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THE PRINCIPAL WORKS
OF
SIMON STEVIN

EDITED BY

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C. V. SWETS & ZEITLINGER

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SIMON STEVIN

VOLUME III

ASTRONOMY

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NAVIGATION

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SIMON STEVIN

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THE ASTRONOMICAL WORKS

OF

SIMON STEVIN

DE HEMELLOOP

THE HEAVENLY MOTIONS

From the *Wisconstighe Gbedachtenisen* (Work XI; i, 3)

INTRODUCTION TO THE WORK

§ 1

In order to understand the place of Stevin's work on astronomical theory it is necessary first to give a short survey of the opinions prevalent among scholars in the second half of the sixteenth century.

Copernicus' great work *De revolutionibus* appeared in 1543 under the more extensive title, due to Osiander: *De revolutionibus orbium coelestium*. Immediately after its publication it became the object of assiduous study, at the main Protestant university of Wittenberg as well as among scholars at other seats of learning. This interest concerned not so much the heliocentric theory, but rather the numerical elements of the orbits. From the first the heliocentric theory was sharply attacked by the Protestant theologians. Melanchthon (the "*praeceptor Germaniae*"), the foremost among the Wittenberg professors, in a series of lectures and in his *Initia doctrinae physicae*, dismissed it as absurd¹). This qualification determined the opinion of contemporary authors. At the same time, however, Copernicus, because of the new basis he afforded for the computation of the celestial motions, was praised as the renovator of astronomy, the first and most famous of astronomers, the new Ptolemy. Several students of the book computed in advance the positions of the planets or the moon with the aid of the new data, and they were able to show that these were in better agreement with the observations than the Alphosine Tables. Foremost among them was Erasmus Reinhold²), professor of mathematics at Wittenberg. First he had to correct several errors in the computations of Copernicus, and in many cases he derived new elements himself. Thus he was able to construct new and better tables, called the Prussian Tables, published in 1551 and reprinted several times afterwards. The new heliocentric world-system, however, is not even alluded to in this work. Reinhold's tables were used by Johannes Stadius³) for the computation of his "Ephemerides" (daily tables) of the celestial bodies. These tables, destined chiefly for use in astrological prognostics, were published in 1556 for the first time, and in later years in five new editions.

In his valuable work on the origin and the extension of the Copernican doctrine, Ernst Zinner⁴) enumerates a number of authors of widely used textbooks on astronomy. Besides Melanchthon's book, mentioned above, with 17 impressions, and Clavius' explanation of the astronomical work of Sacrobosco, which appeared in 1570 and up to 1618 passed through 19 impressions⁵), he mentions Kaspar Peucer, Hartmann Beyer, Michael Neander, Victor Strigel, Heinrich Brucaeus, Georg Bachmann, Alb. Leoninus, Paolo Donati, G. A. Magini, Jean Bodin and others⁶). They all reject the earth's motion or are silent on it. And he remarks:

¹) Ph. Melanchthon, *Initia Doctrinae Physicae*, 1549 (Ed. Bretschneider in *Corpus Reformatorum*, Vol. 13, p. 179). Cf. p. 216 Liber I.

²) Erasmus Reinhold, *Pruenicae Tabulae coelestium motuum* (Tübingen, 1551).

³) Joh. Stadius, *Ephemerides novae et exactae, ab Anno 1554* (Köln, 1556).

⁴) Ernst Zinner, *Entstehung und Ausbreitung der copernicanischen Lehre* (Erlangen, 1943).

⁵) Chr. Clavius, *Opera mathematica V tomis distributa* (Mainz, 1612).

⁶) Kaspar Peucer, *Hypothesen astronomicae* (1571).—Hartmann Beyer, *Quaestiones novae* (1549—1573, 6 impressions).—Michael Neander, *Elementa Sphaericae doctrinae* (1561).—

"Were there any adherents of the new doctrine? We might have doubts, if we consider that until 1590 the work of Copernicus was reprinted only once ⁷⁾, whereas the textbooks mentioned above together saw 62 impressions. In these, the new doctrine either was not mentioned at all or was termed absurd; seldom was its special character set forth" ⁸⁾. Only in England in 1576 an enthusiastic description of the new doctrine was added by Thomas Digges ⁹⁾ to a new edition of a book on prognostics by his father ¹⁰⁾.

Yet the number of adherents slowly increased. Christopher Rothmann, astronomer at the Cassel observatory, in his correspondence with Tycho Brahe, in 1589-90, with very well-chosen arguments defended the motion of the earth ¹¹⁾. Tycho Brahe himself tried to combine the simplicity of the Copernican system with the central position of the earth at rest by a system specially devised — in 1583, as he said —: the planets in describing circles about the sun are carried along with it in its yearly orbit. Though here the motions were only nominally different from Ptolemy's, the Tyconic system found adherence as a symptom of an incipient critical attitude towards the old doctrine. In Italy in 1585 Benedetti spoke of the earth as a subordinate body ¹²⁾. Giordano Bruno, extending the Copernican system into a fantastic conception of a world of innumerable suns and inhabited planets, expounded it during his travels all over Europe. Kepler ¹³⁾ in his "*Mysterium Cosmographicum*", his first work, published in 1596, endeavoured to give the heliocentric system a deeper philosophical basis by explaining the number of the planets (six) and their distances by connecting them with the five regular polyhedra. The English physician William Gilbert ¹⁴⁾ in 1600, in his book on magnetism, introduced the daily rotation of the earth as Copernicus had done, but he did not speak of its yearly revolution.

This enumeration shows that when Simon Stevin, in explaining to his illustrious pupil the motion of the celestial bodies, expounded the Copernican as the true beside the Ptolemaic as the untrue system, he sided with an extremely small group of renovators. Whereas the other adherents had expressed their opinion occasionally, in short remarks or in connection with other subjects, Stevin's book was the first textbook destined to give a simple and easy exposition of the heliocentric theory. Soon after its publication (in Dutch in 1605, Work XI; i, 3) a Latin version appeared in the *Hypomnemata Mathematica* (Work XIIb). A French translation of the *Wisconstighe Gbedachtenissen* was published by Girard in 1633 in his posthumous edition of the Works of Stevin: *Oeuvres Mathématiques de Simon Stevin* (Work XIII).

Victor Strigel, *Epitome doctrinae de primo motu* (1564).—Heinrich Brucaeus, *De motu primo* (1573—1604). Georg Bachman, *Epitome Doctrinae de primo motu* (1591).—Albert Leoninus, *Theoria motuum coelestium* (1583).—Paolo Donati, *Theoriche overo Speculationi intorno alli Moti Celesti* (1575).—J. A. Magini, *Ephemerides coelestium motuum* (1599—1616).—Jean Bodin, *Universae Naturae Theatrum* (1597).

⁷⁾ Basle, 1566.

⁸⁾ Ernst Zinner, *l.c.*, pp. 275-276.

⁹⁾ *Alae seu Scalae Mathematicae* (1573).

¹⁰⁾ Leonard Digges, *A Prognosticon everlastinge. . .* (1576).

¹¹⁾ Cf. Tychonis Brahei Dani *Opera* VI, p. 217.

¹²⁾ J. B. Benedicti *Diversae Speculationes*, p. 195 and 256 (1585).

¹³⁾ Joh. Kepler, *Mysterium Cosmographicum* (1596).

¹⁴⁾ W. Gilbert, *De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure; Physiologia Nova* (1600).

§ 2

In his discussion of the orbits of the heavenly bodies, Stevin follows a special method. He knows that in order to derive these orbits one has to proceed from the observed positions of the sun, moon, and planets in the sky. It would, however, have taken too much time to make such observations with his pupil, Prince Maurice, while this was also beyond the scope of their joint studies. On the other hand, an attempt to derive the orbits from the few observations communicated by Ptolemy and Copernicus would be too difficult. He therefore proceeds from existing printed tables, taking as such the ephemerides computed by Stadius. Since these tables are based upon elements derived from observations, he assumes that they represent the observed motion of the heavenly bodies, and may therefore be used instead of observations for an explanation of the astronomical system. They offer the further advantage that the positions are given for all consecutive days. He calls them "*ervarings dachtafels*" (empirical ephemerides). Though in the Definitions 22 and 23 he distinguishes the empirical ephemerides as determined by instruments, and the computed ephemerides predicted through knowledge of the orbits, he refers to the computed ephemerides as empirical.

Another characteristic of his work is that the theory is presented according to both world-systems. In his First and Second Books the orbits are treated as in the Ptolemaic system, with the earth at rest at the centre; in his Third Book the Copernican theory of the moving earth is introduced. His own opinion on this point is clearly shown by the fact that he calls them the apparent and the true motion (p. 4 of the *Cortbegryp*). The arguments are to be given later on, because the "unknown" motions, to be treated afterwards, show that there is no firm basis as yet for a good theory.

In dealing with the motion of the sun, Stevin first deduces the length of a year by deriving from Stadius' ephemerides the moments of return to the same longitude. From an interval of 52 years between 1554 and 1606 he finds 365d 5h 45m 55s. Finding that this deviates widely from Ptolemy's value (365d 5h 55m 12s, which, however, we know to be too long by 7m), he realizes that the interval of 52 years used was still too short for an exact result. He therefore adopts Ptolemy's value, so that the table of the sun's mean motion, which he adds to his treatment, is identical with Ptolemy's table. Stadius' ephemerides furthermore show that the daily increase of the sun's longitude is smallest in June (57'), greatest in December (1°1'); so the sun moves in an eccentric circle. To find the longitude of its apogee, Stevin determines by trial and error a date in June such that in an equal number of days before and after that day equal arcs of longitude were described. Thus he finds 94°24', holding for 1554. Another method consists in finding two opposite longitudes of the sun, such that each of the semi-circumferences is covered in the same time; the result was 95°14'. From the data of 1594 in the same way he finds 97°53', 3°29' more, from which follows a yearly increase of 5'13". The interval of 40 years of course is too small to give a reliable

result for this increase; hence he compares his longitude of the apogee with Ptolemy's value of $65^{\circ}30'$, 1455 years earlier, from which follows a yearly progress of $1'20''$.

This is the method followed throughout for all the moving celestial bodies. In the case of the moon, in Chapter 2, which has been omitted from this abridged edition, a number of different periods have to be derived. First the moon's motion "in her own orbit"; this is how he denotes its motion from apogee through perigee to the next apogee. Stevin perceives that the greatest daily progress (in perigee) is different for full moon and quarters. In order to avoid such irregularities in the derivation of the period, he makes use of three dates of most rapid motion coinciding with full moon (in 1569, 1572, and 1581), and finds a period (the anomalistic month) of (written in sexagesimals) $27;32,56$ days, corresponding to $27\text{d } 13\text{h } 10\text{m } 24\text{s}$. The corresponding daily motion of $13^{\circ}4'$ is in accordance with Ptolemy's value of $13^{\circ}3'54''56'''$. Thereupon he derives the rapid progress in longitude of the lunar apogee; from an interval of 16,393 days (nearly 45 years), in which the apogee made 5 revolutions, a daily increase of $6'35''$ is found (Ptolemy gave $6'41''$). Adding this motion of the apogee to the first-derived lunar motion relative to the apogee, he gets $13^{\circ}10'35''$ for the daily motion in longitude. From direct comparisons of the longitudes after five intervals of 9 years minus 9 days each, he finds $13^{\circ}9'$ for the moon's daily motion in longitude, which is sufficiently in accordance with the other value.

The mean length of a lunation is derived from two oppositions with an interval of 19 years in which occurred 235 oppositions; the result in days and sexagesimals is $29;32$ days (*i.e.* $29\text{d } 12\text{h } 48\text{m}$). The corresponding daily progress in elongation (called by Stevin the moon's gain) is found to be $12^{\circ}11'25''$, which is hardly different from Ptolemy's value of $12^{\circ}11'27''$. The same progress, when computed by simply subtracting the sun's daily motion from the moon's, is $12^{\circ}11'$. Finally the return of the latitude is derived from the statement that in 1,089 days the same maximum of latitude returned 40 times at the same longitude; consequently, the daily progress is $13^{\circ}13'23''$, and the daily motion of the node is $3'10''$. For all these motions tables are given, "taken from Ptolemy's tables".

The 3rd chapter deals with the motion of Saturn. The retrogradations shown in the ephemerides indicate that Saturn moves on an epicycle, and that the centre of the epicycle describes an eccentric circle (the deferent). In order to eliminate the oscillations due to the epicycle, Stevin only makes use of longitudes in the opposition to the sun, because then the centre of the epicycle is situated behind the planet at the same longitude. To derive the longitude of Saturn's apogee, he makes use of the same method as with the sun; finding a longitude such that the arcs described in the same interval (here about 7 years) before and after this opposition are equal. Thus he finds $268^{\circ}20'$; a special computation is added to make sure that this point of symmetry is the apogee and not the perigee. Tables are then given for the mean motion in longitude of the epicycle's centre, and also of the planet on its epicycle. The latter is found as the difference between the sun's and Saturn's mean motions.

The 4th and the 5th chapter deal with the motions of Jupiter and Mars. The description is said by Stevin to be similar in kind to that for Saturn, and to differ only as to the quantities. Hence we omit them in this edition. The same holds for Venus and Mercury as treated in the 6th and the 7th chapter.

Here, however, some difficulties arise, because there are no oppositions where the epicyclic movement is eliminated. Both coincidences of the planet with the epicycle's centre are conjunctions with the sun, where the planet cannot be observed. What *can* be observed is the greatest elongations on the evening and the morning side. Hence Stevin derives from Stadius' longitudes of the planet and the sun a table of the dates and the values of greatest elongation. Like Ptolemy, he uses elongations from the mean, not from the actual sun. The differences between these values show that the deferent is an eccentric circle. As to the longitudes of apogee and perigee in this circle, he explains that they may be found by looking for a case where at the same longitude the eastern and the western greatest elongations are equal. But he does not pursue this method any further, because he does not have a sufficient number of such data available. Since the construction of such tables, he says, would take more time than it behoves him to spend on it, and the aim is not to derive the planets' orbits with the utmost exactitude, but to understand matters in a general sense, he will use an easier method. From Stadius' tables he derives the conjunctions of the planets with the sun just as if they could have been observed, and he uses them in the same way as Saturn's oppositions. From one case, taken by way of example only, he finds 82° for the longitude of the apogee of the Venus-deferent (which leaves differences partly above 1°). Then he remarks that a more exact determination should have given $76^\circ 20'$, because this value had been used by Stadius as the basis of his tables. Exactly the same method is followed for Mercury, where $59^\circ 51'$ for its apogee is taken from Stadius.

After the planets have been discussed, a short chapter deals with the fixed stars. The constancy of their relative distances and alignments since Ptolemy is stated, and the amount of the precession $1^\circ 29'$ in a century (*i.e.* $53''$ a year) is derived from a comparison of the longitudes of Spica, as determined by Ptolemy and as found in Stadius.

The Second Book, entitled *On the finding of the Motions of the Planets by Means of Mathematical Operations*, extends the previous general knowledge by geometrical computations, resulting in numerical values for the eccentricities and dimensions. It presents the method followed by Ptolemy in computing, from three positions of a planet at known moments of observation, the exact place of the earth within the circular orbit of the planet. There is nothing new or peculiar in Stevin's exposition of the method, so that it was not necessary to include it in this edition. Simpler cases are treated by means of plane trigonometry.

The first application deals with the sun; from three longitudes taken from Stadius, Stevin computes an eccentricity of 0.0325 — which he expresses by 325 parts, 10,000 of which are equal to the radius of the solar circle — and a longitude $95^\circ 41'$ of the apogee. From other such sets of data he finds the values 326 and 318, $95^\circ 9'$ and $95^\circ 14'$. He then explains how from these elements the distance of the sun from the earth is computed, as well as the (negative or positive) correction that must be applied to the longitude of the mean sun to get the sun's true longitude. In translations and older astronomical treatises this correction — our modern *aequatio centri* — is called by the Greek term *prostaphairesis*; Stevin is the only author of the time to render this term by an exact translation into his own language; *voorofachtring*, literally: advance-or-lag.

These results are applied in the derivation of the equation of time. The inequality of the days (intervals between two consecutive meridian passages of

the sun) had not been treated by Ptolemy; the Ancients reckoned with the true solar time. In the 16th century, however, roughly regulated clocks had come into use, and these led to the conception of a "mean time", deviating periodically from the true solar time. Stevin says that he has borrowed the treatment of these concepts from Regiomontanus, who said that he took them from the Arabian Geber: "but they seem not to be Arabian findings, but rather remnants from the Age of the Sages" ¹⁵). Besides the eccentricity of the solar circle producing a yearly periodical inequality in the angular velocity of the sun, there is the obliquity of the ecliptic to the equator causing the equatorial progress to be alternately smaller and larger than the ecliptical progress. He finds from a table by Reinhold that the difference between longitude and right ascension reaches a maximum of $2^{\circ}28'24''$ when the sun is about 46° distant from the equinoxes. He adds a geometrical demonstration that this maximum occurs when the sine of the polar distance is equal to the square root of the cosine of the obliquity, *i.e.* when the ecliptical and the "equatorial" longitude (*i.e.* the right ascension) complement one another to 90° . The results of these computations are used to derive the deviation of the natural from the mean days, for the eccentricity on the one hand and for the obliquity on the other.

Next the distance, the diameter, and the parallax of the sun are dealt with, especially with a view to their subsequent use for the eclipses. For the finding of the parallax he assumes two observers, one at a more northern, one at a more southern latitude, "such as now could easily be done through the great Dutch navigations", if they both measure every day the solar altitude. When afterwards they compare their measurements made on the same days at known latitudes under the same meridian, the parallax can be found and the distance deduced. Though not workable at the time, the principle of later determinations of the parallax is clearly indicated. As a fictitious instance he supposes an observed parallax of $2'$, and derives the sun's corresponding distance to be 1,147 times and its radius 5 times the semi-diameter of the earth.

The 3rd chapter deals with the moon. In the same way as with the sun, the eccentricity and the longitude of the apogee are computed from three observations (*i.e.* from data of Stadius), taking account of the rapid motion of the apogee. Parallax and distance are also deduced. The latitudes of the moon and the motion of the nodes afford the basis for a computation of the eclipses.

In the 4th chapter Saturn is first dealt with in the same way. With this difference, however, that Stevin does not here derive eccentricity and apogee, as Ptolemy was obliged to do, from three oppositions, but takes the latter from the First Book, the derivation of the eccentricity (0.1170) thus being much simpler. The radius of the epicycle is found to be 1,150 when the radius of the deferent is 10,000. He also gives here the demonstration of Apollonius of the condition for retrogradation of a planet. The other planets are treated shortly, since the demonstrations are the same as for Saturn.

The last chapters of this Book deal with the planets' conjunctions and oppositions, *i.e.* chiefly with the eclipses of sun and moon. Since there is nothing of peculiar character in these chapters, the Second Book has been omitted from this edition.

Exception had to be made for the two "Remarks" at the close of this Book —

¹⁵) See Vol. I, p. 7 and 46 and the selection at the end of Vol. III.

which for this reason have been included — because they show Stevin's attitude towards the problems of the heavenly motions. First he remarks that in the last chapters only the conjunctions of sun and moon have been treated, because the motions of the planets are not sufficiently well known for an exact computation of their conjunctions. Then he says that he originally intended to add a sixth chapter on the unknown irregularities detected by Ptolemy and by Copernicus, and a seventh chapter dealing with the motions in latitude. Because of the uncertainty of their explanation he had postponed them till after the Third Book, as being problems that still had to be cleared up. After some time, however, they became so much clearer to him, on the assumption of a moving earth, that he decided to treat them more extensively in a supplement and an appendix to the Third Book.

The Third Book explains the motions of the planets on the assumption of a moving earth. Stevin assumes that this true motion of the celestial bodies had been known in the "Age of the Sages", but that this knowledge was afterwards lost to men, so that Ptolemy did not know of it. Until at last Copernicus had again revealed this system, or a similar one. Stevin's arguments are primarily based on the simplicity and naturalness of the Copernican system: the velocity of the revolutions increases regularly as their size decreases, so that the starry heavens are at rest and the rotation of the small terrestrial globe is the most rapid. The argument is corroborated by the belief that all revolving motions in nature take place in the same direction, from West to East. The prejudice that the heavy earth cannot appear as a luminous star is dispatched by simply assuming that the earth is a heavenly body. Whilst in disproving Ptolemy's fear that buildings will be demolished by the velocity of the motion and the resistance of the air, Copernicus, as a philosophical thinker, stresses the contrast between natural and forcible motions, Stevin, being a practical engineer, refers to everyday experience, such as with a stick in rapidly flowing water.

In the Summary and in the first sections dealing with the general theory of the planets we meet repeatedly with the expression the "heavens" of the planets (the literal translation of Stevin's *hemelen*). This term implies a structure of the planetary system entirely different from that according to our modern ideas. On the outside we have the highest "heaven", that of the fixed stars, which is the immobile sphere assumed by Copernicus. Arranged inside this are the "heavens" of the planets, which are evidently understood as analogous spheres carrying along in their axial rotation the planets themselves. In the same paragraph the planets are said to move in eccentric circles; this is how they appear in the drawing on page 120, which in the wording of Proposition 1 is called the arrangement of the heavens of the planets. The two expressions used indiscriminately in the subsequent sections are sometimes given side by side, as alternatives, *e.g.* the planets revolving "in the largest circles or heavens . . ." (page 125).

The belief that the planets are attached to spheres and are carried along in their orbits by a rotation of these spheres was common among Arabian astronomers in the late Middle Ages. In a sense it was opposed to the epicycle-theory. A system of concentric spheres, detached from one another, could only be constructed on the basis of single circular orbits, without epicycles. By removing the epicycles, Copernicus opened the way for this ambiguous concept, and we find it clearly

described in his works. What Stevin refers to as the "heaven" of a planet, by Copernicus was called its *orbis* ¹⁶⁾.

The system of the world described in the works of Copernicus thus is not identical with our modern heliocentric theory of planets running their course freely through empty space. Their *orbis* was sphere and circle at the same time. In the first of the seven theses of the *Commentariolus*, in the words "*omnium orbium coelestium sive sphaerarum*", the term is explicitly identified with sphere. The seven theses are followed by an exposition of the order of the orbs: *De ordine orbium*, from the immobile *orbis* of the fixed stars down *via* the planets, from Saturn to Mercury. In his great work *De Revolutionibus* this enumeration is repeated (in Book I, *caput X*, just before the figure): "*ordo sphaerarum sequitur in hunc modum*". In this description he says that between the convex *orbis* of Venus and the concave *orbis* of Mars there is space to take up "*orbem quoque sive sphaeram*" for the earth. In his dedication to Pope Paul III he refers to his work as the Books he wrote "*de revolutionibus sphaerarum mundi*". Accordingly, there can be no doubt that the term *orbis* is regularly used by Copernicus to indicate a sphere. At the same time there are numerous instances where it is used for a circular orbit. In the *Commentariolus*, immediately after the seven theses, he says that the magnitudes of the semi-diameters of the *orbis* will be given in the explanation of "the circles themselves". In the same treatise he speaks of the intersections *circulorum orbis et eclipticae*, called the nodes. In his great work, when describing how some authors added more spheres (up to an eleventh) beyond the outer starry firmament, he says that this number of circles (*quem circulorum numerum*) will be shown to be superfluous. Sphere and Circle therefore are both used as synonymous with *orbis* ¹⁷⁾. There is some vagueness about the substance or the substantiality of these spheres called *orbis*. Frisch on this subject observes ¹⁸⁾: "Copernicus nowhere in his work either explicitly asserts or implicitly denies the reality of the spheres".

There can be no doubt that Stevin's "heaven" of a planet is intended to render in the vernacular what Copernicus denoted by *orbis*. There is, however, a difference: the difference between the theoretical philosopher and the practical engineer. What for Copernicus was an ambiguous geometrical concept, to Stevin is a structure of physical objects and materials. In order to emphasize the spatial character of the heaven he sometimes denotes it by *hemelbol*, "celestial sphere". Being natural objects and substances, they must be acting on one another. The structure is a dynamical system. This opinion is not presented as a systematically worked-out theory, complete with proofs and arguments. With Stevin it is rather a picture spontaneously arisen in the background of his mind, a vague feeling appearing in some arguments. The basic idea, *viz.* that bodies contained in another body are bound to share in the movement of the latter, may appear obvious enough. In the world-system it means that outer spheres by their motion

¹⁶⁾ *Orbis* is not identical with the modern concept of orbit; its English equivalent is *orb*, e.g. in the title of O. Mitchell's popular work *The Orbs of Heaven* (London 1853).

¹⁷⁾ Edward Rosen (in the Introduction to the booklet *Three Copernican Treatises*) summarizes the discussion of all these cases as follows: "When he deals with the planetary theory, he uses *orbis* to mean the great circle in the case of the earth and the deferent in the case of the other planets" (p. 21). "But when he is speaking more generally about the structure of the universe or the principles, *orbis* regularly means sphere" (p. 19).

¹⁸⁾ *Johannis Kepleri Opera* III, p. 464.

influence the lower spheres contained within them; the primary force thus derives from the highest sphere, the outer firmament.

At first sight the character of these forces exerted by the higher upon the lower spheres looks very peculiar. The heaven of Mars, rotating in two years, would — Stevin says — in the absence of other forces impose the same two years' period of revolution upon the earth's aphelion. He asserts this without providing any argument or proof. No proof *can* be given, because observation contradicts such a rapid motion of the aphelion. Where the motion of a planetary aphelion could be determined, it was less than one degree in several centuries. Evidently, strong other forces must be at work, which prevent any considerable motion of the aphelia. Stevin's statements about these motions consist entirely in theoretical ideas, which are not clearly formulated or systematically developed. This shows how he is struggling with the problem of understanding causes at a time when there were present as yet only the first traces of a causal natural science. His exposition impresses the modern reader as an entirely artificial and fantastic mechanism: a planet like Mars, attached to a sphere carrying it around in two years, at the same time carries the aphelion of the sphere of the earth along with it in the same period of two years, though this sphere itself revolves in one year. He himself sometimes refers to it as a contradiction, saying (p. 133) that Jupiter's heaven performs a revolution in 30 years, which it receives from Saturn, but at the same time in reality rotates in 12 years about an axis of constant direction; the enforced period relates to the aphelion that protrudes outwards, and it is this bulge which in Stevin's theory is drawn along in a rotation in which the sphere itself does not share. When we call this an attempt to understand something of the mechanism of the world, it should be borne in mind that the physical character of these "heavens" plays no part therein and remains indefinite; it is the geometrical forms — in this case the eccentric circles — which determine the motions. Probably Stevin took this idea from his knowledge of the tides, where, in the abstract simplified case of the absence of continents, a wave crest is drawn along by the moon to pass round the earth in the moon's period of 27 days, whilst the waters of the ocean themselves revolve as one body in 23h 56m.

In the first Proposition of the Third Book Stevin effects a significant improvement in the Copernican theory. Copernicus had assigned three motions to the earth. Besides the axial rotation and the orbital revolution there was a third — annual — motion. He was governed by the classical Greek idea of the revolving body being carried around by its fixed connection with the radius vector, so that the direction in space of the earth's inclined axis of rotation would describe a cone. To explain the constant direction of this axis in space, he was obliged to compensate that motion by another conical motion of the axis in the opposite direction, completed in one year.

Stevin is aware that this is an unnecessary complication. He does not believe that two independent motions in nature can compensate one another so exactly. Copernicus was less rigorous in this: he took the two movements to be independent; their small difference explained the precession. Stevin is of the opinion that their combined result should rather be considered as a single primary phenomenon. He looks upon the constant direction of the axis in space as a fundamental property. In this respect he was guided by the researches of William Gilbert

on magnetism, published in 1600, shortly before his own work. Just as the magnet in the ship's compass continues to point in the same direction in space, notwithstanding the changing course of the ship, the axis of the earth continues to point in the same direction in space during its annual revolution. Stevin therefore calls this property of the earth "*haer seylsteenighe stilstandt*" (literally: "its loadstone standstill"), which has here been translated by "its magnetic rest". This is not a mere analogy; he quotes Gilbert's opinion that the earth itself is a huge magnet¹⁹).

Stevin widens the scope of this idea by applying it to the orbits themselves. From his ideas on the action of the "heavens" of the higher planets on the orbits of the lower planets he had deduced that the aphelia would show rapid rotations. In reality, however, they exhibited only minute, scarcely perceptible displacements. He solved this contradiction by imparting a magnetic character to the orbits. The directions of the planetary aphelia may also be said to be subject to a magnetic constancy. This subjection is the force referred to above as keeping the aphelia at rest. Whence does this force proceed? Arguing by comparison with magnets in closed boxes, Stevin derives that the origin of these forces is situated outside the spheres of the planets, in the sphere of the fixed stars. The matter is different for the moon; its apogee has a rapid daily motion of 6'41" (with a period of nine years), and the origin of the forces is to be sought in the regions of the nearer planets.

It is not only the orbits but the entire spheres constituting the "heavens" of the planets which are subject to a magnetic force. It causes the poles of their axes of rotation (hence also the orbital planes) to keep a constant direction in space. It is the cause that, in spite of the considerable deviation of Mars from the ecliptic, the ecliptic itself (the plane of the earth's motion) keeps its constant position. The constancy of the orbital planes, which the later science of theoretical mechanics styles conservation of moment of momentum, is explained by Stevin as magnetic stability. In his 3rd proposition he speaks of the doubts he had felt with regard to the real cause of the planetary motions; his initial conviction that the motion was transferred from the outer spheres to the inner spheres was disproved by the practical tests. Sometimes he had wondered whether the planets did not run their course freely through empty space "like birds flying around a tower", until finally the principle of magnetic rest suggested itself as the simplest solution of the problem.

Considerable space is devoted by Stevin to the transition from the old to the new system. With regard to the moon the exposition of its course has become more difficult; instead of simply describing a circle about a fixed centre, it now has to revolve about a body which itself revolves in a yearly period. Stevin assumes (in accordance with the Greek epicycle-theory) that in such a case as this the radius vector of the earth to the centre of its circle is the natural zero line for the position of adjacent bodies. Relative to this radius, which changes

¹⁹) It is to be noted that the constant direction of the axis in space had already been mentioned by Copernicus and had even been compared to a magnet in his *Commentariolus*. The *Commentariolus* was not printed in the 16th century, but circulated in a few hand-written copies. It is not probable that Stevin had seen one of these; Gilbert's work is quite sufficient as the source of his theory that the earth itself is a magnet.

its direction in space by progressing 360° a year, *i.e.* $59'8''$ daily, the small advance of the moon's apogee of $6'41''$ daily (360° in 9 years) is a retrogression of $52'27''$ daily. In the same way the retrogradation (in the old system) of the nodes of the moon's orbit by $3'11''$ daily (360° in 18 years), relative to the revolving terrestrial radius vector, is a retrogression of $1^\circ 2'19''$. Formally this contradicts Stevin's view that all motions in the universe have the direction from West to East; but no undue weight is to be attached to this, since apogee and node are no bodies, and may depend on exterior forces in some other way. In his short discussion of these points on page 135 Stevin cannot be said to have succeeded in harmonizing them.

The ensuing method of computing the moon's longitude — by first finding the sun's longitude and then subtracting the lag of the apogee — is demonstrated on page 161 to be right. He remarks that the computation according to the old system (*i.e.* using absolute positions in space) is more direct and rapid.

The orbit of the earth in the new system is exactly identical with what used to be the sun's orbit in the old system, with the same period and the same relative positions. A special figure is given on page 148 to show how the two systems result in the same observed longitudes and the same distances for the sun and the earth. The former longitude of the sun's apogee is identical with the new longitude of the earth's perihelion. The new data of the moon's motion are also shown to correspond directly to the old ones. The numerical data formerly derived for the planets and now transferred to the new system are summarized in a list, in which the earth ranks third in the series of the planets.

It is significant evidence of his practical-mindedness as well as of his pupil's that Stevin did not content himself with presenting the general argument that of course all the relative positions and motions of the planets are the same in the two world-systems. They want to see it in the details of each individual case. For this purpose Stevin gives for each of the planets Mars and Venus (as instances of an outer and an inner planet respectively) a drawing in which the connecting lines and the circles for the two systems are combined. The desire to see their relations illustrated in a geometrical drawing, however, was not the only motive. The interchange of the sun and the earth as central bodies presented certain difficulties. Copernicus neither mentioned nor solved these. In his 18th proposition Stevin states that Copernicus evidently supposed the matter to be so clear that no further proof was needed; but that he himself had met with difficulties, which called for a more detailed investigation.

If we suppose that we shall arrive at the new system by simply interchanging in the old system the positions of the sun and the earth and identifying the old deferent with the new planetary orbit, we are mistaken in the same way as when we sometimes say for the outer planets that the motion on the epicycle reflects the sun's motion about the earth. The perfectly uniform course of the planet on its epicycle, however, is not identical with the sun's course (in the other case) about the earth (which is not uniform), but with the sun's uniform motion about the centre of its circle. Hence, the deferent does not correspond to the planet's orbit about the sun, but to its orbit around the centre of the earth's annual circle. When we pass from the old to the new system, the earth at rest is to be replaced by the centre of its orbit, and not by the sun.

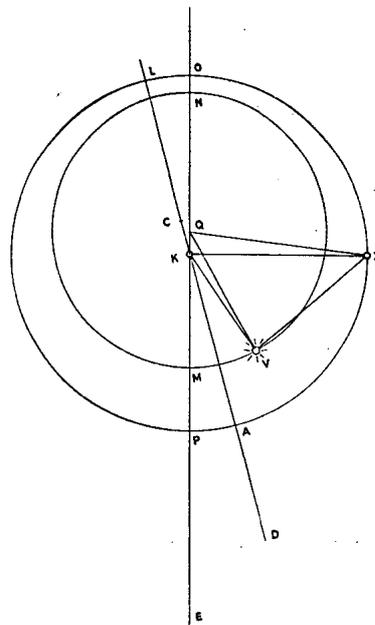
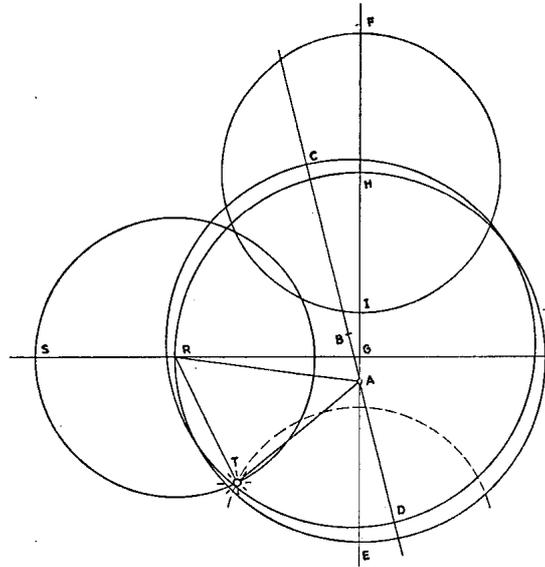
On page 182 Stevin presents the combined drawing for Mars, which is discussed in Proposition 15. Since the reader might easily be thrown into confusion by the

tangle of all these distances and circles, he separates the lines and letters belonging to either system into two drawings on page 192. The positions of the sun and the earth are C and A ; in the old system A is fixed and the sun moves on a circle through C (centre B); in the new system C is fixed and the earth moves on a circle through A (centre K). The fundamental law of the epicycle-theory states that the radius vector of the planet on its epicycle is always parallel to the radius vector of the sun on its circle, so that the arc described by the sun is the sum total of the arcs described by the planet on its epicycle and by the epicycle's centre on the deferent. In the old system Mars' deferent is HRE (centre G), in the new system Mars' orbit is NVM (centre Q). At conjunction the epicycle's centre was at H , Mars at F ; in the new system Mars was at N , the earth at P . At a later time the centre had progressed from H to R , Mars from F to T ; in the new system Mars had passed from N to V , the earth from P to X (arc $PX = \text{arc } HR + \text{arc } ST$). Owing to the equality of circles and arcs, RT and KX as well as AR and KV are seen to be equal and parallel. Then from the equality of triangles ART and KVX , AT and XV are demonstrated to be equal and parallel. This means that Mars is seen from the earth in the same direction and at the same distance according to both systems.

The same demonstration is then given for Venus as an instance of an inner planet. Because the planet's deferent is equal in size to the sun's orbit, with the centres at a small distance, it is more difficult in this case than in that of Mars to disentangle the combined drawing for Venus (on page 196). A separation of the combined drawing into its two components is here all the more necessary; we therefore give these two drawings on page 17.

The positions of the earth and the sun are denoted by A and C ; in the old (geocentric) system, A is fixed and the sun moves on a circle (centre B) through C ; in the new (heliocentric) system, C is fixed and the earth moves on a circle (centre K) through A . The eccentricity of the sun's orbit is $AB = CK$, that of Venus' orbit is $AG = KQ$. For time zero we take an upper conjunction at the apogee; the centre of the epicycle is at H , Venus itself at F ; in the new system Venus is at N , the earth at P . We have to show that at any other time the lines joining the planet to the observer in the old and the new system are equal and parallel. As to their length, we observe that the orbit of the sun and the deferent of the planet (in the old system) are equal in size to the orbit of the earth (in the new system); the planet's epicycle also is equal to the planet's orbit in the new system. This entails the equality of all semi-diameters drawn from any point of such a circle to its centre; $GR = GH = BC = KP = KX$, and $HF = RT = QN = QV$. As to the directions of these lines, we observe that they rotate entirely uniformly. The planet (in the old system) has two motions, one along the epicycle (*e.g.* the arc ST) and another with the epicycle along the deferent (*e.g.* the arc HR). These two motions combined in the heliocentric system convey the planet by its uniform rotation from N to V , so that QV is equal and parallel to RT .

Positions eccentric to these circles (*e.g.* A) do not fall under these headings, so that here an additional computation is needed. For this purpose we compare the acute-angled triangles AGR and QKX . AG and QK (the eccentricity of the planet's orbit) as well as GR and KX are equal and parallel; so the triangles are equal and similar, and consequently their third sides AR and QX are equal and parallel. Since the triangles ART and XQV now have two pairs of sides equal and parallel,



Motion of the planet Venus
 according to the geocentric system (above)
 and to the heliocentric system (below).

this holds also for the third pair AT and XV . This means that the planet is seen from A (the earth in the old system) and from X (the earth in the new system) in the same direction and at the same distance.

For the understanding and the derivation of the motions in longitude Stevin considers the old, untrue system with the earth at rest to be the simplest and most appropriate — probably because it directly represents the observed motion. It is different, however, with the latitudes of the planets. For this reason he omitted the latter from his Second Book, and postponed them until, at the end of the Third Book, he should have treated them according to the true system of the moving earth; for, so he says, in this way we can arrive better at a causal knowledge of the motion in latitude. This reverse way of arguing, *viz.* the derivation of the older imperfect theory from the new, more perfect theory, is an indication of the real underlying character of the problem. Whereas for the three upper planets the epicycle theory was the direct expression of the observed phenomena of the longitudes, this was not the case with the latitudes. Here Ptolemy's theory was an artificial construction; it was complicated because two independent inclinations had to be derived, *viz.* one of the epicycle to the deferent, and one of the deferent to the ecliptic. The new heliocentric system required one angle only, the inclination of the planet's orbit to the ecliptic. Copernicus, assuming that Ptolemy's theory was a good representation of the observed motions in latitude, had to make the inclination variable by assigning an oscillation (between opposition and conjunction) to the orbit. With the epicycle itself, Stevin discarded also its special inclination and stated as the basic structure for the geocentric system: the epicycle in its course along the deferent always has to keep parallel to the plane of the ecliptic.

In dealing first with Saturn, Stevin starts from the values in Ptolemy's tables, which he assumes to represent Ptolemy's observations. In these tables the maximum northern latitude (at a longitude 50° behind the apogee, *i.e.* at longitude 183°) is $3^\circ 2'$, the maximum southern latitude is $3^\circ 5'$, the planet in both cases being at the lowest point of its epicycle. Ptolemy had derived $2^\circ 26'$ for the deferent's inclination, $4^\circ 30'$ for the epicycle's inclination. From the latitude $3^\circ 2'$ and the known distances Stevin finds $2^\circ 43'$ for the inclination of Saturn's orbit to the ecliptic. He shows how easy the computation of Saturn's latitudes is now, since they follow directly from the horizontal distances of Saturn from the earth and the vertical distances of Saturn above or below the ecliptic. Ptolemy's observation that near the nodes Saturn does not show any latitude, which is accidental in his theory, is a necessity in the new theory, because it shows the epicycle at that time to coincide with the ecliptic. In explaining this state of affairs, Stevin cannot refrain from remarking that it forms a strong argument in favour of the moving earth.

The other planets for which the same holds are mentioned in brief statements only, in which their numerical data are given. More space is devoted to Mercury. Stevin begins by expounding Ptolemy's theory of the latitude of Mercury, as an instance of the two inner planets, in the form given by Peurbach and also used by Copernicus. It assumes three oscillations. An oscillation of the deferent about the line of nodes as axis makes its inclination vary between zero and $1^\circ 45'$ to the South. The epicycle has two oscillations about two perpendicular axes, one axis tangential to the deferent's circumference, the other in the radial direction; when the former is zero, the latter reaches its maxima in opposite senses in apogee

and perigee; and conversely the former reaches its maxima in opposite senses at longitudes 90° and 270° , when the latter is zero. Stevin mentions *in margine* their Latin names *deviatio*, *declinatio*, *reflexio*, and himself calls them *afweging*, *afwycking*, and *afkeering*; they have here been translated by "deviation", "declination" and "deflection". Stevin has no use for this complicated theory. The greater simplicity — he says — of the heliocentric system was not realized by Copernicus himself, who slavishly copied Ptolemy's three oscillations, with the numerical values given in the Tables, and incorporated them in his fundamentally different world-system. Stevin draws the former epicycle (representing the planet's orbit) as a small circle at rest about the centre of the larger surrounding circle (the earth's orbit, formerly the deferent) and then has to determine its inclination to the latter. From the "observed" latitudes (in reality, as always in his explanations, taken from Ptolemy's tables) of Mercury at two opposite points at 90° and 270° from the apogee, $1^\circ 45'$ North and $4^\circ 5'$ South, he derives (as illustrated in the lower figure on page 248) an inclination $5^\circ 32'$.

In this short paragraph of Stevin's work the simple construction of the heliocentric system is used as the new basis of computation. Of course it was done in one instance only; much more was not yet possible at the time. His was not the task which Kepler was to accomplish afterwards. He was only interested in explaining the theory of the heavenly motions, not in constructing numerical tables for their computation.

As a Supplement (*Byvough*) Stevin gives what should have been the last chapter of the Second Book, with the treatment of the latitudes in the old geocentric system. He introduces it by presenting this system in a form different from the original one. Originally the two lowest planets, Mercury and Venus, in contrast with the three upper planets have deferents that are completed in a year and thus represent the earth's orbit, whilst the smaller epicycles here represent the planets' orbits. Instead of the size of the circles, Stevin now takes their function in the system to be their specific character. In all cases the circles representing the earth's orbit are to be called epicycles; deferents is to be the name for the planets' circles. Accordingly, for Venus and Mercury the old terms have to be interchanged. He says it in the following way: the circles which are called deferents here are epicycles, and conversely; and he uses these names in the following propositions. Instead of two kinds of planets with different characters, we now have one homogeneous series, with only the size of their orbits regularly decreasing from Saturn to Mercury, the earth finding its place among them. In an illustrative drawing on page 260 Stevin pictures the planetary system in which all the orbits have been provided with terrestrial circles of equal size and all parallel to the ecliptic.

Stevin's task was to show that the phenomena of latitude, too, are the same in the two systems. This was not difficult, since his corrected geocentric system, with the epicycles parallel to the ecliptic, was a formal transformation of the true system. He compares this geocentric system with Ptolemy's and finds that for the upper planets Ptolemy came very near to the truth, since he found the two inclinations (for Saturn $2^\circ 26'$ and $4^\circ 30'$) to be nearly equal, the difference being only $2^\circ 4'$. His criticism (pages 277 and 279) that Ptolemy's tables are not in conformity with his theory is unfounded (*cf.* p. 279, note 3). His statement that for the two lower planets Ptolemy's theory was not successful is true; the basic reason is that the epicycle theory did not fit Mercury.

Stevin knows that his exposition of the motions of the planets is neither exact nor complete. An *Appendix* is therefore added on the "unknown motions" observed by Ptolemy, *i.e.* on motions which formerly were unknown and had not been included in the theory as given in Stevin's three Books. They are Ptolemy's "second inequality" of the moon, and his introduction of a "*punctum aequans*" in the orbits of the planets. Since, in his treatment of the former, Stevin (in his Propositions 2 to 5) renders Ptolemy's discussion rather exactly, it was not necessary to reproduce it here in detail. The other point is dealt with by Stevin in the 6th Proposition of the *Appendix*.

Ptolemy had found that the simple theory of the epicycle's centre uniformly describing an eccentric circle about the earth does not tally with the observed motions of the planets. The motion along the deferent is not uniform, but it seems to be uniform when viewed from the equant (*punctum aequans*) situated at the same distance as the earth from the centre of the deferent, but on the opposite side. This irregularity of the motion along the deferent is the main point in Ptolemy's planetary theory. The radius vector drawn from the equant to the epicycle's centre, though of variable length, rotates perfectly uniformly. Since the planet moves uniformly along the epicycle, the anomaly, reckoned from the apogee of the epicycle as its zero point, also increases uniformly when viewed from the same point. Stevin directs the reader's attention to this point. He first remarks that Ptolemy must have found the simple theory entirely satisfactory as long as he considered only the oppositions to and conjunctions with the sun, *i.e.* the planet in the nearest and the farthest point of the epicycle, where it has the same longitude as the centre of the epicycle. But in the other points it is different. He takes an example where shortly before opposition (anomaly 150°) the planet is observed to have a smaller longitude, hence has progressed farther on its epicycle than had been computed. With the aid of a figure he shows that this can be accounted for if the zero from which the anomaly in the epicycle is reckoned is shifted in a forward direction. Then, in order to see it coincide with the epicycle's centre, Ptolemy had to look not from the centre of the deferent, but from another point, situated nearer to the apogee of the deferent. The precise situation of this point, when computed, would have turned out to be exactly the *punctum aequans*. In this deduction, instead of the most striking property, *viz.* the non-uniform motion of the epicycle along the deferent, the less striking, non-uniform motion of the planet along the epicycle is used to introduce the *punctum aequans*.

This is Stevin's explanation of Ptolemy's theory for the planets Saturn, Jupiter, Mars, and Venus. For the more complicated motion of Mercury he simply reproduces Ptolemy's description of the circles and their motions.

Thereafter Stevin explains how the same "unknown motions", first of the moon (in Proposition 8), then of the planets (in Proposition 9), are dealt with by Copernicus. For the planets Stevin first introduces the "unknown motions" into the geocentric theory. The distance of the earth from the centre of the deferent is diminished by one-fourth of its value; *e.g.* for Saturn, instead of 0.1139, it is taken 0.0854. The one-fourth subtracted (here 0.0285) is taken as the radius of a small circle, which is described by the planet in such a way that in apogee and perigee the effects are subtracted, whereas in a lateral position they are added. Copernicus of course presents this construction on a heliocentric basis; the deferent is now the planet's orbit, with the sun at a distance from

the centre of 0.0854 times the orbit's radius; the planet, in addition, describes the small circle (of radius 0.0285) twice in one revolution. Stevin supposes that Copernicus, though he did not say so, had devised the small circle first in the geocentric, and then transferred it to the heliocentric theory. In any case he himself has chosen this method in order to make the matter clearer to his readers.

In the case of Venus the centre of its circle, of radius 0.7194, situated within the circle of the earth with radius 1.0000, does not have a fixed eccentric position, but moves on a small circle with diameter 0.0208 in such a way that its eccentricity varies between once and double this amount. It is described by the centre of Venus' orbit twice a year in a direct sense; the eccentricity is at its minimum when the earth is in the planet's line of apsides, and at its maximum when it is at a distance of 90° . Stevin, as in all these cases, does not make any comparison with observations; his task is solely to expound the theories of Ptolemy and Copernicus; and he simply adds: "by this means, Copernicus says, the longitudes of Venus are always found in the right way".

The still more complicated system of circles for Mercury as devised by Copernicus is expounded correctly by Stevin in his 12th Proposition. The centre of Mercury's orbit describes a small circle with radius 0.0316 every half year in such a way that the eccentricity is greatest when the earth is in Mercury's line of apsides, and smallest when its longitude is 90° different. In addition, the planet moves to and fro linearly along the radius of its orbit; such a linear movement, as Stevin demonstrates in his 11th Proposition, is produced by two circular movements in opposite directions.

The Book closes with an article on the "unknown motion" of the stars, dealing with the precession of the equinoxes. The difference between Ptolemy's value (1° in a century) and the larger values of later authors is thought by Stevin to be perhaps due to wrong equinoxes caused by irregularities and differences of the refraction. He points to the abnormal phenomenon of the sun observed by the Dutch mariners in Novaya Zemlya in 1596-97, which he also ascribes to a refraction, abnormally large in February, caused by the cold nebulous atmosphere. New measurements of stellar altitudes and refractions in the countries of ancient astronomy as well as at its present centres, he says, will be needed to remove these uncertainties. Moreover, for greater precision in measuring the positions of the planets and the stars better instruments are necessary, such as those constructed and used by Tycho Brahe. The present deviations between observations and theory show that our theory is unsatisfactory and must be improved by means of the best observations available.

Little did Stevin suspect that at the very time when he wrote these words, mapping out the programme in a vague and general way, Kepler was already engaged in establishing the true theory of the planetary motions.

D E R D E
D E E L D E S
W E E R E L T S C H R I F T S
V A N D E N
H E M E L L O O P .

C O R T B E G R Y P

des Hemelloops.



Nc k sal int begin der beschrijving des Hemelloops de saeck nemen al ofter gantsch niet af beken en vware, en daer na den handel met sulcken oirden vervolghen, gelijk daerse haer vermeerdering dadelick meschijnt ghenomen te hebben, daer af beschrijvende drie boucken.

Het eerste bouck vande vinding der dyaelderloopen en der vaste sterren deur ervarings dachtafels met stelling eens vasten Eertcloots.

Het tweede vande vinding der dyaelderloopen deur vvisconstighe vverckingen met stelling eens vasten Eertcloots, en eerste oneventheden.

Het derde vande tweede oneventhedē, vvaer in comt Copernicus stelling eens roerenden Eertcloots.

SUMMARY

OF THE HEAVENLY MOTIONS

In the beginning of the description of the Heavenly Motions I will assume the matter to be altogether unknown, and then I will proceed with the discussion in the same order in which it seems to have taken its progress in actual fact, describing it in three books.

The first book, of the finding of the motions of the Planets and of the fixed stars by means of empirical ephemerides, on the assumption of a fixed Earth.

The second, of the finding of the motions of the Planets by means of mathematical operations, on the assumption of a fixed Earth, and the first inequalities.

The third, of the second inequalities, in which Copernicus' assumption of a moving Earth is set forth.

E E R S T E

BOVCK DES
HEMELLOOPS

VANDE
VINDING DER

*DWAELDERLOOPEN

ender vaststerren deur ervarings

dachtafels, met stelling eens

vasten Eertcloots.

*Motuum
Planetarum*

FIRST BOOK
OF THE HEAVENLY MOTIONS

OF THE FINDING OF THE PLANETS' MOTIONS
AND THE MOTION OF THE FIXED STARS

by means of Empirical Ephemerides,
on the Assumption of a Fixed Earth

CORTBEGRYP

deses eersten Boucx.



E bepalinghen beschreven sijnde soo sal dit eerste bouck acht onderscheytfels hebben, vande vinding deur ervarings dacht afels des loops van Son, Maen, Saturnus, Iupiter, Mars, Venus, Mercurius, en der vaste sterren, alles met stelling eens vasten Eertcloots als vveerelts middelpunt, want hoewwelse eyghentlick in een rondt

Elementa.

draeyt ghelijck d' ander Dwaelders, nochtans leert men de * beginselen deser const lichtelicker verstaen deur het schijnbaer, dan deur het eyghen, soo daer af breeder gheseyt sal worden in des 3 boucx 7 voorstel. Angaende voordere * spiegelinghen waer toe de eyghen stelling des loopenden Eertcloots bequamer is, daer af salick int boveschreven derde bouck handelen.

Speculationes.

