Aerodynamics. — *Spectral analysis of an irregular function.* By J. M. BURGERS. (Mededeling No. 58*b* uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hogeschool te Delft) *).

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5. As a supplement to the result of the preceding part of this article, we will briefly consider the fluctuations which may be expected to be shown by the indication of a galvanometer, connected to a thermo-couple through which is passed the signal $w(t)$, transmitted by the filtering circuit.

We assume that the indication z of the galvanometer depends on the signal w according to the equation:

$$z + \frac{1}{\beta}\frac{dz}{dt} = w^2 - \overline{w^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where $1/\beta$ is an adjustable damping coefficient. The mean value of z will be zero and we are interested in the mean of z^2. The solution of (13) is:

$$z = \beta \int_0^\infty d\tau \, \{ w^2(t-\tau) - \overline{w^2} \} \, e^{-\beta\tau} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and we easily find:

$$\overline{z^2} = \beta^2 \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \, \{ \overline{w^2(t-\tau_1)\, w^2(t-\tau_2)} - (\overline{w^2})^2 \} \, e^{-\beta(\tau_1+\tau_2)}. \quad . \quad (15)$$

Introducing $\sigma = \tau_2 + \tau_1$ and $\delta = \tau_2 - \tau_1$ as new variables, we can bring this integral into the form

$$z^2 = \beta \int_0^\infty d\delta \, \{ \overline{w^2(t)\, w^2(t+\delta)} - (\overline{w^2})^2 \} \, e^{-\beta\delta}. \quad . \quad . \quad . \quad (15a)$$

The mean value $\overline{w^2(t)\,w^2(t+\delta)}$ must be calculated by making use of the integral for w given in (3). Before starting with this calculation, consider a quantity W which is the sum of a large number n of terms of the type: $\varepsilon \cos \theta_k$, where ε is a constant, while the θ_k $(k = 1, 2, \dots n)$ are irregular functions of the time, in such a way that the mean value of each term $\varepsilon \cos \theta_k$ is zero. We easily find:

$$\overline{W} = 0$$

$$\overline{W^2} = n\,\varepsilon^2\, \overline{\cos^2 \theta_k} = \tfrac{1}{2}\, n\, \varepsilon^2$$

$$\overline{W^4} = n\,\varepsilon^4\, \overline{\cos^4 \theta_k} + 3\,n\,(n-1)\,\varepsilon^4\, \overline{\cos^2 \theta_k \cos^2 \theta_l} = \tfrac{3}{8}\, n\, \varepsilon^4 + \tfrac{3}{4}\, n\,(n-1)\,\varepsilon^4.$$

*) Part I has appeared in these Proceedings **51**, 1073 (1948).

Hence if n is very large and ε small, in such a way that $n\,\varepsilon^2$ is finite, we obtain the approximate relation:

$$\overline{W^4} \cong 3\,(\overline{W^2})^2.$$

On the other hand, when an interval of time η is taken, sufficiently long in order that no correlation will exist between $W(t)$ and $W(t+\eta)$, we shall have:

$$\overline{W^2(t)\,W^2(t+\eta)} = (\overline{W^2})^2.$$

From these relations we deduce:

$$\overline{W^2(t)\,W^2(t+\eta)} - (\overline{W^2})^2 = 2\,(\overline{W^2})^2 \qquad \text{for } \eta = 0$$

$$\overline{W^2(t)\,W^2(t+\eta)} - (\overline{W^2})^2 = 0 \qquad \text{for large values of } \eta.$$

We may expect that a similar relation will hold for the function $v(t)$, defining the signal introduced into the filtering system, that is to say, we may suppose that also in the case of this function the combined effect of second order correlations will be more important than that of the fourth order correlations. Then it is possible to calculate the value of $\overline{w^2(t)\;w^2(t+\eta)}$ (see next section), for which the following approximate result is obtained:

$$\overline{w^2(t)\,w^2(t+\eta)} - (\overline{w^2})^2 =$$

$$= (\overline{w^2})^2\,e^{-2p\omega\eta}\,(1+\cos 2\,\omega\,\eta) - p\,\omega\,\frac{d\,(\overline{w^2})^2}{d\omega}\,e^{-2p\omega\eta}\sin 2\,\omega\eta \qquad (16)$$

In deriving this formula it has been assumed that η is several times larger than θ. There is some indication that the domain of validity will extend further downward when p is taken smaller and smaller.

