a prouvé que ceci n'était point le cas dans notre observation. Bien que l'examen expérimental ait montré parfois qu'une influence hypothalamique s'exerce sur les mouvements des globes oculaires, je crois pourtant qu'il n'y a pas, dans ce cas, de raisons suffícantes à l'admission d'un rapport causal entre le procès hypothalamique et les anomalies citées, d'autant plus que l'on n'a pu encore établir avec certitude l'existence d'une connexion entre les centres vestibulaires et cette partie de l'encéphale. Vu l'état actuel de nos connaissances relatives à cette connexion, il me paraît plus juste de rapporter les anomalies des mouvements du regard et l'appareil vestibulaire aux altérations mésodermales de l'entourage de l'aquéduc de Sylvius et les corpora quadrigemina. Bien qu'il n'y eut point ici de perte nerveuse, il reste possible que des troubles circulatoires aient provoqué une diminution de la fonction.

Résumé: Il y avait ici une encéphalite chronique de l'hypothalamus et des territoires avoisinants, laquelle a causé le vaste syndrome clinique décrit au début de cet article. Les différents symptômes ont été vérifiés par l'examen pathologo-anatomique. L'importance de cette observation sera plus amplement exposée ailleurs, comparativement à neuf autre cas qui furent examinés sur des coupes en série.

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Physics. — *Recovery and recrystallization viewed as processes of dissolution and movement of dislocations.* III. By W. G. BURGERS. (Laboratorium voor Physische Scheikunde der Technische Hoogeschool, Delft.) (Communicated by Prof. J. M. BURGERS.)

(Communicated at the meeting of April 26, 1947.)

III. *Dissolution and movement of dislocations during heat treatment.*

III. 1. *Displacements of atoms in the boundary layers.*

We have now to consider what changes will take place in a deformed metal with a structure of the general type discussed in II, when subjected to heat treatment. Here again a precise treatment is wholly impossible. We may best start from BRAGG's conception (21), mentioned in II, 5, according to which the deformed block structure is in a state of dynamic equilibrium, in which the system of boundary layers may be considered as a foam with a definite energy. As the temperature is raised, the mobility of the atoms increases and atomic displacements will take place so that the free energy of the structure diminishes. This, presumably, can best be realized by displacements of the atoms in the dislocated transition layers. Two processes can be discerned, (*a*) such displacements which diminish the "tension" in the layers without displacing them as a whole and (*b*) displacements of the layers themselves. This latter process would be most effective if the layers could be "pushed" and "pulled" up to the boundaries of the test-piece, so that they would finally disappear, leaving in their "wake" an "ideal" single crystal.

We shall consider these two types of displacement somewhat more in detail for the "simplified" block-structure, considered also when discussing the stability in deformed metals in II, 5, to know a "two-dimensional" [18]) block-aggregate with only positive and negative dislocations of "edge-type" in the transition layers separating the blocks.

a. In transition layers containing dislocations of opposite sign, we have discerned between two types of pattern, represented in fig. 10 *a* and *b*: in *a* opposite dislocations have coinciding directions of "easy mobility" (directions of slip), in *b* they lie on alternating lines. In the first case a mutual neutralization of pairs of adjoining dislocations seems possible. A similar process cannot be realized in *b*: here, however, a change in the stress acting parallel to the direction of "easy mobility" over the whole pattern can displace the whole set of positive dislocations with regard to the set of negative dislocations, as discussed in II, 5. Both

[18]) "Two-dimensional' in the sense indicated in footnote [11]).

processes entail no growth of one lattice block at the expense of one of its neighbours. On the contrary, one may expect the involved structural changes to be in general so small, that they are "invisible" by ordinary means of observation (cf. I, 1).

On the other hand the mutual neutralization of opposite dislocations causes an elimination of the stress regions of which they were the centres. The disappearance of these stresses will likewise be noticeable at larger distances. Due to it, for example by a displacement of whole sets of dislocations as occurring in the pattern of fig. 10*b*, a release of stress extending over a whole lattice block may be brought about [19]). This relief of stress is directly visible in transparent crystals, like rocksalt, by means of observation in polarized light (LASCHKAREW and ALICHANIAN (47); BRILLIANTOW and OBREIMOW (48)). Also the decrease of the broadening of DEBYE–SCHERRER lines during annealing is for the greater part due to this process [(37), (38), (49)]. Finally all physical and mechanical properties, which are influenced by the stressed state of the transition layers and the blocks, will be affected. As most probably different properties depend on stresses of different period, it can be foreseen and is experimentally observed that different properties recover in different temperature regions and with different rates (BURGERS (49); (1), § 72). On all these grounds *it seems reasonable to indicate the process of dissolution of pairs of* TAYLOR-*dislocations of opposite sign* ("DEHLINGER–VERHAKUNGEN") *as the fundamental action of what "macroscopically" is called recovery.* This conception shows a close analogy with the ideas developed by KORNFELD (2), DEHLINGER and KOCHENDÖRFER (3), (39) [20]).

