Physiology. — *The problem of the interstitial cells in the nervous endformation* [1]). By J. BOEKE, LL.D., M.D. Utrecht.

(Communicated at the meeting of February 28, 1942.)

In 1894 CAJAL described in the wall of the intestine and different glands a mesh- or network of cells, which he called "neurones sympathiques interstitiels", small triangular or spindleshaped cells having cell processes which seemed to anastomose with each other, and which stained black in his GOLGI preparations. These were found in the sympathetic nervous plexus in the wall of the stomach of frogs and different mammals, and in different glands (pancreas, salivary glands). In 1911 and 1936 he adhered to his original description, only he let fall his original doubt as to their anastomosing with each other and proclaimed them the only neurones showing a syncytial arrangement. They seemed to be connected with the smooth muscle elements ("il résulte de cette description qu'il existe dans les muscles lisses deux sortes d'arborisation nerveuses; les principales qui proviennent des grandes cellules du plexus d'AUERBACH et qui sont en même temps les plus nombreuses, et les accessoires qui émanent des cellules interstitielles". CAJAL, 1894, 1936). CAJAL did not observe a distinct connexion of his interstitial cells with the classic sympathetic elements, and he puts forwards as a very cautious hypothesis the idea that the interstitial elements are influenced by the sympathetic fibres entering the intestinal wall. In the 46 years following the first description by CAJAL these interstitial elements were studied by a number of authors; some regarded them as being of mesenchymatous nature, connective elements, others regard them as being of nervous origin, as CAJAL, LA VILLA a.o. did, but however they regarded them, they all agreed that they form a syncytium (DOGIEL, JOHNSON, COLE, etc.). LA VILLA described them in 1897 as being of nervous nature and origin, and compared them with the primitive ganglion cells of the avertebrates, in which I followed him in 1935; BETHE (1903) described a ground-net of a definitely nervous nature in the mucosa of the frog's mouth, which he homologised with the interstitial cells of CAJAL. ERIK MUELLER described the same thing in 1921, LEONTOWITSCH described a syncytial nervous groundnet in the wall of arteries containing small nerve-cells, which were identical to the interstitial cells (1927), MUENCH and SCHOCK (1905, 1910) described them as occurring in great numbers in the very loose connective tissues of the iris; their description and opinion were followed by WOLFRUM in 1931 and partly by myself in 1933 and 1936. Elements of the same nature were described after staining with methylene blue in the wall of the intestine by OKAMURA in 1935 and 1939, and by SCHABADASCH in 1934, who agreed with my descriptions and suggestions of 1933.

In 1926 LAWRENTJEW investigated these interstitial elements in my laboratory and gave them a central position in his description of the end-formation of the sympathetic nervous system. CAJAL could not find any definite connexion between these cells and the elements of the plexus of AUERBACH or MEISSNER, but supposed that they were under the influence of the real sympathetic ganglion cells. LAWRENTJEW followed up this suggestion and found them to be always lying at the end of the neurofibrillar strands of the plexus and in this way forming an intermediate element between the strands and the smooth muscle-cells.

[1]) The problem will be discussed more fully and with the necessary illustrations in the Acta Neerlandica Morphologiae, of this year (Vol. V), as XII. Innervationsstudie. There the literature bearing on the subject will be more fully accounted for.

VAN ESVELD, who followed him, explained the results of his physiological experiments (which showed him, that even after all the ganglion cells were removed, an isolated strip of the smooth muscle of the intestine still contracted rhythmically and was influenced by drugs influencing the nervous elements), by the presence of these interstitial elements inside the muscle-tissues even when the true ganglion cells had been removed. According to both workers the interstitial elements were arranged syncytially and formed the end of the efferent sympathetic pathway, being connected with the protoplasm of the innervated elements by means of an intermediate network of protoplasmic origin, the "periterminal network", described by BOEKE and by HERINGA.

In 1938 they were described very elaborately by TINEL in his excellent book on the vegetative system (MASSON, 1938), and he put them into the centre of his descriptions of the sympathetic endformation.

In 1937 they were studied as accurately as possible in my laboratory by LEEUWE, as a continuation of my own researches. In the veterinary laboratory of anatomy they have been studied very accurately by MEYLING in the wall of the aorta and in the glomus caroticum. All these authors came to the conclusion that they are nervous in nature. An intermediate position was taken by BLOOM, who admits that they are probably of nervous origin, but that they might possibly be of a microglial nature (1931). This however, is improbable, since CAJAL (and others, as LEEUWE and MEYLING) described a neurofibrillar structure in their protoplasma, which even changed its aspect in hibernating animals (CAJAL, 1911).

