Physics. — *Unitary Quantum Electron Dynamics* I. By H. J. GROENEWOLD.
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1. Introduction.

1.1 *Source particles and carrier particles.* In a previous paper [1]) (here refered to by **1**) we have distinguished source particles (e.g. electrons) and carrier particles (e.g. photons). The carrier particles are emitted and absorbed by source particles and in this way they lead to a (e.g. electromagnetic) interaction between source particles. Because the carrier particles have a finite velocity there will be a time lag in this interaction.

1.2 *Dualistic and unitary theories.* 1.21 *Dualistic theory.* In the usual dualistic theories the two kinds of particles are treated separately without conspicuous relations between the properties of carrier particles and those of source particles.

1.22 *Unitary theories.* Unitary theories intend to give a complete description entirely either in terms of fields of carrier particles (f-theory) or in terms of source particles (p-theory). We shall only be concerned with the latter type and further omit the prefix p.

A unitary theory has to give the complete equations of motion of the interacting source particles. Because of the time lag in the interaction the description will be extremely complicated if not impossible. Not before the theory has been completely established in terms of source particles, fictitious carrier particles which are created and annihilated may be formally introduced in order to simplify the description.

1.23 *Balance.* 1.231 *Dualistic versus unitary theory.* The time lag in the interaction makes that in unitary theory the boundary conditions if they can be stated at all will be frightfully complicated. Further it makes that the energy-momentum which is lost by one source particle in a given region of time-space is gained by another source particle in a different region of time-space. In this way conservation of energy-momentum becomes rather intricate. The same holds for loss and gain of charge in case the carrier particles of the dualistic theory are supplied with some kind of charge.

In the latter case the source particles (e.g. nucleons) emit and absorb in the dualistic theory carrier particles (e.g. charged mesons), which in their turn act as source particles and in this capacity emit and absorb still other kinds of carrier particles (e.g. photons). The latter lead to a (e.g. electromagnetic) interaction between all charged particles. Strictly a somewhat similar situation always exists in general relativity theory in which all particles, source particles as well as carrier particles, have gravitational interaction. In such cases one needs to be highly optimistic in order to have still expectations of a unitary theory.

Altogether if within certain limits a unitary theory might be possible at all it would be extremely complicated and intractable. On the other hand the dualistic theory gives a straightforward and relatively simple description which is continuous in time-space with proper differential equations of motion, clear boundary conditions and differential conservation relations. It is not astonishing that it is always the dualistic theory which is used in practice.

1.232 *Unitary versus dualistic theory.* In spite of the, perhaps unsurmountable difficulties it might still be worth while to deal with some aspects of what might be a unitary theory if ultimately it were feasible. Even an imperfect unitary theory might be able to throw some light on the possibility of

U_2 another outlook on dualistic theory;

U_3 further developments in unitary theory, which are untranslatable into dualistic theory

(there is no U_1; it can be seen in 1.3 why not).

1.2321 *Other outlook.* As long as the unitary and dualistic descriptions are supposed to give the same observable behaviour of source particles they can be translated into each other. Yet each of them may show aspects which are not as easily brought to light in the other description.

In particular a unitary theory would completely derive all properties of carrier particles from the properties of interacting source particles and/or the way in which the carrier particles are introduced.

1.2322 *Untranslatable developments.* It is quite conceivable that in a further development a unitary theory would be modified in a way, which could no longer be translated into a dualistic description. In such a case the dualistic theory could be maintained as an extremely useful approximation, but the unitary theory would become of fundamental importance.

This possibility in particular refers to the divergence difficulties connected with pair processes and self interaction.

1.3 *Queries.* Corresponding to U_2 and perhaps to U_3 there are the two remaining problems (Q_3 slightly extended) of **1**:

Q_2 how are the relations between the properties of carrier particles and those of source particles; can in particular the former be derived from the latter?

Q_3 how shall negative states and pair processes and self interaction be dealt with?

In this paper we shall be concerned with Q_2. Q_3 will only incidentally be touched in connection with the negative states of carrier particles.

1.4 *Conditions.* We mention some of the most striking conditions which a unitary theory would have to satisfy.

1.41 *Opaqueness condition.* There is one condition which can directly be seen to be necessary in order that carrier particles can entirely be eliminated from dualistic theory. Every carrier particle which is absorbed by a source particle must have been emitted by another source particle and every carrier particle which is emitted by a source particle must be absorbed by another source particle. Because identical particles are indiscernible, no carrier particles at all can be allowed to enter into or to escape from the physical universe, not even such ones which never have been emitted and never will be absorbed. If the universe is not opaque for carrier particles, a unitary theory if possible at all cannot be equivalent to the dualistic theory.

