sibility during the first 3 cleavages the eggs show an alternation of phases of increased and lessened sensibility corresponding to definite phases of cleavage.

5. It is evident that the pattern of determination of the organs of the head has not yet been laid down irrevocably at the 24-cell stage in *Limnaea*.

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Physics. — *Recovery and recrystallization viewed as processes of dissolution and movement of dislocations.* II. By W. G. BURGERS. (Laboratorium voor Physische Scheikunde der Technische Hoogeschool, Delft.) (Communicated by Prof. J. M. BURGERS.)

(Communicated at the meeting of March 29, 1947.)

II *Block-structure of the crystalline state.*

Starting from the assumption [LENNARD JONES (13)] that an "ideal" lattice represents the condition of minimal free energy for a crystal, we must conclude that even in the case of a pure element or compound, every crystalline testpiece, independently of its being uni- or polycrystalline, undeformed or coldworked, represents a "metastable" state of thermodynamical equilibrium. The structural differences between these states are merely gradual and not essential. In what follows we shall consider this point somewhat more in detail.

II, 1. *Single crystal.*

According to numerous observations, every "real" crystal, apart from "macroscopic" irregularities ["lineage structure" of BUERGER (14) [5])], has a certain "mosaic" structure, consisting of ideally regular lattice blocks [or lamellae, according to GRAF (16)] with dimensions of the order of magnitude of 0.1—1 micron, the blocks, however, including angles varying from perhaps seconds to minutes of arc. Their presence follows partly from measurements of the intensity of diffracted X-rays [DARWIN (17); EWALD and RENNINGER (18); DEHLINGER and GISEN (19)], partly from microscopic observations of the natural or etched surface of crystals [see in particular GRAF (16)]. Also the "structure-sensitive" character of many physical and mechanical properties [SMEKAL (20)] leads to the same conclusion. Finally the often considerable influence of minute quantities of foreign atoms on the properties of pure metals seems to find a plausible explanation on the assumption that such atoms are preferably "adsorbed" at the boundaries of the lattice blocks and in some way or other exert here their remarkable influence [BRAGG (21)].

As to the "structure" of the block-boundaries, suggestions have been made by various authors [J. M. BURGERS (22); BRAGG (23)]. It is now generally assumed that the deviations of the atoms from their normal positions in these transition layers, which necessarily must occur with regard to the positions of the atoms in both adjoining blocks, are as small as possible. Fig. 4 shows a schematic picture given by J. M. BURGERS (22): here the "fit" between two blocks which include a small angle α, is brought about by a succession of "edge-dislocations" (TAYLOR-dislocations), *all of the same sign,* lying at equal distances h, determined by $tg\ \alpha = \lambda_0/h$, where

[5]) In this connection recent observations by LACOMBE and BEAUJARD (15) of corrosion patterns on aluminium crystals, prepared by recrystallisation, are of great interest.

λ_0 is the lattice constant. [It may be remembered that, by removing the atoms over a certain area of a lattice plane, as indicated in fig. 3 (which is

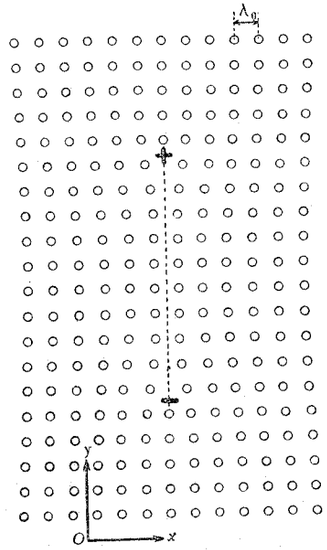

Fig. 3. Lattice region (supposed to extend to infinity in the direction of the z-axis, perpendicular to the plane of the drawing) with two edge-dislocations (TAYLOR-dislocations), one positive (above) and one negative (below). The dislocations are obtained by removing the atoms over a certain area of the lattice perpendicular to the x-axis. The dislocation lines extend at + and —, parallel to the z-axis (after J. M. BURGERS (22)).

supposed to extend to infinity in a direction perpendicular to the plane of the drawing), two edge- or TAYLOR-dislocations are created, one of which (indicated by +) is called positive, the other (—) negative] [6].