b. We now consider transition layers containing only dislocations of one sign, or at least a surplus of them. The most simple type is that shown in fig. 4, where the boundary is built up of a series of equally spaced dislocations of the same sign only. As already remarked while discussing fig. 4 and 5 in II, such a boundary necessarily separates lattice blocks differing in orientation. This type of boundary can never be wholly removed by mutual dissolution of opposite dislocations. The transition region can, however, be displaced as a whole by means of relatively small atomic displacements, as indicated by BRAGG (23) and already mentioned at the end of II, 5. This occurrence entails the enlargement of one lattice block at the expense of a neighbouring block and is *essentially a process of crystal growth.* It sets in when the resultant stress acting on the series of dislocations parallel to the direction of "easy mobility" attains a definite value, this value presumably depending, in the simple case of fig. 4, on the density of succession of dislocations, that is on the angular separation of the adjoining blocks.

[19]) As remarked in footnote [13]), it is also possible to assume sets of dislocations to be present *inside* the blocks.

[20]) As KORNFELD puts it, recovery "mollifies the glue" between the blocks.

In what follows we intend to discuss to what extent some of the characteristics of both recovery and recrystallization can be understood with the help of these types of atomic displacement. Again we must limit our discussion to the "two-dimensional" structures considered so far.

III, 2. *Recovery considered as due to mutual neutralization of dislocations of opposite sign.*

From the foregoing it is at once clear that recovery can be realized at its best in cold-worked test-pieces, where the deformation has produced equal numbers of positive and negative dislocations, as arranged in patterns like those shown in fig. 10. Such arrangements may be produced most exclusively in sheared single crystals, where the presence of adjacent lattice blocks with different orientations is limited to at its most very small regions ["local curvatures" of glide-lamellae: see LEBBINK and BURGERS (34); BARRETT (51), p. 14)] [21]).

It may be expected that in such deformed specimina the mayor part of the stresses can be eliminated by neutralization of pairs of dislocations ("Verhakungen") and by a relative displacement of sets of dislocations, as discussed under *a*) in the preceding section. It is therefore not surprising that, as mentioned in section I, 1, precisely in deformed *single* crystals a far advanced recovery without "visible" recrystallization has been realized.

With increasing inhomogeneity of the deformation process the percentage of blocks in non-parallel positions will increase and the same holds for deformed polycrystalline test-pieces, as a consequence of the hampering effect of the crystal boundaries on the glide-process. Here, therefore, "pure" recovery can never do away with the whole stress system, produced by cold-working. Dissolution of pairs and displacements of whole sets of dislocations must finally be followed or accompanied by a boundary-displacement, as discussed in III, 1 under *b*). That, therefore, pure recovery in such cases is only possible to a limited extent, is not surprising.

Far less understandable is the experimental fact, shown in fig. 1, that annealing at different temperatures both in single- and polycrystals gives rise to different "rest-values". It might be understood to some extent by making the plausible assumption, that the cold-worked test-piece contains groups of dislocations with different "neutralization-energies" Q for individual pairs. Then, even if these energies within each group remain constant during the annealing treatment, the whole course of the recovery might be conceived as composed of a superposition of e-curves of formula [4] in I, 1 with different exponents: dislocation-pairs for which the neutralization-energy is smallest, are eliminated at the lowest temperature and after the shortest time of heating. With suitable values for these

[21]) Their presence follows from "asterism" in Laue-photographs (BURGERS and LEBBINK), and from x-ray microscopic observations by a new method, developed by BARRETT.

energies a course as found experimentally might be realized. It seems, moreover, reasonable to suppose that this effect will be more pronounced if the considerations, developed by DEHLINGER regarding the stability of linear groups of "Verhakungen" (see II, 5), might be applied also to the corresponding two-dimensional arrangements. According to his quantitative estimation, the energy required to dissolve a dislocation-pair decreases with decreasing "density" (that is number of pairs per cm, resp. cm^2). This energy therefore becomes less and less in the course of the dissolution of the group, until finally, when the density has reached a critical value, the remaining group collapses suddenly and completely. It follows that the rate of removal of a given number of dislocation-pairs in the course of the annealing process depends on its *distribution* over the deformed test-piece. The more numerous in membership the groups of which it is built up, the more difficult will be its dissolution: not only the initial value of the dissolution-energy in each group increases with its "size", but also a larger fraction of the total number of dislocations must disappear before collapse of the rest will set in. It seems possible that such circumstances might favour a course of the recovery-isotherms as found experimentally [22]).

