Mathematics. — *On Unities and Dimensions.* II. By H. B. DORGELO and J. A. SCHOUTEN.

(Communicated at the meeting of February 23, 1946.)

If in threedimensional space a negative or positive definite fundamental tensor $g_{\lambda\varkappa}$ is introduced it is possible to consider orthogonal coordinate systems. Denoting orthogonal coordinates by ξ^1, ξ^2, ξ^3 [1]) we have

$$g_{11} = g_{22} = g_{33} = \mp 1 ; \quad g_{ih} = 0 ; \quad i \neq h ; \quad h, i = 1, 2, 3. \quad . \quad (26)$$

The quantity

$$\mathfrak{g} \overset{\text{def}}{=} | \operatorname{Det}(g_{\lambda\varkappa}) | \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

is a scalar density of weight $+2$, having with respect to every *orthogonal* coordinatesystem the value $+1$. For orthogonal coordinatesystems we have $\Delta = \pm 1$ and accordingly the difference between *ordinary quantities and densities vanishes*. This identification can be got by multiplying with $\mathfrak{g}^{1/2}$ or $\mathfrak{g}^{-1/2}$ respectively. Accordingly we get the following table of identification (in orthogonal components)

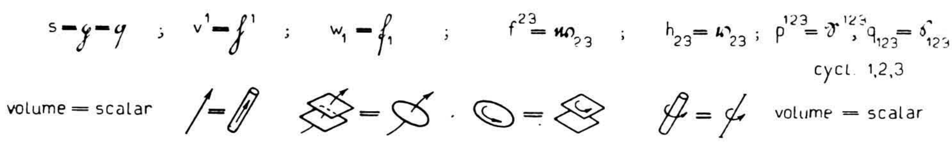

Fig. 2

From fig. 2 we see that the identification is brought about by means of the unity volume fixed by $\mathfrak{g}$. For instance to derive the arrow of $v^\varkappa$ from the tube of $\mathfrak{h}^\varkappa$ we have to construct two parallel planes cutting the unit of volume from the tube. Then $v^\varkappa$ fits precisely between these planes and its direction and orientation is that of the tube. Using the fundamental tensor only as far as the unit of volume is concerned we have got four different quantities. In physical publications these are often called (in the order of fig. 2) *polar vector, polar bivector, axial bivector and axial vector*. But the fundamental tensor fixes also the notion "perpendicular" and this gives rise to more identifications. The fundamental tensor establishes a one to one correspondence between co- and contravariant vectors and bivectors according to the formulas

$$\left. \begin{aligned} w_\lambda &= g_{\lambda\varkappa} v^\varkappa ; \quad & v^\varkappa &= g^{\varkappa\lambda} w_\lambda \\ h_{\lambda\varkappa} &= g_{\lambda\varrho} g_{\varkappa\sigma} f^{\varrho\sigma} ; \quad & f^{\varkappa\lambda} &= g^{\varkappa\varrho} g^{\lambda\sigma} h_{\varrho\sigma} \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

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[1]) We always use greek indices running from *1* to *3* (*4*) for general coordinates and latin indices running from 1 to 3 (4) for orthogonal coordinates.

Hence we get the identifications of fig. 3 (orthogonal components):

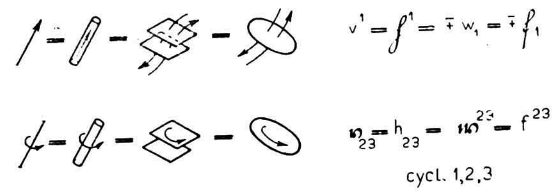

Fig. 3 [2])

Now there remain only two quantities, in physical publications often called *polar vector* and *axial vector* according to their simplest geometrical representation.

