Astronomy. — *Magnitude effects in G-type stars.* By P. J. GATHIER.
(Communicated by Prof. M. G. J. MINNAERT.)

(Communicated at the meeting of March 26, 1949.)

Summary.

Equivalent widths have been measured for Ca I λ 4227 and Sr II λ 4077 in the spectra of 14 stars — supergiants, giants, dwarfs — with all about the same spectral type as the sun. The observed values are satisfactorily explained by the theory of stellar atmospheres. The damping constant is found to be five to ten times larger for dwarfs than for giants and supergiants.

Introduction.

The material consists of 14 stellar spectra, taken by Professor MINNAERT at the McDONALD Observatory with the Cassegrain spectrograph, equipped with two quartz prisms. The focal length of the camera was 50 cm, the dispersion was 40 A/mm near H_γ; the plates cover the region λλ 3500—4700. The stars investigated are dwarfs, giants and supergiants of the spectral types *F*5—*G*5. Nine of the plates carried a calibration, obtained with a tube photometer. Since the exposure times did not differ by more than a factor two and since all plates were developed in the same way, it was possible to construct one mean density curve for the whole material. The spectra have been recorded by means of the Utrecht microphotometer on a 60-fold scale.

We proposed to study for these stars the relation between the observable quantities: the absolute magnitude *M* and the spectral type, and the theoretical parameters: temperature and effective gravitational acceleration. Just because the atmosphere of the sun has been studied so well, these stars of a similar type are well fitted for a comparative theoretical investigation.

A search for luminosity characteristics.

The stars which have been selected are but little different in spectral type; since their absolute magnitudes vary over a considerable range, vid. lic. from $M = +6$ to $M = -6$, it seemed interesting to look in the first place for spectral lines which are very sensitive to variations of the absolute magnitude. With the purpose to find such characteristic lines, the galvanometer deflections on the registrograms were compared for giants and for dwarfs, without converting them into intensities. In figure 1 these values are compared for the dwarf star ζ Her GO IV and for the supergiant β Aqu GO I^b. A hundred points were obtained, scattered about a main line; this is not a straight line under an angle of 45°, because the plates have not the same density. Some points are lying well away from

