Astronomy. - Egyptian "Eternal Tables". I. By B. L. van der Waerden. (Communicated by Prof. A. Pannekoek.)
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## Introduction.

Very little is known about the history of Egyptian astronomy. Were the Egyptians in possession of long observations of movement, periods and stationary points of the planets, so that even the Chaldaeans could learn from them, as Diodoros (I 81) says? Were they able to predict celestial phenomena by graphical or geometrical methods, as Theon of Smyrna (p. 177 Hiller) asserts? Were their astronomy and astrology autochtonous or largely influenced by Greek or Chaldaean ( $=$ Babylonian) ideas? Does Egyptian astronomical science go back to old-Egyptian wisdom, or is it a product of Hellenistic times? ${ }^{1}$ ) What was the nature of the knowledge of the Egyptians about the planets laid down in the "eternal tables", as mentioned by a horoscope ${ }^{2}$ ) for A D 81?

The best method to decide these questions would be the careful analysis of Egyptian astronomical and astrological texts. Only quite recently discussion of the astronomical texts started, mainly through the work of Otto Neugebauer $\left.{ }^{3}\right)^{4}$ ) ${ }^{5}$ ).
I will discuss three of the most important astronomical texts, published by Neugebauer ${ }^{5}$ ), concerning the dates of entrance of the planets into the signs of the zodiac, and I will show that they are calculated entirely by Babylonian methods.

I believe that this fact, combined with other indications of Babylonian influence such as the pictures of the goatfish ( $=$ Capricorn), the archer ( $=$ Sagittarius) and the weigher ( $=$ Libra) on the famous zodiac of Dendera, will help to throw more light on the dark origins of Hellenistic astronomy and astrology in Egypt.

Egyptian Planetary Texts.
The texts will be designed, as in Neugebauer's paper, by the letters $\mathrm{P}, \mathrm{S}$ and T .

Pis the Berlin Papyrus P 8279, first published by Spiegelberg ${ }^{6}$ ).

[^0]S denote the "Stobart tablets", published by Brugsch ${ }^{7}$ ).
T is the Teptunis papyrus II 274, published by Greenfell, Hunt and Goodspeed ${ }^{8}$ ).

Neugebauer re-edited these Egyptian Planetary Texts (1.c.) with an excellent astronomical and historical commentary.

He found that dates in P are given according to the Egyptian calendar, in S according to the Alexandrian calendar. P covers the years 14-41 of the reign of Augustus ( -16 till +11 AD ). S consists of 3 parts:

A : Years Vespasian 4-10 (A D 71-77)
$\mathrm{C}_{1}+\mathrm{C}_{2}$ : Years Trajan 9-Hadrian 3 (A D 105-118)
E : Years Hadrian 11 - 17 (AD 126-132).
The Greek papyrus T gives dates of entrance of planets into the signs for the years $10-18$ Trajan nearly identical with those of S . It contains, furthermore, an interesting additional column giving dates of the new moon and subsequent crescent, calculated according to a simple rule, which can also be found in Papyrus Carlsberg 9, published by Neugebauer and Volten (l.c.).

Comparing the positions given in P and S with modern calculations, Neugebauer found a systematic difference (text minus calculation) of about $4^{\circ}$ in the beginning of the reign of Augustus and decreasing with time. This means that the texts use a fixed origin of the zodiac, connected with the fixed stars, just as Babylonian moon and planetary tables do. If modern longitudes are reduced to the ecliptic of 100 , the difference becomes $5^{\circ}$; hence the origin of the zodiac in our Egyptian texts coincides with that of the Babylonian ephemerides and observation texts of the latest time ${ }^{9}$ ).

Neugebauer has made it highly probable that P and S belong to the so-called "eternal tables" mentioned by Ptolemaios ${ }^{10}$ ) and by a Greek horoscope for A D 81 mentioned before. He could, however, not decide by what means the tables were calculated. He rightly observes that some kind of theory must have been used, for the entrance of a planet into a sign is indicated even if it is invisible. In the main "linear part" of the orbits, where the motion is approximately uniform, he found agreement between text and calculation, but in the retrograde part and immediately before and after there are considerable discrepancies, a fact which excludes

[^1]the use of something like epicycles and excenters. He conjectures that the tables are the result of combined observations and calculations.