Leaving this question as it stands, and assuming the occurrence of "rest-values" as an experimental fact, it is perhaps easier to understand the occurrence, discussed in I, 2, of recovery-isotherms approaching rest-values which lie "higher" for larger degrees of deformation. Reasoning along the lines applied above, this behaviour seems to fit in with the assumption that, taken as a whole, the density of dislocation-pairs, and thus the threshold-values of the energy required for their dissolution, increases with the degree of deformation.

We are well aware that the foregoing considerations are highly speculative. They have only been given with the intention to bring forward some characteristic features of the recovery process, which, as far as known to us, have not been given much attention up to the present [23]).

III, 3. *Recrystallization viewed as a process of displacement of transition-layers.*

We now turn to recrystallization proper and consider as its fundamental process the displacement of a boundary layer between non-parallel lattice

[22]) The picture implies that after *very prolonged* heat-treatment for all temperatures, where broadly speaking recovery is not yet accompanied by recrystallization, the *same* end-value must be reached, only the time to attain this state being different for different temperatures. According to KOCHENDÖRFER (39) (p. 240), this is actually the case. The occurrence of different *approximately constant* "rest-values" for different temperatures remains, however, very remarkable.

[23]) Part of these speculations were given in (1), § 76. Here it was moreover suggested that the observation of KORNFELD, also mentioned in I, 2, of a larger rate of anneal in deformed single crystals as compared with polycrystals, might be "understood" on similar grounds, but the speculations given are too vague to have value.

blocks. Limiting our considerations as before to the simple transition layer shown in fig. 4, such a displacement can only set in when this layer is subjected to a stress of sufficient size, acting in the direction of "easy mobility" of the layer (that is, in the case of fig. 4, in a horizontal direction). It must be assumed that the required stress arises by a "redistribution" of the stress-system, existing in the cold-worked specimen, caused by dissolution of a number of dislocation-pairs (recovery). This leads us to a conception of the nature of "nuclei of recrystallization" and of their "period of incubation", which is essentially that given as long ago as 1929 by DEHLINGER (3), (50). The nuclei in this picture are lattice-blocks already present in the cold-worked state. The time of incubation is merely the time, elapsing from the beginning of the annealing process, until, by the preceding recovery (dissolution of a sufficient number of dislocation-pairs), the critical stress, necessary for movement of the transition layer, has been attained [24]).

It seems natural to suppose that displacement of a transition-layer only occurs if the "structural" conditions prevalent at both sides of the boundary are different. Only under such circumstances the development of a resultant stress acting in one of the two possible directions of mobility can be expected. Otherwise stresses will balance each other and no uni-directional displacement is possible. This conclusion is confirmed by the behaviour of recrystallized copper or nickel-iron sheet with so-called cube-texture: in such material, as is well known, by far the major part of all crystallites lie with a scattering of 5—10° with a cubic-plane and -axis parallel to the plane and direction of rolling. On this and other grounds (for example etch-figures) one feels justified to assume that the foil consists of blocks in nearly parallel and "equivalent" positions with regard to their neighbours, and may be considered as a "pseudo single crystal in 100-position" with an "extreme" mosaic structure. If such material is subjected to a prolonged heating at for example 1000° or 1100°, one never observes (at least as far as we know), that the structure is transformed into one "real" single crystal *of the same position*: elimination of the boundaries between the nearly equivalent blocks appears to be impossible [25]).

If, as often occurs (see (1), § 121), the annealing process gives rise to the formation of large crystals (by what is called "secundary recrystallization" or "exaggerated graingrowth"), these new crystals possess always orientations which *differ considerably* from the 100-orientation. This means, in our conception, that the finely grained material, besides the 100-crystallites, contains a small percentage of blocks with a different

[24]) See, however, p. 727.

[25]) This case thus represents an example of "capillary equilibrium" by the boundaries in the sense as meant by BENEDICKS (44); cf. also BRAGG (21), when discussing the stability of the domain structure, formed when order sets in in the alloy Cu_3Au. There, due to the existence of four equivalent super-lattice arrangements, a relatively stable state of dynamic equilibrium can be realized.

orientation, which, in the course of the annealing process, obtained the faculty to grow at the expense of the 100-crystallites [26]).