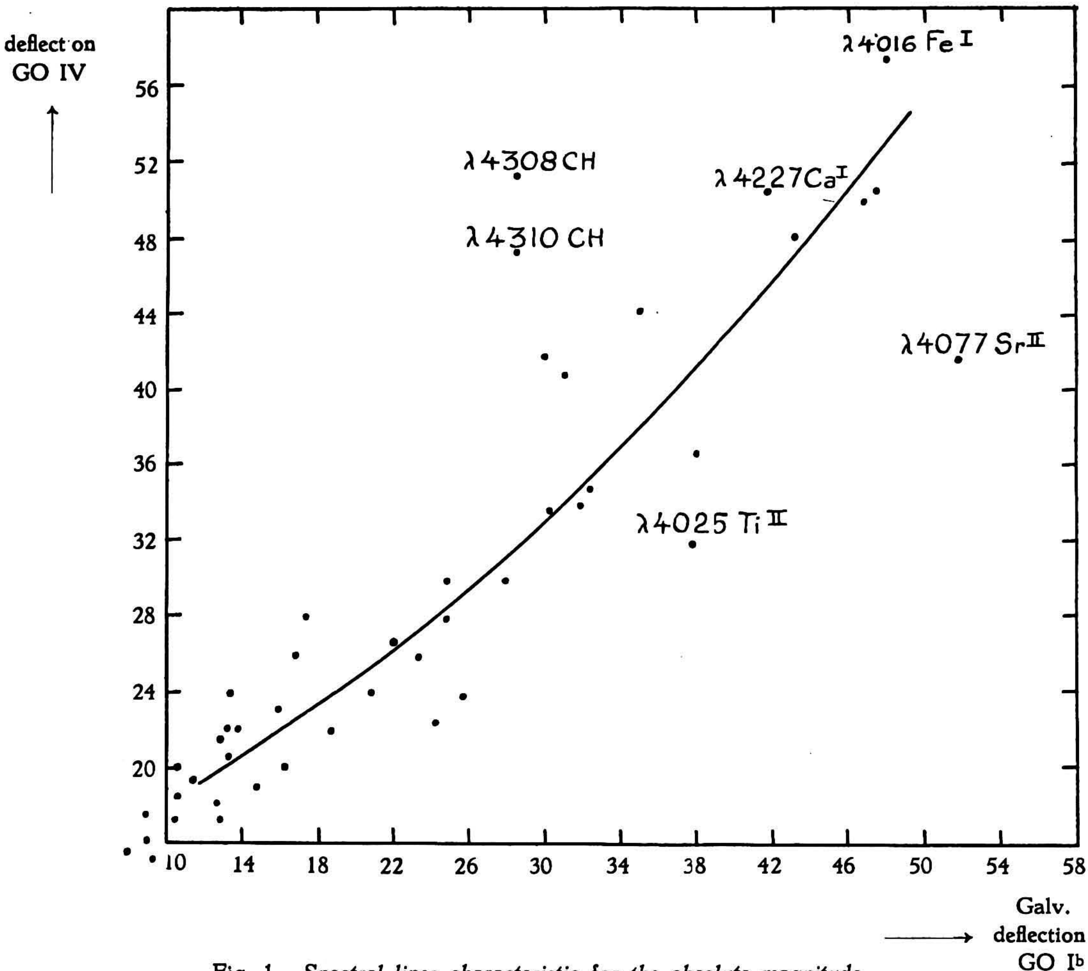

Fig. 1. *Spectral lines characteristic for the absolute magnitude.*
Comparison between the galvanometer deflections for several spectral lines in a dwarf and in a supergiant.

it: these represent lines which are much stronger or much weaker in giants than in dwarfs. Strikingly stronger in giants are for example Sr II λ 4077 and Ti II λ 4025, while the CN-bands and Ca I λ 4227 are conspiciously weaker. These are just the lines, generally used as criteria of the luminosity for G-type stars. For some of these lines the variations of the galvanometer deflection with the spectral type have been determined and the ratio between the deflections for two spectral lines has been investigated as a function of the absolute magnitude (figure 2). Here for the determination of the spectral type, MORGAN's classification [1]) has been

[1]) MORGAN, KEENAN, EDITH KELLMAN, An atlas of stellar spectra.

used (table I). Two interesting lines, namely Ca I λ 4227 and Sr II λ 4077 have been studied into more details, with the purpose to connect the theory of the stellar atmospheres with the observations.