In the following lines it will be shown that the tables $\mathrm{P}, \mathrm{S}$, and T are calculated according to Babylonian methods. The velocities of any planet are assumed to be constant during a definite part of the synodic period and in a definite part of the ecliptic, and to jump suddenly to another constant value if these limits in time or space are exceeded, just as in certain Babylonian procedure texts. As a typical example, I quote the procedure text Rm IV $431{ }^{11}$ ). In this text the ecliptic is divided by the points $2^{\circ}$ 万, $17^{\circ}$ ช, $9^{\circ} 9$, and $9 \mathrm{M}_{\text {into }} 4$ parts, for which different speeds are assumed. On the "fast arcs" from $2^{\circ} \nearrow$ to $17^{\circ} \succ$ the velocity of Jupiter is
during $1^{\mathrm{M}}$ after heliacal rising $15^{\prime}$ per $1^{\mathrm{D}}\left(1^{\mathrm{M}}=30^{\mathrm{D}}\right)$

| $" \quad 3^{M}$ | $8^{\prime}$ per $1^{\text {D }}$ |
| :--- | :--- |
| $5^{\prime}$ per $1^{D}$ |  |

, $4^{\mathrm{M}}$ retrograde $\quad 5^{\prime}$ per $1^{D}$
,. $3^{M} \quad 7^{\prime} 40^{\prime \prime}$ per $1^{D}$
,. $1^{\mathrm{M}}$ until heliacal setting $15^{\prime}$ per $1^{\mathrm{D}}$
", $1^{\mathrm{M}}$ until heliacal rising $15^{\prime \prime}$ per $1^{\mathrm{D}}$
On the "middle arcs" from $9^{\circ} \mathrm{m}$ to $2^{\circ} \mathrm{\gamma}$ and from $17^{\circ}$ ૪ to $9^{\circ}$ g these velocities are multiplied by $\frac{15}{16}$, and on the "slow arcs" from $9^{\circ}$ 6 to $9^{\circ} \mathrm{m}$ by $\frac{5}{6}$.

Hence the velocities show the same ratio as 40: 45: 48. The total synodic arc is $36^{\circ}$ on the "fast arcs", $33^{\circ} 45^{\prime}$ on the "middle arcs" and $30^{\circ}$ on the "slow arcs".

It will be shown, that quite similar schemes are used in our Egyptian "eternal tables"; only the velocities are different from those of the known Babylonian procedure texts.

The purpose of the column of new moons and crescents in T becomes clear too: it allows the reduction of Babylonian dates to the Egyptian or Alexandrian calendar; for the Babylonian month begins with the crescent.

## Venus.

The dates for Venus for the year 9 Trajan in text S are:

| Month | Day | Sign | Difference | Month | Day | Sign | Difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 16 | m | 24 | 7 | 4 | $r$ | 25 |
| 2 | 10 | ي | 24 | 7 | 29 | ૪ | 25 |
| 3 | 4 | $m$ | 25 | 8 | 24 | II | 24 |
| 3 | 29 | $\pi$ | 23 | 9 | 18 | 9 | 25 |
| 4 | 22 | あ | 24 | 10 | 13 | $\Omega$ | 25 |
| 5 | 16 | * | 24 | 11 | 8 | m | 38 |
| 6 | 10 | $x$ | 24 | 12 | 16 | $\underline{1}$ | - |

As one sees from the differences, the planet moves during the first 11 months with a nearly constant velocity. Only at the end of the year, just

[^2]before the retrogradation begins, the motion becomes slower. During the uniform "linear part" of the motion the time needed to pass a sign alternates with few exceptions between 24 and 25 days throughout the whole table.

Two explanations for these constant differences are possible: either some of the positions were observed and the others found by linear interpolation, or all of them were calculated by assuming a constant velocity.

To decide between these two possibilities, I have reduced the dates of the next synodic period (year 11/12) to the year $9 / 10$ by subtracting a synodic period ( 584 days) from the dates and a synodic arc of Venus $\left(360^{\circ}+215 \frac{1}{2}^{\circ}\right)$ from the positions. In the same way, I have reduced the dates of the year $12 / 13$ to the year $9 / 10$ by subtracting 2 synodic periods, etc. for the whole text S . All positions thus obtained are shown as points in diagram 1.

As is seen from this diagram, the linear parts of the 16 synodic periods covered by the text are brought to exact coincidence by our reduction procedure. The deviations seldom exceed 2 days. This proves that all positions are calculated. For if they were partly derived from observation, deviations up to 5 or 6 days ought to occur, owing to the anomalies of the sun and Venus. Observational errors and the difficulty of dividing the ecliptic into 12 exactly equal signs would tend to increase these deviations still more.