1.42 *Asymmetry.* Processes of emission and absorption of carrier particles (radiation processes) are connected with a peculiar kind of asymmetry, which if suppressed at one place peeps up at another place. It appears in relation with retarded or advanced fields or in the properties of radiation damping. Emission of carrier particles occurs spontaneously or induced, absorption only induced. In unitary theory this asymmetry will have to appear in another form. If energy-momentum is transfered from one source particle in a certain region of time-space to another source particle in another region, the energy loss has always to occur earlier in time than the energy gain. This asymmetry, which appears to be rather fundamental, is obviously not an asymmetry in time; loss and gain (like emission and absorption) are interchanged under reversal of time.

This asymmetry condition will be considered in a later paper. That paper (here refered to by **3**) will deal with some unitary aspects of what in dualistic theory are radiation processes.

1.43 *Interference and diffraction.* The periodic wave aspect of carrier particles (e.g. photons) is unrefutably established by interference and diffraction phenomena. In a unitary theory not only the carrier particles, but at the same time their periodic wave aspect will be lost. The periodicity conditions, which are responsible for the observable interference and diffraction effects, have then to be derived from the properties of the interacting carrier particles. Also this point will be considered in **3**.

1.44 *Vacuum effects.* In dualistic theory there are effects of vacuum polarization by virtual creation and annihilation of the same pairs of source particles and of vacuum fluctuations of the carrier field by virtual emission and absorption of carrier particles by the same source particle (self interaction), which in spite of malignant divergences are of fundamental importance [2]). It is not only a task of Q_3 to ask for such a modification of the theory which removes all divergences, but also to take care that the relevant effects are not removed at the same time.

1.5 *Unification and quantization.* One could try two different ways from dualistic classical theory to unitary quantum theory. The one is via unitary classical theory, the other via dualistic quantum theory. We shall rove about a little along the latter way.

1.51 *Dualistic quantum theory.* Those principles of dualistic quantum theory which we shall need for this excursion will be reviewed in 2.

1.52 *Unitary classical theory.* Though we shall not directly be concerned with unification of classical theory, it will be profitable to keep in mind the problems and achievements in this process. The problems are narrowly related to those of the unification of quantum theory. To some extent they have been solved for electromagnetic interaction between electric charges.

As to the classical analogue of Q_2 the equations of motion have already been given by TETRODE [3]) and FOKKER [4]). A thourough physical interpretation and discussion has been given by WHEELER and FEYNMAN [5]).

As to the classical analogue of Q_3 there is a recent attack by FEYNMAN [6]). Also other suggestions could be thought of [7]).

1.6 *Simplifications and limitations.* As our derivations will appear a bit complicated it will be advizable to omit all those complications which are not strictly essential for understanding the fundamental problem. Therefore we shall in the first place restrict the source particles to electrons and the carrier particles to photons. So we shall only consider electromagnetic interaction between electrons. The generalization to meson interaction between nucleons gives no fundamental new difficulties as long as nothing (like disintegration) happens to them but emission and absorption by nucleons.

Further we shall have to resort to very simple models and even then our considerations will remain rather poor. In spite of all this it is hoped that they can throw a bit of light on Q_2.

2. Dualistic theory.

In **1** the interaction between particles of different kinds has hardly been mentioned. In the dualistic part of the present paper the interaction between source particles (electrons) and carrier particles (photons) is of primary importance.

2.1 *Photons.* Before dealing with this interaction, we first recollect some results of **1** for the case of photons (spin $= 1$, $m = 0$).

2.11 *Notation.* For photons we shall now write the time-space vectors, which occur as arguments in the wave functions, as $(cs, y^1, y^2, y^3) =$ $= (y^0, y^1, y^2, y^3)$ $(-g^{00} = g^{11} = g^{22} = g^{33} = 1)$. The wave operators replace some of these sets by other sets, which we now write as $(ct, x^1, x^2, x^3) = (x^0, x^1, x^2, x^3)$. Thus for photons the $(\vec{x}, t)$ and $(\vec{y}, s)$ of **1** are now written as $(\vec{y}, s)$ and $(\vec{x}, t)$ respectively. The reason for this interchange will become clear later on in 2.2122, when the interaction with electrons is introduced.

2.12 *Equations of motion.* The equations of motion for free photons are

$$\mathbf{L}\{\vec{y}_k, s_k\}(\vec{y}_1 s_1, \vec{y}_2 s_2, \ldots |^{\alpha_1 \alpha_2 \cdots} \Phi = 0 \qquad (k = 1, 2, \ldots), \quad . \quad . \quad (1)$$

where

$$\mathbf{L}\{\vec{y},s\} = \left(\frac{\hbar c}{i}\right)^2 \frac{\partial}{\partial y^\alpha}\frac{\partial}{\partial y_\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the absence of electrons the wave functions Φ are subjected to the supplementary conditions

$$\frac{\partial}{\partial y_k^\alpha}(\vec{y}_1 s_1, \vec{y}_2 s_2, \ldots \mid \cdots^{\alpha_k \cdots} \Phi = 0 \qquad (k = 1, 2, \ldots). \quad . \quad . \quad (3)$$

If the conditions (3) are satisfied together with their first order time derivatives at a given set of initial times, (1) makes that they are satisfied at all times.