The difference between these two quantities depending only on the difference between inner and outer orientation, it is possible to get a last identification by introducing next to $g_{\lambda\varkappa}$ a definite screwsense in space. Then we get the identification

$$v^1 = \mathfrak{h}^1 = \mp w_1 = \mp \mathfrak{f}_1 = \mathfrak{v}_{23} = h_{23} = \mathfrak{w}^{23} = f^{23}; \quad \text{cycl. } 1, 2, 3. \qquad (29)$$

leaving only *one* quantity, the "vector", having eight different geometrical representations. It has to be remarked that these different identifications have not to be mixed up (as is often done) but that at each stage it has to be absolutely clear what we have introduced: either only the unit volume or the whole fundamental tensor or the fundamental tensor and a screwsense.

The following algebraic operations (corresponding to the scalar and the vectorial product of vectors in ordinary vectoranalysis) are invariant with *general* coordinate transformations:

1. The transvection of a contravariant and a covariant vector:

$$v \,.\, \overline{w} = v^\lambda w_\varkappa; \quad \lambda = 1, 2, 3; \quad . \; . \; . \; . \; . \; . \; . \qquad (30)$$

2. The alternating product of two contravariant or covariant vectors:

$$\bar{u} = v \times \overline{w}; \quad u^{\varkappa\lambda} = 2\, v^{[\varkappa} w^{\lambda]}; \quad u_{\varkappa\lambda} = 2\, v_{[\varkappa} w_{\lambda]}; \; \varkappa, \lambda = 1, 2, 3; \quad . \qquad (31)$$

In orthogonal components the scalar product can be written in the following ways

$$\left.\begin{aligned} \bar{v} \,.\, \overline{w} &= \mp v^1 w^1 + \text{cycl.} = v^1 w_1 + \text{cycl.} = \mp v_1 w_1 + \text{cycl.} = \\ &= \mp v^{23} w^1 + \text{cycl.} = v^{23} w_1 + \text{cycl.} = \mp v_{23} w^1 + \text{cycl.} = \\ &= v_{23} w_1 + \text{cycl} = \\ &= \mp v^{23} w^{23} + \text{cycl.} = \mp v^{23} w_{23} + \text{cycl.} = \mp v_{23} w_{23} + \text{cycl.} \end{aligned}\right\} \quad (32)$$

For the vectorial product we have in orthogonal components the forms

$$u^1 = u^{23} = v^2 w^3 - v^3 w^2 = v_{31} w^3 - v_{12} w^2 = v^{31} w^{12} - v^{12} w^{31}. \qquad (33)$$

and all the forms that can be deduced from these by raising and lowering of indices with the aid of the fundamental tensor.

[2]) In the figures with two arrows the arrow to the left belongs to the case of a negativ definite fundamental tensor and the arrow to the right to the other case.

The following differential operations (corresponding to the operations grad, div en rot in ordinary vector analysis) are invariant with *general* coordinate transformations:

1. the *gradient* of a scalar s:

$$w_\lambda = \partial_\lambda\, s; \quad \partial_\lambda \overset{\text{def}}{=} \frac{\partial}{\partial\, \xi^\lambda}; \quad \lambda = 1, 2, 3; \quad \ldots \ldots \quad (34)$$

2. the *rotation* of a covariant vector:

$$h_{\mu\lambda} = 2\, \partial_{[\mu}\, w_{\lambda]}; \quad \lambda, \mu = 1, 2, 3; \quad \ldots \ldots \quad (35)$$

3. the *rotation* of a covariant bivector:

$$q_{\omega\mu\lambda} = 3\, \partial_{[\omega}\, h_{\mu\lambda]}; \quad \omega, \mu, \lambda = 1, 2, 3; \quad \ldots \ldots \quad (36)$$

4. the *divergence* of a contravariant vectordensity of weight + 1:

$$\mathfrak{p} = \partial_\mu\, \mathfrak{h}^\mu; \quad \mu = 1, 2, 3; \quad \ldots \ldots \quad (37)$$

5. the *divergence* of a contravariant bivectordensity of weight + 1:

$$\mathfrak{h}^\varkappa = \partial_\mu\, \mathfrak{w}^{\mu\varkappa}; \quad \varkappa, \mu = 1, 2, 3; \quad \ldots \ldots \quad (38)$$

