Astronomy. — *On the Temperature of Cometary Nuclei.* By M. G. J. MINNAERT.

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It is generally accepted, that comets are formed when some blocks of meteoritic matter come in the vicinity of the Sun and are heated by its radiation. The escaping gases are repelled by radiation pressure and form a tail, which scatters the solar light by optical resonance. Along different lines of approach it has been shown that the nuclei are probably composed of a considerable number of blocks, perhaps of the order of 100—200 meter [1]), perhaps of the order of kilometers [2]).

For a good understanding of the processes in comets it is important to ascertain the temperature in their nuclei. ZANSTRA, ORLOV and others have for simplicity pictured each meteoritic block as a sphere, which is in stationary condition at each moment, because it radiates an amount of energy equal to what it receives from the sun. Two extreme cases may be considered. (1). If the conductivity is considerable, the size of the nucleus small, the changes in the radiation slow, the rock will assume one and the same temperature throughout and we have: $T = \frac{276^\circ}{\sqrt{r}}$, r being the distance to the Sun in A.U. (2). If the conductivity is infinitely small: $T = \frac{392^\circ}{\sqrt{r}} (\cos \vartheta)^{1/4}$ in each surface element of which the normal is inclined under an angle ϑ to the rays of the Sun; in taking a mean value of $\cos \vartheta$, it should be remembered that $\cos \vartheta$ should be put equal to zero when $\vartheta > 90^\circ$.

Subsequently WURM showed [2]) that the comet remains such a short time in the vicinity of the Sun, that a stationary condition is not reached; he computed, that a nucleus of 100 km would assume a temperature of 50° K only after 10 years, which of course is very much longer than the few weeks of a perihelion passage; and his conclusion was, that the nucleus will have actually a temperature of about 10° K only. WURM's argument definitely shows that the heating of the nucleus is to be considered as a dynamic process; but his conclusions would only be valid if the heat was instantaneously distributed through the whole of the nucleus. This manifestly is not true, the conductivity of rocks being very small indeed.

In order to get more exact results, we will compute the temperature distribution in the block at each depth and at each point of the orbit. Owing to the uncertainty considering the composition of the nucleus, we consider successively a stone meteorite and a nickel-iron meteorite. Moreover, we distinguish between the case of periodic comets with elliptic orbits and the case of parabolic or quasi-parabolic orbits.

[1]) B. VORONTSOV-VELYAMINOV, Ap. J. **104**, 226, 1947.

[2]) K. WURM, Vierteljahresschrift d. Astr. Ges. **78**, 18, 1943.

1. *A stone meteorite in an elliptic orbit.*

The nucleus is continuously heated by a radiation which, though faint, is at least equal to its aphelion value. In the long run, this heat will penetrate the whole block. Therefore by applying the equilibrium calculation at aphelion distance, the following *minimum temperatures* may be found:

ENCKE	142° K
PONS–WINNECKE	122° K
TUTTLE I	90° K
PONS–FORBES	68° K
HALLEY	49° K

Let us consider HALLEY's comet as a representative case. The elements are: $T = 76^y{,}02$, $e = 0{,}9673$, $a = 17{,}945$ A.U.; the greatest and the smallest radius vector are respectively 35,31 and 0,587 A.U. The nucleus is pictured as a stoney sphere with a diameter of the order of 1 km. We assume that its material has a specific weight 2,5. The heat conductivity of granite varies between 0,003 and 0,008 cal cm^{-1} sec^{-1} $degree^{-1}$; for acid rocks 0,004 seems a suitable value, for basic rocks 0,006; we will assume 0,005. As the conductivity of quartz glass increases with temperature, while that of ordinary quartz decreases, we will consider this coefficient as independant of T. The specific heat is about the same for quartz, quartz glass, ordinary glass and granite, and varies in the same way with temperature. We assume a smooth curve through the points:

$T = 70^\circ$	120°	220°	330°	440°
$c =$ 0,04	0,08	0,14	0,18	0,22

Because of this strong temperature-influence, the equation of heat conduction can not easily be solved by the classical methods. A numerical solution has also the advantage that the results are ready for use and that the tedious substitutions of actual numbers in the analytical formulae are avoided. We will find that the temperature variations are only important up to a depth of some tenths of meters; this being probably only a small fraction of the radius, we will treat the problem as a *plane problem* and apply the ordinary equation of conduction:

$$\frac{\partial T}{\partial t} = \frac{k}{c\varrho} \frac{\partial^2 T}{\partial x^2} \qquad (1)$$