Assuming also β to be small, we find, upon insertion of (16) into (15*a*), the approximate formula:

$$\overline{z^2} = (\overline{w^2})^2 \left\{ \frac{\beta}{\beta + 2\,p\,\omega} + \frac{\beta\,(\beta + 2\,p\,\omega)}{4\,\omega^2} \right\} - \frac{\beta\,p}{2}\,\frac{d}{d\omega}\,(\overline{w^2})^2. \quad . \quad (17)$$

All three terms appearing here have the factor β, which expresses the fact that the fluctuations of the indication of the galvanometer will become smaller and smaller when the damping factor $1/\beta$ is taken larger.

From the first term it will be seen that β must become smaller than $2\,p\,\omega$ in order to make the damping effective. If for a given value of β we should decrease p in order to increase the selectivity of the filter, the ratio of $\overline{z^2}$ to $(\overline{w^2})^2$ will become larger, meaning that larger fluctuations of the galvanometer are to be expected. However, in consequence of the presence of the factor ω, this effect is not much to be feared with a filter of medium or low selectivity (p of the order of 0,1 or more).

A very interesting term is the last one (supposing that p is not very small), as this term contains the derivative of $\overline{w^2}$ with respect to ω, so that it may become significant in regions of the spectrum where the spectral intensity [the magnitude of the Fourier component $I_1(\omega)$] changes rapidly with ω. Such a phenomenon had been observed by BETCHOV, who found that the indication of the galvanometer showed appreciable oscillations of low frequency on both sides of the maximum of the curve giving the spectral energy as function of the frequency, approximately at the places where the curve descended most rapidly. The last term of (17), although it carries the factor p, may give a clue to the appearance of these oscillations (in the arrangement used by BETCHOV the selectivity of the filter corresponded approximately to the value 0,1 for p).

6. *Approximate calculation of* $\overline{w^2(t)\, w^2(t+\eta)}$.

Referring again to equation (3) it will be seen that:

$$\overline{w^2(t)\, w^2(t+\eta)} =$$

$$= 16\,p^2 \iiiint_0^\infty d\tau_1\, d\tau_2\, d\tau_3\, d\tau_4\, V\, e^{-p(\tau_1+\tau_2+\tau_3+\tau_4)} \cdot$$

$$\cdot \cos(\tau_1+\varepsilon)\cos(\tau_2+\varepsilon)\cos(\tau_3+\varepsilon)\cos(\tau_4+\varepsilon) \tag{18}$$

where V has been written for the mean value:

$$V = \overline{v(t-\tau_1/\omega)\, v(t-\tau_2/\omega)\, v(t+\eta-\tau_3/\omega)\, v(t+\eta-\tau_4/\omega)}\ . \tag{18a}$$

while $\sqrt{1-p^2} = \cos\varepsilon$ has everywhere been replaced by unity. An exact calculation of the integral (18) would require knowledge concerning quadruple correlations in the function $v(t)$, which is not available. However, if we restrict ourselves to values of η which sufficiently exceed θ, we can arrive at an approximate evaluation by observing that in this case V will differ from zero only if the four factors occurring in (18*a*) can be arranged into two pairs, in such a way that there exists a certain correlation within each pair, while at the same time the two pairs are sufficiently separated from each other in order that no correlation shall be found between the factors of the first pair and those of the second pair. It is possible that this will not give a full solution of our problem, so that when the formula derived for large values of η is extrapolated downward to $\eta = 0$, the result may be in error. Nevertheless there are indications that, provided p is very small, the approximate method of integration will give the most important part of the quantity to be calculated and we shall try to estimate the possible errors involved.

We introduce the following auxiliary variables:

$$\left.\begin{aligned}\sigma_0 &= \tau_4+\tau_3+\tau_2+\tau_1\\ \sigma &= \tau_4+\tau_3-\tau_2-\tau_1\\ \delta_2 &= \tau_4-\tau_3\\ \delta_1 &= \tau_2-\tau_1\end{aligned}\right\}\ \text{so that:}\ \left\{\begin{aligned}\tau_4 &= \sigma_0/4+\sigma/4+\delta_2/2\\ \tau_3 &= \sigma_0/4+\sigma/4-\delta_2/2\\ \tau_2 &= \sigma_0/4-\sigma/4+\delta_1/2\\ \tau_1 &= \sigma_0/4-\sigma/4-\delta_1/2.\end{aligned}\right.$$