Thus we see that the problem is far from settled, but that the trend of the observations is in the direction of declaring them to be of nervous nature and origin but not stating their function.

LAWRENTJEW declared them to be of lemmoblastic origin, but lying at the end of the sympathetic plexus; from them the ground-bundle of neurofibrils may pass on to other interstitial elements, always maintaining a syncytial arrangement, or give off small endknobs, the motor endings on smooth muscle-fibres. They form the real motor endings of the sympathetic plexus, and from this point of view it would be strange that they were of lemmoblastic nature. According to LEEUWE however they are true ganglion cells, which are in a syncytial connexion with the sheath-cells of the sympathetic plexus.

LEEUWE studied these elements by means of the methylene blue method, and as his work was done in my laboratory and we discussed most of his preparations and studied them together, I will describe here his conclusions more fully, because I am responsible for his work to a certain extent, and fully agree with most of his statements and descriptions, mentioned here. LEEUWE studied the interstitial elements in the enteric plexus of different animals, mammals and frogs, in embryonic tissue and in the full-grown animals, and his methods of staining enabled him to use in-toto-preparations of the thin intestinal wall of the larvae and embryoes, which was of a great advantage in the study of their embryonic development. In the submucous tissue of the frog's mouth, the region where BETHE had described his nervous groundnet, which he homologised with the interstitial elements, LEEUWE succeeded to demonstrate this network with the utmost exactness, contrary to ABRAHAM, who could not find this network (1936) and denied its existence. In the frog's intestine the structure of the enteric plexus LEEUWE found to be similar to that in mammals, and in total preparations of the intestine of frog larvae and small mammals even the development of the interstitial cells could be followed with exactness. They grow out from clusters of the ganglion cells of the developing sympathetic plexus, radiating from them as distinct elements with branching processes, but always in syncytial continuity. They thus spread out into the musculature until they reach the muscle-cells themselves. It was even possible to follow the development of the neurofibrillar structure of these syncytial elements, and it was of interest that the neurofibrillar structure, which appears in these elements, did not begin in the elements of the plexus from which the interstitial syncytium had grown out, but it showed itself first at the terminations of the elements of the end-formation just where the strands came into contact with the muscle-cells. From here it became visible passing backwards, reaching in the end the elements of the primary plexus which were already fibrillated. Thus the end-formation of the syncytial plexus of the developing interstitial elements in which neurofibrillae were visible was connected with the elements of the primary plexus, in which neurofibrillae were also visible, by a series of syncytial non-differentiated elements, which gradually became fibrillated from the periphery towards the centre.

LEEUWE could show, that the interstitial elements possessed NISSL-bodies in their protoplasm, just like ordinary ganglion cells; in the same way they showed other features of ordinary ganglion cells, for instance with regard to the effects of the oxydase- and peroxydase-reaction. The π-granules, which form characteristic elements in the protoplasm of lemmoblasts, could not be detected inside interstitial cells. Thus the so-called interstitial cells belong to the group of ganglion cells and not to that of the lemmoblasts. They are derived from the ganglion cells by a series of intermediate forms. They are always in syncytial connexion with each other and with true ganglion cells of the sympathetic plexus, and must be regarded as a kind of primitive ganglion cells. This same conclusion I had drawn from my own observations some years ago (1935, 1936), though I did not feel entitled to draw such a sharp line of separation between the neuronic elements and the so-called lemmoblasts, or sheath-cells. These too are in syncytial connexion with the ganglion cells (as I showed in the XIth Innervationsstudy, 1941, and in former publications, 1916, 1926). LEEUWE regarded all the sheath elements of the enteric plexus as interstitial elements, in which I could not follow him. There where the interstitial elements appear in the end-formation, they are no more enveloped by lemmoblasts, but they are in syncytial connexion with the lemmoblasts which surround the postganglionic fibres of the ganglion cells of the enteric plexus themselves.

Three questions have to be answered with regard to these interstitial elements: a. are all the interstitial elements lying at the end of the sympathetic endformation of an efferent nature, or is it possible, that afferent elements too belong to the system of the interstitial elements?

b. what are the definite relations of the interstitial elements to the lemmoblasts, the SCHWANN and the REMAK cells?

c. are the interstitial elements to be found exclusively in the endformation of the sympathetic system, or are elements resembling them to be found at the end of the spinal and cerebral nerves too?

