Anatomy. - On spatial relations of the principal skull-measures. By A, de Froe. (Communicated by Prof. M. W. Woerdeman.)
(Communicated at the meeting of June 28, 1947.)

In craniometrics the shape of the skull is usually expressed in a quotient of two of the principal skull measures successively, which quotient, in order to simplify matters, is multiplied by 100 , thus expressing one measure as a percentage of the other.

The short and accurate numerical expression of the form is of importance to the study of the cranium and its main proportions:

1. to the shape-analysis of the cranium in itself.
2. to the study of the variability of the shape of the cranium
3. to the comparative morphology of the cranium.
4. to the research of the deviating skull-measurements, for instance those arising from the premature synostosis of the sutures.
5. to the study of the growth of the skull.
6. to the investigation of sex-differences.
7. to the research of the causalities determining the shape of the cranium: heredity, peristasis.
The expression of a form by a quotient of the principal measures, which quotient has obtained the suggestive title of "Index", is only then permitted:
8. when the choice of the measures is correct.
9. when the measures are highly and positively correlative.
10. when and if the sizes have a constant, structural relation in space, with regard to each-other.
The choice of the correct sizes will not be discussed here, also because the research of both other requirements may contribute to this choice.

As for the second requirement we refer to previous publications.
The mutual relations between these principal skull-measurements are definitely positive. There exists a higher degree of correlation in the cranium of the female than in that of the male.

In woman the correlation figure is:

| between skull-length and width | +0.460 | $\pm 0.085$ |
| :--- | :--- | :--- |
| between skull-length and height | +0.471 | $\pm 0.084$ |
| between skull-width and height | +0.425 | $\pm 0.089$ |
|  |  |  |
| man the correlation-figure is: |  |  |
| between skull-length and width | +0.320 | $\pm 0.063$ |
| between skull-length and height | +0.228 | $\pm 0.068$ |
| between skull-width and height | +0.232 | $\pm 0.067$ |

All values thus found are highly significant. A high correlative value however cannot be observed.

With regard to the third requirement it has to be considered that it would not be justified to demand a completely consistent, structural relationship, as we are not dealing with a product of handicraft but with a biological object, yet.
either 1. the fluctuations may not extensively influence the shape of the cranium;
or 2. if they do, we must accurately know their dispersion and trace the extent of their influence on our judgment;
or 3. we have to attempt to restrict these fluctuations by looking for different sizes.
These, also from a biological viewpoint, reasonable demands, remained hitherto alien to craniometrics.

The research brings about two main difficulties:

1. the covariation in shape and size of the skull, excluding the comparison of the skulls in itself. We can however exclude size by making diagrams and bringing those back to the same scale.
2. the orientation of the skull.

A natural, obvious, self-evident orientation of the skull or of the head cannot be indicated.

Working scientifically, we must limit ourselves to the question:
Is there a position to be found, which enlarges the comparability?
Of all positions we have been examining, that one sufficed best where the Frankforter Horizontal and the orbitalia are running concurrently. Nevertheless one could think of a better position.

Of 200 definitely male and 90 definitely female skulls, the mutual structural relation was examined, according to the following six data.

1. the Frankfurther Horizontal.
2. length of skull: Glabella-opisthocranion.
3. breadth of skull: euryon(d)-euryon(s).
4. height of skull: basion-bregma.
5. form of skull.
6. capacity of skull measured by the seedcorn-method.

To express the form of the skull in a manner, suiting this investigation, no use could be made of one single index, which only represents the skull in one single plane-section.
That is why length, breadth and height were expressed as a percentage of their sum, and according to their percentage-predomination groups were formed like this:
long, broad or high; long $\mathcal{E}$ broad; long $\mathcal{E}$ high; broad $\mathcal{E}$ high etc., etc.
The results of a similar grouping characterize the form of the skull very well, but are exclusively operative with the examined data.

We shall now confine ourselves to a rough reproduction of the definitely positive results of our investigation.
I. The structural relations.

1. apart from asymmetricals the position of the breadth with regard to the length varies in two ways:
$A$. the breadth may a) be situated above the length,
b) be just intersecting the length,
c) be situated under the length.
$B$. the breadth may be found in the course of the length in a more forward or a more rearward position.
A. Of 200 male skulls, 124 have the breadth situated above the length average 9.6 mm maximum 30 mm .
67 skulls had the breadth under the length, on an average of 5.1 mm and to a maximum of 23 mm .
9 skulls had the breadth intersecting the length.
In the female skull the relations are about the same.
$B$. The breadth lies perpendicular-projective to the length, apart from a few cases always nearer to the opisthocranion and at an average distance of $44.4 \%$ of the length, varying from $35-51 \%$. This projectionpoint is always to be found behind the intersectionpoint of length and height. These relations correspond in the male and female skull.
2. The position of the height varies, with regard to that of the length in three ways:
$A$. The angle with which the height intersects the length varies, i.e. the height inclines more or less forward.
B. The point where height and length intersect, varies; this point may be situated more or less to the front.
C. The point where the length intersects the height, may be found to lie higher or lower in the course of the height.
A. The angle between skull-length and height is found to be varying in male skulls from $66 \frac{1}{2}^{-}-93 \frac{1}{2}^{\circ}\left(27^{\circ}\right)$, its average is $83 \frac{1^{\circ}}{}{ }^{\circ}$.
The same average was found in female skulls.
The variation of this angle is greater than that of the angles between height \& Frankfurter Horizontal and between length \& Frankfurter Horizontal.
B. Height and length always intersect in front of the projectionpoint of the breadth.