Furthermore one sees from the diagram, that about the date month 11 day 13 a sudden change in velocity takes place. After this change, the velocity remains constant again during 1 month at least. The law of the retrograde motion is not quite clear, but in any case the retrograde arc is too large (at least $18^{\circ}$, probably $20^{\circ}$ or more), just as in the Babylonian Venus tablet SpI $548+230{ }^{12}$ ). After the retrograde motion the velocity increases again stepwise until it resumes its original value in the linear part.

The highest and second highest velocities, obtained graphically from the diagram, are
$480^{\circ}$ in 392 or 393 days (linear part)
$23^{\circ}$ in 30 days (before and after).
One gets much simpler values, if the velocities are calculated not for 30 days but for one Babylonian month. As I have shown in Eudemus I, the fundamental unit of time in Babylonian planetery texts is the mean synodic month of 29,53 days, which is divided for the purpose of easy calculation into 30 artificial "days". Writing $1^{\mathrm{M}}$ and $1^{\mathrm{D}}$ for the Babylonian month and artificial day, $1^{\text {d }}$ for the real day, we find the two velocities

$$
\begin{aligned}
& 480^{\circ} \text { in } 400^{\text {D}} \text {, i.e. } \frac{6}{5}\left(\left(^{\circ} \text { per }{ }^{\mathrm{D}}\right)\right. \\
& 22 \frac{1}{2}^{\circ} \text { in } 30^{\mathrm{D}} \text {, i.e. } \frac{3}{4}\left({ }^{\circ} \text { per }{ }^{\mathrm{D}}\right)
\end{aligned}
$$

These two values are certain, but the law of motion during the retrograde course and immediately before and after is not quite clear. A reasonable

[^3]conjecture seems to be the following, represented by the broken line in the diagram:


Slight alterations are possible, but in any case the law of motion is quite analogous to those of the Babylonian procedure texts for Jupiter (see Introduction) and Saturn (see hereafter), only in the case of Venus no distinction between different parts of the ecliptic needs to be made, the anomalies of Venus and the sun being small.

Mars.
As yet 4 indications pointed to Babylon:
the fixed origin of the zodiac,
the retrograde arc of Venus of about $20^{\circ}$, which is too large,
the simple values of the velocities in Babylonian units,
the law of motion analogous to Babylonian procedure texts.
We could add that planetary calendars of the same kind as our Egyptian ones, recording the entrance of planets into signs of the ecliptic, exist in cuneiform texts since $200 \mathrm{BC}{ }^{13}$ ). But the definite proof of the Babylonian origin of the Egyptian tables is given by considering the motion of Mars in text $S$.

In this case too we have a "linear part" of the motion and an "irregular part", where it is slower or even retrograde. In the linear part the date differences, that is, the times required for traversing the signs of the ecliptic, are as follows (see following page):

It is seen at once, that the following differences occur most frequently:

| in | $m$ | $n$ | 48 |
| :--- | :--- | :--- | :--- |
| in | $m$ | $\pi$ | 41,42 and 43 |
| in | $\measuredangle$ | $m$ | 38 |
| in | $x$ | $\Upsilon$ | 41 |
| in | $\succ$ | $I I$ | 46 |
| in | $\ddots$ | $\Omega$ | 54 |

[^4]Date differences for Mars in different Signs
( $\sim$ means retrograde motion)

| Text and year |  | 物 | $m$ A |  | $\pm$ |  | $\boldsymbol{r}$ | ૪ | I |  | Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vespasian year | 4/5 |  | $41 \quad 41$ |  | 34 | 39 | 38 | 44 | 46 |  | 55 |
|  | 6 | $\sim 48$ | 4141 |  | 38 | 40 | 38 | 44 | 41 | 54 | 54 |
| A | 7/8 | 48 | $\sim 43$ |  | 38 | 41 | 41 | 46 | 43 | 48 | 53 |
|  | 9 | 4848 | 43 | $\sim$ |  |  |  |  | 42 |  |  |
| Trajan year | 9 |  | 41 |  | 38 | 40 | 40 | 46 | 42 | 51 | 54 |
|  | 10/11 | 48 | $\sim 43$ | 38 | 38 | 41 | 41 | 46 | 44 | 48 | 51 |
| $\mathrm{C}_{1}$ | 12/13 | 4848 | 4452 |  | $\sim$ | 43 | 43 | 48 | 48 | 53 | 49 |
|  | 14/15 | 4548 | $42 \quad 42$ |  | 39 |  | $\sim$ | 46 | 46 | 54 | 54 |
| $\mathrm{C}_{2}$ | 16/17 | 4344 | 4342 |  | 38 | 41 | 39 |  | $\sim$ | 52 | 52 |
|  | 18/19 | 4645 | 3931 | 48 | 38 | 41 | 41 | 46 |  | $\sim$ | 51 |
| Hadrian year | 1/2 | $464614)$ | $41^{14}{ }^{14} 3$ |  | 37 | 39 | 40 | 46 | 4615) | $60^{15}$ |  |
|  | 3 | $\sim 46$ | 4641 |  | 38 | 40 | 35 | 43 | 4614) |  |  |
| Hadrian year <br> E | $\begin{aligned} & 11 \\ & 12 \end{aligned}$ | 4946 | 4242 | 36 | 38 | 41 |  | 46 | 46 | 53 | 48 |