The functional determinant of the set $\sigma_0, \sigma, \delta_2, \delta_1$ with respect to the set $\tau_4, \tau_3, \tau_2, \tau_1$ has the value 8. With the abbreviation: $h = \eta - \sigma/2\,\omega$ the expression for V can be written:

$$V = \overline{v\left(-\tfrac{1}{2}h + \delta_1/2\omega\right) v\left(-\tfrac{1}{2}h - \delta_1/2\omega\right) v\left(\tfrac{1}{2}h + \delta_2/2\omega\right) v\left(\tfrac{1}{2}h - \delta_2/2\omega\right)}. \quad (18b)$$

Putting:

$$\alpha_2 = \tfrac{1}{2}(\sigma_0 + \sigma) + 2\,\varepsilon \quad ; \quad \alpha_1 = \tfrac{1}{2}(\sigma_0 - \sigma) + 2\,\varepsilon,$$

the integral (18) can now be brought into the form:

$$\overline{w^2(t)\,w^2(t+\eta)} =$$

$$= \tfrac{1}{2}p^2 \int d\sigma_0\, e^{-p\sigma_0} \int d\sigma \int d\delta_1 \int d\delta_2\, V(\cos\alpha_2 + \cos\delta_2)(\cos\alpha_1 + \cos\delta_1) \quad (19)$$

The integration with respect to σ_0 can be deferred to the last and a threedimensional picture can be used to illustrate the integrations with respect to σ, δ_1 and δ_2. The domains over which the latter three variables can vary are limited as follows:

$$-\sigma_0 < \sigma < +\sigma_0$$
$$-\tfrac{1}{2}(\sigma_0 - \sigma) < \delta_1 < +\tfrac{1}{2}(\sigma_0 - \sigma)$$
$$-\tfrac{1}{2}(\sigma_0 + \sigma) < \delta_2 < +\tfrac{1}{2}(\sigma_0 + \sigma).$$

When we look for pairs of factors in (18b) between which correlation can exist, it is found that V can differ from zero in the following three cases only:

(a) $|\delta_1| < \omega\,\theta$; $|\delta_2| < \omega\,\theta$, σ having an arbitrary value between $-\sigma_0$ and $+\sigma_0$; in this case:

$$V = A^4 R\left(\frac{\delta_1}{\omega}\right) R\left(\frac{\delta_2}{\omega}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (20a)$$

(b) $|h| < \theta$; $|\delta_2 - \delta_1| < 2\,\omega\theta$; in this case:

$$V = A^4 R\left(h + \frac{\delta_2 - \delta_1}{2\omega}\right) R\left(h - \frac{\delta_2 - \delta_1}{2\omega}\right) \quad . \quad . \quad . \quad (20b)$$

(c) $|h| < \theta$; $|\delta_2 + \delta_1| < 2\,\omega\theta$; in this case:

$$V = A^4 \cdot R\left(h + \frac{\delta_2 + \delta_1}{2\omega}\right) \cdot R\left(h - \frac{\delta_2 + \delta_1}{2\omega}\right). \quad . \quad . \quad . \quad (20c)$$

The three domains have been indicated schematically in fig. 1. Outside of the space thus defined V will be zero. It will be seen that the length of the various domains increases with the value of σ_0. Hence the main contribution to be derived from the triple integration can be expected to be of the order $\sigma_0(\omega\theta)^2$; as this is multiplied by $e^{-p\sigma_0}$, the corresponding contribution to the quadruple integral will be of the order $(\omega\theta)^2/p^2$. It must be observed that the product $\omega\theta$ in general will be a quantity of normal order of magnitude, as the interesting values of ω will be comparable to $1/\theta$. As the quadruple integral according to formula (19) is multiplied by the factor $\frac{1}{2}p^2$, the final result will be of order $(\omega\theta)^2$, as should be expected in connection with the expression (10) for $\overline{w^2}$, given in the preceding part of this paper.