2.13 *Reference functions.* A complete system of orthonormal solutions of the 1-particle equation

$$\mathbf{L}\{\vec{y},s\}(\vec{y}s\mid^\alpha\varphi = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

not subjected to supplementary conditions, is given by

$$(\vec{y}s\mid^\alpha\varphi\mid\vec{\xi}\pm r) = {}^\alpha b\mid\vec{\xi}\pm r)\, e^{-\frac{i}{\hbar c}(\pm\xi cs - \vec{\xi}\vec{y})} / \xi^{\frac{1}{2}}(2h^3c^3)^{\frac{1}{2}} \quad . \quad . \quad . \quad (5)$$

with 4 4-vectors ${}^\alpha b\mid\vec{\xi}\pm r)\,(r = 1, 2, 3, 4)$ for which

$$\left.\begin{aligned} &(\vec{\xi}\pm r\mid b_\alpha^\dagger\, {}^\alpha b\mid\vec{\xi}\pm s) = \delta_{rs}, \\ &\sum_r {}^\alpha b\mid\vec{\xi}\pm r)(\vec{\xi}\pm r\mid b_\beta^\dagger = \delta_\beta^\alpha + 2j^\alpha j_\beta, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where j^α is an arbitrarily chosen normalized time-like vector ($j^\alpha j_\alpha =$ $= -1$), which we take in positive time direction.

2.14 *Positive and negative states.* 2.141 *Density states.* The indefinite density operator is $\left(\frac{\hbar}{i}\frac{\delta}{\delta s} - \frac{\hbar}{i}\frac{\partial}{\partial s}\right)g_{\alpha\beta}$. The positive and negative density states are distinguished by the eigenvalues $+1$ and -1 of the operator η, which is the product of the operator for which the exponentials (5) are eigenfunctions with eigenvalues ± 1 and the operator $(g_{\alpha\beta} + 2\,j_\alpha\,j_\beta)$.

2.142 *Energy states.* The energy operator is $\left(\frac{\hbar}{i}\frac{\delta}{\delta s} - \frac{\hbar}{i}\frac{\partial}{\partial s}\right)/2$. The positive and negative energy states are distinguished by the $+$ and $-$ sign in (5).

2.143 *Charge conjugated states.* The "charge" conjugated state of $(\vec{y}s\mid^\alpha\varphi\mid\vec{\xi}\pm r)$ is $(\vec{y}s\mid^\alpha\varphi\mid\widetilde{\vec{\xi}\pm r}) = (\vec{y}s\mid^\alpha\varphi\mid -\vec{\xi}\pm r)$.

2.15 *Creation and annihilation operators.* The creation and annihilation operators $\mathbf{a}$ and $\mathbf{a}^\dagger$ are defined by

$$\left.\begin{aligned}
&(\vec{\xi}\pm r\,|\,\mathbf{a}_\alpha^\dagger\,|\,\vec{y}s\}\,(\vec{y}_1\,s_1,\ldots\vec{y}_n s_n|^{\alpha_1\ldots\alpha_n}\,\Phi = n^{\frac{1}{2}}\int(d\vec{y}_n)\,(\vec{\xi}\pm r\,|\,\varphi_\beta^\dagger\,|\,\vec{y}_n s_n)\cdot\pm(\delta_\alpha^\beta + 2j^\beta j_\alpha)\\
&\qquad\qquad\left(\frac{\hbar}{i}\frac{\delta}{\delta s_n} - \frac{\hbar}{i}\frac{\partial}{\partial s_n}\right)\mathrm{S}_n\,(\vec{y}_1\,s_1,\ldots\vec{y}_n\,s_n|^{\alpha_1\ldots\alpha_n}\,\Phi,\\
&\}\,\vec{y}s\,|\,\mathbf{a}^\alpha\,|\,\vec{\xi}\pm r)\,(\vec{y}_1\,s_1,\ldots\vec{y}_{n-1}\,s_{n-1}|^{\alpha_1\ldots\alpha_{n-1}}\,\Phi\\
&\qquad\qquad = n^{\frac{1}{2}}\mathrm{S}_n\,(\vec{y}_n\,s_n|^\alpha\varphi\,|\,\vec{\xi}\pm r)\,(\vec{y}_1\,s_1,\ldots\vec{y}_{n-1}\,s_{n-1}|^{\alpha_1\ldots\alpha_{n-1}}\,\Phi
\end{aligned}\right\}\quad(7)$$

if acting to the right and by the HERMITIAN adjoint relation ($7^\dagger$) if acting to the left.