From the foregoing it follows that a lattice-block, which can serve as nucleus for a new crystal, must be in a state of stress different from its neighbours. This condition would be satisfied by a block in a position like *b* in fig. II, which one can consider to lie at the "point of inflexion" of the

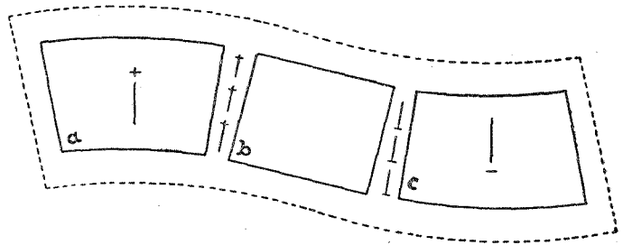

Fig. 11. Three adjoining lattice blocks, which, taken together, can be considered to form a "*S*-curved" lattice region. Block *b* in the "inflexionpoint" can presumably function as a nucleus for recrystallization.

lattice region *a-b-c*. In this block, which is the least curved and thus contains (or is surrounded by: compare footnote [13])) an equal number of dislocations of opposite sign, "true" recovery by dissolution of dislocation-pairs can eliminate a large part of the stresses it contains, so that displacement of the boundaries between this block and its neighbours *a* and *c* can set in. As *b* is least "deformed", we should expect *b* to grow at the expense of *a* and *c*: here again we are in agreement with DEHLINGER's conception of the most probable nuclear spot [27]).

[26]) It is not yet fully understood what factors govern the occurrence of secundary recrystallization in these and other metals and neither what factors determine the orientations of the new crystals formed in this way. From the work of several investigators (for example COOK, MACQUARIE and RICHARDS (52), CUSTERS and RATHENAU (53) and others (see (1), § 121)) definite orientation-relations, partly consisting of twin-positions, have been found. We are inclined to think that the phenomenon is related to that of "stimulated" crystalgrowth mentioned at the end of this paper.

[27]) The problem how actually nuclei for recrystallization are formed, either in a way analogous to nucleus formation in vapour or solution by a favorable thermal fluctuation (in such case the time of nucleation represents something like the time elapsing from the beginning of the annealing before this fluctuation occurs) or by the growth of lattice blocks in favorable state of stress as represented in the foregoing, has not yet been solved and has given rise to much discussion and controversy. DEHLINGER (3), KORNFELD (58) and the author are inclined to consider the second alternative more probable, ANDERSON and MEHL (10) the first one. To decide this question, the experimental data available, especially those relating to the dependence of rate of nucleation and rate of growth on degree of deformation and temperature, must be critically analyzed. Also the influence of a recovery treatment of the deformed test-piece, preceding the recrystallization process, must be taken into account. Most of the data regarding these factors available up to 1940 have been critically considered by the author in (1) § 109; some later considerations are given in ANDERSON and MEHL's paper (10). The question cannot be

III, 4. *Rate of growth of crystals.*

In the foregoing section the growth of one block at the cost of an adjoining block was conceived as a displacement of a transition layer as a whole under the influence of a crtical stress, rather than as a process of individual jumps of atoms. Its rate of propagation probably depends on the size of potential barriers in a similar way as the propagation of a single dislocation along the glide plane in a crystal. As set forth by OROWAN (57), in this latter case the rate is given by a BOLTZMANN-formula if the average stress acting upon the dislocation is less than the stress necessary to set it into movement without the help of the thermal agitation; if, however, the average stress surpasses this critical value, it will attain a constant rate determined by the energy dissipated to the surrounding lattice parts.

As to the displacement of a whole transition layer, it seems natural to assume that the rate so obtained will remain unchanged only, if the "structural characteristics" at both sides of the boundary do not change. These, however, generally change continuously in the course of the devoring of a lattice block, due to a continuous change in its orientation-relations with regard to adjoining lattice blocks. This will occur certainly when a lattice block has been wholly devored by a growing block, as now by a "melting together" of both the moving and the new transition layers, completely different boundary structures (patterns of dislocations) are formed. These may be either such that they require a smaller critical stress for displacement and thus facilitate the displacement, or, what seems perhaps more plausible, the new boundary requires a much larger stress and thus brings the displacement practically to a stop. In that case only dissolution of more dislocation pairs can start the process anew. In this picture the displacement of boundaries is thus conceived to proceed more or less in a jumpy way from boundary to boundary, or from lattice block to lattice block. The final rate is determined by two component processes,

regarded as resolved with any degree of certainty. It is no use to put the suggestions given in this paper to the test with regard to the available data, as these suggestions are vague and moreover the actual structure of the transition layers is far more complicated than the simple type considered here. [Consider for example the influence of a recovery treatment of the deformed test-piece before recrystallization. On the basis of our considerations one might expect this to *shorten* the incubation period, as part of the necessary dissolution of dislocations can take place during this preliminary process. On the other hand, on the basis of the "fluctuation-theory", one might sooner anticipate an *extension* of the incubation period, as recovery will cause a release of stress especially in the points of highest stress concentration, where fluctuations will preferably occur. In reality, in most cases experiments point to an extension (KORNFELD and PAWLOW (54), MATHEWSON and COLLINS (55), although the opposite has apparently also been found (KORNFELD and SCHAMARIN (56); see (1), § 103). — On the other hand, the fact that in rolled sheet the crystal orientations after recrystallization can be found or are in some way connected with those in the cold-worked state of the specimen, is most easily understood on the assumption that the nuclei of the new crystallites formed part of the cold-worked structure.]

one consisting of dissolution of dislocations in order to create after every step the stress required for further displacement, the other that of the displacement proper under influence of these stresses. That process which is the slower determines the final value of the rate of displacement [28]).