Ad a. In my former papers I had drawn the conclusion, that the sympathetic groundplexus must be especially of an efferent nature ("jedenfalls ist er sicher vorwiegend efferenter Art", BOEKE, 1935). Several authors however described small ganglion cells which are undoubtedly of an interstitial nature (LEONTOWITSCH, 1921, 1930; OKAMURA, 1930, 1937; BETHE, 1903; MEYLING, 1938; SMIRNOW, 1895; DOGIEL, 1898) and of an *afferent* sensible nature, and my own observations tend in the same direction; so without doubt we have to distinguish two sorts of interstitial elements, but it seems to me that the sympathetic groundplexus is *for the greater part of an efferent nature.*

Ad b. As I mentioned before, LAWRENTJEW, who in 1926 for the first time described the interstitial cells as lying at the end of the sympathetic plexus, maintained that they formed the real endings of the sympathetic plexus, but nevertheless he identified them with the lemmoblasts ("wir sind berechtigt, die interstitiellen Zellen als Lemmoblasten zu bezeichnen". LAWRENTJEW, 1926). I agree with him fully that the interstitial elements are found lying at the end of the sympathetic plexus, and that they form real synaptic endings on the smooth muscle-fibres, giving off small end-knobs, surrounded with a distinct periterminal network inside the protoplasm of the muscle cells (cf. LEEUWE and MEYLING), but then they must be of the nature of ganglion cells and not of lemmoblasts, although they are everywhere in a syncytial connexion with the lemmoblasts (cf. LEEUWE, PEI-LIN LI, 1940, and many others). We find a whole series of intermediate

forms between these elements and true ganglion cells (BOEKE, 1935, 1937; LEEUWE, 1937). These elements form real synapses (contrary to the theory of the "Terminalreticulum" of STOEHR, which in its original conception formed the connexion with the innervated elements without any trace of a synaptic junctional region where the stimulus was remoulded, polarised, a true "Umwertungsstelle der Erregung"); they are necessary for the humoral transfer of the nervous stimulus (cf. BOEKE, 1937, 1940); there may be found several interstitial elements lying one after the other at the end of the plexus, in syncytial continuity, thus forming a prolonged ending, an elongated "active region", a "plexiform innervation", but they are always of the nature of ganglion cells. Nevertheless they are always in a syncytial connexion with the lemmoblasts and in my opinion we must be very careful, before drawing such a sharp line of demarcation between the elements known as lemmoblasts and the true ganglion cells, especially where these may be of a primitive nature and liable to a very profound differentiation into different forms, and having an intermediate function, form and structure.

Ad c. Are they to be found exclusively at the end of the sympathetic endformation, as is maintained by LEEUWE and by every author who has been studying them, including CAJAL himself, LA VILLA a.s.o.?

In the sympathetic endformation they are connected with the innervated elements by means of a distinct intermediate protoplasmic structure, the periterminal network, which consists of an alveolar structure of the living substance, into which the conducting elements, the neurofibrillar structure of the terminal nervous cells, are radiating. The interstitial cells themselves show a marked alveolar structure of their protoplasma, and they may be regarded as the elements where the humoral energy is produced, the neurohumoral region of the endformation (BOEKE, 1935, 1937; LEEUWE, 1937; MEYLING, 1938). But does this hold true only and exclusively for the sympathetic endformation?

In principle the answer to this question has been given already in a lecture delivered by invitation of the Universities of London and Oxford in the year 1937, which was published with three other lectures by the Oxford Clarendon Press in 1940 [1]). Perhaps I may quote here what I said at the end of this lecture: "the unsolved riddle of the cells of the core of the sensory corpuscles, the tactile cells, the elements surrounding and carrying the terminal arborisation of so many different sensory corpuscles, would perhaps find a solution through the conception of the interstitial elements. These too may perhaps be regarded as interstitial elements, intermediate elements between the conducting nerve elements and the elements receiving sensory excitations" (l.c. page 121).