6. the *divergence* of a contravariant trivectordensity of weight + 1:

$$\mathfrak{w}^{\lambda\varkappa} = \partial_\mu\, \mathfrak{S}^{\mu\lambda\varkappa}; \quad \varkappa, \lambda, \mu = 1, 2, 3 \quad \ldots \ldots \quad (39)$$

The equations

$$2\, \partial_{[\mu}\, w_{\lambda]} = 0 \quad \text{and} \quad \partial_\mu\, \mathfrak{w}^{\mu\varkappa} = 0 \quad \ldots \ldots \quad (40)$$

express the invariant property, that the double-planes of the field (constructed on an infinitesimal scale) fit together to form a system of double-surfaces, filling the whole space.

The equations

$$3\, \partial_{[\omega}\, h_{\mu\lambda]} = 0 \quad \text{and} \quad \partial_\mu\, \mathfrak{h}^\mu = 0 \quad \ldots \ldots \quad (41)$$

express the invariant property, that the tubes of the field (constructed on an infinitesimal scale fit together, filling the whole space.

§ 4. *The electromagnetic equations independent of the choice of the electromagnetic unities.*

The electromagnetic equations are mostly formulated with respect to ordinary orthogonal coordinates and certain well chosen electromagnetic units. In general this is very convenient but the special choice of coordinates and units has as a consequence that the difference between vectors, bivectors and densities gets lost and that the equations do not give a clear insight into the possibilities concerning the choice of the electromagnetic units. In this section the equations will be made independent of the choice of these units. The units of mass, length and time may be arbitrarily chosen. We start from the equations

$$\left.\begin{array}{llll} a) & \nabla \times \mathbf{F} + \alpha \dot{\mathbf{B}} = 0 & e) & \mathbf{K} = \theta\, Q\, \mathbf{F} \\ b) & \nabla \cdot \mathbf{B} = 0 & f) & F = \dfrac{\iota}{\varepsilon} \dfrac{Q}{r^2} \\ c) & \beta \dot{\mathbf{D}} - \gamma \nabla \times \mathbf{H} = -\varrho\, \mathbf{u} & g) & \mathbf{B} = \varphi\, \varepsilon\, \mathbf{F} \\ d) & \zeta \nabla \cdot \mathbf{D} = \mp \varrho\ ^{3)} & h) & \mathbf{B} = \psi\, \mu\, \mathbf{H} \end{array}\right\} \quad . \quad . \quad (4.\,1)$$

where

$$\left.\begin{array}{ll} \mathbf{F} = \text{electric fieldstrength} & \varrho = \text{electric charge density} \\ \mathbf{D} = \text{electric displacement} & Q = \text{electric charge} \\ \mathbf{H} = \text{magnetic fieldstrength} & \varepsilon = \text{dielectric constant} \\ \mathbf{B} = \text{magnetic induction} & \qquad (\varepsilon = \varepsilon_0 \text{ in vacuum}) \\ & \mu = \text{permeability} \\ & \qquad (\mu = \mu_0 \text{ in vacuum}) \\ \multicolumn{2}{c}{\mathbf{K} = \text{force; } \mathbf{u} = \text{velocity of electric charge}} \end{array}\right\} (4.1a)$$

and where α, β, γ, ζ, θ, ι, φ and ψ are eight constant parameters depending on the choice of the electromagnetic units. The fundamental units of mass, length and time being chosen, the electromagnetic units can be fixed by the following seven assumptions:

1. *Unit of* **F**. The unit of F is the field strength exerting the force a on the charge 1.

2. *Unit of* **Q**. Two positive unit charges on a distance 1 exert on each other a force h'^{-1}.

3. *Unit of* **D**. D has the value h on a distance 1 of a unit charge in vacuum.

4. *Unit of* **H**. The unit of **H** is the fieldstrength on a distance 1 of an infinitely long straight conductor carrying a current $\frac{1}{2}\,k$.

5. *Unit of* **B**. If the current of magnetic induction through a closed curve changes with k' units in the unit of time, the electromotoric force in the curve (i.e. the integral of the electric fieldstrength along this curve) has the value 1.