For the differentials we substitute finite differences. Let the nucleus be composed of plane strata, each 20 m thick, their representative central layers being located at depths of 10, 30, 50 ...... meter, and their temperatures being T_1, T_2, T_3, ... We investigate how the temperatures have changed after an interval of one year = 3,15.10⁷ sec.

$$\triangle T_n = \frac{\triangle t}{(\triangle x)^2} \cdot \frac{k}{\varrho c} \cdot (T_{n+1} + T_{n-1} - 2\,T_n) = \frac{a}{c}(T_{n+1} + T_{n-1} - 2T_n), \quad (2)$$

where we have put

$$a = \frac{\triangle t}{(\triangle x)^2} \cdot \frac{k}{\varrho} = \frac{3{,}15.10^7}{4.10^6} \cdot \frac{0{,}005}{2{,}5} = 0{,}016.$$

It would be tedious if we had to follow the comet during many revolutions till stationary conditions have developed. A more practical method is to

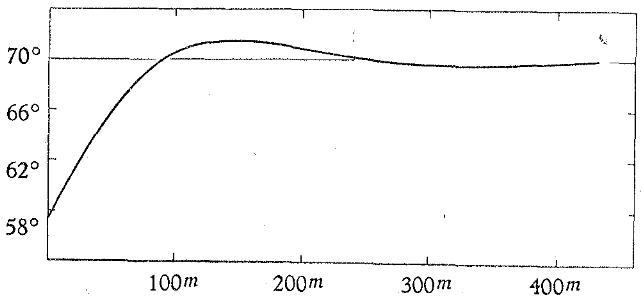

Fig. 1. Temperature distribution in o stone nucleus after one revolution of 76 years.

take as a start a rough estimate of the internal temperature which will be finally approached. This temperature will be somewhat higher than the minimum of 49°, say 70° K.

We assume that, at our start near aphelium, this temperature prevails in all the layers. The exterior layer absorbs solar radiation and radiates infrared radiation to space; let these amounts of energy be exchanged discontinuously at the beginning of each year. The solar radiation at the distance of the earth is known as the solar constant; from this we derive the radiation in our units at a distance r:

$$\frac{1{,}90.3{,}15.10^7}{60.r^2.4} = \frac{250\ 000}{r^2} \text{ cal cm}^{-2} \text{ year}^{-1}.$$

The factor 4 has been introduced because the nucleus is probably in spinning motion, so that the radiation, falling on an effective cross section $\pi \varrho^2$, is actually distributed over an area $4\pi\varrho^2$. The emission of the nucleus amounts to $1{,}37.10^{-12}.3{,}15.10^7\ T^4 = 4330\left(\frac{T_0}{100}\right)^4$ cal cm⁻² year⁻¹,

T_0 being the temperature of the external radiating layer. The difference between the input and the output of radiation is the heat absorbed by the nucleus. We should take into account that such a rock has an appreciable albedo; however, by comparison with the planetoids, we estimate that this will be only of the order of 0,10, and may therefore be neglected.

We illustrate the method of computation by an example (table 1). On

TABLE I. — Temperature distribution in a stone meteorite for $t = 34^y$ and $t = 35^y$.

t	0	10	30	50	70	90	110	130	150	170	190	210 meter
34,0	67,10	65,97 } 68,76 }	64,41	63,76	64,40	65,44	66,51	67,44	68,20	68,78	69,20	69,49 m.
			+1,46	+0,51	+0,16	+0,01	—0,06	0,07	—0,07	—0,06	—0,05	—0,01 m.
35,0	70,70	68,76 } 73,05 }	65,87	64,27	64,56	65,45	66,45	67,37	68,13	68,72	69,15	69,45 m.
			+2,20	+0,74	+0,24	+0,04	—0,03	—0,06	—0,07	—0,06	—0,05	—0,04 m.

the first line we find the mean depth of each layer. On the second line, the temperatures as they are found 34 years after the aphelion. On the third line, the temperature changes during the next year, as computed from equation (2), for all the layers from 40 m on. The fourth line gives the temperatures for the epoch 35 years after the aphelion. The temperature of the first layer is supposed to increase discontinuously by $\triangle T_1$, at the epoch 34,0; if this increase is known, $\triangle T_2$ can also be found. $\triangle T_1$ is determined by the requirement, that the total gain of energy of the block is equal to the excess of the absorption over the emission. This computation will be given now in some detail.