As it was mentioned before, even in the endformation of the sympaticus a synapse, a junctional region, where the nervous stimulus may be remoulded, polarized, "eine Umwertungsstelle der Erregung", is necessary. It is difficult to ascribe this property and the formation of a specific hormone, necessary for the humoral transfer of the nervous stimulus, to a tiny endknob, which is present on only one in hundred smooth muscle-cells, as described by histologists (STOEHR, 1934). The need for a stronger mode of innervation is obvious, more adequate to fulfil the claims of modern physiology of innervation, an active region, a plexiform innervation. But not only in the sympathetic endformation this need is obvious. Even in somatic innervation, in the first place in *sensory innervation*, as soon as a sensory corpuscle has to fulfil a more complicated purpose, that is to say, that by its structure and situation it gives the impression of being able to respond to stimuli of a more delicate nature, such as may be attributed to sensory corpuscles of higher order, it shows an enlargement of the surface of its neurofibrillar structure in order to convert the most delicate impressions to active stimuli. In this way the complicated neurofibrillar structures of the so very delicately sensitive corpuscles of MEISSNER, of KRAUSE, RUFFINI, GRANDRY or HERBST may find an

[1]) J. BOEKE, Problems of nervous anatomy, Oxford, University Press, 1940, 3rd chapter.

explanation. In the endbulbs of KRAUSE for example the whole mass of convolutions of the terminal neurofibrillae inside the bulb may be compared with the "active stretch" of synaptic value. A knob-like ending is not found here, but an "active region", a "wirksame Strecke" of the neurofibrillar endformation, and the same holds true for the MEISSNER corpuscles with their complex neurofibrillar structure, its convolutions with large ribbon-like flattened expansions, gradually breaking up into numerous thin twigs, forming most complicated loops and twists, which are everywhere surrounded by a periterminal network, lying within the protoplasm of the flattened wedge-shaped cells of the core and in continuous connexion with the neurofibrillar endloops. In the corpuscles of GRANDRY the neurofibrillar structure is moulded into a flat disc, but even here this disc is in continuous syncytial connexion with the periterminal network of the protoplasm of the two surrounding tactile cells. The same holds true for the endbulbs of KRAUSE, and for the lamellated corpuscles, as was described already some years ago (2nd Innervations-study, 1933).

Here too the base of the structure called the periterminal network is formed by an alveolar structure of the protoplasm, which indicates a secretory function, the location there of the neuro-humoral region, necessary for the transfer of the nervous stimulus. This is for example emphasized by the mobility of the nucleus of the tactile cells of the corpuscles of GRANDRY following the stimulus and the changes of the mitochondrial apparatus of these cells during the nervous stimulation (SZYMONOWICZ, BOEKE, DIJKSTRA).

The cells of the core of the sensory corpuscles, the tactile cells, connected syncytially with the neurofibrillar structure of the nervous endformation, must be of the nature of a *receptor* of the nervous stimulus. They must have therefore another function than the lemmoblasts, which only conduct the nervous impulse, with which they are however in a true syncytial connexion. We may regard them as the neuro-humoral region of the end-formation, receiving the nervous stimulus, in close connexion with the neurofibrillar apparatus of the nerve-endings, that is to say as the elements of the cerebro-spinal nervous endformation, to which we may ascribe the same function as to the interstitial elements, and however different and changed their form and aspect may be, we must regard them as homologous to the interstitial elements of the sympathetic endformation.

I need not to emphasize here the fact, that such a conception is entirely incompatible with the classical doctrine of the neurone theory, according to which the neurones are and remain independant units without a single syncytial stage either in development or in their fullgrown state.

But if these elements of the core of the sensory corpuscles are to be compared with the interstitial elements of the sympathetic endformation, we may ask, whether it would be possible to analyse even the motor endings, the motor endplates, in this direction. Would it be possible to view even them from the same standpoint?

As is well known, the motor endplate on the cross-striated muscle-fibres is lying hypolemmally, imbedded in the sarcoplasm of the sole-plate, which forms an intermediate structure, the periterminal network, between the neurofibrillar structure of the nerve-ending and the cross-striated myofibrillae themselves. Inside this sarcoplasma are lying three kinds of nuclei, the nuclei of the sarcoplasm itself, large loosely-built nuclei, identical with the other nuclei of the muscle-fibre lying dispersed in the sarcoplasma (fundamental nuclei, noyaux fondamentaux de RANVIER), small darkly-stained nuclei accompanying the nervous arborisations (nuclei of SCHWANN, noyaux de l'arborisation de RANVIER) and nuclei of the sarcolemma, belonging to the sheath of HENLE, lying outside the sarcolemma (noyaux vaginaux de RANVIER). In the developing motor endplates we can state that the small nuclei accompanying the nervous arborisations, are in reality derived from the ingrowing nerve-fibres and identical with their nuclei.

Now it is interesting to note, that as long ago as 1909 and 1910 THULIN and HOLMGREN described what they called "the interstitial cells" of the cross-striated muscle-fibres, but in 1926 it was shown by NOËL, that the so-called sarcosomytes (interstitial

cells) of THULIN and HOLMGREN were nothing else than the nuclei of the muscle-fibre lying directly under the sarcolemma, and I agree entirely with this description of NOËL. The interstitial cells of THULIN and HOLMGREN do not exist.