A jumpy displacement of the boundary of a growing crystal was actually observed in MÜLLER's experiments (11) on the recrystallization of rocksalt. It may be doubted, however, whether this fact may be regarded as confirming the conceptions given above, as the displacement during *one* jump could attain 20—30 μ at 770° C., a displacement much larger than the average size assumed for a mosaic block. (According to MÜLLER, the jumps are due to the accumulation of foreign atoms in the transition layers, and the occasional "piercing" of these layers).

Leaving this question as it stands, it would follow from these considerations, that a constant rate of growth of new crystals can only be attained when the growing nucleus has grown beyound the region of inhomogeneous deformation, where it is supposed to have started (see above fig. 11; this region may be of (sub-) microscopic size) and has entered that part of the test-piece, where the deformation is (quasi)-homogeneous. If, during this first period, the total rate is slower than the final "constant" rate, the curve which gives the rate of growth as a function of time of heating, which is linear in a macroscopically homogeneously deformed specimen, does not

[28]) In this connection we may insert the following remark, to which I am indebted to Prof. MOTT. In ANDERSON and MEHL's paper the rate of growth of a new crystal is given by an expression

$$G = B \cdot e^{-Q_G/RT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

where B is a constant and Q_G an activation energy. The numerical value of B (which depends on the degree of deformation of the matrix) is of the order 10^{14}—10^{16} cm per sec. Now we consider a transition boundary of 1 cm². If a is the atomic distance, than the number of atoms in this surface is $\sim 1/a^2$. The number of these atoms, which undergo displacement by virtue of the activation energy Q_G, will be of the order

$$\nu \cdot 1/a^2 \cdot e^{-Q_G/RT}$$

per second, where ν represents the atomic frequency (which we may put to about 10^{12}). Suppose further that the displacement of every one of these atoms involves, by a kind of "chain reaction" (or gliding), a displacement of N atoms, that is growth over a volume of $N \cdot a^3$ cm³, thus a displacement for a surface of 1 cm² over $N \cdot a^3$ cm, then the displacement of the boundary per second, that is the rate of growth of the recrystallization process, is given by

$$G = N a^3 \cdot \nu \, 1/a^2 \cdot e^{-Q_G/RT} = N \nu a \cdot e^{-Q_G/RT}$$

so that from (α):

$$B = N \nu a \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

With $B = 10^{15}$ cm/sec, $a = 2 \times 10^{-8}$ cm and $\nu = 5 \times 10^{12}$ sec^{-1}, we find for N an order of magnitude of 10^{10}. This corresponds to a volume $N \cdot a^3 = (0.4\,\mu)^3$, that is about the size of a mosaic block, and the above calculation might be considered in favour of the conception that recrystallization proceeds in "steps" of one mosaic block at a time. (See, however, Addendum at the end of this paper.)

pass through the origin (see fig. 12): the "incubation period" thus found may be the *sum* of the "true" incubation period as defined in section III, 3 on p. 723 and the "period of invisible growth" during the consumption of the inhomogeneously deformed region around the nucleus. This conception approaches those of MÜLLER (11) and of KORNFELD (58) (see (1), § 109). The constant rate itself presumably depends in the first place on the mutual orientation of growing and disappearing block. In fact, it seems natural to

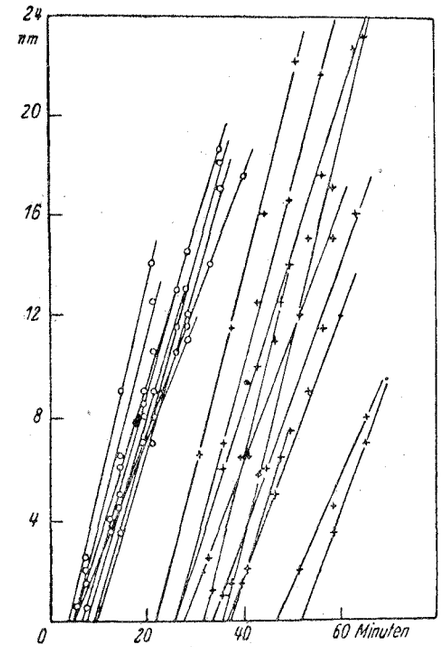

Fig. 12. Constant rate of growth of crystals, formed by recrystallization, in polycristalline aluminium wire (after KORNFELD and PAWLOW (54)). Degree of deformation 3%. Temperature of anneal 450° C.

assume that the critical stress, required for displacement of the boundary layer, depends on its "structure", for example in the simple case of fig. 4, on the density of the dislocation-series, that is on the orientation between the two lattice blocks [29]).