It is, of course, a most important question whether for a definite substance, a pure metal for example, its crystals are always characterized by a definite mosaic structure. This question is closely related to the not yet solved problem of crystal growth. On the one hand it is certain that the degree of imperfection of crystals can be largely influenced by conditions of growth, at least as to their (semi-) macroscopic faults. Moreover it follows from measurements of RENNINGER (18) with rocksalt that also the "size" of the blockstructure, which governs the intensity of reflected X-rays, varies for artificial and natural crystals. ADDINK (18a), from measurements of the specific gravity, concludes that crystals of the alkali-

[6]) In the case, shown in fig. 4, the two blocks are rotated with regard to each other about an axis lying *in* the plane of their common boundary (perpendicular to the plane of the drawing). As is indicated in J. M. BURGERS' paper, the boundary between two blocks, which are rotated with regard to each other about an axis *perpendicular* to this boundary, can be realized by a succession of dislocations of a different type (so-called "screw-dislocations").

chlorides, prepared by melting, are "incomplete" compared with those from a solution. With regard to our subject, it is in this connection of special

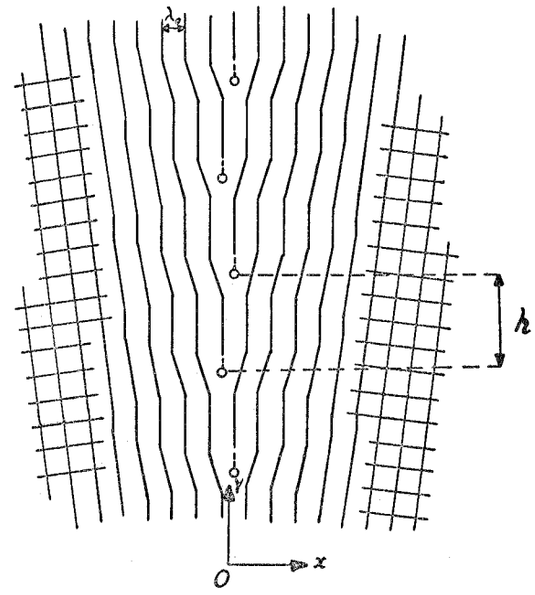

Fig. 4. Schematic picture af a transition surface between two lattice domains, formed by a set of parellel dislocation lines of edge type, all situated in the plane $x = 0$ (after J. M. BURGERS (22)).

interest that measurements by DEHLINGER and GISEN (19) with aluminium have established that crystals formed by recrystallization have a more pronounced mosaic (*viz.* smaller blocks and probably larger angular deviations between them) than crystals obtained from the melt.

On the other hand according to GRAF (16) all crystals, independently of their way of preparation, possess a structure composed of lamellae, the thickness of which varies between narrow limits. In this connection GRAF mentions two papers by HERLINGER (24), according to which at the surface of a growing lattice-block "dislocated" atoms can only be stabilized so long as the block is smaller than a minimum size; whereas for larger blocks the probability of assuming irregular positions increases to such extent that further growth becomes less probable than formation of a new block. The limiting size is estimated as approximately 0.1 micron. A similar process of this nature might lead to a block- or lamellar structure with a size of blocks (lamellae), determined by the lattice forces and thus characteristic for a given lattice. According to M. BORN (24a) a perfect single crystal, owing to the asymmetry of the vibrations of atoms, could not be larger than about a thousand repeat distances in any direction, since at distances greater than this the vibrating atoms would get completely out of step. The figure of 1000 is obtained as a rough mean value of the reciprocal of the

DEBYE temperature multiplied by the thermal expansion coefficient for many substances [7]).

II, 2. *Polycrystal* (crystal aggregate).

