Mathematics. — *Topological characterization of all subsets of the real number system.* By J. DE GROOT. (Communicated by Prof. L. E. J. BROUWER.)

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1. Let M be an arbitrary separable metric space (i.e., a normal space with countable base) and N a subset of M; *under which conditions will N be homeomorphic with a subset of the R_1 (the ordered system of real numbers)?*

The answer will be given in 2., Theorem II.

For *compact* subsets N and a *plane* set M this problem is not difficult and has been solved long ago. To this end a *M.K.-set* (MOORE–KLINE) is defined as a subset K of M, which satisfies the following conditions:

1⁰. K is compact,
2⁰. the (quasi)components [1]) of K are points of (closed) simple arcs,
3⁰. no interior point of a simple arc A is limit-point of the set $K—A$: the arcs are "free".

Now according to MOORE–KLINE [1] (comp. also [2]) a M.K.-set is a subset of a simple arc (belonging to M). In particular K is homeomorphic with a subset of R_1. Generalizations of these problems are treated by ZIPPIN [3].

If, conversely, K has been proved to be homeomorphic with a subset of R_1, then this does not show, however, that it is possible to pass a simple arc, lying in M, through K. On the other hand, the above-mentioned theorem of MOORE–KLINE has been proved only for special spaces M (for instance, M is a plane). Therefore our first step will be to prove that the M.K.-spaces, considered as subsets of an arbitrary separable metric space M, are topologically identical with the compact subsets of R_1.

The proof follows a method which is totally different from the one used for the mentioned MOORE–KLINE problem.

Theorem I. *The M.K.-subsets K of a separable metric space M are topologically identical with the compact subsets of R_1.*

Proof. It is evident that a compact subset of R_1 is a M.K.-space. Conversely, we shall have to prove that a M.K.-set K is topologically equivalent with a subset of R_1. According to 3⁰. K contains only countably many simple arcs A_1, A_2, A_3, ... ad inf. The endpoints of A_i we call a^1_i and a^2_i.

1) In compact separable metric spaces the components are identical with the quasi-components.

The diameters α_i of A_i are converging to zero; this may easily be seen by applying a theorem of ZORETTI, which says that (for a compact set) the limes superior of a sequence of connected sets is connected if the limes inferior is not vacuous. Let the A_i be counted in such a way that $\alpha_{i+1} \leq \alpha_i$ $(i = 1, 2, \ldots)$. We consider the pointset K', which is formed by removing all A_i out of K except the endpoints a^1_i and a^2_i. K is a 0-dimensional compactum. Now we define a system of coverings of K'. The first covering consists of K' itself: $K' = U_1$. Then we divide K' into a finite number of disjoint closed subsets $U_{11}, U_{12}, \ldots, U_{1j_1}$, such that

1⁰. a^1_1 and a^2_1 do not belong to the same U_{1j},
2⁰. a^1_i and a^2_i always belong to the same U_{1j} for fixed $i > 1$,
3⁰. the diameter of every U_{1j} is less than $\alpha_2 + {}^1/_2$.

From the mentioned properties it follows that this covering is possible.

Now every U_{1j} $(j=1,2,\ldots,j_1)$ is subdivided analogically, i.e. a covering of U_{1j} is determined in disjoint closed subsets $U_{1j1}, U_{1j2}, \ldots, U_{1jk_j}$ such that

1⁰. $a^1_{m_j}$ and $a^2_{m_j}$ do not belong to the same U_{1jl}; here m_j is the smallest index for which $a^1_{m_j}$ and $a^2_{m_j}$ belong to U_{1j}. Should U_{1j} contain a^1_1 (respectively a^2_1), then this point is not in the same U_{1jk} with $a^1_{m_j}$ or $a^2_{m_j}$. Should no pair of endpoints belong to U_{1j}, then this condition (and condition 2⁰. as well) is satisfied in a trivial way.

2⁰. If a U_{1jl} contains one endpoint a^1_i, then it also contains a^2_i for every $i > m_j$.

3⁰. The diameter of every U_{1jk} is less than $\alpha_{m_{j+1}} + {}^1/_3$ (should U_{1j} contain no endpoints then we put $\alpha_{m_{j+1}} = 0$).