In the interval considered, the mean distance of the comet is 11,5 A.U. and the irradiation 1890 cal. The emission of the nucleus depends on its surface temperature, which could be taken to be 65°,97. But a better approximation is obtained by extrapolating the temperature till the actual surface. A parabolic formula $T = a + bx + cx^2$ is fitted to the points T_1, T_2, T_3, corresponding with depths of 10, 30, 50 m; we find $T_0 = 1{,}875\ T_1 + 0{,}375\ T_3 - 1{,}25\ T_2 = 67°{,}1$. Moreover, judging from the increase between 33 and 34^y, we estimate that the temperature will rise over 1,9 ° during the next half year. The emission therefore should be computed for $T_0 = 69°{,}1$; it amounts to 990 cal.

So the block will have to absorb $I = 1890 - 990 = 900$ cal., which requires

$$I = \varrho \triangle x\,[c_1 \triangle T_1 + c_2 \triangle T_2 + \sum_3^\infty c_n \triangle T_n] =$$

$$= \varrho \triangle x \left[c_1 \triangle T_1 + c_2 (T_1' + T_3 - 2\,T_2) \frac{\triangle t}{(\triangle x)^2} \frac{k}{\varrho c^2} + \sum_3^\infty c_n \triangle T_n \right].$$

where $T_1' = T_1 + \triangle T_1$ is the temperature of the first layer, increased by $\triangle T_1$, at the beginning of the year. We solve the equation for $\triangle T_1$:

$$\left.\begin{aligned} \triangle T_1 &= \frac{I - \varrho \triangle x\,[a\,(T_1 + T_3 - 2T_2) + \Sigma c_n \triangle T_n]}{\varrho \triangle x\,(c_1 + a)} = \\ &= \frac{I - 5000\,[0{,}016\,(T_1 + T_3 - 2T_2) + \Sigma c_n \triangle T_n]}{5000\,c_1 + 80} \end{aligned}\right\} . \quad (3)$$

In many cases, the specific heat is equal to 0,04 at all depths except in the uppermost two strata; we then use the simpler formula:

$$\triangle T_1 = \frac{I - 200\,[0{,}40\,(T_1 + T_3 - 2T_2) + \sum_n^\infty \triangle Tn]}{5000\,c_1 + 80}.$$

At the epoch $t = 34_y$,

$$\triangle T_1 = \frac{900 - 200\,[0{,}40\,.\,0{,}91 + 0{,}25]}{200 + 80} = 2^\circ{,}79 \text{ and } \triangle T_2 = 1^\circ{,}46\,.$$

For $t = 35^y$ the surface temperature is found to be 70°,70. Apparently we should have computed the radiation for a mean temperature of ½ (67°,1 + 70°,7) = 68°,9 instead of 69°. This difference will be neglected; in other cases it would be possible of course to repeat the computation and to find a better approximation.

Near the perihelion, the temperatures change so rapidly that a subdivision in smaller intervals is required; we simply divide $\triangle t$, I and a by an arbitrary factor. In this way, intervals of ¼ year and ⅛ year were introduced.

The results of the whole computation are summarized in the diagrams 1, 2 and 3. Fig. 1 shows the temperature variation in the uppermost layers