But at the other side there are some facts of the development of the motor endings and of the motor endplates on the cross-striated muscle-fibres, which together with the fact, that the small nuclei accompanying the nervous arborisation of the ingrowing motor nerve-ending are derived from these ingrowing nerve fibres, point in the same direction.

In the first place it is of interest to note that when we study the development of common striated muscle-fibres in the tongue we find the intercalation of a true syncytial stage in their innervation between the first outgrowing process of the motor nerves and the formation of true independent motor end-plates, an initial plexiform innervation and from this the definite individual innervation arises. During this process there is formed on the surface of the muscle-fibre, before a true sarcolemma is developed, a mass of nuclei surrounded by protoplasm, from which the motor end-plate, viz. its sole-plate with the nuclei arises. Nuclei of the ingrowing nerve-endfibres take part in this formation, and become the nuclei of the arborisation, exactly as the cells of the core of the sensory corpuscles, the tactile cells. In the longer muscle-fibres of the muscles of the extremities (according to TELLO, who studied these developing fibres in 1917) we find at the top of the ingrowing nerve-fibres always one or two nuclei, which do not belong to the muscle-fibre, but to the ingrowing nerve-fibres themselves (cf. TELLO, 1917, fig. 33, page 173). Where a nucleus is to be seen, it must be surrounded by protoplasm [1]) (which in the preparations of TELLO was not stained and therefore invisible), and we see how this mass of protoplasm and nuclei is incorporated in the sarcoplasma of the growing sole-plate. From these nuclei are derived the nuclei of the arborisation, and in my opinion these nuclei with their surrounding protoplasma are homologous with the syncytial cells of the core of the sensory corpuscles and with the interstitial elements of the sympathetic endplexus. They form together with the sarcoplasma of the sole-plate and its nuclei (noyaux fondamentaux de RANVIER) the neuro-humoral region of the nerve-ending, in which the periterminal network is formed and in which the humoral energy is produced, which is necessary for the transmission of the nervous stimulus.

Thus it seems to me that there is every reason to suppose that we have to do here with the selfsame elements. In evertebrates they form the sympathetic ganglion cells, bearing the neuroplasm and taking over or giving off the nervous impulses. In amphioxus they still appear as anastomosing elements, primitive ganglion cells, which are in connexion with the smooth muscle cells of the intestine (BOEKE, 1935), and they often show their connection with them in the form of the dendritlamellae, demonstrated in the sympathetic ganglion cells of the vertebrates by LAWRENTJEW, and which form in Amphioxus the common mode of motor ending (BOEKE, Anat. Anz. Vol. XXXIII., 1908). In the vertebrates they appear as the interstitial cells of the sympathetic endformation, anastomosing with each other and with the true ganglion cells, which are here superimposed on them as controlling elements in the course of evolution. They are in a syncytial arrangement with each other and with the conducting nerve fibres and give off the terminal synapses [2]). In the peripheral synapses of the cerebro-spinal nerves, the sensory corpuscles and the motor endings they are still further specialised; here they are no more recognisable as ganglion cells. In the sensory corpuscles they form the elements of the core or the tactile cells, syncytially arranged and in living connexion with the lemmoblasts, the conducting elements, and with the neurofibrillar structure of the nerve termination; they form the neuro-humoral region of the nerve terminations, in which the nervous stimulus

[1]) In the more elaborate paper, which is to be published in the Acta Neerlandica Morphologiae as the XII. Innervationsstudie, these observations will be discussed more fully and with illustrations.

[2]) In this connexion it is interesting to note, that even in 1895 BERKLEY suggested this, and that CAJAL sometimes speaks of "la cellule interstitielle ou terminale".

is transformed and the humoral energy is produced. In the motor endings, in which the motor end-plate is lying hypolemmally inside the sarcoplasma, they are entirely irrecognizable as distinct elements, and appear simply as the nuclei of the arborisation inside the sarcoplasma of the sole-plate, surrounded by their protoplasma, showing the periterminal network, the "receptive substance" of LANGLEY, and in continuous connexion with the sarcoplasma of the sole-plate of the muscle fibre itself. Here too they transform the nervous stimulus and produce the necessary humoral energy, even here building up the neuro-humoral region of the terminal formation.