Except a few cases the intersectionpoint in male skulls lies closer to the glabella, and at an average distance of $46 \%$ of the length, varying from 42-51 \%.
In female skulls this intersectionpoint lies distinctly more to the front, the average of $45 \%$, as well as the minimum and maximum 40 and $48 \%$.

In the female skull this point and the breadthpoint have a greater distance between them, than in the male skull.
C. The length intersects the height always closer to the basion than to the bregma.
In men the intersection-point lies at an average distance of $39.9 \%$ from the height of the basion, varying from $27.5-48.3 \%$.
In women this average is $40.3 \%$, varying from $32.6-48.1 \%$.
II. The connections.

1. A connection of these data with the capacity of the skull generally fails. In male skulls the breadthpoint lies more to the front in larger skulls.
2. A connection with the form of the skull is generally present and often even evident, meaning that, in short, narrow, but high skulls (the fémale type):
3. the breadth often lies above the length and also occasionally higher.
4. the height is more inclining to the front.
5. the intersectionpoint of length and height lies more in front.
6. it is not so evident that the breadthpoint lies more backward.
7. The mutual connection is as follows:
8. higher situated breadth correlates with more forward inclined height.
9. higher situated breadth correlates with more backward-situated breadth. This corresponds to the correlation of the form.
10. When a great angle exists between the length and Frankfurter Horizontal:
a. the breadth lies distinctly higher above the length
$b$. the height inclines more to the front we find this angle to be $1^{\circ}$ higher on an average, in the female, than in the male.
c. this angle evidently correlates with the angles between length and F.H. and between height and F.H.
d. this angle correlates milaty with the form of the skull.
e. this angle correlates with the capacity of the skull in women, a greater angle in larger skulls.

Conclusion.
We are negative in our answer to the question whether, as regards to the principal skull-measures, there might exist a reasonable, consistent, structural relation
But not only an ample variation of the mutual positions exists, the variation of the positions itself shows a distinct influence upon the form of the skull.

This influence shows determinedly systematical, distinct connection

1. Especially the height of the skull is involved.
2. Distinct sex-differences can be observed in it.
3. It bore the view that the orientation of the skull to the F.H. is not the same for each skull.
From this inquiry may finally be shown:
4. that the height may not be disregarded as a form-distinguishing measure.
5. that the problem of sex-dimorphism of the skull has not been solved by far.
6. that the results of the hitherto customary, craniometrical investigation should most certainly be replenished, according to the directions indicated above.

## REFERENCES.

Dr. A. De Froe, Meetbare variabelen van den menschelijken schedel en hun onderlinge correlaties in verband met leeftijd en geslacht. Thesis, 1938.
Dr. A. DE Froe, De beteekenis van het correlatie-onderzoek voor de cramiometrie, Mensch en Maatschappij 1939.
Dr. A. de Froe. Het verband tusschen indices en correlaties. Nederl. Tijdschr, voot Geneeskunde, 84, 572-574 (1940).
Dr. A. De Froe, Die Bestimmung der Kopthöhe am lebenden Menschen. Acta Neerlandica Morphologiae, 1942.

Physics. - A calculation of the viscosity and the sedimentation velocity for solutions of large chain-molecules taking into account the hampered flow of the solvent through each chain molecule. By H. C. Brinkman.

## ERRATUM.

To our regret the above-mentioned calculation contains some numerical errors. The following corrections should be made.

Formula (17a) for the sedimentation constant should read:

$$
\begin{equation*}
f_{\mathrm{t}}(\lambda R)=\frac{1-\frac{T}{\lambda R}}{1+\frac{3}{2 \lambda^{2} R^{2}}-\frac{3 T}{2 \lambda^{3} R^{3}}} \tag{17a}
\end{equation*}
$$

Formula (18a) for the viscosity:

$$
\begin{equation*}
f_{2}(\lambda R)=\frac{1-\frac{3 C}{\lambda R}+\frac{3}{\lambda^{2} R^{2}}}{1+\frac{10}{\lambda^{2} R^{2}}-\frac{30 C}{\lambda^{3} R^{3}}+\frac{30}{\lambda^{4} R^{4}}} \cdot \cdots \cdot \tag{18a}
\end{equation*}
$$

These results agree with those obtained by Debije (private communication to the author by prof. Debije)
For the viscosity the correction has only a slight influence on the numerical results. In the interval $30<N<200$ the function $f_{2}(N) \sqrt{N}$ becomes larger by an approximately constant amount of 0,04 . The exponent in the modified Staudinger law (24) becomes $\alpha=0,71$.
In the same interval of $N$ the function $\sqrt{N} / f_{1}(N)$ for the sedimentation should be multiplied approximately by a factor 0,7 . In the applications this smaller value for the sedimentation results in a smaller value of the length $A$ of a statistical chain-element. For nitrocellulose $A=42$, for cellulose-acetate $A=50$.