In an annealed polycrystalline test-piece the "block-system" is somewhat more complicated in character. Here, superimposed as it were on the mosaic structure inside the individual crystallites, is a system of intercrystalline boundaries, where much larger angular deviations between the lattices of adjacent blocks have to be bridged. Also these "true" grain boundaries most probably are not "amorphous", but possess a definite structure, dependent on the relative orientation of adjacent grains. This conception finds experimental support in experiments by CHALMERS (28) on the strength of double-crystals of tin; by SNOEK (29) on the intercrystalline oxydation of polycrystalline nickeliron, with- and without preferential orientation of the crystallites; and by LACOMBE and BEAUJARD (30) on the corrosion of high-purity aluminium. In these latter experiments, the chemical reagent ($HNO_3 + HCl + HF$), applied after electropolishing, produced, apart from corrosion figures inside the individual crystallites, boundary lines between neighbouring crystallites which were clearer developed the larger their difference in lattice orientation.

On the basis of such experimental evidence it seems justified to assume [J. M. BURGERS (22); W. L. BRAGG (23); LENNARD JONES (13)] that also

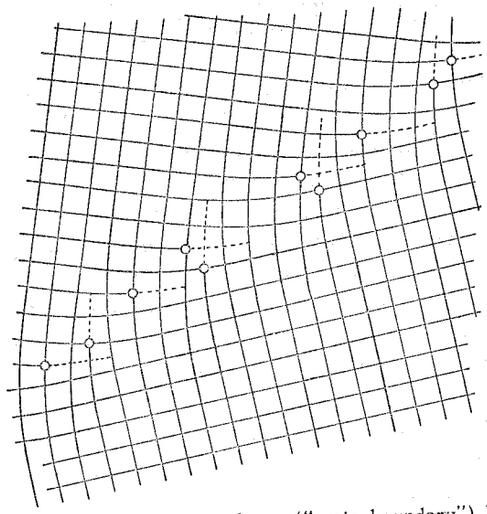

Fig. 5. Schematic picture of a transition layer ("grain boundary") between two lattice blocks, including a large angle. The layer is built up of *two* sets of dislocation lines of the edge type (compare fig. 4) (after J. M. BURGERS).

[7]) As is well known, ZWICKY (25) has postulated the existence of a stable mosaic-structure on energetic grounds. Later criticism of his conceptions by OROWAN (26) and BUERGER (27) have shown that his conclusions cannot be maintained.

the grain boundary constitutes the best possible fit between adjacent grains, affecting a minimal number of lattice planes in the transition region. As pointed out in (22), a transition surface between lattice blocks including an arbitrary angle might be built up in an analogous way, as is shown in fig. 4, now, however, introducing a double set of dislocation lines [8]). Fig. 5, for which I am indebted to J. M. BURGERS, shows this schematically.

The conception of a grain boundary, built up of dislocations, is furthermore illustrated in a very striking way by BRAGG's experiments with soap bubbles (31), as fig. 6 may show.

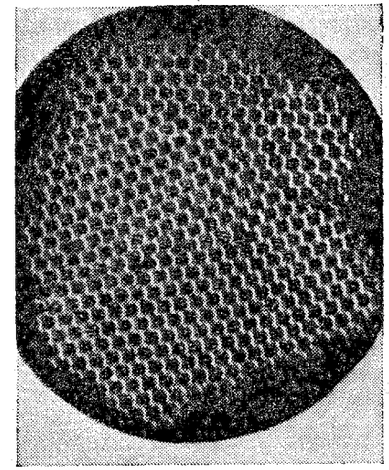

Fig. 6. "Intercrystalline boundary", as shown by floating soap bubbles (after W. L. BRAGG (31)).

II, 3. *Cold-worked state.*

The experimental fact, first observed by VAN ARKEL (32) and by DAVEY (33), that the DEBYE–SCHERRER pattern of a cold-worked metal differs from that of the annealed state at most by a slight broadening or a decreased intensity of the interference lines [9]), is proof that even here by far the major part of the atoms form still coherent lattice regions (lattice blocks). So once more, according to the current view [DEHLINGER (3); KORNFELD (2); BRAGG (21); (1), §§ 52—54] the resulting structure, even in the most severely worked condition, is conceived as consisting of lattice-blocks, held together by systems of dislocated atoms. The main difference between the cold-worked and the annealed state consists in the circumstance that cold-working, as a consequence of alternation and mutual hindrance of

[8]) As remarked in (22), in this case a certain relation must be fulfilled between the spacings of the two sets and the angle of inclination of the transition plane, in order that at great distances from this plane the lattices shall be free from stress.