In the middle of the linear part, near the conjunction, deviations from these normal values occur frequently, just as if after the heliacal setting or rising the dates were corrected in order to restore the exact synodical period, but near the beginning and the end of the linear part, the normal values prevail.

I shall call these normal values $t$, taking in the second case $\left(\prod_{\neq}\right) t=42$ as a mean value.

Now, in a Babylonian table for Mars exactly the same division of the ecliptic into 6 parts of 2 signs each occurs. The rule for calculation of the synodical arc of Mars in AO 6481 is as follows ${ }^{16}$ ):
if the arc begins in $\mathbb{T O}$ or $\underline{n}$ it is $360^{\circ}+40^{\circ}$, but if the arc of $40^{\circ}$ exceeds $30^{\circ}$ 上, then for every degree one half degree is added;
if it begins in $m$ or $\nexists$ the arc is $360^{\circ}+60^{\circ}$, but if it exceeds $30^{\circ} \not \approx$, for every degree one-half degree is added;
if it begins in $\overline{6}$ or $\approx$, the arc is $360^{\circ}+90^{\circ}$, but if it exceeds $30^{\circ} \approx$, for every degree $15^{\prime}$ are subtracted;
if it begins in $\gamma$ or $\Upsilon$, the arc is $360^{\circ}+67 \frac{1}{2}^{\circ}$, but if it exceeds $30^{\circ} \Upsilon$, for every degree $20^{\prime}$ are subtracted;
if it begins in $\succ$ or $\mathbb{I}$, the arc is $360^{\circ}+45^{\circ}$, but if it exceeds $30^{\circ}$ II, for every degree $20^{\prime}$ are subtracted;
if it begins in $\sigma$ or $\Omega$, the arc is $360^{\circ}+30^{\circ}$, but if it exceeds $30^{\circ} \Omega$, for every degree $20^{\prime}$ are added.

[^5]Hence the velocity of Mars is, just as in our Egyptian text, largest in $\zeta$ and $\approx$, smaller in $\mathcal{H}$ and $\Upsilon$, still smaller in $\eta$ and $\not \approx$, much smaller in $\succ$ and $\mathbb{I}$, still smaller in $m$ and $\underline{\varrho}$, and smallest in $\otimes$ and $\Omega$. This qualitative correspondence can even be formulated quantitatively: if $360+u$ is the synodical arc of Mars according to AO 6481, and $t$ as before the time necessary to traverse a sign of the ecliptic in the linear part of the motion according to S , we have the following corresponding values

| Signs | u | $360^{\circ}$ : u | t |
| :---: | :---: | :---: | :---: |
| mp | $40^{\circ}$ | 9 | 48 |
| m | $60^{\circ}$ | 6 | 42 |
| $\chi$ - | $90^{\circ}$ | 4 | 38 |
| $\cdots r$ | $67^{\circ} 30^{\prime}$ | $5 \frac{1}{3}$ | 41 |
| $\bigcirc$ ¢ | $45^{\circ}$ | 8 | 46 |
| (8) $\Omega$ | $30^{\circ}$ | 12 | 54 |

Hence a linear relation

$$
t=2 \cdot \frac{360}{u}+30
$$

holds. This means that the velocity of Mars in the different signs of the ecliptic is given by

$$
\begin{equation*}
\frac{30^{\circ}}{\mathbf{t}}=\frac{30 \mathbf{u}}{720+30 \mathbf{u}}=\frac{\mathbf{u}}{24+\mathbf{u}}\left({ }^{\circ} \text { per }{ }^{\mathrm{d}}\right) . \tag{1}
\end{equation*}
$$

Theoretical deduction of (1).
A relation like (1) could be expected a priori, if the motion of Mars is represented by a scheme like that of the Babylonian procedure texts.