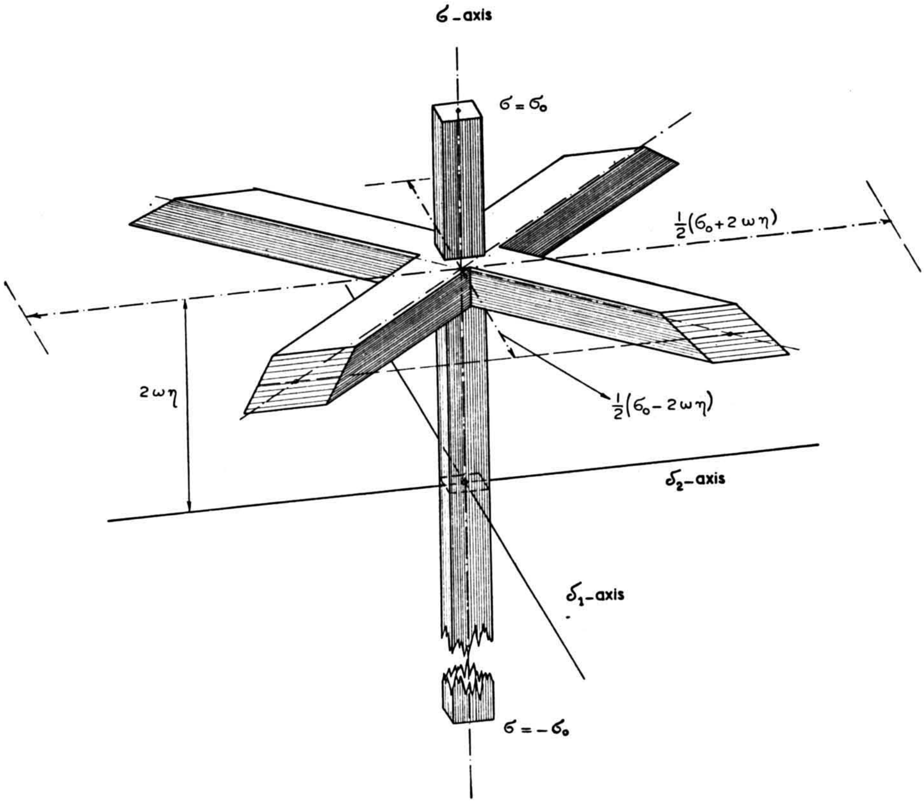

Fig. 1.

In calculating the triple integral we shall attempt also to take care of certain contributions of the order $(\omega\theta)^2$, giving contributions of the order $(\omega\theta)^2/p$ to the quadruple integral and of order $p(\omega\theta)^2$ in the final result. Although the presence of the factor p detracts somewhat from the importance of these contributions, they have been retained on account of the interesting form which they present in the final result.

The domains (a), (b), (c) overlap more or less in a region having a volume of the order $(\omega\theta)^3$. The fact that a contribution derived from the region of overlapping is counted in all three domains, is correct, as the three different forms of correlation indicated by the expressions (20a) — (20c) are actually to be found in this region and must be taken into account, each for itself. It can be brought forward that more intimate (fourfold) correlations between the four factors of the expression V might occur in this region. As nothing has been given concerning fourfold correlations in the course of the function $v(t)$, we have no data on this matter. However, if we suppose that $v(t)$ presents a character analogous to that of the quantity W, mentioned by way of example in the preceding section, we may expect that the contribution due to fourfold correlations will be small in comparison with that derived from the three pairs of double correlations. In that case the contribution from these fourfold correlations to the triple integral would be small in comparison with $(\omega\theta)^3$ and the corresponding contribution to the final result would be small in comparison with $p(\omega\theta)^3$. It seems possible therefore to leave this matter out of account. At any rate we may be sure that the main terms depending upon second order correlations have been collected without omissions.

It must further be observed that the domains (b) and (c) will have their complete forms only provided $\sigma_0 > 2\,\omega(\eta+\theta)$, while they will not exist at all when $\sigma_0 < 2\,\omega(\eta-\theta)$. In calculating the contributions from (b) and (c) we shall take $2\,\omega\,\eta$ as the lower limit in the integration with respect to σ_0 and shall reckon as if (b) and (c) are complete for all $\sigma_0 > 2\,\omega\,\eta$. The error introduced into the quadruple integral in this way is of the order $(\omega\,\theta)^4$ (see below), which can be neglected.

In consequence of the symmetry of the situation, the contributions from the domains (b) and (c) will be equal, so that we can restrict to the calculations for (b).

Contribution obtained from the domain (a).

As $R(\delta/\omega)$ becomes zero for $|\delta| > \omega\theta$, the integrations with respect to δ_1 and δ_2 can be written as if the limits are $-\infty$, $+\infty$.