In this manner the coverings

$$\{U_1\}\ ,\ \{U_{1r_1}\}\ ,\ \{U_{1r_1r_2}\}, \ldots$$

of K' are formed. It is clear how by means of induction the covering $\{U_{1r_1r_2\ldots r_{n+1}}\}$ is defined. In doing this we take care that a $U_{1a_1a_2\ldots a_n}$ is divided into finitely many disjoint closed sets $U_{1a_1a_2\ldots a_n a_{n+1}}$ $(a_{n+1}=1,2,\ldots,s)$, such that a^1_p, a^2_p (where p is the smallest index for which a^1_p and a^2_p belong to $U_{1a_1a_2\ldots a_n}$) and a possibly appearing point a^1_q (resp. a^2_q) (where a^2_q (resp. a^1_q) does not belong to $U_{1a_1\ldots a_n}$) are lying in three separate sets $U_{1a_1a_2\ldots a_n a_{n+1}}$, such that further a^1_t and a^2_t $(t > p)$ are in the same $U_{1a_1a_2\ldots a_n a_{n+1}}$, while, lastly, the diameter of the sets $U_{1a_1a_2\ldots a_n a_{n+1}}$ is less than $\alpha_{p+1} + \dfrac{1}{n+2}$ $\left(\text{eventually } \dfrac{1}{n+2}\right)$. Such a division is apparently possible.

Now we are going to map K' topologically on a subset of the segment [01] of the real axis. Therefore we let K' correspond with $[01] = I_1$. Consider the second covering $\{U_{1r_1}\}$ $(r_1 = 1, 2, \ldots, j_1)$.

We consider j_1 subsequent closed intervals $I_{11}, I_{12}, \ldots, I_{1j_1}$, lying disjoint on [01], each of which has a length $< 1/2$. We map

$$U_{1j} \longleftrightarrow I_{1j}.$$

Let U_{11} contain a point a_1^1 and U_{12} a point a_1^2; then we map a_1^1 on the right endpoint of I_{11} and a_1^2 on the left endpoint of I_{12}.

Now we consider, whithin I_{1j}, k_j disjoint closed intervals $I_{1j1} \ldots I_{1jk_j}$, each of a length $< 1/3$, where and endpoint, which is already an image, is contained (as an endpoint) in an interval I_{1jl}.

Let U_{111} contain point a_1^1, U_{112} point $a_{m_1}^1$, and U_{113} point $a_{m_1}^2$, then we let the sets U_{11s} and I_{11s} $(s = 1, 2, \ldots k_1)$ correspond one-to-one, in such a manner that the interval I_{11t}, which contains the image of a_1^1, corresponds with U_{111}, while U_{112} and U_{113} correspond with two immediately subsequent intervals I_{11s_1} and I_{11s_2}, where, moreover, $a_{m_1}^1$ is mapped on the right endpoint of I_{11s_1} and $a_{m_1}^2$ on the left endpoint of I_{11s_2}. Analogically the sets U_{12s} and I_{12s} are mapped on each other, where the points a_1^2, $a_{m_2}^1$ and $a_{m_2}^2$ appear. In general the sets U_{1jk} $(k = 1, 2, \ldots, k_j$; j fixed) are corresponding one-to-one with the intervals I_{1jk}, in such a way that the two U-sets containing $a_{m_j}^1$ and $a_{m_j}^2$ correspond with immediately subsequent I-intervals, where $a_{m_j}^1$ and $a_{m_j}^2$ are mapped on the two middle endpoints of the I-intervals. By means of induction the system of intervals $\{I_{1r_1r_2 \ldots r_n}\}$ of a length $< 1/n$ is generally defined in analogous way and corresponding one-to-one with the sets of $\{U_{1r_1r_2 \ldots r_n}\}$. It is essential that the already mapped endpoints a_i^1 and a_i^2, which appear successively while the construction is made, all belong to disjoint U-sets and therefore that the images belong to disjoint I-intervals, while moreover there is no I-interval or part of an I-interval between the images of a_i^1 and a_i^2. Every point of K' is defined as an intersection of exactly one sequence

$$U_1 \supset U_{1a_1} \supset U_{1a_1a_2} \supset \ldots \ldots \qquad (1)$$

while, conversely, every such sequence (1) defines exactly one point of the compact set K'.