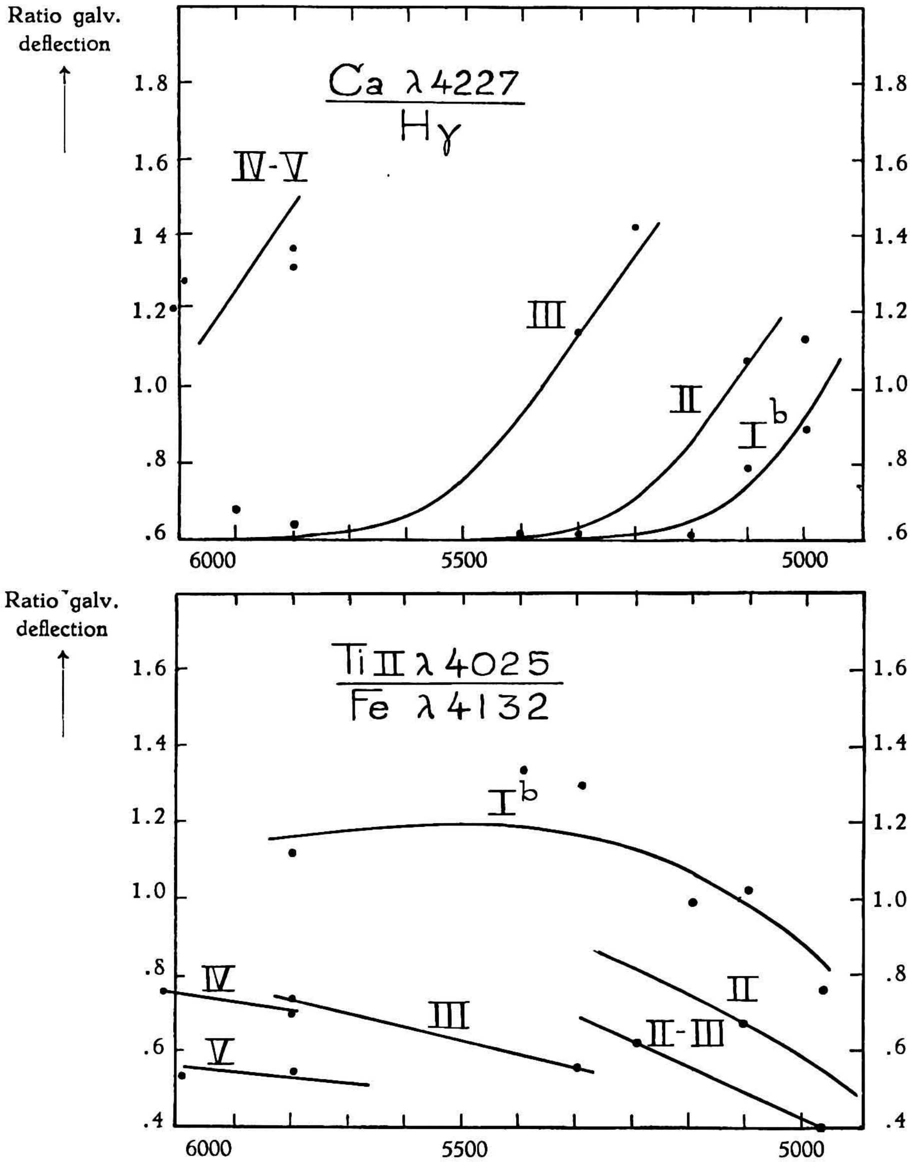

Fig. 2. The ratio of the galvanometer deflections:
1) for Ca I λ 4227 and H_γ;
2) for Ti II λ 4025 and Fe I λ 4132.
Variation of these ratio's in stars of different absolute magnitudes and effective temperatures.

Ca I λ 4227.

The equivalent width W_λ was measured on the available 14 stellar spectra.

TABLE I. Stars observed for spectral line intensities.

Star	Spectrum	M	W_{4077}	W_{4227}	T_{eff}° K
β Aqu	G0 I^b	−3	2.0	1.4	4950
ζ Her	G0 IV	+3.2	1.0	1.4	5800
α Per	F5 I^b	−3	1.55	0.95	5800
$\varkappa$ Cet	G2 V	+5.0	0.77	1.5	5900
δ C Ma	F8 I^a	−6	2.3	1.5	5000
9 Peg	G2 I^b	−2.3	1.65	2.0	4700
$\varkappa$ Cet	G2 V	+5.0	0.8	1.5	5900
β Lep	G2 II	−2.0	1.0	1.4	5000
η Cas A	G0 V	+4.9	0.6	0.9	6050
υ Peg	F8 III	+0.7	0.9	0.9	5700
ϱ Pup	F6 II	−0.3	1.3	1.1	5800
γ Cyg	F8 I^b	−4.3	1.8	1.1	5300
α Sag	G1 II	−1.9	1.35	1.35	5100
η Peg	G2 II—III	−1.0	1.25	1.5	5100
9 Peg	G2 I^b	−2.3	1.7	1.9	4700
υ And	F8 IV	+4.9	0.75	1.2	6000

Because λ 4227 is a resonance line and all Ca-atoms are in the fundamental level, the number of absorbing atoms is determined by the ionisation temperature T_{ion}. According to investigations on spectra taken with a considerable dispersion T_{ion} will be very much the same as the effective temperature T_{eff}.

With the help of the data of BECKER [2]) and KUIPER [3]) the relation between the effective temperature and the spectral type has been determined for each of MORGAN's luminosity classes (table I).

In figure 3 the measured W_λ-values are plotted as a function of T_{eff}, values of W_λ determined by other writers being also included. Two different sequences are clearly shown, one for the dwarfs and another one for the giants and supergiants.

The following simple theory was used for the calculation of the equivalent widths of Ca I λ 4227 in stellar atmospheres, characterized by different values of T and the effective gravitational acceleration g. The total number of calcium-atoms in a stellar atmosphere is determined by:

1. The total number of all atoms or ions in that part of the atmosphere which can be considered to be transparent;
2. the Ca-abundance, A_{Ca}, defined as the fraction of the atoms or ions in the atmosphere which are either Ca I or Ca II;
3. the equilibrium between the number of Ca-atoms and Ca-ions, as given by SAHA's law for every value of T and p_e, the electron pressure.

For the calculation in these three steps, the numerical values of the different quantities will be chosen as well as possible in accordance with

---

[2]) BECKER, Zs. f. Ap. 25, 145 (1948).

[3]) KUIPER, Ap. J. 88, 429 (1938).