6. *Unit of* ε. The unit of ε is $\dfrac{\varepsilon_0}{p}$

7. *Unit of* μ. The unit of μ is $\dfrac{\mu_0}{p'}$

Every one of these assumptions contains one constant, that can be fixed in an arbitrary way. In consideration of (4. 1) we get from them

$$\theta = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.\,2)$$

$$\iota = \frac{\varepsilon_0}{a\,h'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.\,3)$$

[3]) The sign — belongs to a negative definite fundamental tensor and the sign + to a positive definite one: $\mathbf{v} \cdot \mathbf{w} = \mp v^1 w^1 \mp v^2 w^2 \mp v^3 w^3$.

$$\zeta=\frac{1}{4\pi h};\quad \varphi\,\varepsilon_0=a\,h\,h';\quad \beta=\frac{1}{4\pi h}\quad \ldots\ldots \quad (4.\ 4)$$

$$\gamma=\frac{k}{4\pi}\quad \ldots\ldots \quad (4.\ 5)$$

$$\alpha=\frac{1}{k'}\quad \ldots\ldots \quad (4.\ 6)$$

$$\varepsilon_0=p\quad \ldots\ldots \quad (4.\ 7)$$

$$\mu_0=p'\quad \ldots\ldots \quad (4.\ 8)$$

From the fact that the propagation of electromagnetic phenomena has the velocity c in vacuum it can be deduced that

$$c^2=\frac{h\,k\,k'}{\varphi\,\varepsilon_0\,\psi\,\mu_0}\quad \ldots\ldots \quad (4.\ 9)$$

or

$$\psi=\frac{k\,k'}{a\,h'\,p'\,c^2}.\quad \ldots\ldots \quad (4.\ 10)$$

Hence the eight parameters α, β, γ, ζ, θ, ι, φ and ψ and the values of ε_0 and μ_0 can now be expressed in terms of the seven constants a, k, k', h, h', p and p':

$$\left.\begin{aligned}
\alpha&=\frac{1}{k'} & \iota&=\frac{p}{a\,h'}\\
\beta&=\frac{1}{4\pi h} & \varphi&=\frac{a\,h\,h'}{p}\\
\gamma&=\frac{k}{4\pi} & \psi&=\frac{k\,k'}{a\,h'\,p'\,c^2}\\
\zeta&=\frac{1}{4\pi h} & \varepsilon_0&=p\\
\theta&=a & \mu_0&=p'
\end{aligned}\right\}\quad \ldots\ldots \quad (4.\ 11)$$

That implies that there have to exist just three relations between the eight parameters and ε_0 and μ_0. In fact it is readily proved that

$$\beta=\zeta=\frac{1}{4\pi\,\iota\,\varphi}\quad;\quad \varphi\,\psi=\frac{\gamma}{\alpha\,\beta\,\varepsilon_0\,\mu_0\,c^2}.\quad \ldots \quad (4.\ 12)$$

The equations (4. 1) now take the form

$$\left.\begin{aligned}
&a)\quad k'\,\nabla\times\mathbf{F}+\dot{\mathbf{B}}=0 && e)\quad \mathbf{K}=a\,Q\,\mathbf{F}\\
&b)\quad \nabla\,.\,\mathbf{B}=0 && f)\quad F=\frac{p}{a\,h'}\,\frac{1}{\varepsilon}\,\frac{Q}{r^2}\\
&c)\quad \frac{1}{4\pi h}\,\dot{\mathbf{D}}-\frac{k}{4\pi}\,\nabla\times\mathbf{H}=-\,\varrho\,\mathbf{u} && g)\quad \mathbf{D}=\frac{a\,h\,h'}{p}\,\varepsilon\,\mathbf{F}\\
&d)\quad \frac{1}{4\pi h}\,\nabla\,.\,\mathbf{D}=\mp\,\varrho && h)\quad \mathbf{B}=\frac{k\,k'}{a\,h'\,p'\,c^2}\,\mu\,\mathbf{H}
\end{aligned}\right\}.\quad (4.\ 13)$$

The choice of the seven constants a, k, k', h, h', p and p' is entirely free. It is allowed to give them a dimension, i.e. to take them dependent on the choice of the fundamental units of mass, length and time. To every choice belongs a system of electromagnetic units.