The influence of a difference of orientation of neighbouring blocks upon the rate of displacement of the common boundary can be clearly demonstrated in recrystallization experiments with slightly deformed single

[29]) In this simple case one might perhaps expect this stress to be larger for larger deviation [at least as long as the difference in orientation increases and does not, as a consequence of symmetry relations, becomes less pronounced again (see footnote [12])], as displacement of a "boundary" in an ideal lattice (with density of dislocation zera) may be said to require no force at all.

crystals: here new grains often show more or less straight boundaries, parallel to certain directions. Fig. 13 gives a striking example, it shows the progressive growth of a new crystal in the unicrystalline matrix parallel to a definite direction. Other examples are given in experiments by KORNFELD and RYBALKO (59), also with aluminium, where the new crystals are of lozengeshape [30]).

In a different way the same result is confirmed by recrystallization of polycrystalline material. If such material has no preferential orientation, the new grains have approximately a circular appearance (fig. 14a), showing that the average rate of growth is the same in all directions, so that individual differences, which may be expected due to the different orientation between growing crystal and disappearing crystallites (see fig. 15) are effaced. That, however, such individual differences are actually

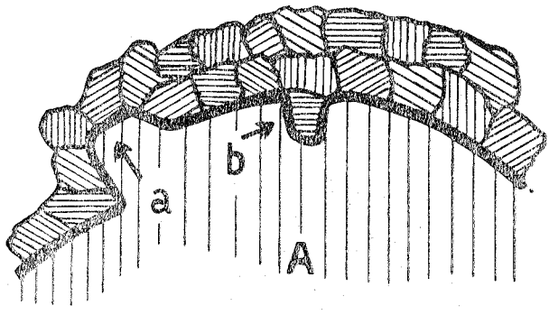

Fig. 15. Schematic representation of the growth of a large crystal at the cost of small surrounding crystals. Growth may proceed "from below" (at *a*) or "sideways" (at *b*). Due to differences in mutual orientation, differences in rate of consumption of individual grains are to be expected.

present is clear when the mother-material is rather coarsely-grained: the corrugated character of the boundaries of the new grains, as shown in fig. 14*b*, is due to this effect. On the basis of this "effacing-effect", one might expect that new crystals, formed in homogeneously deformed finely grained material (without any preferential orientation) *all* show the same rate of growth, this rate being the mean value of the rates between the growing crystal and the surrounding individual crystallites, which are being devored [31]).

Direct determinations of the growth of crystals under such circumstances in various metals by KARNOP and SACHS (60) and by KORNFELD and

[30]) The exact nature of the directions of preferential growth have still to be found as determinations of various authors gave no corresponding results (see fig. 13). According to KORNFELD and RYBALKO they are related to octahedral planes, which would not seem surprising. Further work on this question is necessary.

[31]) Moreover, the rate can be averaged still more due to the possibility (see fig. 15) that the growing crystal approaches the small crystallites not "directly" but via an adjoining crystallite.

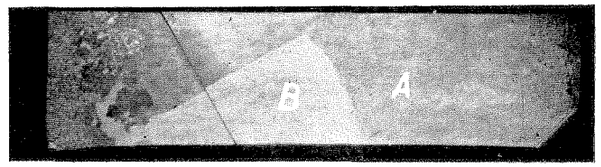

Fig. 13. Directional growth of new crystal (*B*) in a stretched single crystal (*A*) of aluminium. The boundary is indicated after 15 and 25 minutes of heating at 600° C. The rate of displacement in a direction perpendicular to the oblique boundary was zero. This boundary was parallel to the trace of a 111-plane of the growing crystal (*not* of the mothercrystal *). Nat. size.

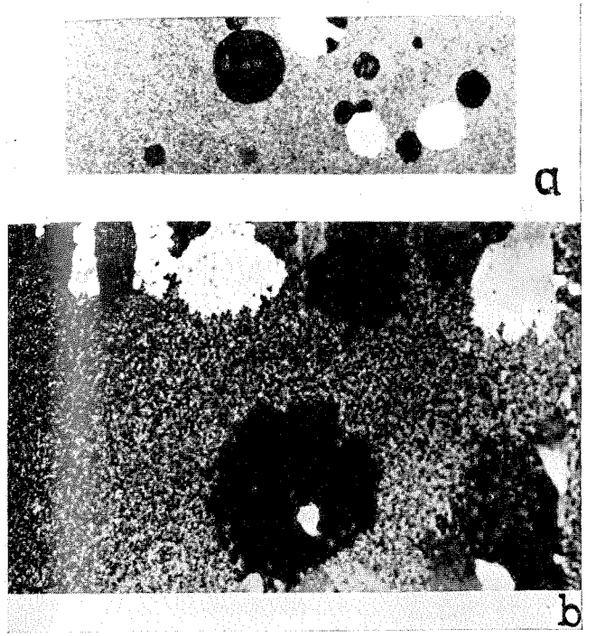

Fig. 14. Formation of circular crystals in quasi-isotropic, finegrained aluminium sheet, which has been subjected to a (macroscopically) homogeneous stretching of a few percent. *The mean rate of growth is equal in all directions.*

a. original material finegrained: *smooth* boundaries.