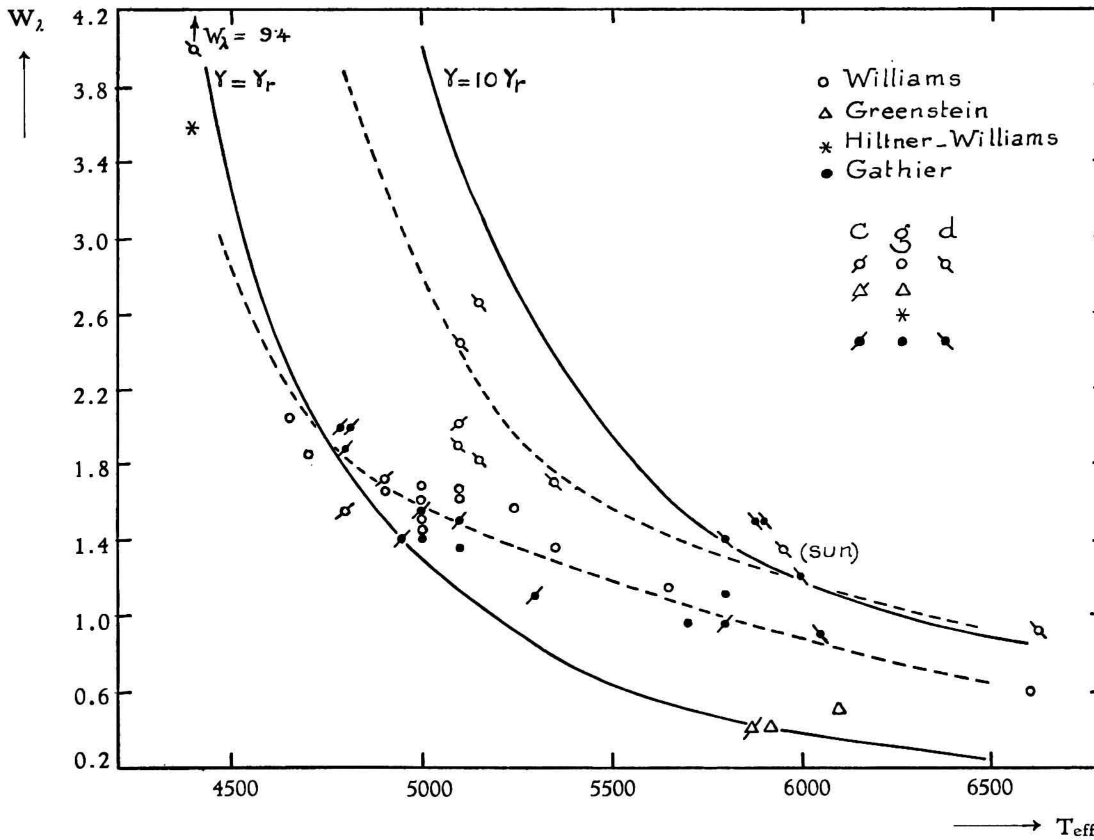

Fig. 3. *Equivalent widths of Ca I λ 4227.*
Fulldrawn lines: theoretical equivalent widths for $\gamma = \gamma_r$ and for $\gamma = 10\,\gamma_r$.
Dotted lines: observed values of W_{4227} as a function of temperature, for giants-supergiants (left line) and for dwarfs (right line).
The dispersions used are (near H_γ):
WILLIAMS [4]) 17 A/mm; GREENSTEIN [5]) 3 A/mm;
HILTNER–WILLIAMS [6]) 3 A/mm; GATHIER 40 A/mm.

earlier results of other authors. If N is the average number of atoms or ions per cm³ and H is the effective height of the atmosphere, we have:

$$NH = \frac{\tau_0}{\mu\, m_H\, \varkappa}$$

where τ_0 is the effective optical depth, μ the mean atomic weight, m_H the mass of the H-atom and $\varkappa$ the continuous absorption coefficient [7]).

We assume [8]) $\mu = 1.5$ and provisionally $\tau_0 = 0.7$, which is a convenient value for the sun [9]).

[4]) WILLIAMS, P. A. S. P. **48**, 113 (1936).
[5]) GREENSTEIN, Ap. J. **107**, 151 (1948).
[6]) HILTNER–WILLIAMS, A Photometric Atlas of Stellar spectra.
[7]) UNSÖLD, Physik der Sternatmosphären 1938, Chapter XII.
[8]) Miss ROSA, Zs. f. Ap. **25**, 9 (1948).
[9]) TEN BRUGGENCATE, Veröff. Univ. Sternw. Göttingen **76** (1947).