We introduce:

$$\left.\begin{aligned} \int_{-\infty}^{+\infty} d\delta\, R(\delta/\omega) &= F \\ \int_{-\infty}^{+\infty} d\delta\, R(\delta/\omega)\cos\delta &= G \end{aligned}\right\} \quad \ldots\ldots \quad (21)$$

For small values of p the quantity G is nearly equal to $2\,I_1$, where I_1 has been defined by $(9a)$, the error being of the order p^2 in consequence of the symmetry of $R(\delta)$. Hence making use of (10) we shall assume:

$$A^2\, G \cong \overline{w^2}.$$

When the integrations with respect to δ_1 and δ_2 are carried out, the integrand becomes:

$$A^4\,(F\cos a_2 + G)\,(F\cos a_1 + G).$$

After working out the product and performing the integration with respect to σ, between the limits $-\sigma_0$ and $+\sigma_0$, the following result is obtained:

$$A^4\,[F^2\,\{\sigma_0\cos(\sigma_0+4\varepsilon)+\sin\sigma_0\} + 4\,FG\,\{\sin(\sigma_0+2\varepsilon)-\sin 2\varepsilon\} + 2\,\sigma_0\,G^2].$$

This expression must be multiplied by $e^{-p\sigma_0}$ and integrated with respect to σ_0 from 0 to ∞. It is found that the last term of the preceding expression gives a result of the order p^{-2}, whereas the others give results of order unity or of order p^2. Hence the last term is the only important one and it is found that the contribution derived from the domain (a) has the approximate value:

$$\tfrac{1}{2}\, p^2 \cdot 2 A^4 G^2/p^2 = A^4 G^2 \cong (\overline{w^2})^2. \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

Contribution obtained from the domain (b).

We again start with the integrations with respect to δ_2 and δ_1. The domain of integration for a given value of σ has been represented separately in fig. 2. As $h = \eta - \sigma/2\,\omega$ must be confined between $-\theta$ and $+\theta$, we

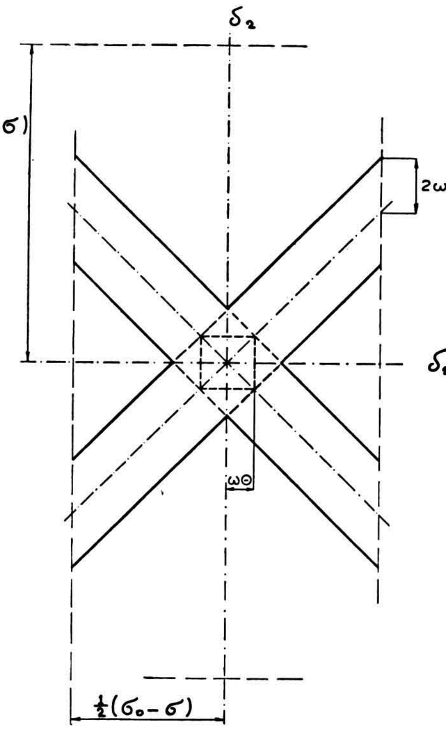

Fig. 2.

can restrict to the consideration of positive values of σ between $2\,\omega(\eta - \theta)$ and $2\,\omega(\eta + \theta)$, and it follows that the limits for δ_2 are wider than those for δ_2.

We take the integration with respect to δ_2 first and write $\delta_2 = \delta_1 + \delta'$; δ' will be taken as integrational variable instead of δ_2 (for constant δ_1). According to $(20b)$ the value of V will be zero as soon as $|\,\delta'\,| > 2\,\omega\theta$; hence we may write $-\infty$, $+\infty$ as limits for the integration with respect to δ'.

We introduce:

$$\left.\begin{aligned}\int_{-\infty}^{+\infty} d\delta'\, R\,(h+\delta'/2\,\omega)\, R\,(h-\delta'/2\,\omega) = P\,(h)\\ \int_{-\infty}^{+\infty} d\delta'\, R\,(h+\delta'/2\,\omega)\, R\,(h-\delta'/2\,\omega)\cos\delta' = L\,(h)\end{aligned}\right\} \quad . \; . \; . \quad (23)$$

