed by its response curve [5]), to a certain band of frequencies. As yet, the

[5]) The different wording "response curve" and "excitation curve" is not immaterial. As *response curve* we define the response of one receptor to different frequencies. As *excitation curve* we define the excitation of different receptors by one frequency. Thus if the receptors were to respond uniformly up to various frequency limits, the response curves would extend *below*, the excitation curves *above* those limits. In our case, where narrow curves and a continuous set of receptors are considered, the difference is often negligible.

width of this band is unknown, since different methods lead to estimations of widely different order. We may, however, assume that the half-value width is at least half a tone.

The lower harmonics, being an octave, a fifth, a fourth etc. apart, will chiefly excite narrow bands of receptors divided by regions in which the response is comparatively small. Conversely the receptors in those regions will respond almost exclusively to particular harmonics. For higher harmonics, however, which, in relative measure, come closer together, the receptors will be excited in comparable measure by several harmonics at the time. Hence, those receptors will not perform a sinusoïdal oscillation, but will respond with a complex wave form *revealing the periodicity of the total wave form*. The overlapping of the excitation curves of various harmonics necessarily leads to the introduction of the fundamental *period* in the response of the receptors.

If thus the ear were to perceive pitch by virtue of the periodicity of the excitation of the receptors, the existence of the residue would be explained.

It should be remembered that the only reason why it was originally assumed that pitch is determined by the place of the excited receptor is for the sake of simplicity, so as not to make a complicate assumption if a simpler one serves as well. The simpler assumption now being proved untenable, it is necessary to make further assumptions as to the nature of the transmitting mechanism. In the "colour" theory of pitch perception it is assumed that from each receptor not only a quantitative but also a qualitative message is transmitted towards higher centres.

This may take place in two ways which we shall call the one-fiber and the multiple-fiber principle. In the one-fiber principle the clue as to the frequency has to be transmitted by the quality of discharge of the nerve fiber (telephone-theories), e.g. by the number of discharges per second. Another possibility, however, is given by the multiple-fiber principle which is generally assumed to account for the colour sensitivity of the eye. It is supposed, there, that the clue as to the colour is transmitted by the relative amount in which several fibers are excited by the same receptor. Two fibers would thus account for dichromatic vision, three fibers or two pairs of fibers for trichromatic vision.

We mention these possibilities merely to show that the "telephone" principle is by no means the only principle accounting for a colour theory of pitch perception and that, in view of our knowledge of the eye, the multiple-fiber principle is well worth considering too.

In fig. 1 an illustration is given of the response of a set of mechanical resonators to a driving force given by a "periodic impulse". Although the spatial analysis in the inner ear may very well be accounted for by other principles than that of mechanical resonance, the mechanical model, yet, in its essential features, displays the properties to be expected from all systems having a limited resolving power.