2.151 *Wave operators.* For photons we shall use dynamical wave operators (representation s_2). In the distinction made in **1** between DIRAC's 1942 theory (*D*-modification) and the current hole theories (*H*-revision), there remained an ambiguity just in the photon case (cf. **1** 4.3224). In that case we had to do with positive and negative energy functions and with positive and negative vectors. Each of them can be treated either by *D*-modification or by *H*-revision. That gives (writing small types for the vector treatment) the 4 combinations *Dd*, *Dh*, *Hd* and *Hh*. We shall consider them all.

The dynamical wave operators are defined by

$$\left.\begin{aligned}
(\vec{x}t|^\alpha\,\varphi'_\beta\,|\,\vec{y}s\} &= \sum_r\sum_\pm\int(d\vec{\xi})\,(\vec{x}t|^\gamma\,\varphi\,|\,\vec{\xi}\pm r)\,(\vec{\xi}\pm r\,|\,\mathbf{a}_\beta^\dagger\,|\,\vec{y}s_0\}\,|\,\lambda_{\pm\gamma}^{\ \alpha}\\
&+\sum_r\sum_\pm\int(d\vec{\xi})\,(\vec{x}t|^\gamma\varphi\,|\,\vec{\xi}\pm r)\,\{\vec{y}s_0|^\beta\mathbf{a}\,|-\vec{\xi}\mp r)\,\mu_{\pm\gamma}^{\ \alpha},\\
\{\vec{y}s|^\beta\,\varphi_\alpha'^\dagger\,|\,\vec{x}t) &= \sum_r\sum_\pm\int(d\vec{\xi})\,\{\vec{y}s_0|^\beta\mathbf{a}\,|\,\vec{\xi}\pm r)\,(\vec{\xi}\pm r\,|\,\varphi_\gamma^\dagger\,|\,\vec{x}t)\,\lambda_{\pm\alpha}^{\dagger\gamma}\\
&+\sum_r\sum_\pm\int(d\vec{\xi})\,(-\vec{\xi}\mp r\,|\,\mathbf{a}_\beta^\dagger\,|\,\vec{y}s_0\}\,(\vec{\xi}\pm r\,|\,\varphi_\gamma^\dagger\,|\,\vec{x}t)\,\mu_{\pm\alpha}^{\dagger\gamma}.
\end{aligned}\right\}.\quad(8)$$

The λ's and μ's are for the 4 combinations given by

	Dd	*Dh*	*Hd*	*Hh*	
$\lambda_{\pm\gamma}^{\ \alpha}$	δ_γ^α	$\delta_\gamma^\alpha + j^\alpha j_\gamma$	$\frac{1\pm1}{2}\delta_\gamma^\alpha$	$\frac{1\pm1}{2}\delta_\gamma^\alpha + j^\alpha j_\gamma$	
$\mu_{\pm\gamma}^{\ \alpha}$	0	$\mp j^\alpha j_\gamma$	$\frac{1\mp1}{2}(\delta_\gamma^\alpha + 2j^\alpha j_\gamma)$	$\frac{1\mp1}{2}\delta_\gamma^\alpha + j^\alpha j_\gamma$	(9)
$\lambda_{\pm\gamma}^{\dagger\alpha}$	$\pm(\delta_\gamma^\alpha + 2j^\alpha j_\gamma)$	$\pm(\delta_\gamma^\alpha + j^\alpha j_\gamma)$	$\frac{1\pm1}{2}(\delta_\gamma^\alpha + 2j^\alpha j_\gamma)$	$\frac{1\pm1}{2}\delta_\gamma^\alpha + j^\alpha j_\gamma$	
$\mu_{\pm\gamma}^{\dagger\alpha}$	0	$-j^\alpha j_\gamma$	$\frac{1\pm1}{2}\delta_\gamma^\alpha$	$\frac{1\mp1}{2}\delta_\gamma^\alpha + j^\alpha j_\gamma$	

if acting to the right. If they act to the left λ and $\lambda^\dagger$ have to be interchanged in (9) and also μ and $\mu^\dagger$.

2.152 *Field operators.* From the mutually HERMITIAN adjoint wave operators φ and $\varphi^{\dagger}$ we form for later use the self-adjoint field operators Φ_c and Φ_s

$$\left.\begin{aligned} \Phi_c(\vec{x}t) &= (2\pi)^{\frac{1}{2}}\, \hbar c\, (\{\vec{y}s_0|^{\beta}\varphi_{\alpha}'^{\dagger}|\vec{x}t) + (\vec{x}t|_{\alpha}\varphi_{\beta}'|\vec{y}s_0\}), \\ \Phi_s(\vec{x}t) &= (2\pi)^{\frac{1}{2}}\, \hbar c\, (\{\vec{y}s_0|^{\beta}\varphi_{\alpha}'^{\dagger}|\vec{x}t) - (\vec{x}t|_{\alpha}\varphi_{\beta}'|\vec{y}s_0\})/i. \end{aligned}\right\} \quad . \; . \; (10)$$

The numerical constant has been inserted for later purpose of normalization.