[9]) We do not consider here the changes due to the formation of preferential orientations of the crystallites.

various active glide-combinations, has caused a slight elastic stressing and bending of the blocks, as is evident from the above-mentioned changes in line-width and intensity of the DEBYE–SCHERRER lines [10]. Simultaneously the essential characteristic feature of the annealed polycrystalline state, namely the presence of *groups* of nearly parallel lattice blocks *inside* the original grains, which, taken together, include much larger angles with adjoining groups (grains), is more or less effaced.

As WOOD (35) pointed out, an estimate of the minimum size of the blocks may be obtained by assuming that the broadening of the DEBYE–SCHERRER lines is wholly due to their *smallness*. In this way sizes of the order of 0.1 μ are calculated for most metals (in aluminium, where the broadening is very slight, this figure comes out as approx. 1 μ). DEHLINGER and KOCHENDÖRFER (36), taking into account that part of the line-broadening is due to the stressed state of the blocks [VAN ARKEL and BURGERS (37); MEGAW, LIPSON and STOKES (38)], give the size of mosaic-blocks in rolled copper as 0.4 μ.

In fig. 7 we give a schematic picture of the deformed state, in which the

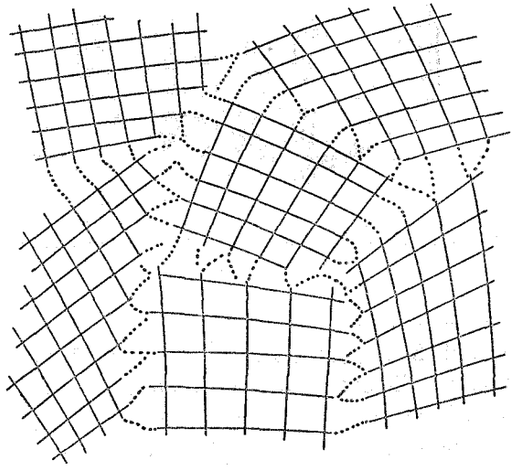

Fig. 7. Schematic representation of the cold-worked state: elastically stressed and bent "mosaic-blocks" connected by transition layers of dislocated atoms (taken from (1)).

stressed and bent state of the blocks is largely exaggerated. The transition layers have only been indicated by dots. We may consider them as built up of a complicated system of various types of dislocations. To give a more

[10]) The exceptional case of aluminium, where line-broadening after cold-work is practically absent or in any case much less pronounced then with most other metals, might be understood if an alternation of the active glide-planes were more easily brought about in this metal than in the others [DEHLINGER (3) p. 755; see also references in (1), § 41, § 47]. Experiments with sheared aluminium crystals support this view [BURGERS and LEBBINK (34)].

detailed picture is, considering all the different possibilities [see for example the cases treated in (22)], not well possible.

II, 4. *"Twodimensional" block-structure.*

For the case however, that we limit ourselves to a "twodimensional" aggregate [11]) and take into account only the presence of dislocations of the edge type, as shown in fig. 4, we can say for certain that the "structure" of the layer is essentially dependent on the angle of misfit between adjoining blocks: there, where large angles have to be bridged, a concentration of dislocations of the same sign (see fig. 3) is unavoidable (fig. 4 and 5). The "density" of the concentration of either positive or negative dislocations increases generally speaking with this angle [12]).

It must not be left out of sight that also lattice blocks which are in parallel or nearly parallel positions can be separated by systems of dislocations; when this is the case these systems must contain an equal (or nearly equal) number of dislocations of both signs. These may alternate either "individually" or in groups. Some arrangements, which can be envisaged, have been indicated in a purely schematic way in fig. 8 [13]).