SUMMARY OF THIS FIRST BOOK

After the definitions have been described, this first book is to comprise eight sections, of the finding by means of empirical ephemerides of the motion of Sun, Moon, Saturn, Jupiter, Mars, Venus, Mercury, and of the fixed stars, all this on the assumption of a fixed Earth as centre of the universe; for though in reality it revolves in a circle, like the other Planets, nevertheless it is easier to understand the elements of this science from the apparent than from the true motion, as will be set forth more in detail in the 7th proposition of the 3rd book. As to further theories, for which the true assumption of the moving Earth is better suited, I will deal with those in the third book referred to above.

BEPALINGHEN.



Ngheſien in des driehouckhandels 4 bouck, beſchreven ſijn de bepalinghen der Hemelſche ronden boghen en punten die wy daer behoufden, ſoo houden wy de ſelve hier voor bekent: Sulcx dat nu gheſtelt ſullen worden de reſterende bepalinghen int volghende noodich.

1 BEPALING.

Den tijt van dat de Sonnens middelpunt uyt het middachront gaet, tot dattet vveder daer in comt, is een *natuerlicken dach. En die natuerlicke dagen vvorden ooc oneven gheſeyt. En ſulcke tijt int ghemeen *oneven tijt.

Dies naturalis.
Dies inaequales.
Tempus inaequale.

Deſe tijt eens natuerlicken dachs is van een keer des *evenaers, met noch ſulcken boochſken des ſelfden, alſſer deutlijdt te wijle de Son met haer eygen loop daerentuffchen te rugh ghegaen is. Nu by aldien al die boochſkens evegroot waren, ſoo ſouden de natuerlicke daghen al evelanck ſijn, t'welck niet en ghebeurt. De reden van dier boochſkens onevenheyt is tweederhande: Ten eerſten deur dien de Sonwechboochſkens op dien tijt vande Son beſchreven niet evegroot en ſijn, om de *uytmiddelpunticheyts wille. Ten anderen al warens evegroot, nochtans ſoo en gaens met gheen even boochſkens des evenaers deur *t'middachront, om de ſcheefheyt of afwijcking des *Duyſteraers. En hoe wel dit verſchil op een dach ongevoelelick is, nochtans op veel daghen t'ſamen cant merckelick ſijn.

Aequatorii.

Excentricitatem.

Meridiani, Zodiaci.

2 BEPALING.

*Natuerlick jaer is dē tijt der Sonnens omloop, vviens begin en einde een ghenomen punt is dat altijts evevvijte vande lentsne blijft, gheduerende 365 daghen 5 uyren, met noch een onſeker ghedeelte.

Annus naturalis.

Ptolemeus heef dat onſeker ghedeelte boven de 5 uyren, gheſtelt op 5 ① 12 ②: *Albaregni* op 46 ① 24 ②. Ander hebben ander uytcomſt bevonden.

3 BEPALING.

Egips jaer is dat 365 daghen begrijpt.

Deſe Egipsche jaren verleken by de natuerlicke verlopen alle vier jaren bycans een dach.

A 3 4 BEPA.

DEFINITIONS

Since in the 4th book of the treatise on trigonometry ¹⁾ the definitions of the heavenly circles, arcs, and points which we there needed have been described, we here assume them to be known, so that the remaining definitions needed hereafter will now be given.

1st DEFINITION.

The time from the moment when the centre of the Sun leaves the meridian up to the moment when it returns thereto is a natural day. And these natural days are also said to be unequal. And such time in general: unequal time.

This time of a natural day is of one revolution of the equator, together with an arc thereof such as it passes through while the Sun with its proper motion has meanwhile gone back. If all these arcs were equal, all the natural days would have the same length, which does not happen. The reason of the inequality of these arcs is twofold. Firstly, that the arcs of the Sun's orbit described in that time by the Sun are not equal, because of eccentricity. Secondly, even if they were equal, yet they would not pass with equal arcs of the equator through the meridian, because of the obliquity or deviation of the Ecliptic. And although this difference is insensible in one day, yet it may be perceptible in a large number of days taken together.

2nd DEFINITION.

A natural year is the time of the Sun's revolution, whose beginning and end is an adopted point which always remains at the same distance from the vernal equinox, this being 365 days and 5 hours, with an uncertain fraction in addition.

Ptolemy ²⁾ has put this uncertain fraction over and above the 5 hours at 55m 12s; *Albategni* ³⁾ at 46m 24s. Others have found other results.

3rd DEFINITION.

An Egyptian year is one that comprises 365 days.

These Egyptian years, when compared with the natural years, shift nearly one day in every four years.

¹⁾ Work XI; i, 14. See Vol. II B, p. 755.

²⁾ *Syntaxis* III, 1 (Translation of Manitius, I, p. 146).

³⁾ Abu-'Abdallāh Muḥammad ibn Jābir ibn Sinān al-Battāni, al-Harrāni, al-Sābi (born near Harran in Mesopotamia, 850?—Damascus 928?). His principal work, *al-Zīj*, translated into Latin in the early part of the 12th century, and into Spanish somewhat later, was printed at Marburg (*De Scientia Stellarum*, 1536) and at Nuremberg (*De Motu Stellarum*, 1537).

4 BEPALING.

Iuliaensche jaren sijn vvelcker drie 365 dagen vervan-
ghen, het vierde 366, hebbende dan de maent Februarius
29 daghen, daerse anders maer 28 en heeft.

Om t'verloop der Egiptsche jaren weerom te innen, en t'naturlickjaer naer-
der te commen, soo heeft *Iulius Cesar* den voorschreven dach alle vier jaren in
Februario veroirdent, welck gedaente van jaren Iuliaensche genaemt werden.

5 BEPALING.

Soo men neemt het naturlickjaer te hebben soo veel
even dagen en even uyren met haer gedeelte, alser oneven
dagen en oneven uyren met haer gedeelte in sijn: Sy vvor-
den * evedaghen oock middeldagen ghenoeemt. En sulc-
ken tijt int ghemeen * eventijt ende middeltijt.

*Dies aequales
& dies medij.
Tempus
aequale Tem-
pus medium.*

Doitfaeck des naems middeldaghen, is van weghen datse middelmatich sijn
tusschen de langher en corter naturlicke daghen der 1 bepaling.

6 BEPALING.

Planete.

* Dvvaelders sijn seven vveereltlichten, die int uyer-
lick ansien schijnen sonder regel te loopen al offe dvvael-
den, met naem Saturnus, Iupiter, Mars, Son, Venus, Mer-
curius, Maen.

Hoewel den Eertclood eyghentlick me een Dwaelder is, nochtans angho-
sien wy om de redenen vooren int Cortbegrijp verhaelt, die nemen vast te
staen, soo en wortse hier onder de Dwaelders niet ghetelt.

7 BEPALING.

*Excentricus
circulus.
Concentri-
cus.*

* Uytmiddelpuntichront noemtmen diens middel-
punt buyten den Eertclood staet. Maer * middelpun-
tichront diens middelpunt oock des Eertcloots middel-
punt is.

Lact

4th DEFINITION.

Julian years are such that three of them contain 365 days, the fourth 366, the month of February then having 29 days, whereas otherwise it has only 28.

In order to recover the shifting of the Egyptian years and come nearer to the natural year, *Julius Caesar* ordained the above-mentioned day to be added every four years in February, which kind of years were called Julian years.

5th DEFINITION.

If the natural year is taken to have as many equal days and equal hours with their fraction as there are unequal days and unequal hours with their fraction in it, they are called equal days and also mean days. And such time in general: equal time and mean time.

The origin of the name mean days is that they are the mean between the longer and shorter natural days of the 1st definition.

6th DEFINITION.

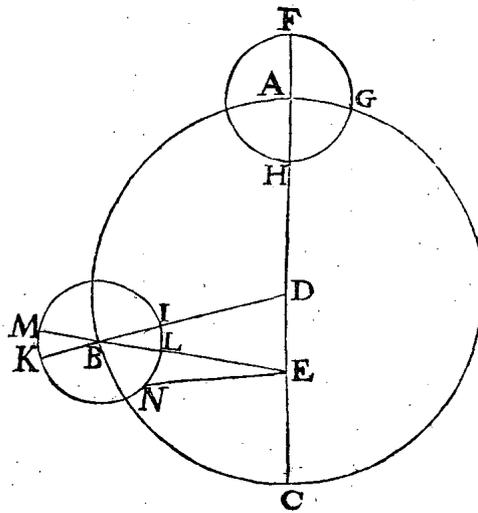
Planets are seven luminaries which in outward appearance seem to move without rule, as if they were wandering; their names are Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon.

Although the Earth is in reality also a Planet, yet because for the reasons related hereinbefore in the Summary we assume it to be fixed, it is not reckoned here among the Planets.

7th DEFINITION.

Eccentric circle we call a circle whose centre is outside the Earth. But concentric circle, a circle whose centre is also the centre of the Earth.

Laet A B C een rondt
sijn diens middelpunt D,
en E den Eertclood: Dit
foo wesende, A B C heet
uytmiddelpuntichront.
Maer soo men naem het
punt D den Eertclood te
sijn, t'soude dan middel-
puntich heeten.



8 BEPALING.

De lini van des uytmiddelpuntich ronts middelpunt
totten Eertcloots middelpunt heet * uytmiddelpuntich- *Linea excess-*
heytlijn. *tricitati.*

Als inde form der 7 bepaling de lini D E.

9 BEPALING.

Uytmiddelpuntichronts * verstepunt noemtmen dat *Apogeeum,*
verft vanden Eertclood is: En * naestepunt dat den Eert- *Perigeum.*
clood naest staet.

Laet inde form der 7 bepaling de uytmiddelpunticheyt D E op beyden sij-
den voortghetrocken worden tot inden omtreck an A, en C, Twelck soo sijnde,
want het punt A, inden omtreck A B C alderverft vanden Eertclood E is,
en C aldernaest, soo wert A des uytmiddelpuntichronts A B C verstepunt ge-
noemt, en C het naestepunt.

10 BEPALING.

Dvvaelder vvech is een uytmiddelpuntichront daerin *Deferens-voel*
een Dvvaelder loopt. *Via Planetae.*

11 BEPALING.

Wesende opeen punt als middelpunt, inden omtreck
A 4 cens

Let ABC be a circle, whose centre shall be D , and E the Earth. This being so, ABC is called eccentric circle. But if the point D were taken to be the Earth, it would be called concentric.

8th DEFINITION.

The line from the centre of the eccentric circle to the centre of the Earth is called line of eccentricity.

Thus, in the figure of the 7th definition the line DE .

9th DEFINITION

Apogee of the eccentric circle we call the point farthest from the Earth, and perigee the point nearest to the Earth.

In the figure of the 7th definition let the line of eccentricity DE be produced on either side to the circumference, at A and C . This being so, because the point A on the circumference ABC is farthest from the Earth E , and C nearest to it, A was called the apogee of the eccentric circle ABC , and C the perigee.

10th DEFINITION.

Planet's orbit is an eccentric circle in which a Planet moves.

eens uytmiddelpuntichronts, beschreven een cleen ront, daer in men neemt een Dvvaelder te loopen: Dat cleender heet * inront, en t'grooter * inrontvvech.

*Epiroclytus
Deferens
epirotulum.*

Laet inde form der 7 bepaling opeenich punt als A des uytmiddelpuntichronts A B C beschreven worden het cleender ront F G H daer in men neemt een Dwaelder te loopen, t'felvecleender heet inront, en t'grooter A B C inrontwech.

12 BEPALING.

Inronts verstepunt noemtmen dat verft vanden Eertclood is: En * naestpunt dat die naest is: Maer * inronts middelverstepunt, dat verft van sijn vvechs middelpunt is: En * middelnaeste punt dat die naest is.

*Apogeeum
epirotuli vel
aux vera.
Perigeum
vel oppositum
aux vera.
Aux epirotuli
media.
Oppositum
aux media.*

Laet inde form der 7 bepaling de middellijn C A voortghetrocken worden tot F inden omtreck des inronts: Tselve punt F boven alle punten des inronts ten verften vanden Eertclood E sijnde, men noemet inronts verstepunt, en H naest wefende heet naestpunt. Maer om nu te verclare de gedaente vant middelverstepunt, en middelnaeste punt, soo laet het inront ghecommen sijn tot een ander plaats sijns wechs, ick neem met sijn middelpunt an B, daer na sy getrocken de rechte DBK, sijnende den omtreck ten naesten an I, en ten verften an K: Sghelijcx de rechte E L B M, sijnende den omtreck ten naesten an L, en ten verften an M: Dit soo wefende, M is om de voorgaende redenen verstepunt, en L naestpunt: Maer K verft van D is middelverstepunt, en I naest D middelnaestpunt: Sulcx dat dese twee verstepunten M, K, het inront daer wefende, verscheyden plaetsen hebben: Maer als het inronts middelpunt an des wechs verstepunt A, of naestpunt C is, dan sijn verstepunt en middelverstepunt een selve, alsoo oock sijn naestpunt en middelnaestpunt. D'oirsaek des naems van middelverstepunt en middelnaestpunt is dese: Want K een verstepunt is, t'welck uyt D ghesien altijt eenvaerdelick everas voorgaet, maer M al sijn rasscher dan tragher, tusschen welck de loop van K als middelmaetich sijnde, wort middelverstepunt gheheeten.

13 BEPALING.

Des inronts boochsken tusschen de tvveeverstepunten noem ick * verstepuntensbooch: T'ander tusschen de tvvee naeste punten, naestpuntensbooch.

*Aequatio
centri in epi-
rotulo.*

Als t'boochsken K M inde form der 7 bepaling tusschen t'verstepunt K, en het middelverstepunt M heet verstepuntensbooch, maer I L naestpuntensbooch.

11th DEFINITION.

When about a point as centre, on the circumference of an eccentric circle, there be described a small circle, on which a Planet is taken to move, the smaller circle is called epicycle and the larger, deferent.

In the figure of the 7th definition, let there be described about any point of the eccentric circle ABC , such as A , the smaller circle FGH , on which a Planet is assumed to move; this smaller circle is called epicycle, and the larger ABC , deferent.

12th DEFINITION.

The epicycle's apogee we call the point which is farthest from the Earth, and perigee that which is nearest to it. But the epicycle's mean apogee: the point which is farthest from the centre of its orbit, and the mean perigee: that which is nearest to it.

In the figure of the 7th definition let the diameter CA be produced to F on the circumference of the epicycle. This point F being farthest of all points of the epicycle from the Earth E , it is called the epicycle's apogee, and H , being nearest, is called perigee. But in order to explain the nature of the mean apogee and the mean perigee, let the epicycle have arrived at another point of its orbit, I assume with its centre in B . Thereafter let there be drawn the straight line DBK , intersecting the circumference at the nearest point in I and at the farthest in K ; likewise the straight line $ELBM$, intersecting the circumference at the nearest point in L and at the farthest in M . This being so, for the above reasons M is the apogee and L the perigee. But K , being farthest from D , is the mean apogee, and I , being nearest to D , is the mean perigee, so that these two apogees M, K , when the epicycle is in this position, are in different places. But when the epicycle's centre is in the orbit's apogee A or perigee C , the apogee and the mean apogee are one and the same point, and so are the perigee and the mean perigee. The origin of the name mean apogee and mean perigee is as follows: because K is an apogee which, viewed from D , always moves at uniform velocity, but M moves now faster, now slower, between which the motion of K , as being a medium value, is called the mean apogee.

13th DEFINITION.

The epicycle's arc between the two apogees I call arc of the apogees, the other between the two perigees, arc of the perigees.

Thus, the arc KM in the figure of the 7th definition, between the apogee M ¹⁾ and the mean apogee K ¹⁾, is called arc of the apogees, but IL , arc of the perigees.

¹⁾ In the Dutch text, for K read M and for M read K .

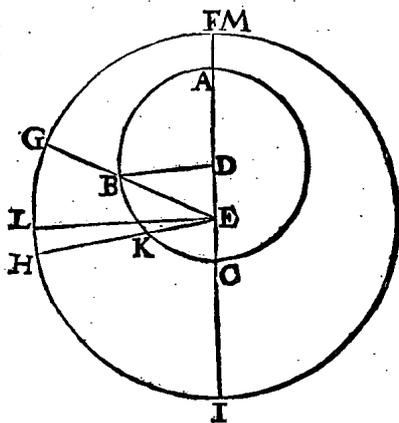
14 BEPALING.

Wechs eerste halffront noem ick de booch vant verstepunt tottet naestepunt, na'tvervolgh der trappen, dats van't begin totten 180, d'ander booch tvveede halffront.

15 BEPALING.

Schijnbaer Dwaelder is een punt daer hy schijnt te vvesen, doch eygentlick niet en is: En Schijnbaerloop een booch die hy schijnt te loopen, doch eyghentlick niet en loopt.

Laet $A B C$ den Dwaelderwech beteyckenē, diens middelpunt D , en E den Eertcloop, op welcke als middelpunt beschrevē sy den Duysteraer $F G H$. Voort sy ghetrocken deur $E D$ des duysteraers middellini $F I$, sniende den Dwaelderwech in A als verstepunt, en in C



als naestepunt: En den Dwaelder sy inde wech ant punt B , deur'twelck uyt E getrocken is de rechte lini $E B G$. Dit soo wesende, den Dwaelder eyghentlick an B sijnde, sal uyt den Eertcloop E gesien, inden duysteraer schijnen te wesen tusschen de vaste sterren an G , en daerom heet selve punt G schijnbaer Dwaelder. Maer so hy van B voortloopt, ick neem tot K , en deur K ghetrocken de rechte lini $E K H$, den Dwaelder die eyghentlick gheloopt heeft den booch $B K$, sal uyt den Eertcloop E gesien schijnen gheloo-

pen te hebben den booch $G H$, die metten eyghen loop $B K$ onghelijck is: En daerom heet de selve booch $G H$ (daermen anders oock wel voor neemt den houck $B E K$ of $G E H$) des Dwaelders schijnbaerloop.

V E R V O L G H.

Deur dit voorbeeld van schijnbaer Dwaelder en sijn schijnbaer loop met stelling eens uymiddelpuntighe wechs, is oock dergelijcke openbaer met stelling eens inronts.

Voort, deur't'ghene hier gheseyt is van schijnbaer Dwaelder en schijnbaer loop, is kennelick genouch watmen verstaen sal met schijnbaer booch, schijnbaer verstepunt als F , schijnbaer naestepunt als I , en dierghelijcke, welke hier na alst de sake vereycht ghenoecht sullen worden.

16 BEPA.

14th DEFINITION.

The first semi-circle of the orbit I call the arc from the apogee to the perigee, in the order of the degrees, *i.e.* from the beginning to 180 degrees; the other arc, second semi-circle.

15th DEFINITION.

Apparent Planet is a point where it appears to be, but is not in reality, and Apparent Motion is an arc through which it appears to move, but does not move in reality.

Let ABC denote the Planet's orbit, whose centre shall be D , and E the Earth, about which as centre let there be described the Ecliptic FGH . Further let there be drawn through ED the ecliptic's diameter FI , intersecting the Planet's orbit in A as apogee and in C as perigee. And let the Planet be in the orbit at the point B , through which is drawn from E the straight line EBG . This being so, the Planet, which is in reality at B , will, viewed from the Earth E , appear to be in the ecliptic between the fixed stars at G , and on this account this point G is called apparent Planet. But if it moves on from B , I assume to K , and through K there be drawn the straight line EKH , the Planet, which has in reality moved through the arc BK , will, viewed from the Earth E , appear to have moved through the arc GH , which is different from the proper motion BK . And on this account this arc GH (for which is sometimes also taken the angle BEK or GEH) is called the Planet's apparent motion.

SEQUEL.

From this example of an apparent Planet and its apparent motion on the assumption of an eccentric orbit the same is also evident on the assumption of an epicycle.

Furthermore, from what has been said here about the apparent Planet and apparent motion it is sufficiently obvious what is to be understood by apparent arc, apparent apogee, *e.g.* F , apparent perigee, *e.g.* I , and the like, which will be mentioned hereinafter, when required.

16 BEPALING.

Middeldvvaelder is een verdocht punt inden duyfteraer eenvaerdelick voortgaende, en altijt ant schijnbaer verstepunt vvesende, als den vwaren Dvvaelder gheen inront hebbende an sijn vvechs verstepunt is: Maer een inront hebbende, als des selfden inronts middelpunt ant verstepunt is: En sijn loop heet middelloop.

Als by voorbeelt wanneer den Dwaelder inde form der 15 bepaling is ant verstepunt A, soo isser een verdocht punt ant schijnbaer verstepunt F alijt eenvaerdelick voortgaende, sulcx dat wanneer den waren Dwaelder wecrom ghecommen is ant verstepunt A, ghedaen hebbende een volcommen keer, soo sal dat punt commen sijn an t'schijnbaer verstepunt, oock ghedaen hebbende een volcommen keer, met soo veel meer als daerentusschen het schijnbaer verstepunt geloope heeft: Sulck punt heet Middeldwaelder, en sijn loop middelloop: De selve hier verclaert sijnde op den tijt eens volcommen keers des waren Dwaelders, wy sullen om alles noch breeder uyt te legghen, voordere verclaring doen op deel eens keers; tot welcken einde ick aldus segh: Den waren Dwaelder ghecommen sijnde ick neem van A tot B, soo sal den middeldwaelder volghende t'ghestelde daerentusschen ghelopen hebben een booch ghelijck met A B, welcke sy F L (te weten L soo ghestelt, dat E L ewewijdeghe sy met D B) en noch soo veel meer als daerentusschen den loop des schijnbaer verstepunts bedraecht, t'welck sy van M tot F: Inder voughen dat M L is den middelloop overcommende in tijt met des waren Dwaelders wechloop A B, en heet middelloop, uyt oirsaeck dat soodanighe even loopen middelmatig sijn tusschen de schijnbaer rasscher en tragher.

Deur dit voorbeelt van Middeldwaelder en middelloop met stelling eens dwaelderwechs, is oock dergheelijcke openbaer met stelling eens inronts, want nemende des inronts middelpunt als voor Dwaelder, soo sal alt'ghene vooren gheseyt is daer op overcommen.

17 BEPALING.

*Additio.**Diminutio.**Propha-
refis.*

Voordering is t'ghene den vwaren Dvvaelder in den duyfteraer schijnbaerlick voordere is dan den Middeldvvaelder: En *achtering t'gene hy schijnbaerlick meer achtervvaert is. En deser tvvee ghemeene naem vvort *voorofachteringh gheseyt.

Als by voorbeelt den waren Dwaelder inde form der 15 bepaling gelooopen hebbende van A tot B, en schijnbaerlick sijnde an G, soo sal daerentusschen den Middeldvvaelder gelooopen hebbende booch F L gelijk met A B: Tweelck soo wese, de Dwaelder is in de duyfteraer an G schijnbaerlick meer achterwaert dan den Middeldwaelder L, soo veel als de booch G L, en daerom heet de selve booch G L achtering, welcke achtering overal int eerste halfont des wechs

16th DEFINITION.

Mean Planet is an imagined point which moves uniformly in the ecliptic and is always at the apparent apogee when the true Planet, if it has no epicycle, is at its orbit's apogee; but if it has an epicycle: when the centre of this epicycle is at the apogee; and its motion is called mean motion.

Thus, when in the figure of the 15th definition the Planet is at the apogee A , there is an imagined point in the apparent apogee F which always moves uniformly, so that when the true Planet has returned to the apogee A , and has performed a complete revolution, this point will have arrived at the apparent apogee, having also performed a complete revolution, with the addition of the distance meanwhile passed through by the apparent apogee. This point is called Mean Planet, and its motion mean motion. This having been expounded here with regard to the time of a complete revolution of the true Planet, in order to set forth everything even more fully we shall give a further explanation with regard to part of a revolution, to which end I say as follows. When the true Planet has come, I assume, from A to B , according to the supposition the Mean Planet will meanwhile have passed through an arc similar to AB , which shall be FL (to wit, L being so placed that EL shall be parallel to DB), with the addition of the distance meanwhile passed through by the apparent apogee, which shall be from M to F . In such a manner that ML is the mean motion corresponding in time to the distance AB passed through by the true Planet, and it is called mean motion because such uniform motions are the mean between the apparently faster and slower ones.

From this example of the Mean Planet and mean motion on the assumption of a Planet's orbit the same is also evident on the assumption of an epicycle, for when we take the epicycle's centre for the Planet, all that has been said above will apply thereto.

17th DEFINITION.

"Voordering" (advance) is the amount by which the true Planet is apparently in advance of the Mean Planet in the ecliptic, and "achtering" (lag) is the amount by which it apparently lags behind. And the common name of these two is "voorofachtering" (advance-or-lag¹).

Thus, when in the figure of the 15th definition the true Planet has moved from A to B and is apparently at G , meanwhile the Mean Planet will have passed through the arc FL , similar to AB . This being so, the Planet in the ecliptic at G apparently lags behind the Mean Planet L , as much as the arc GL , and for this reason this arc GL is called lag, which lagging takes place all through

¹) This expression, being the English equivalent of Stevin's Dutch word, will be used here instead of Ptolemy's Greek word *prosthaphairesis*.

wechs ghebeurt. Maer als sulcx met voordering valt, t'welck overall int tweede halfrondt gheschiet, men noemet voordering, en soodanighe twee int ghegemeen voorofachtering.

Merckt noch datmen soo wel den houck D B E, als de booch G L, achtering des Dwaelders noemt, om dat de ghetalen haerder tr. even sijn, uyt oirfaeck dat G L booch is des houcx G E L, even sijnde met D B E.

Tot hier toe is voorbeeldt beschreven van een Dwaelder in een wech loopende, maer bovē dien een inrondt hebbende, hy heeft tweederley voorofachtering, d'eene van wegen sijn inronnds middelpunt, t'welck een voorofachtering ontfangt van ghedaente ghelijck de boveschreven des Dwaelders in sijn wech, d'ander van wegen sijn loop int inrondt, en de voorofachtering veroirfaeckt deur dese twee voorofachteringen t'samen, wort int gemeen des Dwaelders voorofachtering gheseyt.

Laet tot breeder verclaring inde form der 7 bepaling den Dwaelder int inrondt sijn an N, alwaer hy benevens de achtering die hy van weggen sijn inronnds middelpunt ontfangt, voordering crijcht des houcx B E N, welck by d'ander D B E vervought (te weten vergaert alffe beyde cennamich sijn maer van malcander ghetrocken verscheennamich wesende als hier) datter uyt comt is des Dwaelders voorofachtering

Merckt noch t'gheral der trappen des verstepuntenbooch als hier M K, of des naestepuntenbooch als I L, alijt t'overcommen met het getal der voorofachtering vant middelpunt des inronnds als D B E, om dat I L beschreven is opt middelpunt B.

18 BEPALING.

Eerste voorofachtering is die des middelpunts vant inrondt. Twvede voorofachtering die des Dwaelders int inrondt. Gheeffende voorofachtering die veroirfaeckt vwort deur t'vermenghen van d'eerste en twvede.

Den Dwaelder sonder inrondt maer een voorofachtering hebbende, te weten die hy door de middeluytpunticheyt sijns wechs crijcht, en behouft gheen onderscheyden namen van eerste tweede noch gheeffende voorofachtering deser bepaling, welke alleen de Dwaelders toecommen in inronden loopende, wiens eerste en tweede beyde voordering sijnde, haer somme is gheeffende voordering, ende beyde achtering wesende, haer somme is gheeffende achtering, maer d'een voordering d'ander achtering sijnde, soo is haer verschil mette naem van t'grootste ghetal der twee de gheeffende voorofachtering.

19 BEPALING.

Tghene d'een Dwaelder opeen ghestelde tijt voorder loopt als d'ander, of daer op vvint, vwort * Dwaelder-
 vvinst gheheeten. *Morus B. stavia.*

Defen tijt wort dickwils verstaen te beghinnen op de saming twee Dwaelders, sulcx dat mette schijnbaerbooch na t'vervolgh der trappen vandē tracchen

the first semi-circle of the orbit. But if this is positive, which happens all through the second semi-circle, it is called advance, and these two together, advance-or-lag.

Note also that both the angle DBE and the arc GL are called lag of the Planet, because the numbers of their degrees are equal, owing to GL being the arc of the angle GEL , which is equal to DBE .

Up to this point the example has been described of a Planet moving in an orbit; but if it also has an epicycle, it has two kinds of advance-or-lag, one owing to its epicycle's centre, which receives an advance-or-lag of a similar nature to that described above with regard to the Planet in its orbit, the other owing to its motion on the epicycle, and the advance-or-lag caused by these two together is generally called the Planet's advance-or-lag.

With a view to a fuller explanation, in the figure of the 7th definition let the Planet be on the epicycle at N , where, in addition to the lag it receives owing to its epicycle's centre, it receives the advance of the angle BEN ; when the latter is combined with the other, DBE (to wit, added when they have the same sign, but one subtracted from the other when they have the opposite sign, as here), the result is the Planet's advance-or-lag.

Note also that the number of degrees of the arc of the apogees, such as here MK , or of the arc of the perigees, such as IL , always corresponds to the amount of the advance-or-lag of the centre of the epicycle; such as DBE , because IL has been described about the centre B .

18th DEFINITION.

The first advance-or-lag is that of the centre of the epicycle. The second advance-or-lag is that of the Planet on the epicycle. The total advance-or-lag is that which is caused by combining the first and the second.

The Planet without an epicycle, having only one advance-or-lag, to wit, that which it receives owing to the eccentricity of its orbit, does not need the different names of first, second or total advance-or-lag of this definition, which are only to be assigned to the Planets which move on epicycles; when its first and second are both advances, their sum is the total advance, and when both are lags, their sum is the total lag; but when one is an advance and the other a lag, their difference, with the sign of the larger of the two, is the total advance-or-lag.

19th DEFINITION.

That by which one Planet moves in advance of another in a given time, or gains thereon, is called the Planet's gain.

This time is frequently understood to begin at the conjunction of two Planets,

I V O O R S T E L.

Te verclaren hoet schijnt dat de Menschen eerst begosten tot kennis vande loop der * Dwaelders te gheraken, *Planetarum.* of daer toe souden meugen beginnen te commen, sooder gantsch gheen af en vvaer.

WAnt het kennelick is datmen om een const wel en grondelick te verstaen, behoort an te vanghen met haer uyerste beginselen: Soo sal ick mijn ghevoelen segghen hoet schijnt dat sy deden die eerst begosten de ghedaenten des Dwaelderloops te leerén, of hoemen soude meughen doen datter gantschelick niet af beschreven en ware. Om dan van dese Hemelsche stof eerst deur aertsche by voorbeelt te spreken, ick seggh dat gelijk ymant die in Caert wil brenghen een Lantschap dat noyt caertsche wijze gheteyckent en was, of daer hem gheen teyckening noch onderrichting af ter handt ghecommen en is, soude moeten het Landtschap of self dadelick besien, of seker onderricht hebben vande gene diert dadelick gesien hadden: Alsoo eenen die de manier des loops der Dwaelders wil verstaen en beschrijven, moet eerst haer loop of self ghesien hebben, of daer af sekerlick onderricht sijn van hemlien diert deur dadelicke ervaring weten: En sulcx hebben de voorganghers ghedaen, welcke alsoof eertijts ernstelick galsoughen de plaetsen der Hemelsche lichten, ende siende tusschen de groote menichte der vaste sterren, beneven Son en Maen noch seker vijf beweeghlicke, diens loop int uyerlick an sien seer ongheregelt scheen, nu ras, dan slap, somwijlen stil staen, en ettelicke mael te rugh keeren, sy hebben hun begeven tottet ondersoucken der oirsaken dier ongereltheyt, beginnende met daghelicx seer nau ga te slaen, en op te teyckenen haer schijnbaer plaetsen tusschen de vaste sterren, oock Maen duysterin ghen, en Sonduysterin ghen, met haer omstandighen, als tijt van haer begin tottet eynde, hoe groot t' verduysterde deel was, en op welcke sijde verduyftert, onder wat duysteraerlangde en breede t' middel der duystering gheschiede, en tot wat plaets des Eertcloots sy dat gageslaghen hadden. Het streckte oock tot noch meerder sekerheyt des handels, op te teyckenen den tijt met ander omstandighen vande duysteringhen der vaste sterren, te weten als sy die vande Dwaelders bedeckt saghen, en oock de duysteringhen van d'ander Dwaelders onder malcander. vut de boveschreven schijnbaer plaetsen der Dwaelders, merckten sy haer daghelicksche veranderingen in langde en breede, welcke sy dachtafelsche wijze opteyckenden, daer by noch voughende de boveschreven anclevinghen van der Dwaelders duysteringen: En creghen alsoo de nacommelinghen metter tijt, benevens haer eygen gageslagen dachtafels, oock die haerder voorgangers van seer veel jaren: Inde selve hadden sy bequame middel om de ghedaente des eyghen loops der Dwaelders t' ondersoucken, ghelijck oock souden ghehadt hebben *Hyparchus*, *Ptolemeus*, en hun nacommers, by aldien se t' haerder handt ghecommen waren.

Maer verloren blijvende, en sedert gheen ander soo ghemaect wesende als de saeck vereyscht, ick sal om desen handel te verclaren, in die plaets nemen eenighe berekende nu in druck uytgaende, als die van *Stradius*, want hoewelfe op effentijt berekent sijn: daermen de ervarings dachtafels op oneven of natuerlicke maect, en dat se daer benevens niet genouch mette saeck en overcommen,

so that by the apparent arc, in the order of the degrees, from the slowest Planet to the fastest the Planet's gain is generally denoted, which is calculated with mean as well as true Planets.

20th DEFINITION.

When two luminaries have the same apparent ecliptical longitude, they are said to be in conjunction. But when they differ by an apparent semi-circle: in opposition.

21st DEFINITION.

The Mean Planets' conjunctions and oppositions are called the Planets' mean conjunctions and mean oppositions.

22nd DEFINITION.

Empirical ephemerides of the Planets are tables in which are described in regular order, from day to day, the positions of the Planets as they have been found in practice by means of suitable mathematical instruments, with the times of their conjunctions, both with the fixed stars and among themselves, further their related phenomena, such as eclipses, size of the eclipsed part, on which side they are eclipsed, when the eclipse begins, when it ends, and the like.

23rd DEFINITION.

Calculated ephemerides of the Planets are tables which are calculated by means of knowledge of the Heavenly Motions, in which are described in regular order, from day to day, the positions of the Planets as they are expected to be in the future; further eclipses of Sun and Moon, with their related phenomena.

Such calculated ephemerides are now frequently printed, *e.g.* those of *Johannes Stöfflerus, Erasmus Rheinoldus, Leovitius, Stadius, Maginus, Martinus Everarti,* and the like ¹⁾.

¹⁾ Johannes Stöffler (Blaubeuren 1452—*ibid.* 1531), professor of mathematics at Tübingen. Author of the *Tabulae Astronomicae* (Tübingen 1500 and 1514). Jointly with Pflaum he published ephemerides of the planets for 1499—1531: *Almanach Nova* (Ulm, 1499). Later he published alone the sequel: *Ephemeridum opus . . . a capite anni 1532 in alios 20 proxime subsequentes. . . elaboratum* (Tübingen, 1531 and subsequent editions in 1533 and 1548).

Erasmus Reinhold the Elder (Saalfeld 1511—*ibid.* 1553), professor of mathematics at Wittenberg. Shortly after his ephemerides for 1550—1551 (Tübingen, 1550) appeared his celebrated *Prutenicae Tabulae coelestium motuum*. (Tübingen 1551 and several subsequent editions up to 1585), dedicated to the Duke Albrecht von Preussen. They were based on the work of Copernicus and remained the best until the publication of the Rudolphine tables; they formed the basis for the Gregorian reform of the calendar.

Cyprian Leowitz (Leowitz in Bohemia 1524—Lauingen 1574), mathematician to Count Otto Heinrich, author of *Ephemeridum novum atque insigne opus ab A. 1556—1606 accuratissime supputatum* (Augsburg 1557).

Johannes Stadius (Leonhout near Antwerp 1527—Paris 1579), professor of mathematics at Louvain and Paris, author of *Ephemerides novae et exactae ab A. 1554 ad A. 1570* (Cologne 1556; later editions up to 1591 contain an extension up to the year 1606). They were based on the *Prutenicae Tabulae*.

Giovanni Antonio Magini (Padua 1555—Bologna 1617), professor of mathematics, physics and astronomy at Bologna, author of *Ephemerides coelestium motuum ab A. 1581—*

E E R S T E
O N D E R S C H E Y T
D E S E E R S T E N
B O V C X , V A N D E V I N -
ding des Sonloops deur
ervarings dachtafels.

*Eer ick komme totte Sonloop int besonder, sal beschrijven
t'volghende eerste Voorstel vande vinding der Dwaelder-
lopen haer en d'ander int ghemeen angaende.*

24th DEFINITION.

The period of revolution of a Planet we call the time in which from a great multitude of observed revolutions is found the velocity of the motion in each circle in a known time.

NOTE.

The ecliptic, like the equator and other circles, will here be divided into 360 degrees, without using the twelve signs, each of 30 degrees, or their names as used of old, the reason of which I will set forth in the *Appendix* to these Heavenly Motions ¹).

1620 (Venice, 1582), later extended to 1608—1630 (Frankfurt 1608 and 1610); a supplement appeared in 1614 (Venice) and 1615 (Frankfurt).

Martin Everart or Everaerts (born at Bruges), mathematician and surgeon. *Ephemerides meteorologicae anni 1583* (Antwerp 1582); *Ephemerides novae et exactae* (Leiden 1597), relating to the years 1590—1610. In new editions, they were extended up to 1615 (Heidelberg, 1600 and 1602).

¹) This explanation is not found in the *Appendix*.

ronden, gevonden voort de rasheyte van yder rondts loop op bekende tijt.

M E R C K T.

Den duyfteraer sal hierghelijck den eyenaer en meer ander ronden, gedeelt worden in 360 trappen, sonder twaelf teyckens te ghebruycken elck van 30 tr. of namen van dien na d'oude ghewoonte, waer af ick de reden inden Anhang deses Hemelloops verclaren sal.

B E E R S T E

FIRST CHAPTER

OF THE FIRST BOOK

Of the Finding of the Sun's Motion by Means of Empirical Ephemerides

Before I come to the Sun's motion in particular, I will describe the following first Proposition of the finding of the Planets' motions, relating to her and the others in common.

sten Dwaelder totten snelsten, int ghemeen Dwaelderwinft beteyckent wort, welcke men soo wel berekent met middeldwaelders als ware.

20 B E P A L I N G.

*Conjunctio-
ne.
Oppositione.* Tvvee vveereltlichten een selve schijnbaer duyfteraer-
langde hebbende vvorden in *saming gheseyt. Maer een
schijnbaer half rondt verschillende, in *teghestant.

21 B E P A L I N G.

*Conjunctio-
nes et oppo-
sitiones me-
diae.* Der Middeldvvaelders saminghen en teghestanden
vvorden haer *middelsaminghen ende middeltegestan-
den gheseyt.

22 B E P A L I N G.

*Instrumenta
mathemati-
ca.* Ervarings dachtafels der Dvvaelders sijn vvaer in oir-
dentlick van dach tot dach beschreven staende plaetsen
der Dvvaelders soomense deur *vvifconstuygen daer toe
bequaem dadelick bevonden heeft, met tijt haerder sa-
minghen, soo mette vaste sterren als onder malcander,
voort haer anlevende als duyfteringhen, grootheyts des
verduyftert deels, over vvelcke sijde verduyftert, vvan-
neer beginnende, vvanneer eindende, endierghelijcke.

23 B E P A L I N G.

Ephemeris. Berekenende dachtafels der dvvaelders, sijn die deur ken-
nis des Hemelloops berekent vvordē, vvaer in oirdentlick
van dach tot dach beschreven staen, der Dvvaelders plaet-
sen, soo men meynt datse in toecommende tijden sijn
sullen: Voort duyfteringhen van Son en Maen, met haer
anlevende.

Sulcke berekenende dachtafels gaen nu veel in druck nyt, als van *Ioannes Stoffle-
rus, Erasmus Rheinoldus, Leovitiu, Stadius, Maginus, Martinus Everarti,* en
dierghelijcke.

24 B E P A L I N G.

*Tempus pe-
riodicum.* Keertijt eens Dvvaelders noemtmen, in vvelcke deur
een groote menichte der gagheslaghen keeren van sijn
ron-

1st PROPOSITION.

To set forth how man first seems to have begun to acquire knowledge of the motions of the Planets, or might begin to acquire it, if there were none at all.

Because it is obvious that, in order to understand a science well and thoroughly, one should start with its first principles, I will give my opinion how it seems that those proceeded who first began to learn the nature of the planetary motions, or how one might set about it if there were no description of it at all. In order first to speak of these heavenly matters by means of terrestrial examples, I say that just as a man who wishes to map out a region of which no map has ever been drawn or of which no drawing or report has come into his hands, would either have to inspect the region actually himself or to have reliable information from those who had actually inspected it: thus a man who wants to understand and describe the manner of the motion of the Planets must either first have seen their motion himself or must have been reliably instructed on it by those who know it from practical experience. And this is what our predecessors did, who — when formerly they seriously observed the positions of the heavenly luminaries and saw, among the great multitude of the fixed stars, besides the Sun and the Moon at least five more moving stars, whose motion to all outward appearance seemed very irregular, now fast, now slow, sometimes stopping, and several times returning — started to examine the causes of this irregularity, beginning by observing every day very closely and noting their apparent positions among the fixed stars, also Lunar eclipses and Solar eclipses, with their circumstances, such as the time from their beginning to their end, how large was the eclipsed part and on which side it was eclipsed, at what ecliptical longitude and latitude the middle of the eclipse took place, and in what place on the Earth they had observed this. It also tended to greater certainty in the treatment to note the time and other circumstances of the eclipses of the fixed stars, to wit, when they saw them covered by the Planets, and also the eclipses of the other Planets among themselves. From the apparent positions of the Planets described above they noted their daily changes in longitude and latitude, which they recorded in the manner of ephemerides, adding thereto the circumstances described above, of the Planets' eclipses. And thus, in due time, the successors obtained, besides the ephemerides observed by themselves, also those of their predecessors of a great many years. In these they had suitable means for examining the nature of the true motion of the Planets, as *Hipparchus*, *Ptolemy*, and their successors would also have had, if these had come into their hands.

But because these remained lost and no others have since been made such as the matter requires, in order to set forth this subject I will take instead some calculated tables, now printed, namely those of *Stadius*¹⁾; for though they are calculated for exact time, whereas empirical ephemerides are made for unequal or natural time, and though moreover they do not agree sufficiently with the facts,

¹⁾ See note p. 45.

ERVARINGS DACHTAFELS. 19
 MAECKSEL DES TAFELS
 VANDE MIDDELLOOP
 DER SON.

VANDE boveschreven loop eens dachs, dats van 24 uyren doende
 59 ① 8. 17. 13. 12. 31.
 Ghenomen het $\frac{1}{24}$, comt voor 1 uyr 2 ① 27. 50. 43. 3. 1.
 Diens dobbel voor 2 uyr 4 ① 55. 41. 26. 6. 2.
 Ende soo voortgaende men crijcht den loop van al'd'ander uyren tot 23 toe:
 Daer na tot den loop van een dach, vergaert den loop van noch een dach,
 men heeftse van twee: Ende soo vervolghende men crijchtse van meer dagen,
 oock van Egipsche jaren tot 810, ghelijckse inde nabeschreven tafel staen.

B 4

on the one hand owing to miscalculations, on the other hand because the Heavenly Motions are not sufficiently known, yet by way of example it will be permissible to explain therewith my intention: to wit, how firstly the Planets are supposed to move on eccentric circles, and some moreover on epicycles. Secondly, how thus the Planets' mean motions are found. Further the apparent ecliptical longitudes of the apogees or perigees of the orbits and epicycles.

The Planets' motions thus far being roughly known by means of simple intelligible reasons, I will thereafter, in the second book and the others, proceed to examine this matter by mathematical means, in the order set forth in the Summary of the Heavenly Motions. For though the above things are now at once examined after the manner of *Hipparchus*, and as they came into *Ptolemy's* hands and were left to us by them (for which we owe them thanks), to wit by certain mathematical operations, based on three observed apparent ecliptical longitudes of a Planet, it is natural to suppose that the first investigators did not begin in this way, but that many other things had to precede, from which people learned that the observation of this would tend to produce a sure basis of knowledge of the Heavenly Motions, further that those mathematical operations were thereupon added to the others for greater completeness and certainty. And thus I will also describe them, after the matter has first been understood by means of empirical ephemerides. Note also that since to outward appearance the Earth seems to stand still and the beginning of the knowledge of the Heavenly Motions started from this assumption, and also begins thus more suitably and understandably, we shall imitate this here, describing on this basis the first and the second book, but in the third the Heavenly Motions on the natural assumption of a moving Earth.

Further it is to be noted that the description of the Heavenly Motions, as it came into *Ptolemy's* hands, according to his own reports was very simple, namely, the motion of the Moon, like that of the Sun, in an eccentric circle and the other Planets only in eccentric epicycles. But *Ptolemy*, being of opinion that this was not sufficiently in agreement with his actual experience, combined the theory devised by him for this purpose with the above-mentioned simple assumption and wrote a treatise about it. The same was also done by *Copernicus* on the assumption of a moving Earth. But I will not here follow this mixed method, but will describe the Heavenly Motions only on the first simple assumption, so that we may first have it as it came into *Ptolemy's* hands; and with those inequalities devised afterwards, which are unnaturally obscure, and are erroneous, I will deal in particular in an Appendix, in order that it may appear all the more clearly to the pupil what is sought for in the improvement of the Heavenly Motions, thus to be able to strive on a firmer basis after more suitable theories, because this appears to me the most suitable method for making his PRINCELY GRACE understand in the shortest possible time and with the greatest clearness what is known to me about the motions of the Planets, and also for showing manifestly how needful are empirical ephemerides and Observers for acquiring such knowledge of this science as existed in the Age of the Sages.

NOTE.

Before I go on, I will also say this: to wit, that my intention is not to prove with great certainty the Planets' future true positions, but only to set forth the

brack den tijt des loops van 29 ① : Om welke te vinden, ick sie inde dachtafel dat de Son doen op een dach liep 59 ①, daerom segh ick, 59 ① gheven 24 uyren, wat de boveschreven 29 ① comt alfooren 11 uyren 47 ① 48 ②. Nu vanden 11 maerte int jaer 1554, totten 10 Maerte 11 uyr 47 ① 48 ② int jaer 1606, sijn gheschiet 52 keeren, waer over de Son gheloopen heeft 18992 daghen 11 uyren 47 ① 48 ② (te weten 52 mael 365 min 1 (ick segh min 1 om dattet is vanden 11 Maerte totten 10 Maerte) dats 18979, mer noch 13 dagen der 13 schrickeljaren, die in Februarius vervought worden) Dit soo sijnde, ick segh, 52 keeren duynen 18992 dagen 11 uyren 47 ① 48 ②, hoc langh sal 1 keer duynen? Comt voor de begheerde lanckheyt des jaers na dese rckening, 365 daghen 5 uyren 45 ① 55 ②.

T B E S L V Y T. Wy hebben dan deur ervarings dachtafels de lanckheyt des natuerlick jaers ghevonden, na den eysch.

3 V O O R S T E L.

De Sonnens middelloop op een ghegeven tijt te vinden, en daer afeen tafel te beschrijven.

1 M E R C K.

Int 2 Voorstel is gheseyt het jaer bevonden te wesen van 365 daghen 5 uyren 45 ① 55 ②, waer op men als gront soude moghen voortvaren, int maken van nieuwe tafels des middelloops der Son: Doch want my r'selve moeylick soude vallen, dat oock daerbenevens dit besluyt vande lanckheyt des jaers (ghelijck oock alle ander na den wijstijt) weynich sekerheys heeft, en dat alles maer voorbeeltsche wijze en gheschiet, om de redenen van dies breeder verclaert int 1 voorstel, soo sal ick om sulcke moeyte te schuwen, nemen de lanckheyt des jaers by *Ptolemus* beschreven, en de tafelen by hem daer op berekent. Dese lanckheyt des jaers heeft hy na de wijze alfooren bevonden van 365 daghen 5 uyren 55 ① 12 ②, die in ander verdeeling sonder uyren te noemen, doen 365 daghen 14 ① 48 ②, ofte anders 365 $\frac{37}{170}$ daghen.

En sulcx als hier in dit merck gheseyt is vande Sonloop, dergelijcke sal int volghende oock alsoo ghedaen worden met *Ptolemus* tafels der middelloopen van d'ander Dwaelders, die ick nemen sal in plaets van nieuwe te maken.

T G H E G E V E N. Het is den tijt van een dach. T B E G H E E R D E. Men wil daer op de Sonnens middelloop gevonden hebben. T W E R C K. Ick segh, op 365 $\frac{37}{170}$ daghen, loopt de Son 360 tr. deur het 1 merck deses voorstels, wat op 1 dach? Comt voor t'begheerde 59. ① 8. 17. 13. 12. 31.

2 M E R C K.

Wy hebben hier een voorbeelt ghestelt, int welke de Sonnens eyghenloop eens dachs ghevonden wert door een reghel van drien, waer me kennelick is hoemen deur dergelijcke wercking, de Sonloop seude vinden van alle ghegeven tijt, maer want die wercking moeyelick valt, ende dat boven diens ons sulcx in desen handel dickwils te vooren comt, soo wordender tafels ghemacckt, om van alle ontmoetende tijt den loop met lichticheyt te vinden, welck maecksel ick beschrijven sal als volght.

M A E C K.

manner of their motion, taking the examples that are most appropriate, either certain or uncertain, because in my opinion the treatment as a whole does not have a sufficiently firm basis, on account of the unknown second inequalities, and requires a new and more certain foundation, which it will not be possible to lay so soon, on the one hand because there is no nation whose members together practise this science very earnestly in their native language, and further because there cannot be found as many observers as are required for the matter, as has been stated more fully in the 6th definition of the first book of Geography. Secondly, because such an improvement also takes time.

CONCLUSION. We have thus set forth how men first seem to have begun to acquire knowledge of the motion of the Planets, or might begin to acquire it, if there were none at all; as required.

2nd PROPOSITION.

To find the length of the natural year by means of empirical ephemerides.

The work is suitably started by investigating the length of the natural year, because a certain well-defined time is needed in which the motions of all the Planets and the Heavenly Bodies are calculated. To come to the matter, it is necessary, in order to understand the sequel easily, to have the above-mentioned ephemerides of *Stadius*, or others instead for that matter, which we use as if they were empirical ephemerides, for without them everything would be more obscure. These tables therefore being to hand, I seek in some one year — I take the first, which is the year 1554 — at what noon the Sun was nearest to the Vernal Equinox, and find on 11th March, for then it was at $359^{\circ}59'$, which is only $1'$ from the Vernal Equinox. I then seek in the following year 1555 when the Sun was again at the above $359^{\circ}59'$, and find 11th March, after noon at 5h 36m, for at noon according to the ephemeris it was at $359^{\circ}45'$, so that the time of the motion of $14'$ is still wanting. In order to find this, I see in the ephemeris that the Sun then moved in one day 1° , that is $60'$; therefore I say: $60'$ give 24 hours, what do the above $14'$ give? This gives, as above, 5h 36m. But from 11th March 1554 to 11th March 1555 there are 365 days; therefore, according to this calculation the length of the year would be 365d 5h 36m.

This has thus first been calculated by way of example for the motion of one revolution of the Sun in order that everything may be understood more clearly and thoroughly. But since the certainty is greater for many revolutions or years than for one or few (for one hour's error on a thousand years only makes $1/1000$ hour on a year, whereas one hour's error on one year amounts to a whole hour for every year), we shall now take for this as many years as there are in the ephemerides.

I then seek in the last year, which is 1606, when the Sun was again at $359^{\circ}59'$, and find 10th March, after noon, at 11h 47m 48s, for at noon it was at $359^{\circ}30'$, so that the time of the motion of $29'$ was still wanting. In order to find this, I see in the ephemeris that the Sun then moved in one day $59'$; therefore I say: $59'$ give 24 hours, what do the above $29'$ give? This gives, as above, 11h 47m 48s. Now from 11th March of the year 1554 to 10th March at 11h 47m 48s of the year 1606, 52 revolutions have taken place, which the

de ware plaetsen, maer alleen te verclaren de manier des loops, nemende voorbeelden die best te pas commen, ghewis of onghewis, uyt oirsaeck dat den handel int gheheel na mijn gevoelen om der onbekende tweede oneventheden wille, als nu gheen ghenouchsaem vaste gront en heeft, en een nieuwe ghewisser vereyscht, die soo haest niet gheleyt en sal connen worden, eensdeels om datter geen geslacht van volck en is die in haer aengeboren tael hun heel ernstelick daer in t'samen oeffenen, en vervolghens niet soo veel Gaslaghers en connen ghevonden worden als de saeck vereyscht, ghelijck daer af breeder gheseyt is onder de 6 bepaling des eersten boucx van Eertclootschrift. Ten anderen dat bovendien sulcke verbetering oock tijt vereyscht.