I need not to emphasize here the fact, that this conception is entirely irreconcilable with the classic neurone theory and that the identification of these interstitial elements with ganglion cells applies exclusively to their function as bearers of the neuroplasma and not to their form.

Utrecht, February 1942.

Mathematics. — *Ueber die M_3^3 dreier Ebenen im R_5.* Von R. WEITZENBÖCK.

(Communicated at the meeting of February 28, 1942.)

Drei Ebenen E_1, E_2 und E_3 im fünfdimensionalen projektiven Raume R_5 spannen eine dreidimensionale Punktmannigfaltigkeit dritten Grades M_3^3 auf, die der Klasse der sogen. SEGRE'schen Mannigfaltigkeiten angehört [1]). Für die projektive Geometrie dieser M_3^3 bildet die Theorie der binär-ternären Bilinearform die natürliche Grundlage und wird also beherrscht durch die Theorie der Punktreihen der Ebene, die durch E. A. WEISS ausführlich dargestellt wurde [2]).

Im Bereiche der senären Formen, wenn die M_3^3 durch die drei Ebenen a, α, p (oder 1, 2, 3) gegeben ist, entsteht die Frage nach jenen projektiven Komitanten dieser drei Ebenen, die, gleich Null gesetzt, als „Gleichung der M_3^3" bezeichnet werden können.

Drei Ebenen im R_5 besitzen keine projektiven Invarianten [3]) und haben auch, wie ich unlängst bewiesen habe [4]), keine Komitanten mit nur einer Reihe Punktkoordinaten x oder nur einer Reihe R_4-Koordinaten u'. Es ist daher nicht möglich die M_3^3 durch eine einzige Gleichung in x_i bzw. in u'_i darzustellen. Dagegen ist dies möglich mit einer Reihe Linienkoordinaten π_{ik} (dual mit einer Reihe R_3-Koordinaten π'_{ik}) und auch mit einer Reihe Ebenenkoordinaten π_{ijk}. Die dabei auftretenden Komitanten sollen hier ermittelt werden.

§ 1.

Am einfachsten kommt man bei Ebenenkoordinaten zum Ziel. Die allgemeine erzeugende Ebene der M_3^3 wird nämlich gegeben durch

$$E_\delta = E_1 A_{23} + 9\delta J_{\pi 123} - 9\delta^2 J_{\pi 213} + \delta^3 E_2 A_{31} = 0 \quad . \quad . \quad . \quad (1)$$

Dabei bedeuten:

$$A_{23} = (2^3\, 3^3) = (\alpha^3\, p^3) \qquad J_{\pi 123} = (\pi^3\, 1^2\, 2)\,(1 2^2\, 3^3)$$
$$E_1 = (1^3\, \pi^3) = (a^3\, \pi^3)$$

und δ ist das Doppelverhältnis der vier Punkte in denen eine erzeugende Gerade von den Ebenen E_1, E_2, E_δ und E_3 getroffen wird [5]).

Bei Benützung der in allen drei Ebenen symmetrischen Komitante

$$S = \tfrac{1}{24} \Sigma J_{\pi 123}$$

kann man (1) auch so schreiben:

$$\left.\begin{aligned} E_\delta = E_1 A_{23} + \delta\,(-\tfrac{3}{2} E_1 A_{23} + \tfrac{1}{2} E_2 A_{31} - \tfrac{1}{2} E_3 A_{12} + 9S) + \\ + \delta^2 (\tfrac{1}{2} E_1 A_{23} - \tfrac{3}{2} E_2 A_{31} - \tfrac{1}{2} E_3 A_{12} - 9S) + \delta^3 E_2 A_{31} = 0 \end{aligned}\right\} \quad . \quad (2)$$

Ist in (1) oder (2) π_{ijk} gegeben, so sind die drei Wurzeln δ die Doppelverhältnisse, die auf den drei Transversalen der vier Ebenen E_1, E_2, π_{ijk} und E_3 bestimmt werden. Die

[1]) C. SEGRE, Rendic. di Palermo **5**, 192—204 (1891) und Mathem. Enzyklopädie III C 7, 828.

[2]) E. A. WEISS, Punktreihengeometrie, Teubner (1939). Insbes. S. 45 ff.

[3]) Proc. Kon. Akad. v. Wetensch., Amsterdam, **35**, 1026—1029 (1932).

[4]) Proc. Ned. Akad. v. Wetensch., Amsterdam, **45**, 139—141 (1942).

[5]) Proc. Ned. Akad. v. Wetensch., Amsterdam, **44**, 907—913 (1941).