The conception of the cold-worked state as built up of elastically stressed lattice-blocks, connected by transition layers of dislocated atoms, is, for the "two-dimensional" structure considered here, to some extent confirmed by the amount of energy taken up in the course of the deformation process. As pointed out in (1) (§§ 55, 56), and by BRAGG (40) and others, the elastic energy in the blocks is only a small fraction (of the order of 1 %) of the total energy of cold-work. This latter is, according to the measurements of TAYLOR and QUINNEY (41) for copper maximally about 10 calories ($\sim 4.10^8$ erg) per cm^3. If we put the energy of a dislocation line (as defined in fig. 3) at about 10^8 electron volt ($\sim 10^{-4}$ erg) per cm [SEITZ and READ (42); KOEHLER (43)], then the number of dislocation-lines per cm^2 is of the order 10^{12}. If all these were arranged along lines 0.2 micron (2.10^{-5} cm) apart (that is, at the sides of mosaic blocks of this width), this number would amount to 2.10^7 dislocation lines per cm, or about 1 dislocation line per 5A^0: this comes near to one dislocation line per every two and a half atomic plane. This is, as BRAGG (40) remarks, equivalent to

[11]) "Two-dimensional" is here taken in this sense that the pattern extends without change of structure in a direction perpendicular to the plane considered in the drawings.

[12]) At least up to a certain extent: in the cubic system for example, due to symmetry relations, the angle $45° - \alpha$ may be equivalent to $45° + \alpha$.

[13]) One might, of course, introduce the assumption of dislocations *inside* the blocks: in a stressed block, which, taken as a whole, is *not* bent, we could have an equal number of positive and negative dislocations; and an excess of either positive or negative in an elastically bent part. In how far this is justified depends to some extent on what one considers as a "block" [compare our fig. 11; also KOCHENDÖRFER (39) pp. 119 ff.]. In any case we do not think this additional assumption could affect our reasoning in the following paragraphs in an essential way.

an interface between two-dimensional closed packed arrangements, meeting at 30°, which is the largest angle at which they can meet (since 60° is

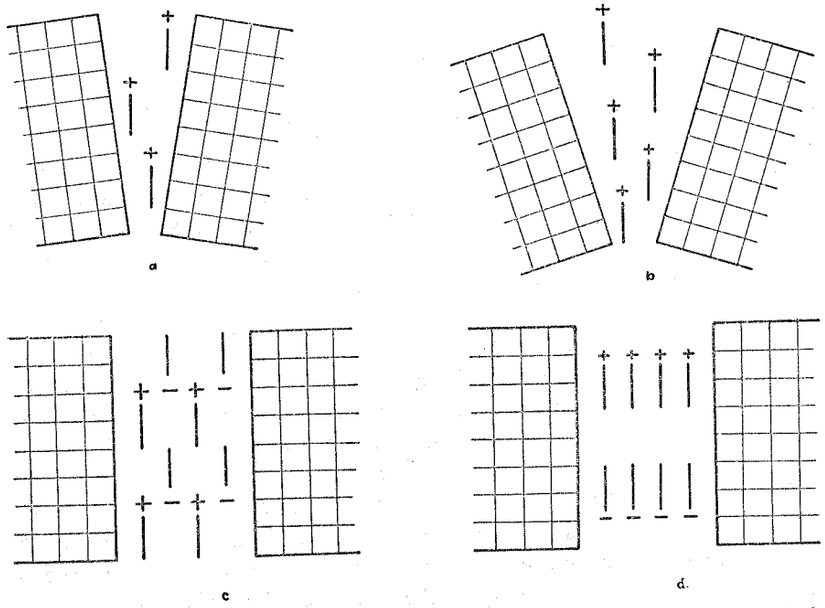

Fig. 8. Schematic representation of transition layers between neighbouring lattice blocks, built up of systems of dislocations of edge-type (TAYLOR-dislocations). Here and in figures 10 and 11 such dislocations have been indicated by $\overset{+}{|}$ (positive dislocation) and $\underset{-}{|}$ (negative dislocation): the | represents the "direction" of the layer of atoms removed from the lattice (compare fig. 3). The direction of "easy mobility" of the dislocations lies in the plane of the drawing perpendicular to the direction of the removed layer.
a, b: adjoining blocks non-parallel: surplus of dislocations of same sign.
c. d: adjoining blocks parallel: equal numbers of + and — dislocations.

equivalent to 0° owing to symmetry). In this simplified picture, this actually represents the maximum of crowding of dislocations, in agreement with the maximal degree of cold-work of the metal [14]).

