multiplied in order to bring it in accordance with the protein percentage found by the gravimetric method. Slight deviations from case to case of factor 6.30 which we have taken now are possible, but a value of 7 is certainly too high. The too high value of the total protein percentage gives much too high values for the globulin percentage (this was then determined as the difference between total protein and albumin percentage) and consequently the spreading factor is much too high. This cause, however, is not sufficient to bring the factor found for globulin to the value of 0.93 found now.

Summary.

Serum albumin and globulin were determined by means of nitrogen determinations according to the KJELDAHL method and by means of spreading. Average spreading factors of 0.93 for globulin, 1.04 for albumin and 1.01 for total protein were found.

Physics. — *Meson theories in five dimensions.* By L. ROSENFELD. (Communicated by Prof. H. A. KRAMERS.)

(Communicated at the meeting of January 31, 1942.)

In spite of the attractiveness of its basic idea, the meson field theory of nuclear systems cannot be said to be firmly established in any definite form. Quite apart from the convergence difficulties inherent in any quantum field theory, one is here confronted from the start with a choice between four *a priori* possible types (1) of meson fields: scalar, vector, and the two dual types with respect to spatial reflexions, pseudoscalar and pseudovector. One may then try to examine which choice provides the widest scope for the theory, including not only an account of properties of nuclear systems, but also a theory of β-disintegration, which in particular involves a definite relation between β-decay constants and the mean life time of free mesons. From this point of view, it appears necessary to adopt a particular combination of a pseudoscalar and a vector meson field, characterized by a simple relation between the constants which define the intensities of the nuclear sources of the meson fields (2) (3).

Recently, MøLLER (4) has pointed out that this "mixed theory" presents itself in a very natural way as a *single* type of meson field in a five-dimensional (pseudo-euclidian) space, *viz.* as a five-vector with respect to the group of ordinary five-dimensional "rotations" (of determinant $+1$) [1]). Moreover, such a representation of the mixed theory leads to an essential reduction of the number of arbitrary constants in the source densities of the meson field. The physical interpretation of the fifth coordinate introduces, however, an element of arbitrariness in the theory. One might, as originally proposed by MøLLER, identify the five-dimensional space with DE SITTER's universe, thus suggesting a somewhat unexpected connexion between nuclear forces and cosmological features. An alternative interpretation consists in considering the five-dimensional space as a projective one, according to VEBLEN's original suggestion (5): this has the advantage of permitting a straightforward treatment of the interaction of the mesons and nucleons with the electromagnetic field; a detailed discussion of this possibility has recently been carried out by PAIS (6).

The special position, thus recognized, of the mixed theory as a fundamental type of five-dimensional meson field raises at once the question as to which other types of such fields would also be possible *a priori*. A convenient starting point for discussing this question is provided by the so-called "particle aspect" of meson theory, *i.e.* a linearized form of the field equations, involving a system of matrices subjected to suitable commutation rules (7). In fact, the different possible types of meson fields are then immediately given by the inequivalent irreducible representations of the algebra of these matrices. Thus, in four dimensions, we have essentially [2]) two irreducible representations, of degree 5 and 10 respectively, to which correspond the scalar and the vector type of mesons, or the two dual types, according to the reflexion properties imposed on the wave function (7). Such considerations are readily extended to five dimensions (8), with the following result: there are essentially [2]) four inequivalent irreducible representations of the extended algebra, of degrees 6, 10, 10 and 15, corresponding to a five-scalar, two distinct five-pseudovector and a five-vector type of meson field respectively.

[1]) This group includes in fact both the Lorentz group and the spatial reflections, provided the latter are associated with a change of sign of the fifth coordinate. More accurately, the "mixed theory" appears as some degenerate or approximate form of the five-vector theory.

[2]) *i.e.* apart from a trivial representation of degree 1.