The corresponding I-sequence

$$I_1 \supset I_{1a_1} \supset I_{1a_1a_2} \supset \ldots \ldots \qquad (2)$$

determines exactly one point of the interval [01] as intersection. The set of all those points obviously is homeomorphic with K'. If we add to this set all the open intervals between the images of a_i^1 and a_i^2 then the thus formed subset of [01] is homeomorphic with K, since every A_i may be mapped topologically on the corresponding interval of [01] connecting

the images of a_i^1 and a_i^2), which mapping together with the mapping of K' gives the required topological mapping of K.

2. Our proper purpose is the characterization of arbitrary pointsets of the R_1.

To this end we define a *generalized M.K.-space* as a separable metric space K, which satisfies the conditions:

1⁰. the quasicomponents of K are points or open, half open or closed simple arcs (i.e., homeomorphic with the sets $\{x\}$ defined by $0 < x < 1$, or $0 \leq x < 1$, or $0 \leq x \leq 1$);

2⁰. the arcs are free; no interior point of a simple arc A is limitpoint of the set $K - A$.

A topological space is, as is known, *semicompact* if every point has arbitrarily small neighbourhoods with compact boundaries. Now our contention is that the semicompact generalized M.K.-spaces K are homeomorphic with the subsets of R_1.

Remark. The condition of semicompactness cannot be omitted, as will be shown by the following examples.

Example 1. Let P be a subset of the plane containing the following points (x, y):

(o, o)

$\left(x, \frac{1}{n}\right)$, where $(0 \leq x \leq 1)$, $(n = 1, 2, \ldots)$.

P is a generalized M.K.-space, but not semicompact (in point $(0, 0)$).

P is not homeomorphic with a subset of R_1, as may easily be seen.

The condition of semicompactness cannot be omitted even in the case that all quasicomponents consist of one point, as is shown by spaces constructed (to another purpose, however) by SIERPIŃSKI [8] and MAZURKIEWICZ [9]:

Example 2. For every natural number n there exist totally disconnected (i.e., the quasicomponents consist of one point) n-dimensional separable spaces.

These spaces are obviously not homeomorphic with a subset of R_1.

The theorem becomes untrue if (in 1⁰.) components are considered instead of quasicomponents, as is shown by

Example 3. Let S be a subset of the plane containing the following points (x, y):

$\left(x, \frac{1}{n}\right)$ where $(0 \leq x \leq 1)$, $(n = 1, 2, \ldots)$,

$(x', 0)$ where $(0 \leq x' \leq 1$; x' rational$)$.

S is a semicompact space satisfying the modified condition 1⁰. (components are considered instead of quasicomponents) and condition 2⁰. Yet apparently S is not homeomorphic with a subset of R_1.

Theorem II. *The semicompact generalized M.K.-spaces are — topologically spoken — identical with the subsets of the* R_1 [2]).

2.1. **Preliminary remarks.**

The proof may be given by a — be it far-reaching — generalization of the proof of Theorem I. This is, however, a very long and complicated way.

Therefore we shall give a proof, which implies a direct use of Theorem I: we shall embed the given semicompact generalized M.K.-space in a M.K.-space; from the homeomorphism of this last space with a subset of R_1 it will follow immediately that the former space is also homeomorphic with a subset of R_1.

For our proof we shall use a number of different theorems and notions which — in order to keep the proof brief — will be mentioned here.

(α) Every semicompact separable metric space M may be compactified to a compact separable metric space $\overline{M}$ by an 0-dimensional set $\overline{M}-M$ (where M is everywhere dense in $\overline{M}$); conversely, every subset M of a compactum $\overline{M}$, for which $\overline{M}-M$ is 0-dimensional, is semicompact (see [5], p. 45).