II, 5. *Stability of the block structure.*

In the preceding considerations we have left aside the question in how far a deformed block-structure of the type we have in mind will be stable (or, rather, metastable). With regard to this problem only a few indications can be found in the current literature. It may be useful to recapitulate the basic conceptions which should be kept in mind in discussing this subject. The cohesive forces in the lattice are derived from the electric and other short range forces which neighbouring atoms exert upon each other.

[14]) In (40), this conclusion is reached in a somewhat different way.

Leaving aside the normal thermal vibration, every atom is in equilibrium under the forces acting upon it; in other words the forces upon every atom must have zero resultant. In a perfectly regular lattice the system of forces exhibits exact periodicity. Hence when we imagine a cut to be made in the lattice by an element of surface extending over a whole number of periods, the forces exerted by the atoms lying on one side of this element upon the atoms lying on the other side, will be zero, provided the system is not acted upon by exterior forces. In that case we say that the lattice is unstressed. Every misfit, however, produces some irregularity, which will have its influence both upon the positions of neighbouring atoms and upon the forces experienced by them. This influence theoretically can extend over infinite distances, although in most cases the presence of irregularities of more or less opposite character has a compensating effect, which materially reduces the action radius of a single irregularity. When we now again imagine a cut to be made in the lattice by means of a small element of surface, extending over a whole number of periods, the forces exerted by the atoms lying on one side of the element upon the atoms lying on the other side, may differ from zero, so that we can say that there are stresses in the lattice.

When the lattice irregularities are of a well defined character, as is the case with dislocations, we may consider those dislocations as the seats of forces, producing the observed stresses in the lattice. Some authors, like KORNFELD (2), therefore use the expression that a system of elastically stressed blocks is kept in equilibrium by forces which have their seat in the transition layers (where the dislocations are to be found). In the same connection BRAGG (21) speaks of a "dynamic stability by the boundaries" and compares the cold-worked state to a foamstructure [compare also BENEDICKS (44)].

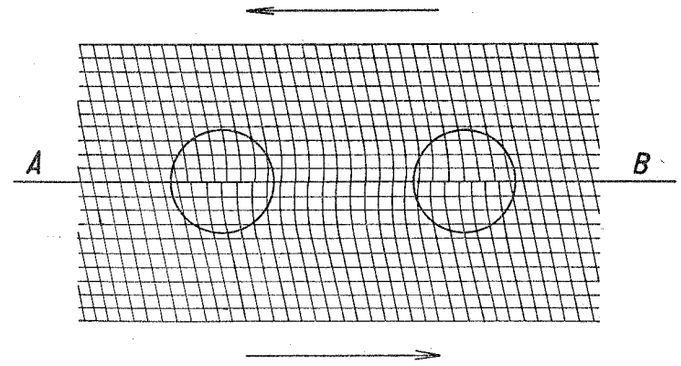

Fig. 9. Pair of dislocations, one positive (at the left) and one negative (at the right), formed as the result of a local glide jump (after OROWAN (45). In the "abridged" way of representation, described in fig. 8, such a pair of dislocations ("Verhakung" as called by DEHLINGER) is indicated with $\overset{+}{|}\,\underset{-}{|}$.

Now the concept of stability enters into the picture when we see the possibility that a number of atoms in the neighbourhood of a dislocation may be able to find more than one position of equilibrium without being obliged to pass through a high energy barrier. In such a case a shift from one position of equilibrium to a neighbouring one may initiate a displacement of the irregularity, and in particular the displacement of a dislocation line. As soon as such possibilities are taken in view a degree of stability can be defined, depending upon the energy which must be communicated to certain atoms in order to initiate a change in the configuration of the system of dislocations.

DEHLINGER (3) has been the first to attack this question in a quantitative way. His considerations are limited to a series of "Verhakungen", a "Verhakung" being approximately equivalent to the combination of a positive and a negative TAYLOR-dislocation, as can be formed in a perfect lattice by a so-called glide jump, in the way indicated in figure 9, due to OROWAN (45).