Though it is quite a relief that the cumbersome notation for the wave operators has to be abandoned, the role of the omitted variables and dashes should continually be kept in mind. We shall further write the argument $(\vec{x}t)$ in general as (x).

2.16 *D-functions.* The asymmetrical and symmetrical invariant *D*-functions are (beause of $m = 0$ degenerated into)

$$\left.\begin{aligned} D_a(x) &= -\frac{i}{hc}\,\delta(x^{\alpha}x_{\alpha}) = -\frac{i}{hc}\,(\delta(x-ct)-\delta(x+ct))/2x, \\ D_s(x) &= \frac{1}{\pi hc}\,\frac{1}{x^{\alpha}x_{\alpha}} = \frac{1}{\pi hc}\left(\frac{1}{x-ct}-\frac{1}{x+ct}\right)/2x. \end{aligned}\right\} \quad . \; (11)$$

We shall also need their linear combinations

$$D_{\pm}(x) = D_s(x) \pm D_a(x). \quad . \; . \; . \; . \; . \; . \; . \; . \; (12)$$

2.17 *Commutation relations.* We check that for each kind of wave operators we do obtain the required commutation relations

$$\left.\begin{aligned} &[(\vec{x}t|_{\alpha}\varphi_{\beta}'|\vec{y}s_0\}\,,\,\{\vec{y}s_0|^{\beta}\varphi_{\alpha'}'^{\dagger}|\vec{x}'t')]^{-} \\ &= \sum_{r}\sum_{\pm}(\lambda_{\pm\alpha\gamma}\lambda^{\dagger}_{\pm\alpha'\gamma'} - \mu_{\pm\alpha\gamma}\mu^{\dagger}_{\pm\alpha'\gamma'})\int(d\vec{\xi})\,(\vec{x}t|^{\gamma}\varphi|\vec{\xi}\pm r)\,(\vec{\xi}\pm r|\varphi^{\dagger\gamma'}|\vec{x}'t') \\ &= g_{\alpha\alpha'}\,D_a(x-x'). \end{aligned}\right\} \quad (13)$$

The commutators which give zero have not been written down.

The field operators satisfy the commutation relations

$$\left.\begin{aligned} &[\Phi_{c\alpha}(x)\,,\,\Phi_{c\alpha'}(x')]^{-} = [\Phi_{s\alpha}(x)\,,\,\Phi_{s\alpha'}(x')]^{-} = 2\pi(\hbar c)^2 g_{\alpha\alpha'}\,2D_a(x-x'), \\ &[\Phi_{c\alpha}(x)\,,\,\Phi_{s\alpha'}(x')]^{-} = 0. \end{aligned}\right\} \quad (14)$$

2.18 *Empty-empty part.* 2.181 *Empty states.* With a view to unification we shall be particularly interested in states in which no photons are present (empty states). The wave function of such a state contains no sets $(\vec{y}s_0)$. If it contains no variables of particles of another kind, it is a constant. The empty state projection operator **E** has the eigenvalue 1 in the empty states, 0 in all other states. As the empty-empty part of an operator $\mathbf{Q}\{\vec{y}_1 s_{10}, \vec{y}_2 s_{20}, \ldots\}$ we define the operator

$$\mathbf{E}\,\mathbf{Q}\{\vec{y}_1 s_{10}, \vec{y}_2 s_{20}, \ldots\}\,\mathbf{E}. \quad . \; . \; . \; . \; . \; . \; . \; . \; (15)$$

It is that part of **Q**, which gives an empty state function if it operates upon an empty state function and zero otherwise.

2.182 *Order in time.* Suppose we have a product of creation and annihilation operators, each containing a set $(\vec{x}t)$. We shall call the product well ordered in positive/negative time direction if the values of the successive t's do not increase/decrease if one goes from on factor to its neighbour in the direction to which the product operates.

2.183 *A product of exponentials.* We shall later need the empty-empty part of a product of exponentials in the field operators of the form

$$\prod_{k=1}^{n} e^{\frac{ie}{\hbar c} s_k^{\alpha} \Phi_{\alpha}(x_k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

in which the n factors are well ordered in positive time direction. Because according to (14) the factors commute in world points with a space-like connection, the product (16) is independent of the choice of the time axis. Therefore we can order (16) with respect to the direction of the time-like vector j^{a} used in (6), even if the time axis has a different direction.