In a non-projective interpretation of the five-dimensional formalism, it is found (8) that these four types of meson fields uniquely reduce to only three types of four-dimensional theories; *viz.* the five-scalar is equivalent to the four-scalar theory, both five-pseudovector types give rise to the same four-pseudovector theory, while the five-vector type is just equivalent to the mixed theory with the reduced number of source constants. In fact, in each theory suitable covariant source densities can be defined in the usual way by means of Dirac matrices. The projective interpretation, on the other hand, leads to essentially different conclusions. The discussion of this case, which has recently been worked out by PAIS (9), starts from the basic correspondence established in a well-defined way (5) between any five-projector and a set of four-tensors of all lower and equal degrees (*e.g.* a projective five-vector defines a four-vector and a four-scalar). It is, however, possible to define in a projective way the universal four-pseudoscalar $\varepsilon_{ijkl} = \pm |\det g_{mn}|^{1/2}$ and by means of this so to modify the correspondence just mentioned that any member of the set of four-tensors be replaced by its dual with respect to spatial reflections (thus, instead of a four-vector and a four-scalar, one may, from a projective five-vector, also get a pseudovector and a scalar, or a vector and a pseudoscalar, or a pseudovector and a pseudoscalar). It then follows that from the four irreducible types of projective theories for *free* mesons any one of the four-dimensional types can be derived, as well as any combination of vector or pseudovector with scalar or pseudoscalar. But the number of possibilities is greatly reduced when due account is taken of the definition of the source densities by means of Dirac matrices. If one adopts for these sources the familiar definitions, eventually modified with respect to reflection properties by multiplication with the pseudoscalar ε_{ijkl}, it is readily seen that in every irreducible type of projective theory all different four-dimensional possibilities obtained in the way indicated above lead just to the same physical theory[1]). So far we thus get exactly the same result as with the non-projective interpretation, *viz.* the scalar, the pseudovector and the mixed theory.

Still, the projective interpretation allows of a greater freedom in the definition of the source densities than the non-projective standpoint, because it involves a universal projector, *viz.* the coordinate vector x^μ, which may be combined in a covariant way with the Dirac matrices. While this circumstance does not give rise to any essentially new possibility in the five-scalar and five-pseudovector theories, it leads for the five-vector type, in addition to the mixed theory, also to a pure vector and a pure pseudoscalar field. Summing up, we see that the five-dimensional point of view, in its widest interpretation, does not exclude any one of the four-dimensional types of meson theories, but singles out the mixed theory as the only combination of four-dimensional types which can be derived from an *irreducible* five-dimensional type of field[2]).

Whatever the formal aspect of the problem may be, the adoption of some particular form of meson theory (if any) can of course only be decided on physical arguments. If we first consider the application of meson theory to the phenomena of β-disintegration, an essential requirement in this respect is to avoid the difficulty, pointed out by NORDHEIM (11), of reconciling on such a theory the empirical value of the mean life time of the mesons with the β-decay constants of light elements. This may be achieved[3])

[1]) For the cases of the four-scalar and four-vector theories, a similar conclusion has also been reached by M. SCHÖNBERG in a recent note (10). He therefore proposes to include any pair of dual cases (scalar-pseudoscalar, vector-pseudovector) in a single type of meson theory. It would seem more practical, however, to retain the usual classification.

[2]) The reduction of the number of source constants in the mixed theory, which was stressed by MØLLER (4) as an important feature of the non-projective point of view, is not strictly implied in the projective interpretation, though it still appears as a consequence of the *simplest* choice of source densities in this case.

[3]) A quite different possibility, involving, however, the cutting-off of a divergent expression, has been pointed out by S. SAKATA, Proc. phys.-math. Soc. Japan **23**, 283 (1941).

either by adopting a purely pseudoscalar theory or by introducing two independent kinds of mesons of very different life times (3). The latter case may just be provided by the mixed theory; more precisely (3), one has here to assume, taking the five-dimensional form of the theory with the reduced number of source constants, that the pseudoscalar mesons have a much longer mean life than the vector mesons. Either one of these two possibilities thus leads to the conclusion that cosmic ray mesons observed at sea-level, being of pseudoscalar type, should have zero spin, — a conclusion strikingly supported by the analysis (12) of recent cosmic ray observations.

While such phenomena therefore appear to be in harmony with the consequences of meson theory, they do not permit to decide between pseudoscalar or mixed theory. The adoption of the latter seems to be claimed, however, for a rational treatment of nuclear forces (2). It is true that the issue in this respect is somewhat obscured by the inevitable occurrence of the well-known divergences inherent in any quantum field theory. Still, adopting a point of view analogous to the "correspondence" method of quantum electrodynamics, it is possible first to discuss the convergence of the "classical" meson theory obtained by neglecting all quantum effects of the meson field, and then to examine how the validity of such classical calculations has to be restricted in order to keep off quantum singularities. The "classical" interaction potential between a pair of nucleons at (mean) distance r from each other is thus found to consist of a "static" potential and a series of non-static terms, the order of magnitude of which, in comparison with the static potential, is given by some power of the parameter $\Gamma/(\varkappa r)^n$, where $\varkappa^{-1}$ denotes the range of nuclear forces and $\Gamma \sim g^2/4\pi\hbar c \sim 0.065$ the intensity of nuclear sources of meson fields, while the exponent n depends on the type of meson theory considered. On pseudoscalar as well as vector meson theory, there occurs in the static potential a dipole interaction term in r^{-3}, which must be cut off at some distance smaller than the range; owing to this singular term, one has in this case $n = 3$, from which it follows that the static potential in no way approximates the interaction in the region comprised between the cut-off distance and the range, where a quantitative expression for this interaction is at all of any significance. The mixed theory, on the other hand, is just defined in such a way that the singular dipole interaction term is eliminated from the static potential; one has then $n = 1$ and the inconsistency just mentioned disappears[1]). Of course, the divergences arising from the quantization of the meson field severely restrict the domain of validity of the mixed theory; the critical distance for which it breaks down, however, may, according to Heisenberg, be defined by $\Gamma/(\varkappa r_0)^2 = 1$, so that there still remains a region, between r_0 and $\varkappa^{-1}$, where — in contrast to pseudoscalar or vector meson theory — it yields unambiguous results.