At present it seems well established that the continuous absorption for stars of the spectral type investigated is due to the H^--ions; the absorption coefficient $\varkappa$ is known from theory [10].

For A_{Ca} a value 1.45×10^{-6} has been taken. The electron pressure p_e can be expressed as a function of T and the gas pressure p. With the aid of the relation $p = \frac{\tau}{\varkappa} g$ we introduce g as parameter instead of p_e [11]. So, if N_a is the number of active Ca-atoms, it is possible to calculate $N_a H$ for every value of T and g (fig. 4). From this figure one can see that in

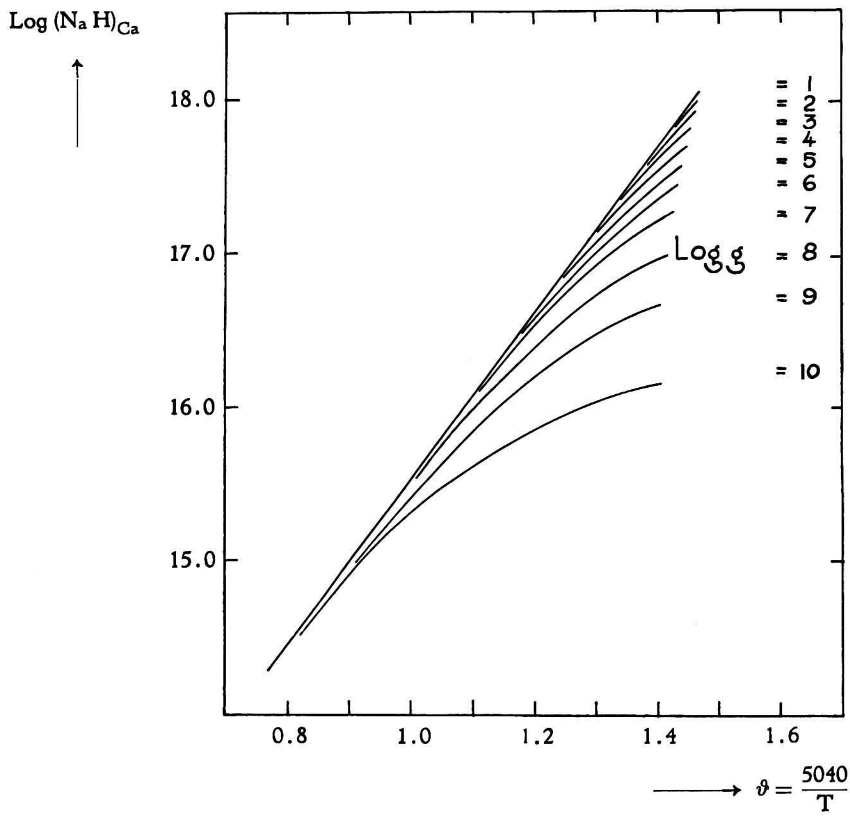

Fig. 4. *The theoretical number of active Ca-atoms,* for different temperatures and effective gravitational accelerations.

the temperature range here investigated, since log g is always < 5, the values of $N_a H$ cannot be expected to be different for dwarfs and for giants.

The Ca-line at λ 4227 is a strong line and so its equivalent width can be calculated by the relation:

$$W_\lambda = \frac{\lambda^2}{c} \sqrt{\frac{e^2}{mc} N_a H \gamma f}.$$

10) CHANDRASEKHAR and MÜNCH, Ap. J. **104**, 446 (1946).

11) Compare also Miss ROSA, Zs. f. Ap. **25**, 4 (1948).

The best value for f is 1.2. About the damping there is always the uncertainty how much of it is due to the radiation and how much to collisional damping. Therefore W_λ has been calculated for two different cases:

1. for pure radiation damping, $\gamma = \gamma_r = 1.6 \times 10^8$; and
2. for the damping as derived from the solar spectrum, which is ten times the damping caused by radiation. In figure 3 both theoretical cases have been considered.

In comparing the theoretical curves with the observations, it is found that the E.W., measured on small dispersion plates, are in general too great; GREENSTEIN, working with the Coudé-spectrograph, finds lower values which are much closer to the theoretical ones. This effect has been found many times before; it may be due to blends, which are not well discerned if the dispersion is small; or perhaps to photographic effects. We conclude that probably some correction of the order of —0,4 A will have to be applied to our measurements, but that the main differences between the luminosity classes will subsist and that the values of the damping constants will not materially be altered.