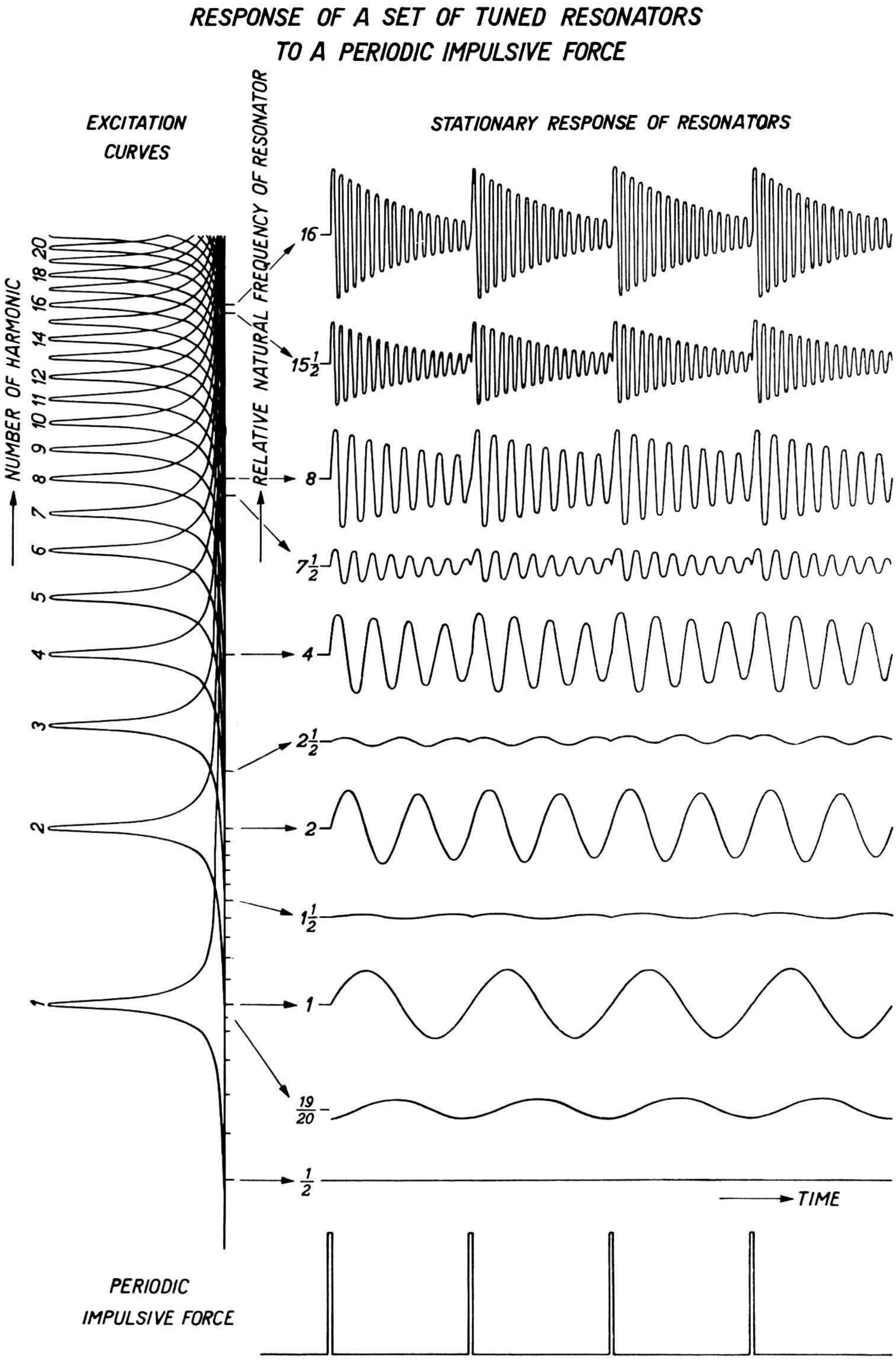
RESPONSE OF A SET OF TUNED RESONATORS
TO A PERIODIC IMPULSIVE FORCE
EXCITATION CURVES
RELATIVE NATURAL FREQUENCY OF RESONATOR
STATIONARY RESPONSE OF RESONATORS
NUMBER OF HARMONIC
1 2 3 4 5 6 7 8 9 10 11 12 14 16 18 20
16
15 1/2
8
7 1/2
4
2 1/2
2
1 1/2
1
19/20
1/2
TIME
PERIODIC
IMPULSIVE FORCE

Fig. 1.

Graph showing the response of a set of tuned mechanical resonators to a periodic impulsive force.

The graph gives the amplitude x of the system:

$$m\ddot{x} + r\dot{x} + sx = F(t)$$

in which the constants are so adjusted that the sensitivity of the resonators and the relative width of the excitation curves are independent of the natural frequency. This is performed by taking s constant, $r \sim \frac{1}{\nu}$ and $m \sim \frac{1}{\nu^2}$ if ν is the natural frequency of the resonator $\left(\frac{r}{2m} = \frac{\nu}{10}\right)$.