Let us suppose that this scheme was as follows:
In 9 and $\Omega$, where the velocities are smallest, let $\mathrm{v}_{1}$ be the velocity of fast motion (valid in the linear part of the course), $\mathrm{v}_{2}$ that of the following slower motion, - $\mathrm{v}_{3}$ that of retrograde motion, $\mathrm{v}_{4}$ that of the slow motion following it. Let, further, $\tau_{2}, \tau_{3}, \tau_{4}$ be the numbers of days during which the velocities are $\mathrm{v}_{2},-\mathrm{v}_{3}$, and $\mathrm{v}_{4}$.

In $\mathbb{m}$ and $\stackrel{\varrho}{\varrho}$, let all these velocities be multiplied by a factor $c_{1}$, in $m$ and $\nRightarrow$ by $c_{2}$, in $\nearrow$ and $\approx$ by $c_{3}$, in $\mathcal{r}$ and $\gamma$ by $c_{4}$, in $\measuredangle$ and $I f$ by $c_{5}$, the times $\tau_{2}, \tau_{3}, \tau_{4}$ remaining unaltered, just as in the procedure texts for Jupiter and Saturn.

Now what is the time necessary to traverse the two signs $\mathcal{G}$ and $\Omega$, if the planet enters the sign 9 with velocity $\mathrm{v}_{1}$, remaining within these two signs during the times $\tau_{2}, \tau_{3}$ and $\tau_{4}$ (with velocities $\mathrm{v}_{2},-\mathrm{v}_{3}$ and $\mathrm{v}_{4}$ ) and leaving the sign $\Omega$ with velocity $\mathrm{v}_{1}$ again?

During the time $\tau_{2}+\tau_{3}+\tau_{4}$ the travelled distance is $\tau_{2} \mathbf{v}_{2}-\tau_{3} \mathbf{v}_{3}+\tau_{4} \mathbf{v}_{4}$.

The remaining distance $60^{\circ}-\tau_{2} \mathrm{v}_{2}+\tau_{3} \mathrm{v}_{3}-\tau_{4} \mathrm{v}_{4}$ is traversed with velocity $\mathrm{v}_{1}$, hence the time is

$$
\begin{aligned}
& \tau_{2}+\tau_{3}+\tau_{4}+ \frac{60^{\circ}-\tau_{2} v_{2}+\tau_{3} v_{3}-\tau_{4} v_{4}}{v_{1}} \\
&= \\
& \frac{60}{v_{1}}+\tau_{2}\left(1-\frac{v_{2}}{v_{1}}\right)+\tau_{3}\left(1+\frac{v_{3}}{v_{1}}\right)+\tau_{4}\left(1-\frac{v_{4}}{v_{1}}\right)=\frac{60}{v_{1}}+\tau
\end{aligned}
$$

where $\tau$ depends only of $\tau_{2}, \tau_{3}, \tau_{4}$ and the ratios of the velocities. $\tau$ is the delay caused by retardation and retrogradation, for without this delay the time would be $\frac{60}{v_{1}}$.

Now $\tau$ remains the same if the velocities $\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \mathrm{v}_{4}$ are multiplied by a common factor $c_{i}$. Hence the delay $\tau$ is independent of the assumed signs $9 \Omega$. Thus we find for the time necessary to traverse any pair of signs like $\mathbb{T}$ N

$$
\frac{60}{c_{1} v_{1}}+\tau
$$

the factor $c_{i}$ depending on the chosen signs, but $\tau$ being always the same.
By a slight modification of the argument one can even see that the delay $\tau$ remains the same if the retrograde arc belongs to two adjacent signs having different velocity factors $c_{i}$. The times $t_{2}, t_{3}$, and $t_{4}$ must in this case be split up into terms $\mathrm{t}_{2}^{\prime}+\mathrm{t}_{2}^{\prime \prime}$, etc., belonging to these signs, but the result remains the same. In such a case, the time necessary to traverse e.g. the four signs $9 \Omega \mathbb{M}$ will be

$$
\frac{60}{v_{1}}+\frac{60}{c_{1} v_{1}}+\tau
$$

where $\tau$ is again the delay due to retardation and retrogradation:

$$
\tau=\tau_{2}\left(1-\frac{v_{2}}{v_{1}}\right)+\tau_{3}\left(1+\frac{v_{3}}{v_{1}}\right)+\tau_{4}\left(1-\frac{v_{4}}{v_{1}}\right)
$$

Now what is the time necessary to traverse the whole ecliptic, beginning with normal velocity $\mathrm{v}_{1}$ or $\mathrm{c}_{\mathrm{i}} \mathrm{v}_{1}$, next proceeding slower, going back, then proceeding forward slowly and fast again? If we call this time $T$, we have obviously

$$
T=\frac{60}{v_{1}}+\frac{60}{c_{1} y_{1}}+\frac{60}{c_{2} v_{1}}+\frac{60}{c_{3} v_{1}}+\frac{60}{c_{4} v_{1}}+\frac{60}{c_{5} v_{1}}+\tau
$$

Thus it is seen that $T$ is independent of the sign or signs in which retardation or retrogradation takes place ${ }^{17}$ ).