T B E S L V Y T. Wy hebben dan verclaert hoet schijnt dat de menschen eerst begosten tot kennis vande loop der Dwaelders te gheraken, of daer toe souden meughen beghinnen te commen, sooder gantsch gheen af en waer, na den cysch.

2 V O O R S T E L.

Deur ervarings dachtafels de lanckheyd des natuerlick jaers te vinden.

Men beghint billichlick mettet ondersoucken der lanckheyd des natuerlick jaers, omdatmen een seker bepaelde tijt behouft, waer in alder Dwaelders en Hemelen loopen berekent worden. Op dat wy dan totte saeck commen, t'is noodich datmen om t'volghende lichtelick te verstaen, hebbe de voorschreven dachtafels van *Stadius*, of immers ander in haer plaets, welcke wy nemen al oft ervarings dachtafels waren, want sonder die soude alles duyfster vallen. De selve dan by der hand wefende, ick souck in eenich jaer, ick neem het eerste, wefende t'jaer 1554 op welcke middach de Son de Lentne ten naefsten was, en bevinde opten 11 Maerte, want doen wasse onder den 359 tr. 59 ①, t'welck alleenlick 1 ① vande Lentne is. Ick souck daer na opt volghende jaer 1555, wanneer de Son weerom was inden voorschreven 359 tr. 59 ①, en bevinde den 11 Maerte na middach te 5 uyren 36 ①, want op den middach wasse na ruytwijfen des dachtafels inden 359 tr. 45 ①, sulcx datter noch gebreect den tijt des loops van 14 ①: Om welcke te vinden ick sie inde dachtafel dat de Son doen op een dach liep 1 tr. dais 60 ①, daerom segh ick, 60 ① geven 24 uyren, wat de boveschreven 14 ① comt allvooren 5 uyr 36 ①. Maer vanden 11 Maert 1554, totten 11 Maert 1555, sijn 365 daghen, daerom t'jaer soude na die rekening dueren 365 daghen 5 uyr 36 ①.

Dit is aldus eerst voorbeeltliche wijze metten loop van een Son keer berekent, op dat alles claerder en grondelicker verstaen worde. Maer anghesien men op veel keeren of jarē, meer sekerheyd heeft dan op een of weynich (want op duyfent jaren een uyre ghemist, en maect op een jaer maer $\frac{1}{1000}$ uyr, daer anders op een jaer een uyr ghemist, voor t'felve een yder jaer een heele uyr bedraecht) soo sullen wy nu daer toe soo veel jaren nemen alster inde dachtafels sijn.

Ick souck dan opt laetste jaer, wefende het 1606, wanneer de Son weerom was inden 359 tr. 59 ①, en bevinde den 10 Maerte na middach 11 uyr 47 ① 48 ①, want op den middach wasse inden 359 tr. 30 ①, sulcx datter noch ghe-

B 3 brack

Sun performed in 18,992d 11h 47m 48s (to wit: 52×365 minus 1) (I say minus 1, because it is from 11th March to 10th March), that is 18,979 plus 13 days of the 13 leap years, which are added in February). This being so, I say: 52 revolutions take 18,992d 11h 47m 48s, how long will 1 revolution take? According to this calculation the required length of the year is 365d 5h 45m 55s.

CONCLUSION. We have thus found the length of the natural year by means of empirical ephemerides; as required.

3rd PROPOSITION.

To find the Sun's mean motion in a given time, and to describe an ephemeris thereof.

1st NOTE.

In the 2nd Proposition it has been said that the year has been found to be 365d 5h 45m 55s, on which basis we might proceed to make new ephemerides of the Sun's mean motion. But because this would be difficult for me, while moreover there is little certainty in this conclusion as to the length of the year (just as in all other things subsequent to the Age of the Sages), and everything is only done by way of example, for the reasons set forth more fully in the 1st Proposition, I will, in order to eschew this trouble, take the length of the year described by *Ptolemy*, and the tables calculated by him thereon. This length of the year was found by him in the above manner to be 365d 5h 55m 12s, which by another division, without mentioning hours, makes 365;14,48,1) or otherwise $365\frac{37}{150}$ days.

And the same as has been said in this note for the Sun's motion will also be done hereafter for *Ptolemy's* tables of the mean motions of the other Planets, which I will take instead of making new ones.

SUPPOSITION. The time is one day. WHAT IS REQUIRED. The Sun's mean motion is required to be found. PROCEDURE. I say: in $365\frac{37}{150}$ days the Sun moves 360° , by the first note of this proposition; what does it move in one day? The required value is $0^\circ;59,8,17,13,12,31^2$).

2nd NOTE.

We have here given an example in which the Sun's proper motion of one day was found by the rule of three, from which it is evident how by a similar operation the Sun's motion might be found in any given time, but because this operation is difficult, while moreover we shall often meet with it in this work, tables are made for easily finding the motion for any time that may occur, the construction of which tables I will describe as follows.

¹⁾ In sexagesimals of a day, hence meaning $365 + \frac{14}{60} + \frac{48}{3600}$ days.

²⁾ In angular (or analogous) quantities written sexagesimally, the successive sexagesimal subdivisions (which Stevin indicated by writing the digits 1, 2, . . . in small circles) in accordance with modern practice are separated by commas. The whole numbers are followed by a semi-colon. If there are only few subdivisions, the ordinary notation in degrees, minutes, and seconds is used.

eenſdeels deur miſrekeninghen dieder vallen; ten anderen om dat den Hemelloop nu niet ghenouch bekend en is, doch ſal̄ voorbeeldiſche wiſſe meughen beſtaen, om daer me mijn voornemen te verclaren: Te weten hoemen voor t'eerſte vermoedt de Dwaelders te loopen in uytmiddelpuntige ronden, en eenighe boven dien noch in inronden. Ten anderen hoemen daer deur vindt der Dwaelders middelloopen: Voort de ſchijnbaer duyſteraerlangden der verſtepunten of naeſtepunten van weggen en inronden.

Den Dwaelderloop dus verre deur platte verſtanelicke redenen uyt den rouwen bekend sijnde, ſoo ſal ick daer na int tweede bouck en d'ander volghende voorder commen tottet onderſoucken deſer ſtof deur wiſconſtighen handel, na d'oirden int Cortbegrijp des Hemelloops verclaert: Want datmen de boveſchreven dinghen ten eerſten begint te ſoucken na de wiſſe van *Hypparchus*, en *Ptolemus* ter handt ghecommen, en deur hemlien ons achtergelaten (daer wy hun danck af ſchuldich ſijn) te weten deur ſeker wiſconſtighe wercking, ghegront op een Dwaelders drie gagheſlaghen ſchijnbaer duyſteraerlangden, de natuerlicke reden ſchijnt te willen dat d'eerſte onderſouckers daer me niet en begoften, maer dat veel andere dinghen voor moeſten gaen, deur welcke men leerde datter gaſlaen van ſulcx tot ſoo ſeker gront vande kennis des Hemelloops ſtrecken ſoude, voort dat die wiſconſtighe werckingen daer na tot meerder overvloet en ſekerheyt by d'ander vervougt wierden. En alſoo ſal ick die oock beſchrijven, na dat de ſaek eerſt deur wercking met ervarings dachtafels verſtaen ſal ſijn: Merckt noch dat nadien den Eertcloot int uyerlick anſien ſchijnt ſlil te ſtaen, en dat den anyang der leering des Hemelloops op ſulcke ſtelling begoft heeft, oock alſo bequamelicker en verſtaenlicker begint, ſoo ſullen wy t'ſelve hier na volghen, beſchrijvende op ſulcken gront het eerſte en tweede bouck, maer int derde den Hemelloop mette natuerlicke ſtelling eens roerenden Eertcloots.

Voort iſte te weten dat de beſchrijving des Hemelloops *Ptolemus* ter handt gecommen, na ſijn eygen ſeggen ſeer eenvoudich was, namelick de Maenloop, gelijk vande Son in een uytmiddelpuntichront, en d'ander Dwaelders alleene-lick in uytmiddelpuntige inronden: Maer *Ptolemus* achtende dat ſulcx niet ghenouch met ſijn dadelicke ervaringē overeen en quam, heeft de ſpiegeling deur hem daer toe verdocht, ghemengt mette voorchreven eenvoudige ſtelling, en daer af een werck gemaeckt: Sgelijcx heeft oock *Copernicus* op de ſtelling eens roerendē Eertcloots gedaen: Doch ick en ſal hier die vermengde wiſſe niet volgen, maer den Hemelloop met d'eerſte eenvoudige ſtelling alleē beſchrijvē, op dat wy die voort'eerſte alſoo hebbē, gelijkſe *Ptolemus* ter handt quam, en van die naverdochte oneventhedē weſende onnatuerlick duyſter en gemiſt, ſal ick in een Anhang beſonderlick handelen, op dat voor den leerlinck te claerlicker blijcke watter inde verbetering des Hemelloops gefocht wort, om alſoo op een vaſter voet na bequamer * ſpiegelinghen te meughen trachten, want dit de bequaemſte wech iſ die my nu te voren comt, om op den cortſten tijt mette meeſte claerheyt, ſijn VORSTELICKE GHENADE te doen verſtaen t'ghene my vandē loop der Dwaelders bekend iſ, oock mede om opentlick te doē blijcken, hoe noodich ervarings dachtafels en * Gaſlagers ſijn, om tot ſulcken kennis deſer conſt te gheraken, alſer inden Wijſentijt af gheweest heeft.

Theoria.

*Observa-
res.*

M E R C K T.

Eer ick voorder comme ſal noch dit ſegghen: Te weten dat mijn voornemen niet en iſ met groote ſekerheyt te bewijſen der Dwaelders toecommen-
de ware

CONSTRUCTION OF THE TABLE
OF THE SUN'S MEAN MOTION.

If of the above-mentioned motion of one day, that is 24 hours,
being $0^{\circ};59, 8,17,13,12,31$

there be taken $\frac{1}{24}$, we get, for 1 hour $0^{\circ}; 2,27,50,43, 3, 1$

The double of this is, for 2 hours $0^{\circ}; 4,55,41,26, 6, 2$

And, proceeding in this way, we get the motion of all the other hours, up to 23.
If thereupon to the motion of one day there be added the motion of another
day, we have the motion of two days. And, proceeding in this way, we get it
of more days, also of Egyptian years up to 810, as they appear in the
following table.

[The following table (pp. 20-21) has not been reproduced here]

SONLOOPS VINDING DEVR GHEBRVYCK DES TAFELS.

Laet begheert sijn de Sonnens eyghen loop op 879 Egipsche jaren 66 dagen 2 uuren. T'WERCK. Soo inde tafel de 879 Egipsche jaren 66. dag. 2 uyr. metten middelloop van dien al in een reghel ghevonden wierden, wy soudeten eersten den begheerden middelloop hebben, sonder eenighe vergaring van ghedeelten te behouven: Maer dat niet wesende, wy moeten verscheyden stikken by malcander voughen die t'amen dat heel maken: Tot defen eynde neem ick ten eersten uyt de tafel de jaren die de begheerde 879 naect sijn, als 810, die stellende met haer middelloop alleenlick tot ② toe (als totte saeck ghenouch sijnde, om dat de rest dient tottet maecksel der tafel, niet tottet ghebruyck van dien, soo int laetste deel der nabeschrevē waerschouwing opt maecksel der tafel, breeder verclaert sal worden) En de ghestalt der wercking sal dusdanich sijn:

810. jaren. 163 tr. 4. 12.

Nu ghebreken my noch 69 jaren, daer toe neem icker (hoe wel men die noch andersins soude meughen nemen) 45 en 15: Daer na 60 dag. en 6 dag. ende ten laetsten de twee uyr: De selve altemael oiridentlick vervought onder de boveschreven 810 jaer, ende alles vergaert na t'behooren, soo sal de ghestalt der wercking sijn als hier onder.

810 jaer.	163 tr.	4.	12.
54 jaer.	346 tr.	52.	17.
15 jaer.	356 tr.	21.	11.
60 dag.	59 tr.	8.	17.
6 dag.	5 tr.	54.	50.
2 uyr.	0 tr.	4.	56.
<hr/>			
931 tr. 25. 43.			

vyt dese 931 tr. ghetrocken al de heele ronden dieder in sijn elck van 360 tr. comt twee ronden, die ick verlaet, ende rest 211 tr. welcke mette rest doen 211 tr. 25 ① 43 ②, doch verlatende de 43 ②, comt ten naecten voor den begheerden loop op den ghegevenen tijt 211 tr. 26 ③.

T'BSLVYT. Wy hebben dan de Sonnens middelloop op een ghegevenen tijt ghevonden, en daer afeen tafel beschreven, na den eyfch.

WAERSCHOVING OPT MAECKSEL DER TAFEL.

Hier valt te bedencken dat de tafel van een dach vermeerderende, vereyscht ghemaect te worden alijt deur menichvuldig of vergaring, ende niet deur deeling of afrecking: Als by voorbeelt, my bekent geworden sijnde den loop van 18 jaren, doende 168 tr. 49. 52. 9. 9. 45. 0. soo ick daer me wil vinden den loop van 4 mael 18, dats van 72 jaren, ick menichvuldighe den voorschreven loop met 4, en comt (heele ronden verlaten) 315 tr. 9. 28. 36. 39. 0. 0. T'welck alsoo deur menichvuldig wel gaet, ghelijck boven gheseyt is: Maer soo ons deur ander wech eerst bekent hadde geweest de selven loop van 72 jaren, ende datmen deur verkeerde wech der voorgaende, dats deelende dien loop 315 tr. 9. 28. 36. 39. 0. 0. deur 4, soude meenen te vinden den loop van 18 jaren, ten soude niet volgen, als blijktt, want sulck vierendeel den loop van 18 jaren niet uyt en brengt, ende dat om bekende oirfaken, te weten datmen al menichvuldighen-

USE OF THE TABLE.

Let it be required to find the Sun's proper motion in 879 Egyptian years 66 days and 2 hours. PROCEDURE. If in the table the 879 Egyptian years 66 days and 2 hours, with the mean motion thereof, were all found in one line, we should at once have the required mean motion, without having to add together any parts. But this not being so, we have to add different parts which together form the whole. To this end I first take from the table the years which are nearest to the required 879, namely 810, recording them with their mean motion only to seconds (this being sufficient for the matter, because the rest serves for the construction of the table, not for its use, as will be set forth more fully in the last part of the caution to be given hereafter about the construction of the table). And the form of the procedure will be as follows:

810 years	163° 4' 12"
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Now there are still 69 years short; for this I take (though one might also take them differently) 54¹⁾ and 15. Thereafter 60 days and 6 days, and finally the two hours. When these are all placed in the right order below the above-mentioned 810 years and everything is properly added together, the form of the operation will be as shown below.

810 years	163° 4' 12"
54 years	346° 52' 17" 2)
15 years	356° 21' 11"
60 days	59° 8' 17"
6 days	5° 54' 50"
2 hours	0° 4' 56"
931° 25' 43"	

When from these 931° I take all the complete circles which are contained therein, each of 360°, I get two circles, which I discard, and the remainder, 211°, which together with the rest make 211°25'43"; but when I discard the 43", I get approximately 211°26' for the required motion in the given time.

CONCLUSION. We have thus found the Sun's mean motion in a given time, and described a table thereof; as required.

CAUTION ABOUT THE CONSTRUCTION OF THE TABLE.

It should be borne in mind that the table, progressing by days, requires to be made always by multiplication or addition, and not by division or subtraction. For example, when I have found the motion of 18 years, making 168;49,52,9,9,45,0³⁾, when I wish to find by means of this the motion of 4 times 18, that is of 72 years, I multiply the above-mentioned motion by 4, and this makes (whole circles being discarded) 315;19,28,36,39,0,0. Which is thus done correctly by multiplication, as has been said above. But if by other means this motion of 72 years had first been known to us, and if by the converse method to the above, *i.e.* dividing that motion of 315;19,28,36,39,0,0 by 4, we thought we might find the motion of 18 years, this would not follow, as is

¹⁾ For 45 in the Dutch text read 54.

²⁾ For 146 in the Dutch text read 346.

³⁾ This example has been taken from the tables of the moon (Book I, second chapter, p. 47 of the Dutch text).

dighende heele ronden verlaet, diemen al deelende daer by soude moeten doer, maer de selve onbekent sijnde en can niet bequamelick te wegheghebrocht worden. Tis oock openbaer waerom datter werck deur de boveschreven deeling gheen hindernis en crijcht deur deeling des dach in uyren, ghelijckt oock en soude in de ghedeelten des tijts van 365 daghen, namentlick om dat daer in gheen heel ront en can wesen.

VOORT, anghesien wy van cleender ghedeelten dan ① als ② ③ &c. in dadelicke ervaringhen weynich of gheen sekerheyt en hebben, als t' sijnder plaets verclaert is, so mocht ymant dencken, waerom dese Sonloopen inde tafel cleender dan met ① beschreven sijn tot ⑥ toe, te weten waerom datmen niet en seyde den loop eens dachs te wesen alleenelick van 59 ①, achterlatende de rest: Ofte waerom datmen even tot ⑥ comt, ende niet tot noch cleender gedeeltē: De reden daer af can deur t' voorgaende openbaer sijn: Want na dien der ouden voornemen was, te beschrijven een tafel van 810 jaren, begrijpende 295650 dagen, ende dat de Sonloop der selve gevonden wort deur verlaming van soo veel dagelicke Sonloopen (want van elcke tijt des tafels haer loop deur reghel van drien te vinden, als eens gedaen is int vinden des loops van 1 dach, men soude alsoo wel een tafel meughen maken alleen tot ③ toe, maer sulck maecksel soude veel te moeyelick vallen, als boven gheseyt is) daer uyt soude volghen datter inde Sonloop vande voorschreven 810 jaren, gemist soude sijn over de 295650 mael 8 ② (dieder boven de 59 ① sijn, bedragende over de 657 tr. Daerom de Sonloop der tafels in ① te einden en waer uyt oirsack van sulcx niet behoerlick. Ende om de selve reden salmen oock verstaen, dattet niet behoerlick en waer die te einden in ②, ③, of ④, want in ④ eindende, ende eenighe ⑤ daer an ghebrekende, ofte overschietende, het soude inde groote tijden eenighe ② feyls connen geven, want 30 ④ genomen 295650 mael, maken over de 40 ②, welke deur vergaring van verscheyden gedeelten, feyl van eenighe ① soude connen veroirsaken. Inder voughen dat in sulck ansien, de Sonloopen des tafels tot ③ souden moeten commen, om int gebruyck van dien sekerheyt van ① te hebben: Doch tot noch meerder gewisheyt, ende om Sonloopen te meughen berekenen op grooter tijden dan 810 jaren, so hebben de ouden, ghelijckt schijnt, tot ⑥ ghecommen, ende met goede reden tot gheen cleender ghedeelten, als onnoodich sijnde.

M E R C K T.

Ghelijck hier een tafel gemaect is mette gevonden eygenloop der Son eens dachs; Alsoo sullen int volgende tafels gemaect worden mette gevonden loop eens dachs van d'ander dwaelders, ende haer Hemelen: Doch wy en sullen aldaer noch manier des maeckfels, noch des ghebruycx van dien, beschrijven, als gelijk sijnde ande voorgaende. Men sal oock verstaen de boveschreven waerichouwing over de ghelijcke volghende tafelen ghemeen te wesen.

4 V O O R S T E L.

Deur ervarings dachtafels te vinden de schijnbaer
 ★ duysteraerlangde van des Sonvvechs verstepunt en
 naestepunt

*Longitudinē
 Zodiaci apo-
 gei & perigaei.*

Alfmen

apparent, because such fourth part does not give the motion of 18 years, such from well-known causes, to wit that, in multiplying, whole circles are discarded, which, in dividing, would have to be added thereto; but since they are unknown, this cannot be brought about adequately. It is also evident why the procedure of the above-mentioned division is not impaired by the division of the day into hours, just as it would not be in parts of the time of 365 days, namely because there cannot be a whole circle therein.

Further, as we have little or no certainty in practical experience of parts smaller than 1', such as higher-order sexagesimals, as has been set forth in its proper place, one might think why these Sun's motions have been described in the table in parts smaller than 1', down to sixth-order sexagesimals, to wit, why it was not said that the motion of one day was only 59', discarding the rest. Or why we go precisely up to sixth-order units, and not to even smaller parts. The reason for this may be evident from what precedes. For as it was the intention of the Ancients to describe a table of 810 years, containing 295,650 days, and the Sun's motion therein is found by addition of as many daily Sun's motions (for by finding for each time of the table its motion by the rule of three, as has once been done in finding the motion of one day, one could thus make a table only up to 1'', but the construction of this would be much too difficult, as has been said above), it would follow therefrom that in the Sun's motion in the above-mentioned 810 years there would be an error of more than 295,650 times 8'' (which there are in addition to the 59'), which is more than 657°. On this account it would not be right to end the Sun's motion in the tables at 1'. And for the same reason it is also to be understood that it would not be right to end them at the second, third or fourth order of fractions, for if we stopped at the fourth and there should be some of the 5th order short or in excess, this might produce an error of some seconds in long periods, for if we take 295,650 times 30 fifth-order units, this makes more than 40'', which through the addition of different parts might cause an error of several times 1'. In such a way that in this respect the Sun's motions in the table would have to go up to the 5th order if we are to have a certainty of 1' in using it. But for the sake of even greater certainty, and in order that they might be able to calculate Sun's motions in periods greater than 810 years, the Ancients apparently went up to the 6th order, and with good reason not to smaller parts, this being unnecessary.

NOTE.

Just as here a table has been made by means of one day's proper motion of the Sun as found, thus tables will be made hereafter by means of the motion of one day, as found for the other planets and their Heavens¹⁾. But we shall there describe neither the manner of making the table nor its use, since they are identical with the preceding case. It is also to be understood that the above-mentioned caution applies to the similar following tables as well.

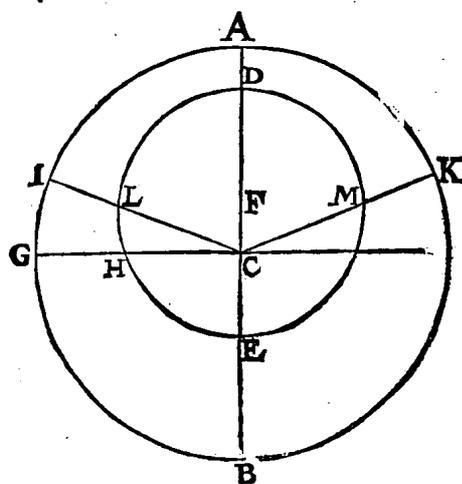
4th PROPOSITION.

To find, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of the Sun's orbit.

¹⁾ Cf. Third Book, Chapter I; also *Introduction*, p. 11.

24 SONLOOPS VINDING DEVR

Alfmen de dachtafels van *Stadius* (die wy om de redenen des 1 voorstels nemen al offe uyt ervaringhen beschreven waren) wel deursiet, men bevint onevenheyt inde schijnbaer Sonloop, te wetē op d'een tijt wel 3 of 4 ① sdaechs meer als op d'ander, en daerbenevens dattet een jaer voor t'ander na, den loop ontrent het middel van Junius altijt ten slapsten is, daghelix van 57 ①. Als by voorbeelt op den 15 Junius int jaer 1555, bevinde ick de Son onder den 92 tr. 39 ①, en op den 16 daer na onder dē 93 tr. 36 ①: Sulcx datse op dien dach vanden 15 Junius totten 16, gelooopen heeft 57 ①: en so veel loopise oock van yder 15 Junius totten 16 op elck der volgende ende voorgaende jaren. Maer ontrent half December isse alle jare ten snelsten van ontrent 1 tr. 1 ①. Nudit aldus jaerlicx ghebeurende, het gheeft den eersten ondersouckers der oirsaken deser dinghen billichlick vermoen, dat de Son in een rondt draeyt, diens middelpunt des Eertcloots middelpunt niet en is, maer daer buyten. Om t'welck deur een form



breeder te verclaren, laet A B den duyfteraer sijn, diens middelpunt dats dē Eertclood C, en D E de Sonwech, diens middelpunt F, t'verstepunt D, t'naestepunt E: Laet daer na van C totten duyfteraer an G, getrockē worden C G rechthouckich op A B, en sijnende de Sonwech in H. Dit soo wefende, men siet d'oirsaeck waerom de Son in haer wech loopende van D tot H meer dan eē vierendeel ronts, nochtans inden duyfteraer schijnbaerlick van A tot G al-

leenelick een vierendeelronts: Sghelijcx waerom sy in haer wech eyghentlick loopende van H tot E min dan een vierendeelronts, nochtans inden duyfteraer schijnbaerlick van G tot B een volcommen vierendeelronts. Men siet oock de oirsaeck waerom de Son over even schijnbaer boghen als A G en G B oneven tijden loopt, te weten langer van D tot H, dats de schijnbaerloop A G, dan van H tot E, dats de schijnbaerloop G B. Maer in des schijnbaerloops twee half ronden A G B, B A, loopise even soo lang als in des eyghen loops twee half ronden D H E, E D. Hier uyt is openbaer dat de Son ant verstepunt D wefende, int middel van haer schijnbaer traechste loop moet sijn, en an t'naestepunt E int middel van haer schijnbaer snelste. Maer om dat middel, t'welck t'begeerde deses voorstels is, op een seker voert te foucken, ick salder eerst dese verclaring af doen. Laet inden duyfteraer gheteyckent worden de twee punten I en K, alsoo dat de booch A I even sy an A K: Daer na sy ghetrocken van C tot I inden duyfteraer de lini C I, sijnende de Sonwech in L, sghelijcx C K sijnende de Sonwech in M. Dit soo sijnde, tis openbaer dat ghelijck de schijnbaer booch A I over d'een sijde, even is mette schijnbaer booch A K over d'ander, alsoo is de Sonwechbooch D L over d'een sijde, oock even mette Sonwech booch D M over d'ander: En vervolghens alle twee schijnbaer boghen die over beyde sijden van

If we look through the ephemerides of *Stadius* (which for the reasons mentioned in the 1st proposition we use as if they had been described from experience) attentively, we find inequality in the Sun's apparent motion, to wit at one time as much as 3' or 4' a day more than at another, and moreover that year in and year out its motion is always slowest about the middle of June, being 57' daily. Thus, for example, on 15th June of the year 1555 I find the Sun at $92^{\circ}39'$, and on the 16th after that at $93^{\circ}36'$, so that in that day, from 15th June to 16th, it moved 57'; and the same is also its motion from every 15th June to the 16th in each of the following and preceding years. But about the middle of December its motion is fastest every year, namely about $1^{\circ}1'$. Since this happens every year, it gives the first investigators of the causes of these things reasonable conjecture that the Sun revolves in a circle the centre of which is not the centre of the Earth, but outside it. In order to explain this more fully by means of a figure, let AB be the ecliptic whose centre is the Earth C , and DE the Sun's orbit, whose centre is F , its apogee D , its perigee E . Thereafter let there be drawn from C to the ecliptic, at G , CG at right angles to AB and intersecting the Sun's orbit in H . This being so, we see the cause why the Sun in its orbit, while moving from D to H more than a quarter circle, yet in the ecliptic appears to move from A to G only a quarter circle. Likewise why, moving in reality in its orbit from H to E less than a quarter circle, yet in the ecliptic it appears to move from G to B a complete quarter circle. We also see the cause why the Sun passes through equal apparent arcs such as AG and GB in unequal times, to wit, longer from D to H , *i.e.* the apparent motion AG , than from H to E , *i.e.* the apparent motion GB . But through the two semi-circles AGB , BA of the apparent motion it moves just as long as through the two semi-circles DHE , ED of the proper motion. From this it is evident that the Sun, when it is at the apogee D , must be in the middle of its apparently slowest motion, and at the perigee E in the middle of its apparently fastest motion. But in order to seek this middle — which is the thing required in this proposition — on a sure basis, I will first give the following explanation of it. Let there be marked in the ecliptic the two points I and K , in such a way that the arc AI shall be equal to AK . Thereafter there shall be drawn from C to I in the ecliptic the line CI , intersecting the Sun's orbit in L ; likewise CK , intersecting the Sun's orbit in M . This being so, it is evident that just as the apparent arc AI on the one side is equal to the apparent arc AK on the other side, the arc of the Sun's orbit DL on the one side is also equal to the arc of the Sun's orbit DM on the other side. And consequently the two apparent arcs which are equally large

den van A evegroot sijn, gheven oock twee Sonwechbogen over beyde sijden vā D evegroot. Hier uyt volght dat als men inde ervarings dachtafels een schijnbaer plaets sulcx vint, dat haer twee loopen op even tijden over beyde sijden evegroot sijn, de selve plaets de Sonnens schijnbaer verstepunt moet wesen.

Het foucken van dien gaet aldus toe: Ick verkies eenighe maent van Iunius (daer in men den schijnbaerloop over al ten traechsten vindt) als neem ick Iunius int jaer 1554, ondersouck daer me het voornemen op eenighen dach die by raming ontrent het middel des traechsten loops der Son is, ick neem op dē 10, alwaer de Son bevonden wort onder den 88 tr. 2 ①: Maer binnē drie maenden daer te vooren (welcke drie maenden, of tijt waer me de Son ontrent een vierendeel haers wechs loopt, ick liever neem als ander, om de redenen die daer af int volgende Merck verclaert sullen worden) te weten den 10 Maerte, wasse onder den 359 tr. doende die booch 89 tr. 2 ①: En binnen drie maenden na den 10 Iunius, dats totten 10 September, alwaer de Son is onder den 176 tr. 39 ①, wort sulcken booch bevonden van 88 tr. 37 ①, welcke niet even sijnde mette 89 tr. 2 ①, soo en was onder den 88 tr. 2 ① het verstepunt niet.

Daerom dit alsoo ondersocht op een ander dach, ick neem op den 15 Iunius, alwaer de Son bevonden wort onder den 92 tr. 49 ①, ick vinde de booch op drie maenden daer te vooren van 88 tr. 53 ①, en op drie maenden daer na van 88 tr. 46 ①, welcke twee noch niet even sijnde, en de laetste booch cleender dan d'eerste, ghelijck inde voorgaende ondersoucking, tis teycken daer men noch voordr in Iunius moet comen: Later ondersocht worden op den 19 dach, alwaer ick de Son vinde onder dē 96 tr. 39 ①, en den booch op drie maenden daer te voor van 88 tr. 46 ①, maer op drie maendē daer na van 88 tr. 51 ①, welcke twee noch niet even sijnde, en de laetste nu grooter dan d'eerste, tis teycken dat ick te verre in Iunius gecommen ben, en deysen moet: Daerom ondersouck ick dergelijcke op dē 16, alwaer ick de Son bevinde onder dē 93 tr. 46 ①, en de booch op d'eerste drie maenden van 88 tr. 51 ①, maer op de laetste drie van 88 tr. 48 ①: Twelck noch niet effen commende, ick versouck dergelijcke op dē 17 van Iunius, en bevinde dan de Son indē 94 tr. 43 ①, en d'eerste booch op drie maenden van 88 tr. 48 ①, de laetste van 88 tr. 50 ①.

Sulcx dat ick tot hier toe bevonden heb t' verstepunt te moeten wesen tusschen den 93 tr. 46 ①, en den 94 tr. 43 ①. Maer om nu noch naerder te comen, ick sie dat de laetste booch op den 16 van Iunius te cleen bevondē wiert, en op den 17 te groot, t'welck teycken is, t'begheerde tusschen beyden te moeten schuylen, te weten op den 16 Iunius met eenighe uyren: Daerom dit ondersouckende eerst neem ick op den 16 Iunius met noch 12 uyren, daer na met meer, ick vinde mette 16 uyren de Son te wesen onder den 94 tr. 24 ①, en d'eerste booch op drie maenden van 88 tr. 49 ①, maer de laetste van 88 tr. 49 ① 20 ②, t'welck weynich verschilt, doch diet noch nauwer begeerde, mocht met gedeelten van uyren wercken. Dit soo sijnde, ick houde den boveschreven 94 tr. 24 ① voor de begheerde schijnbaer duyfteraerlangde des verstepunts: Waer toe vergaert 180 tr. comt voor begheerde schijnbaer duyfteraerlangde des naestepunts 274 tr. 24 ①. Merckt dat Copernicus dit verstepunt t' sijnder tijt stelde onder den 96 tr. 40 ①: Doch dit comt aldus na t'inhoudt deser dachtafels op die manier ghewrocht.

Maer beneffens de selve, soo isser noch een ander die tot proef van d'eerste verstrecken can, welcke ick oock verclaren sal als volght:

Hier vooren is geseyt de Son schijnbaerlick even soo lang te loopen int een half front A G B, als int ander B A, waer uyt volght, dat als men inde dachtafels

C twee

on either side of A also give two arcs of the Sun's orbit which are equally large on either side of D . From this it follows that if we find in the empirical ephemerides an apparent point such that the motions on both sides of it in equal times are equally large, this point must be the Sun's apparent apogee.

The seeking of this takes place as follows. I choose some month of June (where the apparent motion is always found to be slowest), for example June of the year 1554, investigate therewith the object in view on some day which is estimated to be near the middle of the slowest motion of the Sun, for example on the 10th, when the Sun is found at $88^{\circ}2'$. But three months previously (which three months, or time in which the Sun moves about one-fourth of its orbit, I take rather than any other, for the reasons that will be set forth about it in the succeeding Note), to wit 10th March, it was at 359° , that arc being $89^{\circ}2'$. And three months after 10th June, *i.e.* on 10th September, when the Sun is at $176^{\circ}39'$, this arc is found to be $88^{\circ}37'$; and this not being equal to $89^{\circ}2'$, the apogee was not at $88^{\circ}2'$.

Therefore, when this is examined on another day, for example, 15th June, when the Sun is found to be at $92^{\circ}49'$, I find the arc three months previously to be $88^{\circ}53'$ and three months afterwards $88^{\circ}46'$, and these two not yet being equal and the last arc smaller than the first, as in the preceding investigation, this is a sign that we have to be further still in June. Let it be examined on the 19th day, where I find the Sun at $96^{\circ}39'$, and the arc three months previously $88^{\circ}46'$, but three months afterwards, $88^{\circ}51'$; and these two still not being equal, and the last now larger than the first, this is a sign that I have come too far in June and have to go back. I therefore examine the same thing on the 16th, where I find the Sun at $93^{\circ}46'$, and the arc three months previously, $88^{\circ}51'$, but three months afterwards, $88^{\circ}48'$, and this still not being equal, I examine the same thing on 17th June, and then find the Sun at $94^{\circ}43'$, and the first arc three months previously $88^{\circ}48'$, and the last $88^{\circ}50'$.

Thus I have hitherto found that the apogee must be between $93^{\circ}46'$ and $94^{\circ}43'$. But in order to come nearer still, I see that the last arc on 16th June was found to be too small and on the 17th too large, which is a sign that the required point must be between the two, to wit on 16th June with a few hours added. Therefore, investigating this, I first take 16th June with 12 hours in addition, thereafter with more; I find that with 16 hours the Sun is at $94^{\circ}24'$, and the first arc three months previously $88^{\circ}49'$, but the last $88^{\circ}49'20''$, which is a slight difference, but who should wish it even closer might operate with parts of hours. This being so, I take the above-mentioned $94^{\circ}24'$ to be the required apparent ecliptical longitude of the apogee. When to this is added 180° , we get for the required apparent ecliptical longitude of the perigee $274^{\circ}24'$ (Note that *Copernicus* in his day put this apogee at $96^{\circ}40'$). But this results from the data of these ephemerides, made in the said manner.

But in addition to this there is still another manner that may serve as proof of it; which I will also set forth as follows:

It has been stated above that the Sun apparently moves as long in the one semi-circle AGB as in the other BA , from which it follows that when we have found in the ephemerides two apparent places of the Sun which are 180° apart,

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twee schijnbaer Sonplaetsen ghevonden heeft 180 tr. van malcander, sulcx dat de Son over elck halffront evevel tijts geloopt heeft, d'een dier twee plaetsen t'schijnbaer verstepunt d'ander t'schijnbaer naestepunt te moeten wesen. Om deur sulcke wech t' verstepunt te vinden, ick vergaer 180 tr. totten voorschreven 94 tr. 24 ① des schijnbaer verstepunts, comt gelijk bovē gefeyt is 274 tr. 24 ① voor schijnbaer naestepunt: Ick souck daer na inde dachtafels wanneer tot die plaets de schijnbaer Son was, en bevinde int selve jaer 1554 in December dē 16 dach 8 uyr 16 ①: By aldiē nu den tijt des Sonloops dier 180 tr. te weten vandē 16 Junius 16 uyr, tottē 16 December 8 uyr 16 ①, even waer ande Sonloop des ander halffronts: Of anders gefeyt dat den selven loop duerde den helft des jaers doende 182 daghen 14 uyren 58 ①, soo soude d'eerstghevonden plaets des verstepunts de begheerde wesen: Maer vanden boveschreven 16 Junius 16 uyr totten 16 December 8 uyr 16 ①, sijn 182 daghen 16 uyr 16 ①, t'welck alleene-lick 1 uyr 18 ① te veel sijnde, soo en ist niet verre van daer. Doch om t'begeerde naerder te commen, ick versouck dergelijcke tot een ander plaets, nemende t' verstepunt te sijn daer de Son schijnbaerlick was op den 18 Junius, te weten inden 95 tr. 41 ①, en volghende daer me de boveschreven manier van wercking, vinde over den loop des halffronts o uyr 25 ① te luttel, sulcx dat na dese wijze t' verstepunt moet wesen tusschen den 94 tr. 24 ①, en den 95 tr. 41 ①: Maer om de saeck noch naerder te commen, ick versouck dergelijcke tot een plaets tusschen de twee voorschreven, ick neem op den 17 dach 13 uyren van Junius, op welcken tijt ick inde dachtafels merck de Son geweest te hebben onder den 95 tr. 14 ①, en volghende daer me de boveschreven manier van wercking, vinde den loop des halffronts van o uyr 2 ① te cleen, t'welck als voor even meughende genomen worden, soo soude na dese wijze t' verstepunt sijn onder den 95 tr. 14 ①, dat nu d'ander bevonden wiert onder den 94 tr. 24, ① die alleene-lick verschillen 50 ①, wese in desen gevalle van cleenderacht, en con-nende spruyten uyt der dachtafels onvolcommenheyt.

VERVOLGH.

Ghelijck hier de twee boghen elck van drie maenden loops evegroot vallen, alsoo moeten om de voorgaende redenen (de dachtafels wel sijnde) alle ander foodanighe twee boghen op even tijden gheloopt oock evegroot sijn, als by voorbeelt op 1 maent doende 31 dagen vooren na den 17 Junius 1554, was de Son bycans eveverre vant verstepunt, te weten over d'een sijde 29 tr. 37 ①, over d'ander 29 tr. 39 ①. Ende op twee maenden van 61 daghen wasse over d'een sijde 58 tr. 31 ①, over d'ander 58 tr. 33 ①. Angaende dit verschilken van 2 ①, dat spruyt openbaerlick uyt onvolcommenheyt der tafels. Maer tot ander plaetsen can sulck verschil groot ghevonden worden: Als by voorbeelt op den 14 September 1554, was de Son onder den 180 tr. 36 ①, en op 3 maenden daer te vooren, hadse 3 tr. 5 ① schijnbaerlick min geloopt dan op 3 maenden daer na, want op den 14 Junius, te weten daer te vooren 3 maenden doende 92 daghen, wasse onder den 91 tr. 51 ①: Maer 92 daghen na den 14 September, te weten op den 15 December wasse onder den 273 tr. 2 ①: Deerste schijnbaer-loop doende 89 tr. 21 ①, is als vooren 3 tr. 5 ① cleender dan de tweede, doende 92 tr. 26 ①.

MERCKT.

Hier boven is gheseyt, dat ick verclaren soude de reden waerom het nemen des tijts waer me de Son een vietendeelronts van t' verstepunt comt, sekerder besluyt

in such a way that the Sun has passed through each semi-circle in the same time, one of these two places must be the apparent apogee, the other the apparent perigee. In order to find the apogee in this way, I add 180° to the above-mentioned $94^\circ 24'$ of the apparent apogee, which makes, as said above, $274^\circ 24'$ for the apparent perigee. I then look up in the ephemerides when the apparent Sun was in that place and find this to be in the said year 1554 in December, the 16th day, at 8h 16m. If the time of the Sun's motion through those 180° , to wit from 16th June at 16h to 16th December at 8h 16m, were equal to the Sun's motion of the other semi-circle; or in other words: if the said motion took half a year, making 182d 14h 58m, the place of the apogee first found would be the one required. But from the above-mentioned 16th June at 16h to 16th December at 8h 16m there are 182d 16h 16m, which, being only 1h 18m too much, is not far amiss. But in order to come nearer to the required value, I examine the same thing in another place, taking the apogee to be where the Sun was apparently on 18th June, to wit at $95^\circ 41'$, and carrying out therewith the above method of operation, I find the interval over the semi-circle to be 0h 25 m short, so that the apogee must be between $94^\circ 24'$ and $95^\circ 41'$. But in order to come nearer still to the matter, I examine the same thing in a place between the two mentioned above, for example, on 17th June at 13h, at which time I find in the ephemerides that the Sun was at $95^\circ 14'$, and carrying out therewith the above method of operation, I find the interval over the semi-circle to be 0h 2m short, and since this can be considered equal, in this way the apogee would be at $95^\circ 14'$, so that, since the other was found to be at $94^\circ 24'$, they differ only by $50'$, which in this case is insignificant and may be due to the imperfection of the ephemerides.

SEQUEL.

Just as here the two arcs, each of the motion of three months, are equal, for the above reasons (the ephemerides being correct) all other two such arcs passed through in equal times must also be equal; for example, one month — making 31 days — before and after 17th June 1554 the Sun was almost the same distance from the apogee, to wit, on the one side $29^\circ 37'$, on the other side $29^\circ 39'$. And at two months — 61 days — (before and after the said date) it was on the one side $58^\circ 31'$, on the other $58^\circ 33'$. As to this small difference of $2'$, this is evidently due to the imperfection of the ephemerides. But in other places this difference may be found to be large. For example, on 14th September 1554 the Sun was at $180^\circ 36'$, and 3 months before it had moved apparently $3^\circ 5'$ less than 3 months after, for on 14th June, to wit, 3 months — making 92 days — before, it was at $91^\circ 51'$ ¹⁾. But 92 days after 14th September, to wit, on 15th December, it was at $273^\circ 2'$. The first apparent motion, being $89^\circ 21'$, as before is $3^\circ 5'$ less than the second, being $92^\circ 26'$.

NOTE.

Above it has been said that I would set forth the reason why a more certain conclusion is gained by taking the time in which the Sun has reached a point

¹⁾ Instead of $91^\circ 51'$, a value of $91^\circ 15'$ has been used in the subsequent computations.

besluit gheeft als ander tijt waer me sy naerder of verder vant verstepunt is: Om nu daer toe te commen ick segh by voorbeelt aldus: Op den 12 Maerte 1555, was de Son onder den 0 tr. 45 ①, en drie maenden daer te vooren, te weten 90 dagen (tis wel waer dat d'alder meeste sekerheyt het recht vierendeel jaers soude sijn, doch t'voornemē sal hier me genouch connē verclaert wordē) hadse 3 tr. 7 ① schijnbaerlick meer gheloopen, dan op 3 maenden daer na. Dit groot verschil ons meer versekerende dan cleen dat den 180 tr. 36 ① wijt vant schijnbaer verstepunt is, wort bemerct deur de boveschreven neming des tijts waer me de Son een vierendeelronts loopt: Want soo wy, by voorbeelt geseyt, maer ghenomen en hadden 2 daghen voor en na den 14 September, wy souden den loop sulcker twee boghen evegroot bevinden, elck van 1 tr. 58 ①, sonder kenn is of wy ant verstepunt waren of niet. Maer dattet nemen van twee even tijden waer me de Son verder dan een vierendeelronts vant verstepunt comt, ooc achterlick sijn, wort daer deur verstaen, datmen daer me comt tot twee cleene boochkens niet wijt van t'naestepunt, waer af de reden de selve is als vant verstepunt.

Noch is te weten, dat al hebben wy hier boven gheseyt van t'nemen des tijts waer me de Son een vierendeelronts vant verstepunt is, daer by verstaemen sulcx niet alleenlick te wesen het vierendeel jaers als boven ghenomen wiert, maer alle tijden die de Son daer brengen, als een of meer heele jaren met noch een vierendeel daer toe: Oock drie vierendeelen jaers alleen, of met een of ettelicke heele jarē daer toe, op alle welcke de Son een vierendeelronts van t'verstepunt comt, uytghenomen soo veel als t'verstepunt daerentusschen mocht verlopen sijn, t'welck op weynich jaren van gheender acht en is.

Noch staet te ghedencken dat t'ghene hier in dit merck geseyt is de Sonloop angaende, dergheelijcke oock plaets te houden met d'ander Dwaelers daer int volghende afghehandelt sal worden, wantmen daer int soucken van haer verstepunten, om de boveschreven redenen oock de meeste sekerheyt heeft met t'nemen des tijts waer me den Dwaeler een vierendeelronts vant verstepunt comt.

T B E S L V V T. Wy hebben dan deur ervarings dachtafels ghevonden de schijnbaer duystraerlangde vande Sonwechs verstepunt en naestepunt, na den eyfch

5 V O O R S T E L.

Deur ervarings dachtafels den loop vande Sonwechs
★ verstepunt te vinden.

Apogee.

Eerst gesocht hebbende gelijk int 4 voorstel de schijnbaer duystraerlangde van des Sonwechs verstepunt, en die int jaer 1554 bevonden inden 94 tr. 24 ①, ick souck op de selve wijze waer die op eenich volghende jaer, ick neem 1594 gheweest heeft, en bevindese inden 97 tr. 53 ①; waer onder de Son bevonden wiert op den 20 Junius, want drie maenden daer te vooren wasse onder den 9 tr. en drie maenden daer na onder den 186 tr. 46 ①, wiens twee boghen even sijn doende elck 88 tr. 53 ①. Sulcx dat vant jaer 1554, tot 1594, makende 40 jaren, so is t'verstepunt op dien tijt verloopē volgende dese dachtafels en voor t'begeerde 3 tr. 29 ①, want so verre ist vanden 94 tr. 24 ① totten 97 tr. 53 ①. En hier me wort bekent des verstepunts loop op alle ghegheven tijt: Als

C 2 by voor.

a quarter circle away from the apogee than by taking another time in which it is nearer to or further away from the apogee. To arrive at this explanation, I say, for example, as follows: On 12th March 1555 the Sun was at $0^{\circ}45'$, and in three months before this, to wit, 90 days (it is true indeed that the greatest certainty would be the exact quarter of a year, but the intention can be sufficiently set forth in this way), it had apparently moved $3^{\circ}7'$ more than in three months after. That this large difference gives us greater certainty than the small one does, about this longitude of $180^{\circ}36'$ being far away from the apparent apogee, is perceived from the above-mentioned ascertaining of the time in which the Sun moves through a quarter circle. For if, for example, we had taken only 2 days before and after 14th September, we should find the motion of two such arcs to be equal, each $1^{\circ}58'$, without knowing whether we were at the apogee or not. But that the taking of two equal times in which the Sun reaches two points further than a quarter circle away from the apogee would also be less accurate, is understood from the fact that thus we get two small arcs not far away from the perigee, the cause of which is the same as with the apogee.

It is also to be noted that, though we have spoken above of the taking of the time in which the Sun has reached a point a quarter circle away from the apogee, it should be understood that this is not only the quarter of a year as was taken above, but all those times which bring the Sun there, *e.g.* one or more whole years with a quarter added. Also three quarters of a year alone, or with one or several whole years added, at all of which times the Sun reaches a point a quarter circle away from the apogee, except for the amount the apogee should meanwhile have moved, which is of no significance in a few years.

It is also to be borne in mind that what has been said in this note with regard to the Sun's motion also applies to the other Planets which will be dealt with hereafter, for there, in seeking their apogees, the greatest certainty is for the above-mentioned reasons also reached by taking the time in which the Planet has reached a point a quarter circle away from the apogee.

CONCLUSION. We have thus found, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of the Sun's orbit; as required.

5th PROPOSITION.

To find, by means of empirical ephemerides, the motion of the apogee of the Sun's orbit.

First having sought, as in the 4th proposition, the apparent ecliptical longitude of the apogee of the Sun's orbit, and having found it in the year 1554 at $94^{\circ}24'$, I look up in the same way where it was in some later year — for example, 1594 — and find it at $97^{\circ}53'$, at which the Sun was found to be on 20th June, for three months before it was at 9° and three months after at $186^{\circ}46'$, which two arcs are equal, since they are each $88^{\circ}53'$. Thus, according to these ephemerides, from the year 1554 to 1594, making 40 years, the apogee in that time (as required) has moved $3^{\circ}29'$, for that is the distance from $94^{\circ}24'$ to $97^{\circ}53'$. And in this way the motion of the apogee in any given time becomes known. For example, in

by voorbeelt om die te hebben op een jaer, ick seggh 40 jaren gheven 3 tr. 29 ①, wat een jaer? comt 5 ① 13 ②.

Merckt dat int berekenen deser dachtafels heeft moeten ghemist sijn, doch het voorbeelt can dienen om te betoonen de wijze hoemen met beter ervarings doen sal, maer om de saeck naerder te comen, men soude d'eerste ervaring meughen nemen over veel langher tijt geschiet, als die *Prolemeus* beschrijft int 4 hoofstuck sijns 3 boucx 463 jaren na t'overlijden vanden grootē *Alexander*, alwaer hy seggh des Sonwechs verstepunt ghevonden te hebben onder des duysteraers 65 tr. 30 ① (en hoe wel ick volghende sijn ghestelde vinde 65 tr. 30 ①, doch latet voorbeeltsche wijze sijn soo hy seggh) Voor d'ander ervaring ghenomen den boveschreven 97 tr. 53 ① ghebeurt op den 20 Junius 1594, t'welck was 1918 jaren na *Alexander*, soo heeft het verstepunt van d'een tijt tot d'ander bedraghende ontrent 1455 jaren, ghelooopen 32 tr. 23 ①, want soo verre ist vanden 65 tr. 30 ① totten 97 tr. 53 ①: En als men hier me wil vinden den loop eens ghegheven tijts, ick neem eens jaers, men seggh 1455 jaren gheven 32 tr. 23 ①, wat 1 jaer? comt 1 ① 20 ②: En dit laetste getal (in plaats vant eerste 5 ① 13 ②) sal ick ghebruycken in voorbeelden daer van desen loopgehandelt wort.

T B E S L V Y T. Wy hebben dan deur ervarings dachtafels den loop vande Sonwechs verstepunt ghevonden, na den eyfch.

6 V O O R S T E L.

Deur ervarings dachtafels de Sonnens loop in haer vvech te vinden.

Dē loop vande Sonwechs verstepunt wort op een jaer bevonden deur het 5 voorstel van

1 ① 20 ②.

Die ghetrocken vande middelloop der Son oock op een jaer, te weten een Egips doende deur het 3 voorstel

359 tr. 45 ①.

Blijft voor begheerde loop der Son in haer wech op een Egips jaer

359 tr. 43 ① 40 ②.

Ende is openbaer dat alsoo ghevonden sal worden den loop van alle voorgestelden tijt.

T B E W Y S. Angefien de Son op een jaer inden duysteraer schijnbaerlick een keer doet, en dat daerentusschen haer wech self 1 ① 20 ② gelooopen heeft, soo moetsie in haer wech die 1 ① 20 ② min ghelooopen hebben.

T B E S L V Y T. Wy hebben dan deur ervarings dachtafels de Sonnens loop in haer wech ghevonden, na den eyfch.

7 V O O R S T E L.

Deur ervarings dachtafels de Sonvvechs afvvijsking vanden ^{Aequatore.} * evenaer te vinden.

De Sonnens dagelicksche schijnbaer duysteraerbreden, en worden in dese dachtafels nevens haer schijnbaer duysteraerlangden niet beschreven, maer in sommige dachtafels deur gemeene reghel met een tafel gevonden: Doch by aldiensfer stonden, ghelijckie in ware ervarings dachtafels sijn (t'welckmen leerings halven sich mach inbeelden soo te welen) men soude die grootste breedden

order to have that in one year, I say: 40 years give $3^{\circ}29'$; what does one year give? It gives $5'13''$.