The graph shows that in the low frequency region the resonators almost exclusively respond to one harmonic (1, 2, 3 etc.). The regions of very strong excitation (1, 2, 3 etc.) are separated by regions ($\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$) in which the excitation is very small. In the high frequency region the response of the resonators increasingly shows a periodicity equal to that of the fundamental tone. Spatial analysis becomes poorer and poorer (compare $7\frac{1}{2}$ with 8, $15\frac{1}{2}$ with 16).

§ 4. *On beats.*

It is curious to consider that our present interpretation of the residue amounts to the supposition that this residue is *the perception as a separate component of the beats of the higher harmonics.*

That brings us back to a controversy even older than that between SEEBECK and OHM. In 1800 THOMAS YOUNG [6]) supposed that the beats of two pure tones might be heard as a separate tone: "The greater the "difference in the pitch of two sounds, the more rapid the beats, till at last, "like the distinct puffs of air in the experiments already related, they "communicate the idea of a continued sound; and this is the fundamental "harmonic described by TARTINI."

This interpretation was strongly contested by HELMHOLTZ and was since abandoned, at least in physical and technical circles, the argument being, that beats have no physical existence as a separate tone and hence will never excite the corresponding receptor. If, none the less, a tone of corresponding pitch is heard, it *must* be due (according to the old extension of the place theory proper) to the presence of a pure tone of that frequency and, hence, to non-linear distortion.

Thorough studies of the properties of beats and beat-notes were carried out by KÖNIG, LORD KELVIN and TER KUILE [7]). HERMANN [8]) went in detail

[6]) THOMAS YOUNG, Outline of Experiments and Inquiries respecting Sound and Light. Phil. Trans. Roy. Soc. London, I, 106—150 (1800).

[7]) R. KÖNIG, Ueber den Zusammenhang zweier Töne, Pogg. Ann. Physik, **157**, 181—237 (1876).

SIR W. THOMSON, On Beats of Imperfect Harmonics, Proc. Roy. Soc., **9**, 602—612 (1878).

TH. E. TER KUILE, Over phasen, zwevingen en klankaard. Ac. Thesis Amsterdam (1904).

[8]) L. HERMANN, Zur Theorie der Combinationstöne, Pfl. Arch. f. Physiol., **49**, 499—518 (1891).

into the theoretical consequences of these phenomena. The essence of these studies is the following. Slow beats will be heard not only if the two frequencies are almost identical but also on approximation to each of the harmonies 1 : 2, 1 : 3, 2 : 3, 3 : 4, 3 : 5, 4 : 5 etc. This was confirmed lately by LOTTERMOSER [9]). If the frequency of the beats increases the beats will change into beat-notes and thus into additional components of definite pitch. HELMHOLTZ' view, consisting in the supposition that the beats should be considered as beats between harmonics of almost common frequency and that the beat-notes should be considered as difference tones, is contested. According to KÖNIG (l.c. p. 235): "the origin of the beat-notes "is simply the periodic coincidence of similar maxima of the two wave "forms". He does not consider this to be in conflict with the place theory, although his arguments are not quite clear. To HERMANN we owe a very thorough investigation of the applicability of non-linear distortion to the explanation of beats and beat-notes. He finally, almost reluctantly, finds himself compelled (l.c. p. 517) "to drop HELMHOLTZ' hypothesis of "resonators in the ear, elegant as it might be" and (p. 514) "to return "to the old, so simple and natural, derivation of difference tones from beats, "viz. to ascribe to the ear the property to respond to any periodicity, within "certain limits of frequency, with a sensation of tone".

Once it is realized that HELMHOLTZ' theory consists of *two* assumptions: the place theory of the analysing mechanism (place theory proper) and the place theory of the transmitting mechanism, it is seen that HERMANN's criticism applies to the latter theory only. If the latter is replaced by the periodicity theory, the place theory proper even gains in probability since beat-notes, of which the residue is the most striking example, are most pronounced in those sounds where, according to the place theory proper, the different excitation curves show the highest grade of overlapping.