If the fundamental units of mass, length and time are multiplied by m^{-1}, l^{-1} and t^{-1} and the constants a, k, k', h, h', p and p' by α', $\varkappa$, $\varkappa'$, χ, χ', ω and ω' respectively, it follows from (4. 13) that the units of **F**, **D**, **H**, **B**, **Q**, ε and μ get the following factors:

$$\left.\begin{array}{llll} \mathbf{F}: & m^{-1/2}\, l^{1/2}\, t\, \chi'^{1/2}\, \alpha' & \mathbf{Q}: & m^{-1/2}\, l^{-3/2}\, t\, \chi'^{-1/2} \\ \mathbf{D}: & m^{-1/2}\, l^{1/2}\, t\, \chi^{-1}\, \chi'^{-1/2} & \varepsilon: & \omega^{-1} \\ \mathbf{H}: & m^{-1/2}\, l^{-1/2}\, t^{2}\, \varkappa\, \chi'^{-1/2} & \mu: & \omega'^{-1} \\ \mathbf{B}: & m^{-1/2}\, l^{3/2}\, \varkappa'^{-1}\, \chi'^{-1/2}\, \alpha' & & \end{array}\right\} \quad . \quad . \quad (4.\,14)$$

If the factor of the unit of charge is denoted by q^{-1}, it is possible to eliminate χ' and to express the factors in terms of m, l, t, q, α', $\varkappa$, $\varkappa'$, χ, ω and ω':

$$\left.\begin{array}{llll} \mathbf{F}: & m^{-1}\, l^{-1}\, t^{2}\, q\, \alpha' & \mathbf{Q}: & q^{-1} \\ \mathbf{D}: & l^{2}\, q^{-1}\, \chi^{-1} & \varepsilon: & \omega^{-1} \\ \mathbf{H}: & l\, t\, q^{-1}\, \varkappa & \mu: & \omega'^{-1} \\ \mathbf{B}: & m^{-1}\, t\, q\, \varkappa'^{-1}\, \alpha' & & \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4.\,15)$$

and these expressions do not contain any fractional exponents. Instead of the unit of charge it would have been possible to take the unit of **F**, **D**, **H** or **B** to get rid of fractional exponents.

In the following four wellknown systems the seven constants have the values:

	Electromagnetic *c.g.s*	*Electrostatic* *c.g.s.*	*Gauss* *c.g.s.*	*Giorgi* *m. kg. s*
a	1	1	1	1
k	1	1	1	1
k'	1	1	c	1
h	1	1	c	1
h'	$1/c^2$	1	1	$10^7/c^2$
p	$1/c^2$	1	1	$10^7/c^2$
p'	1	$1/c^2$	1	10^{-7}
	(c in cm/s)	(c *in* cm/s)	(c in cm/s)	(c in M/s).

(4. 16)

In each of these systems the constants are chosen in such a way that φ en ψ in (4. 1, g, h) have dimension [1]:

$$\varphi = \frac{a\,h\,h'}{p} = 1\,; \qquad \psi = \frac{k\,k'}{a\,h'\,p\,c^2} = 1 \quad . \quad . \quad . \quad . \quad (4.\,17)$$