Note that in the calculation of these ephemerides errors must have been made, but yet the example may serve to show the way in which to proceed with better experiences. But in order to get nearer to the matter, we might take the first experience a much longer time ago, such as *Ptolemy* describes in the 4th chapter of his 3rd book, 463 years after the death of the great *Alexander*, where he says he has found the apogee of the Sun's orbit at $65^{\circ}30'$ of the ecliptic (and though, following his supposition, I find $65^{\circ}30'$, let it be as he says, by way of example 1)). When for the other experience the above-mentioned $97^{\circ}53'$ is taken, which happens on 20th June 1594, which was 1918 years after *Alexander*, the apogee has moved $32^{\circ}23'$ from one time to the other — amounting to about 1455 years — for this is the distance from $65^{\circ}30'$ to $97^{\circ}53'$. And if from this we wish to find the motion in a given time — for example, in one year — we say: 1455 years give $32^{\circ}23'$; what does 1 year give? It gives $1'20''$. And it is this latter value (instead of the first $5'13''$) which I shall use in examples in which this motion is dealt with.

CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the apogee of the Sun's orbit; as required.

6th PROPOSITION.

To find, by means of empirical ephemerides, the Sun's motion in its orbit.

The motion of the apogee of the Sun's orbit is found to be in one year, by the 5th proposition, $1'20''$

This being subtracted from the mean motion of the Sun, also in one year, to wit an Egyptian year, making by the 3rd proposition $359^{\circ}45'$

There remains, for the required motion of the Sun in its orbit in an Egyptian year $359^{\circ}43'40''$

And it is evident that thus the motion in any suggested time will be found.

PROOF. Since in one year the Sun apparently performs one revolution in the ecliptic, while its orbit itself has meanwhile moved $1'20''$, it must have moved those $1'20''$ less in its orbit.

CONCLUSION. We have thus found, by means of empirical ephemerides, the Sun's motion in its orbit; as required.

7th PROPOSITION.

To find, by means of empirical ephemerides, the deviation of the Sun's orbit from the equator.

The Sun's daily apparent equatorial latitudes²⁾ are not described in these ephemerides in addition to its apparent ecliptical longitudes, but in some ephemerides are found by a common rule with a table. But if they were there, as they are in true empirical ephemerides (which we may imagine to be the case for the sake of instruction), the greatest latitudes would be found twice a year,

¹⁾ The sense of the passage in parentheses is not clear; there must be a clerical error in it. The quotation refers to *Syntaxis* III, 4 (Manitius, I, p. 167).

²⁾ Stevin's *duysteraerbreden* (ecliptical latitudes) is an error for *evenaerbreden*.

den jaerlicx tweemacl vinden, d'eene ontrent den 12 van Junius na t'Noorden, d'ander ontrent den 12 van December na t'Zuyden, en dat in dese dachtafels volghende *Copernicus* stelling van 23 tr. 28 ①. En datmen hier op noch vorder lette, men soude mercken sulcx altijts te gebeuren wesende de Son schijnbaerlick 90 tr. vanden Lentine, waer uyt men beslyt de Sonwechs afwijcking te wesen vande selve 23 tr. 28 ①, om dat sulcke breetheyts booch ghetrocken op de lanckheys booch doende een vierendeelronds, voor grootheyt haers tegenoverhoucx verstreckt, als blijkt inden handel der clootische driehoucken.

T B E S L V Y T. Wy hebben dan deur ervarings dachtafels de Sonwechs afwijcking vanden evenaer ghevonden, na den eyfch.

8 V O O R S T E L.

Deur ervarings dachtafels te makē berekende * dach-^{Ephemerides.} tafels des Sonloops van toecommende tijden.

Als men in *Stadius* dachtafels (die wy om de boveschreven redenē hier houden al offe deur ervaringhen ghevonden waren) siet na t'ghelijck vervolgh der schijnbaer Sonplaetsen van d'een tijt by d'ander verlekē, om deur de Sonplaetsen des voorleden tijts, te oordeelen vande Sonplaetsen des toecommenden, men merckt datmen overal van vier tot vier jaren, de Son op ghelijcke daghen bycans onder een selve duysteraerlangde vint. Als by voorbeeld int jaer 1554 den 1 van Ianuarius was de Son onder den 290 tr. 36 ①. En vier jaer daer na, te weten den 1 Ianuarius 1558 onder den 290 tr. 37 ①. Voort inde jaren 1562, 1566, 1570, 1574, 1578, telcken op den 1 Ianuarius wiertse bevonden onder dē 290 tr. 37 ①, 290 tr. 36 ①, 290 tr. 37 ①, 290 tr. 37 ①, 290 tr. 38 ①, en alsoo met ander dierghelijcke.

Hier uyt machmen beslyuten, dat by aldien de dachtafels niet voordere gageslagen waren dan neem ick tottē 1 van Ianuarius 1578, en datmē na dien voorleden tijt wilde maken dachtafels van toecommenden tijt, mē soude de 4 volghende jaren meughen maken met sulcken voortganck, als de vier voorgaende, seggende de Son op den 2 van Ianuarius int Jaer 1574 te sullen sijn, tot sulcken schijnbaer plaets alsse was den 2 Ianuarius int jaer 1570, te weten onder den 291 tr. 38 ①: Maer op dē 3 Ianuarius onder den 292 tr. 40 ①, en so voort, welke dachtafelsche wijze opgeteyckent, men crijcht berekende dachtafels des Sonloops van toecommende tijden, na t'begheerde.

1 M E R C K.

By aldien de lancheyt des natuerlic jaers effen waer vā 365 dagē 6 uyren, gelijck hei Iuliaens jaer inhout, daer en soude gantschelick geen verandering vallen van 4 tot 4 jaren: Maer t'wiert in dese dachtafels deur het 2 voorstel bevonden alleenlick van 365 dagen 5 uyren 45 ① 55 ②, daerom gebreeft alle jare 14 ① 5 ② van een uyr, op welke de Sonloop bedraecht nagenouch 35 ②, die doen te vier jaren 2 ① 20 ②, en so veel soudemen na die rekening alle vier jarē tot yder dach der voorgaende moeten toedoen. Maer de langde des jaers genomē na *Ptolemeus* rekening op 365 dagen 14 ① 48 ②, waer an totte 356 $\frac{1}{4}$ dagē alleenlick gebreken 12 ② eens daechs, op welke de Sonloop bedraecht nagenouch oock 12 ②, die doen te vier jaren 48 ②, en so veel soudmē na *Ptolemeus* rekening des jaers, alle vier jaren tot yder dach der voorgaende moetē toedoē.

C 3 2 M E R C K

one about 12th June towards the North, the other about 12th December towards the South, and such in these ephemerides according to *Copernicus'* assumption of $23^{\circ}28'$. And if we observed this further, we should perceive that this always happens when the Sun is apparently 90° from the Vernal Equinox, from which we conclude that the deviation of the Sun's orbit is the said $23^{\circ}28'$, because the arc of the circle of latitude at 90° longitude represents the magnitude of its opposite angle, as appears from the work on spherical triangles.

CONCLUSION. We have thus found, by means of empirical ephemerides, the deviation of the Sun's orbit from the equator; as required.

8th PROPOSITION.

To make, by means of empirical ephemerides, calculated ephemerides of the Sun's motion in future times.

If in *Stadius'* ephemerides (which for the above-mentioned reasons we here use as if they had been found by experience) we look at the similar sequence of the apparent positions of the Sun of one time as compared with another, in order to judge from the positions of the Sun in the past what will be the positions of the Sun in the future, we perceive that every four years the Sun is found on similar days almost at the same ecliptical longitude. Thus, for example, in the year 1554, the 1st of January, the Sun was at $290^{\circ}36'$. And four years later, to wit, on 1st January 1558, it was at $290^{\circ}37'$. Further in the years 1562, 1566, 1570, 1574, 1578, each time on 1st January, it was found at $290^{\circ}37'$, $290^{\circ}36'$, $290^{\circ}37'$, $290^{\circ}37'$, $290^{\circ}38'$, and the same in other similar years.

From this it may be concluded that if the ephemerides did not contain observations beyond, for example, 1st January 1578, and if from this past time it was desired to make ephemerides of the future, the 4 following years could be made by the same procedure as the four preceding years, saying that the Sun would be on 2nd January of the year 1574 in the same apparent place as it was on 2nd January of the year 1570, to wit, at $291^{\circ}38'$. But on 3rd January at $292^{\circ}40'$, and so on; and when these values are recorded in the manner of ephemerides, we get calculated ephemerides of the Sun's motion in future times; as required.

1st NOTE.

If the length of the natural year were exactly 365d 6h, as the Julian year comprises, there would be no change at all every four years. But in these ephemerides it was found by the 2nd proposition to be only 365d 5h 45m 55s; therefore every year the hour is short of 14m 5s, in which the Sun's motion amounts to almost $35''$, which makes in four years $2'20''$, and this amount according to this calculation we should have to add every four years on every day to the preceding value (of the Sun's longitude). But if the length of the year were taken according to *Ptolemy's* calculation to be $365;14,48^1$, where in $365 \frac{1}{4}$ days a day is short of only $12 \times \frac{1}{3600}$ of a day, in which the Sun's motion also amounts to almost 12, in four years this makes 48, and this amount would according to *Ptolemy's* calculation of the year have to be added every four years to the longitudes of the preceding years.

¹⁾ Ptolemy here uses sexagesimals of a day: $0;0,12 = \frac{12}{3600} = 4 \text{ m } 48\text{s}$.

2. M E R C K.

Merckt wijder datmen hier fiet de oirfaeck van t'verloop des tijts, te weten waerom den 1 Maerte die nu int voorjaer comt, met lancheyt van tijt inde former foude vallen, daer na inden Herbst, en so voorts, ten waer dat somwijlen voorcommen wierde met afcorting van daghen ghelijck int jaer 1582 met 10 daghen ghedaen is, want volgende de rekening der lanckheyd des jaers van *Prolemus*, soo beeloper op de 300 jaren een dach, t'welck na de rekening des jaers van anderen meer bedraecht.

3. M E R C K.

Soo yemant de dachtafels van *Stadium*, voorder onderfocht op de voet als boven, hy foude faute bevinden: I act by voorbeeld hier ghenomen worden al de Sonplaetien op den 1 Ianuarius van 4 tot 4 jaren, als hier na volght:

1554.	290 tr.	36	①
1558.	290 tr.	37	①
1562.	290 tr.	37	①
1566.	290 tr.	36	①
1570.	290 tr.	37	①
1574.	290 tr.	37	①
1578.	290 tr.	38	①
1582.	290 tr.	39	①
1586.	290 tr.	39	①
1590.	290 tr.	40	①
1594.	290 tr.	40	①
1598.	291 tr.	31	①
1602.	291 tr.	32	①
1606.	291 tr.	7	①

Alwaermen een tamelick vervolgh fiet tot opt jaer 1594 (doch niet ghenouch vermeerderende) maer van daer voort opt jaer 1598, is by de 51 ① onbehoorlick verschil, en by de 25 ① vant jaer 1602 tot 1606. Doch ten is gheen feyl inde boveschreven ghemeenheit der reghel, maer openbaerlick deur misrekening of misdrukking.

T B E S L V Y T. Wy hebben dan deur ervarings dachtafels ghemaect berekende dachtafels des Sonloops van toecommende tijden, na den eysch.

Tot hier toe is beschreven mijn voorghenomen anvang vande kennis des Sonloops diemen deur ervarings dachtafels crijcht, op welke men als geleyde gront nu voorder met wisconftighe stof foude meughen handelen, maer ick sal eerst verclaren dergelijcken anvang deur ervarings dathtafels van d'ander Dwaelders, en daer na het 2 bouck beschrijven vande wisconftighen handel der Dwaelders int ghemeen, volgende t'voornemen verhaelt int Corrbegrijp des Hemelloops.

T W E E.

2nd NOTE.

Note further that we here see the cause of the shifting of time, to wit, why the 1st of March, which now comes in spring, would in the course of time fall in summer, thereafter in autumn, and so on, unless this were prevented sometimes by an omission of days, as was done in the year 1582 by 10 days, for according to the calculation of the length of the year by *Ptolemy* this runs into one day in 300 years, which, however, is more according to the calculation of the year by others.

3rd NOTE.

If anyone were to examine the ephemerides of *Stadius* further on the same basis as above, he would find errors. Let there, for example, be taken here all the Sun's positions on the 1st of January every four years, as follows below:

1554	290°36'	1582	290°39'
1558	290°37'	1586	290°39'
1562	290°37'	1590	290°40'
1566	290°36'	1594	290°40'
1570	290°37'	1598	291°31'
1574	290°37'	1602	291°32'
1578	290°38'	1606	291° 7'

In the above we see a fair sequence up to the year 1594 (but insufficiently increasing), but thence to the year 1598 there is an inadmissible difference of 51', and of 25' from the year 1602 to 1606. But this is not an error in the above-mentioned general rule, but is evidently due to miscalculation or misprints.

CONCLUSION. We have thus made, by means of empirical ephemerides, calculated ephemerides of the Sun's motion in future times; as required.

Up to this point have been described my intended beginnings of the knowledge of the Sun's motion which is acquired by means of empirical ephemerides, upon which as foundations we might now proceed further with mathematical material. But I will first set forth similar beginnings by means of empirical ephemerides of the other Planets, and thereafter describe the 2nd book, of the mathematical treatment of the Planets in general, according to the intention related in the Summary of the Heavenly Motions.

[The second chapter of the First Book, on the finding of the Moon's motion by means of empirical ephemerides, has not been reproduced]

D E R D E
O N D E R S C H E Y T
D E S E E R S T E N
 B O V C X, V A N D E V I N -
 ding van Saturnusloop deur
 ervarings dachtafels.

15 V O O R S T E L.

Te verclaren hoemen deur ervarings dachtafels uyt den rouvven merckt den tijt van Saturnus omloop: Mette ghedaente van sijn deysing en stilstant: Oock dat hy in een inront drayt.

Tvoornemen sijnde t'ondersoucken de ghedaente van Saturnusloop deur ervarings dachtafels (in wiens plaets wy de berekende van *Stadius* gebruycken, om de redenē verclaert int 1 voorstel) ick let op de derde pilaer hem angaende. Ghenomen dan dat my ten eersten voorvalt den 1 Ianuarius vant jaer 1554, op welcken dach ick hem vinde onder den 342 tr. 27 ①: Op een jaer daer na, te weten den 1 Ianuarius 1555, vinde hem gheweest te hebben onder den 353 tr. 53 ①, t'welck op dat jaer 11 tr. 26 ① ghevoordert is. En sghelijcx vinde ick hem den 1 Ianuarius opt volghende jaer 1556 ghevoordert te sijn noch 11 tr. 51 ①. En soo voortgaende tot opt jaer 1584, bevinde hem dan op den 1 Ianuarius weerom gecomē te wesen wat over de plaets daer hy int jaer 1554 begoft, te weten onder den 347 tr. 54 ①, hebbende over dien keer ghedaen by de dertich jaren. En sghelijcx ondersouckende tot ander plaetsen, bevinde hem overal ontrent de 30 jaren een keer te doen.

Daer na voorder acht nemende op de ghedaente sijns loops in yder jaer, ick bevinde hem d'tenmael te verrasschē, d'andermael te verslappen, ja somwijlen stil te staen, en dat noch meer is ettelickmael te deysen. En op de saeck nauwer lettende, men bevint het middel dier deysingen altijt te gebeuren wesende Saturnus ontrent teghestant der Son, en hoe hy de Son naerder comt, hoe hy met voortganck meer versnelt. Om hier afby voorbeelt te spreken, int jaer 1569 int begin van Iunius, siemen hem daghelix 1 ① loopē, daer na dagelick 2 ①, daer na noch meer, tottet begin vā September, loopende daer dagelick 8 ①, en daer ontrent ten snelstē geweest hebbende, begint int laetste desselven maēts weerom te vertragen dagelick meer en meer, ja sulcx dat hy eintlick vanden 18 tottē 24 Ianuarius 1570 stille staet onder den 202 tr. 20 ①. En daer na begint hy te deysen, t'welc eygentlick anvangende op dē 21 Ianuarius, duert totten 11 Iunius, vervanghende 141 dagen, diens helft wesende 70 dagen, soo valt het middel op

THIRD CHAPTER
OF THE FIRST BOOK
of the Finding of Saturn's Motion
by Means of Empirical Ephemerides

15th PROPOSITION.

To set forth how the time of Saturn's motion is roughly found by means of empirical ephemerides; with the nature of its retrogradation and standstill; also that it moves on an epicycle.

The intention being to investigate the nature of Saturn's motion by means of empirical ephemerides (instead of which we use the calculated ephemerides of *Stadius*, for the reasons set forth in the 1st proposition), I note the third column relating thereto. Let us therefore assume that I chance first upon 1st January of the year 1554, on which day I find it at $342^{\circ}27'$. One year later, to wit on 1st January 1555, I find it to have been at $353^{\circ}53'$, which is an advance of $11^{\circ}26'$ in that year. And in the same way I find it on 1st January of the following year 1556 to have advanced by $11^{\circ}51'$ more. And continuing like this up to the year 1584, I find that on 1st January it has come back to a little beyond the place where it started in the year 1554, to wit at $347^{\circ}54'$, having performed this revolution in about thirty years. And when I investigate this similarly in other places, I find that it always performs one revolution in about thirty years.

When thereafter I further attend to the nature of its motion in every year, I find it to move faster at one time, slower at another, nay sometimes to stop and, what is more, to retrograde several times. And when we observe the matter more closely, we find that the middle of these retrogradations always occurs when Saturn is nearly in opposition to the Sun, and the nearer it comes to the Sun, the faster becomes its motion. To give an example thereof: in the year 1569, at the beginning of June, we see it move 1' daily, thereafter 2' daily, thereafter even more, up to the beginning of September, when it moves 8' daily, and having moved fastest at about that time, it begins to move slower again in the latter part of this month, more and more with every day, even in such a way that at last from 18th to 24th January 1570 it stands still at $202^{\circ}20'$. And thereafter it begins to retrograde, which, starting in reality on 21st January, continues to 11th June, which makes 141 days, one half of which is 70 days; thus the middle falls on 1st April, and not long from that date, to wit, on 30th March, only 2 days before, Saturn and the Sun were in opposition, so that,

del op den 1 April, en niet seer verre van daer, te wetē den 30 Maerte, allene-
lick 2 dagen daer te voorē, waft tegestant van Saturnus en de Son. Sulcx dat ge-
lijck gheseyt is, het middel der deysing gebeurt alijt wefende Satūtnus ontrent
tegestant der Son. Maer hoe hy de Son naerder comt, hoe hy meer versnelt,
inder voughen dat de snelste loop alijt ghebeurt ontrent saming, als inde bove-
schreven snelste loop van September, daer was hy in saming (genomen datmen
hem deur ervaring hadde connen sien) op den 25 der selver maent. Men siet
oock dat vap yder teghestant tot teghestant, van saming tot saming, van begin
der deysing tot begin der deysing, is over al een jaer met ontrent noch een halve
maent: Als vande teghestant op den 17 Maerte 1569, totten eerstvolgenden te-
ghestant ghebeurende op den 30 Maerte 1570, is een jaer met ontrent een hal-
ve maent, op welcken tijt overal een deysing gheschiet, geduerende als vooren
ontrent de 140 daghen, te weten twee of drie daghen meer of min: Wiens ghe-
daente wy deur de dachtafels verclaren wilden.

Dit boveschreven gheeft vermoeden Saturnus in gheen wech te loopen als
de Son, waer uyt foodanighe deysing niet volghen en can, maer in een inront:
Sulcx dat hy ontrent teghestant der Son alijt is an des selven inronts naeste-
punt, alwaer hy om sijn loop int inront meer achterwaert gaende, dan hem den
loop vant middelpunt des inronts inden inrontwech voorwaert brengt, soo
wort daer uyt de deysing veroirsaect. Maer ontrent saming mette Son ant
verstepunt wefende, sal daer soo veel rasscher moeten loopen dan des inronts
middelpunt, als sijn voorwaert looping int inront veroirsaect.

Saturnus dan om sulcke oirsaken d'eenmael voorwaert d'ander achterwaert
loopende, daer volghet uyt datter by foodanighe verandering een tijt van stilstant
moet wesen: En weerom verkeert gheseyt, nadienmen dese voortlooping, stil-
stant, en deysing siet, soo merckten d'oirsaeck waerom datter een inront ge-
seyt wort: T B E S L V Y T. Wy hebben dan verclaert hoemen deur ervarings
dachtafels uyt den rouwen merckt den tijt van Saturnus omloop, mette ghe-
daente van sijn deysing en stilstant: Oock dat hy in een inront draeyt, na den
eych.

16 VOORSTEL.

Te verclaren hoemen deur ervarings dachtafels merckt
Saturnus inronts vvech uyt middelpuntich te vvesen.

By aldien Saturnus inronts wech middelpuntich waer, daer uyt soude vol-
ghen dat hy op alle twee even tijden, d'een voor ontrent sijn tegestant der Son,
d'ander daer na, evegroote schijnbaer bogen moest loopen, dat teghen d'erva-
ring strijt. Om t'welck deur een form te verclaren, laet het ront ABC Satur-
nus inronts wech beteyckenē waert meugelick middelpuntich sijnde, te weten
soo dat sijn middelpunt D des Eertcloots middelpunt is, daer na sy op A als
middelpunt beschreven het inront EF, diens verstepunt E, en naestpunt F,
voort op D als middelpunt den duyfteraer GH, en G sijn schijnbaer verstepunt
van E, Daer na sijn gheteyckent de twee punten I, K, inden inrontwech eve-
wijt van A, en de twee punten van L, M, int inront ewewijt van F, voort sy
van D deur L tot inden duyfteraer ghetrocken de lini D L N, sghelijcx van D
deur M tot inden duyfteraer de lini D M O. Dit soo wefende, ick sal hier me
t'voornemen verclaren.

E 4 Het

as has been said, the middle of the retrogradation always occurs when Saturn is nearly in opposition to the Sun. But the nearer it comes to the Sun, the faster it moves, in such a way that the fastest motion always occurs nearly at conjunction; thus, in the above-mentioned fastest motion of September, it was in conjunction (assuming that we could have seen it by observation) on the 25th of this month. We also see that from opposition to opposition, from conjunction to conjunction, from the beginning of one retrogradation to the beginning of the next retrogradation, it is always one year with about half a month in addition. Thus from the opposition on 17th March 1569 to the next opposition, occurring on 30th March 1570, it is one year and about half a month, in which time a retrogradation always occurs, which takes, as before, about 140 days, to wit, two or three days more or less; the nature of which we wished to set forth by means of the ephemerides.

The above makes us suspect that Saturn does not move in an orbit like that of the Sun, from which such retrogradation cannot follow, but on an epicycle, in such a way that when it is nearly in opposition to the Sun, it is always at the perigee of this epicycle; and because its motion on the epicycle is more backwards than the motion of the centre of the epicycle on the deferent takes it forward, the retrogradation is caused by this. But when it is at the apogee, nearly when in conjunction with the Sun, it will have to move so much faster there than the epicycle's centre as is caused by its progress on the epicycle.

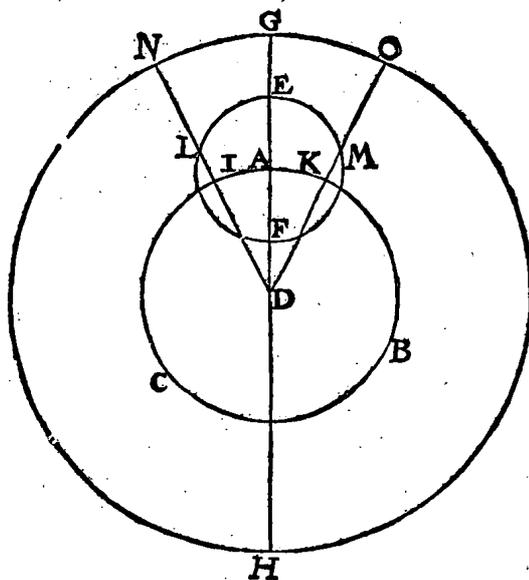
Since from these causes Saturn thus moves now forwards, now backwards, it follows therefrom that with such change there must be a time of standstill. And, *vice versa*, because we see this forward motion, standstill, and retrogradation, we perceive the reason why it is said that there is an epicycle. **CONCLUSION.** We have thus set forth how the time of Saturn's motion is roughly found by means of empirical ephemerides, with the nature of its retrogradation and standstill; also that it moves on an epicycle; as required.

16th PROPOSITION.

To set forth how it is found, by means of empirical ephemerides, that Saturn's deferent is eccentric.

If Saturn's deferent were concentric, it would follow therefrom that in any two equal times, one before it is nearly in opposition to the Sun and the other after this, it must move through equal apparent arcs, which is contrary to experience. In order to set this forth by means of a figure, let the circle *ABC* designate Saturn's deferent, being — if possible — concentric, to wit, so that its centre *D* is the centre of the Earth. Thereafter let there be described about *A* as centre the epicycle *EF*, whose apogee is *E* and perigee *F*; further about *D* as centre the ecliptic *GH*, and *G* the ¹⁾ apparent apogee of *E*. Thereafter let there be marked the two points *I, K*, on the deferent at equal distances from *A*, and the two points *L, M*, on the epicycle at equal distances from *F*. Further let there be drawn from *D* through *L*, up to the ecliptic, the line *DLN*; likewise from *D* through *M*, up to the ecliptic, the line *DMO*. This being so, I shall herewith set forth my intention.

¹⁾ For *syn* in the Dutch text read *het*.



Het inronts middelpunt eerst geweest hebbende an I, en alsdan Saturnus an M, en schijnbaerlick an O, soo sy het inronts middelpunt daer na gecommen van I tot A, en op den selven tijt Saturnus van M tot F ant naestepunt, alwaer hy dan om de voorgaende redenen sijn sal in teghestant der Son: Daer na sy gheleden een ander tijt even mette voorgaende, en sal daeren tusschen het inronts middelpunt moeten gecommen sijn van A tot K, een booch

even an A I, en Saturnus van F tot L, een booch even an F M, en N sal dan Saturnus schijnbaer plaats wesen, soo verre van G, als O van G, om de evenheyt der houcken G D O, G D N: Sulex dat ghelijck wy verclaren wilden, by aldien Saturnus inronts wech middelpuntich waer, als dese, daer uyt soude volgen dat hy op alle twee even tijden, d'een voor sijn teghestant der Son, d'ander daer na, evegrootte schijnbaer boghen moest loopen, Maer dat strijt teghen d'ervaring, daerom den inrontwech is voor uytmiddelpuntich te houden: Teghen d'ervaringh te strijden wort aldus betoont: Laet ons tot voorbeeld nemen eenighe teghestant als die gebeurt is int jaer 1569 den 17 Maerte, wesende Saturnus onder den 186 tr. 29 ☉: Maer op eenighen tijt daer te vooren, ick neem 7 jaren, te weten den 17 Maerte int jaer 1562, was hy onder den 88 tr. 14 ☉, tusschē welke een booch is van 98 tr. 15 ☉: Maer 7 jaren daer na, te weten op dē 17 Maerte 1576, was hy onder den 271 tr. 44 ☉, waer op den booch (te weten vanden 186 tr. 29 ☉ af) doet 85 tr. 15 ☉, die 13 tr. verschilt vande eerste booch 98 tr. 15 ☉. T B E S L V Y T. Wy hebben dan verclaert hoemē deur ervarings dachtafels merckt Saturnus inrontswech uytmiddelpuntich te wesen, na den eych.

17 VOORSTEL.

Deur ervarings dachtafels te vinden de schijnbaer
 * *Longitudinem zodiaci.* duysteraerlangde van Saturnus inrontyvechs verstepunt, en naestepunt.

Deur het 16 voorstel bemerckt sijnde dat den inrontwech middelpuntich is, daer reste doen voorder te soucken sijn verstepunts schijnbaer duysteraerlangde: Tor desen einde heefsimen verdocht, dat gebeurende een teghestant van Saturnus en de Son, als des inronts middelpunt is in sijn wechs verstepunt of naeste-

The epicycle's centre first having been in I , and then Saturn in M , and apparently in O , let the epicycle's centre thereafter have come from I to A , and in the same time Saturn from M to F at the perigee, where it will then for the above reasons be in opposition to the Sun. Thereafter let another time equal to the preceding have elapsed, then the epicycle's centre must meanwhile have come from A to K , an arc equal to AI , and Saturn from F to L , an arc equal to FM , and N must then be Saturn's apparent position, as far from G as O from G , because of the equality of the angles GDO , GDN ; in such a way that, as we wished to set forth, if Saturn's deferent were concentric, as this one, it would follow therefrom that in any two equal times, one before its opposition to the Sun and the other thereafter, it must move through equal apparent arcs. But this is contrary to experience, therefore the deferent is to be assumed to be eccentric. That it is contrary to experience is proved as follows: Let us take as example an opposition such as took place in the year 1569 on 17th March, when Saturn was at $186^{\circ}29'$. But some time before, for example 7 years, to wit, on 17th March in the year 1562, it was at $88^{\circ}14'$, between which there is an arc of $98^{\circ}15'$. But 7 years thereafter, to wit, on 17th March 1576, it was at $271^{\circ}44'$, so that the arc (to wit: from $186^{\circ}29'$) makes $85^{\circ}15'$, which differs 13° from the first arc of $98^{\circ}15'$.

CONCLUSION. We have thus set forth how it is found, by means of empirical ephemerides, that Saturn's deferent is eccentric; as required.

17th PROPOSITION.

To find, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of Saturn's deferent.

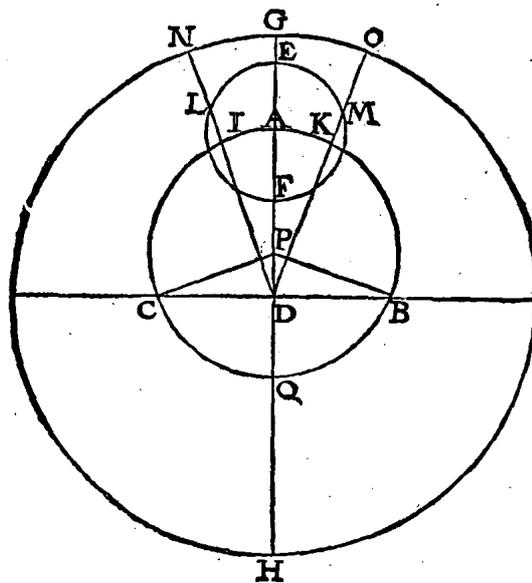
It having been found from the 16th proposition that the deferent is eccentric ¹⁾, it remained to find further the apparent ecliptical longitude of its apogee. To this end it was imagined that, Saturn and the Sun being in opposition, when the

¹⁾ For *middelpuntich* in the Dutch text read *uytmiddelpuntich*.

naestepunt, dat hy dan op alle twee even tijden, d'ecne voor sijn teghestant, d'ander daer na, evegroote schijnbaer boghen moet loopen: En weerom verkeert, als men inde ervarings dachtafels een teghestant sulcx vint, dat hy op alle twee even tijden, d'ecne voor die teghestant, d'ander daer na, evegroote bogen loopt, dat des inronts middelpunt in die teghestant an sijn wechs verstepunt of naestepunt moet wesen, waer me de schijnbaer duystraerlangde des selvē verstepunts of naestepunts bekend is.

Maer om van t'gene tot hier toe geseyt is breeder verclaring te doen, soolaet de volghende form ghelijck sijn met die des 15 voorfels, en de letters vande selve beteyckening, uytghenomen dat dese wech A B C uyt middelpuntich is, te weten dat des Eertcloots middelpunt D, nu niet en sy des inrontwechs middelpunt, maer P: En des inronts middelpunt A is an sijn wechs verstepunt, wesende Saturnus an des inronts naestepunt F in teghestant der Son. Dit soo sijnde, ick sie de twee schijnbaer loopen G N, G O, op even tijden noch evegroot te moeten wesen, om de evenheyt der twee houckē G D N, G D O, veroirsaect deur de evenheyt van F L met F M. Oock mede dat dergelijcke ghebeuren moet, wesende des inronts middelpunt met sulcke gedaente an sijn wechs naestepunt gheteyckent met Q.

Merckt noch dat hier vooren geseyt is, Saturnus altijt ant inronts naestepunt te wesen ontrent sijn teghestant der Son: Maer om hier nu eygentlicker te spre-



ken, soo is te wetē dat hy an sijn inronts naestepunt niet heel volcommelick en is dan in teghestant wesende des inronts middelpunt niet alleene-lic an sijn wechs naestepunt, maer boven dien noch de Son in haer wechs verstepunt of naestepunt, want nadiense inden duystraer schijnbaerlick oneventlic loopt, en Saturnus in sijn inront eenvaerdich, soo en can hy op alle teghestant der Son niet ant inronts naestepunt wesen, maer wel in alle teghestant der middelson, die deur de be-

paling eenparich loopt: En daerom sullen wy hier na om eyghentlicker te spreken, hem segghen altijt ant naestepunt of verstepunt te weten, in sijn teghestant of saming der middelson.

Dit soo sijnde, en om nu te commen tottet soucken der schijnbaer duystraerlangde van Saturnus inrontwechs verstepunt of naestepunt, t'is kennelick deur de voorgaende redenen, dat ick inde ervarings dachtafels een sijnder teghestanden der middelson soodanich moet vinden, dat hy op alle twee even tijden, d'een

epicycle's centre is at the apogee or perigee of its deferent, it must move through equal apparent arcs in any two equal times, the one before its opposition, the other thereafter. And conversely: if we find in the empirical ephemerides an opposition such that in any two equal times, the one before that opposition, the other thereafter, it moves through equal arcs, the epicycle's centre at that opposition must be at the apogee or perigee of its deferent, with which the apparent ecliptical longitude of this apogee or perigee is known.

But to set forth more fully what has been said up to this point, let the following figure be similar to that of the 16th¹⁾ proposition, the letters having the same meaning, except that here the deferent ABC is eccentric, to wit, that the centre of the Earth D shall not now be the deferent's centre, but P . And the epicycle's centre A is at the apogee of its deferent whilst Saturn is at the epicycle's perigee F in opposition to the Sun. This being so, I see that the two apparent motions GN , GO must still be equal in equal times, because of the equality of the two angles GDN , GDO , caused by the equality of FL and FM . Also that it must happen similarly when the epicycle's centre with similar figures is situated at the perigee of its deferent denoted by Q .

Note also that it has been said above that Saturn is always at the epicycle's perigee when it is nearly in opposition to the Sun. But to speak more truly, it is to be known that it is not perfectly at its epicycle's perigee except when not only the epicycle's centre is in opposition to the perigee of its deferent, but moreover the Sun is at the apogee or perigee of its orbit; for since in the ecliptic it moves apparently non-uniformly, and Saturn on its epicycle uniformly, it cannot at every opposition to the Sun be at the epicycle's perigee, but it can at every opposition to the Mean Sun, which by the definition moves uniformly. And therefore, to speak more truly, we shall hereinafter say that it is always at the perigee or apogee when it is in opposition to or conjunction with the Mean Sun.

This being so, and to come now to the finding of the apparent ecliptical longitude of the apogee or perigee of Saturn's deferent, it is evident for the above reasons that I must find in the empirical ephemerides one of its oppositions to the mean sun such that in two equal times, the one before and the other

¹⁾ For 15th in the original read 16th.

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d'een daer voor d'ander daer na, evegroote schijnbaer boghen loopt. Om die te foucken, ick slae het bouck open, en valt my ten eerste voor, neem ick, het jaer 1669, waer in ick sijn teghestant der Son bevinde geschiet te sijn op den 17 Maerte, wefende Saturnus onder den 186 tr. 29 ① (t'is waer datmen in plaets der ware Son soude behooren de middelson te nemen, volghende de voorgaende redenen, maer t'is tijts genouch daer me te wercken int laetste, als men siet datmen deur neming der ware Son na ghenouch by t'begheerde comt) Hier me versouck ick hoe de bovesch revē twee bogen op even tijden (tot welke tijden ick om de redenen vant merck des 4 voorstels, verkies 7 jaer, wefende byna het vierendeel eens keers duerende ontrent 30 jaren deur het 14 voorstel) d'een voor, d'ander na teghestant, met malcander overcommen, en bevinde hem op 7 jaren daer te vooren (dats vanden 17 Maerte 1562 onder den 88 tr. 14 ①, totten 17 Maerte 1569 onder den 186 tr. 29 ①) gheloopen te hebben 98 tr. 15 ①: Maer op 7 jaren daer na (dats vanden 17 Maerte 1569 onder den 186 tr. 29 ① totten 17 Maerte 1576 onder den 271 tr. 44 ①) vinde ick hem geloopt te hebbē alleenelick 85 tr. 15 ①, t'welck veel verschillende vande eerste booch 98 tr. 15 ①, te weten 13 tr. soo is t'schijnbaer verste of naestepunt wijt vande boveschreven 186 tr. 29 ①, oock van sijn teghepunt den 6 tr. 29 ①: Doch een van beyden en can ten hoochsten maer 90 tr. van daer sijn, daerom salmen derghelijcke tusschen sulcke bekende palen menighen onderfoucken op de teghestanden vande volghende of voorgaende jaren, voor of na het jaer 1569: En gecommen sijnde, neem ick, opt jaer 1572, sie aldaer teghestant te gheschien op den 23 April, wefende Saturnus onder den 222 tr. 51 ①, alwaer de rekening gemaectt als boven op 7 jaer voor en na, vinde eintlick noch verschil der twee boghen van 9 tr. 42 ①: Daerom onderfouck ick der ghelijcke tot een ander plaets, te weten meer voorwaert, om dat d'eerste booch te groot was, als by de teghestant dieder gebeurde int jaer 1576 op dē 10 Iunius onder dē 268 tr. 20 ①, alwaer de rekening gemaectt als boven, op 7 jaer voor en na, vinde eintlick genouchsaem evenheyt, te weten alleenelic verschil vā 4 ①, hier van cleēder acht.

Dit aldus soo na commende mette teghestant der eyghen Son, ick fouck nu waer op dien tijt de middelson was, om op haer teghestant nauwer rekening te maken: Neem tot dien einde de Sonwechs verstepunt na luyt des 4 voorstels te wesen onder den 94 tr. 24 ①, alwaer de schijnbaer Son en de middelson deur de 16 bepaling t'samen commende, en dat op den 16 Iunius, sy krijgen soo veel verschil als in 6 daghen loops vanden 10 totten 16 Iunius veroirsaectt wort, welck verschil ick aldus vinde: De schijnbaer Son heeft inde dachtafels op de 6 daghen gheloopen 5 tr. 44 ①, maer de middelson deur het 3 voorstel 5 tr. 55 ①, die alleenelick 11 ① verschillende, en hier van gheender acht sijnde, ick later by de voorgaende rekening blijven. Diet nauwer begheerde, soude dese 11 ① trecken vande Sonnens schijnbaer duysteraerlangde 88 tr. 20 ①, welke sy hadde op den 10 Iunius, t'ghene datter blijft, te weten 88 tr. 9 ①, is voor plaets der middelson, om daer op te berekenen Saturnus teghestant die wat vrougher valt als vande ware Son, en om voort de rekeninghen te maken als boven.

Van dese schijnbaer duysteraerlangde des verstepunts of naestepunts, mach voordere proufghedaen worden op ander even tijden dan 7 jaren: Als by voorbeelt, op 10 jaren voor en na den 10 Iunius 1576, vinde ick verschil alleenelick van 5 ①: En op 20 jaren alleenelick van 3 ①, waer uyt ick den boveschreven 268 tr. 20 ① houde voort t'begheerde.

Tot hier toe ghevonden sijnde onder den 268 tr. 20 ① het verste of naestepunt te wesen, daer rest noch te foucken wat elck van beyden is: Om daer toe te com-

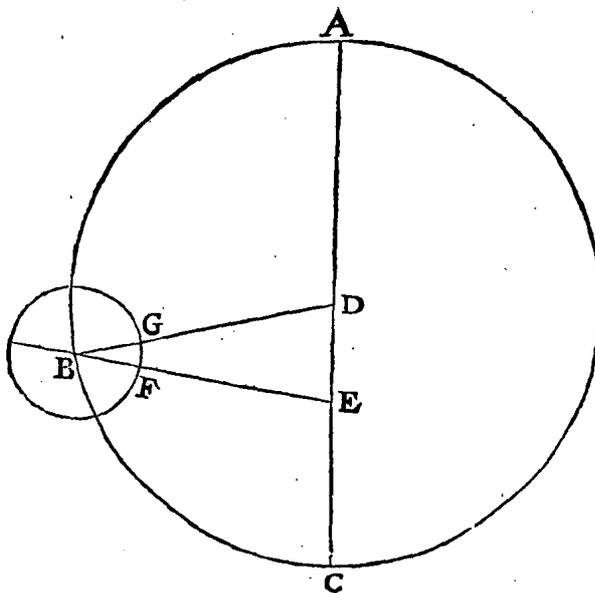
after it, it moves through equal apparent arcs. In order to find that, I open the book and I chance first, for example, upon the year 1569, where I find its opposition to the Sun to have occurred on 17th March, when Saturn was at $186^{\circ}29'$ (it is true that instead of the true Sun we ought to take the mean sun, for the above reasons, but it is time enough to operate therewith at the end, when we see that by taking the true Sun we come near enough to what is required). Herewith I examine how the above-mentioned two arcs correspond in equal times (for which times, for the reasons given in the Note to the 4th proposition, I choose 7 years, which is nearly the fourth part of a revolution taking about 30 years by the 14th proposition), the one before and the other after the opposition, and find that in 7 years previously (that is from 17th March 1562 at $88^{\circ}14'$ to 17th March 1569 at $186^{\circ}29'$) it has moved $98^{\circ}15'$. But in 7 years subsequently (that is from 17th March 1569 at $186^{\circ}29'$ to 17th March 1576 at $271^{\circ}44'$) I find it has moved only $85^{\circ}15'$; and since this differs much from the first arc of $98^{\circ}15'$, to wit: 13° , the apparent apogee or perigee is far away from the above-mentioned $186^{\circ}29'$, also from its opposite point, $6^{\circ}29'$. But only one of the two can at most be only 90° away therefrom; therefore the same thing can be examined between such known limits at the oppositions of the subsequent or preceding years, before or after the year 1569. And when I have come, for example, to the year 1572, I see that opposition then occurred on 23rd April, when Saturn was at $222^{\circ}51'$; if I there make the calculation as above, for 7 years before and after, I at last still find the difference between the two arcs to be $9^{\circ}42'$. Therefore I examine the same thing in another place, to wit, further forward, because the first arc was too large, for example, at the opposition that occurred in the year 1576, on 10th June, at $268^{\circ}20'$; and when the calculation is made there as above, for 7 years before and after, I at last find sufficient equality, to wit, a difference of only $4'$, which is insignificant here.

This coming thus so near the opposition to the true Sun, I now find where the Mean Sun was at that time, in order to make a more accurate calculation on its opposition. To this end I take the apogee of the Sun's orbit, according to the 4th proposition, to be at $94^{\circ}24'$; and since the apparent Sun and the mean sun coincide there by the 16th definition, such on 16th June, they will differ as much as is caused by 6 days' motion, from 10th to 16th June, which difference I find as follows: In the ephemerides the apparent Sun in 6 days has moved $5^{\circ}44'$, but the Mean Sun by the 3rd proposition $5^{\circ}55'$, and since these values differ only $11'$, which is insignificant here, I leave it at the preceding calculation. Who should require it more accurately, would have to subtract these $11'$ from the Sun's apparent ecliptical longitude of $88^{\circ}20'$, which it had on 10th June; what remains, namely $88^{\circ}9'$, is the position of the Mean Sun, from which we may calculate Saturn's opposition, which falls slightly earlier than that of the true Sun, and further make the calculations as above.

With regard to this apparent ecliptical longitude of the apogee or perigee, further attempts may be made in equal times other than 7 years. Thus, for example, ten years before and after 10th June 1576 I find a difference of only $5'$, and in 20 years of only $3'$; from which I assume the above-mentioned $268^{\circ}20'$ to be the required value.

It having been found thus far that the apogee or perigee is at $268^{\circ}20'$, it remains to be found which of these has this value. To arrive at this, for the sake of a clearer exposition thereof let ABC be Saturn's deferent, its centre D , eccentric

commen, laet tot opentlicker verclaring van diē A B C Saturnus inrontswech
sijn, diens middelpunt D, uymiddelpuntichpunt, of den Eertcloot E, verfte-



punt A, nae-
stepunt C, en
op B als mid-
delpunt sy be-
schreven het
inront F G,
diens naeste-
punt F, en
middelnac-
stepunt G.
Dit soo we-
sende, t'lijct
dat deur dien
A het verfte-
punt is, soo
moet dē mid-
delloop A D
B int eerste
halfront A B
Coveral gro-
ter sijn dan
dē schijnba-

ren A E B. En deur t'verkeerde van dien is openbaer, dat als des inronts mid-
delpuntsloop of Saturnus middelloop, cp een gestelden tijt int eerste halfiont
grooter bevonden wort dan de schijnbaer, soo moet punt van daer de telling
begint het verstepunt wesen, maer t'verkeerde ghebeurende, als dan het naeste-
punt. Dit soo sijnde, ick souck inde dachtafels eenighe tegestant verre genouch
van A of C, dats verre ghenouch vanden 268 tr. 20 ① of haer tegenoverpunt,
en comt my te vooren, neem ick, de tegestant gebeurt int jaer 1584 den 15 Sep-
tember, wesende Saturnus onder den 2 tr. 44 ①, op welcke sijn inronts mid-
delpunts schijnbaer loop vanden 268 tr. 20 ① af, doet 94 tr. 24 ① voor den
houck A E B: Maer desselven inronts middelpunts eyghenloop op dien tijt, te
weten vandē 10 Junius 1576 tot desen 15 September 1584, geduerende 8 Egip-
sche jaren 99 daghen, doet deur het volghende 17 voorstel 101 tr. 6 ① voor den
houck A D B, welke grooter sijnde dan A E B 94 tr. 24 ①, soo moet om de
voorgaende redenen A dats onder des duystraers 268 tr. 20 ① het verstepunt
sijn, en C onder des duystraers 88 tr. 20 ① het naestepunt.

Angaende ymant nu mocht segghen dat de rechte lini die ten tijde der tege-
stant strecke vanden Eertcloot E deur Saturnus F, niet nootsakelick des inronts
middelpunt B en ontmoet, om bekende oirsaken, te weten de uymiddelpun-
tigheden vanden inronts wech en Sonwech, over sulcx twijffelende of t'bove-
schreven besluyt vast gaet: Hier op wort geantwoort, datmen daer af volcom-
men sekerheyt can hebben in deser voughen: T'ghetal des inrontboochs tus-
schen Saturnus en t'middelnacstepunt, even bevondē sijnde mette naestepun-
tensbooch dieder ten tijde der teghestant behoort te wesen, soo moet Saturnus
dan openbaerlick ant naestepunt wesen.

Om t'welck t'ondersoucken, ick treck den boveschreven houck

A D B 101 tr. 6 ① van 180 tr. blijft den houck E D B

78 tr. 54 ①.

En den

point, or the Earth E , apogee A , perigee C , and about B as centre let there be described the epicycle FG , its perigee F and mean perigee G . This being so, it appears that because A is the apogee, the mean motion ADB in the first semi-circle ABC must always be larger than the apparent motion AEB . And conversely it is evident that if the motion of the epicycle's centre or Saturn's mean motion is found to be greater than the apparent motion in a given time in the first semi-circle, the point from which counting starts must be the apogee, but if the converse happens, then the perigee. This being so, I seek in the ephemerides some opposition far enough away from A or C , *i.e.* far enough away from $268^{\circ}20'$ or its opposite point, and I chance, for example, upon the opposition that occurred in the year 1584, on 15th September, when Saturn was at $2^{\circ}44'$, where the apparent motion of its epicycle's centre from $268^{\circ}20'$ makes $94^{\circ}24'$ for the angle AEB . But the proper motion of this epicycle's centre in that time, to wit, from 10th June 1576 to this 15th September 1584, taking 8 Egyptian years and 99 days, by the following 17th proposition makes $101^{\circ}6'$ for the angle ADB , and because this is greater than AEB ($94^{\circ}24'$), for the above reasons A , *i.e.* at $268^{\circ}20'$ of the ecliptic, must be the apogee, and C , at $88^{\circ}20'$ of the ecliptic, the perigee.

If anyone now should say that the straight line which at the time of the opposition passed from the Earth E through Saturn F does not necessarily meet the epicycle's centre B , from known causes, to wit, the eccentricities of the deferent and the Sun's orbit, doubting whether the above-mentioned conclusion is quite certain, to this it is replied that we can have perfect certainty about this in the following manner: Since the amount of the epicycle's arc between Saturn and the mean perigee has been found equal to the perigee's arc that ought to be there at the time of opposition, Saturn must evidently be at the perigee.

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En den houck AEB dats oock DEB doet als vooren 94 tr. 24 ①.

Dese twee houcken EDB, DEB, des driehoucx EDB, bedra-
ghen t'samen 173 tr. 18 ①, welcke ghetrocken van 180 tr.
blijft voor den houck DBE, t'welck oock is voor de naeste-
puntensbooch FG 6 tr. 42 ①.

Nu segh ick dat by aldien de inrontsbooch vant middelnaeste-
punt G tot Saturnus, ten tijde deser teghestant oock bevon-
den wierde van 6 tr. 42 ①, dat aldan Saturnus an t'naeste-
punt F, en vervolghens inde rechte lini EB soude moeten ge-
weest hebben, maer sulcke booch wiert bevonden van 5 tr.
27. ①, want Saturnus heeft op de boveschreven 8 Egipsche ja-
ren 99 daghen gheloopt int inront deur het volghende 18
voorstel 354 tr. 33 ①, welcke ghetrocken van 360 tr. blijft als
boven 5 tr. 27 ①.

Die vande 6 tr. 42 ① alleenelick verschillen 1 tr. 15 ①.

Die naerder overeencoming begheert, soude dergheelijcke wercking meugen
ondersoucken met een tegestant voor of na de boveschreven tot dat hye von-
de. Als by voorbeelt, sulcx ghedaen mette tegestant int jaer 1583 den 3 Sep-
tember, vinde daer me te luttel 1 tr. 13 ①. Maer mette tegestant int jaer 1579
dē 15 Iulius, viel DEB van 33 tr. 55 ①, BDE 142 tr. 9 ①, en EBD 4 tr. 4 ①,
welcke drie t'samen maken 180 tr. 8 ①, dat allcenelick 8 ① te veel is, en voor
ghenouchsaem overcomming mach ghehouden worden. Inder voughen dat
A het begheerde verstepunt blijft als boven onder den 268 tr. 20 ①, na ghe-
nouch overcommende metten 268 tr. van *Rheindus in Prutenicis tabulis*.

Belanghende dat de volcommenheyt soude vereyschen met teghestant der
Middelson te wercken, sulcx schijnt in dese anvangsche wijse der leering on-
noodich, te meer dat wy tot so nauwe ondersoucking veel langdueriger dach-
tafels souden moeten hebben dan dese. T B E S L V Y T. Wy hebben dan deur
ervarings dachtafels gevonden de schijnbaer duystraerlangde van Saturnus in-
rontwechs verstepunt en naestepunt, na den eyfch.

18 VOORSTEL.

Deur ervarings dachtafels Saturnus middelloop op
een ghegheven tijt te vinden, en daeraf een tafel te be-
schrijven.