[1]) Explicit calculations of non-static interaction terms, which very instructively illustrate the general argument here summarized, have been published by E. STUECKELBERG and J. PATRY (13) and E. STUECKELBERG (14). As regards the numerical results given there, it must be observed that, owing to the assumption $\Gamma = 0.1$ instead of ~ 0.065, they perhaps convey an overpessimistic impression of the convergence of the mixed theory. The main interaction terms arising from the quantization of the meson fields have also been calculated by several authors; see especially E. STUECKELBERG and J. PATRY, *loc. cit.* (13), § 7 and H. BETHE, *loc. cit.* (15), p. 272; the calculations of MØLLER and ROSENFELD quoted by BETHE (from a verbal communication) have, however, not been published. For the vector theory, the ratio of the quantum interaction terms of order Γ^2 to the static potential is found, as mentioned by BETHE, to be of the order of magnitude $\Gamma/(\varkappa r)^2$; for the mixed theory, however, according to the unpublished calculations just referred to, this ratio becomes $\Gamma/(\varkappa r)^4$. According to the "correspondence" interpretation, all such terms have to be discarded.

REFERENCES.

1. N. KEMMER, Proc. Roy. Soc. **A 166**, 127 (1938).
2. C. MøLLER and L. ROSENFELD, Proc. Copenh. **17**, no. 8 (1940).
3. C.MøLLER, L. ROSENFELD and S. ROZENTAL, Nature **144**, 629 (1939); in this note, the possibility of a consistent account of β-disintegration and meson decay on a purely pseudoscalar theory is erroneously disregarded.
   S. ROZENTAL, Proc. Copenh. **18**, no. 7 (1941); in this paper, the first paragraph on p. 42 must be cancelled; see a forthcoming note by S. ROZENTAL in Phys. Rev.
   See also S. SAKATA, Proc. phys.-math. Soc. Japan **23**, 291 (1941).
4. C. MøLLER, Proc. Copenh. **18**, no. 6 (1941).
5. See especially W. PAULI, Ann. d. Phys. **18**, 305, 337 (1933).
6. A. PAIS, Thesis, Utrecht (1941), Physica **8**, 1137 (1941), and other forthcoming papers in Physica.
7. N. KEMMER, Proc. Roy. Soc. **A 173**, 91 (1939).
8. J. LUBANSKI and L. ROSENFELD, Physica **9**, 117 (1942).
9. A. PAIS, Physica, in the press.
10. M. SCHÖNBERG, Phys. Rev. **60**, 468 (1941).
11. L. NORDHEIM, Phys. Rev. **55**, 506 (1939).
12. R. CHRISTY and S. KUSAKA, Phys. Rev. **59**, 405, 414 (1941).
    J. OPPENHEIMER, Phys. Rev. **59**, 462 (1941).
13. E. STUECKELBERG and J. PATRY, Helvet. Phys. Acta **13**, 167 (1940).
14. E. STUECKELBERG, Helvet. Phys. Acta **13**, 347 (1940).
15. H. BETHE, Phys. Rev. **57**, 260, 390 (1940).

Geophysics. — *On the* STONELEY-*wave equation*. II. By J. G. SCHOLTE. (Communicated by Prof. J. D. v. d. WAALS.)

(Communicated at the meeting of November 29, 1941.)

§ 3. *Discussion of the* STONELEY *equation*.

In the preceding paragraph we found that the roots ζ of this equation must be less than 1; we shall now prove that these roots cannot be negative.