The units belonging to these four systems are

	Electromagnetic c.g.s	*Electrostatic* c.g.s.	*Gauss* c.g.s.	*Giorgi* m. kg. s.
Force	1 dyne	1 dyne	1 dyne	1 Newton $= 10^5$ dyne
Work	1 erg	1 erg	1 erg	1 Joule $= 10^7$ erg
Q	10 coulomb	$^{10}/_c$ coulomb	$^{10}/_c$ coulomb	1 coulomb
$\mathbf{F}$	10^{-8} $^{\text{volt}}/_{\text{cm}}$	$10^{-8} \cdot c$ $^{\text{volt}}/_{\text{cm}}$	$10^{-8} \cdot c$ $^{\text{volt}}/_{\text{cm}}$	1 $^{\text{volt}}/_{\text{m}}$
$\mathbf{D}$	$\frac{1}{4\pi} \cdot 10$ $^{\text{coulomb}}/_{\text{cm}^2}$	$\frac{1}{4\pi} \cdot \frac{10}{c}$ $^{\text{coulomb}}/_{\text{cm}^2}$	$\frac{1}{4\pi} \cdot \frac{10}{c}$ $^{\text{coulomb}}/_{\text{cm}^2}$	$\frac{1}{4\pi}$ $^{\text{coulomb}}/_{\text{m}^2}$
$\mathbf{H}$	1 oerstedt	c^{-1} oerstedt	1 oerstedt	10^{-3} oerstedt
$\mathbf{B}$	1 gauss	c gauss	1 gauss	10^{-4} gauss
ε	$c^2 \varepsilon_0$	ε_0	ε_0	$10^{-7} c^2 \varepsilon_0$
μ	μ_0	$c^2 \mu_0$	μ_0	$10^7 \mu_0$
E	10^{-8} volt	$10^{-8} \cdot c$ volt	$10^{-8} \cdot c$ volt	1 volt
I	10 ampère	$^{10}/_c$ ampère	$^{10}/_c$ ampère	1 ampère
R	10^{-9} ohm	$10^{-9} \cdot c^2$ ohm	$10^{-9} \cdot c^2$ ohm	1 ohm
	(c in $^{\text{cm}}/_{\text{s}}$)	(c in $^{\text{cm}}/_{\text{s}}$)	(c in $^{\text{cm}}/_{\text{s}}$)	(c in $^{\text{m}}/_{\text{s}}$)

. (4. 18)

and the equations (4. 13) take the form

	Electromagn., electrostat., Giorgi	*Gauss*
a)	$\nabla \times \mathbf{F} + \dot{\mathbf{B}} = 0$	$c \nabla \times \mathbf{F} + \dot{\mathbf{B}} = 0$
b)	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
c)	$\frac{1}{4\pi} \dot{\mathbf{D}} - \frac{1}{4\pi} \nabla \times \mathbf{H} = -\varrho \mathbf{u}$	$\frac{1}{4\pi} \dot{\mathbf{D}} - \frac{c}{4\pi} \nabla \times \mathbf{H} = -\varrho \mathbf{u}$
d)	$\frac{1}{4\pi} \nabla \cdot \mathbf{D} = \mp \varrho$	$\frac{1}{4\pi} \nabla \cdot \mathbf{D} = \mp \varrho$
e)	$\mathbf{K} = Q \mathbf{F}$	$\mathbf{K} = Q \mathbf{F}$
f)	$F = \frac{1}{\varepsilon} \frac{Q}{r^2}$	$F = \frac{1}{\varepsilon} \frac{Q}{r^2}$
g)	$\mathbf{D} = \varepsilon \mathbf{F}$	$\mathbf{D} = \varepsilon \mathbf{F}$
h)	$\mathbf{B} = \mu \mathbf{H}$	$\mathbf{B} = \mu \mathbf{H}$

(4. 19)

In every one of these four systems it is inconvenient that there occurs a factor 4π in (c) and (d). Comparing (4. 19) with (4. 13) we see that it is possible to get rid of this factor by starting from (4. 19) and choosing α', $\varkappa$, $\varkappa'$, χ, χ', ω and ω' in such a way that

$$a) \quad \varkappa = 4\pi; \qquad b) \quad \chi = \frac{1}{4\pi}. \qquad \qquad (4.20)$$

But from (4. 13) we see that then (g) and (h) also change. Now we make the condition that the factor 4π vanishes in all formulas but for (f) and that in (f) a factor 4π comes in the denominator of the right member. Then from (4. 13, a, c, d, e) it follows that

$$\alpha' = 1; \quad \varkappa = 4\pi; \quad \varkappa' = 1; \quad \chi = \frac{1}{4\pi}. \quad . \quad . \quad . \qquad (4.\ 21a)$$

and from (4. 13, f, g, h) that

$$\chi' = 4\pi\omega; \quad \omega\omega' = 1. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4.\ 21b)$$