The giants and supergiants are pretty well in agreement with the theoretical curve for $\gamma = \gamma_r$. Evidently γ is greater for dwarfs, because the pressures in their atmospheres and the collisional damping are much higher than in giants or even supergiants. To the change from giants to supergiants corresponds no further decrease of γ; this seems to show that we have already reached a minimum value for γ in the giants, which must evidently be γ_r. Thus for stars with $M < 3$ or $\log g < 2$, we have $\gamma = \gamma_r$, while for larger values of M the damping increases rapidly. For the sun ($M = 5$, $\log g = 4.4$) usually $\gamma = 10\ \gamma_r$ is adopted; WRIGHT supposes that this applies to all dwarfs [12]).

From figure 3 a somewhat smaller value for γ is deduced but this may be due to our assumption, that the effective optical depth is the same in dwarfs and in giants. Moreover, the material is still too small to derive more definitive values.

Sr II λ 4077.

Also for this spectral line the equivalent widths have been measured on the available plates (figure 5). The theoretical values of λ 4077 have been calculated in the same way as for λ 4227. For the Sr-abundance we assumed 2.35×10^{-9} [13]), while for all other quantities the same numerical values as for λ 4227 have been used. In figure 5 on the left side the theoretical values of W_λ have been plotted for $\gamma = \gamma_r$, and at the right for $\gamma = 10\,\gamma_r$. The assertion that $\gamma = \gamma_r$ for giants and supergiants and $\gamma = 10\,\gamma_r$ for dwarfs seems well-confirmed; the g-values from figure 5

[12]) WRIGHT, Dom. Ap. Obs. **8**, 1 (1948).

[13]) UNSÖLD, Zs. f. Ap. **24**, 306 (1948).

agree in a satisfactory way with those, mentioned in the literature for supergiants, giants and dwars [14], [15], [16].

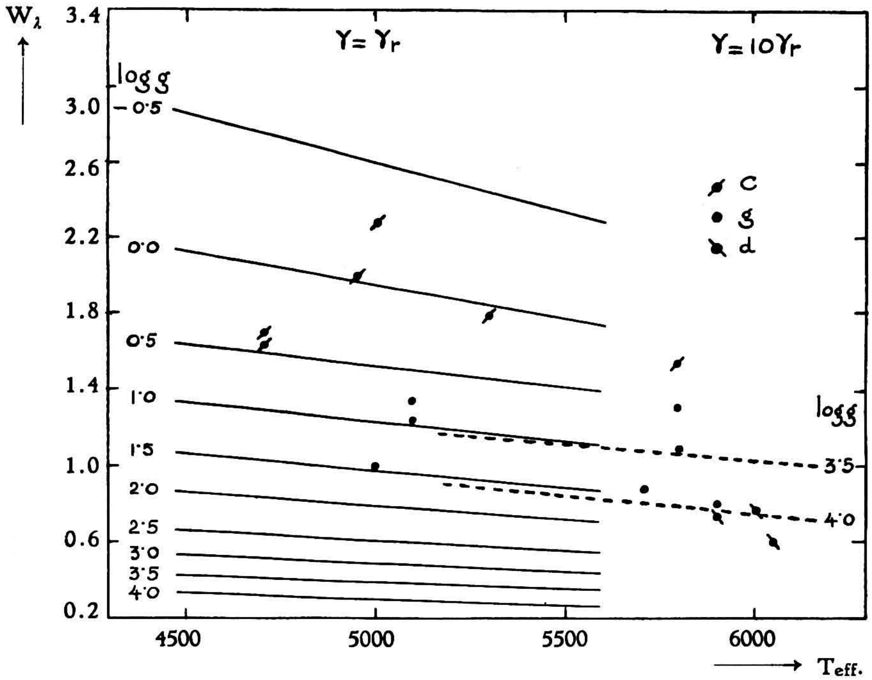

Fig. 5. *Equivalent widths of Sr II λ 4077.*
Fulldrawn lines: the theoretical values of W_{4077} for different values of log g, computed for $\gamma = \gamma_r$.
Dotted lines: the same for $\gamma = 10\,\gamma_r$. The points represent the measured W_{4077}.

I am much indebted to Professor M. MINNAERT, who suggested this investigation and gave me his continuous help.

---

[14]) GREENSTEIN, Ap. J. **107**, 151 (1948).
[15]) WRIGHT, Dom. Ap. Obs. 8, 1 (1948).
[16]) PANNEKOEK and VAN ALBADA, Publ. Astr. Inst. Univ. Amsterdam, **6** (1946).