b. original material somewhat coarser than *a*): *corrugated* boundaries, due to differences in the rate of consumption of individual crystallites (compare fig. 15). Nat. size.

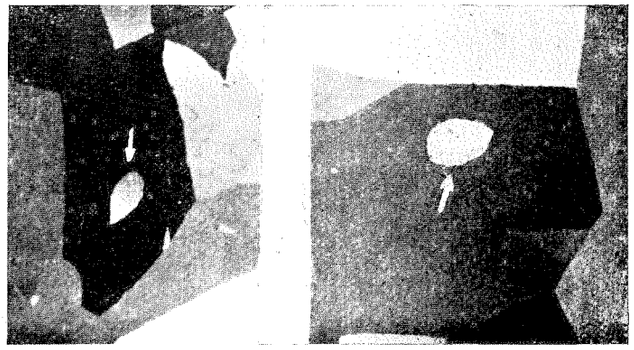

Fig. 16. Aluminium plates at approximately natural size, showing large crystals, formed by recrystallization of finegrained material. The presence of pear-shaped crystals, inside larger surrounding crystals, is due to the smaller rate of growth of the former with regard to the latter: The ratio is given by the cosine of half the angle at the pointed and of the enclosed crystal.

*) The contrary was found in KORNFELD and RYBALKO's experiments, mentioned in the text.

co-workers (54) (61) carried out by measuring the diameter of the crystals as a function of time of heating (see example in fig. 12), are not sufficiently accurate for this purpose: the difference in inclination of the lines of growth can easily be due to the impossibility to measure the size of the crystals with sufficient precision.

III, 5. *Occurrence of crystals with different rates of growth.*

Indirectly, however, the presence of considerable differences follows from an analysis of the *form* of the boundaries of the new crystals in recrystallized aluminium plates, as done first by SANDEE (62), and afterwards by BURGERS and MAY (63). Considering the problem as "two-dimensional" and assuming a constant rate of growth in all directions, SANDEE showed that special types of boundaries and especially the presence of pear-shaped crystals *within* larger surrounding crystals (examples are given in fig. 16) can only be explained by assuming a constant ratio between the rates of growth of both crystals during the whole course of their growth. From the analysis it follows that this ratio is given by the cosine of half the angle at the pointed end of the enclosed crystal. As angles of up to 80° occur, the ratio can assume values up to about $\frac{3}{4} : 1$.

The occurrence of these differences in rate of growth between various crystals growing at the expense of the same matrix may mean that the "effacing-effect", discussed in the foregoing section, is not so completely realized as we expected there, so that crystals with different orientations have still different rates of growth. So far, however, we have not succeeded in finding a definite relation between crystal-orientation and rate of growth. This suggests the possibility that the observed differences find their origin in the internal state of the growing crystals themselves. In view of the mechanism of growth discussed above, this would lead to the hypothesis that besides the block structure of the disappearing crystallites *also that of the growing crystal* has an influence on the growing process. It is not easy to understand in what way this might be possible. It might imply that some imperfection, created in the very beginning of the growth, would enforce itself upon the whole future crystal. A schematic picture as given in fig. 17 (which is given by TAYLOR (46) in another connection) might be suggestive here: suppose two blocks are growing while including a small angle, then perhaps each time the gap between them attains one atomic distance, a break in the growth process may occur, which influences its final rate. To explain the difference in rate of growth encountered with different crystals, it would moreover be necessary to assume that different crystals are characterized by different "imperfections". Considering the findings of DEHLINGER and GISEN (19), mentioned in II, 1, as to the presence of differences between aluminium crystals grown by recrystallization and from the melt, this possibility need not be rejected a priori.

For recrystallization-crystals, it must be kept in mind that they start from nuclei which certainly are in different states of stress with regard to their immediate surroundings.