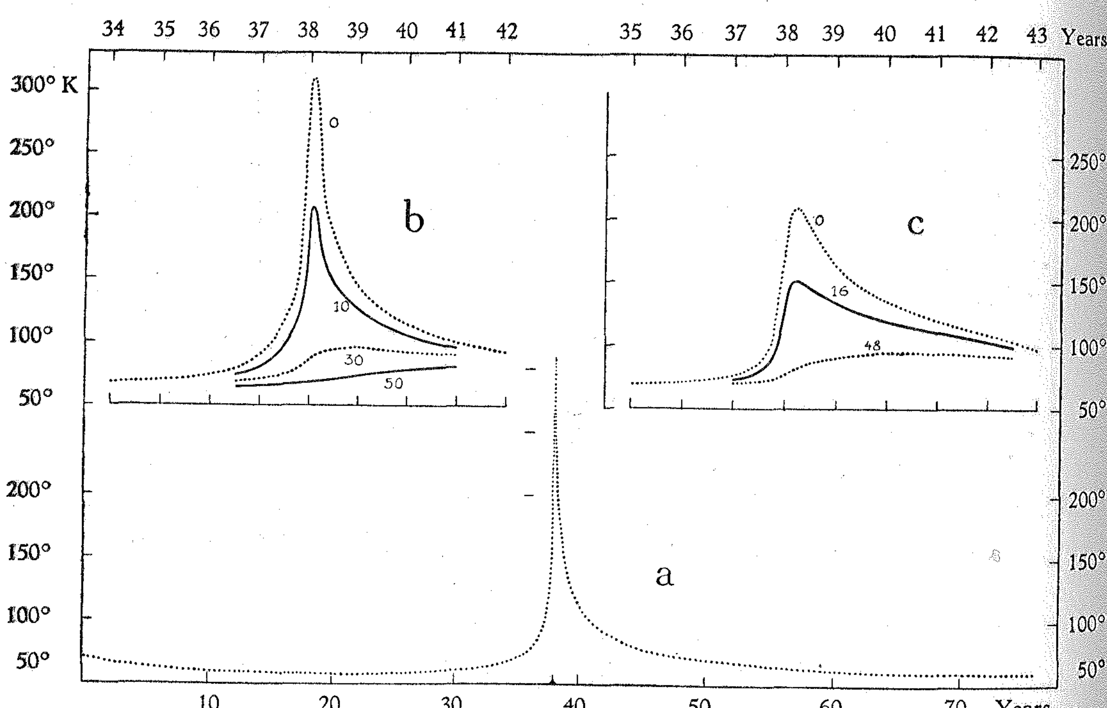

Fig. 2. Temperature distribution in a nucleus, following the orbit of HALLEY's comet. $T = 76$ years; perihelion distance = 0,59 A.U. *a*) Surface temperature of stone nucleus; *b*) temperature in deeper layers of a stone nucleus near perihelion; *c*) the same for an iron nucleus.

after one revolution. It appears that at the end the temperature distribution is not completely identical to that from which we started. In the outer layer the temperature is lower by about 10°; the difference decreases inward, and between 100 and 250 m the temperature is slightly higher than it was originally; this is the heat wave which originated near perihelion and which gradually reached greater depths. Altogether the heat content has decreased a little; apparently the temperature of 70° from which we started was somewhat too high, 60° would have been better. We will see later that such a difference has no influence at all on the surface temperature.

This surface temperature reaches much higher values than WURM had assumed (fig. 2a). Near the perihelion it rises to 305° K; of course this maximum value is subject to a considerable uncertainty, because the changes in radiation are so quick. The peak looks very sharp; but fig. 2b, where it is reproduced on an elarged scale, shows that during more than 8 months the surface temperature exceeds 200°, the temperature at 10 m exceeding 150°. The peak is asymmetric; the penetration of the heat wave has a velocity of the order of 10 m a year near the surface, 5 m a year at a depth of 100 meter. It is clear that a long time will elapse before a stationary condition is reached.

In fig. 3 we see how the radiation absorbed and the rediation emitted vary during a perihelion passage. The absorbed radiation is symmetric with

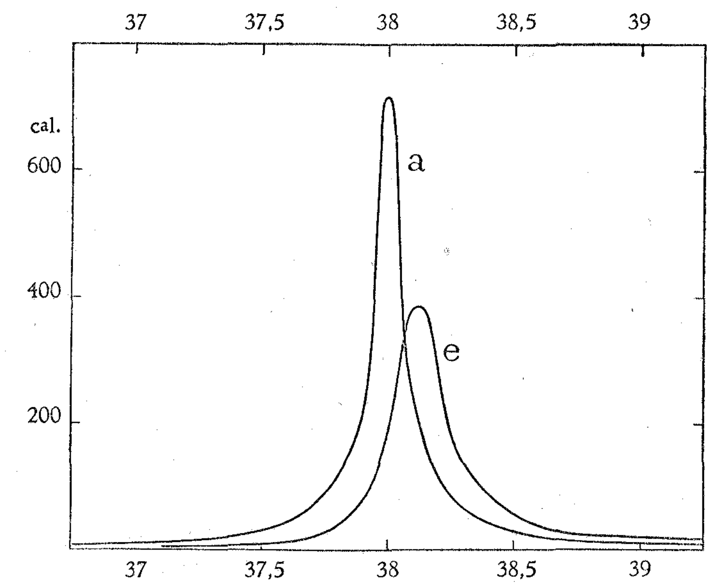

Fig. 3. Radiation absorbed (*a*) and emitted (*e*) during a perihelion passage by a stone nucleus. Unit: 1000 cal cm^{-2} year^{-1}.

respect to the perihelium epoch, the emitted radiation shows a lag of about ⅛ year.