We do not know the exact value of $T$, because we ignore the exact

[^6]Babylonian law of motion. In our text S , the time T necessary to traverse the whole ecliptic varies between $698^{d}$ and $712^{\text {d }}$, the mean value being about $707^{\text {d }}$. The deviations from this mean value are probably due to casual corrections of the dates, just as the deviations from the normal values of $t$ we previously observed. It turns out that the best agreement is reached by adopting the value

$$
\mathrm{T}=1 \frac{14}{15} \text { years }=706^{\mathrm{d}}
$$

Now let v be the sun's velocity. In one synodic period Mars covers a distance $360+\mathbf{u}$, and the sun a distance $720+\mathbf{u}$, hence the synodic period is

$$
\frac{720+u}{v}
$$

But the time necessary for Mars to proceed $360^{\circ}$, including the retardated and retrograde part of the motion, is T , hence the time necessary to cover the remaining distance $u$ in the linear part of the motion is

$$
\frac{720+\mathfrak{u}}{v}-T .
$$

So the velocity $\frac{30}{t}$ is

$$
\begin{equation*}
\frac{30}{t}=\frac{u}{\frac{\mathbf{7 2 0}+\mathrm{u}}{\mathbf{v}}-\mathrm{T}}=\frac{\mathbf{u}}{720-\mathrm{Tv}+\mathbf{u}} \cdot \mathbf{v} . \tag{2}
\end{equation*}
$$

Now Tv is the arc which the sun covers in $\mathrm{T}=\frac{29}{15}$ years, vid

$$
\mathrm{Tv}=\frac{29}{15} \cdot 360^{\circ}=720^{\circ}-24^{\circ}
$$

hence (2) yields

$$
\frac{30}{\mathrm{t}}=\frac{\mathrm{u}}{24+\mathrm{u}} \mathrm{v}
$$

which is nearly the same thing as (1), since $v$ is nearly 1 ( ${ }^{\circ}$ per ${ }^{d}$ ).
The exactitude of our determination of the constants in (1) is of course not sufficient to decide which exact values of T and v were adopted. But in any case the form of the relation (1) is just what might be expected from Babylonian planetary theory, and also the order of magnitude of the constants involved agrees with expectation.

## Jupiter.

Kugler has analyzed 3 types of Babylonian Jupiter tables. In the tables of the first kind the ecliptic is divided into 2 parts: from $30^{\circ} \mathrm{m}$ to $25^{\circ}$ II the synodic arc of Jupiter is $36^{\circ}$, and from $25^{\circ}$ II to $30^{\circ} \mathrm{m}$ it is $30^{\circ}$. In the tables of the second kind the ecliptic is divided into 4 parts: from $9^{\circ}$ g to $9^{\circ} \mathrm{m}$ the synodic are is $30^{\circ}$, from $9^{\circ} \mathrm{m}$ to $2^{\circ}$ 万 and from 17 ช to $9^{\circ}$ g it is $33^{\circ} 45^{\prime}$, and from $2^{\circ} \measuredangle$ to 17 ૪ it is $36^{\circ}$, just as in the
procedure text Rm IV 431. The time intervals necessary to cover these distances of $30^{\circ}, 33^{\circ} 45^{\prime}$ and $36^{\circ}$ are nearly $13^{\mathrm{M}} 12^{\mathrm{D}}, 13^{\mathrm{M}} 16^{\mathrm{D}}$, and 13M18D (see Eudemus I, p. 34).


Diagram 1.


If our Egyptian tables are composed according to the same scheme, we should expect that in the "slow part" ( $\Omega \mathbb{M P}$ ㅇ, perhaps also 9 and $\mathbb{M}$ ) the time necessary to traverse a sign of the ecliptic $\left(30^{\circ}\right)$ should be
$13^{\mathrm{M}} 12^{\mathrm{D}}=395^{\mathrm{d}}$, and the retrograde motion too should be reproduced exactly in the next sign after that lapse of time.