If a space M is the sum of a collection of disjoint closed sets A:

$$M=\Sigma A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

a new space $^*M$ — *the decomposition space* of M corresponding with the decomposition (1) — is formed by identifying (in the well-known way) each set A to one point $^*a$. — The decomposition (space) is called *upper semi-continuous* [3]) (comp. for instance WHYBURN [7], p. 122 a.f.), provided that for every neighbourhood $U=U(A/M)$ of an arbitrary A in M may be found a neighbourhood $V=V(A/M)$ such that any A-set intersecting V is totally contained in U:

$$A\cdot V\neq 0\rightarrow A\subset U.$$

Now

(β) Every upper semi-continuous decomposition space $^*M$, corresponding with a decomposition (1) of a separable metric space M in disjoint compact sets A, is a separable metric space (comp. for instance WHYBURN [7], p. 123, 124).

We also use another partition-space of M, the so-called *space of quasicomponents* $Q(M)$ of M, corresponding with that partition (1) of M for which the sets A are exactly the quasicomponents of M.

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[2]) This theorem was mentioned in [4] without proof. In that paper was, in defining semicompact generalized M. K.-spaces, erroneously spoken of components instead of quasicomponents.

[3]) The upper semi-continuous decomposition of MOORE is identical with the "continuous decomposition" (stetige Zerlegung) of ALEXANDROFF (comp. [10], p. 66—67).

The neighbourhoods in $Q(M)$ of a point $^*a$ are defined as follows: consider the quasicomponent A of M, corresponding with point $^*a$ in $Q(M)$; take an arbitrary set S, both open and closed, with $M\supset S\supset A$; the set of points $^*a$ corresponding with all quasicomponents $A\subset S$ is a neighbourhood of $^*a$ in $Q(M)$. Therefore $Q(M)$ is always 0-dimensional.

(γ) There be given a separable metric space M and the partition of M in its quasicomponents: $M=\Sigma A$. If for every neighbourhood $U=U(A/M)$ of an arbitrary A in M may be found a set S, both open and closed, such that

$$A\subset S\subset U, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

then the decomposition space $^*M$, corresponding with $M=\Sigma A$, is upper semi-continuous and $^*M$ and $Q(M)$ are identical.

Further we apply a simple but useful theorem (first used by LEFSCHETZ):

(δ) If f is a continuous mapping of a separable metric space M on a separable metric space $f(M)$, then the subset

$$\sum_{m\subset M} m\times f(m)$$

of the topological product $M\times f(M)$ is homeomorphic with M.

2.2. **Proof of Theorem II.**

It is evident that every subset of R_1 is a semicompact generalized M.K.-space. Conversely, let K be a semicompact generalized M.K.-space. — Every open arc O of K is mapped topologically within an arbitrary closed arc $O'\subset O$. Every half-open arc H of K is mapped topologically on a proper subset H' of H such that the endpoint belonging to H is mapped on itself. Now K is homeomorphic with

$$K'=K-\Sigma O-\Sigma H+\Sigma O'+\Sigma H'.$$

The above defined mapping of ΣO on $\Sigma O'$ and ΣH on $\Sigma H'$ and the identical mapping of the remaining points of K apparently give a topological mapping of K on K'.

Now apparently $K'\subset K$. Consider

$$M=K'+\Sigma\overline{O}+\Sigma\overline{H} \text{ (closures with regard to } K).$$

M is formed out of K' by adding all endpoints of simple arcs (as far as they were not already contained in K').

M satisfies conditions 2⁰. and 3⁰. of 1. Moreover M is semicompact, for M is semicompact in the points of K' as well as K, while M is, apparently, also semicompact in the added set of endpoints $M-K'$. It is not necessary, however, that M is compact.

Consider the decomposition of M in its quasicomponents (points or closed simple arcs): $M=\Sigma A$. We shall prove that for every neighbourhood $U=U(A/M)$ of an arbitrary A there may be found a both open and closed set S such that $A\subset S\subset U$.

Within U is a neighbourhood V with compact boundary such that $A \subset V \subset U$; if the quasicomponent A consists of one point this follows immediately from the semicompactness of M; if A is a simple closed arc, and therefore compact, there may be found a finite number of neighbourhoods O_i (open in M) of points of A, each of which has a compact boundary, such that ΣO_i covers the set A. Now the boundary of the neighbourhood $V = \Sigma O_i$ of A is apparently compact.