The product (16) can be expanded into a power series in the Φ's. Each Φ is according to (10) a linear combination of a creation and an annihilation operator. Each creation operator adds a set $(\vec{y}s_0)$, each annihilation operator takes away such a set. If no sets are left to be taken away (empty states) the annihilation operator yields zero. Only those terms of (16) can contribute to the empty-empty part, which are a product of a number of creation operators and an equal number of annihilation operators and in which no factor is preceded by more annihilation than creation operators. Each of these products begins with a creation operator, which is supposed to act on an empty state function and ends with an annihilation operator, which than produces an empty state function again. The creation and annihilation operators of such a product can be dovetailed together with the help of (8) and (7), observing that the product is well ordered in the time direction of j^{a}. The empty-empty part of (16) is finally obtained by summing all these terms. It is most easily seen by means of complete induction that the result is

$$\prod_{kl,=1}^{n} e^{\frac{ie^2}{\hbar c} \frac{1}{2} s_k^{\alpha} s_l^{\alpha'} W_{\alpha\alpha'}(x_k, x_l)} \quad \mathbf{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The cross factors $(k \neq l)$ are counted twice, the "self" factors $(k = l)$ once. Whether Φ_c or Φ_s is used in (16), the function $W_{\alpha\alpha'}(x, x')$ is determined by

$$\frac{i}{hc} W_{\alpha\alpha'}(x, x') = \sum_{r} \sum_{\pm} \lambda_{\pm\alpha\gamma} \lambda^{\dagger}_{\pm\alpha'\gamma'} \int (d\vec{\xi}) (\vec{\xi} \pm r | \varphi^{\dagger\gamma''} | x'') (x''' |^{\gamma'''} \varphi | \vec{\xi} \pm r)$$

$$+ \sum_{r} \sum_{\pm} \mu_{\pm\alpha\gamma} \mu^{\dagger}_{\pm\alpha'\gamma'} \int (d\vec{\xi}) (x'' |^{\gamma''} \varphi | \vec{\xi} \pm r) (\vec{\xi} \pm r | \varphi^{\dagger\gamma'''} | x'''). \quad (18)$$

2 and 3 dashes stand for 0 and 1 dash or crosswise in that way for which with respect to the time direction of j^a x'' is later than x'''. We regard to this order in time we write

$$\sigma(x) = -\frac{j_\alpha x^\alpha}{|j_\alpha x^\alpha|};\ f^\sigma(x^\alpha) = f(x^\alpha + j^\alpha(j_\beta x^\beta + |j_\beta x^\beta|)), \quad \ldots \quad (19)$$

so that for even and odd functions $f(x)$

$$f^\sigma_{\text{even}}(x) = f_{\text{even}}(x),\ f^\sigma_{\text{odd}}(x) = \sigma(x) f_{\text{odd}}(x). \quad \ldots \quad (20)$$

Then (18) reads

$$\frac{i}{hc} W_{\alpha\alpha'}(x,x') = \sum_r \sum_\pm (\lambda_{\pm\alpha\gamma} \lambda^\dagger_{\pm\alpha'\gamma'} + \mu_{\mp\alpha\gamma} \mu^\dagger_{\mp\alpha'\gamma'})(g^{\gamma\gamma'} + 2j^\gamma j^{\gamma'}) D^\sigma_\pm(x-x'). \ (21)$$

The 4 combinations *Dd*, *Dh*, *Hd* and *Hh* give different results

$$\left.\begin{array}{l|l} & \frac{i}{hc} W_{\alpha\alpha'}(x,x') \\ \hline Dd & g_{\alpha\alpha'} D^\sigma_a(x-x') \\ Dh & (g_{\alpha\alpha'} + 2j_\alpha j_{\alpha'}) D^\sigma_a(x-x') \\ Hd & g_{\alpha\alpha'} D^\sigma_+(x-x') \\ Hh & (g_{\alpha\alpha'} + j_\alpha j_{\alpha'}) D^\sigma_+(x-x') + j_\alpha j_{\alpha'} D^\sigma_-(x-x'). \end{array}\right\} \quad \ldots \quad (22)$$

The evaluation of the numerical coefficients in the expansion of (17) into a power series in the W's is a matter of combinatorics. The corresponding coefficients in the dovetailed expansion of (16) are only partially a result of combinatorics, for the rest they are produced at the dovetails by the square roots which occur in (7). At this point the EINSTEIN–BOSE statistics of the photons play a decisive part.

The functions $D^\sigma_s(x)$ and $D^\sigma_a(x)$ are not only relativistic invariant but even independent of j^a, because $D_s(x)$ is even and $D_a(x)$, which is odd, vanishes outside the light cone.

2.2 *Electrons and photons.* Now turning to the electrons and their interaction with photons we first consider a single electron.