For the moment, in absence of further experimental work, this question must be left as it stands. Perhaps it will be possible by applying special methods of x-ray research (for example Mrs. LONSDALE's "divergent beam"

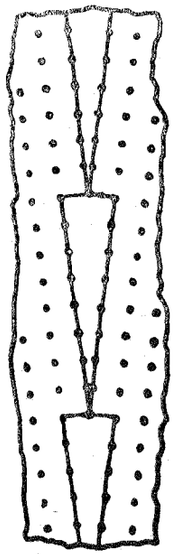

Fig. 17. Boundary of two crystals at slightly different orientations, showing boundaries of separate crystalblocks and a regular repetition of "surface of misfit" (after TAYLOR (46)). The figure is given here as an endeavour to understand the repetition, in a growing crystal, of a definite imperfection.

method (64) or BARRETT's "x-ray microscope" (51)) to get some further experimental evidence regarding the presence or absence of differences in mosaic character of different crystals [32]).

[32]) Perhaps recrystallization of silverhalide crystals can also be of help while investigating this problem. Preliminary experiments by BURGERS and TAN KOEN NIOK (65) have shown that such crystals, after irradiation with ultraviolet light, show an optical "etching effect" parallel to cube-planes, which seem to form an indication of the presence of a mosaic structure with blocks parallel to these planes. Differences might be found in this "optical" way between various crystals. [In (65) it was suggested that the diffuse bands on Laue-photographs of these crystals was partly due to this supposed mosaic character. Further work has shown, however, that the bands are of thermal nature, similar to those found for other crystals by PRESTON (66), LONSDALE and SMITH (67, GUINIER (68), KRONING and ARLMAN (69) and others.]

(To be concluded.)

Addendum (June 1947).

With regard to the remark in footnote [28]), Mr. F. R. N. NABARRO drew my attention to the fact that measurements of rates of growth by other investigators [KORNFELD and PAWLOW (9) with aluminium; MÜLLER (11) with rocksalt] lead with the same reasoning to much smaller values of the number of atoms (N), involved in a "chain of jumps". In this connection it might be remarked that the mosaic character of a crystal, even of the same substance, may be largely dependant on its mode of formation or on its degree of purity: cf. section II, 1. The question, however, may be raised whether the introduction of a quantity N, as done in footnote [28]), can be maintained.

Mathematics. — *Einige Anwendungen des Dualitätsprinzips in topologischen Strukturen.* By J. RIDDER. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of May 31, 1947.)

I. Charakterisierung des offenen Kernes durch ein einziges Axiom.

§ 1 [1]). Wir betrachten eine BOOLEsche Algebra S, definiert durch die nachfolgenden Axiome 1°—7°; ihre Elemente wollen wir *Somen* nennen.

Axiom 1°. α) $a \subset a$; β) aus $a \subset b$ und $b \subset c$ folgt $a \subset c$.

Definition. $a = b$, falls $a \subset b$ und $b \subset a$.

Definition. Ein Soma $\prod_{a_\iota \in K} a_\iota$ wird *Produkt der zu der Klasse K gehörenden Somen* (a_ι) genannt, falls: α) $\prod_{a_\iota \in K} a_\iota$ jedes a_ι; β) aus $b \subset a_\iota$ für jedes $a_\iota \in K$ folgt $b \subset \prod_{a_\iota \in K} a_\iota$.

Axiom 2°. Für jedes Somenpaar a, b gibt es ein Produkt ab.

Definition. Ein Soma $\sum_{a_\iota \in K} a_\iota$ wird *Summe der zu einer Klasse K gehörenden Somen* (a_ι) genannt, falls: α) jedes $a_\iota \subset \sum_{a_\iota \in K} a_\iota$; β) aus $a_\iota \subset b$ für jedes $a_\iota \in K$ folgt $\sum_{a_\iota \in K} a_\iota \subset b$.

Axiom 3°. Für jedes Somenpaar a, b gibt es eine Summe $a + b$.

Axiom 4°. Es gibt ein kleinstes (leeres) Soma 0; das soll heissen: $0 \subset a$ für jedes $a \in S$.

Axiom 5°. Es gibt ein grösstes Soma 1; das soll heissen: $a \subset 1$ für jedes $a \in S$.

Axiom 6°. $ac + bc = (a + b)c$.

Axiom 7°. Zu jedem Paar von Somen a, b, mit $a \subset b$, gibt es ein Soma $b - a$ derart, dass $a + (b - a) = b$ und $a \cdot (b - a) = 0$ ist.

Definition. *Das Komplement von a, a'*, sei das Soma $1 - a$.

§ 2 [1]). Jedem Soma $a \in S$ seien in eindeutiger Weise zugeordnet das Soma $\bar{a}$, *die abgeschlossene Hülle von a*, und $\underline{a}$, *der offene Kern von a*; zwischen diesen soll folgender Zusammenhang bestehen:

$$\underline{a} = (\overline{a'})', \text{ oder } \bar{a} = (\underline{a'})' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

[1]) Siehe J. RIDDER, Verhand. Ned. Akad. v. Wetensch., Amsterdam, Sekt. 1, 18, Nr 4 (1944), 43 Seiten.