For a fixed but arbitrary point p lying in the boundary $R(V)$ of V and for A may be found a division of M into disjoint closed sets M_1 and M_2, such that

$$M = M_1 + M_2 \qquad p \in M_1 \quad , \quad A \in M_2.$$

This is possible because p and A belong to different quasicomponents. M_1 is obviously open and a neighbourhood of p not containing A. For every $p \in R(V)$ may be found such a M_1 and M_2. Since V is compact, V is covered by a finite number of such sets M_1. Then the intersection I of the corresponding M_2-sets is a both open and closed set, just like the intersection $I \cdot U = S$.

Now, however, $A \subset S \subset U$; q.e.d.

Therefore we may apply (γ), from which follows that $^*M = Q(M)$ and therefore upper semi-continuous. Since the sets A are compact we may apply (β), from which it is evident that $^*M$ is separable and metric. Since $Q(M)$ is always 0-dimensional, $^*M$ is a 0-dimensional separable metric space [4]).

According to (α) the semicompact set M may be compactified to a compactum $\overline{M}$ by an 0-dimensional set $\overline{M} - M$. According to a well-known theorem of SIERPIŃSKI the 0-dimensional set $^*M$ is homeomorphic with a subset of the discontinuum D of CANTOR, which we also denote by $^*M$, such that $\overline{^*M}$ (closure in D) is a compact 0-dimensional set.

The mapping $A \to {}^*a$ of M on $^*M$ is an apparently continuous mapping f of M on $^*M = f(M)$.

Consider in the topological product $\overline{M} \times \overline{^*M}$ the subset

$$M' = \sum_{m \subset M} m \times f(m).$$

M' is homeomorphic with M according to (δ).

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[4]) *Remark.* From this part of the proof it follows that we have proved, in general, the following theorem:

Theorem. The space of quasicomponents of a semicompact separable metric space of which the quasicomponents are compact, is an 0-dimensional metric space.

In this theorem we meet two conditions: the semicompactness of the space in question and the compactness of the quasicomponents. If one of these conditions is omitted the theorem becomes untrue. Example 2 gives an instance of a separable metric space with quasicomponents consisting of one point (therefore certainly compact), the space of quasicomponents of which is not a separable metric space, according to [6], p. 134. Semicompact separable metric spaces of which the space of quasicomponents is not a separable metric space, may also be constructed.

We shall prove that $\overline{M'}$ (closure in $\overline{M} \times \overline{^*M}$) is a M.K.-space. Then we shall have attained our object, for the original set K is, as follows from what precedes, homeomorphic with a subset of $\overline{M'}$. By applying Theorem I on the M.K.-space $\overline{M'}$, K becomes homeomorphic with a subset of R_1; q.e.d.

$\overline{M'}$ is obviously a M.K.-space, if we can prove that a quasicomponent of $\overline{M'}$ is exactly a quasicomponent of M' or a point of $\overline{M'} - M'$ [5]), for in that case the conditions 1⁰. and 2⁰. from 1. will be satisfied, and also, since M' is everywhere dense in $\overline{M'}$, condition 3⁰.

The projection, i.e., the mapping g

$$m \times \overline{^*M} \to m \qquad \text{for every } m \in \overline{M}$$

of $\overline{M} \times \overline{^*M}$ on $\overline{M}$ is obviously continuous. g is even topological on M' and its image $g(M') = M$. — Further $g(\overline{M'}) = \overline{M}$.

A point $p \in \overline{M'} - M'$ has an image $g(p)$ belonging to $\overline{M} - M$, for suppose $g(p)$ should belong to M; consider a sequence of points $m'_i \in M'$, converging to p; then $g(m'_i)$ converges to $g(p)$ of M; since g is topological on M', $m'_i \to m'$, where m' is that point of $g^{-1}(p)$ which belongs to M'. This gives a contradiction with $m'_i \to p$.