Putting

$$\sqrt{(1-\zeta)(1-\nu_1\zeta)}=1-\varepsilon_1\zeta, \qquad \sqrt{(1-\alpha\zeta)(1-\omega\zeta)}=1-\omega\varepsilon_2\zeta,$$
$$\sqrt{(1-\nu_1\zeta)(1-\omega\zeta)}=1-\varphi_1\zeta, \qquad \sqrt{(1-\alpha\zeta)(1-\zeta)}=1-\varphi_2\zeta,$$

we have $1>\varepsilon_1>\nu_1$, $1>\varepsilon_2>\nu_2$, $1>\varphi_2>\nu_2$. Equation (2) takes the form:

$$8-4\zeta\left\{\frac{1-\varrho_2/\varrho_1}{1-\mu_2/\mu_1}+\varepsilon_1+\omega\varepsilon_2\right\}+\zeta^2\left\{\left(\frac{1-\varrho_2/\varrho_1}{1-\mu_2/\mu_2}\right)^2+4\omega\varepsilon_1\varepsilon_2\right\}=$$
$$8-4\zeta\left\{\frac{1-\varrho_2/\varrho_1}{1-\mu_2/\mu_1}+\varepsilon_1+\omega\varepsilon_2\right\}+\zeta^2\left\{\left(\frac{1+\varrho_2/\varrho_1}{1-\mu_2/\mu_1}\right)^2+\frac{4\omega\varepsilon_2-4\varrho_2/\varrho_1\varepsilon_1}{1-\mu_2/\mu_1}\right\}-$$
$$-\zeta^3\cdot\frac{(\varrho_2/\varrho_1)^2\varepsilon_1+\omega\varepsilon_2+\varrho_2/\varrho_1\varphi_2+\varrho_2/\varrho_1\varphi_1}{(1-\mu_2/\mu_1)^2}$$

or

$$\zeta\left\{\frac{\mu_1}{\mu_1}\varepsilon_2+\frac{\varrho_2}{\varrho_1}\varepsilon_1+\varphi_1+\varphi_2\right\}=4\left\{1-\varepsilon_1\left(1-\frac{\mu_2}{\mu_1}\right)+\right.$$
$$\left.+\varepsilon_2\left(\frac{\mu_1}{\mu_2}-1\right)-\varepsilon_1\varepsilon_2\frac{\mu_1}{\mu_2}\left(1-\frac{\mu_2}{\mu_1}\right)^2\right\}$$

hence $\zeta=\dfrac{4\{1-\varepsilon_1(1-\mu_2/\mu_1)\}\{1-\varepsilon_2(1-\mu_1/\mu_2)\}}{\varrho_2/\varrho_1\varepsilon_1+\mu_1/\mu_2\varepsilon_2+\varphi_1+\varphi_2}$, which is positive, ε_1 and ε_2 being less than 1.

As we have now proved that $0<\zeta<1$, it follows that $sin\ r_1$ is real and greater than $1\left(\zeta=\dfrac{1}{sin^2 r_1}\right)$, hence the cosines of the angles occurring in equation (1) are imaginary, and the wave function

$F(pt-h_1 x\ sin\ i_1-h_1 z\ cos\ i_1)$ becomes $F(pt-h_1 x\ sin\ i_1-i\,h_1 z\sqrt{sin^2 i_1-1})$.

The waves of the system $\{A_r\ \mathfrak{A}_r\ A_d\ \mathfrak{A}_d\}$ are therefore exponentially damped in the z direction.

It is convenient once more to choose a new variable, namely $\eta=\dfrac{1}{\zeta}=sin^2 r_1$. Equation (2) is then:

$$\left.\begin{aligned}&\left(2\eta-\frac{1-\varrho_2/\varrho_1}{1-\mu_2/\mu_1}\right)^2+4\sqrt{(\eta-1)(\eta-\nu_1)(\eta-\alpha)(\eta-\omega)}=\\&\sqrt{\frac{(\eta-\alpha)(\eta-\omega)}{\eta^2}}\cdot\left(2\eta-\frac{1}{1-\mu_2/\mu_1}\right)^2+\sqrt{\frac{(\eta-\nu_1)(\eta-1)}{\eta^2}}\cdot\left(2\eta+\frac{\varrho_2/\varrho_1}{1-\mu_2/\mu_1}\right)^2+\\&+\frac{\varrho_2/\varrho_1}{(1-\mu_2/\mu_1)^2}\cdot\left(\sqrt{\frac{(\eta-\alpha)(\eta-1)}{\eta^2}}+\sqrt{\frac{(\eta-\nu_1)(\eta-\omega)}{\eta^2}}\right)\end{aligned}\right\}\quad(3)$$