This expectation is fulfilled in both tables $P$ and $S$, for the dates of entrance into the signs and the time intervals to the next similar entrance are:

| Text P | forward | retrograde | forward | Time intervals |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Augustus year | $\begin{array}{ccc} 2 & 20 & m p \\ 3 & 20 & \underline{\underline{n}} \\ 4 & 22 & m \\ {[5]} & 23 & \pi \end{array}$ | $\begin{array}{ccc} 8 & 5 & \Omega \\ {[9]} & 9 & \underline{2} \\ 10 & 12 & \underline{\underline{\omega}} \end{array}$ |  | $\begin{gathered} 395 \mathrm{~d} \\ 397 \\ {[396]} \end{gathered}$ | $\begin{aligned} & \text { 399d } \\ & 398 \end{aligned}$ | $\begin{aligned} & {[394]} \\ & {[394]} \end{aligned}$ |
| year $״$ " $"$ " |  |  |  | $\begin{aligned} & 393 \\ & 400 \\ & 400 \\ & 400 \\ & 404 \end{aligned}$ |  |  |
| year | $\begin{array}{rrr}1[1] & 14 & 6 \\ 12 & 12 & \Omega \\ 1 & 12 & \mathrm{~m}\end{array}$ |  |  | $\begin{aligned} & 393 \\ & 400 \end{aligned}$ |  |  |
| Text S | forward | retrograde | forward |  | e inte | vals |
| $\begin{array}{lc}\text { Vespasian year } \\ \text { A } \\ & \prime \prime\end{array}$ | $\begin{array}{lrr}4 & 6 & \mathrm{~m} \\ 4 & 21 & \text { 不 }\end{array}$ | $9 \quad 16$ ํํ | $\begin{array}{lll}11 & 24 & \underline{\text { m }} \\ 12 & 16 & \mathrm{~m}\end{array}$ | 380 |  | 387 |
| $\begin{array}{lc}\text { Trajan } & \text { year } \\ & \text { " } \\ C_{1} & " \\ & "\end{array}$ | $\begin{array}{rrr}12 & 28 & m \\ 1 & 22 & \underline{\sim} \\ 2 & 21 & \text { m } \\ 3 & 18 & \star\end{array}$ |  |  | $\begin{aligned} & 395 \\ & 394 \\ & 392 \end{aligned}$ |  |  |
| Trajan year <br> Hadrian $"$ <br> $C_{2}$ $"$ <br>  $"$ | $\begin{array}{ccc} 10 & 15 & Q \\ 11 & 13 & \Omega \\ 11^{19} & 16 & \mathrm{mp} \\ \text { no } & \text { entrance } \end{array}$ |  |  |  | correc | $\text { d } 398$ |
| Hadrian year $\mathbf{E}$ | $\left[\begin{array}{ccc} 10 & 3 & \ddots \\ {[11]} & 1 & \Omega \\ 12 & 4 & \mathrm{~m} \\ 1 & 1 & \underline{\varrho} \\ 2 & 2 & \mathrm{~m} \\ 6 & 29 & \approx \end{array}\right.$ |  |  | $\begin{aligned} & 393 \\ & 398 \\ & 393 \\ & 396 \\ & 147 \end{aligned}$ |  |  |

I cannot explain why several time intervals in P are about $5^{\mathrm{d}}$ too large. Those in $S$ are quite correct: the deviations do not exceed $4^{d}$, except twice in A. Even in the signs 9 and $\eta$ the correct intervals appear; only in Hadrian year 17 the dates are irregular. In all other cases the signs 9 and
${ }^{18}$ ) Restored by interpolation.
$\left.{ }^{19}\right)$ Presumably an error: should be 12.
$m$ are obviously included in the region of slow motion, just as in the Jupiter tables of the first kind.

Next we seek to determine the law of motion in the "fast past" of the ecliptic, consisting at least of the signs $\varnothing \approx \mathcal{N})(\succ$. Assuming a synodic arc of $36^{\circ}$ and a synodic period of $18^{M} 18^{D}=401^{\text {d }}$ in this region, one can reduce all positions of Jupiter to one synodic period, just as it was done in the case of Venus. I have chosen the year 15 Trajan and reduced all other positions of C in the fast region to it by adding or subtracting multiples of $36^{\circ}$ to the positions and of $401^{\text {d }}$ to the dates. From the positions of E in the fast region I first subtracted $5^{\circ}$ and from the data 12 years $5^{\mathrm{d}}$ according to Babylonian planetary theory, and likewise I added $15^{\circ}$ to the positions of A and 36 years $15^{\text {d }}$ to the data; after this operation these positions too could be reduced to the year 15 Trajan. The result is shown in Diagram 2. The law of motion in the irregular and especially in the retrograde part of the motion is uncertain, but in the linear part a straight line fits very well. As is seen from the diagram, the velocity during 3 months in the neighbourhood of the conjunction is

$$
27^{\circ} \text { in } 90^{\mathrm{D}} \text {, i.e. } 18^{\prime} \text { per } 1^{\mathrm{D}} .
$$