(the corresponding integral with sin δ' would be zero). The integration with respect to δ' then turns the integrand of (19) into:

$$A^4\,(P\cos a_2 + L\cos\delta_1)\,(\cos a_1 + \cos\delta_1).$$

A further integration, with respect to δ_1 between the limits $\pm\frac{1}{2}(\sigma_0 - \sigma)$, gives the result:

$$\left.\begin{aligned}A^4\,[(\sigma_0-\sigma)\,(P\cos a_1\cos a_2 + \tfrac{1}{2}\,L) + \tfrac{1}{2}\,L\sin(\sigma_0-\sigma) + \\ + 2\,(P\cos a_2 + L\cos a_1)\sin\tfrac{1}{2}\,(\sigma_0-\sigma)]\end{aligned}\right\} \quad (24)$$

This expression must next be integrated with respect to σ. We put $\sigma = 2\,\omega\eta - \sigma'$; the limits for σ' can be taken as $-\infty$, $+\infty$, as $P(h)$ and $L(h)$ both vanish when $|h| = |\eta - \sigma/2\,\omega| = |\sigma'/2\,\omega|$ exceeds θ [this supposes that $\sigma_0 > 2\,\omega(\eta+\theta)$]. We shall not write down the full result, but restrict to those terms which appear to furnish the main contributions in the integration with respect to σ_0 which is still to follow. These terms are:

$$\tfrac{1}{2}\,A^4\left[(\sigma_0 - 2\,\omega\,\eta)\,Q\,(1+\cos 2\,\omega\,\eta) + (Q^* - 2Q)\sin 2\,\omega\,\eta\right] \quad . \; . \quad (25)$$

where:

$$\left.\begin{aligned}Q &= \int_{-\infty}^{+\infty} d\sigma'\,P\,(\sigma'/2\,\omega)\cos\sigma' = \int_{-\infty}^{+\infty} d\sigma'\,L\,(\sigma'/\,2\,\omega)\\ Q^* &= \int_{-\infty}^{+\infty} d\sigma'\,P\,(\sigma'/2\,\omega)\,\sigma'\sin\sigma'\end{aligned}\right\} \quad . \; . \; . \; . \quad (26)$$

The expression (25) is not exact when $\sigma_0 < 2\,\omega(\eta+\theta)$. Hence when the lower limit $2\,\omega\,\eta$ is used in the integration with respect to σ_0, we make an error of the order $(\omega\,\theta)^2\,Q$, that is of the order $(\omega\,\theta)^4$, in the full quadruple integral, as had been mentioned before.

After multiplication by $e^{-p\sigma_0}$ and integration with respect to σ_0 from $2\,\omega\,\eta$ to ∞ (it is convenient to put $\sigma_0 = 2\,\omega\,\eta + \sigma^*$) and having regard to the factor $\frac{1}{2}\,p^2$ before the integral in (19) the following expression is obtained for the contribution from the domain (b):

$$\tfrac{1}{4}\,A^4\,Q\,e^{-2p\omega\eta}\,(1+\cos 2\,\omega\,\eta) + \tfrac{1}{4}\,pA^4\,(Q^* - 2\,Q)\,e^{-2p\omega\eta}\sin 2\,\omega\,\eta. \quad (27)$$

Now if we write $s = \frac{1}{2}(\sigma' + \delta')$; $r = \frac{1}{2}(\sigma' - \delta')$, the integrals (26), defining Q and Q^*, can be transformed as follows:

$$Q = \int_{-\infty}^{+\infty} d\sigma' \int_{-\infty}^{+\infty} d\delta' \, R\left(\frac{\sigma' + \delta'}{2\omega}\right) R\left(\frac{\sigma' - \delta'}{2\omega}\right) \cos\delta' =$$

$$= 2 \int_{-\infty}^{+\infty} ds \int_{-\infty}^{+\infty} dr \, R(s/\omega) \, R(r/\omega) \, (\cos s \cos r - \sin s \sin r) =$$

$$= 2 \left\{ \int_{-\infty}^{+\infty} ds \, R(s/\omega) \cos s \right\}^2 = 2\, G^2 \qquad (28a)$$

while Q^* becomes:

$$Q^* = 4\, G\, G^* \quad . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (28b)$$

where:

$$G^* = \int_{-\infty}^{+\infty} ds \, R(s/\omega) \, s \sin s \; . \; . \; . \; . \; . \; . \; . \; . \qquad (29)$$

It is not difficult to prove the relation:

$$G^* = G - \omega \cdot dG/d\omega,$$

so that:

$$Q^* - 2\,Q = 4\,G\,(G^* - G) = -4\,\omega\, G \cdot dG/d\omega. \; . \; . \; . \qquad (30)$$

Addition of the contributions from all three domains gives the final result:

$$\overline{w^2(t)\, w^2(t+\eta)} = A^4 G^2 \{ 1 + e^{-2p\omega\eta} (1 + \cos 2\,\omega\eta) \} -$$
$$- 2\, p\, \omega\, A^4\, G \frac{dG}{d\omega} e^{-2p\omega\eta} \sin 2\,\omega\eta. \qquad (31)$$

The second term has the factor p and consequently will be of subordinate importance. Nevertheless it may become influential in cases where G steeply changes with ω at some point of the spectrum.