In order to investigate the effect of a more detailed subdivision of the layers on the numerical integration, the computation was repeated for the years near the perihelion with layers only 10 meters thick. A maximum temperature of 310° was found, in good agreement with our first result.

2. *An iron-nickel meteorite in an elliptic orbit.*

In order to investigate the influence of the composition, we assume the same orbit as for the stone meteorite. The conductivity of pure iron would be of the order of 0,200, about 40 times higher than that of stone; but for allois it is considerably less: for nickel-steel with 30 % nickel, the coefficient is 0,030. We will adopt this last figure, but a better value could be determined by direct measurements on meteorites.

The specific heat of iron allois differs little from that of pure iron; this one we will assume, because its temperature dependence is better known.

$T = 60°$	70°	80°	100°	125°	150°	200°	250° K
$C =$ 0,025	0,032	0,039	0,050	0,065	0,078	0,095	0,103

The density is taken to be 8. The fraction $\frac{k}{c\varrho}$ is $\frac{0{,}030}{0{,}032 . 8} = 0{,}121$ at 70°, thus about 2,5 times as great as for stone.

In order to abbreviate the computation, we remark that the important temperature variations occur near the perihelion. We start from a rough estimate for the temperature distribution at the epoch 34^y, and apply our integration method to the years 34—44.

From the form of the differential equation, it is seen that the same relation holds, if $\frac{c}{k\varrho}$ is multiplied by n and the scale of depths by $\sqrt{n}$. Therefore the temperature distribution of the stone meteorite would apply to iron, if the strata were taken to be $20 \sqrt{2{,}5} = 32$ m thick. This however would not satisfy the boundary conditions: the irradiation has been the same as for the stone meteorite, the difference in emission is negligible, so the heat content must be also the same. Now this is equal to $\Sigma c\varrho\,(T - 70)\,\triangle x = 0{,}032.8.3200.$ $(T - 70) = 820\,(T - 70)$ i.e. 3,9 times that of the stone meteorite. We therefore divide all temperature differences by 3,9, the relations still satisfying the differential equation. The estimated temperatures at 34^y deviate so little from 70° that an uncertainty in our approximations is immaterial for the further results.

The temperature peak is now computed as before, the results being summarized in fig. 2c. As a consequence of the greater heat capacity, all temperatures are lower than for stone. The surface temperature reaches now a maximum of 210°; during 8 months it exceeds 175°. Due to the greater conductivity, the temperature penetrates more rapidly in deeper layer; this is especially clear from a comparison between the layer at 50 m for stone and the layer at 48 m for iron. The asymmetry and the after-action of the heating are very conspicuous, because the deeper layers play the role of a heat reservoir.

3. *A stone meteorite in a parabolic orbit.*

For a comparison with elliptic orbit, we select a parabola having the same perihelion distance as the ellipse of HALLEY's comet (0,587 A.U.).

At an infinite distance from the Sun, the nucleus will have a temperature near the absolute zero. It receives a very faint radiation but during a considerable time, because the motion of the comet is very slow.

We will assume that the energy absorbed until 40^y before the perihelion has communicated to the whole of the nucleus a temperature of 10° K. Doing this, we certainly will have rather exaggerated the difference between the elliptic and the parabolic case. We now compute the temperature distribution as in the parabolic orbit. At first the specific heat is very low and we have to consider very thick strata; 4 years before the perihelion the temperatures are already so high, that layers of normal thickness (20 m) can be introduced.

The results are surprising. Though we started from a much lower temperature, the surface temperature near the perihelion has nearly reached that of the elliptic orbit, and 1^y after the perihelium the difference has practically disappeared (fig. 4). This is understandable, because near the

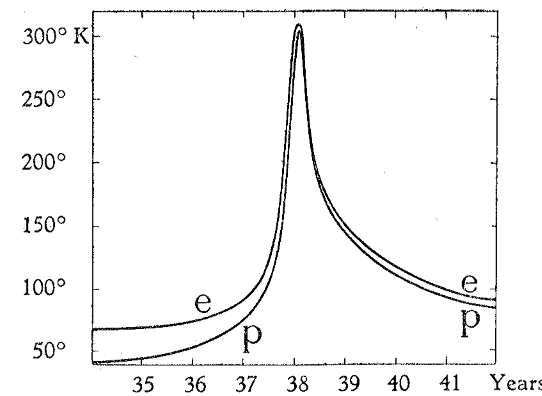

Fig. 4. Surface temperatures in an elliptic and in a parabolic orbit with the same perihelion distance.