In what follows we shall schematize a TAYLOR-dislocation by means of a plus or minus sign, accompanied by a vertical dash,

$$+\ \text{or}\ \vert\ -$$

the dash indicating the "removed" lattice plane (compare fig. 3). As is well known, the direction of easy mobility of such a dislocation lies perpendicular to the removed lattice plane, thus in the case indicated in the horizontal direction, in the plane of the paper. A "Verhakung" will be indicated by $+\ \vert\ -$.

According to DEHLINGER's calculation [15]) an isolated "Verhakung" is unstable: there is a strong tendency for the occurrence of such shifts that the two dislocations of opposite sign move towards each other and mutually "neutralize" as soon as they have come together. Following TAYLOR (46) we may consider this as a consequence of a certain "attraction" between dislocations of opposite sign. Now the important feature pointed out by DEHLINGER is that in the case of a large number of "Verhakungen" lying in a single row [16]), the system will become stable when the density (that is,

$$+\ \vert\ -\qquad +\ \vert\ -\qquad +\ \vert\ -\qquad +\ \vert\ -$$

the number of "Verhakungen" per unit length) surpasses a certain value. By stability is meant that a positive threshold energy must be introduced

15) I am indebted to Mr. F. R. N. NABARRO (Bristol) for a closer elucidation of DEHLINGER's conclusions.

16) Such series may be created if a large number of glide jumps of the type shown in fig. 9 all took place at different points along the same glide plane [cf. KOCHENDÖRFER (39), loc. cit.].

into the system, before dislocations can be dissolved, whereas in the case of a single "Verhakung" (one pair of dislocations only) the thermal energy of the atoms is sufficient to bring about mutual approach and neutralization of the two dislocations.

The "critical" density of the series depends upon the elastic constants of the metal; for a common metal as copper or silver DEHLINGER estimates it to be reached for one "Verhakung" per every 5 or 10 atoms. For larger densities the threshold energy ("activation energy for dissolution") increases, which means that the system of dislocations becomes more difficult to dissolve and thus is more stable.

Wo do not know of similar calculations for more complicated sets of dislocations. For example the question has not been considered whether an arrangement as indicated in fig. 10*a*, where every horizontal row carries an alternating system of positive and negative dislocations, which thus could neutralize each other, is stable in DEHLINGER's sense.

In the calculations referred to it has been assumed that the lattice is not affected by exterior forces. We can, however, also put the question of stability for a lattice which is stressed by the application of exterior forces. Such cases have been treated by TAYLOR (46), who has considered the stability of certain two-dimensional arrangements, likewise containing an equal number of positive and negative dislocations of edge-type, in the

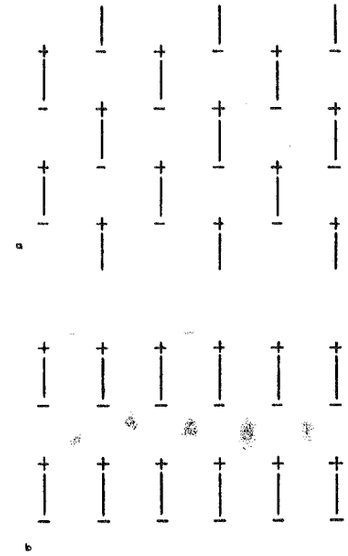

Fig. 10. Two-dimensional arrangements of alternating positive and negative TAYLOR-dislocations. The direction of "easy mobility" ("direction of slip") is supposed to run horizontally in the figure; the dislocations can be formed by slip parallel to this direction.
a. positive and negative dislocations alternate along the same "direction of slip".
b. positive and negative dislocations lie on alternating "directions of slip".

case when an exterior shearing stress acts in a direction parallel to the direction of easy mobility. An example of such an arrangement is shown in fig. 10*b*, which differs from that depicted in fig. 10*a*, by the fact that in fig. 10*b* dislocations of opposite sign lie on alternating rows. TAYLOR has shown that in the absence of exterior stress the system of fig. 10*b* is stable. If, however, an external shear stress acts parallel to the glide plane, a displacement of *all* the positive with regard to *all* the negative dislocations is brought about. Starting from the arrangement of fig. 10*b*, every value of the shear stress which lies below a certain maximum value, gives a new stable equilibrium with a definite displacement of the positive with regard to the negative dislocations. As soon as the stress surpasses a certain value, the two sets of opposite sign migrate steadily in opposite directions (until they are stopped by some obstacle, for example a transition layer between two mosaic blocks). The magnitude of this critical stress increases with the "density" of the dislocation lines per unit surface.