§ 5. *Consequences.*

An interesting consequence may be drawn with regard to persons having so-called "tonal lacunae" or "islands of deafness". If a tone to which such person is normally deaf is made loud enough the unsevered regions above and below the deaf region will respond to it. According to our theory such a person should still correctly perceive the pitch of the tone, whereas, according to the principle of maximal stimulation, he should perceive two pitches. According to TROLAND [10]) such persons indeed perceive the pitch correctly.

A paradoxical consequence may be drawn with regard to *high tone deafness*. In a periodic sound all high components are perceived together

[9]) W. LOTTERMOSER, Bemerkung zu den subjektiven harmonischen Teiltönen. Ak. Zeitschr., 2, 148—149 (1937).

[10]) L. T. TROLAND, The Psychophysiology of Auditory Qualities and Attributes. J. Gen. Psych., 2, 28—58 (1929). In this paper a very clear distinction is made between theories of the analysing and the transmitting mechanism.

as a residue. Hence *high* tone deafness should chiefly affect one of the *lowest* components (the residue) of a complex sound.

A third consequency must be drawn with regard to the lower frequency limit of sound perception. One must distinguish between a pure tone of low pitch and a residue of that pitch. The loudness of the former will be determined by the sensitivity of the receptors to that frequency but not the loudness of the latter. Many contradictory results as regards this lower limit may be due to a residue having been listened to instead of a pure tone.

As a fourth consequence we must realize that the determination of periodicity is by no means always unambiguous. In those cases the pitch should show the same ambiguity.

§ 6. *Summary.*

The inner ear mechanism can be divided into the analysing mechanism within the cochlea and the transmitting mechanism from the cochlea towards higher centres.

The analysing mechanism consists of a set of receptors. Any one frequency will excite a number of receptors to a various degree expressed by the excitation curve. This leads to a spatial analysis on the basilar membrance of a complex sound into its sinusoïdal components. The analysis is limited by the width of the excitation curves. If the distance of two or more sinusoïdal components is small compared to the width of the curves, these will materially overlap and hence the receptors in that region will respond to several frequencies at the time.

The transmitting mechanism consists of the nerve fibers. The nerve fibers transmit both the quantity and the quality (periodicity) of excitation of the receptors. The former will determine loudness, the latter pitch, similar to the perception of brightness and colour in the eye. The locus of stimulation (determining pitch in the place-place theory) will most probably affect the tone quality (sharpness).

To account for the distinctness of the pitch of a pure tone the "principle of maximal stimulation" is no longer necessary, since this distinctness will, independent of the width of the stimulated area, be determined by the distinctness and the equality of the periodicity of the response of the receptors. Both will be ideal in the case of pure tones.

For complex sounds the number of pure tones perceptible in it will be determined by the efficiency of the spatial analysis and, hence, by the resolving power of the analysing mechanism. As soon as the excitation curves overlap beats or even (with increasing frequency) beat-notes (residues) will arise at the cost of the constituting pure tones. The pitch of those components is determined by the *periodicity* of the response of the receptors. The most striking example is found with the high harmonics of a periodic sound. All receptors in that region will respond to several harmonics at the time, thus revealing in their response a periodicity equal to

that of the fundamental tone. Hence, those high harmonics, which are not separately perceptible, will be heard together as a separate component with a pitch equal to that of the fundamental tone. This component, the "residue", is distinguishable from the fundamental tone by its sharpness.

It thus remains right to say that the ear performs a Fourier analysis of the sound, but for the imperfections due to the limited resolving power of the analysing mechanism. At the same time it is also right to say, with the old investigators, that any periodicity may be heard as a separate note, provided that, as mostly will be the case, the analysing mechanism at some region responds with that periodicity.

For determining the mere components of a complex sound it is adequate to look at the *Fourier spectrum* of the sound. For determining the pitch of each separate component it is adequate to look at the *wave form* of the response of the receptors. That is the answer to the final query in our first paper (l.c. p. 1092) and the reconciliation of the "spectrum" and "periodicity" theories of hearing.

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