Considering only simple values of ω, ω' and χ' there are only two possibilities:

First method of rationalization (GAUSS):

$$\alpha' = \varkappa' = 1; \ \varkappa = 4\pi; \ \chi = \frac{1}{4\pi}; \ \chi' = 4\pi; \ \omega = \omega' = 1. \qquad (4.\ 22a)$$

Second method of rationalization:

$$\alpha' = \varkappa' = 1; \ \varkappa = 4\pi; \ \chi = \frac{1}{4\pi}; \ \chi' = 1; \ \omega = \frac{1}{4\pi}; \ \omega' = 4\pi. \ (4.\ 22b)$$

With both methods the values of φ and ψ in (4. 1 g, h) remain 1. With the method of GAUSS all units except those of ε and μ get a factor $\sqrt{4\pi}$ or $1 : \sqrt{4\pi}$. This is highly objectionable. Using the second method, only the units of **D**, **H**, ε and μ change and the unit of **H** gets the factor 4π. Because of

$$4\pi \text{ oerstedt} = 10 \,\frac{\text{ampère windings}}{\text{cm}}. \quad . \quad . \quad . \quad . \quad . \qquad (4.\ 23)$$

the new unit of **H** is certainly better than the old one for all practical purposes. For the new units of **D**, **H**, ε and μ we get for the second method of rationalization

	Electromagnetic c. g. s.	*Electrostatic* c. g. s.	*Gauss* c. g. s.	*Giorgi* m. kg. s.	
D	$10\ \text{coulomb}/\text{cm}^2$	$\frac{10}{c}\ \text{coulomb}/\text{cm}^2$	$\frac{10}{c}\ \text{coulomb}/\text{cm}^2$	$1\ \text{coulomb}/\text{m}^2$	
H	$10\ \text{amp. w.}/\text{cm}$	$\frac{10}{c}\ \text{amp. w.}/\text{cm}$	$10\ \text{amp. w.}/\text{cm}$	$1\ \text{amp. w}/\text{m}$	(4. 24)
ε	$4\pi c^2 \varepsilon_0$	$4\pi\varepsilon_0$	$4\pi\varepsilon_0$	$4\pi \,.\, 10^{-7} c^2 \varepsilon_0$	
μ	$\frac{1}{4\pi}\mu_0$	$\frac{1}{4\pi} c^2 \mu_0$	$\frac{1}{4\pi}\mu_0$	$\frac{1}{4\pi} \,.\, 10^7 \mu_0$	
	(c in cm/s)	(c in cm/s)		(c in m/s)	

and for both methods of rationalization the equations take the form

	Electromagn., electrostat., Giorgi	*Gauss*	
a)	$\nabla \times \mathbf{F} + \dot{\mathbf{B}} = 0$	$c \nabla \times \mathbf{F} + \dot{\mathbf{B}} = 0$	
b)	$\nabla . \mathbf{B} = 0$	$\nabla . \mathbf{B} = 0$	
c)	$\dot{\mathbf{D}} - \nabla \times \mathbf{H} = -\varrho \mathbf{u}$	$\dot{\mathbf{D}} - c \nabla \times \mathbf{H} = -\varrho \mathbf{u}$	
d)	$\nabla . \mathbf{D} = \mp \varrho$	$\nabla . \mathbf{D} = \mp \varrho$	(4. 25)
e)	$\mathbf{K} = Q \mathbf{F}$	$\mathbf{K} = Q \mathbf{F}$	
f)	$F = \frac{1}{\varepsilon} \frac{Q}{4 \pi r^2}$	$F = \frac{1}{\varepsilon} \frac{Q}{4 \pi r^2}$	
g)	$\mathbf{D} = \varepsilon \mathbf{F}$	$\mathbf{D} = \varepsilon \mathbf{F}$	
h)	$\mathbf{B} = \mu \mathbf{H}$	$\mathbf{B} = \mu \mathbf{H}$	