Het vinden des loops vant inronts middelpunt deur de ervarings dachtafels
gaet aldus toe: Men siet dat Saturnus ontrent de 30 jaren een schijnbaer keer
des duystraers doet, daerom souck men in welck jaer en dach ontrent 30 jaren
(of ontrent ettelicke 30 jaer als de dachtafels lang genouch duyre) voor of nadē
boveschrevē 10 Junius 1576, dat ontrent dē 10 Junius Saturnus onder dē 268 tr.
20 ① (wesende deur het 16 voorstel des inrontwechs verstepunt) in tegestant
der Son is, en den keer of keeren aldan gheschiet, sijn na ghenouch volcom-
men, om vande uymiddelpunticheden des inrontwechs en Sonwechs gheen
hinderlicke dwaling te krijghen: Voort is den tijt van d'een tegestant tot d'an-
der bekent, waer deur mē oock vindt den loop eens dachs, en tafelen maect, die
gelijck inde Sonloop geseft is daer na op langer en langer tijden meer en meer
verbetert worden. Maer om van t'ghene tot hier toe int ghemeen gheseyt is,
nu by

To investigate this, I subtract the above-mentioned angle ADB of $101^{\circ}6'$ from 180° ; there remains the angle EDB of $78^{\circ}54'$

And the angle AEB , *i.e.* also DEB , as before makes $94^{\circ}24'$

These two angles EDB , DEB of the triangle EDB are together $173^{\circ}18'$, and when this is subtracted from 180° , there remains for the angle DBE , which is also the value of the perigees' arc FG , $6^{\circ}42'$

Now I say that if the epicycle's arc from the mean perigee G to Saturn at the time of this opposition were also found to be $6^{\circ}42'$, Saturn must then have been at the perigee F , and consequently on the straight line EB ; but this arc was found to be $5^{\circ}27'$, for Saturn has moved, in the above-mentioned 8 Egyptian years and 99 days, by the following 18th proposition, $354^{\circ}33'$ on the epicycle, and if this is subtracted from 360° , there remains, as above, $5^{\circ}27'$
which differ from $6^{\circ}42'$ only $1^{\circ}15'$

Who should wish for closer agreement, might make a similar investigation with an opposition before or after the one described above until he found it. Thus, for example, when I do so with the opposition in the year 1583, on 3rd September, I find therewith $1^{\circ}13'$ short. But with the opposition in the year 1579, on 15th July, DEB was $33^{\circ}55'$, BDE $142^{\circ}9'$, and EBD $4^{\circ}4'$, which three together make $180^{\circ}8'$, which is only $8'$ too much and may be considered sufficient agreement; so that A remains the required apogee, as above, at $268^{\circ}20'$, which agrees substantially with the 268° of *Rheindus* in *Prutenicis tabulis* ¹⁾.

As to the fact that perfection would require us to operate with opposition to the Mean Sun, this seems unnecessary in this elementary instruction, the more so because for such accurate investigations we should need ephemerides covering much longer periods than the present. CONCLUSION. We have thus found, by means of empirical ephemerides, the apparent ecliptical longitude of the apogee and perigee of Saturn's deferent; as required.

18th PROPOSITION.

To find, by means of empirical ephemerides, Saturn's mean motion in a given time, and to describe a table thereof.

The finding of the motion of the epicycle's centre, by means of the empirical ephemerides, takes place as follows. We see that Saturn performs in approximately 30 years one apparent revolution in the ecliptic; therefore we seek what year and day approximately 30 years (or several times 30 years, if the ephemerides cover a long enough period) before or after the above-mentioned 10th June 1576 Saturn is about 10th June at $268^{\circ}20'$ (which by the 16th proposition is the deferent's apogee) in opposition to the Sun, and the revolution or revolutions that have then taken place are sufficiently complete not to get any disturbing error due to the eccentricities of the deferent and the Sun's orbit. Further the time from one opposition to the other is known, by means of which we also find the motion of one day and make tables which, as has been said in the Sun's motion, are thereafter improved more and more in longer and longer times. But now to give an example of what has so far been said in general, I seek in the ephemerides

¹⁾ See note p. 45.

nu by voorbeeld te spreken, ick soucke inde dachtafels ontrent 30 jaren (of ontrent ettelicke 30 jaren als de dachtafels langhe ghenouchduyren) voor of na de boveschreven teghestant op den 10 Junius 1576, een teghestant wesende Saturnus ten naesten by den 268 tr. 20 ①, wort bevonden op den 4 Junius 1605 onder den 263 tr. 35 ①, t'welck van d'ander 4 tr. 45 ① verschilt, maer op soo cleenen boochsken en connen de uytmiddelpunticheden gheen hinderlick verschil by brenghen, immers op een voorbeeldsche leering als dese.

Dit soo wesende, ick sie dat Saturnus, en vervolghens sijn inronts middelpunt vanden 10 Junius 1576, totten 4 Junius 1605, doende 10586 daghen, geloopen heeft 355 tr. 15 ①, te weten vanden 268 tr. 20 ①, totten 263 tr. 35 ①: Daerom siegh ick, 10586 daghen, gheven 355 tr. 15 ①, wat 1 dach? Comt 2 ① 0 ② 49 ③: T'welck luttel schilt van *Copernicus* ghestelde middelloop doende 2 ① 3 ② 36 ③, daer *Ptolemeus* voor nam by de 2 ① 0 ② 34 ③. En waer de boveschreven teghestandt (die 3 tr. 18 ① van rechte teghestandt schilt) naerder rechte teghestandt gheweest (naerder soudemense connen vinden in dachtafels van langher jaren ghelijckmen in den Wijsentijt had) dese loop eens dachs mocht oock naerder t'ghestelde van *Copernicus* ghevallen hebben.

Nu dan de loop eens dach wesende als boven, t'is kennelick hoemen daer me soude connen maken, ghelijck vande Sonloop int 3 voorstel ghedaen is, tafels om daer deur met lichticheyt te vinden Saturnus inronts middelpunts loop, anders gheseyt Saturnus middelloop, op alle ghegheven tijt inde *dact *Praxi* ghemeenelick te voeren commende.

Maer anghesien wy *Ptolemeus* tafels voorbeeldsche wijze ghebruycken sullen, om de redenen van diergelijcke inde Sonloop en Maenloop verclaert, so sal ickse stellen als volght:

F TAFEL

about 30 years (or about several times 30 years, if the ephemerides cover a long enough period) before or after the above-mentioned opposition on 10th June 1576 an opposition with Saturn being nearest to $268^{\circ}20'$; this is found to be on 4th June 1605, at $263^{\circ}35'$, which differs $4^{\circ}45'$ from the other, but on such a small arc the eccentricities cannot cause any disturbing difference, especially in instruction by means of an example, such as the present.

This being so, I see that Saturn, and consequently its epicycle's centre, from 10th June 1576 to 4th June 1605, which makes 10,586 days, has moved $355^{\circ}15'$, to wit, from $268^{\circ}20'$ to $263^{\circ}35'$. I therefore say: 10,586 days give $355^{\circ}15'$; what does 1 day give? This gives 2;0,49 minutes, which differs little from the mean motion given by *Copernicus*, which is 2;3,36 minutes, for which *Ptolemy* took about 2;0,34 minutes. And if the above-said opposition (which differs $3^{\circ}18'$ from exact opposition) had been nearer to exact opposition (they could be found nearer in ephemerides covering more years, such as people had in the Age of the Sages), this motion of one day might also have been nearer the value given by *Copernicus*.

The motion of one day thus being as above, it is evident how one might make tables therewith, as has been done for the Sun's motion in the 3rd proposition, by means of which to find easily the motion of Saturn's epicycle's centre, in other words Saturn's mean motion, as it usually appears in practice in any given time.

But since we shall use *Ptolemy's* tables by way of example, for the reasons set forth for a similar matter in the Sun's motion and the Moon's motion, I will give them as follows.

TAFEL VAN SATVRNVS

uyren	tr.	①	②	③	④	⑤	⑥	Dagē	tr.	①	②	③	④	⑤	⑥
1	0	0	5	1	23	48	42	10	0	20	5	35	14	48	30
2	0	0	10	2	46	37	24	11	0	22	6	8	46	17	21
3	0	0	15	4	11	26	6	12	0	24	6	42	17	46	12
4	0	0	20	5	35	14	48	13	0	26	7	15	49	15	3
5	0	0	25	6	59	3	31	14	0	28	7	49	20	43	54
6	0	0	30	8	22	52	13	15	0	30	8	22	52	12	45
7	0	0	35	9	46	40	55	16	0	32	8	56	23	41	36
8	0	0	40	11	10	29	37	17	0	34	9	29	55	10	27
9	0	0	45	12	34	18	19	18	0	36	10	3	26	39	18
10	0	0	50	13	58	7	1	19	0	38	10	36	58	8	9
11	0	0	55	15	21	55	43	20	0	40	11	10	29	37	0
12	0	1	0	16	45	44	25	21	0	42	11	44	1	5	51
13	0	1	5	18	9	33	8	22	0	44	12	17	32	34	42
14	0	1	10	19	33	21	50	23	0	46	12	51	4	3	33
15	0	1	15	20	57	10	32	24	0	48	13	24	35	32	24
16	0	1	20	22	20	59	14	25	0	50	13	58	7	1	15
17	0	1	25	23	44	47	55	26	0	52	14	31	38	30	6
18	0	1	30	25	8	36	38	27	0	54	15	5	9	58	57
19	0	1	35	26	32	25	20	28	0	56	15	38	41	27	48
20	0	1	40	27	56	14	2	29	0	58	16	12	12	56	39
21	0	1	45	29	20	2	43	30	1	0	16	45	44	25	30
22	0	1	50	30	43	51	27	60	2	0	33	31	28	51	0
23	0	1	55	32	7	40	9	90	3	0	50	17	13	16	30
Dagē								120	4	1	7	2	57	42	0
1	0	2	0	33	31	28	51	150	5	1	23	48	42	7	30
2	0	4	1	7	2	57	42	180	6	1	40	34	26	33	0
3	0	6	1	40	34	26	33	210	7	1	57	20	10	58	30
4	0	8	2	14	5	55	24	240	8	2	14	5	55	24	0
5	0	10	2	47	37	24	15	270	9	2	30	51	39	49	30
6	0	12	3	21	8	53	6	300	10	2	47	37	24	15	0
7	0	14	3	54	40	21	57	330	11	3	4	23	8	40	30
8	0	16	4	28	11	50	48	360	12	3	21	8	53	6	0
9	0	18	5	1	43	19	39								

MIDDELLOOP.

63

laten.	tr.	①	②	③	④	⑤	⑥
1	12	13	23	56	30	30	15
2	24	26	47	53	1	0	30
3	36	40	11	49	31	30	45
4	48	53	55	46	2	1	0
5	61	6	59	42	32	31	15
6	73	20	23	39	3	1	30
7	85	33	47	35	33	31	45
8	97	47	11	32	4	2	0
9	110	0	35	28	34	32	15
10	122	13	59	25	5	2	30
11	134	27	23	21	35	32	45
12	146	40	47	18	6	3	0
13	158	54	11	14	36	33	15
14	171	7	35	11	7	3	30
15	183	20	59	7	37	33	45
16	195	34	23	4	8	4	0
17	207	47	47	0	38	34	15
18	220	1	10	57	9	4	30
36	80	2	21	54	18	9	0
54	300	3	52	51	27	13	30
72	160	4	43	48	36	18	0
90	20	5	54	45	45	22	30
108	240	7	5	42	54	27	0
126	100	8	16	40	3	31	30
144	320	9	27	37	12	36	0
162	180	10	38	34	21	40	30
180	40	11	49	31	30	45	0
198	260	13	0	28	39	49	30
216	120	14	11	25	48	54	0
234	340	15	22	22	57	58	30
252	200	16	33	20	7	3	0

laten.	tr.	①	②	③	④	⑤	⑥
270	60	17	44	17	16	7	30
288	280	18	55	14	25	12	0
306	140	20	6	11	34	16	30
324	0	21	17	8	43	21	0
342	220	22	28	5	52	25	30
360	80	23	39	3	1	30	0
378	300	24	50	0	10	34	30
396	160	26	0	57	19	39	0
414	20	27	11	54	28	43	30
432	240	28	22	51	37	48	0
450	100	29	33	8	46	52	30
468	320	30	44	45	55	57	0
486	180	31	55	43	5	1	30
504	40	33	6	40	14	6	0
522	260	34	17	37	23	10	30
540	120	35	28	34	32	15	0
558	340	36	39	31	41	19	30
576	200	37	50	28	50	24	0
594	60	39	1	25	59	28	30
612	280	40	12	23	8	33	0
630	40	41	23	20	17	37	30
648	0	42	34	17	26	42	0
666	220	43	45	14	35	46	30
684	80	44	56	11	44	51	0
702	300	46	7	8	53	55	30
720	160	47	18	6	3	0	0
738	20	48	29	3	12	4	30
756	240	49	40	0	21	9	0
774	100	50	50	57	30	13	30
792	320	52	1	54	39	18	0
810	180	53	12	51	48	22	30

F 2 T B E,

64 SATVRNVSLOOPS VINDING DEVR

T B E S L V Y T. Wy hebbē dan deur ervarings dachtafels Saturnus middelloop op een ghegheven tijt ghevonden, en daer afeen tafel beschreven, na den eysch.

19 V O O R S T E L.

Deur ervarings dachtafels Saturnus loop in sijn inront op een ghegheven tijt te vinden, en daer afeen tafel te beschrijven.

T G H E G H E V E N. Laet te vinden sijn Saturnus loop in sijn inront op een dach.

M E R C K T.

Nadien ick t'werck defes voorstels beschreven hadde na de ghemeene wijze, treckende Saturnus middelloop vande Sonnens middelloop, ghelijck hier onder blijcken sal, so heeft sijn V O R S T E L I C K E G H E N A D E tot grontlicker kennis der oirsaken (soo wel van d'ander volghende Dwaelders, alwaer der ghelijcke sal ghedaen worden, als van dese) daer by noch begeert en vervought een werck op Saturnus eyghen gront ghebout, als volght.

1 W E R C K.

Hy heeft hier toe inde dachtafels gesocht twee van Saturnus tegestandē met de Son, lange tijt van malcander gheschiet, byna en onder een selve plaets des duyfteraers, want daer me moeten soo wel sijn keeren int inront als de Sonkeeren volcommen wesen, sonder van eenighe uytmiddelpunticheden hinder te krijghen, en oock onnoodich sijnde rekening te maken mette Middelfon, om bekende reden. D'eerste van sulcke twee teghestanden nam hy dieder viel op den 10 Junius 1576, d'ander den 4 Junius 1605; tusschen welcke sijn by de 29 jaren, doende 10586 daghen, waer op Saturnus int inront soo veel keeren ghedaen heeft alser teghestanden gebeurt sijn, welcke bevonden wierden 28, elcke van 360 tr. maken 10080 tr. Hier me segghende, 10586 dagen geven 10080 tr. wat 1 dach? Comt voor t'begheerde 57 ① 7. 55. weynich verschillende van Copernicus en Ptolemus besluyt int volghende.

2 W E R C K.

Vande Sonnens middelloop eens dachs, doende deur het 3 voorstel	0 tr. 59. 8. 17. 13. 12. 31.
Getrocken Saturnus middelloop eens dachs, doende deur het 17 voorstel	0 tr. 2. 0. 33. 31. 28. 51.
Blijft voor begheerden Saturnusloop in sijn inront op een dach	0 tr. 57. 7. 43. 41. 43. 40.

T B E W Y S.

Anghesien Saturnus altijt ant inronts naestepunt is in sijn teghestant der Middelfon deur t'voorgaende, soo moet nootfakelick dien inrontsloop even sijn ande middelloop der Son, min soo veel als bedraecht des inronts middelpuntsloop.

Om

CONCLUSION. We have thus found, by means of empirical ephemerides, Saturn's mean motion in a given time, and described a table thereof; as required.

19th PROPOSITION.

To find, by means of empirical ephemerides, Saturn's motion on its epicycle in a given time, and to describe a table thereof.

WHAT IS REQUIRED ¹⁾. Let Saturn's motion on its epicycle in one day have to be found.

NOTE.

After I had described the procedure of this proposition in the usual manner, subtracting Saturn's mean motion from the Sun's mean motion, as will appear below, for a more thorough knowledge of the causes (both of the other — following — Planets, where a similar operation will be carried out, and of this one) His PRINCELY GRACE required in addition and added a method based on data from Saturn's tables alone, as follows.

1st METHOD.

To this end he sought in the ephemerides two of Saturn's oppositions to the Sun, which had occurred a long time apart, nearly at the same place of the ecliptic, for therewith its revolutions on the epicycle as well as the Sun's revolutions must be complete, without being disturbed by any eccentricities, and it also being unnecessary to take account of the Mean Sun, for known reasons. For the first of these two oppositions he took the one that fell on 10th June 1576, for the other 4th June 1605, between which there are nearly 29 years, making 10,586 days, in which Saturn has performed so many revolutions on the epicycle as there have been oppositions, which were found to be 28, each of 360°, making 10,080°. Saying now: 10,586 days give 10,080°, what does 1 day give? The value required is 57; 7, 55 minutes. which differs little from *Copernicus'* and *Ptolemy's* results in the following.

2nd METHOD.

When from the Sun's mean motion in one day, making	
by the 3rd proposition	0°; 59, 8, 17, 13, 12, 31
there be subtracted Saturn's mean motion in one day,	
making by the 18th proposition ²⁾	0°; 2, 0, 33, 31, 28, 51
there remains for Saturn's required motion on its epicycle	
in one day	0°; 57, 7, 43, 41, 43, 40

PROOF.

Since Saturn by the above is always at the epicycle's perigee in its opposition to the Mean Sun, this epicyclic motion must necessarily be equal to the mean motion of the Sun, *minus* the amount of the motion of the epicycle's centre.

¹⁾ Evident mistake in the Dutch text. Instead of *Tghegheven* read *Tbegheerde*.

²⁾ In the Dutch text, read 18 instead of 17.

ERVARINGS D A C H T A F E L S. 65

Om hier af met breeder reden te verclaren, ick segh aldus: Soo by voorbeelt het inronts middelpunt gheen loop en hadde, maer vast bleve, en dat Saturnus in saming dan ewewel altijd in sijn inronts verstepunt waer, t'is kennelick dat Saturnusloop in sijn inront dan even soude moeten sijn ande Sonnens middelloop: Maer het inronts middelpunt heeft daerentusschen een loop gedaen, daerom soo veel die is, soo veel moet Saturnus inrontloop cörter dueren dan de middelloop der Son, en vervolghens treckende Saturnus inronts middelpunts loop, vande Sonnens middelloop, de rest is de begheerde loop van Saturnus in sijn inront.

V E R V O L G H.

De loop eens dachs wesende als boven, t'is kennelick hoemen daer me sal maken, ghelijck vande Sonloop int 3 voorstel ghedaen is, tafelen als de navolghende, om daer me met lichticheyt Saturnus loop in sijn inront te vinden op alle ghegheven tijt inde * daer ghemeenelick te vooren commende. *Pract.*

Merckt noch datmen tot prouf der voorschreven werckinghen mach sien soo dickwils alsment oirboir verstaet, of het laest ghevonden ghetal even is mettet ghetal daer me overcommende: Als by voorbeelt van t'ghetal des Sonloops eens jaers, ghetrocken t'ghetal van Saturnus inrontloop eens jaers, of die rest overcomt met foodanich ghetal anders ghevonden deur ervaring.

F 3 TAFEL

To set this forth more fully, I say as follows: If, for example, the epicycle's centre did not move, but remained fixed, and Saturn in conjunction were yet always in its epicycle's apogee, it is evident that Saturn's motion on its epicycle would then have to be equal to the Sun's mean motion. But the epicycle's centre has meanwhile moved on; therefore, as much as that is, by so much must the motion of Saturn's epicycle be less than the mean motion of the Sun¹⁾, and consequently, when the motion of Saturn's epicycle's centre is subtracted from the Sun's mean motion, the remainder is the required motion of Saturn on its epicycle.

SEQUEL.

The motion in one day being as above, it is evident how tables such as the following will be made therewith — as was done for the Sun's motion in the 3rd proposition — by means of which to find easily Saturn's motion on its epicycle as it usually appears in practice in any given time.

Note also that, to test the aforesaid methods, we may ascertain as often as we deem suitable whether the value last found is equal to the value corresponding thereto. Thus, for example, when from the value of the Sun's motion in one year there is subtracted the value of Saturn's motion on its epicycle in one year, whether the remainder corresponds to a similar value found in another manner, by experience.

¹⁾ The words *corter dueren* in the Dutch text should be *corter sijn*, which means just the reverse.

TAFEL VAN SATVRNVS LOOP

uren	tr.	①	②	③	④	⑤	⑥	Dagē	tr.	①	②	③	④	⑤	⑥
1	0	2	22	49	19	14	19	10	9	31	17	16	57	16	40
2	0	4	45	38	28	28	38	11	10	28	25	0	39	0	20
3	0	7	8	27	57	42	57	12	11	25	32	44	20	44	0
4	0	9	31	17	16	57	17	13	12	22	40	28	2	27	40
5	0	11	54	6	36	11	36	14	13	19	48	11	44	11	20
6	0	14	16	55	55	25	55	15	14	16	55	55	25	55	0
7	0	16	39	45	14	40	14	16	15	14	3	39	7	38	40
8	0	19	2	34	33	54	33	17	16	11	11	22	49	22	20
9	0	21	25	23	53	8	52	18	17	8	19	6	31	6	0
10	0	23	48	13	12	23	12	19	18	5	26	50	12	49	40
11	0	26	11	2	31	37	31	20	19	2	34	33	54	33	20
12	0	28	33	51	50	51	50	21	19	59	42	17	36	17	0
13	0	30	56	41	10	6	9	22	20	56	50	1	18	0	40
14	0	33	19	30	29	20	28	23	21	53	57	44	59	44	20
15	0	35	42	19	48	34	47	24	22	51	5	28	41	28	0
16	0	38	5	9	7	49	7	25	23	48	13	12	23	11	40
17	0	40	27	58	27	3	26	26	24	45	20	56	4	55	20
18	0	42	50	47	46	17	45	27	25	42	28	39	46	39	0
19	0	45	13	37	5	32	4	28	26	39	36	23	28	22	40
20	0	47	36	26	24	46	23	29	27	36	44	7	10	6	20
21	0	49	59	15	44	0	42	30	28	33	51	50	51	50	0
22	0	52	22	5	3	15	2	60	57	7	43	41	43	40	0
23	0	54	44	54	22	29	21	90	85	41	35	32	35	30	0
Dagē								120	114	15	27	23	27	20	0
1	0	57	7	43	41	43	40	150	142	49	19	14	19	10	0
2	1	54	15	27	23	27	20	180	171	23	11	5	11	0	0
3	2	51	23	11	5	11	0	210	199	57	2	56	2	50	0
4	3	48	30	54	46	54	40	240	228	30	54	46	54	40	0
5	4	45	38	38	28	38	20	270	257	4	46	37	46	30	0
6	5	42	46	22	10	22	0	300	285	38	38	28	38	20	0
7	6	39	54	5	52	5	40	330	314	12	30	19	30	10	0
8	7	37	1	49	33	49	20	360	342	46	22	10	22	0	0
9	8	34	9	33	15	33	0								

INT INRONT.

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laten.	II.	①	②	③	④	⑤	⑥	laten.	II.	①	②	③	④	⑤	⑥
1	347	32	0	48	50	38	20	270	234	3	39	47	52	30	0
2	335	4	1	37	41	16	40	288	9	39	54	27	4	0	0
3	322	36	2	26	31	55	0	306	145	16	9	6	15	30	0
4	310	8	3	15	22	33	20	324	280	52	23	45	27	0	0
5	297	40	4	4	13	11	40	342	56	28	38	24	38	30	0
6	285	12	4	53	3	50	0	360	192	4	53	3	50	0	0
7	272	44	5	41	54	28	20	378	327	41	7	43	1	30	0
8	260	16	6	30	45	6	40	396	103	17	22	22	13	0	0
9	247	48	7	19	35	45	0	414	238	53	37	1	24	30	0
10	235	20	8	8	26	23	20	432	14	29	51	40	36	0	0
11	222	52	8	57	17	1	40	450	150	6	6	19	47	30	0
12	210	24	9	46	7	40	0	468	285	42	20	58	59	0	0
13	197	56	10	34	58	18	20	486	61	18	35	38	10	30	0
14	185	28	11	23	48	56	40	504	196	54	50	17	22	0	0
15	173	0	12	12	39	35	0	522	332	31	4	56	33	30	0
16	160	32	13	1	30	13	20	540	108	7	19	35	45	0	0
17	148	4	13	50	20	51	40	558	243	43	34	14	56	30	0
18	135	36	14	39	11	30	0	576	19	19	48	54	8	0	0
36	271	12	29	18	23	0	0	594	154	56	3	33	19	30	0
54	46	48	43	57	34	30	0	612	290	32	18	12	31	0	0
72	182	24	58	36	46	0	0	630	66	8	3	51	42	30	0
90	318	1	13	15	57	30	0	648	201	44	4	30	54	0	0
108	93	37	27	55	9	0	0	666	337	21	2	10	5	30	0
126	229	13	42	34	20	30	0	684	112	57	16	49	17	0	0
144	4	49	57	13	32	0	0	702	248	33	31	28	28	30	0
162	140	26	31	52	43	30	0	720	24	9	46	7	40	0	0
180	276	2	26	31	55	0	0	738	159	46	0	46	51	30	0
198	51	38	41	11	6	30	0	756	295	22	15	26	3	0	0
216	187	14	55	50	18	0	0	774	70	58	30	5	14	30	0
234	323	51	10	92	29	30	0	792	206	34	44	44	26	0	0
252	98	27	25	8	41	0	0	810	342	10	59	23	37	30	0

F 4 T B E

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T B E S L V Y T. Wy hebben dan deur ervarings dachtafels gevonden Saturnus loop in sijn inront op een gheghen tijt, en daeraf een tafel beschreven, na den eyfch.

M E R C K T.

By aldien dese dachtafels van *Stadius* soo lang duerden, en soo veel Saturnus keeren hadden, datmen daer in mocht vinden twee sijnder tegheftanden mette Son onder een selve duyfteraerlangde, wy souden dan hier meughen een voorstel beschrijven, gelijk inde Sonloop en Maenloop gedaen is, te weten: Deur ervarings dachtafels te maken berekende dachtafels van Saturnus loop op toecommende tijden: Maer angesien hy daer in alleenelick een volcommen keer ghedaen heeft, soo en wil sulcke vinding hier niet vallen: Doch hoemen deur wilconftige rekeningen vindt van hoe langen tijt de dachtafels souden moeten beschreven sijn om sulcken selfheyte te krijgen, dat sal elders verclaert worden.

20 V O O R S T E L.

Deur ervarings dachtafels den loop van Saturnus inront vvechs verstepunt te vinden.

Angesien de dachtafels van *Stadius* (die wy nemen al offe deur ervarings beuonden waren, om de redenen verclaert int 1 voorstel) te cort sijn om daer deur bequamelick t'begheerde te vinden, soo sal ick tot hulp nemen de ervarings van *Ptolemeus*, welke int 5 hoofstlick sijns 11 boucx Saturnus inront wechs verstepunt bevondē heeft onder des duyfteraers 233 tr. maer int 16 voorstel bevintment onder den 268 tr. 20 ①, T'welck 35 tr. 20 ① voorder sijnde, het verstepunt is na dese rekening soo veel verlopen op dien tijt, bedraghende ontrent 1450 jaren, te weten vant jaer 127 tottet jaer deser ervaring 1576. En hier me wort bekend des verstepunts loop op alle ghegheven tijt: Als by voorbeelt, om die te hebben op een jaer, ick segh 1450 jaren gheven 35 tr. 20 ①, wat 1 jaer: Comt 1 ① 28 ②. T B E S L V Y T. Wy hebben dan deur ervarings dachtafels gevonden den loop van Saturnus inront wechs verstepunt, na den eyfch.

21 V O O R S T E L.

Deur ervarings dachtafels Saturnus inronts middelpunts loop in sijn vvech te vinden.

Den loop van Saturnus inrontwechs verstepunt wort op een jaer beuonden deur het 18 voorstel van 1 ① 28 ②.
 Die ghetrocken van Saturnus middelloop oock op een jaer, te weten een Egips doende deur het 17 voorstel 12 tr. 13 ①.
 Blijft voor begheerde loop des inronts middelpunts in sijn wech op een Egips jaer 12 tr. 12 ①.
 En is openbaer dat alsoo ghevonden sal worden den loop van alle voorghefelde tijt: Waeraf t'bewijs deur t'werck openbaer is. T B E S L V Y T. Wy hebben dan deur ervarings dachtafels Saturnus inronts middelpunts loop in sijn wech ghevonden, na den eyfch.

VIERDE

CONCLUSION. We have thus found, by means of empirical ephemerides, Saturn's motion on its epicycle in a given time, and described a table thereof; as required.

NOTE.

If these ephemerides of *Stadius* covered so long a time and contained so many revolutions of Saturn that we might find therein two of its oppositions to the Sun at identical ecliptical longitude, we could here describe a proposition as has been done in the Sun's motion and the Moon's motion, to wit: To make, by means of empirical ephemerides, calculated ephemerides of Saturn's motion in future times. But since it has performed only one complete revolution therein, such finding cannot here take place. But it will be set forth elsewhere how it is found by mathematical calculations what length of time the ephemerides would have to cover in order to obtain such identity.

20th PROPOSITION.

To find, by means of empirical ephemerides, the motion of the apogee of Saturn's deferent.

Since the ephemerides of *Stadius* (which we use as if they had been found by experience, for the reasons set forth in the 1st proposition) cover too short a period to find therewith easily what is required, I will have recourse to the experiences of *Ptolemy*, who in the 5th chapter of his 11th book has found the apogee of Saturn's deferent at 233° of the ecliptic; but in the 16th proposition we find it at $268^\circ 20'$. This being $35^\circ 20'$ ahead, according to this calculation the apogee has moved this amount in that time, being about 1,450 years, to wit, from the year 127 to the year of the present observation, 1576. And in this way the motion of the apogee in any given time becomes known. Thus, for example, to have it in one year, I say: 1,450 years give $35^\circ 20'$, what does one year give? This gives $1'28''$. CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the apogee of Saturn's deferent; as required.

21st PROPOSITION.

To find, by means of empirical ephemerides, the motion of the centre of Saturn's epicycle on its deferent.

The motion of the apogee of Saturn's deferent is found, by the 18th proposition, to be in one year 1'28''

This being subtracted from Saturn's mean motion in one year,
to wit, an Egyptian year, making by the 17th proposition 12°13'
there remains for the required motion of the epicycle's centre on
its deferent in an Egyptian year 12°12'

And it is evident that thus the motion in any suggested time can be found, the proof of which is evident from the procedure. CONCLUSION. We have thus found, by means of empirical ephemerides, the motion of the centre of Saturn's epicycle on its deferent; as required.

[*Chapters 4 up to and including 7, dealing with Jupiter, Mars, Venus, and Mercury, have not been reproduced here*]

A C H T S T E
O N D E R S C H E Y T
D E S E E R S T E N
 B O V C X , V A N D E V I N -
 ding des loops der
 vaste Sterren.

50 V O O R S T E L .

Te verclaren hoemen deur ervarings dachtafels merct de vaste Sterren in haer hemel vast te vvesen, en den hemel te drayen op den as des duyfteraers.

Anghesien dit roersel soo traech is, datmen daer afweynich bescheyts can mercken op den tijt deser dachtafels van *Stadius* geduerende 52 jaren, soo sullen wy hier en int volgende 51 voorstel tot hulp gebruycken *Ptolemeus* nagelaten schriften vande plaetsen der vaste sterren, nemende voorbeeltische wijse al of die mer dese t' samen ons ervarings dachtafels maeckten.

Dit soo wesende, angesien de vaste sterren nu in *Stadius* dachtafels met sulcken verheyte van malcander ghevonden worden als ten tijde van *Ptolemeus*, na luyt des 5 hoofsticx sijns 7 boucx, en dat die doen in rechte linien stonden, nu daer in noch sijn, niet teghenstaende t'selve over de 1400 jaren gheleden is, soo besluytmen daer uyt haer vasticheyt.

Maer want haer duyfteraerbreeden de selve blijven, en haer evenaerbreeden met duyfteraerlangden veranderen, soo besluytmen den heelen hemel te moeten draeyen op den as des duyfteraers. Als by voorbeelt de * are des Maechts diens Zuydersche duyfteraerbreede *Ptolemeus* stelt op 2 tr. 10 ①, wort alsoo oock in *Stadius* dachtafels beschreven: Maer de duyfteraerlangde die *Ptolemeus* stelde op 176 tr. 40 ①, is by *Stadius* van 197 tr. 38 ①: En haer evenaerbreede welcke *Ptolemeus* int 3 hoofsticx sijns 7 boucx stelt op 30 ① na het Zuyden, die comt, volghende *Stadius* beschrijving 8 tr. 56 ① Zuytwaert, want hoewelle in sijn dachtafels niet en staet, soo volght sulcx uyt de boveschreven duyfteraerlangde en breede deur het 9 werckstuck der Hemelclootsche werckstucken: En want dit roersel gheschiet na t' vervolgh der trappen, soo moetet sijn van Westen na Oosten. T B E S L V Y T. Wy hebben dan verclaert hoemen deur ervarings dachtafels merckt de vaste Sterren in haer hemel vast te wesen, en den hemel te drayen op den as des duyfteraers, na den eyfch.

51 V O O R -

EIGHTH CHAPTER

OF THE FIRST BOOK,

Of the Finding of the Motion of the Fixed Stars¹⁾

50th PROPOSITION.

To explain how it is found by means of empirical ephemerides that the fixed Stars are fixed in their heaven and that this heaven rotates on the axis of the ecliptic.

Since this motion is so slow that little information can be obtained about it in the time of these ephemerides of *Stadius*, which cover 52 years, here and in the subsequent 51st proposition we will use as aids the writings left by *Ptolemy* on the positions of the fixed stars, assuming by way of example that those together with these constitute empirical ephemerides to us.

This being so, since now in *Stadius'* ephemerides the fixed stars are found to be at the same distances from one another as at the time of *Ptolemy*, according to the 5th chapter of his 7th book, and that those which then were in straight lines now still are, notwithstanding the fact that this was more than 1400 years ago, it is concluded from this that they are fixed.

But because their ecliptical latitudes remain the same and their equatorial latitudes vary with the ecliptical longitudes, it is concluded that the whole heaven must rotate on the axis of the ecliptic. Thus, for example, *Spica Virginis*, whose Southerly ecliptical latitude *Ptolemy*²⁾ puts at $2^{\circ}10'$, is also described thus in *Stadius'* ephemerides. But the ecliptical longitude, which *Ptolemy* put at $176^{\circ}40'$, is $197^{\circ}38'$ in *Stadius*. And its equatorial latitude, which *Ptolemy* in the 3rd chapter of his 7th book puts at $30'$ to the South, according to *Stadius'* description comes at $8^{\circ}56'$ to the South, for though it does not occur in his ephemerides, this follows from the above-mentioned ecliptical longitude and latitude by the 9th problem of the problems on Heavenly Spheres. And because this motion takes place in the order of the degrees, it must be from West to East.

CONCLUSION. We have thus explained how it is found by means of empirical ephemerides that the fixed Stars are fixed in their heaven and that this heaven rotates on the axis of the ecliptic; as required.

¹⁾ By a printing error in the original text the preceding page 114 has been called 104.

²⁾ *Syntaxis* VII, 5 (*Manitius* II, p. 48).

116 VASTE STERRENS LOOPS VINDING.

51 VOORSTEL.

Deur ervarings dachtafels te vinden den eyghen loop der vaste Sterren.

Anghesien de Arc des Maechts ten tijde van *Ptolemeus* int jaer 139 was in des duystraers 176 tr. 40 ①, maer ten tijde van *Stadius* int jaer 1554 inden 197 tr. 38 ②, dats 20 tr. 58 ③ voorder, soo heeftse de selve booch gelooopen op den tijt tusschen beyden bedraghende 141 ½ jaren, daerom segh ick, 141 ½ jaren gheven 20 tr. 58 ④, wat 100 jaren? Comt 1 tr. 29 ⑤. En (ghelijcx can openbaerlick ghevonden worden den loop op alle ghegheven tijt, waer af 'bewijs openbaer is: T B E S L V Y T: Wy hebben dan deur ervarings dachtafels ghevonden den eyghen loop der vaste Sterren, na den eyfch.

DES EERSTEN BOVCX

EINDE.

51st PROPOSITION

To find, by means of empirical ephemerides, the proper motion of the fixed Stars.

Since at the time of *Ptolemy* in the year 139 *Spica Virginis* was at $176^{\circ}40'$ of the ecliptic, but at the time of *Stadius* in the year 1554 at $197^{\circ}38'$, *i.e.* $20^{\circ}58'$ farther, it has passed over this arc in the time between these two, which is 1415 years. I therefore say: 1415 years give $20^{\circ}58'$; what do 100 years give? This gives $1^{\circ}29'$. And in the same way the motion in any given time can clearly be found, the proof of which is clear. CONCLUSION. We have thus found, by means of empirical ephemerides, the proper motion of the fixed Stars; as required.

END OF THE FIRST BOOK.

T W E E D E
BOVCK DES
HEMELLOOPS
VANDE
DVAELDERLOOP
DEVR WISCONSTIGHE
vverckinghegront op de oney-
ghen stelling eens vasten
Eertcloots

L

SECOND BOOK OF THE HEAVENLY MOTIONS

OF THE MOTIONS OF THE PLANETS

by Means of Mathematical Operations,
Based on the Untrue Theory¹⁾ of a Fixed Earth

¹⁾ The word *stelling*, as used by Stevin, corresponds literally to *supposition, assumption*. In the following we have translated it by *theory*, because of the vast scope of the Ptolemaic or Copernican conceptions, their numerous consequences, and the great amount of work which has been devoted to their development. In treatises on the history of Astronomy, the expression "planetary theories" is commonly used.

C O R T B E G R Y P

deses tyveeden Boucx.



Net eerste bouck des Hemelloops deur ervaringhen uyt den rouwen bemerckt sijnde, de Dwaelders in uytmiddelpuntighe ronden en inronden te loopen, met ander omstandighen dies angaende, waer me onsghedacht een gront heeft, om deur wisconstighe werkinghen veel nauwer en sekerder t'onderfoucken oft het oock volcommen ronden sijn, en offe daer in altyt eenvaerdich everas loopen, voort wat reden haer halfmiddellijnen en uytmiadelpunticheytlijnen tot malcander hebben, tot wat plaetsen sy in toecommende tijden sijn sullen, boven dien om te gheraken tot kennis der oneven daghen, der stilstanden en der sijnghen, der grootheyt vande verduyftere deelen die Son en Maen in haer duysterighen cryghen, der verheden daerse vanden Eertcloodt in sijn, der grootheden diese hebben teghen den Eertcloodt verleken, met meer ander dierghelijcke daer an clevende, soo sullen wy nu tottet tweede bouck commē, inhoudende der selve wisconstige werkinghen, en dat noch al op de ghemiste stelling eens vasten Eertcloots, gelijk de beschrijving van dien Ptolemeus eerst ter handt quam, te weten sonder daer in vermengt te wesen sijn verdochte vonden der onbekende roersels (als de tweede onevenheyt der Maen, mette derde onevenheden van Saturnus, Iupiter, Mars, Venus en Mercurius, metsgaders het onbekent breederoersel der vijf laetste, welke men int gemeen der Dwaelders onbekent loop noemt) die ick daer uyt scheyden sal, en daer na besonderlick beschrijven, op dat elck alsoo claerlicker siende watter deur stelling eens vasten Eertcloots ghebreëckt, te bequamelicker na ander beter vonden trachten mach. En sal hier affeven onderscheytsels maken.

Het 1 vande wisconstighe uytmiddelpunticheyts handel int gemeen.

Het tweede vande Sonloop.

Het 3 vande Maenloop.

Het 4 van Saturnus, Iupiters, Mars, Venus en Mercurius loop.

Het 5 van der Dwaelders saminghen, teghenstanden, en duysterighen.

Het 6 van Ptolemeus verdochte tweede onevenbede der Maen, en derde onevenheden van Saturnus, Iupiter, Mars, Venus en Mercurius.

Het 7 van Ptolemeus verdochte breedeloop der vijf Dwaelders, Saturnus, Iupiter, Mars, Venus en Mercurius.

M E R C K T.

Ick noem dese wercking wisconstich, tot onderscheyt der wercking ghetrocken uyt ervatinghen int eerste bouck, en hoewelse dickwils deur rekeningen met tafels in geen heele volcommenheyt der getalen en besaet, gelijk in volcommen wisconstighe wercking vercyscht wort, nochtans anghesien daer in is een voet van oneindelicke naerdering, mettet wisconstich groote ghemeeschap hebbende, en de bewijfen oock wisconstich sijnde, soo schijnet datmen se om t'boveschreven onderscheyts wille wisconstich noemen mach.

1 O N D E R.

SUMMARY OF THE SECOND BOOK

As it has been roughly perceived in the first book of the Heavenly Motions by experience that the Planets move in eccentric circles and epicycles, with other circumstances relating thereto, so that our thoughts have a basis on which to examine much more accurately and surely, by means of mathematical operations, whether the circles are perfect circles and whether they always move therein with uniform velocity, further what proportion their semi-diameters and lines of eccentricity have to one another, in what places they will be in future times, and further to acquire knowledge of the unequal days, of the stations and retrogradations, of the magnitude of the eclipsed parts of Sun and Moon during their eclipses, of their distances from the Earth, of their magnitudes as compared with the Earth, with more such related phenomena, we shall now come to the second book, containing their mathematical treatment, such all on the incorrect theory of a fixed Earth, as the description thereof first came into the hands of *Ptolemy*, to wit, without being mingled with his supposed discoveries of the unknown motions (such as the second inequality of the Moon, with the third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury, as also the unknown motion in latitude of the five last-mentioned planets, which in general is called the unknown motion of the Planets), which I will remove therefrom and describe thereafter separately, so that everybody, thus seeing all the more clearly what defect is involved in the theory of a fixed Earth, may strive the more easily to make other and better discoveries. And I will make thereof seven chapters.

The first of the mathematical treatment of eccentricity in general.

The second of the Sun's motion.

The third of the Moon's motion.

The fourth of the motions of Saturn, Jupiter, Mars, Venus, and Mercury.

The fifth of the Planets' conjunctions, oppositions, and eclipses.

The sixth of *Ptolemy's* supposed second inequalities of the Moon, and third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury.

The seventh of *Ptolemy's* devised motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury.

NOTE.

I call this operation mathematical to distinguish it from the procedure based on experiences, in the first book; although it often operates, through computations with tables, with numbers which are lacking the perfect accuracy required in perfect mathematical operations; nevertheless, since there is in it a basis of infinite approximation, having great similarity to the mathematical, and since the proofs are also mathematical, it seems that because of the above distinction it may be called mathematical.

[For reasons, set forth in the Introduction, the Second Book has not been reproduced, with the exception of the last paragraph only]

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seyt is, soude connen verstrecken tot ghemeene reghel van al d'ander, overmidts men mette bekende winst van d'een Dwaelder boven d'ander soo soude wercken, als vooren gedaen is: Sulcx datter niet en gebreeckt dan sekerder kennis van der Dwaelders loop in langde en breedde.

2 M E R C K.

Mijn voornemen was int beschrijven des Cortbegrijps deses 2 boucx als blijkt, hier noch by te voughen een seste Onderscheyt van *Ptolemus* verdochte tweede oneventheden der Maen, en derde oneventheden van Saturnus, Iupiter, Mars, Venus, en Mercurius in langdeloop: Metsgaders een sevende Onderscheyt van *Ptolemus* verdochte breedeloop der vijf Dwaelders, Saturnus, Iupiter, Mars, Venus, en Mercurius: Maer nadien ick volghens mijn voornemen, int eerste en tweede bouck beschreven hadde den Hemelloop met stelling eens vasten Eertcloots, sonder daer in te vermenghen *Ptolemus* vonden der boveschreven onbekende roersels, die ick besonderlick alleen ghestelt hadde: Sghelijcx oock gedaen hebbende met *Copernicus* beschrijving eens roerenden Eertcloots, die sijn eyghen vonden der selve onbekende roersels daer oock in vermengde, welcke ick mede daer uyt liet, en alleen beschreef, om daer deur de foucking des onbekenden handels voor yghelick claerder en verstaenlicker te maken, en na beter spiegeling te meugen trachten: Soo ist gebeurt dat ghene ick aldus voor ander bereyt hadde, my self tot inleyding verstrekte, om tot spiegeling te geraken die my beter docht, want alsoo ick eens quam te oversien mijn geschreven derde bouck na *Copernicus* wijze (dat een tijt lang stil gelegen hadde) omdat inden druck te brengen, ick quam tot ander kennis des breedeloops der Dwaelders, Saturnus, Iupiter, Mars, Venus, en Mercurius, sulcx dat my docht de selve gheen onbekende roersels meer en behooren te heeten, en vervolghens oock onnoodich te wesen het voornoemde sevende Onderscheyt te beschrijven, want alle ramingen der mensche int foucken der Dwaelders loop, seer nauwe te willen deurgonden, het schijnt datmē den tijt beter soude connen besteden met ghewisse dinghen te leeren. Nu dan angesien de breedeloop om de voorgaende reden comen sal int derde bouck met stelling eens roerenden Eertcloots, soo is dit d'oirsaeck waer deur ick het boveschreven sevende Onderscheyt hier uyt laet. Angaende het 6 Onderscheyt, dat heb ick oirboir verstaen inden Anhang des Hemelloops te brengen, om de redenen welcke aldaer van dies sullen verclaert worden.

Noch is te ghedencken, dat soo ymant int voorgaende of volghende deser wisconstige ghedachtenisse, quaem t'ontmoeten woorden of redenen in welke de boveschreven roersels der breedeloop onbekent genoemt worden, die ick noch tans nu segh bekent te sijn, d'oirsaeck daer af te wesen dat het volghende derde bouck t'laetste was, datter gedruckt wiert, hoewel ander stoffen t'bouck gebonden sijnde daer achter volghen, en dat ick als gheseyt is, int oversien des selven derden boucx eerst tot breeder kennis gherocht.

DES TWEEDEN BOVCX
EYNDE.

FIFTH CHAPTER OF THE SECOND BOOK

63rd PROPOSITION

2nd NOTE

When writing the Summary of this 2nd book, I meant to add — as appears — a sixth Chapter on *Ptolemy's* devised second inequalities of the Moon and third inequalities of Saturn, Jupiter, Mars, Venus, and Mercury in the motion in longitude; as also a seventh Chapter on *Ptolemy's* imagined motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury. But after, according to my intention, in the first and the second book I had described the Heavenly Motions on the theory of a fixed Earth, without including therein *Ptolemy's* discoveries of the above-mentioned unknown motions, which I had placed apart, while I had also done similarly with *Copernicus'* description of a moving Earth, who also included therein his own findings of the said unknown motions, which I also left out and described separately, in order thus to make the search for the unknown phenomena clearer and more intelligible for everyone and to be able to strive after a better theory, it so happened that what I had thus prepared for others, served as a preliminary for myself to arrive at a theory which seemed better to me. For when I went through my third book as written after the manner of *Copernicus* (which had been laid aside for some time) in order to prepare it for publication, I arrived at other knowledge of the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, so that it appeared to me that they ought no longer to be called unknown motions, and that consequently it was also unnecessary to describe the aforesaid seventh Chapter, because it seems that we might spend our time better teaching things that are certain rather than wish to fathom exactly all men's estimates in the search after the Planets' motions. Since therefore the motion in latitude for the aforesaid reason is to come in the third book, on the theory of a moving Earth, this is the cause why I here omit the above-mentioned seventh Chapter. As to the 6th Chapter, I thought it appropriate to put this in the Supplement of the Heavenly Motions, for the reasons that will there be given for it.

It is further to be borne in mind that if in the preceding or following parts of these mathematical memoirs anyone should come across words or arguments in which the above-mentioned motions in latitude are called unknown, which nevertheless I now say to be known, the cause of this is that the subsequent third book was the last to be printed, though, after the book was bound, other subjects follow thereafter, and that, as has been said, I arrived at greater knowledge while going through the said third book.

END OF THE SECOND BOOK.

D E R D E
BOVCK DES
HEMELLOOPS,
VANDE
VINDING DER DVVAEL-
DERLOOPEN, DEVR WIS-
constighe vvercking ghegront opde
wesentlicke stelling des roe-
renden Eertcloots.

THIRD BOOK
OF THE HEAVENLY MOTIONS

OF THE FINDING OF THE MOTIONS OF THE PLANETS

by Means of Mathematical Operations,
Based on the True Theory of the Moving Earth

C O R T B E G R Y P D E S E S D E R D E N B O V C X.



M de somme van desen te verclaren, soo staet te gbedencken dat de Dwaelders tweeley loop hebben, d'een in langde, de ander in breedte: Angaende de langdeloop, die sal hier wisconstelick bevvesen worden met stelling eens roerenden Eertcloots i' selve besluyt te cruygen, dat se met stelling eens vasten heeft, alleenelick daer in verschillende, dat i' ghene mette stelling eens vasten voor vreemt en met verwondering angefsien wort, deur d'ander sonder wonder is, als gegront sijnde op t' gene wesentlick inde natuer bestaet. Vyt het voornomde bevvijs, te wetten dat d'een en d'ander stelling een selve besluyt voortbrengt, sal dese beschrijving seer cort valle, vwant ick op dese roerende stelling geen nieuwe vercstucken maken en sal van t'winden der Hemelloopsche dinghen, maer voor gemeene regel nemen, dat al i' gene ons vande langdeloop inde daet voorvalt te berekenen, gedaen sal worden deur de verckstucken gegront op stelling eens vasten Eertcloots, en beschreven inde voorgaende twee eerste boucken: Ick sal oock in een besonder voorstel verclaren mijn gevoelen, vvaerom ick sulcke vercking gedaen op een oneygen stelling, voor bequamer houde dan die op de eygen vware gegront is. Na de langdeloop sal de breedeloop volgen, die ick oock met d'een en d'ander stelling een selve besluyt voortbrengende beschrijven sal.

Dit inhoudt int gemeē soodanich sijnde sal vijf onderscheyt sels hebben. Het eerste van der Dwaelders Hemelē gedaente, so veel noodich schijnt tot verclaring haers loops met stelling eens roerenden Eertcloots. Daer na sullen volgen drie onderscheyt sels vande langdeloop der Dwaelders met stelling eens roerendē Eertcloots: Te wetten het tweede vande Eertclood: Het derde vande Maē: Het vierde van Saturnus, Iupiter, Mars, Venus, en Mercurius. Daer na sal volgen t'laetste onderscheyt des breedeloops met stelling eens roerenden Eertcloots.

E E R-

SUMMARY OF THIS THIRD BOOK

To expound all this, it should be borne in mind that the Planets have two kinds of motion, one in longitude and the other in latitude. As regards the motion in longitude, this will here be proved mathematically to lead to the same conclusion on the theory of a moving Earth as on the theory of a fixed Earth, with the only difference that what is considered strange and gives rise to astonishment in the theory of a fixed Earth does not give rise to astonishment in the other theory, because it is based on what actually happens in nature. On account of the aforesaid proof, to wit, that the one as well as the other theory leads to the same conclusion, this description will be very short, for on this theory of a moving Earth I will not make any new problems of the finding of things connected with the Heavenly Motions, but I will make it a general rule that all that we have to compute in practice with regard to the motion in longitude must be done by means of the propositions based on the theory of a fixed Earth and described in the preceding first two books. In a special proposition I will also set forth my opinion why I regard such a procedure based on an untrue theory as more convenient than that which is based on the true theory. The motion in longitude is to be followed by the motion in latitude, which I will also describe as leading to the same conclusion with one theory and with the other.

The contents of this book, which are in general as given above, are divided into five chapters. The first of the figure of the Planets' Heavens, as much as seems necessary to explain their motion on the theory of a moving Earth. This is to be followed by three chapters on the motion in longitude of the Planets, on the theory of a moving Earth: to wit, the second of the Earth, the third of the Moon, the fourth of Saturn, Jupiter, Mars, Venus, and Mercury. This is to be followed by the last chapter, of the motion in latitude on the theory of a moving Earth.

E E R S T E
O N D E R S C H E Y T
 DES DERDEN BOVCX VAN
 der Dvvaelders Hemelen gedaente,
 soo veel noodich schijnt tot vercla-
 ring haers loops met stelling eens
 roerenden Eertcloots.

C O R T B E G R Y P D E S E S
 E E R S T E N O N D E R S C H E Y T S.

N Adien t'voornemen is hier te beschrijven der Dvvaelders loop met stelling eens roerenden Eertcloots, soo schijnt oirboir vant wesen en gedaente des overelts, als gront daer t'voorgaende op ghebout wort, soo veel te verclaren als wy daer af wveten of vermoeden, en totte kennis des voorghenomen loops behulpich is: Tot dien einde sal ick in dit eerste Onderscheyt ses voorstellen beschrijven.

Het eerste van d'oirden der hemelen vande Dvvaelders, met stelling eens roerenden Eertcloots.

Het tweede van des Eertcloots roersel in plaets, en haer seylsteenighe stilstandt.

Het derde vande seylsteenighe stilstant der Dvvaeldervveggen en haer hemelen.

Het vierde vande plaets des crachts die den Eertclood en vveggen der Dvvaelders in haer seylsteenighe stilstandt houdt.

Het vijfde dattet niet nootsakelicken blijktt de Son middelpunt te wvesen vanden vaste sterrens hemel, maer met goede reden daer toe vercoren wort.

Het sesste vande verwonderinghen sonder wvonder der ghene die een vaste Eertclood stellen.

I V O O R S T E L.

Te beschrijven d'oirden der hemelen vande Dvvaelders met stelling eens roerenden Eertcloots.