Note. — It was pointed out to me that investigations on problems of similar nature have been published by S. G. RICE in two extensive papers: *"Filtered thermal noise — fluctuation of energy as function of interval length"*, Journ. Acoustical Society of America **14**, 216—227 (1943) and *"Mathematical analysis of random noise"*, The Bell System Technical Journal **23**, 282—332 (1944) and **24**, 46—156 (1945). The starting point of RICE's deductions and the mathematical methods applied by him differ greatly from those used in the article above, which makes a comparison difficult. A formula resembling the first term of (31), but without the exponential factor, is given in the second paper (Bell System Technical Journal **24**, p. 89, equation (3. 9—7)). I did not find an indication that RICE's formulae might explain the appearance of a high value of $\overline{z^2}$ when ω has such a value that $|dI_1/d\omega|$ is large, as signaled by BETCHOV. This may be connected with the circumstance that RICE's formulae apparently refer to a case for which p is zero.

Résumé.

Le but de cet article était d'obtenir une formule approchée pour la distribution de l'énergie dans le spectre d'une fonction aléatoire, sans qu'il soit nécessaire de supposer que cette fonction puisse être représentée par une intégrale de Fourier. On suppose qu'un signal électrique proportionnel à la fonction considérée est passé à travers d'un circuit de filtrage; il est possible alors de calculer le carré moyen du courant sortant du filtre, pourvu que le carré moyen et le coefficient de corrélation de la fonction aléatoire soient donnés. La formule obtenue, qui dépend encore d'un paramètre définissant le degré de sélectivité du filtre, tend vers la formule donnée par TAYLOR, quand la sélectivité devient infinie.

Dans la seconde partie de l'article on a étudié le carré moyen de l'indication d'un galvanomètre mesurant le courant donné par une thermocouple, auquel on a appliqué le courant sortant du filtre. L'expression trouvée donne une relation entre ce carré moyen et les paramètres définissant la sélectivité du filtre et l'amortissement du système thermocouple-galvanomètre; elle montre que pour un filtre de grande sélectivité et un système à amortissage faible ce carré moyen peut devenir relativement grand. On peut attendre des oscillations spéciales auprès des valeurs de la fréquence pour lesquelles l'intensité spectrale change rapidement.

Resumo.

Ĉi tiu artikolo celas trovi proksimuman formulon por la distribuo de la energio en la spektro de funkcio de hazarda karaktero, ne enkondukinte la supozon ke tiu funkcio povas esti prezentata per integraĵo laŭ Fourier. Oni supozas ke elektra signalo proporcia al la funkcio ekzamenata estas sendata tra filtro-cirkvito; tiuokaze estas eble kalkuli la mezan kvadraton de la kurento eliranta el la filtro, kondiĉe ke la meza kvadrato kaj la koeficiento de korelacio de la funkcio de hazarda karaktero estas konataj. La formulo trovita dependas de parametro difinanta la gradon de selektiveco de la filtro; se ĉi tiu selektiveco fariĝas infinita, la formulo egaliĝas al la formulo donita de TAYLOR.

En la dua parto de la artikolo oni studas la mezan kvadraton de la indiko de galvanometro mezuranta la kurenton produktatan de varmopilo al kiu oni aplikas la kurenton elirantan el la filtro.

La trovita esprimo donas rilaton inter ĉi tiu meza kvadrato kaj la parametroj difinantaj la selektivecon de la filtro kaj la amortizon de la sistemo varmopilo-galvanometro; ĝi montras ke por filtro tre selektiva kaj amortizo malgranda, ĉi tiu meza kvadrato povas fariĝi relative granda.

Specialaj osciloj povas aperi plue apud tiaj frekvencoj apud kiaj la spektra energio rapide modifiĝas kun la frekvenco.