2.21 *1 electron.* 2.211 *Notation.* For electrons we shall write the time-space vectors which occur as arguments in the wave functions as $(ct, x^1, x^2, x^3) = (x^0, x^1, x^2, x^3)$. In this paper we shall not use creation and annihilation operators of electrons so that we need no other sets than (x). Further we write the spin matrices $(1, \alpha^1, \alpha^2, \alpha^3) = (\alpha^0, \alpha^1, \alpha^2, \alpha^3)$. ($\alpha^\alpha$ transforms as a 4-vector density, $\beta\alpha^\alpha$ as a 4-vector).

2.212 *Equations of motion.* 2.2121 *No photons.* We put the electron with charge e and mass m in a given 4-potential field (A^0, A^1, A^2, A^3). This field only serves to determine the unperturbed state of the electron and will not be quantized. The unperturbed equation of motion is

$$\mathbf{K}^0\{x\}(x|\psi^0 = 0 \quad \ldots\ldots\ldots \quad (23)$$

with

$$\mathbf{K}^0\{x\} = \alpha^\alpha \left(\frac{\hbar c}{i}\frac{\partial}{\partial x^\alpha} + e A_\alpha(x)\right) + \beta mc^2. \quad . \quad . \quad . \quad . \quad (24)$$

The density operator is **1**, the inner product of $\psi^\dagger \mid x)$ and $(x \mid \psi'$ is

$$\int (d\vec{x})\, \psi^\dagger \mid x)\,(x \mid \psi'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The wave function at a time t is determined by the wave function at a time t' according to the integral equation

$$(x \mid \psi^0 = \int (dx')\, \mathbf{N}(x; x')\,(x' \mid \psi^0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where the nucleus $\mathbf{N}(x; x')$ satisfies the differential equation

$$\mathbf{K}^0\{x\}\,\mathbf{N}(x;\, x') = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

with the initial condition

$$\mathbf{N}(x; x')_{t=t'} = \delta(\vec{x} - \vec{x}'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

If $(x \mid \psi^0 \mid \mu)$ is a complete orthonormal system of solutions of (23), $\mathbf{N}(x; x')$ is given by

$$\mathbf{N}(x; x') = \sum_\mu (x \mid \psi \mid \mu)\,(\mu \mid \psi^\dagger \mid x'). \quad . \quad . \quad . \quad . \quad . \quad (29)$$

2.2122 *With photon interaction.* In a photon field the equation of motion is

$$(\mathbf{K}^0\{x\} + \mathbf{K}^1(x))\,(x; y_{10}, \ldots \mid \psi = 0 \quad . \quad . \quad . \quad . \quad . \quad (30)$$

with the interaction operator

$$\mathbf{K}^1(x) = \alpha^\alpha\, e\, \Phi_\alpha(x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The wave functions are taken up to date in the electron coordinates (x), which stand for $(\vec{x}t)$, and at the beginning in the photon coordinates (y_{k0}), which stand for $(\vec{y}_k\, s_{k0})$, $(k = 1, \ldots\ldots)$. Speaking about the wave function at a certain time will therefore always refer to electron time. The 4-potential operator Φ_α may be either $\Phi_{c\alpha}$ or $\Phi_{s\alpha}$ of (10), the remaining one is redundant [8]) and further on denoted by $\Phi_{rd\alpha}$. Φ_α in the equation of motion describes creation and annihilation of photons. It is formed from the wave operators of free photons. That means that apart from creation and annihilation the photons behave dynamically just as in the absence of charges.

In order to determine the wave function at a time t from the wave function at a time t' we shall prefer to use a not full-grown calculus introduced by FEYNMAN [9]) without taking over his interpretation. This calculus which is mathematically in a rather undeveloped state provides a very apt tool for handling with the general formulation of our present problems.

We divide the time interval $t'\,t$ into a large number p of infinitesimal intervals $t'\,t''$, $t''\,t'''$,... $t^{(p)}\,t^{(p+1)}$ with $t^{(p+1)} = t$. Our expressions are hoped to be valid in the limit $p \to \infty$, max $| t^{(k+1)} - t^{(k)} | \to 0$. Then we write

$$\left.\begin{array}{l}(x; y_{10}, \ldots | \psi = \lim \prod\limits_{m=1}^{p} (\int (\overrightarrow{dx^{(m+1)}})\, \mathbf{N}(x^{(m+1)}; x^{(m)})) \\ e^{\frac{ie}{\hbar c}\sum\limits_{m=1}^{p}(x^{(m+1)\alpha} - x^{(m)\alpha})\Phi_\alpha(x^{(m')})} \qquad (x'; y_{10}, \ldots | \psi.\end{array}\right\} \quad . \quad . \quad (32)$$

$x^{(m')\alpha}$ lies in the interval $x^{(m)\alpha}\, x^{(m+1)\alpha}$. In order to get later on an easier notation we have symbolically written a sum in the exponent rather than writing the exponential under the product sign. That makes that whereas otherwise the product of exponentials had to be well ordered in time, this order has now to be observed in the sum of exponents. This comment has to be well observed for all following exponential operators, which without it would have to be understood in a different way. Also the N's have to be well ordered in time. They commute with the exponentials.