Finally BRAGG (23) has considered the *mobility of a series of dislocation lines of the same sign* which form the boundary between two mosaic blocks as shown in fig. 4 (also in fig. 8*a* and 8*b*). Such a series can be moved by a relatively small force acting in a direction perpendicular to itself (thus horizontal in the figure), which is the direction of "easy-mobility" of each dislocation separately. In this process the individual atoms in the transition layer suffer only slight displacements. The resultant effect is equivalent to the growth of one block at the cost of the other, the disappearing block suffering as it where a rotation about an axis perpendicular to the plane of the drawing, so that its orientation gradually coincides with that of the growing blocks [17].

(To be continued.)

[17] In (13) LENNARD JONES considers the movement of two adjoining lattice blocks, arranged in a similar way as shown in fig. 4, in a direction *parallel* to the transition plane (vertical direction in the drawing).

Mathematics. — *Eine Bemerkung über das Mass in Strukturen.* By J. RIDDER. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of May 31, 1947.)

Die Bemerkung, um welche es sich hier handelt, ist die Konstatierung, dass die in den Theoremen A und B enthaltene Bedingung notwendig und hinreichend ist für die Messbarkeit der abgeschlossenen Somen sowohl beim beschränkt- wie beim total-additiven Mass.

I. *Beschränkt additives Mass.*

§ 1 [1]. *Eine Struktur* S sei aufgebaut aus Elementen, *Somen* genannt, die im folgenden mit kleinen Buchstaben angedeutet werden, und den folgenden Axiomen genügen:

Axiom 1°: $\alpha)\ a \subset a$; $\beta)$ aus $a \subset b$ und $b \subset c$ folgt $a \subset c$.

Definition. $a = b$, falls $a \subset b$ und $b \subset a$.

Definition. Ein Soma ab wird *Produkt* des Somenpaares a, b genannt, falls:

$\alpha)\ ab \subset a$; $\beta)\ ab \subset b$; $\gamma)$ aus $c \subset a$ und $c \subset b$ immer folgt $c \subset ab$.

Axiom 2°: Für jedes Somenpaar a, b gibt es ein Produkt ab.

Definition. Ein Soma $a + b$ wird *Summe* der Somen a, b genannt, falls: $\alpha)\ a \subset a + b$; $\beta)\ b \subset a + b$; $\gamma)$ aus $a \subset c$ und $b \subset c$ immer folgt $a + b \subset c$.

Axiom 3°: Für jedes Somenpaar a, b gibt es eine Summe $a + b$.

Axiom 4°: Es gibt ein Soma 0 mit $0 \subset a$ für jedes Soma $a \in S$.

Axiom 5°: Es gibt ein Soma 1 mit $a \subset 1$ für jedes Soma $a \in S$ [1a].

Axiom 6°: $ac + bc = (a + b)c$.

Definition. Ist $a \subset b$, so wird durch $b - a$ angedeutet jedes Soma x, das den Bedingungen genügt:

$$ax = 0, \quad a + x = b.$$

Axiom 7°: Zu jedem Paar von Somen a, b, mit $a \subset b$, gibt es ein Soma $b - a$.

Eine derartige Struktur ist eine BOOLEsche Algebra.

§ 2 [1]. Für jedes Soma $x \in S$ sei eine (*reellwertige*) *Massfunktion* $m^\circ(x)$ definiert.

Definition. Ein Soma a heisse $m^\circ(x)$*-messbar oder messbar in bezug auf* $m^\circ(x)$, wenn für jedes Soma w, mit $m^\circ(w)$ endlich,

$$m^\circ(w) = m^\circ(wa) + m^\circ(w - wa)$$

ist.

[1] Siehe J. RIDDER, Acta math. **73** (1941), S. 131—173.

[1a] Auch ohne Annahme dieses Axioms behalten die nachfolgenden Sätze und Theoreme ihre Gültigkeit.