De Menschen certijts deur stelling eens vasten Eertcloots gecommen sijnde tot stelling van sommige inronden, waer in der Dvvaelders keering in tijt

Y effen

FIRST CHAPTER

OF THE THIRD BOOK

of the Figure of the Planets' Heavens,
as Much as Seems Necessary to Expound Their
Motion on the Theory of a Moving Earth

SUMMARY OF THIS FIRST CHAPTER

Since the object is here to describe the Planets' motions on the theory of a moving Earth, it seems appropriate to set forth as much about the nature and figure of the world — being the basis on which the foregoing is built — as we know or surmise and is conducive to the knowledge of the motion in view. To this end I will describe six propositions in this first Chapter.

The first of the arrangement of the Heavens of the Planets, on the theory of a moving Earth.

The second of the motion of the Earth in its place and its magnetic rest.

The third of the magnetic rest of the Planets' orbits and their Heavens.

The fourth of the place of the force keeping the Earth and the orbits of the Planets in their magnetic rest.

The fifth that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason.

The sixth of the wondering at what is no wonder, of those who assume a fixed Earth.

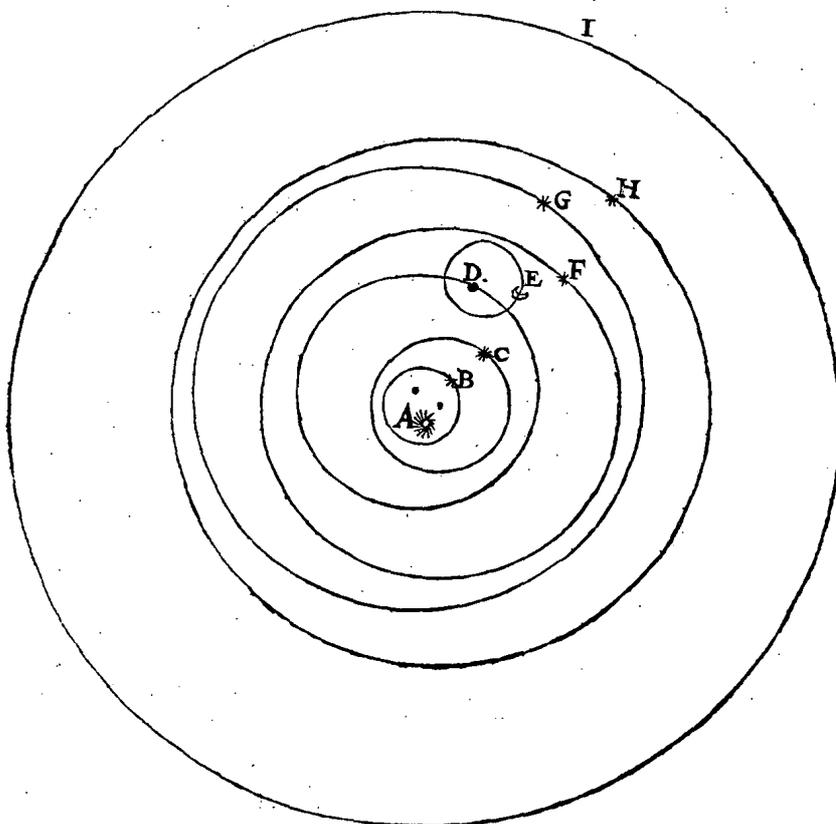
1st PROPOSITION.

To describe the arrangement of the Heavens of the Planets on the theory of a moving Earth.

Whereas formerly, on the theory of a fixed Earth, man came to assume some epicycles on which the revolution of the Planets corresponded exactly in time

250 DER HEMELN GHEDAENTE MET

effen overquam metter overschot des Sonloops boven den loop vant inron-
 t middelpunt in haer wegen, als van Saturnus, Iupiter en Mars: Voort ander in-
 rond: n diens middelpuntens loop effen mette Sonloop overquam, ghelijck int
 eerste bouck gheseyt is, sulcx en docht ander menschen die dese saken gronde-
 licker begosten t'onderfoucken, alsoo inde natuer niet te bestaen: Overdenc-
 kende daer na osmen die inronden met haer versierde roersels niet en soude
 meugen weeren, stellende den Eertclood in een rondt te loopen, bevonden dat
 ja, ghelijck by euelicke ouden betuycht wort eertijts smenschen ghevoelen van
 een loopenden Eertclood gheweest te hebben, maer de eyghentlicke form en
 ghedaente van hun beschrijving en is na den ondeiganck des Wijsentijts ter
 handt van *Ptolemus* noch ymannt anders ghecommen, datmen weet, dan met
 veel ander dinghen verloren bleven. Doch is ten laetsten *Nicolaus Copernicus*
 verschenen, welke de selve stelling, of een ander die groote gemeenschap daer
 me schijnt te hebben, weerom int licht gebrocht heeft, welke ick in dit voor-
 stel beschrijven sal, en tot dien einde aldus segghen: Binnen den Hemel der
 vaste sterren by *Copernicus* onbeweeghlick gestelt, sijn oirdentlick vervolghen-
 dede hemelen van Saturnus, Iupiter, en Mars, daer na des Eertcloods opt na-
 tuerlick jaer een keer volbrenghende, met noch twee roersels in plaats, waer af



de verclaring int 2 Hoofstlick ghedaen sal worden wefende de middellijn deses
 hemels sonder gevoelicke reden tegen de halfmiddellijn des Hemels der vaste
 sterren

to the surplus of the Sun's motion over the motion of the epicycle's centre in their orbits, as of Saturn, Jupiter, and Mars — while further the motion of the centres of other epicycles (Venus, Mercury) corresponded exactly to the Sun's motion, as has been said in the first book — it appeared to other men, who began to inquire into these matters more thoroughly, that this did not exist in nature. Considering thereafter whether those epicycles with their imagined motions could not be eliminated, assuming the Earth to move in a circle, they found that indeed, as is stated by several ancient writers, man formerly held the opinion that the Earth moved; but the true form and appearance of their description did not, after the end of the Age of the Sages, come into the hands of either *Ptolemy* or anyone else, as far as we know, but they were lost together with many other things. But at last *Nicolaus Copernicus* appeared, who has brought to light again this theory, or one that seems to resemble it closely, which I shall describe in this proposition, and to this end I say: Inside the Heaven of the fixed stars, assumed by *Copernicus* to be immovable, there are arranged in due order the Heavens of Saturn, Jupiter, and Mars, thereafter that of the Earth, performing one revolution in the natural year, with two more motions in its place, the exposition of which will be given in the 2nd Chapter, the diameter of this Heaven having no perceptible ratio to the semi-diameter of the Heaven of the fixed

sterren: Rontom den Eertcloot draeyt de Maen in een uytmiddelpuntichront, daer na volghen Venus en Mercurius, loopende in uytmiddelpuntighe ronden om de Son, die als weerelts middelpunt onbeweeghlick blijft. De loopen der boveschrevē ses Dwaelers ende des Eertcloots sijn altemael na t'vervolgh der trappen, dat is van Westen int Oosten.

Om t'boveschreven deur een form opentlicker te verclaren, laet den tip A de vaststaende Son beteyckenen, rontom welcke beschreven sijn ses uytmiddelpuntighe ronden: Int eerste van diengaet Mercurius an B: Int tweede Venus an C: Int derde den Eertcloot an D, en daerom in een uytmiddelpuntichront de Maen an E: Int vierde vijfde en sefte volghen oirdentlick Mars, Iupiter, Saturnus gheteyckent F, G, H, altemael sonder inront, I beduyt den Hemel der vaste sterren beschreven opt middelpunt A, t'welck als vooren de Son is.

De boveschreven oirden aldus eenvoudelick int corte gheseyt sijnde sonder eenich bewijs, soo sullen wy nu daer af wat breeder verclaring doen. De reden waer uyt men merckt Venus en Mercurius binnen den Eertclootwech te loopen, is, datse nummermeer tot teghestant der Son en gheraken: Maer Mercurius loop binnen Venuswech begrepen te wesen, blijkt an sijn cleender afwijckinghen daer t'sijnder plaets int eerste bouck deur ervaringen afgheseyt is.

Belanghende dat ymant twijffelen mocht en segghen, Venus en Mercurius wegen binnen den Eertcloot te connen wesen, sonder nochtans de Son te vervanghen, dat en mach soo niet sijn, om dat haer meeste verheden vanden Eertcloot veel langher bevonden worden dan des Eertclootwechs halfmiddellijn, na t'inhoudt der rekening die daer af int volghende ghedaen sal worden. Angaende Saturnus, Iupiter en Mars, ghemerckt die tot teghestant der Son gheraken, soo moeten haer wegghen den Eertclootwech vervanghen, anders waert onmeughelick. De reden waer uyt men merckt Saturnus de verste te wesen, daer na Iupiter, en Mars de naeste, can tweederley sijn: Ten eersten dat den Dwaeler der twee die in saming bedeckt wort, de verste moet sijn, doch dat ghebeurt seer selden, oock sijnder inde werelt te luttel gaffaghers dieder oppassen. Ten anderen dat sulcx deur stelling eens loopenden Eertcloots (anders dan deur stelling eens vasten Eertcloots, waer me dit onbekent blijft) dadelick ghemeten can worden, ghelijckmen opt landt meet welcke sichtbaer torre wijft van ons is, want de verheyte van twee standen die den Eertcloot tot verscheyden plaetsen haers wechs heeft, verstreckt ons voor driehoucx * gront, hebbende te. *Basis.* ghen haer sijden seer ghevoelelicke reden, ghelijck daer af t'sijnder plaets eyghentlicker gheseyt sal worden. Angaende de reden waer uyt men besluyt de Maenloop te moeten wesen ghelijck de voorgaende form anwijft, is dusdanich: By aldien se liepe ghelijck een van d'ander Dwaelers, dat soude moeten sijn buyten den Eertclootwech, ghelijck Saturnus, Iupiter en Mars, of daer binnen ghelijck Venus en Mercurius. Buyten en cant niet wesen, om datter alsdan nummermeer Sonduytering vallen en soude. Binnen en macht oock niet sijn om tweederley reden: Ten eersten datse nummermeer ghelijck Venus en Mercurius, in teghestant der Son en soude gheraken. Ten anderen datse soude connen comen achter de Son, sulcx dat al waren dan haer beyder middelpunten en het ooggh des sienders, alle drie in een rechte lini, soo en souder nochtans gheen Sonduytering wesen, strijdende tegghen d'ervaring.

De redenen waerom gheloofte wort de stelling des loopenden Eertcloots lijckformich te wesen mettet ghene inde natuere bestaet, en niet de stelling des vasten Eertcloots, sijn dusdanich: Ten eersten datmen daer deur besluyt, ghelijck int volghende breeder blijcken sal, de Dwaelers eenvoudelick te drayen

stars. Round the Earth moves the Moon in an eccentric circle; after this follow Venus and Mercury, moving in eccentric circles round the Sun, which remains immobile, as being the centre of the world. The motions of the above-mentioned six Planets and of the Earth are all in the order of the degrees, *i.e.* from West to East.

To explain the above more clearly by means of a figure, let the dot *A* denote the fixed Sun, about which have been described six eccentric circles. In the first of these moves Mercury at *B*, in the second Venus at *C*, in the third the Earth at *D*, and round this in an eccentric circle the Moon at *E*. In the fourth, fifth, and sixth follow successively Mars, Jupiter, Saturn, drawn at *F*, *G*, *H*, all without any epicycle; *I* denotes the Heaven of the fixed stars, described about the centre *A*, which, as said above, is the Sun.

After the above-mentioned arrangement has thus been simply stated in brief, without any proof, we shall now give a more detailed exposition of it. The reason from which it is perceived that Venus and Mercury move within the Earth's orbit is that they never come in opposition to the Sun. But the fact that Mercury's motion is contained within Venus' orbit becomes apparent from its smaller deviations, which have been referred to in their place in the first book by means of experiences.

As for the case that someone should doubt and say that Venus' and Mercury's orbits may be within that of the Earth, but without containing the Sun, this cannot be so because their greatest distances from the Earth are found to be much greater than the semi-diameter of the Earth's orbit, according to the contents of the computation that is to be made thereof in the following. As to Saturn, Jupiter, and Mars, since they come in opposition to the Sun, their orbits must contain the Earth's orbit, otherwise it would be impossible. The reason from which it is perceived that Saturn is the most distant, then Jupiter, and Mars the nearest may be twofold. Firstly, that that Planet of the two that is covered in conjunction must be the most distant, but this happens very seldom, and there are also too few observers in the world who attend to it. Secondly, that on the theory of a moving Earth (unlike on the theory of a fixed Earth, when this remains unknown) this can be measured in practice, as we measure on the land which visible tower is most distant from us, for the distance of two positions which the Earth has in different places of its orbit serves us as base of a triangle, whose ratio to the sides is very perceptible, as will be described more truly in its place. As for the reason from which it is concluded that the Moon's motion must be as denoted in the foregoing figure, this is as follows. If it moved like one of the other Planets, this would have to be outside the Earth's orbit, like Saturn, Jupiter, and Mars, or within it, like Venus and Mercury. It cannot be outside, because there would then never occur a Solar Eclipse. It cannot be within either, for two reasons. Firstly, that it would never, like Venus and Mercury, come in opposition to the Sun. Secondly, that it might come behind the Sun, so that, even if the centres of both and the eye of the observer were all three in a straight line, there would yet be no Solar Eclipse, which is contrary to experience.

The reasons why it is believed that the theory of the moving Earth is in accordance with what happens in nature, and not the theory of the fixed Earth, are as follows. Firstly, that it is concluded therefrom, as will become more fully apparent in the following, that the Planets simply revolve in circles, without

in ronden, sonder stelling van inronden met haer versierde loopen inde selve, welcke daer deur al verlaten worden.

Ten tweeden, angese het inde natuer soo veroident is, dat de Dwaelers die inde grootste ronden of hemels draeyen, slappelicxt ommeloopen, soo strijdet tegen dese gemeene oirden als men den aldergrootsten Hemel der vaste sterren stelt aldersnelst te draeyen, te weten alle daghe een keer: Ende is daerom de natuerlike reden lijckformigert te gelooven en te stellen sulck aldersnelste roersel het cleenste rondt toe te commen, te weten t' rondt des Eertcloots in sijn plaets.

Ten derden, na dient int wesen soo vervought is, dat al de loopē der Dwaelers sijn van Westen in Oosten, soo strijdet tegen dese gemeene oirden dat men dē loop der vaste sterren verkeerdelick stelt van Oostē in Westen, en is daerom de natuerlike reden lijckformigert te ghelooven en te stellen sulcke loop den Eertcloon toe te commen, oock van Oosten na Westen, gelijk van al d'ander.

Angaende mē de swaerheit des Eertcloots houdt voor oirsaeck van haer onbeweeghlicheyt, men mocht daer op aldus antwoordē: Angesen dē Eertcloon openbaerlick eē Hemels licht is, ontfangende vande Son haer claerheit gelijk de Maen, ten is niet buyten natuerlike reden, toe te staen dat de stof deser twee lichten, en oock van al d'ander sterren, deur gelijcke genegentheyt by malcander gehouden worden, te weten ghelijck de stoffen daer den Eertcloon uyt bestaet, gheneycht sijn na heur middelpunt te strecken, en derwaert te vallen soo lange tor datse niet voorderen connē, en dā geduerlick derwaert druckē, datte alsoo oock toegaet mette stoffen daer d'ander voorschrevē lichten uyt bestaen, welcke ghemeene ghenegentheyt om natuerlike redenen oock nootfakelick schijnt, want by aldient soo niet en waer, het sandt, water, en ander Eertse stoffen, soudē van malcander scheidē sonder een clootsche form te blijvē, genouchsaem als eē sichtbaer hoop aschē die deur de wint int wilde wechvlieght, verspreyt en onzichtbaer wort: Nu dan de natuerlike reden willende dat men roelate, soodanighe ghelijcke lichten ghelijcke eyghenschappen te hebben, en dat men voor feker houdt de swaricheyt der Dwaelers hemlien gheen onbeweeghlicheyt te veroirsaken, men behoort derghelijcke vande swaricheyt des Eertcloots oock te oirdeelen, en sulcx schijnt oock t'ghevoelen van *Copernicus* int 9 Hoofstuck sijns eerste boucx: Maer om hier af by voorbeelt noch wat claerder te spreken, ghenomen dat eenighe menschen verre vanden Eertcloon waren, en siende die blincken als een Dwaeler tusschen d'ander hemelsche lichten (welcke claerheit voornamelick sijn moet ter plaets daer t' water vande Son beschenen is) dat ymant van hemlien seyde die sterre boven d'ander wonderlick swaer te wesen, t'is kennelick dat d'ander menschen (sulcx sonder bewijs gheseyt wesende, en gheen teycken van meerder swaricheyt int een licht dan int ander siende) dien eenen niet ghelooven en souden: Ende alsoo en behooren wy die op den Eertcloon sijn, hem noch al de ghene die hier alsoo sonder bewijs spreken, oock niet te ghelooven.

Belangende t'ghene *Prolemus* teghen de stelling eens roerenden Eertcloots voorwent, te weten by aldien sy draeyde dat torren en ghestichten souden omvallen, van weghen de gheweldighe strijcking diese teghen de locht souden doen: Voort dat yet opspringhende, niet ter selve plaets neer en soude vallen, maer soo verre van daer als den Eertcloon daerentusschen verlopen waer: Al dit en mach niet bestaen uyt oirsaeck dat na sulck ghestelde de selve ongevallen souden moeten volgen met stelling eens vasten Eertcloots, want nadiē mē hier neemt dat de locht of den hemel die dē Eertcloon vervangt, niet en draeyt metten Eertcloon, maer d'een alleen sonder d'ander, soo sal de locht om den vasten Eert-

the assumption of epicycles with their imagined motions therein, which are thus all eliminated.

Secondly, since it is so arranged in nature that the Planets revolving in the largest circles or heavens revolve slowest, it is contrary to this general arrangement if the greatest Heaven of all, that of the fixed stars, is assumed to move fastest of all, to wit, every day one revolution. And therefore it is more in accordance with natural reason to believe and to assume that this fastest motion of all is to be assigned to the smallest circle, to wit, the circle of the Earth in its place.

Thirdly, since matters are so arranged that all the motions of the Planets are from West to East, it is contrary to this general arrangement that the motion of the fixed stars should inversely be assumed to be from East to West, and therefore it is more in accordance with natural reason to believe and to assume that this motion is to be assigned to the Earth, also from West to East ¹⁾, like all the others.

If the heaviness of the Earth is held to be the cause of its immobility, to this the following answer may be given: Since the Earth is evidently a Heavenly luminary, receiving from the Sun its light, like the Moon, it is not beyond natural reason to admit that the matter of these two luminaries, and also of all the other stars, is kept together by a similar affinity, to wit, as the substances of which the Earth consists tend towards its centre and to fall in that direction until they cannot get any further and then continually press in that direction; and that it also happens thus with the substances of which the other aforesaid luminaries consist, which general affinity also seems necessary for natural reasons; for if it were not so, the sand, water, and other Earthy substances would fall apart without the spherical form being preserved, somewhat like a visible heap of ashes which flies away at random on the wind, is dispersed, and becomes invisible. Natural reason therefore requiring it to be admitted that such similar luminaries have similar properties, and it being considered certain that the heaviness of the Planets does not cause their immobility, it should also be judged similarly of the heaviness of the Earth, and this also seems to be the opinion of *Copernicus* in the 9th Chapter of his first book. But to speak a little more clearly about this by means of an example: let us assume that some people were far from the Earth, and saw it shine like a Planet among the other heavenly luminaries (which brightness must be mainly in the places where the water is lit up by the Sun), and that one among them should say that this star was singularly heavy, more so than the others, it is evident that the other people (since this was said without any proof and they saw no sign of greater heaviness in one luminary than in another) would not believe that one man. And in the same way, we who are on the Earth ought not to believe him nor all those who here thus speak without any proof.

As to what *Ptolemy* ²⁾ advances against the theory of a moving Earth, to wit, that if it moved, towers and buildings would collapse, on account of their enormous friction against the air; further, that something which leaps up would not fall down in the same place, but so far thence as the Earth had meanwhile moved — all this cannot be so, because on this supposition the same accidents would have to happen on the theory of a fixed Earth; for since it is here assumed that the

¹⁾ For *van Oosten na Westen* in the Dutch text read *van Westen na Oosten*.

²⁾ *Syntaxis* I, 7 (Manitius p. 19).

Eertcloot vlieghende even soo stijf teghen de ghestichten strijcken, als den loopenden Eertcloot teghen de stilstaende locht, want ghenomen dat een stock overeinde ghesteken sy in een loopende rivier, en een ander stock deur een stilstaende water over einde voortghedronghen worde, soo ras als t'water vande rivier loopt, t'is toe te laten dattet water teghen d'een en d'ander stock evestijf sal drucken: En alsoo soudet oock openbaerlick toegaen mette locht teghen de ghestichten, of de ghestichten teghen de locht. Vyt het ghene voorseyt is soude volghen, dat yet op een vasten Eertcloot opspringhende, niet ter selver plaets weerom neer en soude vallen, maer soo verre van daer als hem de locht daerentusschen wech dronghe. Doch sulcx niet ghebeurende, soo ist nootfaekelick dat de lochtcloot en den Eertcloot van hem vervanghen, t'samen een cloot maken, die mette stelling eens vasten Eertcloots int gheheel stil staet, of mette stelling eens draeyenden Eertcloots int gheheel draeyt: En wijder die sulcke vereening des cloots vyt aerde en locht, seggen soo te wesen als men van een vasten Eertcloot spreeket, maer soo niet te sijn als t'verschil van een loopende is, met hemlien die alsoo haer selven tegenspreken, soudet ander swaer vallen te overcomen. T B E S L V Y T. Wy hebben dan beschreven d'oirden der hemelen vande Dwaelers met stelling eens roerende Eertcloots, na dē eysch.

2 V O O R S T E L

Te verclaren des Eertcloots roersel in plaets, en haer seylsteenighe stilstandt.

Benevens den loop des Eertcloots van plaets tot plaets in haer wech, doende alle jare een keer vyt Westen na Oosten, daer int 1 voorstel afgheseyt is, soo heeftser noch twee in plaets: D'een loop is den daghelickschen keer op haer as van Westen in Oosten, maer om dit roersel in plaets by voorbeelt wat breeder te verclaren, men mocht segghen dattet is ghelijck een draeyenden slijpsteen in een varende schip, welcke deur t'schip een roersel ontfangt van plaets tot plaets, maer heur draeying op den as blijft daerentusschen int schip op een selve plaets, en alsoo metten Eertcloot nieuwelini. D'ander loop (na dē sin van *Copernicus* int 11 Hoofstuck sijns 1 boucx) is dusdanich: Te wijle den Eertcloot haer jaerlickschen keer doet van Westen na Oosten daer int 1 Hoofstuck afgheseyt is, soo doetsē daerentusschen op de selve tijt teghen den voorschreven loop een keer in plaets van Oosten na Westen, sulcx dat hier deur den as gheduerlick na een selven oirt streckt. Maer om dit roersel en strekking vanden as gheduerlick na een selven oirt, te verclaren mette bequaemste gelijckenis die my nu te vooeren comt, ick segh aldus: Ghenomen dat ymant opt middelpunt des ront papiers van een seecompas stack een stroyken, streckende ewewijdich metten as des Eertcloots, dat seecompas staende in een schip, varende neem ick in een ronde gracht van een Slot of Schans, t'is kennelick dattet selve schip een keer ghedaen hebbende van plaets tot plaets na d'een sijde, soo sal daerentusschen t'compas in sijn plaets oock eens ghekeert sijn na d'ander sijde, dats teghen den keer vant schip, en sulcken ghedeelte eens keers t'selve schip ghedaen heeft na d'een sijde, foodanich ghedeelte sal oock het seecompas ghedaen hebben na de ander sijde, blijvende het boveschreven stroyken gheduerlick ewewijdich metten as des Eertcloots: En alsoo salmen derghelijcke oock verstaen vanden loop des Eertcloots in haer wech, welcke te wijle sy daer een keer doet, soo dracytē een keer in haer plaets teghen den voorgaenden keer, en blijvende den as altijt

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air or the heaven that contains the Earth does not move with the Earth, but the one alone without the other, the air flying about the fixed Earth will press just as closely against the buildings as the moving Earth against the stationary air; for if we assume that a stick be put up vertically in a flowing river, and another stick be drawn vertically through stagnant water as fast as the water of the river flows, it must be admitted that the water will press equally against one stick and the other. And the same would evidently happen with the air against the buildings or the buildings against the air. From what has been said before it would follow that something leaping up on a fixed Earth would not fall down again in the same place, but so far thence as the air drew it away meanwhile. But since this does not happen, it is necessary that the sphere of air and the Earth contained by it together form one sphere which on the theory of a fixed Earth stands still in its entirety or on the theory of a rotating Earth rotates in its entirety. And further, as for those who say that this composition of the sphere of earth and air is thus when a fixed Earth is spoken of, but not when there is the difference that it moves, it would be difficult for others to agree with those who thus contradict themselves. CONCLUSION. We have thus described the arrangement of the Heavens of the Planets on the theory of a moving Earth; as required.

2nd PROPOSITION.

To expound the motion of the Earth in its place, and its magnetic rest.

In addition to the motion of the Earth from place to place in its orbit, performing every year one revolution from West to East, as has been described in the 1st proposition, it has two more motions in its place: one motion is the daily rotation on its axis from West to East, but to explain this motion in its place somewhat more fully by means of an example, it might be said to be like a turning grindstone in a vessel sailing, which from the ship receives a motion from place to place, but its rotation on the axis meanwhile remains in the ship in the same place; and the same is the case with the Earth. The other motion (according to *Copernicus* in the 11th Chapter of his 1st book) is as follows: While the Earth performs its annual revolution from West to East, which has been described in the 1st Chapter, in the same time it meanwhile performs, against the aforesaid motion, a rotation in its place from East to West, in such a way that the axis constantly tends in the same direction. But to explain this motion and constant tendency of the axis in the same direction by means of the most convenient comparison that now occurs to me, I say as follows; if a man were to put in the centre of the round paper of a mariner's compass a straw parallel to the axis of the Earth, this compass standing in a ship sailing, I assume, in a circular moat of a Castle or Entrenchment, it is evident that when this ship has made one turning from place to place towards one side, in the mean time the compass in its place will also have made one turning towards the other side, *i.e.* against the turning of the ship; and such part of a turning as the ship shall have made towards one side, the same part the compass will also have made towards the other side, the aforesaid straw constantly remaining parallel to the axis of the Earth. And it must be similarly understood also with regard to the motion of the Earth in its orbit: while it performs one revolution, it rotates once in its place against the former revolution, the axis always tending in the same direction. This is, I think,

na een selven oirt streckende. Dit meyn ick te wesen den rechtē sin vant roersel des Eertcloots deur *Copernicus* beschreven, en met een figure verclaert int 11 Hoofstlick sijns eersten boucx. Maer want dit roersel van hem aldus eenvoudelick gheseyt wort sonder eenighe natuerlicke reden of bewijs, soo heeft my dese stelling langhen tijt int ghedacht ghequollen, overmidts alle Hemelsche roersels diemen everas versiert en op malcander doet passen, ghelijckmen de rayers van een uyrwerck doet over een commen, my niet en bevallen, als niet schijnende inde natuer te bestaen: Nochtans moest dit roersel soo toeghelaten sijn, om al d'ander natuerlicke overeencomminghen die uyr stelling des roerenden Eertcloots volghen een seker gront te gheven: Doch is daer na int licht ghecommen het bouck vanden grooten * Eertclootschen seylsteen, beschreven deur *Guilielmus Gilbertus*, waer in de natuerlicke oirfaeck deses roersels, mijns bedunckens ghetrossen en gheopenbaert is, waer af ick hier de somme int corte stellen sal: Men bevint inden Eertcloon soo groote menichte van seylsteen, en ander stoffen met seylsteensche cracht (als yfergheberchten die overal seer menichvuldich sijn, en altemael dien aert hebbē) datse als een grooten seylsteen in heur de eyghenschappen heeft diemen vint inde cleene clootkens van seylsteen gemaect, want sulck een met corck clootsche wijze soo beset wesende, dattet int water hangt gelijk dē Eertcloon inde locht, alsdan en sal den noortschen aspunt niet alleen na t'Noorden wijfen, gelijk mette bestrecken seylnaeldē toegaet, maer den heelen as stelt heur ewewijdich met des Eertcloots as, te weten den Noorderschen aspunt des steens na t'Noordē, den Zuyderschen na t'Zuyden: Voort soomen opt seylsteenclootken stelt seylnaeldekens, ghelijck ghemeenelick inde dragelicke Sonwijferkens sijn, men bevintse daer op sulcke rijcinghen, afwijckinghen, beweginghen, en eyghenschappen te hebben, als op den grooten Eertcloon. Noch heeft dese groote gemeenschap gheleert d'oirfaeck vande ongeregelde wijckinghen der seylnaelde vant Noorden, daer veel menschen al verwonderende hun gedachten dus langhe me becommert hebben, want almen neemt een cloot van seylsteen met putten daer in, en datmen een seylnaeldecken stelt niet boven t'middel des puts daert rechte Noortwijfing hebben can, maer by de kant na t'Oosten, het sal Oostelicken, maer ghestelt sijnde by de cant na Westen, sal Westelicken, nu want den Eertcloon oock sulcken seylsteencloot is met diepe puttē der zee, in wiens beweghende water de boveschreven seylsteenschen aert niet sijn en can, soo bevintmen wel rechte Noortwijfing ontrent het middel der groote zee, als in de Oceane tusschen America en Eurcpa, maer op de oostzijde na Europa commende de naelde oostelickt, en westwaert na America sy westelickt: De selve reden is oock voor groote uytstekende punten van Landen, als Cabo de Buna Esperanca en meer ander, waer af den Schrijver inde laefste Hoofstlicks sijns vierden boucx verscheyden voorbeelden stelt.

Nu dan den Eertcloon in heur hebbende desen seylsteenschen aert, soo moetet boveschreven roersel (te weten eens jaerlickschē keers in plaets van Oosten na Westen, sulcx dat den asgheduerlick na een selven oirt streckt) alsoo nootfelick wesen.

Tot hier toe is dese seylsteenigen aert des Eertcloots genoemt een draeyende roersel, om mijn voornemen beter te verclaren, maer acht nemende op t'geene ick voorder segghen wil, vinde bequamer dat te heeten seylsteenighe stulstandt, om dese reden: Ick heb hier vooren gheseyt dat hemelsche roersels diemen everas versiert, en op malcander doet passen, my niet en gevallen, als niet schijnende inde natuer te bestaen, nochtans, mocht ymant segghen, sulcx nu te blijc-

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the correct meaning of the motion of the Earth described by *Copernicus* and explained by means of a drawing in the 11th Chapter of his first book. But since this motion is thus simply described by him, without any natural argument or proof, this supposition long troubled me in my mind, since the notion that all the Heavenly motions are imagined equally fast and made to fit into one another, as the wheels of a timepiece are made to fit together, does not satisfy me, as not seeming to happen in nature. Nevertheless this motion had to be admitted in order to give a sure basis for all the other natural correspondences that follow from the theory of a moving Earth. But thereafter there was published the book about the great terrestrial magnet, described by *Guilelmus Gilbertus* ¹⁾, in which the natural cause of this motion in my opinion is hit off and revealed, which I will here summarize as follows. In the Earth there is found such a large amount of loadstone and other substances with magnetic force (such as mountains containing iron, which are of frequent occurrence everywhere and all have that property) that, like a big loadstone, it has in itself the properties that are found in small spheres made of loadstone: for if such a small sphere is covered with cork in the form of a sphere, so that it hangs in the water like the Earth in the air, not only will the northerly pole point towards the North, as happens with magnetized needles, but the whole of the axis takes up a position parallel to the axis of the Earth, to wit, the Northerly pole of the stone pointing towards the North, the Southerly towards the South. Further, if on the sphere of loadstone we put magnetic needles, as they are generally present in portable Sun-dials, they are there found to have the same elevations, deviations, movements, and properties as on the big Earth. This great similarity has also taught the cause of the irregular deflections of magnetic needles from the North, about which people have wondered and puzzled so long. For if we take a sphere of loadstone with pits therein and we put a magnetic needle, not over the centre of the pit, where it can point straight towards the North, but sideways towards the East, it will deviate towards the East, but if it is put sideways towards the West, it will deviate towards the West. Now since the Earth is also such a sphere of loadstone with deep pits, namely, the seas, in whose moving water the above-mentioned magnetic character cannot be present, straight Northward pointing indeed is found about the middle of the great seas, such as in the Ocean between America and Europe, but when we get to the East, towards Europe, the needle deviates towards the East, and when we get to the West, towards America, it deviates towards the West. The same reason also holds for large protruding points of land, such as Cabo de Bona Esperança, and others, of which the Author gives several examples in the last Chapters of his fourth book ²⁾.

The Earth therefore having this magnetic character, the motion described above (to wit, that of an annual rotation in its place from East to West, such that the axis constantly tends in the same direction) must be necessary.

Up to this point this magnetic character of the Earth has been called a rotary motion, in order to set forth my intention the better, but with a view to what I further intend to say, I find it more suitable to call it magnetic rest, for the following reason. Above I have said that the notion that heavenly motions are

¹⁾ William Gilbert (Colchester 1544—London 1603), physician to Queen Elizabeth in London, author of *De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure; Physiologia Nova* (London, 1600).

²⁾ This refers probably to *The Haven-Finding Art* (Work XI, i 25).

te blijcken met des Eertcloots boveschreven soodanige twee even roersels. Ick antwoorde hier op datmen dit twee even roersels noemen mach die op malcander passen, om, als voorseyt is, de voorghestelde saeck bequamelicker me te verclaren, maer om eyghentlicker te spreken soo soudement beter des Eertcloots seylsteenighe stilstant heeten, sulcx datse heur de keering haers wechs niet an en treckt: Als by voorbeelt, de casse daer een seylnaelde in staet, een keer omghedraeyt sijnde na de rechterzijde, datmen soude seggen de seylnaelde daerentusschen een keer gedaen te hebben na de sijncker zijde, ten waer niet soo eyghentlick ghesproken als te segghen datse stil staet, sonder heur de keering der casse an te treckē: En alsoo oock metten Eertcloon in haer wech: Sulcx dat t'gene ick tot hier toe genoemt hebbe des Eertcloots-tweede loop in plaets, dat heet ick na het inhoudt vant opschrift deses voorstels seylsteenighe stilstant, als wesende een woort dat ghenouch schijnt uyt te beelden de manier van stilstant dieder int voorgaende ghemeeent is, en int volghende ghemerckt sal worden. Merckt noch dat als men dese seylsteenighe stilstandt roersel noemde, soo soudet een roersel sijn teghen t'ervolgh der trappen, t'welck deur sich selfs bestaende ick niet en sie in eenighe hemelen te ghebeuren.

T B E S L V Y T. Wy hebben dan verclaert des Eertcloots roersel in plaets, en haer seylsteenighe stilstant, na den eysch.

3 V O O R S T E L.

Te verclaren de seylsteenighe stilstant der Dwaelders vveghen en haer hemelen.

Dese eyghenschap van seylsteenighe stilstandt en is niet alleen inden Eertcloon als vooren, maer oock (dienende t'een tot breeder bevesting vant ander) in haer wech, als opentlick blijkt deur t'verstepunt des selfden, t'welck om dē loop van Mars hemels wille daer den Eertcloon af vervangen is, en in gediegen wort, te twee jaren een keer soude moeten doen, dat nochtans niet en ghebeurt soo de ervaring leert, deurwelcke men bevint datter veel jaren verlooppen eert 1 tr. voordert, en is bycans al oft stil stonde, waer uytmen besluyten mach niet alleen den Eertcloon, maer oock haer heelen wech den seylsteenschen aert te hebben: Ia dattet om dergelijcke reden alsoo toegaet met elcken wech van d'ander Dwaelders, diens verstepunten geen roersel en krijgen vande hemelen haer omvangende, so merckelicxt blijkt in Mercurius wech, diens verstepunt uyt oirfaeck van Venus wechs keering seer snellick soude moeten ommedraeyen, te 225 daghen een keer doende, en noch soo veel rassicher als dan veroirfaeckt wierde deur al d'ander Dwaelders hemelen die daer boven d'een d'ander begrijpen, al t'welck niet en ghebeurt, want Mercurius wechs verstepunt soo slappen voortganck heeft als al d'ander.

Noch is te weten desen aert der seylsteenighe stilstandt niet alleen te sijn in der Dwaelders platte weggen als boven, maer oock inde heele hemelsche clooten daerse in ghedreghen worden, sulcx dat haer assen (gelijck vooren int tweede voorstel vanden Eertcloots as gheseyt is) geduerlick na een selven oirt strecken, want sooder die seylsteenighe stilstandt niet en waer, de twee aspunten des Hemelcloots die den Eertcloon draecht, en souden niet op een selve plaets blijven, maer te twee jaren sulcken rondt moetē beschrijvē, als deur Maenswechs afwijking vanden duysteraer veroirfaeckt wierde: Of anders gheseyt, den Eertclooten soude niet gheduerlick sonder breede blijven int plat datmen voor

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imagined equally fast and made to fit into one another does not satisfy me, as not seeming to happen in nature; yet someone might say that this now is evident from the above-mentioned two equal motions of the Earth. To this I reply that we may call them two equal motions which fit together, in order — as has been said before — to set forth the object in view more properly therewith; but to speak more truly it would be better to call it the magnetic rest of the Earth, meaning that it does not take account of the revolution in its orbit. For example, if, the box in which a magnetic needle is contained having performed one revolution to the right, it should be said that the magnetic needle had meanwhile performed a revolution to the left, this would not be as true a statement as saying that it stands still without taking account of the revolution of the box. It is the same with the Earth in its orbit, so that what I have hitherto called the second motion of the Earth in its place, according to the wording of the heading of this proposition I call magnetic rest, this being a word that seems to denote sufficiently the kind of rest that is meant in the foregoing and is to be noted in the following. Note also that if this magnetic rest were called motion, it would be a motion contrary to the order of the degrees, which as existing in itself I do not see happening in any heavens.

CONCLUSION. We have thus expounded the motion of the Earth in its place and its magnetic rest; as required.

3rd PROPOSITION.

To expound the magnetic rest of the Planets' orbits and their heavens.

This property of magnetic rest resides not only in the Earth, as said above, but also (the one serving to confirm the other more fully) in its orbit, as is evident from the apogee of the latter, which — because of the motion of Mars' heaven, within which the Earth is contained and carried — would have to perform one revolution in two years, which nevertheless does not happen, as experience teaches, by which it is found that many years elapse before there is a progress of 1° ; and it is almost as if it stood still; from which it may be concluded that not only the Earth, but also its whole orbit has a magnetic character, nay, that for similar reasons the same holds for any orbit of the other Planets, whose farthest points do not acquire a motion from the heavens containing them, as is most apparent in Mercury's orbit, whose farthest point on account of Venus' orbital revolution would have to revolve very fast, performing one revolution in 225 days, and even so much more fast as would be caused by the heavens of all the other Planets which are above and contain one another — all of which does not happen, for the farthest point of Mercury's orbit has just as slow a progress as all the others.

It should also be known that this character of magnetic rest resides not only in the plane orbits of the Planets, as said above, but also in the entire heavenly spheres in which they are carried, such that their axes (as has been said above in the second proposition of the Earth) constantly tend in the same direction. For if this magnetic rest were not present, the two poles of the Heavenly Sphere carrying the Earth would not remain in the same place, but in two years would have to describe a circle such as would be caused by the deviation of Mars' ¹⁾ orbit from the ecliptic. Or in other words, the Earth would not constantly remain

¹⁾ For *Maenswechs* in the Dutch text read *Marswechs*.

duyfteraer houdt, maer somwijlen daer af soo groote afwijckingen hebben als Marswech vanden duyfteraer heeft. Ende sulcx als hier gheseyt is vanden Hemelsclood die den Eertclood draecht, sal mē oock verstaen op de Hemelclooden van al d'ander Dwaelders, onder welcke dese seylsteenighe stilstandt inde Manens hemel seer merckelick is, overmidts het verstepunt hem dē jaerlickschen keer die den hemel int volgen of leyden des Eertcloots niet an en treckt. Oock is te gedencken dat inde breedeloop vande onderste Dwaelders sulcken ongheregheltheit soude moeten wesen als veroirsaeckt wierde uyt de mengeling van al de verscheyden afwijckingen der Dwaelderweghen die boven hemlien sijn: Al t welck niet ghebeurende (ghelijck grondelicker sal blijcken deur de volghende beschrijving vande eenvoudighe oirden diese in breedeloop houden) soo valter uyt te beslyten, desen ghemeenen aert der seylsteenighe stilstandt niet alleen te wesen in der Dwaelders weggen, maer oock geheelick in haer Hemelclooden.

Merckt noch dit: Alsmen mette beschrijving der Dwaelders Hemelen sulcken oirden wilde volghen als *Copernicus* int 11 Hoofstuck sijns eersten boucx mette beschrijving des Eertcloots ghedaen heeft, welcke hier verhaelt wiert in her 2 voorstel, men soude moeten aldus segghen: Te wijle Jupiters hemel heur dertichjarige keer doet van Westen na Oosten diese van Sarurnus hemel ontfangt, soo doerse daerentusschen op de selve tijt teghen den voorschreven loop een keer in plaets van Oosten na Westen, sulcx dat hier deur den as geduclick na een selven oirt streckt. Maer my dunckt om de voorgaende redenen verstaenlicker en nauwerlicker, dit in plaets van soodanich roersel te noemen seylsteenighe stilstandt, te meer dat opt roersel van d'onderste Dwaelders als neem ick van Mercurius, meer soude moeten geseft sijn dan van Venus loop, ghemerckt het soude moeten wesen de somme der loopen van al de Dwaelders dieder boven sijn.

Noch moet ick seggen dat ick over een tijt van desen handel onbesloten gedachten hadde, houdende ter eender sijde voor ghemeene reghel, dat alle begrepen die wech henen moet daer hem sijn begrijpende draecht, waer uyt volghen soude dat elck Dwaelder een roersel moest hebben ghemengt uyt al de roersels der Dwaelders die boven hem sijn: Ter ander sijde sach ick metter daer t verkeerde ghebeuren: Dit dede my dencken oft soude meughen sijn dat de Dwaelders niet en waren in Hemelen ghehecht, maer deur de locht vloghen ghelijck de voghelen om een torre, sonder het roersel van d'een, ant roersel van d'ander eenige beweeghnis te veroirsaken, waer tegen ander redenen my weerom anders deden vermoeden: Maer gecommen sijnde ter kennis vande voorgaende eyghenschap die ick seylsteenighe stilstandt noem, die twijfelachtighe ghedachten namen daer me een einde. T B E S L Y T. Wy hebben dan verclaert de seylsteenighe stilstandt der Dwaelders weggen en haer Hemelen, na den eysh.

4 V O O R S T E L.

Te seggen vande plaets der crachten die den Eertclood, vveghen, en Hemelen der Dwaelders in haer seylsteenighe stilstandt houden.

De crachten die den Eertclood, weggen, en Hemelen der Dwaelders in haer seylsteenighe stilstandt houden (welcke crachten men by verstaenlicker gelijk-

kenis

with latitude zero in the plane that is taken for the ecliptic, but would sometimes have as great deviations therefrom as Mars' orbit has from the ecliptic. And the same that has here been said of the Heavenly Sphere carrying the Earth is also to be understood for the Heavenly Spheres of all the other Planets, among which this magnetic rest is very considerable in the Moon's heaven, since the apogee does not take account of the annual revolution which the heaven performs ¹⁾ in following or leading the Earth. It should also be remembered that in the motion in latitude of the lowermost Planets there would have to be such irregularity as would be caused by the combination of all the different deviations of the Planets' orbits that are above them. Since all this does not happen (as will become more fully apparent from the following description of the simple order they keep in their motion in latitude), it may be concluded therefrom that this general character of magnetic rest resides not only in the Planets' orbits, but also altogether in their Heavenly Spheres.

Note also the following. If in the description of the Planets' Heavens we wished to follow the same order as *Copernicus* has done in the 11th Chapter of his first book with the description of the Earth, which has here been related in the 2nd proposition, we should have to say as follows: While Jupiter's Heaven performs its thirty years' revolution from West to East, which it receives from Saturn's Heaven, it performs meanwhile in the same time, contrary to the aforesaid motion, a rotation in its place from East to West, so that the axis thus tends constantly in the same direction. But for the above reasons it seems to me more intelligible and more natural to call this, instead of such a motion, magnetic rest; the more so because with regard to the motion of the lowermost Planets, such as, for example, Mercury, more would have to be said than on Venus' motion, because it would have to be the sum total of the motions of all the Planets that are above it.

I also have to say that for some time I was undecided in my mind about this matter, holding it on the one hand a general rule that all bodies contained by other bodies must take the course in which their containing bodies carry them, from which it would follow that every Planet must have a motion consisting of a combination of all the motions of the Planets that are above it. On the other hand, in practice I saw the reverse happening. This caused me to think whether it could be possible that the Planets were not attached to Heavens, but were flying through the air like birds about a tower, without the motion of the one causing any change in the motion of the other; but other reasons again made me think differently. But when I had gained knowledge of the foregoing property, which I call magnetic rest, my doubts were resolved. CONCLUSION. We have thus expounded the magnetic rest of the Planets' orbits and their Heavens; as required.

4th PROPOSITION.

To speak of the place of the forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest.

The forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest (which forces, by an intelligible comparison, might be

¹⁾ This or some such word must have been omitted in the Dutch text.

kenis yders seylsteen mocht noemen) schijnen altemael, uytghenomen vande Maenwech, te moeten wesen buyten de Hemelen der Dwaelders. Om van t'welck by voorbeeld te spreken, soo ymant een seylsteen (als treckende cracht der seylnaelde) leyde inde casse daer een seylnaelde op de pinne in rust, de selve casse ghekeert wesende, soo en soude de naelde niet gheduerlick na een selven oirt wijfen, maer altyt na den seylsteen gheneycht sijn, om dat de treckende cracht self me draeyt: Ende alsoo oock by aldien de treckende crachten die den Eertcloon en haer heelen Hemel in die stant houden, waren binnen haer casse, dat Mars Hemel, en daer in me voortghedregen wierden, des Eertclootwechs verstepunt soude den loop van Marswech krijghen te twee jaren eens ommeloopende. Doch want sulcx niet en ghebeurt, soo ist daer voor te houden dat die treckende cracht in Mars Hemel niet en is, noch om derghelijcke redenen in den Hemel van eenighe van d'ander Dwaelders: Maer anghesien haer verstepunt soo veel men deur ervaring bemerckt, altyt streckt na een selve plaets tusschen de vaste sterren, volghende den tragen loop der selve, soo machment daer voor houden die treckende cracht inde casse of Hemel der vaste sterren te wesen, en dit niet alleen vandē Eertcloon en haer Hemel, maer oock vande Hemelen van al d'ander Dwaelders, uytghenomen soo gheseyt is de Maen, diens wechs verstepunt ontrent de negen jaren eens ommeloopende deur het 9 voorstel des 1 boucx, haer treckende cracht soude in een ander casse of leeger hemel moeten draeyen, en dat soot schijnt tusschen Mars en Iupiter. Maer te wijle ick an dese stof ghecommen ben, sal daer af met eene noch dit segghen: Alsoo ick bevonden hadde de Maenwechs eyghentlicke verstepunts loop niet te wesen van 6 ① 41 ② sdaechs na t'vervolgh der trappen, gelijkmen deur stelling eens vasten Eertcloots besluyt, maer als uyt stelling eens roerenden Eertcloots volght van 52 ① 27 ② sdaechs teghen t'vervolgh der trappen, ghelijck t'sijnder plaets bewesen sal worden, soo gafi my vrecmt de eyghentlicke stelling des roerenden Eertcloots me te brenghen, yet ghevonden te worden dat teghen t'vervolgh der trappen liep: Maer overdenckende daer na dat haer treckende cracht in eenighen anderen hemel ontrent de neghen jaren als gheseyt is een keer dede na t'vervolgh der trappen, soo docht my dat hier niet teghen de regel en ginck, maer eyghentlick alles van Westen na Oosten te loopen, en sulck schijnfel sijn bekende oirsaken te hebben; Dergelijcke oock vermoedende vande duysteringne loop die niet sdaechs 3 ① 11 ② volghens de stelling eens vasten Eertcloots, maer eyghentlick als uyt stelling eens roerenden Eertcloots volght sdaechs 1 tr. 2. 19. tegen t'vervolgh der trappen, wiens treckende cracht schijnt ghedreghen te sijn in een hoogher tragher hemel dan de voorgaende, doende ontrent de 18 jaren een keer na t'vervolgh der trappen.

T B E S L V Y T. Wy hebben dan gheseyt vande plaets der crachten die den Eertcloon, wegghen, en Hemelen der Dwaelders in haer seylsteenige stilstandt houden, na den eysch.

5 V O O R S T E L.

Te verclaren dattet niet nootfakelick en blijktt de Son middelpunt te vvesen vanden vaste sterrens hemel, maer met goede reden daer toe vercoren vvort.

Ghelijck int Eertclootschrift oirboir is op den Eertcloon eenich halfmid-dachront te verkiesen, dat by al de ghene die vande selve stof handelen int ghe-meen

called the loadstone of each) all seem to have to be outside the Heavens of the Planets, with the exception of the Moon's orbit. To give an example of this, if someone laid a magnet (as attractive force of the magnetic needle) in the box in which a magnetic needle rests on the pin, then, if the box were turned, the needle would not constantly tend in the same direction, but would always incline towards the magnet, because the attractive force itself takes part in the revolution. And in the same way, if the attractive forces which keep the Earth and its entire Heaven in that position were within its box, *i.e.* Mars' Heaven, and were carried along in it, the apogee of the Earth's orbit would receive the motion of Mars' orbit, revolving once in two years. But because this does not happen, it is to be concluded that this attractive force does not reside in Mars' Heaven, nor for similar reasons in the Heaven of any of the other Planets. But since its apogee, as far as is noted by experience, always tends in the same direction among the fixed stars, following the slow motion of the latter, it may be assumed that this attractive force resides in the box or Heaven of the fixed stars, such not only for the Earth and its Heaven, but also for the Heavens of all the other Planets, except, as has been said, for the Moon; since the apogee of its orbit revolves once in about nine years, by the 9th proposition of the 1st book, its attractive force would have to revolve in another box or lower Heaven, such apparently between Mars and Jupiter. But now that I have arrived at this subject, I will at the same time also say the following about it. As I had found the true apogee's motion of the Moon's orbit not to be 6'41" a day, in the order of the degrees, as is concluded from the theory of a fixed Earth, but, as follows from the theory of a moving Earth, 52'27" a day, against the order of the degrees, as will be proved in its place, it seemed strange to me that the true theory of the moving Earth should involve that something was found that moved against the order of the degrees. But reflecting thereafter that its attractive force in any other heaven, as has been said, would perform one revolution in about nine years according to the order of the degrees, it seemed to me that this was not contrary to the rule, but that in reality everything moved from West to East, and that this apparent state of affairs has its known causes. And I suspected the same for the motion of the line of nodes, which is not 3'11" a day, as according to the theory of a fixed Earth, but in reality, as follows from the theory of a moving Earth, 1°2'19" a day, against the order of the degrees, its attractive force seeming to reside in a higher, slower Heaven than the preceding, performing one revolution in about 18 years according to the order of the degrees ¹⁾).

CONCLUSION. We have thus spoken of the place of the forces which keep the Earth, the orbits and the Heavens of the Planets in their magnetic rest; as required.

5th PROPOSITION.

To expound that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason.

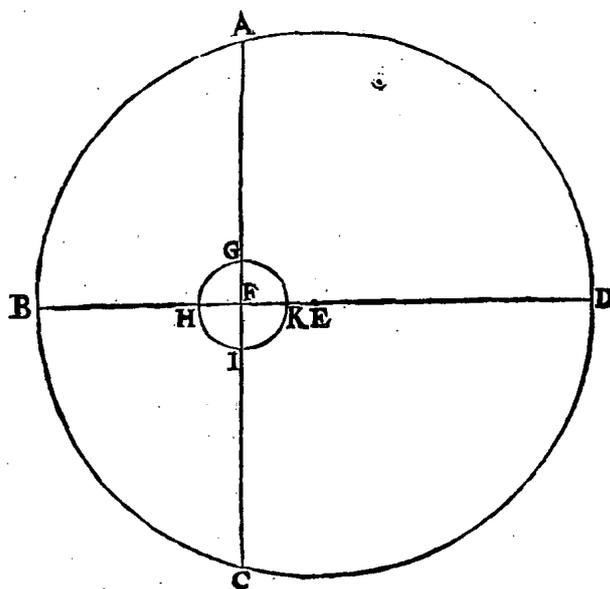
Just as it is expedient in Geography to choose on the Earth some half-meridian which is generally looked upon as the starting point by all those who deal with the

¹⁾ This statement is wrong, though it is not a clerical or a printer's error.