It is easily seen that (32) satisfies the equation (30) and the initial condition $(x | \psi = (x' | \psi$ for $t = t'$. This remains true if everywhere in (32) $(x^{(m+1)\alpha} - x^{(m)\alpha})$ is replaced by $\alpha^\alpha c\, (t^{(m+1)} - t^{(m)})$, provided the ordered N's are properly sandwiched between the ordered exponentials with which they no longer commute.

It would of course also be possible to treat the interaction with the field A_α in the same way as that with the field Φ_α. Doing so one could write

$$\mathbf{N}(x^{(m+1)}; x^{(m)}) = \mathbf{N}^0(x^{(m+1)}; x^{(m)})\, e^{\frac{ie}{\hbar c}(x^{(m+1)\alpha} - x^{(m)\alpha}) A_\alpha (x^{(m')})}, \quad . \quad (33)$$

where $\mathbf{N}^0$ refers to free electrons. Dealing with $\mathbf{N}^0$ becomes urgent if one is interested in the self interaction of the electron. That belongs to Q_3.

2.22 *Many electrons*. 2.221 *Description*. In describing a system of many electrons we shall use many-times theory [10] [11].

We do not consider creation and annihilation of electrons, their number n is taken fixed. We shall not use electron creation and annihilation operators. Further in order to avoid unessential complications we shall not be concerned with electron exchange effects. Therefore we treat the electrons as if they were discernible and only account for the FERMI–DIRAC statistics by PAULI's exclusion principle.

2.222 *Equations of motion*. 2.2221 *No photons*. The unperturbed field A_k will be taken different for different electrons. The unperturbed equations of motion

$$\mathbf{K}_k^0 \{x_k\} (x_1, \ldots x_n | \Psi^0 = 0 \qquad (k = 1, \ldots n) \quad . \quad . \quad . \quad (34)$$

are independent of each other. There is no interaction between the electrons. The wave function at a set of times $(t_1, \ldots t_n)$ is determined by the wave function at a set of times $(t_1', \ldots t_n')$ according to

$$(x_1, \ldots x_n | \Psi^0 = \int \ldots \int (\overrightarrow{dx_1'}) \ldots (\overrightarrow{dx_n'})\, \mathbf{N}_1(x_1; x_1') \ldots \mathbf{N}_n(x_n; x_n')\, (x_1', \ldots x_n' | \Psi^0. \quad (35)$$

The nuclei $\mathbf{N}_k$ and $\mathbf{N}_l$ for different electrons ($k \neq l$) commute with each other.

2.2222 *With photon interaction.* In a photon field the equations of motion are

$$(\mathbf{K}_k^0 \{x_k\} + \mathbf{K}^1(x_k))(x_1, \dots x_n; y_{10}, \dots | \Psi = 0 \qquad (k = 1, \dots n), \tag{36}$$

where $\mathbf{K}^1(x)$ is given by (31). The commutation relations (14) make that the n equations (36) are only compatible if all world points x_k and x_l ($k, l = 1, \dots n$; $k \neq l$) lie outside each others light cone [12], i.e. if $c \,|\, t_k - t_l \,| < |\, \vec{x}_k - \vec{x}_l \,|$. The wave function at a set of times ($t_1, \dots t_n$) is determined by the wave function at a set of times ($t'_1, \dots t'_n$) according to

$$\left.\begin{aligned} &(x_1, \dots x_n; y_{10}, \dots | \Psi = \lim \prod_{k=1}^{n} \prod_{m=1}^{p} \left(\int (d\vec{x}_k^{(m_k)}) \, \mathbf{N}_k (x_k^{(m_k+1)}; x_k^{(m_k)}) \right. \\ &e^{\frac{ie}{\hbar c} \sum_{k=1}^{n} \sum_{m=1}^{p} (x_k^{(m_k+1)\alpha} - x_k^{(m_k)\alpha}) \Phi_\alpha (x_k^{(m'_k)})} \qquad (x'_1, \dots x'_n; y_{10}, \dots | \Psi. \end{aligned}\right\} \tag{37}$$

For each k ($k = 1, \dots n$) the product of the $\mathbf{N}_k$'s and the (symbolical) sum of exponents each have to be well ordered in all electron times. Everywhere in (37) $(x_k^{(m_k+1)\alpha} - x_k^{(m_k)\alpha})$ can again be replaced by $\alpha_k^\alpha \, c \, (t_k^{(m_k+1)} - t_k^{(m_k)})$, provided the ordered N's are properly sandwiched between the ordered exponentials.

(37) can be maintained for electron world points which lie inside each others light cone.

2.23 *Supplementary conditions.* The wave functions are still subjected to supplementary conditions. They will be dealt with in 4.12.

(*To be continued.*)