Sun the parabolic motion is not very different from the elliptic motion; moreover, the emission is so sensitive to the temperature, that already small differences in T_0 are sufficient to compensate for differences in absorption. For the same reasons, a metallic meteorite would show the same surface temperatures near its perihelion in a parabolic as in an elliptic orbit.

Our result is also important, because it shows that the surface temperatures for an elliptic or parabolic orbit would have been found practically the same if we had assumed a different temperature at aphelion.

4. The desorption process by which the gases are liberated from the nucleus has been treated in a brilliant paper by LEVIN [3]). According to well-known laws, the quantity of gas liberated amounts to $n = n_0 \sqrt{\frac{kT}{2\pi m}}\, e^{-\frac{L}{RT}}$, L being the heat of adsorption. From provisional considerations on the

[3]) Russ. A. J. **21**, 48, 1943.

temperature of the nucleus, he derives that $T = \frac{350^\circ}{\sqrt{r}}$ and writes:

$$n = n_0 \sqrt{\frac{kT_0}{2\pi m}}\, r^{-\frac{1}{4}}\, e^{-\frac{L\sqrt{r}}{350R}} \quad (3)$$

The brightness of the comet, being due mainly to scattering of these gases, can be easily shown to be proportional to the number of gas molecules. By investigating the brightness of the comet at different distances from the Sun, the heat of adsorption is directly determined from the observations.

By applying this theory to ENCKE's comet, values are found of about 7000 cal./mol., in excellent general agreement with laboratory results; for HALLEY's comet a somewhat higher value of 8500 cal./mol. is derived.

It is now possible to improve these interesting results by making use of our computations. There is some doubt whether the gases are liberated only at the surface itself, or whether a layer of some depth, say the first 20 m layer, is involved. We compute for both cases the product $T\sqrt{r}$ about the perihelion:

t	r	T_0	$T_0\sqrt{r}$	T_1	$T_1\sqrt{r}$
37	5,12	95	214	84	191
37,50	2,87	131	222	110	186
38	0,59	298	229	203	154
38,50	2,87	175	296	141	239
39	5,12	130	294	117	204
		mean:	251		208

It is clear that the product $T\sqrt{r}$ should be taken rather equal to 200 or 250 than to 350. This gives an improved estimate for the adsorption heat, which proves to be 5000—6100 cal./mol. for HALLEY's comet. It would be 4000—5000 cal. for ENCKE's comet, if the same value of $T\sqrt{r}$ may be applied to an orbit so different in size.

5. The asymmetry in the temperature curve suggests, that in studying the brightness of comets as a function of the radius vector, we should carefully distinguish between observations before and after the perihelion. Unfortunately this has been often neglected; even the classic book of HOLETSCHEK does not make the difference. For ENCKE's comet, a provisional comparison is possible, and it is found that the brightness after the perihelion is rather less than before. This proves that the temperature is not the only controlling factor, but that there is also the effect of exhaustion, which is already apparent in the increased adsorption heat after the perihelion.

6. WURM required very low temperatures of the nucleus in order to explain the extraordinary sharp accessory rays which are observed in many comets. According to his calculations, any appreciable thermal motion should produce a blur.

Since however the surface temperature of the nucleus appears to be so much higher than he expected, we must look to another explanation for these rays. It looks very probable that their gases have considerably greater velocities than these in the tail itself; this would be in excellent agreement with their straight delineation. A factor 5 in the velocity would not seem impossible and would increase the allowed temperature limit by a factor 25.

An additional consideration in favour of the high nuclear temperatures is the observed liberation of gases itself. This could never take place at 10° K.

Summary.

The nucleus of a comet is pictured as a swarm of meteoritic blocks. For such a block the temperature is calculated by numerical methods as a function of the position in the orbit and for layers at different depths. We consider successively: 1. a stone meteorite in the elliptic orbit of HALLEY's comet; 2. an iron meteorite in the same orbit; 3. a stone meteorite in a parabolic orbit with the same perihelion distance. The maximum surface temperature near the perihelion proves to be 310° K.

Making use of this result, the adsorption heat of cometary gases is found to be about 5000 cal./mol. The sharp accessory rays observed in cometary tails cannot be attributed to low surface temperatures of the nucleus.