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meen voor begin gehouden wort, daer eyghentlicker af gheseyt is inde 4 bepaling vant 1 bouck des Eertclootschrifts, alsoo ist int Hemelloopschrift oirboir inde werelt eenich punt te verkiefen, dat al de gene die hemlien inde selve s'lof oeffenen voor haer middelpunt annemen: Hier toe wort met stelling eens vasten Eertcloots billichlick den Eertcloot vercoren, want als men besluyt den hemel der vaste sterren op haer middelpunt te draeyen, soo moet den Eertcloot daer an sijn, of anders en soude d'een helft des ghesceerenden hemels niet boven den sichteinder noch d'ander helft daer onder sijn, strijdende teghen d'ervaring, ghelijck *Prolemus* dat verclaert int 5 Hoofstuck sijns 1 boucx.

Angaende des werelts middelpunt met stelling eens roerenden Eertcloots, daer wort billichlick de Son toe ghenomen, om datse ghenouchsaem het middelpunt is vande ronden beschreven deur de verstepunten der Dwaelersweghen, maer t'middelpunt vanden hemel der vaste sterren te wesen machmen vermoeden, dan men cant, soo ick meen, niet volcommelick bewijsen. Laet tot verclaring van dien A B C D den ghesceerenden hemel beteycken, diens



middelpunt E, waer deur ghetrocken is de middellijn B E D, en A C rechthouckich op B D, sniende de selve in F buyten t'middelpunt E, en op F als middelpunt sy beschreven den Eertcloot wech G H I K. Dit soo sijnde t'is kennelick dat hoewel de booch A B C cleender is dan C D A, nochtans anghesien het rondt G H I K altijt tot die plaets blijft, en het rondt A B C D stil staet, soo schijnt d'een en d'ander booch uyt des Eertclootwechs middelpunt F ghesien, altijt een half front te wesen, ghelijck A B of den houck A F B altijt een vierendeel schijnt: T'selve heeft hem oock alsoo ghesien uyt den Eertcloot tot yder plaets des omtreux G H I K om dat de heele middellijn H K gheen ghevoelicke reden en heeft teghen de halfmiddellijn B E of teghen B H: Men mocht oock aldus segghen; nadien den heelen Eertcloots hemel verleken by den hemel der vaste sterren, maer en is als een punt, soo en connen wy niet bewijsen de Son meer middelpunt des vastesterrens hemel te wesen als het verstepunt des Eert-

same subject matter, as is described more properly in the 4th definition of the 1st book of Geography, so it is expedient in Astronomy to choose in the world some point which all those who pursue this subject take for its centre. For this, on the theory of a fixed Earth, the Earth is justly chosen, for if it is decided that the Heaven of the fixed stars rotates about its centre, the Earth must be there; otherwise one half of the starry Heaven would not be above the horizon, nor the other half below it, which is contrary to experience, as *Ptolemy* sets it forth in the 5th Chapter of his 1st book.

As to the world's centre on the theory of a moving Earth, for that the Sun is justly taken, because it is sufficiently near the centre of the circles described through the apogees of the Planets' orbits; but that it is the centre of the Heaven of the fixed stars can be surmised, but in my opinion it cannot be fully proved. By way of explanation let *ABCD* denote the starry Heaven, whose centre be *E*, through which is drawn the diameter *BED*, and *AC* at right angles to *BD*, intersecting the latter in *F* outside the centre *E*; and about *F* as centre let there be described the Earth's orbit *GHIK*. This being so, it is evident that, though the arc *ABC* is smaller than *CDA*, yet since the circle *GHIK* always remains in that place and the circle *ABCD* stands still, one arc as well as the other, seen from the centre of the Earth's orbit *F*, seems always to be a semi-circle, just as *AB* or the angle *AFB* always seems to be a quarter circle. The same is also the case, when seen from the Earth in any place on the circumference *GHIK*, because the entire diameter *HK* has no perceptible ratio to the semi-diameter *BE* or to *BH*. We might also say as follows: Since the entire Heaven of the Earth is but as a point in comparison with the Heaven of the fixed stars, we cannot prove that the Sun rather than the apogee of the Earth's orbit or any other point contained therein is the centre of the heaven of the fixed stars. Nay, it

clootwechs, of eenich ander punt daer in begrepen: Ia men macht daer voor houden, dat self Saturnus hemel verleken by den hemel der vaste sterren maer en is als een punt, om dese reden: Haer halfmiddellijn is ontrent de negen mael soo lanck als des Eertcloot hemels halfmiddellijn, ghelijck int voorstel deses 3 boucx blijcken sal: Hier uyt volght dat de vaste sterren ghesien uyt Saturnus wech, een verscheensicht ofte voor of achtring krijghen, negen mael soogroot als hemlien verscheensicht uyt den Eertclootwech ghesien, dat negen mael een onbemerckelicke sake, welcke oock of onbemerckelick is, of immers seer cleen moet sijn: Nu dan Saturnus hemel als middelpunt schijnende des hemels der sterren, soo sal elck punt in Saturnus hemel begrepen, meughen ghenomen worden voor middelpunt des vaste sterrens hemel, sonder uyt middelpunticheyt te connen bemerckt worden, en vervolghens soo en schijnet niet bewijfelick de Son meer haer eygentlick middelpunt te wesen dan t'verstepunt van Saturnus wech, of eenich ander in sijn hemel begrepen.

Angaende *Copernicus* int 10 Hoofstuck sijns 1 boucx vraecht, wie in dese schoonste kercke die lampe in een ander beter plaets soude stellen dan int middel, van daer sijt over al t'lamen mach lichten? t'sijn wel beweeghlicke natuerlike redenen, maer op gheen Meetconstich bewijs ghegront. Soo veel isser af, by aldienmen eenich ander puut dan de Son, ick neem des Eertclootwechs middelpunt, wilde houden voor werelts middelpunt, stellende de Son daer rontom te draeyen, met een halfmiddellijn even an des Eertclootwechs uytmiddelpunticheytlijn, men soude daer op een beschrijving des Hemelloops connen doen sonder dwaling, maer de Son voor werelts middelpunt te nemen valt gherievigher, soo wel om bequamelick te leeren de overeencomminghen der stellinghen eens vasten en roerenden Eertcloots, daer hier na afgheschreven sal worden, als om meer ander ontmoetende saken die aldus lichter en verstaenlicker sijn. **T B E S L V Y T.** Wy hebbē dan verclaert datter niet nootfakelick en blijckt de Son middelpunt te wesen vanden vastesterrens hemel, maer met goede reden daer toe vercoren wort, na den eych.

6 VOORSTEL

Te segghen vande verwonderinghen sonder vvonder der ghene die een vasten Eertcloot stellen.

Ettelicke der gene die *Ptolemæus* beschrijving der Dwaelderloopen met een vasten Eertcloot verstaen, en voor recht houden, verwonderen hun in sommige eyghenschappen dieer in mercken: Ten eersten dat Saturnus, Iupiter en Mars in teghestant der Son alijt ten naesten by den Eertcloot commen, maer in saming ten versten. Ten tweeden dat haer loop int inront alijt effen overcomt metter overschot des Sonloops boven den loop van haer inronts middelpunten. Ten derden dat Venus en Mercurius t'verkeerde ghebeurt, want haer loop int inront en heeft mette Son sulcke overcomming niet, maer den loop van haer inronts middelpunt isser even me: Dit houden sy voor een teycken van besonderheyt des weerdichsten Dwaelders de Son, na wiens roersel d'ander als na een Koninck opsicht nemen en haer loop vervoughen: Doch men macht houden voor ghedwaelde* spieghelinghen, volghende uyt ghemiste Theorij. stelling eens vasten Eertcloots. Maer want dit groote ghelijckheyt heeft met luyden die het sloopvaren onghewoon sijnde, ghemeenlick het roersel van haer schip ander schepen toeschrijven, als wanneer sy die teghencommen en beneen

may be assumed that even Saturn's Heaven, in comparison with the Heaven of the fixed stars, is but as a point, for the following reason. Its semi-diameter is about nine times the length of the semi-diameter of the Earth's Heaven, as will appear in the [13th]¹⁾ proposition of this book. From this it follows that the fixed stars, seen from Saturn's orbit, receive a parallax or advance-or-lag nine times greater than their parallax when seen from the Earth's orbit, that is nine times an imperceptible amount, which is also either imperceptible or at any rate must be very small. If therefore Saturn's Heaven appears as centre of the Heaven of the fixed stars, any point contained in Saturn's Heaven can be taken for centre of the Heaven of the fixed stars, without eccentricity being perceptible, and consequently it does not seem possible to prove that the Sun is its true centre rather than the apogee of Saturn's orbit, or any other point contained in its Heaven.

As to the fact that *Copernicus* asks in the 10th Chapter of his 1st book who would in this beautiful church place that lamp in any other, better place than in the middle, from where it can illuminate everything: these are moving, natural reasons indeed, but not based on a geometrical proof. So much is true that if one wished to take any point other than the Sun, for example the centre of the Earth's orbit, for the centre of the world, assuming the Sun to revolve about it, with a semi-diameter equal to the line of eccentricity of the Earth's orbit, one might base on this a description of the Heavenly Motions without any error; but it is more convenient to take the Sun for the centre of the world, both in order to learn properly the correspondences of the theories of a fixed and a moving Earth, which are to be described hereinafter, and on account of other matters that may arise, which are easier and more intelligible in this way. CONCLUSION. We have thus expounded that it appears not to be necessary that the Sun should be the centre of the Heaven of the fixed stars, but that it is chosen for this with good reason; as required.

6th PROPOSITION.

To speak of the wondering at what is no wonder, of those who assume a fixed Earth.

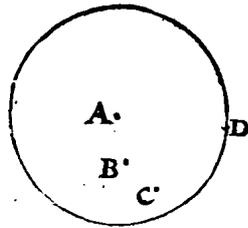
Some of those who understand *Ptolemy's* description of the Planetary Motions based on a fixed Earth and consider it correct, are astonished at some properties they perceive therein. Firstly, that Saturn, Jupiter, and Mars, when in opposition to the Sun, always come nearest to the Earth, but when in conjunction, farthest from it. Secondly, that their motion on the epicycle always corresponds exactly to the surplus of the Sun's motion over the motion of the centres of their epicycles. Thirdly, that with Venus and Mercury the converse takes place, for their motion on the epicycle has no such correspondence to the Sun's motion, but the motion of the centre of their epicycle is equal to it. They take this for a sign of the special character of the worthiest Planet, the Sun, from whose motion the others take their guidance as from a King and move accordingly. But these may be considered erroneous speculations, resulting from the incorrect theory of a fixed Earth. But since this is greatly similar to people who, not being used to sailing, generally ascribe the motion of their ship to other ships, — such as,

¹⁾ Apparently this number has been inadvertently omitted in the original text.

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beneen boort ligghen, sonder water of landt te sien, segghen, hoe ras vaert dat schip buyten t'onse. Of hun schip een keer doende, segghen t'ander dat mischien stil light rontom het haer te draeyen, soo sal ick dit als voorbeeld ghebruycken tot verclaring deser stof.

Laet dese seven punten A, B, C, D, E, F, G, seven schepen in zee beteycken waer af A den Admirael sijnde stil light: Maer t'schip D vaert gheduerlick in een rondt, daer de drie schepen A, B, C, binnen sijn, en de drie E, F, G,



buytē. Dit soo sijnde, en ymant in het schip D wesende, meynt na de boveschreven ghemeene wijze dattet stille light, en d'ander al rontom hem onghereghelt draeyen, En volghende sulck gestelde neemt acht op de ghedaente des loops, en seght met

een verwonderen aldus: Wat een vreemde saeck ist, dat telckmael als een der drie schepen E, F, G, comt in een rechte lini van hem over ons totten Admirael A, soo is dan elck van dien ons altijt ten naesten: En ten versten, wesende in de selve rechte lini over d'ander sijde vanden Admirael, hoe onghereghelt oock hun gheduerighe vaert is: Hier uyt besluyt hy elck dier drie schepen noch te draeyen in een cleender rondt, daer deur sy naerderen en afwijcken, hem verwonderende waerom sulcken keer in tijt seker overcomming heeft metten keer vanden Admirael: Sghelijcx voor een vreemdicheyt houdende waerom de twee schepen CB, oock een regel houden metten Admirael, doch verkeert vande voorgaende, te weten dat den keer des groote ronts dese doen om t'schip D draeyende, in tijt effen overcomt met een keer des Admirael, seght voort dat sulcx is een teycken van eerbieding die den Admirael van d'ander schepen anghedaen wort.

Dit soo sijnde, ghenomen nu dat een ervaren Schipper wetende hoet mette saeck ghestelt is, tegen sulck een aldus seyde: Ghy breeckt u hoofd met voor wonder te houden daer geen wonder en is, want ons schip t'welck ghy meent stil te legghen, vaert gheduerlick rontom de drie schepen A, B, C, waer uyt nootfakelick volghet, dat soo dickwils wy sijn tusschen den Admirael A en een der drie E, F, G, soo moet ons dan elck van dien ten naesten sijn, en ten versten als A tusschen ons en een van hemlien is: Inder voughen dat die schepen niet en varen in ronden, met soodanighen versierden loop, die hun doet naerderen en afwijcken, noch oock de twee schepen B, C, in sulcke ronden, effen overcommende metten loop van A, soo ghy meent: Maer men mochtet voor onnatuerlick houden dat t'ghene voor de onervarenen alsoo schijnt, cyghentlick niet anders en waer.

Ende even eens soude een ervaren Hemelmeeter tot een onervaren meughen segghen: Ghy breeckt u hoofd met voor wonder te houden daer gheen wonder en is, want ons weereltlicht dats den Eerteloot die ghy meent stil te ligghen, draeyt gheduerlick rontom de drie Dwaelers, Son, Venus, Mercurius, waer uyt nootfakelick volghet dat soo dickwils wy sijn tusschen de Son en een

when they meet them while lying below deck without seeing water or land, they say: how fast that ship outside ours sails, or, if their ship makes a turning, say that the other, which perhaps lies still, moves round theirs — I will use this as an example to illustrate this subject matter.

Let these seven points *A, B, C, D, E, F, G* denote seven ships at sea, of which *A*, being the Admiral, lies still. But the ship *D* continually sails in a circle, within which are the three ships *A, B, C*, while the three *E, F, G* are on the outside. This being so, a man being in the ship *D* is of opinion, according to the above, that it lies still and all the others move irregularly about it. And according to this supposition he observes the nature of the motion and, wondering, says as follows: How strange it is that whenever one of the three ships *E, F, G* comes in a straight line from it *via* us to the Admiral *A*, each of them is always nearest to us; and farthest, when it is in this straight line to the other side of the Admiral, however irregular their continual sailing may be! From this he concludes that each of those three ships also moves in a smaller circle, in consequence of which they approach and withdraw, wondering why this turning has an exact correspondence in time to the turning of the Admiral. Likewise, considering it strange why the two ships *C, B* also have a connection with the Admiral, but contrary to the foregoing, to wit, that the motion of the large circle they perform in moving round the ship *D* corresponds exactly in time to one turning of the Admiral, he says further that this is a sign of homage paid to the Admiral by the other ships.

This being so, let us assume that a skilled Skipper, knowing what is the matter, said to such a person as follows. You rack your brains in wondering where there is no wonder, for our ship, which you think is lying still, is continually sailing round the three ships *A, B, C*, from which it follows necessarily that whenever we are between the Admiral *A* and one of the three *E, F, G*, each of those must be nearest to us, and farthest from us when *A* is between us and one of them; so that those ships do not sail in circles, with such an imagined motion as makes them approach and withdraw, nor do the two ships *B, C* sail in such circles, corresponding exactly to the motion of *A*, as you think. But it might be considered unnatural that what seems thus to the inexperienced should not in reality be otherwise.

And in the same way an experienced Astronomer might say to an inexperienced one: You rack your brains in wondering where there is no wonder, for our luminary, *i.e.* the Earth, which you think lies still, travels continually round the three Planets Sun, Venus, Mercury, from which it follows necessarily that whenever we are between the Sun and one of the three: Saturn, Jupiter, Mars,

EEN ROERENDE EERTCLOOT. 261

een der drie Saturnus, Iupiter, Mars, soo moet ons dan elck van dien ten naefsten sijn, en ten versten als de Son tusschen ons en een van hemlien is: Inder voughen dat die drie Dwaelders niet en draeyen in rondon met foodanighen versterden loop die hun doet naerderen en afwijcken, noch oock de twee Venus en Mercurius in sulcke rondon, effen overcommende mette Sonloop soo ghy meent, maer men mochtet voor onnatuerlick houden dat t'ghene voor d'onervaernen soo schijnt niet eyghentlick anders en waer.

Noch isser by veelen een verwonderen vande seltsaem haspeling des breedeloops der Dwaelders Saturnus, Iupiter, Mars, Venus, en Mercurius, ghegront op stelling eens vasten Eertcloots: Maer volghens de stelling eens roerenden Eertcloots, soo en isser gheen wonder, dan sijn eenvoudelicke weggen afwijckende vanden duysteraer, ghelijck de Maenwech, waer uyt rekeninghen der breedte volghen met kennis der oirsaken, als t'sijnder plaets blijcken sal.

T B E S L V Y T. Wy hebben dan gheseyt vande verwonderinghen sonder wonder der ghene die een vasten Eertclood stellen.

Dit eerste Onderscheyt van der Dwaelders Hemelen ghedaente ten einde sijnde, ick sal nu tottet beschrijven des loops commen, en eerst vande langde-loop.

Z V A N

each of them must be nearest to us, and farthest from us when the Sun is between us and one of them; so that those three Planets do not move in circles with such an imagined motion as makes them approach and withdraw, nor do the two, Venus and Mercury, move in such circles, corresponding exactly to the Sun's motion, as you think. But it might be considered unnatural that what seems thus to the inexperienced should not in reality be otherwise.

Many people also wonder at the curious jumbling of the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, based on the theory of a fixed Earth. But according to the theory of a moving Earth there is nothing astonishing, but they are simple orbits deviating from the ecliptic, like the Moon's orbit, from which follow computations of the latitudes with knowledge of the causes, as will appear in its place.

CONCLUSION. We have thus spoken of the wondering at what is no wonder, of those who assume a fixed Earth.

This first Chapter of the figure of the Planets' Heavens being at an end, I am now coming to the description of their motion, and first of their motion in longitude.

EERTCLOOTLOOPS VINDING
**VANDE LANG-
 DE LOOP.**

TWEEDE
ONDERSCHEYT
 DES DERDEN BOVCX VAN
 des Eertcloots loop en de Son-
 nens schijnbaer roerfel.

CORTBEGRYP DESES
 TWEEDEN ONDERSCHEYTS.

 *A vier bepalinghen sullen volgen vier voorstellen: T eerste, wesenende in d'oirden het 7 vanden loop des Eertcloots in haer vech.*

Het vveede, wesenende in d'oirden het 8, dat de Son met stelling eens roerenden Eertcloots, de selve schijnbaer duysteraerlangde, verbeert vanden Eertclood, en voorofachtring ontfangt, diese heeft met stelling eens vasten Eertcloots.

Het derde, wesenende in d'oirden het 9, dat des Eertcloodvachs naeste punt onder de selve duysteraerlangde is, daer de Sonvachs verstepunt met stelling eens vasten Eertcloots onder is.

Het vierde, t'welck in d'oirden is het 10, dat wesenende de Son met stelling eens vasten Eertcloots in haer vechs verstepunt of eerste halfront, den Eertclood met haer roerende stelling oock in haer vechs verstepunt of eerste halfront is, en sulcken langde en voorofachtring de Son in haer vech heeft, dergbelijcke langde en voorofachtring oock den Eertclood inde hare te hebben.

I BEPALING.

Wesende de Son ghenomen vast te staen als vveerelts middelpunt, en den Eertclood daer rontom te draeyen, in een rondt en met een roerfel even ant rondt en roerfel datmen de Son met stelling eens vasten Eertcloots toeschrijft: Sulcx heet stelling eens roerenden Eertcloots.

2 BE-

OF THE MOTION IN LONGITUDE

SECOND CHAPTER

OF THE THIRD BOOK

Of the Motion of the Earth and the Sun's Apparent Motion

SUMMARY OF THIS SECOND CHAPTER

After four definitions there are to follow four propositions: The first, which in the sequence is the 7th, of the motion of the Earth in its orbit.

The second, which in the sequence is the 8th, that on the theory of a moving Earth the Sun acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth.

The third, which in the sequence is the 9th, that the nearest point (perihelion) of the Earth's orbit is at the same ecliptical longitude where the farthest point (apogee) of the Sun's orbit is on the theory of a fixed Earth.

The fourth, which in the sequence is the 10th, that when, on the theory of a fixed Earth, the Sun is at the farthest point (apogee) of its orbit or its first semi-circle, the Earth, when assumed to be moving, is also at the farthest point (aphelion) of its orbit or its first semi-circle, and that such longitude¹⁾ and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit.

1st DEFINITION.

When it is assumed that the Sun is fixed as the world's centre and that the Earth moves round it, in a circle and with a motion equal to the circle and the motion ascribed to the Sun on the theory of a fixed Earth, this is called the theory of a moving Earth.

¹⁾ The longitude "in its orbit" is reckoned from the apogee.

2 B E P A L I N G.

Wefende de Maen ghefelt te draeyen rontom den roerendē Eertclood, gelijk mēse andersins neemt te draeyen rontom een vasten: Sulcx heet Maenloop met stelling eens roerenden Eertcloots.

3 B E P A L I N G.

Wefende gestelt den Eertclood te loopen in een vech even ant inrondt van Saturnus, Jupiter of Mars, en de selve Dvvaelders niet in haer inrondt als met stelling eens vasten Eertcloots, maer ter plaets van des inronts middelpunt: Sulcx heet haer loop met stelling eens roerenden Eertcloots.

4 B E P A L I N G.

Wefende gestelt den Eertclood te loopen in een vech even an den inrontvvech van Venus of Mercurius, en de selve Dvvaelders in haer inrondt binnen den Eertcloodvvech: Sulcx heet haer loop met stelling eens roerenden Eertcloots.

7 V O O R S T E L.

Te beschrijven den loop des Eertcloots in haer vech op een ghegeven tijt.

De boucken des Hemelloops *Ptolemus* ter handt ghecommen, betuyghen dat de Ouden voor hem een ghebruyck hadden, inde beschrijving van yder Dwaelder te beginnen met sijn daghelicksche loop, of anders gheseyt met sijn loop op een bekenden tijt deur ervaring bevonden, welcke wijze in reden ghegront schijnende, ick salse in dit derde bouck met stelling eens roerenden Eertcloots soo navolgen, ghelijck ick int eerste mette stelling eens vasten gedaen heb. Het is dan te weten dat yder Dwaelder een ander wesentlicke loop heeft dan men hem deur de stelling eens vasten Eertcloots toeschrijft: T'ghene daer af vanden Eertclood te segghen valt, t'is dat hy in hem heeft den wesentlicken loop diemen in d'ander stelling de Son-toeschrijft, welcke int 3 voorstel des 1 bouck berekent is sdaechs op 59 \odot 8. 17. 13. 12. 31. en int natuerlick jaer een keer te doen. T' B E S L V Y T. Wy hebben dan beschreven den loop des Eertcloots in haer wech op een ghegeven tijt, na den eyfch.

2nd DEFINITION.

When the Moon is assumed to move round the moving Earth, as it is otherwise taken to move round a fixed Earth, this is called motion of the Moon on the theory of a moving Earth.

3rd DEFINITION.

When it is assumed that the Earth moves in an orbit equal to the epicycle of Saturn, Jupiter or Mars, and the said Planets do not move on their epicycles, as on the theory of a fixed Earth, but are at the place of the epicycle's centre, this is called their motion on the theory of a moving Earth.

4th DEFINITION.

When it is assumed that the Earth moves in an orbit equal to the deferent of Venus or Mercury, and the said Planets move on their epicycles within the Earth's orbit, this is called their motion on the theory of a moving Earth.

7th PROPOSITION.

To describe the motion of the Earth in its orbit in a given time.

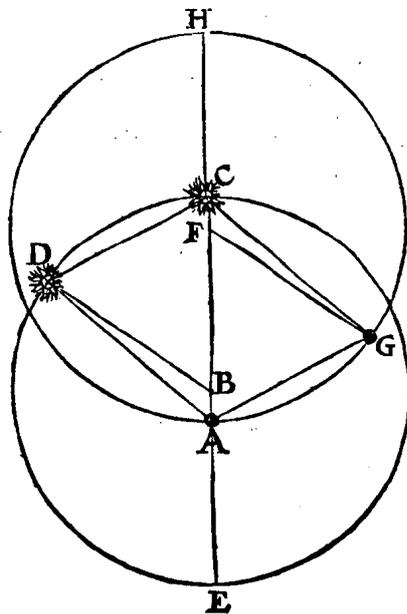
The books on the Heavenly Motions that came into *Ptolemy's* hands declare that the Ancients before him were accustomed to begin in the description of each Planet with its daily motion, in other words: with its motion in a given time found by experience, which method, seeming to be based on good reasons, I will follow in this third book on the theory of a moving Earth, as I have done in the first on the theory of a fixed Earth. It should be known that each planet has a true motion different from the one ascribed to it on the theory of a fixed Earth. What can be said in this respect of the Earth is that it has in it the true motion which is ascribed to the Sun on the other theory, which in the 3rd proposition of the 1st book has been computed to move 59;8,17,13,12,31 minutes a day and to perform one revolution in the natural year. CONCLUSION. We have thus described the motion of the Earth in its orbit in a given time; as required.

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VERTOCCH. 8 VOORSTEL.

De Son ontfangt met stelling eens roerenden Eertcloots, de selve schijnbaer duyfteraerlangde, verheyt vanden Eertcloon, en voorofachtring, diese heeft met stelling eens vasten Eertcloots.

T'GHEGHEVEN. Laet voor eerste stelling ghenomen worden t'punt A een vasten Eertcloon te beteykenen, van A tot B sy de Sonwechs uytmiddelpunticheytlijn, doende na *Ptolemeus* rekening sulcke 417, alser de halfmiddellijn die BC sy 10000 doet, mette selve BC sy op B als middelpunt beschreven de Sonwech CDE, waer in CA voortgetrocken tot E, soo is E t'nacstepunt, C t'verstepunt, waer an ick voor t'eerste neem de Son te wesen, welcke van daer ghecommen sy tot D.

Laet voor tweede stelling ghenomen worden den Eertcloon A te loopen,



en de Son C vast te staen: Tot desen einde teycken ick in CA t'punt F, alsoo dat de uytmiddelpunticheytlijn CF evē syan AB, en beschrijf op F als middelpunt, mette halfmiddellijn FA, die evē moet sijn mette halfmiddellijn BC, het ronr AGH als Eertcloonwech, diens naestpunt H, verstepunt A, waer in ick neem dē Eertcloon van A gecommen te sijn tot G, sulcx dat de booch AG even is met de booch CD: Laet daer na ghetrocken worden de ses rechtelinien AD, GC, CD, GA, BD, FG.

T'BEGERDE. Wy moeten bewijsen dat de vaste Son an C, gesien uyt den loopenden Eertcloon an G, de selve schijnbaer duyfteraerlangde

en verheyt heeft der loopende Son an D, gesien uyt den vasten Eertcloon an A.

T'BEWYS. Ten eersten segh ick dat wesende de Son an C, en den Eertcloon an A, t'sy datmen neemt den Eertcloon A vast te staen, en de Son C loopende, of den Eertcloon A loopende, en de Son C vast, dat gheeft openbaerlick int punt des tijts datse alsoo elck an des anders verstepunt sijn een selve schijnbaer plaets, en verheyt der Son vanden Eertcloon: Maer om te bewijsen dat sulcx overal gheschiet, ick segh aldus: Anghesien de booch AG even en ghelijck is mette booch CD, soo is den * evebeenighen driehouck BCD, even en ghelijck metten evebeenighen FAG, en daerom den houck BCD, even met tē houck FAG, dats oock ACD met CAG, waer deur de twee even rechte linien DC, AG ewewijdeghe sijn, en de twee rechte AD, GC daer tusschen ghetroc-

Uytsaet.

THEOREM.

8th PROPOSITION.

On the theory of a moving Earth the Sun acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth.

SUPPOSITION. Let it be assumed first that the point A denotes a fixed Earth; let the line from A to B be the line of eccentricity of the Sun's orbit, which according to *Ptolemy's* computation makes 417 if the semi-diameter, which shall be BC , makes 10,000. With this BC let there be described about B as centre the Sun's orbit CDE , in which, if CA is produced to E , E is the perigee, C the apogee, at which I first take the Sun to be, which shall have travelled thence to D .

Let it be assumed secondly that the Earth A moves and the Sun C is fixed. To this end I mark on CA the point F so that the line of eccentricity CF be equal to AB , and I describe about F as centre, with the semi-diameter FA , which must be equal to the semi-diameter BC , the circle AGH as the Earth's orbit, its perihelion being H , its aphelion A , in which I take the Earth to have travelled from A to G , so that the arc AG is equal to the arc CD . Thereafter let there be drawn the six straight lines AD , GC , CD , GA , BD , FG .

WHAT IS REQUIRED. We have to prove that the fixed Sun at C , seen from the moving Earth at G , has the same apparent ecliptical longitude and distance as the moving Sun at D , seen from the fixed Earth at A .

PROOF. Firstly I say that when the Sun is at C and the Earth at A , whether the Earth A is taken to be fixed and the Sun C moving or the Earth A moving and the Sun C fixed, this gives evidently as regards time that thus each is at the other's farthest point at the same apparent place and distance of the Sun from the Earth. But to prove that this happens always, I say as follows: Since the arc AG is equal and similar to the arc CD , the isosceles triangle BCD is equal and similar to the isosceles triangle FAG , and therefore the angle BCD is equal to the angle FAG , *i.e.* also ACD to CAG , in consequence of which the two equal straight lines DC , AG are parallel, and the two straight lines AD , GC drawn between

ghetrocken moeten oock even en ewewijdeghe wesen : Maer A D ewewijdege sijnde met G C , soo moet de vaste Son C ghesien uyt den loopenden Eertclood an G , op de selve plaets schijnen der loopende Son an D ghesien uyt den vasten Eertclood an A , om dat A G noch oock de heele middellijn des Eertcloodwechs gheen ghevoelicke reden en heeft totte halfmiddellijn des vaste sterrecloots. Oock ist openbaer de verheyte G C even te sijn mette verheyte A D , ghemerckt het twee ewewijdeghe sijn tusschen de twee ewewijdege D C , A G . Angaende het derde punt , te weten dat de voorofachtring van d'eenen d'ander stelling de selve is , blijkt aldus : De roerende Son an D wort uyt den vasten Eertclood A , soo veel in den duystraer schijnbaerlick meer achterwaert ghesien dan de middelson (die anghewesen is mette lini welke vande Sonwechs middelpunt B deur D na den duystraer streckt) als den houck A D B bedraecht : Maer soo veel wort de vaste Son uyt den roerenden Eertclood G oock inden duystraer schijnbaerlick meer achterwaert ghesien dan de Middelson (die anghewesen is mette lini welke vanden Eertclood G deur des Eertcloodwechs middelpunt F na den duystraer streckt) om dat A D ewewijdege is met G C , en B D met G F , en vervolghens den houck A D B even metten houck C G F , waer deur de voorofachtring , t'welck hier achtring valt , van d'een en d'ander stelling de selve is. T' B E S L V Y T . De Son dan ontfangt met stelling eens roerenden Eertcloots , de selve schijnbaer duystraerlangde , verheyte vanden Eertclood , en voorofachtring , die se heeft met stelling eens vasten Eertcloots , t'welck wy bewijfen moesten.

9 V O O R S T E L .

Des Eertcloodwechs naestepunt onder de selve duystraerlangde te wesen daer des Sonwechs verstepunt met stelling eens vasten Eertcloots onder is.

T' G H E G H E V E N . Laet de form des 8 voorstels andermael voor t'ghegeven ghenomen worden , alwaer blijkt H naestepunt des Eertcloodwechs onder de selve duystraerlangde te wesen , daer de Sonwechs verstepunt C met stelling eens vasten Eertcloots onder is , want ghetrocken van C (t'welck is de Son als weerelts middelpunt met stelling eens roerenden Eertcloots) een rechte lini deur haer naestepunt H , sy wijft inden duystraer t' selve punt der langde die anghewesen wort mette rechte lini ghetrocken van A (t'welck is den Eertclood als weerelts middelpunt met stelling eens vasten Eertcloots) deur C verstepunt des Sonwechs. T' B E S L V Y T . Des Eertcloodwechs naestepunt dan is onder de selve duystraerlangde daer des Sonwechs verstepunt met stelling eens vasten Eertcloots onder is , t'welck wy bewijfen moesten.

V E R T O O C H . 10 V O O R S T E L .

Wesende de Son met stelling eens vasten Eertcloots in haers vvechs verstepunt , of eerste halffront , den Eertclood is met haer roerende stelling oock in haer vvechs verstepunt , of eerste halffront , en sulcken langdeen voor-

Z 3 ofach-

them must also be equal and parallel. But when AD is parallel to GC , the fixed Sun C , seen from the moving Earth at G , must appear to be in the same place as the moving Sun at D , seen from the fixed Earth at A , because neither AG nor the entire diameter of the Earth's orbit has any perceptible ratio to the semi-diameter of the sphere of the fixed stars. It is also evident that the distance GC is equal to the distance AD , since they are two parallel lines between the two parallel lines DC , AG . As to the third point, to wit, that the advance-or-lag is the same for one theory and for the other, this becomes apparent as follows: The moving Sun at D is seen from the fixed Earth A as much apparently more backwards in the ecliptic than the mean sun (which is denoted by the line which extends from the centre of the Sun's orbit B through D to the ecliptic) as is the amount of the angle ADB . But this amount also the fixed Sun is seen from the moving Earth G apparently more backwards in the ecliptic than the Mean Sun (which is denoted by the line which extends from the Earth G through the centre of the Earth's orbit F to the ecliptic) because AD is parallel to GC , and BD to GF , and consequently the angle ADB is equal to the angle CGF , in consequence of which the advance-or-lag, which is lag in this case, is the same for one theory and for the other. **CONCLUSION.** The Sun therefore on the theory of a moving Earth acquires the same apparent ecliptical longitude, distance from the Earth, and advance-or-lag which it has on the theory of a fixed Earth; which we had to prove.

9th PROPOSITION.

That the perihelion of the Earth's orbit is at the same ecliptical longitude where the apogee of the Sun's orbit is on the theory of a fixed Earth.

SUPPOSITION. Let the figure of the 8th proposition be taken once more for the supposition, where it appears that H , the perihelion of the Earth's orbit, is at the same ecliptical longitude where the apogee of the Sun's orbit C is on the theory of a fixed Earth; for when from C (which is the Sun as the world's centre on the theory of a moving Earth) a straight line is drawn through its perihelion H , it indicates in the ecliptic the same point of longitude that is indicated by the straight line drawn from A (which is the Earth as the world's centre on the theory of a fixed Earth) through C , the apogee of the Sun's orbit. **CONCLUSION.** The perihelion of the Earth's orbit is therefore at the same ecliptical longitude where the apogee of the Sun's orbit is on the theory of a fixed Earth; which we had to prove.

THEOREM.

10th PROPOSITION.

When the Sun, on the theory of a fixed Earth, is at the apogee of its orbit, or its first semi-circle, the Earth, when assumed to be moving, is also at the farthest point (aphelion) of its orbit, or its first semi-circle, and such longitude

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ofachtring de Son in haer vvech heeft, dergelijcke langde en voorofachtring heeft oock den Eertclood inde hare.

Laet de form des 8 voorstels andermael voor t'ghegheven verstrecken, waer me ick aldus segh: Wefende de Son met stelling eens vasten Eertcloots in haer wechs C D E verstepunt C, t'is openbaer dat den Eertclood dan met haer roerende stelling oock is in haer wechs A G H verstepunt A.

Maer wefende de Son met stelling eens vasten Eertcloots in haer wechs C D E eerste halffront C D E an D, t'is openbaer dat den Eertclood dan met haer roerende stelling oock is in haer wechs A G H eerste halffront A G H.

Voort sulcken langde de Son an D heeft in haer wech C D E, te weten de booch C D, dergelijcke langde heeft oock den Eertclood an G in haer wech A G H, te weten den booch A G, want die even is met C D deur t'ghefelde.

Ten laetsten sulcken achtring A D B de Son D, heeft in haer wech C D E, dergelijcke achtring heeft oock den Eertclood G, in haer wech A G H, want den houck der achtring A D B, is even metten houck C G F.

T B E S L V Y T. Wefende dan de Son met stelling eens vasten Eertcloots in haer wechs verstepunt, of eerste halffront, den Eertclood is met haer roerende stelling oock in haer wechs verstepunt of eerste halffront, en sulcken langde en voorofachtring de Son in haer wech heeft, dergelijcke langde en voorofachtring heeft oock den Eertclood inde hare, t'welck wy bewijfen moesten.

V E R V O L G H.

Anghesien den Eertclood mette roerende stelling alijt tot sulcken plaets haers wechs is, als de Son met stelling eens vasten Eertcloots inde hare, soo volght daer uyt dat tot sulcke drie plaetsen als men de Son in haer wech neemt te wesen om daer deur de uymiddelpunticheyt te berekenen, tot sulcke drie plaetsen moet oock den Eertclood wesentlick sijn, sulcx datmen met haer loopende stelling soude moghen berekenen de selve uymiddelpunticheyt, metsgaders de effening der daghen, en alles watter int tweede bouck deur stelling eens vasten Eertcloots berekent wort, maer sich in sulcke rekeningen een vasten Eertclood int ghedacht te prenten valt gherievigher, om de redenen die t'haerder plaets breeder verclaert sulcken worden.

DERDE

and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit.

Let the figure of the 8th proposition serve once more for the supposition, so that I say as follows. When the Sun, on the theory of a fixed Earth, is at the farthest point (apogee) C of its orbit CDE , it is evident that the Earth, when assumed to be moving, is also at the farthest point (aphelion) A of its orbit AGH .

But when the Sun, on the theory of a fixed Earth, is in the first semi-circle CDE of its orbit CDE at D , it is evident that the Earth, when assumed to be moving, is also in the first semi-circle AGH of its orbit AGH .

Further, such longitude as the Sun has at D in its orbit CDE , to wit, the arc CD , the same longitude the Earth also has at G in its orbit AGH , to wit, the arc AG , because this is equal to CD by the supposition.

Lastly, such lag ADB as the Sun D has in its orbit CDE , the same lag the Earth G also has in its orbit AGH , because the angle of the lag ADB is equal to the angle CGF .

CONCLUSION. When the Sun therefore, on the theory of a fixed Earth, is at the apogee of its orbit, or its first semi-circle, the Earth, when assumed to be moving, is at the aphelion of its orbit, or its first semi-circle, and such longitude and advance-or-lag as the Sun has in its orbit, the same longitude and advance-or-lag the Earth also has in its orbit; which we had to prove.

SEQUEL.

Since the Earth, when assumed to be moving, is always in the same place of its orbit as the Sun in its orbit on the theory of a fixed Earth, it follows therefrom that in such three places as the Sun is taken to be in its orbit in order to compute the eccentricity therefrom, in the same three places the Earth also must be in reality, so that when it is assumed to be moving, this eccentricity might be computed, as well as the equation of time, and all that is computed in the second book on the theory of a fixed Earth; but it is more convenient in such computations to impress on one's mind a fixed Earth, for the reasons to be set forth in their place.

D E R D E
O N D E R S C H E Y T
 D E S D E R D E N B O V C X V A N
 d e M a e n e n s l a n g d e l o o p m e t s t e l -
 l i n g e e n s r o e r e n d e n
 E e r t c l o o t s .

C O R T B E G R Y P D E S E S
 D E R D E N O N D E R S C H E Y T S .



It derde Onderscheyt sal twee voorstellen hebben: T'eerste. wvessende in d'oirden het 11, om op een ghegeven tijt den loop des Maenwechs verstepunts en der duystering sne te vinden, deur vvisconstige vvercking ghegront op stelling eens roerenden Eertcloots.

Het tweede, wvessende in d'oirden het 12, dat de Maen met stelling eens roerenden Eertcloots de selve schijnbaer duyster aerlangde en verheyt vanden Eertcloodt ontfangt, diese heeft met stelling eens vasten Eertcloots.

I I V O O R S T E L .

Te vinden op een gegeven tijt den loop van des Maenwechs versteput, en der duystering sne, deur vvisconstige vvercking ghegront op stelling eens roerendē Eertcloots.

1 Voorbeelt van t'vinden des Maenwechs verstepunts middelloop.

T'GHEGHEVEN. Het is den tijt eens dachs. **T'BEGHEERDE.** Men wil daer op ghevonden hebben des Maenwechs verstepunts middelloop in schijnbaer duysteraerlangde, ghegront op stelling eens roerenden Eertcloots.

T' W E R C K .

Des Eertcloots middelloop doet deur het 3 voorstel des 1 boucx (welverstaende dat de getalē des Sonloops aldaer beschrevē hier om bekende reden voor Eertcloots middelloop ghenomen worden) daechs **011.59. 8.17.13.12.31.**
 Daer af ghetrocken de middelloop der voordering diemen des Maenwechs verstepunt met stelling eens vasten Eertcloots bevint te voorderen in schijnbaer duysteraerlangde op 1 dach, bedraghende deur het 11 voorstel des 1 boucx **011. 6.41. 2.15.38.31.**
Z 4 Blijft

THIRD CHAPTER
OF THE THIRD BOOK
Of the Moon's Motion in Longitude
on the Theory of a Moving Earth

SUMMARY OF THIS THIRD CHAPTER

This third Chapter is to contain two propositions: the first, which in the sequence is the 11th, to find in a given time the motion of the apogee of the Moon's orbit and of the nodes, by mathematical operations based on the theory of a moving Earth.

The second, which in the sequence is the 12th, that on the theory of a moving Earth the Moon acquires the same apparent ecliptical longitude and distance from the Earth that it has on the theory of a fixed Earth.

11th PROPOSITION.

To find in a given time the motion of the apogee of the Moon's orbit and of the nodes, by mathematical operations based on the theory of a moving Earth.

1st Example, of the Finding of the Mean Motion of the
Apogee of the Moon's Orbit.

SUPPOSITION. Let the time be one day. **WHAT IS REQUIRED.** It is required to find in this time the mean motion of the apogee of the Moon's orbit in apparent ecliptical longitude, based on the theory of a moving Earth.

PROCEDURE.

By the 3rd proposition of the 1st book the mean motion of the Earth (it being understood that for known reasons the figures of the Sun's motion there described are taken for the mean motion of the Earth) in one day is 0°;59, 8,17,13,12,31

When from this is subtracted the mean amount of the advance which the apogee of the Moon's orbit is found to make, on the theory of a fixed Earth, in apparent ecliptical longitude in 1 day, which by the 11th proposition of the 1st book is 0°; 6,41, 2,15,38,31

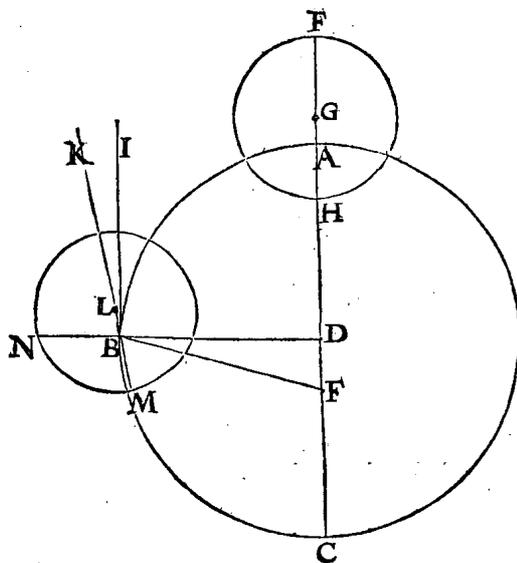
Blijft des Maenwechs verstepunts begeerde middel-

loop teghen t'vervolgh der trappen op een dach 0 tr. 52. 27. 14. 57. 34. 8.
Tot hier toe is voorbeelt ghegheven op den loop eens dachs, waer deur men
sekerder sien can t'groot verschil deses eyghen loops, teghen den oneygen met
stelling eens vasten Eertcloots, dan deur langhe tijden daer heele ronden in
commen diemen verlaet: Maer want de boochskens eensdachs seer cleen sijn,
sulcx dattet volghende bewijs daer deur soo claer niet vallen en soude als op
meerder, soo sal ick tot dien einde andermael nemen den tijt van 90 daghen.
Hier op doet den Eertcloots middelloop deur het 3 voorstel des
1 boucx 88 tr. 42.

Daer afgetrocken de middelloop der voordering diemen des Maen-
wechs verstepunt met stelling eens vasten Eertcloots bevint te
voorderen in schijnbaer duystraerlangde op 90 daghen, bedra-
ghende deur het 11 voorstel des 1 boucx 10 tr. 2.

Blijft des Maenwechs verstepunts begeerde middelloop tegen t'ver-
volgh der trappen op 90 daghen 78 tr. 40.

BEREYTSSEL VAN T'BEWYS. Om van dit bewijs wat int ghemēten te
segghen eer ick totte besonder verclaring commē, soe is voor al kennelick dat
des Maenwechs verstepunts loop die wy schijnbaerlick inden duystraer mer-
ken, ghemēgt is van haer eygen mette ghene die se vanden loop des Eertcloots
ontfangt, welke om tot rechte spiegeling tegeraken nootfakelick moeten on-
derscheyden sijn, ghelijckt int wesen toegaet, want by aldien men sonder daer
opacht te nemen, des Maenwechs verstepunt met een roerenden Eertcloot
sdaechs voordering gawe van 6 ① 41 ②, sulcke teyckening en rekening en sou-
de mette sake niet overeencommen. Dit verstaen sijnde soo laet ABC den



Eertclootwech be-
teyckenē, diēs mid-
delpunt D, de Son
E, en deur de twee
punten D, E, ghe-
trocken sijnde de
rechte lini ADEC,
so beteycken A het
verstepunt, an het
welck ick ten eerstē
neem dē Eertcloot
te wesen, daer na
DA voort getrockē
sijnde tot F, ick teye-
kē inde lini AF het
punt G, beschrijf
daer op als middel-
punt het rondt FH
bediedēde dē Maen-
wech, diens verste-
punt ick neem te
wesen F, daer na sy

den Eertcloot op de boveschreven 90 dagen gecommen van A tot B, waer op
haer middelloop doet deur t'werck 88 tr. 42 ① voor den houck ADB: Ick
treck daer na de lini BI ewijdege met DA, en BK alsoo dat den houck IBK
doe de

There remains for the required mean motion of the apogee of the Moon's orbit, against the order of the degrees, in one day

$0^{\circ};52,27,14,57,34, 8$

Hitherto an example has been given of the motion of one day, from which the great difference between the true motion and the untrue motion on the theory of a fixed Earth can be seen with greater certainty than from long times, in which there are whole circles, which are discarded. But since the arcs of one day are very small, such that the following proof would not be as clear as for a longer time, for this purpose I will take next the time of 90 days.

In this time, by the 3rd proposition of the 1st book, the mean motion of the Earth is

$88^{\circ}42'$

From this is subtracted the mean amount of the advance which the apogee of the Moon's orbit is found to make on the theory of a fixed Earth in apparent ecliptical longitude in 90 days, which by the 11th proposition of the 1st book is

$10^{\circ} 2'$

There remains for the required mean motion of the apogee of the Moon's orbit, against the order of the degrees, in 90 days

$78^{\circ}40'$

PRELIMINARY TO THE PROOF. To make a general statement about this proof before I come to the particular explanation, it is especially evident that the motion of the apogee of the Moon's orbit, which we apparently observe in the ecliptic, is a combination of its own motion with that which it receives from the motion of the Earth, which must necessarily be separated to reach a right theory, as happens in reality; for if, without heeding this, one should give the advance in one day of the apogee of the Moon's orbit, on the theory of a moving Earth, as $6'41''$, neither the figure nor the computation would be in agreement with the true state of affairs. This being understood, let ABC denote the Earth's orbit, its centre being D , the Sun E ; then, the straight line $ADEC$ being drawn through the two points D and E , A denotes the apogee, where I first assume the Earth to be. Thereafter, DA being produced to F , I mark on the line AF the point G and describe about this as centre the circle FH , which denotes the Moon's orbit, whose apogee I assume to be F . Thereafter let the Earth have moved in the above-mentioned 90 days from A to B , in which its mean motion, according to the procedure, is $88^{\circ}42'$ for the angle ADB . Thereafter I draw the line BI parallel to DA , and BK so that the angle IBK is the

doe de voordering diemen de middelloop van des Maenwechs verstepunt op de 90 daghen bevinde ghevoordert te sijn in schijnbaer duyfteraerlangde, bedraghende deur t'werck 10 tr. 2 ①: Ick stel daer na in B K t'punt L, sulcx dat B L syde uytmiddelpunticheytlijn, en beschrijf op L als middelpunt de Maenwech K M, diens verstepunt K, en naestepunt M, treck oock D B voorwaert tot inden omtrekan N. T' B E W Y S. By aldien het verstepunt als F gheen eyghen roersel ghehadt en hadde, t'soude, den Eertclood ghecommen wesende an B, dan sijn inde voortghetrocken D B deur N, maer het is van daer achterwaert ghecommen tot K, als hebbende tot die plaets onder den duyfteraer bevonden gheweest deur t'werck, daerom moeten wy bewijfen den houck N B K te doen de boveschreven 78 tr. 40 ①, t'welck aldus toegact: Anghesien B I ewwijdege is met D A, sulcx datmen van B deur I t'selve punt des duyfteraers siet, datmen uyt D deur A sach, soo moet den houck I B N even sijn metten houck A D B, en doen als die 81 tr. 42 ①, daer af ghetrocken den houck K B I doende deur t'werck 10 tr. 2 ①, blijft voor den houck N B K 78 tr. 40 ①: Daerom op de 90 daghen in welke den Eertclood ghecommen is van A tot B na t'vervolgh der trappen, heeft het verstepunt in sijn eyghen roersel geloopt tegen t'vervolgh der trappen den houck N B K, doende gelijck wy bewijfen moeste 78 tr. 40 ①.

2 Voorbeelt van t'vinden des duyftering snees loop.

T'GHEGHEVEN. Het is den tijt eens dachs. T'BEGHEERDE. Men wil daer op gevonden hebben des duyftering snees middelloop in schijnbaer duyfteraerlangde, ghegront op stelling eens roerenden Eertcloots.

T' W E R C K.

Des Eertcloots middelloop doet deur het 3 voorstel
des 1 boucx sdaechs OIT. 59. 8.17.13.12.31.
Daer toe vergaert de middelloop der achtring diemen
de duyftering snee met stelling eens vasten Eertcloots
bevinde te verachten in schijnbaer duyfteraerlangde
op 1 dach, bedraghende deur het 11 voorstel des
1 boucx OIT. 3.10.41.15.26. 7.
Comt de duyftering snee begheerde middelloop tegen
t'vervolgh der trappen op een dach 1 tr. 2.18.58.28.38.38.
Waer af t'bewijs deur t'voorgaende bewijs des 1 voorbeelts als daer groote
ghelijckheit me hebbende kennelick ghenouch is. T'BESELYT. Wy heb-
ben dan gevonden op een gegeven tijt den loop van des Maenwechs verstepunt,
en der duyftering snee, deur wisconftighe wercking ghegront op stelling
eens roerenden eertcloots, na den eysch.

V E R V O L G H.

Anghesien deur het 31 voorstel des 2 boucx bekent is des Maenwechs verstepunts schijnbaer duyfteraerlangde op den anvangtijt, soo ist openbaer hoe-
men die sal vinden op alle ghegeven tijt, want totte plaets des anvangtijts, ver-
vough den eyghen loop van daer af totten ghegeven tijt na de leering deses
voorstels, en daer toe noch ghedaen den Eertcloots loop op den selven tijt,
men heeft het begheerde. En sghelijcx is oock te verstaen mette duyftering-
snee.