Therefore

$$g(\overline{M'} - M') = \overline{M} - M.$$

Therefore $\overline{M'} - M'$ totally belongs to the topological product of the 0-dimensional sets $\overline{M} - M$ and $\overline{^*M}$; this product is, however, 0-dimensional, according to a well-known theorem, and therefore $\overline{M'} - M'$ is also 0-dimensional.

Consider, on the other hand, the projection, i.e., the continuous mapping h

$$\overline{M'} \times {}^*a \to \overline{^*M} \qquad \text{for every } {}^*a \in \overline{^*M}$$

of $\overline{M'} \times \overline{^*M}$ on $\overline{^*M}$. Apparently $h(\overline{M'}) = \overline{^*M}$, h being continuous! Since $\overline{M'}$ is compact the quasicomponents of $\overline{M'}$ are identical with its components. By a continuous mapping a connected set is mapped on a connected set. A component C of $\overline{M'}$ is mapped by h, since $\overline{^*M}$ is 0-dimensional, on exactly one point $^*m$ of $\overline{^*M}$. The set C therefore belongs to the intersection

$$h^{-1}({}^*m) \cdot \overline{M'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If this intersection (3) contains a point of M', then it contains exactly one quasicomponent of M', since different quasicomponents of M (and also of the homeomorphic set M') are mapped (by f) on different points

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[5]) This is by no means the case in $\overline{M}$; $\overline{M}$ may even be connected. For this reason it is necessary to apply the method of topological products.

of $^*M$. Besides this possible quasicomponent Q of M', (3) may only contain points of $\overline{M'}-M'$, therefore an 0-dimensional pointset N. The intersection (3) therefore is the sum of a *compact* quasicomponent Q (point or simple closed arc) and an *0-dimensional* set N (where Q or N may be vacuous).

A component of $Q+N$, however, is apparently either Q or a point of N; therefore it follows from what precedes that every component C of $\overline{M'}$ is either a quasicomponent of M' or one point of $\overline{M'}-M'$, which is what we had to prove.

2.3. **Remark.** From Theorem II it is particularly evident that in a semicompact generalized M.K.-space every point has an order 2, 1 or 0. Naturally we might have conditioned this property instead of the semicompactness; but then our theorem would have been less general.

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Mathematics. — *On the existence and uniqueness of the solution of the fundamental equation in the theory of metallic conduction.* By L. J. F. BROER. (Communicated by J. D. VAN DER WAALS.)

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1. *The fundamental equation.* According to the standard theories of electrons in metals [1]) the state of the electrons in the conduction band can be described by the discrete distribution function f_k. The argument k refers to the L^3 different values of the reduced wave vector $\vec{k}$, where L is the number of atoms per unit length. It is assumed that f_k can change only by the quantum jumps caused by the interaction with the lattice and by the acceleration due to the applied electrical field, not by mutual interaction of the electrons. When we denote the total number of electron transitions from the state k to the state k' per unit time by $I_{kk'}$, the stationary states of the system are found from the fundamental LORENTZ–BLOCH equation:

$$\left(\frac{\partial f_k}{\partial t}\right)_{\text{field}} - \sum_{k'} I_{kk'} + \sum_{k'} I_{k'k} = 0 \; . \quad . \quad . \quad . \quad . \quad (1)$$

The set (1) consists of L^3 equations in the L^3 unknowns f_k. However, if we suppose that the field is not so strong as to cause ionisation, the set (1) is interdependent, as summation of all equations yields $0=0$. If we now add the condition:

$$\sum_k f_k = N \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $N < L^3$ is the number of electrons, we can expect that (1) and (2) together allow an unique solution under suitable conditions. The object of this note is to supply these conditions, which apparentely never have been stated in literature. (This problem constitutes the so called third fundamental problem of the theory of metallic conduction [2])).

For our purposes we can assume that the electric field F and the temperature of the lattice are constant throughout the metal. In this case it is shown in the current treatments that:

$$\left(\frac{\partial f_k}{\partial t}\right)_{\text{field}} = -\frac{eF}{h}\frac{\partial f_k}{\partial k_x}$$

[1]) See e.g. A. SOMMERFELD and H. BETHE, Handbuch der Physik, Vol. 24, part 2, (Berlin 1930).

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