This is larger than the velocity of ${15^{\prime}}^{\prime}$ per $1^{\mathrm{D}}$ assumed in Rm IV 431 (see Introduction), but the time of $90^{\mathrm{D}}$ (probably again $1^{\mathrm{M}}$ before heliacal setting, $1^{\mathrm{M}}$ invisible and $1^{\mathrm{M}}$ after rising) is quite the same. The value of $15^{\prime}$ is better than $18^{\prime}$, just as the division of the ecliptic into 4 parts assumed in Rm IV 431 is better than the division into 2 parts used by the more ancient kind of tables. Hence it seems that the Babylonian source of S follows a more primitive system than that of Rm IV 431. Also the retrograde arc seems to be larger (perhaps $12^{\circ}$ in the "fast part", $10^{\circ}$ in the "slow part" of the ecliptic).


[^0]:    ${ }^{1}$ ) A thorough discussion of these questions is given by O. NeugebauER: Egyptian planetary texts, Trans. Amer. Philos. Soc. 32 (1942) p. 235.
    ${ }^{2}$ ) Published by Kenyon, Greek Papyri in the British Museum, Cat. I (1893) p. 133.
    ${ }^{3}$ ) O. Neugebauer and A. Volten, Ein demotischer astronomischer Papyrus (Pap. Carlsberg 9). Quellen u. Studien Gesch. Math. B 4 (1938) p. 383.
    $\left.{ }^{4}\right)$ O. Lange and O. Neugebauer, Papyrus Carlsberg No. 1, ein hieratisch demotischer kosmologischer Text, Kgl. Danske Vid. Selsk. Hist.-fil. Skrifter 1, no. 2 (1940).
    $\left.{ }^{5}\right)$ O. Neugebauer, Eg. Plan. Texts, Trans. Amer. Philos. Soc. 32 (1942), p. 209.
    $\left.{ }^{6}\right)$ W. Spiegelberg, Demotische Papyrus aus Berlin, Leipzig 1902.

[^1]:    ${ }^{7}$ ) H. Stobart, Egyptian Antiquities, Paris and Berlin 1855. - H. Burgsch, Nouvelles Recherches sur la division de l'année, suivies d'une mémoire sur des observations planétaires, Berlin (F. Schneider) and Paris (P. Duprat) 1856.
    ${ }^{8}$ ) B. P. Greenfell, A. S. Hunt and E. J. Goodspeed, The Teptunis Papyri II, Univ. of California Publ. 2, London 1907.
    ${ }^{9}$ ) Kugler, Sternkunde und Sterndienst in Babel II, p. 520. - B. L. v. D. Waerden, Babylonische Planetenrechnung, Eudemus I (1941) p. 47. I found for earlier texts ( $210-160 \mathrm{BC}$ ) a difference of about $3^{\circ} 30^{\prime}$, for $160-130 \mathrm{BC}$ about $4^{\circ} 10^{\prime}$, and for $110-60 \mathrm{BC}$ exactly $5^{\circ} 20^{\prime}$, compared with the ecliptic of -100 .
    ${ }^{10}$ ) Ptolemaios, Syntaxis IX 2 (p. 211 Heiberg).

[^2]:    $\left.{ }^{11}\right)$ See Kugler, Sternkunde I, p. 136, and V. D. WaERDEN, Eudemus I, p. 35. KUGLER calls the procedure texts "Lehrtexte".

[^3]:    12) Kugler, Sternkunde und Sternendienst I, p. 205.
[^4]:    ${ }^{13}$ ) See Kugler, Sternkunde und Sternendienst I, p. 92-107, II 470-513 and 3. Ergänzungsheft p. 358-374. In the planetary tables for the years 178,234 and 236 of the Seleucid era (Kugler II, p. 496, 490 and 474) the velocity of Venus in the linear part of the motion is $30^{\circ}$ in 25 D , just as in our Egyptian tables.

[^5]:    14) Following Neugebauer, I have corrected $11 \mathrm{~m}\left(\mathrm{C}_{2}\right.$, year 1) into 130 , and 12309 (year 3) into 121.
    $\left.{ }^{15}\right)$ I have corrected 1214 g6 ( $\mathrm{C}_{2}$, year 1) into 1224.
    $\left.{ }^{16}\right)$ Kugler, Sternkunde und Sternendienst in Babel II, p. 580.
[^6]:    ${ }^{17}$ ) The time $T$ is not identical with the orbital period, which is the time in which Mars in mean motion would traverse the ecliptic.