According to (4. 24) with the second method of rationalization ε_0 and μ_0 take the values

	Electromagnetic c. g. s.	*Electrostatic* c. g. s.	*Gauss* c. g. s.	*Giorgi* m. kg. s.	
ε_0	$^1/_{4\pi c^2}$	$^1/_{4\pi}$	$^1/_{4\pi}$	$^{10^7}/_{4\pi c^2}$	. (4. 26)
μ_0	4π	$^{4\pi}/_{c^2}$	4π	$4\pi . 10^{-7}$	
	(c in $^{cm}/_s$)	(c in $^{cm}/_s$)		(c in $^{m}/_s$)	

Comparing (4. 18) and (4. 19) with (4. 24) and (4. 25) we see that the system of GIORGI, especially in the second rationalized form is the most recommendable for all practical purposes.

§ 5. *The relative dimensions independent of the choice of the electromagnetic units.*

The table (4. 14) or (4. 15) is the base of all dimension formulas. The constants a, h, h', k, k', p and p' can be chosen in such a way that all or a part of them depend on some natural unit (cf. § 2). In fact this is done in the four practical systems mentioned above by relating the constants with the velocity of light (cf. (4. 16)). There would be no objection to the use of other natural units as the mass of the electron, the charge of the electron or the elementary quantum.

In order to deduce from (4. 14) or (4. 15) the dimensions in m, l, t or m, l, t, q of the units of a definitely given system it is not allowed to drop only the factors α', $\varkappa$, $\varkappa'$, χ, χ', ω and ω' because among the constants a, k, k', h, h', p and p', fixing the system, there may be some having a dimension. This dimension has to be taken into account. Then we get the following *general dimension formulas,* valid for vectors only as far as *orthogonal components* are concerned ($[\varphi] = [\psi] = [1]$):

$$\left.\begin{array}{lll}
\mathbf{Q}: & [m^{1/2}\, l^{3/2}\, t^{-1}]\, [h'^{1/2}] & = [q] \\
\mathbf{F}: & [m^{1/2}\, l^{-1/2}\, t^{-1}]\, [h'^{-1/2}\, a^{-1}] & = [m\, l\, t^{-2}\, q^{-1}]\, [a^{-1}] \\
\mathbf{D}: & [m^{1/2}\, l^{-1/2}\, t^{-1}]\, h\, h'^{1/2}] & = [l^{-2}\, q]\, [h] \\
\mathbf{H}: & [m^{1/2}\, l^{1/2}\, t^{-2}]\, [k^{-1}\, h'^{1/2}] & = [l^{-1}\, t^{-1}\, q]\, [k^{-1}] \\
\mathbf{B}: & [m^{1/2}\, l^{-3/2}]\, [k'\, h'^{-1/2}\, a^{-1}] & = [m\, t^{-1}\, q^{-1}]\, [k'\, a^{-1}] \\
\varepsilon: & [p] = [h\, h'\, a] & = [m^{-1}\, l^{-3}\, t^2\, q^2]\, [h\, a] \\
\mu: & [p'] = [l^{-2}\, t^2]\, [k\, k'\, h'^{-1}\, a^{-1} & = [m\, l\, q^{-2}]\, [k\, k'\, a^{-1}] \\
E: & [m^{1/2}\, l^{1/2}\, t^{-1}]\, [h'^{-1/2}\, a^{-1}] & = [m\, l^2\, t^{-2}\, q^{-1}]\, [a^{-1}] \\
I: & [m^{1/2}\, l^{3/2}\, t^{-2}]\, [h'^{1/2}] & = [t^{-1}\, q] \\
R: & [l^{-1}\, t]\, [h'^{-1}\, a^{-1}] & = [m\, l^2\, t^{-1}\, q^{-2}]\, [a^{-1}]
\end{array}\right\} \quad . \quad . \quad (5.1)$$