V E R

advance which the mean motion of the apogee of the Moon's orbit is found to make in 90 days, in apparent ecliptical longitude, which according to the procedure amounts to $10^{\circ}2'$. I then take in BK the point L such that BL be the line of eccentricity and I describe about L as centre the Moon's orbit KM , whose apogee is K and whose perigee is M , and I also produce DB to the circumference, to N . PROOF. If the apogee (F) had not had any motion of its own, it would — when the Earth had arrived at B — be in DB produced through N , but it has moved backwards from there to K , for in that place in the ecliptic it was found according to the procedure; we therefore have to prove that the angle NBK makes the aforesaid $78^{\circ}40'$, which is done as follows. Since BI is parallel to DA , so that from B through I the same point of the ecliptic is seen that was seen from D through A , the angle IBN must be equal to the angle ADB and like the latter must make $81^{\circ}42'$. When from this is subtracted the angle KBI , which according to the procedure makes $10^{\circ}2'$, there remains for the angle NBK . $78^{\circ}40'$. Thus, in the 90 days during which the Earth has moved from A to B in the order of the degrees the apogee in its own motion against the order of the degrees has moved the angle NBK , which makes $78^{\circ}40'$, as we had to prove.

2nd Example, of the Finding of the Motion of the Nodes.

SUPPOSITION. Let the time be one day. WHAT IS REQUIRED. It is required to find in this time the mean motion of the nodes in apparent ecliptical longitude, based on the theory of a moving Earth.

PROCEDURE.

By the 3rd proposition of the 1st book the mean motion of the Earth in one day is $0^{\circ};59, 8,17,13,12,31$

To this is added the mean amount of the lag which the nodes are found to lag in apparent ecliptical longitude in one day, on the theory of a fixed Earth, which, by the 11th proposition of the 1st book, is $0^{\circ}; 3,10,41,15,26, 7$

The required mean motion of the nodes, against the order of the degrees, in one day becomes $1^{\circ}; 2,18,58,28,38,38$

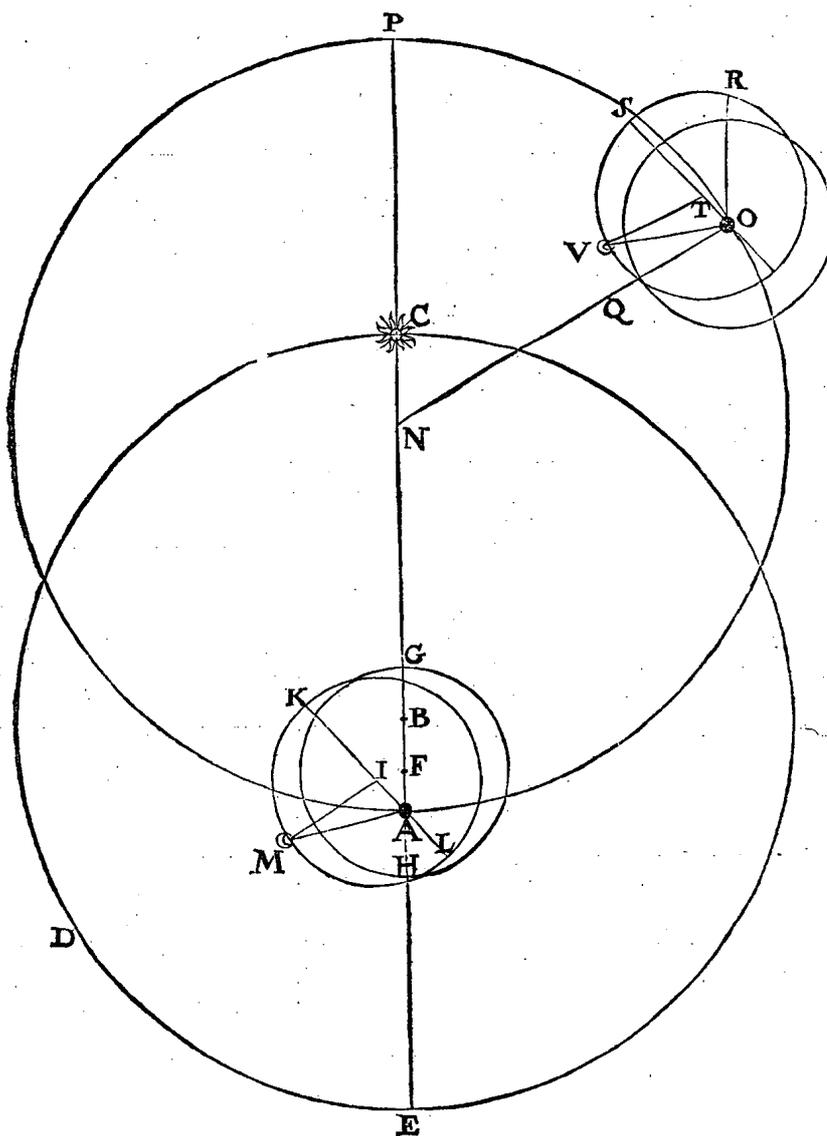
The proof of which is evident enough from the foregoing proof of the 1st example, as being greatly similar thereto. CONCLUSION. We have thus found the motion of the apogee of the Moon's orbit and of the nodes in a given time, by mathematical operations, based on the theory of a moving Earth; as required.

SEQUEL.

Since from the 31st proposition of the 2nd book the apparent ecliptical longitude of the apogee of the Moon's orbit is known at the initial moment, it is clear how it is to be found at any given time, for when to the position of the initial moment we add its own motion from there to the given time, according to the present proposition, and to this we further add the motion of the Earth in the said time, we have found what was required. And the same is also to be understood for the nodes.

De Maen ontfangt met stelling eens roerenden Eertcloots, de selve schijnbaer duyfteraerlangde en verheynt vanden Eertcloodt diese heeft met stelling eens vasten Eertcloodts.

T'GHEGHEVEN. Laet voor eerste stelling ghenomen worden 'punt A een vasten Eertcloodt te beteykenen, van A tot B sy de Sonwechs uytmiddel.



THEOREM.

12th PROPOSITION.

On the theory of a moving Earth the Moon acquires the same apparent ecliptical longitude and distance from the Earth which it has on the theory of a fixed Earth.

SUPPOSITION. For the latter theory ¹⁾, let the point A be assumed to denote a fixed Earth, let the distance from A to B be the line of eccentricity of the Sun's orbit, and with the semi-diameter BC let there be described about B as

¹⁾ For *eerste stelling* in the Dutch text read *tweede stelling*.

punticheytlijn, en mette halfmiddellijn BC sy op B als middelpunt beschreven de Sonwech CDE , waer in CA voortgetrocken tot E , soo is E t'naestepunt, C t'verstepunt, daer na sy tusschē den vasten Eertcloon A en B gestelt het punt F als Maenwechs middelpunt, en daer op mette halfmiddellijn FG beschreven den Maenwech GH , sniende A in H als naestepunt, en AC in G als verstepunt, an t'welck ick voor begin neem de Maen te wesen. Op een tijt lanck daer na sy de Maenwechs middelpunt gecommen van F tot I , waer op ick mette halfmiddellijn IK , even an FG , beschrijf de Maenwech KL , diens verstepunt K , sulcx dat op den boveschreven tijt des Maenwechs verstepunt gecommen is van G tot K , en daerentusschen is de Maen van het verstepunt ghecommen soo verre als van K tot M (met soo veel heele ronden daer toe alst wesen mocht, diemen hier om bekende redenen verlaet) daer na ghetrocken AM , soo sal de Maen uyt A gesien schijnbaerlick soo verre sijn van C , wesende onder des duystraers 65 tr. 30 D , als den houck CAM mebrengt.

Dese teyckening des Maenwechs met stelling eens vasten Eertcloots aldus ghedaen sijnde, wy sullen totte teyckening van de ander stelling commen. Tot desen einde stel ick in CA t'punt N , alsoo dat de uytmiddelpunticheytlijn CN even sy an AB , en beschrijf op N als middelpunt mette halfmiddellijn NA die even moet sijn mette halfmiddellijn BC het rondt AOP als Eertcloonwech, diens naestepunt P , verstepunt A , waer in ick neem den Eertcloon A mette Maenwech daer rontom ghecommen te sijn tot O , ghelooopen hebbende den booch AO , of houck ANO . By aldien nu des Maenwechs verstepunt gheen eyghen roerfel ghehadt en hadde te wijle den Eertcloon ghecommen is van A tot O , maer altijt ghebleven hadde tusschen den Eertcloon en t'punt N , t'is kennelick dattet soude wesen inde lini ON an t'punt Q , soo verre van O als van A tot G : Maer het heeft deur het 11 voorstel deses 3 boucx een roerfel teghen t'vervolgh der trappen even an den Eertcloonloop ANO , soo veel min als sijn schijnbaer voordering inden duystraer bedraecht, dats den houck GAK , daerom treck ick de lini OR even en ewewijdeghe met AG , en soude des Maenwechs verstepunt moeten sijn an R , ghelooopen hebbende teghen t'vervolgh der trappen den houck QOR , waerder niet noch af te trecken een houck even ande schijnbaer voordering GAK , daerom treck ick de lini OS even an AG , en alsoo dat den houck ROS even sy metten houck GFK , teycken daer in t'punt T , soo dat OT even sy met AF , en beschrijf op T als middelpunt, mette halfmiddellijn TS , den Maenwech SVR , diens verstepunt S : Maer want op den boveschreven ghestelden tijt de Maen met stelling eens vasten Eertcloots, ghecommen is van t'verstepunt K tot M , soo stel ick inde Maenwech SVR t'punt V , soo dat de booch SV of houck STV , even is metten houck KIM , en stel de Maen te wesen an t'punt V . Dit soo sijnde, ick segh de Maen V uyt den roerenden Eertcloon O , ghesien te worden onder de selve schijnbaer duystraerlangde daer de Maen an M onder ghesien wort uyt den vasten Eertcloon A , en dat de verhey OV even is mette verhey AM , om dese reden. $T'BEWYS$. Anghesien OR ewewijdeghe is met AG , en den houck ROS even metten houck GAK , soo moet OS ewewijdeghe sijn met AK . Voort anghesien den houck STV , even is metten houck KIM , soo is de lini TV ewewijdeghe met IM , boven dien soo heeft den driehouck TOV twee sijden TO , TV , even en ewewijdeghe met des driehouck IAM twee sijden IA , AM , waer deur haer derde sijden OV , AM oock even en ewewijdeghe sijn, en daerom wort de Maen an V uyt den roerenden Eertcloon O , ghesien onder de selve schijnbaer duystraer-

centre the Sun's orbit CDE , in which, when CA is produced to E , E is the perigee, C the apogee. Thereafter let there be marked between the fixed Earth A and B the point F for the centre of the Moon's orbit, and about this, with the semi-diameter FG , let there be described the Moon's orbit GH , intersecting AE in H for the perigee and AC in G for the apogee, at which I assume the Moon to be, to begin with. At a time long after, let the centre of the Moon's orbit have moved from F to I , about which with the semi-diameter IK , equal to FG , I describe the Moon's orbit KL , whose apogee is K , so that in the above-mentioned time the apogee of the Moon's orbit has moved from G to K , and meanwhile the Moon has moved from the apogee as far as from K to M (with as many whole circles as may be, which are here discarded for known reasons). When thereafter AM is drawn, the Moon, as seen from A , will apparently be as far from C , which is at $65^{\circ}30'$ of the ecliptic, as the angle CAM implies.

This drawing of the Moon's orbit on the theory of a fixed Earth thus having been made, we shall proceed to draw the situation on the other theory. To this end I mark in CA the point N such that the line of eccentricity CN be equal to AB , and I describe about N as centre, with the semi-diameter NA (which is to be equal to the semi-diameter BC), the circle AOP for the Earth's orbit, whose perihelion is P and whose aphelion is A , in which I assume the Earth A with the Moon's orbit around it to have arrived at O , having moved the arc AO or the angle ANO . If the apogee of the Moon's orbit had had no motion of its own while the Earth moved from A to O , but had always remained between the Earth and the point N , it is evident that it would be in the line ON at the point Q , as far from O as from A to G . But according to the 11th proposition of this 3rd book it has a motion against the order of the degrees, equal to the motion of the Earth ANO , as much less as its apparent advance in the ecliptic amounts to, *i.e.* the angle GAK . I therefore draw the line OR equal and parallel to AG ; then the apogee of the Moon's orbit would have to be at R , having moved, against the order of the degrees, the angle QOR , if it were not that an angle equal to the apparent advance GAK has to be subtracted from it. I therefore draw the line OS equal to AG and such that the angle ROS be equal to the angle GAK ¹⁾, mark therein the point T such that OT be equal to AF , and describe about T as centre, with the semi-diameter TS , the Moon's orbit SVR , whose apogee is S . But because in the above-mentioned time the Moon has moved from the apogee K to M on the theory of a fixed Earth, I mark in the Moon's orbit SVR the point V such that the arc SV or the angle STV is equal to the angle KIM , and assume the Moon to be at the point V . This being so, I say that the Moon V is seen from the moving Earth O at the same apparent ecliptical longitude at which the Moon at M is seen from the fixed Earth A , and that the distance OV is equal to the distance AM for this reason. PROOF. Since OR is parallel to AG and the angle ROS is equal to the angle GAK , OS must be parallel to AK . Further, since the angle STV is equal to the angle KIM , the line TV is parallel to IM . Moreover the triangle TOV has two sides (TO , TV) equal and parallel to the two sides IA , IM ²⁾ of the triangle IAM , in consequence of which their third sides OV , AM are also equal and parallel, and therefore the Moon at V is seen from the moving Earth O at the same apparent ecliptical longitude at

¹⁾ For GFK in the Dutch text read GAK .

²⁾ For AM in the Dutch text read IM .

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raerlangde daer de Maen an M onder gesien wort uyt den vasten Eertcloot A, en de verheyd O V is even mette verheyd A M. T B E S L V Y T. De maen dan ontfangt met stelling eens roerenden Eertcloots de selve schijnbaer duyfteraerlangde en verheyd vanden Eertcloot die se heeft met stelling eens vasten Eertcloots, t'welck wy bewijfen moesten.

V E R V O L G H.

T'is kennelick, datmen om op een ghegheven tijt te vinden de Manens schijnbaer duyfteraerlangde gegront op stelling eens roerenden Eertcloots, sal foucken des Maenwechs verstepunts schijnbaer duyfteraerlangde na de manier vant vervolgh des 11 voorstels, en daer na de rest ghelijck met stelling eens vasten Eertcloots, maer t'eenmael te rekenen op de stelling eens vasten Eertcloots valt gherievigher, om de redenen die t'haerder plaets breeder verclaert fullen worden.

VIERDE

which the Moon at M is seen from the fixed Earth A , and the distance OV is equal to the distance AM . CONCLUSION. On the theory of a moving Earth the Moon thus acquires the same apparent ecliptical longitude and distance from the Earth that it has on the theory of a fixed Earth; which we had to prove.

SEQUEL.

It is evident that in order to find at a given time the Moon's apparent ecliptical longitude, based on the theory of a moving Earth, we have to find the apparent ecliptical longitude of the apogee of the Moon's orbit in the manner of the sequel to the 11th proposition, and thereafter the rest in the same way as on the theory of a fixed Earth, but it is more convenient to make the computation at once on the theory of a fixed Earth, for the reasons to be explained more fully in the proper place.

VIERDE
ONDERSCHEYT
 DES DERDEN BOVCX VAN
 Saturnus, Iupiters, Mars, Venus
 eu Mercurius langdeloop met
 stelling eens roerenden
 Eertcloots.

CORTBEGRYP DESES
 VIERDEN ONDERSCHEYTS.

De vierde onderscheyt sal seven voorstellen hebben.
 Het eerste wvessende in d' oirden het 13, om te wvindē de half-
 middellijnen der wvvegghen, de uyt middelpunticheytlijnen, met
 meeste en minste verheden van de Dvvaelders, in sulcke dee-
 len alsser des Eertclootwvechs halfmiddellijn 10000 doet, deur wvvisconslighe
 wvvercking ghegront op stelling eens roerenden Eertcloots.

Het wvveede wvvesende in d' oirden het 14, van den loop der drie boven-
 ste Dvvaelders Saturnus, Iupiter en Mars in haer wvvegghen op een ghege-
 ven tijt, met stelling eens roerenden Eertcloots.

Het derde wvvesende in d' oirden het 15, dat de drie bovenste Dvvaelders
 Saturnus, Iupiter en Mars, met stelling eens roerenden Eertcloots de sel-
 ve schijnbaer duysteraerlangde en verheyt vanden Eertcloot ontfanghen,
 diese hebben met stelling eens vasten Eertcloots.

Het vierde wvvesende in d' oirden het 16, vanden loop der wvvee onder-
 ste Dvvaelders Venus en Mercurius in haer wvvegghen op een ghegeven
 tijt, met stelling eens roerenden Eertcloots.

Het wvvijde wvvesende in d' oirdē het 17, dat de wvvee onderste Dvvael-
 ders Venus en Mercurius met stelling eens roerenden Eertcloots, de selve
 schijnbaer duysteraerlangde en verheyt vanden Eertcloot ontfanghen die-
 se hebben met stelling eens vasten Eertcloots.

Het sefte wvvesende in d' oirden het 18, inhoudende wvverclaring der reden
 wvvaerom ick inde 8.12.15. en 17 voorstellen, bevvesen hebbe de Dvvaelders
 deur stelling eens roerenden Eertcloots bevonden te wvworden totte selve
 schijnbaer plaetsen en verheden van malcander, diemensē met stelling eens
 vasten Eertcloots bevint, mette omstandighen van dien.

Het sevende wvvesende in d' oirden het 19, inhoudende wvverclaring op
 Aa wvvelcke

FOURTH CHAPTER
 OF THE THIRD BOOK
 Of Saturn's, Jupiter's, Mars', Venus', and
 Mercury's Motion in Longitude, on the
 Theory of a Moving Earth

SUMMARY OF THIS FOURTH CHAPTER

This fourth Chapter is to contain seven propositions.

The first, which in the sequence is the 13th, to find the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth.

The second, which in the sequence is the 14th, of the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits at a given time, on the theory of a moving Earth.

The third, which in the sequence is the 15th, that the three upper Planets Saturn, Jupiter, and Mars on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

The fourth, which in the sequence is the 16th, of the motion of the two lower Planets Venus and Mercury in their orbits in a given time, on the theory of a moving Earth.

The fifth, which in the sequence is the 17th, that the two lower Planets Venus and Mercury on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

The sixth, which in the sequence is the 18th, containing an explanation of the reason why in the 8th, 12th, 15th, and 17th propositions I have proved that on the theory of a moving Earth the Planets are found at the same apparent places and mutual distances that they are found on the theory of a fixed Earth, with the circumstances relating thereto.

The seventh, which in the sequence is the 19th, containing an exposition on which theory — to wit, the untrue theory of a fixed Earth or the true one of a moving Earth — it seems most suitable to make the computations of the motion in longitude of the Planets.

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welcke stelling, te weten de oneyghen met een vasten Eertcloot, of de eyghen met een roerende, oirboirft schijnt de rekeninghen te maken vande langdeloop der Dwaelders.

13 VOORSTEL.

Te vinden de halfmiddellijnen der vvegghen, de uytmiddelpunticheytlijnen, met meeste en minste verheden vande Dwaelders, in sulcke deelen alser des Eertclootwechs halfmiddellijn 1000 doet, deur vvisconstighe vvercking gegront op stelling eens roerenden Eertcloots.

Mijn voornemen is hier te ghebruycken de ghetalen by *Ptolemaeus* gevonden, want hoe wel de verstepunten sedert seer verlopen sijn, en dat voorbeelden des teghenwoordighen tijts oirboirder souden moghen wesen om te sien de overeencumminghen deser sloten mette dadelicke ervaringhen, nochtans ghemerckt in *Ptolemaeus* beschrijving gevonden worden veel bequame voorbeelden, soo wel des breedeloops als langdeloops van al de Dwaelders, om daer uyt te bevestighen de voorstellen ghegront op stelling eens roerenden Eertcloots, soo heb ick die vercoren voor ander.

Om dan totte saeck te commen, 't is te anmercken dat de stelders eens vasten Eertcloots, niet werende dat der drie bovenste Dwaelders schijnbaer in ronden daerfe in schijnen te loopen, sijn in plaets des Eertclootwechs, en datse daerom mette selve evegroot behooren te wesen, soo en hebben se haer halfmiddellijnen gheen selve ghetal ghegheven, ghelijck sy souden meughen doen die sulcx beken is. Maer want hier uyt volght dat de ghetalen der halfmiddellijnen van wegghen en uytmiddelpunticheytlijnen versheydener Dwaelders, niet everedelick en sijn mette wesentlicke langden, ghelijck nochtans de sake vereycht om bequamelick te wercken met stelling eens roerenden Eertcloots, en *Hemelloopstuyck der Dwaelders te meughen teyckenen diens deelen everedelick sijn met haer overeencommende deelen des wecelts, soo sal ick op allen een ghemeene maet stellen, te weten des Eertclootwechs halfmiddellijn in 1000 ghedeelt, welcke met reden daer toe vercoren wort, om datse in de rekeningen van elck der ander Dwaelders comt, en dat haer ghetal van 1000 lichticheyt int werck veroirsaect.

*Instrumenta
Astronomi-
ca.*

SAMING

*der halfmiddellijnen en uytmiddelpunticheytlijnen door Ptolemaeus
gevonden en beschreven op stelling eens vasten Eertcloots.*

Sonwechs halfmiddellijn	60 deel.
Sonwechs uytmiddelpunticheytlijn	2 deel 30 ①.
Maenwechs halfmiddellijn	60 deel.
Maenwechs uytmiddelpunticheytlijn (die <i>Ptolemaeus</i> inronts halfmiddellijn noemde)	5 deel 14 ①.
Anderfins heeft <i>Ptolemaeus</i> int 13 voorstel sijns 5 boucx, in plaets van die twee ghetalen 60 deel en 5 deel 14 ①, ghenomen twee inde selve reden, te weten 59 deel en 5 deel 10 ①, wesen	

13th PROPOSITION.

To find the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth.

My intention is here to use the values found in *Ptolemy*, for though the apogees have shifted a good deal since then and examples of the present time might be more suitable to see the correspondences of these results with practical experience, nevertheless, seeing that in *Ptolemy's* description many suitable examples are found both of the motion in latitude and of the motion in longitude of all the Planets, with which to confirm the propositions based on the theory of a moving Earth, I have chosen them in preference to others.

To come to the matter, it is to be noted that those who assume a fixed Earth, not knowing that the apparent epicycles of the three upper Planets, in which they seem to move, have come instead of the Earth's orbit, and that they ought therefore to be of the same magnitude as the latter, have not given their semi-diameters the same value, as those to whom this is known might do. But since it follows from this that the values of the semi-diameters of the orbits and the lines of eccentricity of different Planets are not proportional to the actual lengths, as nevertheless is required for a convenient treatment on the theory of a moving Earth and for making it possible to draw astronomical instruments of the Planets ¹⁾, whose parts are proportional to the corresponding parts of the world, I shall set a common measure for all, to wit, the semi-diameter of the Earth's orbit divided into 10,000, which is chosen for it with good reason because it occurs in the computations of each of the other Planets and because its value of 10,000 facilitates the procedure.

COMPILATION

of the Semi-diameters and Lines of Eccentricity
Found and Described by *Ptolemy* on the Theory of
a Fixed Earth.

Semi-diameter of the Sun's orbit	60 ^P 2)
Line of eccentricity of the Sun's orbit	2 ^P 30'
Semi-diameter of the Moon's orbit	60 ^P
Line of eccentricity of the Moon's orbit (which <i>Ptolemy</i> called epicycle's semi-diameter)	5 ^P 14'

¹⁾ Stevin does not here allude to instruments for observing the stars, but to structures, mostly of cardboard and parchment, such as were often made in those days to facilitate the understanding of the planetary orbits.

²⁾ Sexagesimal division was used in ancient science for linear distances as well. The unit of distance was put 60 parts, and the first- and second-order subdivisions were indicated by the symbols now used for minutes and seconds of arc.

MET EEN ROERENDEN EERTCLOOT. 275

wefende Eertcloots halfmiddellijnen, fulcke alſer de Sonwechs halfmiddellijn 1210 doet.

Saturnus inrontwechs halfmiddellijn	60 deel.
Saturnus inronts halfmiddellijn	6 deel 30 ①.
Saturnus uytmiddelpunticheytlijn	3 deel 25 ①.
Iupiters inrontwechs halfmiddellijn	60 deel.
Iupiters inronts halfmiddellijn	11 deel 30 ①.
Iupiters uytmiddelpunticheytlijn	2 deel 45 ①.
Mars inrontwechs halfmiddellijn	60 deel.
Mars inronts halfmiddellijn	39 deel 30 ①.
Mars uytmiddelpunticheytlijn	6 deel.
Venus inrontwechs halfmiddellijn	60 deel.
Venus inronts halfmiddellijn	43 deel 10 ①.
Venus uytmiddelpunticheytlijn	1 deel 15 ①.
Mercurius inrontwechs halfmiddellijn	60 deel.
Mercurius inronts halfmiddellijn	21 deel 26 ①.
Mercurius uytmiddelpunticheytlijn	5 deel 41 ①.

Om deſe ghetalen ghevonden met ſtelling eens vaſten Eertcloots, te verkeeren in getalen met ſtelling eens roerenden Eertcloots, diens wechs halfmiddellijn 10000 doet als voorſeyt is, ick ſal metten Eertcloon beginnen, diens wechs halfmiddellijn de ſelve wefende diemen anders de Sonwech toefchrijft, ſy is deur de voorgaende SAMING tot haer uytmiddelpunticheytlijn, in ſulcken reden als 60 deel tot 2 deel 30 ①, maer om die te hebben in ſulcke deelen alſer des Eertcloonwechs halfmiddellijn 10000 doet, ick ſegh 60 deelen gheven 2 deel 30 ①, wat 10000? comt 417, ſulcx dat doende des Eertcloonwechs halfmiddellijn

10000.

De uytmiddelpunticheytlijn doet

417.

Ende want *Ptolemeus* int 4 Hoofſtick ſijns 3 boucx de Sonwechs verſtepunt ſtelt onder des duyſteraers 65 tr. 30 ①, in wiens plaets nu des roerenden Eertcloonwechs naeſtepunt comt deur het 9 voorſtel deſes 3 boucx, ſoo valt des ſelfden verſtepunt onder des duyſteraers

245 tr. 30.

Angaende de Maenwechs halfmiddellijn, anghesien die totte halfmiddellijn des Eertcloonwechs in ſulcken reden is deur de voorgaende SAMING als 1210 tot 59, ick ſegh 1210, gheeft 59, wat 10000? comt voor de Maenwechs halfmiddellijn

488.

Om te hebben de Maenwechs uytmiddelpunticheytlijn, ick ſegh Maenwechs halfmiddellijn 59 deel, gheeft uytmiddelpunticheytlijn 5 deel 10 ① deur de voorgaende SAMING, wat Maenwechs halfmiddellijn 488 vierde in d'oirden? comt Maenwechs uytmiddelpunticheytlijn

43.

Die vergaert totte Maenwechs halfmiddellijn 488 vierde in d'oirden, comt voor de meeſte verheyte vanden Eertcloon totte Maen

531.

Ende die 43 vijfde in d'oirden, ghetrocken vande Maenwechs halfmiddellijn 488 vierde in d'oirden blijft de minſte verheyte

445.

De Maenwechs verſtepunts ſchijnbaer duyſteraerlangde op corten tijt groote verandering krijghende en vereyſcht hier niet te commen.

A a 2 Om te

On the other hand, in the 13th proposition of his 5th book, *Ptolemy* took, instead of those two values of 60^p and $5^p14'$, two values in the same ratio, to wit, 59^p and $5^p10'$, the units being the semi-diameter of the Earth, of which the semi-diameter of the Sun's orbit has 1,210.

Semi-diameter of Saturn's deferent	60^p
Semi-diameter of Saturn's epicycle	$6^p30'$
Saturn's line of eccentricity	$3^p25'$
Semi-diameter of Jupiter's deferent	60^p
Semi-diameter of Jupiter's epicycle	$11^p30'$
Jupiter's line of eccentricity	$2^p45'$
Semi-diameter of Mars' deferent	60^p
Semi-diameter of Mars' epicycle	$39^p30'$
Mars' line of eccentricity	6^p
Semi-diameter of Venus' deferent	60^p
Semi-diameter of Venus' epicycle	$43^p10'$
Venus' line of eccentricity	$1^p15'$
Semi-diameter of Mercury's deferent	60^p
Semi-diameter of Mercury's epicycle	$21^p26'$
Mercury's line of eccentricity	$5^p41'$

In order to convert these values, found on the theory of a fixed Earth, into values on the theory of a moving Earth, the semi-diameter of whose orbit makes 10,000, as has been said above, I shall begin with the Earth, and since the semi-diameter of its orbit is the same as that otherwise ascribed to the Sun's orbit, it is according to the above Compilation to its line of eccentricity in the ratio of 60^p to $2^p30'$; but to have it in such parts as the semi-diameter of the Earth's orbit has 10,000, I say:

60^p give $2^p30'$; what does 10,000 give? It makes 417, so that if the semi-diameter of the Earth's orbit makes

10,000
417

the line of eccentricity makes

And since *Ptolemy* in the 4th Chapter of his 3rd book puts the apogee of the Sun's orbit at $65^\circ30'$ of the ecliptic, which is now replaced by the perihelion of the moving Earth's orbit, by the 9th proposition of this 3rd book, the aphelion of the latter now falls in the ecliptic at

$245^\circ30'$

As regards the semi-diameter of the Moon's orbit, since this is to the semi-diameter of the Earth's orbit in the ratio of 1,210 to 59, according to the above Compilation, I say: 1,210 gives 59; what does 10,000 give? It gives for the semi-diameter of the Moon's orbit

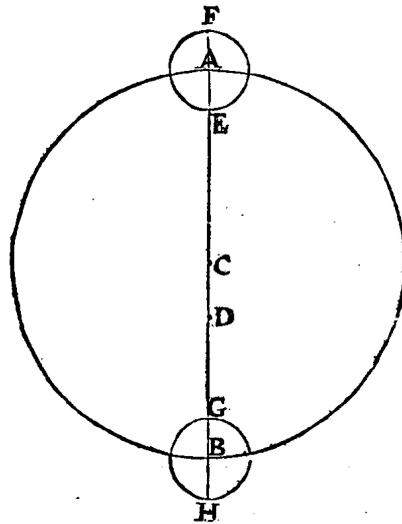
488

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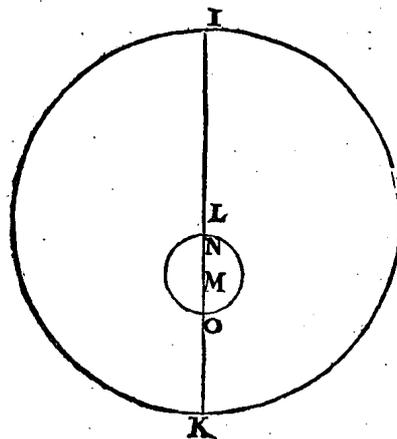
Om te hebben Saturnus wechs halfmiddellijn, ick segh Saturnus inronts halfmiddellijn 6 deel 30 ①, gheeft sijn inrontwechs halfmiddellijn 60 deel deur de voorgaende SAMING, wat 10000? comt voor Saturnuswechs halfmiddellijn 92308.

Om te hebben Saturnus uytmiddelpunticheytlijn, ick segh sijn inronts halfmiddellijn 6 deel 30 ①, gheeft sijn uytmiddelpunticheytlijn 3 deel 25 ① deur de voorgaende SAMING, wat 10000? comt Saturnus uytmiddelpunticheytlijn 5256.

Om te hebbē Saturnus meeste en minste verhedē, ick sal op dat alles claerder sy teyckenē twee formē, d'eerste met stelling eens vasten Eertcloots, d'ander eens roerenden. Laet A B Saturnus inrontwech sijn diēs middelpunt C, en de halfmiddellijn C A doet 92308 achtste in d'oirdē, D is den vasten Eertcloor, C D de uytmiddelpunticheytlijn doende 5256 negende in d'oirden, A des inrontwechs



ick trek D C 5256 met G B 10000 t'samen 15256, van C B



verstepunt, B naestepūt, voort y op A als middelpunt beschreven het inront E F, diens halfmiddellijn A E doet 10000 deur t'ghestelde, en des inronts verstepunt is F, naestepunt E, daer na sy op B als middelpunt beschrevē het inront G H even an E F, diens naestepunt G. Dit so sijnde, en om nu te hebben de meeste verhey D F, soo vergaerick tot C A 92308 achtste in d'oirden, de uytmiddelpunticheytlijn D C 5256 neghende in d'oirden, mette halfmiddellijn A F 10000, comt t'samen voor de meeste verhey D F 107564. Ende om te hebbē de minste verhey D G,

92308, en blijft voor de minste

verhey D G 77052.

Maer om nu te siē sulcke overeencoming met een roerendē Eertcloor, laet het ront I K sijn Saturnus wech, diens middelpunt L, en halfmiddellijn I L evē an C A doet als die 92308, en t'middelpunt des Eertclootwechs sy M, uytmiddelpunticheytlijn L M doende als C D 5256, Saturnuswechs verstepunt sy I, naestepunt K, voort sy op M als middelpunt beschreven dē Eertclootwech N O, diēs halfmiddellijn M N even

In order to have the line of eccentricity of the Moon's orbit, I say: semi-diameter of the Moon's orbit 59^P gives line of eccentricity $5^P10'$ according to the above Compilation; what does the semi-diameter of the Moon's orbit 488 give, the fourth in the list? The line of eccentricity of the Moon's orbit becomes

43

When this is added to the semi-diameter of the Moon's orbit 488 (the fourth in the list), the greatest distance of the Earth from the Moon becomes

531

And when 43 (the fifth in the list) is subtracted from the semi-diameter of the Moon's orbit 488 (the fourth in the list), the least distance is

445

Since the apparent ecliptical longitude of the apogee of the Moon's orbit varies widely in a short time, this is not required to be discussed here.

In order to have the semi-diameter of Saturn's orbit, I say: the semi-diameter of Saturn's epicycle $6^P30'$ gives the semi-diameter of its deferent 60^P according to the above Compilation; what does 10,000 give? The semi-diameter of Saturn's orbit becomes

92,308

In order to have Saturn's line of eccentricity, I say: the semi-diameter of its epicycle $6^P30'$ gives its line of eccentricity $3^P25'$ according to the above Compilation; what does 10,000 give? Saturn's line of eccentricity becomes

5,256

In order to have Saturn's greatest and least distances, I will — to make everything clearer — draw two figures, the first on the theory of a fixed Earth, the other on that of a moving Earth. Let AB be Saturn's deferent, whose centre be C , and the semi-diameter CA makes 92,308 (the eighth in the list), D is the fixed Earth, CD the line of eccentricity making 5,256 (the ninth in the list), A the deferent's apogee, B its perigee; further let there be described about A as centre the epicycle EF , whose semi-diameter AE makes 10,000 by the supposition, then the epicycle's apogee is F , its perigee E ; thereafter let there be described about B as centre the epicycle GH equal to EF , whose perigee is G . This being so, and in order to have the greatest distance DF , I add to CA 92,308 (the eighth in the list) the line of eccentricity DC 5,256 (the ninth in the list), with the semi-diameter AF 10,000; this makes together 107,564 for the greatest distance DF . And in order to have the least distance DG , I subtract DC 5,256 with GB 10,000, together making 15,256, from CB 92,308; then there is left for the least distance DG

77,052

But in order to see now such correspondence with a moving Earth, let the circle IK be Saturn's orbit, whose centre be L and whose semi-diameter IL equal to CA , like the latter, makes 92,308, and let the centre of the Earth's orbit be M , its line of eccentricity LM making (like CD) 5,256; let the apogee of Saturn's orbit be I , its perigee K ; further let there be described about M as centre the Earth's orbit NO ,

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even sijnde met A E doet als die 10000. Dit soo wefende de meeste verhey O I, moet even sijn mette boveschreven D F, en de minste O K even mette boveschreven minste D G, want tot I L 92308, vergaert L M 5256 met M O 10000, comt voor O I (ghelijck boven quam voor D F) als Saturnus meeste verhey	107564.
Ende L M 5256 met M O 10000 t'samen 15256 ghetrocken van L K 92308, blijft voor O K (ghelijck boven quam voor D G) als Saturnus minste verhey	77052.
Ende Saturnuswechs verstepunt was ten tijde van <i>Ptolemus</i> soo hy seght int 5 Hoofstuck sijns 11 boucx onder des duystraers	233 tr.
En ghedaen sijnde derghelijcke rekeninghen met Iupiter en Mars, men bevint d'uytcomst als volght	
Iupiters wechs halfmiddellijn	52174.
Iupiters uytmiddelpunticheytlijn	2391.
Iupiters meeste verhey	64565.
Iupiters minste verhey	39783.
Iupiters wechs verstepunt was ten tijde van <i>Ptolemus</i> soo hy seght int 1 Hoofstuck sijns 11 boucx onder des duystraers	161 tr.
Marswechs halfmiddellijn	15190.
Mars uytmiddelpunticheytlijn	1519.
Mars meeste verhey	26709.
Mars minste verhey	3671.
Marswechs verstepunt wasten tijde van <i>Ptolemus</i> soo hy segt int 7 Hoofstuck sijns 10 boucx onder des duystraers	115 tr. 30.
Om te hebbē Venuswechs halfmiddellijn met stelling eens roerenden Eertcloots, soo is voor al te weten dat haer wech met stelling eens vasten Eertcloots even ghenomen sijnde metten Eertclootwech 10000, soo doet haer inronts halfmiddellijn dā sulcke 7194, want segghende Venus inrontwechs halfmiddellijn 60 deel, geeft haer inronts halfmiddellijn 43 deel 10 ⊕ deur de voorgaende SAMING, wat 10000? comt voor haer inronts halfmiddellijn als vooren 7194. Maer t'gene men met stelling eens vasten Eertcloots noemt Venus inrontwech, is met stelling eens roerenden Eertcloots voor des selfden roerenden Eertclootwech: Ende het ghene men met stelling eens vasten Eertcloots noemt Venus inront, is met stelling eens roerenden Eertcloots voor Venus wech, daerom Venus wechs halfmiddellijn met stelling eens roerenden Eertcloots doet	7194.
Om te hebben Venus uytmiddelpunticheytlijn, ick seggh haer inrontwechs halfmiddellijn 60 deel, gheeft haer uytmiddelpunticheytlijn 1 deel 15 ⊕ deur de voorgaende SAMING, wat 10000? comt Venus uytmiddelpunticheytlijn	208.
Om te hebben Venus meeste en minste verheden, ick sal op dat alles clarder sy, teyckenen twee formen, d'eerste met stelling eens vasten Eertcloots, d'ander eens roerenden. Laet A B Venus inrontwech sijn, diens middelpunt C, en de halfmiddellijn C A doet 10000 deur t'gestelde, D is den vasten Eertcloot, C D de uytmiddelpunticheytlijn doende 208 vierentwintichste in d'oirdē, A des inrontwechs verstepunt, B naestepunt, voort sy op A als middelpunt beschreven het inront EF, diens halfmiddellijn A E doet	

whose semi-diameter MN , being equal to AE , like the latter makes 10,000. This being so, the greatest distance OI must be equal to the above-mentioned DF , and the least OK equal to the above-mentioned least distance DG , for if to IL 92,308 is added LM 5,256 with MO 10,000, this makes for OI (as was found above for DF), as Saturn's greatest distance,

107,564

And if LM 5,256 with MO 10,000, making together 15,256, is subtracted from LK 92,308, there is left for OK (as was found above for DG), as Saturn's least distance,

77,052

And the apogee of Saturn's orbit was at the time of *Ptolemy*, as he says in the 5th Chapter of his 11th book, in the ecliptic at

233°

And when similar computations are made with Jupiter and Mars, the result is found as follows:

Semi-diameter of Jupiter's orbit

52,174

Jupiter's line of eccentricity

2,391

Jupiter's greatest distance

64,565

Jupiter's least distance

39,783

The apogee of Jupiter's orbit was at the time of *Ptolemy*, as he says in the 1st Chapter of his 11th book, in the ecliptic at

161°

Semi-diameter of Mars' orbit

15,190

Mars' line of eccentricity

1,519

Mars' greatest distance

26,709

Mars' least distance

3,671

The apogee of Mars' orbit was at the time of *Ptolemy*, as he says in the 7th Chapter of his 10th book, in the ecliptic at

115°30'

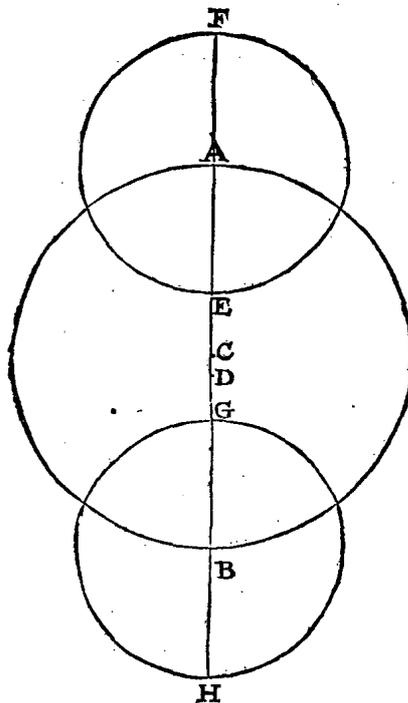
In order to have the semi-diameter of Venus' orbit on the theory of a moving Earth, it is to be noted first of all that if its orbit on the theory of a fixed Earth is taken equal to the Earth's orbit 10,000, the semi-diameter of its epicycle makes 7,194, for if I say: the semi-diameter of Venus' deferent 60^p gives its epicycle's semi-diameter 43^p10' according to the above Compilation, what does 10,000 give? Its epicycle's semi-diameter becomes, as above, 7,194. But what on the theory of a fixed Earth is called Venus' deferent, on the theory of a moving Earth is the orbit of this moving Earth. And what on the theory of a fixed Earth is called Venus' epicycle, on the theory of a moving Earth is Venus' orbit; therefore the semi-diameter of Venus' orbit on the theory of a moving Earth makes

7,194

In order to have Venus' line of eccentricity, I say: its deferent's semi-diameter 60^p gives its line of eccentricity 1^p15' according to the above Compilation; what does 10,000 give? Venus' line of eccentricity becomes

208

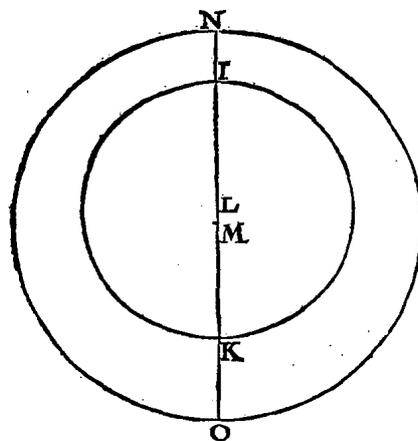
In order to have Venus' greatest and least distances, I will — in order to make everything clearer — draw two figures, the first on the theory of a fixed Earth, the second on that of a moving Earth. Let AB be Venus' deferent, whose centre is C , and the semi-diameter makes 10,000 by the supposition, D is the fixed Earth, CD the line of eccentricity, which makes 208 (the twenty-fourth in the list), A the



7194 driewintichste in de oirden, en des inronts verstepunt is F, naestepunt E, daer na sy op Bals middelpūt beschrevē het inront GH evē an EF, diēs naestepunt G: Dit so sijnde, en om nu te hebbē de meeste verhey D F, so vergaet ick tot CA 10000, de uytmiddelpunticheytlijn DC 208 vierētwintichste in d'oirden, mette halfmiddellijn AF 7194 driewintichste in d'oirdē, comt t'samē voor de meeste verhey D F 17402: Ende om te hebbē de minste verhey D G, ick tteck DC 208 met GB 7194 t'samen 7502, van CB 10000, en blijft voor de minste verhey D G 2598.

Maer om nu te sien sulcke overeencomming met eē roerendē Eertclood, laet het ront I K sijn Saturnus wech, diens middelpunt L, en de halfmiddellijn I L evē an A F doet als

die 7194, en t' middelpunt des Eertcloodwechs sy M, uytmiddelpunticheytlijn L M, doende als CD 208, Venuswechs verstepunt van M sy I, naestepunt K,



voort sy op M als middelpunt beschreven den Eertcloodwech NO, diens halfmiddellijn M N evē met CA doet als die 10000. Dit so wesende, de meeste verhey O I moet even sijn mette boveschreven DF, en de minste verhey N I, evē mette boveschreven minste DG, want tot I L 7194, vergaert L M 208 met M O 10000, comt voor O I (ghelijck bovē quam voor DF) als Venus meeste verhey diese vandē Eertclood hebben can

17402.

De selve getrocken vande heele middellijn NO 20000, blijft voor

2598.

de minste verhey die Venus vanden Eertclood hebben can
 En Venuswechs verstepunt was ten tijde van Ptolemeus, soo hy seght in 2 Hoofstlick sijns 10 boucx, onder des duyfteraers
 Ende ghedaen sijnde dergelijcke rekeninghen met Mercurius, men bevint d'uytcomst als volght :

Mercur.

deferent's apogee, B its perigee. Further let there be described about A as centre the epicycle EF , whose semi-diameter AE makes 7,194 (the twenty-third in the list), then the epicycle's apogee is F , its perigee E . Thereafter let there be described about B as centre the epicycle GH equal to EF , whose perigee be G . This being so, and in order to have the greatest distance DF , I add to CA 10,000 the line of eccentricity DC 208 (the twenty-fourth in the list), with the semi-diameter AF 7,194 (the twentythird in the list); this makes together for the greatest distance DF 17,402. And in order to have the least distance DG , I subtract DC 208 with GB 7,194, making together 7,402¹⁾, from CB 10,000; then there is left for the least distance DG 2,598.

But in order to see such correspondence with a moving Earth, let the circle IK be Venus' ²⁾ orbit, whose centre is L , then the semi-diameter IL equal to AF , like the latter, makes 7,194; and let the centre of the Earth's orbit be M , the line of eccentricity LM , making (like CD) 208; let the apogee of Venus' orbit from M be I , its perigee K . Further let there be described about M as centre the Earth's orbit NO , whose semi-diameter MN equal to CA , like the latter, makes 10,000. This being so, the greatest distance OI must be equal to the above-mentioned DF , and the least distance NI equal to the above-mentioned least distance DG , for if to IL 7,194 is added LM 208 with MO 10,000, this makes for OI (as was found above for DF), as the greatest distance from the Earth that Venus can have

17,402

When this is subtracted from the whole diameter NO 20,000, there is left for the least distance from the Earth that Venus can have

2,598

And the apogee of Venus' orbit was at the time of *Ptolemy*, as he says in the 2nd Chapter of his 10th book, in the ecliptic at

55°

And when similar computations are made with Mercury, the result is found to be as follows:

¹⁾ For 7,502 in the Dutch text read 7,402.

²⁾ For *Saturnus* in the Dutch text read *Venus*.

MET EEN ROERENDEN EERTCLOOT. 279

Mercurius wechs halfmiddellijn	3572.
Mercurius uytmiddelpunticheylijn	947.
Mercurius meeste verhey	14519.
Mercurius minste verhey	5481.
Mercurius wechs verstepunt was tē tijde van <i>Ptolemeus</i> , soo hy seght int 7 Hoofstick sijns 9 boucx, onder des duyfteraers	190 tr.

M E R C K T.

De ghetalen der halfmiddellijnen, uytmiddelpunticheylijnen, verheden, en schijnbaer duyfteraerlangden hier boven beschreven gelijckse gevonden wierden, die sal ick nu andermael int corte oirdentlick by een vervoughen, op dat daer uyt int volghende ghebruyck de begheerde ghetalen te gherievelicker gevonden meughen worden.

B Y E E N V O V G I N G V A N D E

Dwaaelders halfmiddellijnen der vveghen, uytmidderpunteylijnen, meeste en minste verheden vanden Eertcloon, altemael in sulcke deelen alffer des Eertcloonwechs halfmiddellijn 10000 doet, metsgaders der verstepuntens schijnbaer duyfteraerlangden ten tijde van Ptolemeus.

	Mercur.	Venus.	Eertcloon.	Maen.	Mars.	Iupiter.	Saturnus.
Vvechs halfmiddellijn.	3572	7194	10000	488	15190	52174	92308
Uytmiddelpunticheylijn.	947	208	417	43	1519	2391	5256
Meeſte verhey.	14519	17402	10417	531	26709	64565	107564
Minſte verhey.	5481	2598	9583	445	3671	39783	77052
Verstepuntens schijnbaer duyſteraerlangde.	190 tr.	55 tr.	245 tr. 30	- -	115 tr. 30	161 tr.	233 tr.

T B E S L V Y T. Wy hebben dan gevondē de halfmiddellijnē der wegen, de uytmiddelpunticheylijnen, met meeste en minste verhedē vande Dwaaelders, in sulcke deelen alffer des Eertcloonwechs halfmiddellijn 10000 doet, deur wiſconſtige wercking gegront op ſtelling eens roerendē Eertcloots, na den eyſch.

V E R V O L G H.

Aldus bekent sijnde de reden deser linien vande Dwaaelders weghen en haer inronden, in sulcke deelen alffer des Eertcloots halfmiddellijn 10000 doet, en dat des Eertcloonwechs halfmiddellijn in haer begrijpt 1210 halfmiddellijnen des Eertcloots, soo ist openbaer hoemen sal connen beschrijven sulcke voorstellen, als daer vermaen afgedaen is int 2 merck des 49 voorstels vant 2 bouck met ſtelling eens vasten Eertcloots, te weten sulcke voorstellen als het 23 en 24 vande Son, oock ghelijck het 39 en 40 vande Maen: En boven dien hoemen vinden sal alle linien daer voorvallende, in sulcke deelen alffer des Eertcloonwechs halfmiddellijn 10000 doet.

14 V O O R S T E L.

Te beschrijven den loop der drie bovenſte Dwaaelders Saturnus, Iupiter en Mars in haer vveghen, opeen ghegeven tijt, met ſtelling eens roerenden Eertcloots.

A a 4 T G H E.

Semi-diameter of Mercury's orbit	3,572
Mercury's line of eccentricity	947
Mercury's greatest distance	14,519
Mercury's least distance	5,481
The apogee of Mercury's orbit at the time of <i>Ptolemy</i> was, as he says in the 7th Chapter of his 9th Book, in the ecliptic at	190°

NOTE.

I will sum up once more briefly and orderly the values of the semi-diameters, lines of eccentricity, distances, and apparent ecliptical longitudes described above, as they have been found, so that the required values may be found more conveniently in the subsequent application.

LIST

of the semi-diameters of the Planets' orbits, lines of eccentricity, greatest and least distances from the Earth, all in such parts as the semi-diameter of the Earth's orbit has 10,000, as also the apparent ecliptical longitudes of the aphelia at the time of *Ptolemy*.

	Mercury	Venus	Earth	Moon	Mars	Jupiter	Saturn
Semi-diameter of the orbit	3,572	7,194	10,000	488	15,190	52,174	92,308
Line of eccentricity	947	208	417	43	1,519	2,391	5,256
Greatest distance	14,519	17,402	10,417	531	26,709	64,565	107,564
Least distance	5,481	2,598	9,583	445	3,671	39,783	77,052
Apparent ecliptical longitude of aphelion	190°	55°	245°30'	—	115°30'	161°	233°

CONCLUSION. We have thus found the semi-diameters of the orbits, the lines of eccentricity, with the greatest and least distances of the Planets, in such parts as the semi-diameter of the Earth's orbit has 10,000, by means of mathematical operations based on the theory of a moving Earth; as required.

SEQUEL.

The ratio of these lines of the Planets' orbits and their epicycles thus being known, in such parts as the semi-diameter of the Earth's orbit has 10,000, and the semi-diameter of the Earth's orbit comprising 1,210 semi-diameters of the Earth, it is evident how it is possible to describe propositions such as have been announced in the 2nd note to the 49th proposition of the 2nd book, on the theory of a fixed Earth, to wit, propositions such as the 23rd and 24th on the Sun, as also the 39th and 40th on the Moon ¹⁾; and moreover how any lines that may occur can be found, in such parts as the semi-diameter of the Earth's orbit has 10,000.

14th PROPOSITION.

To describe the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits in a given time, on the theory of a moving Earth.

¹⁾ These propositions, on the pages 171, 201 and 221 of the Dutch original, concern the distances of the Earth to the Moon, to the Sun and to Saturn, all expressed in the semi-diameter of the Earth.

280 SAT. IVP. MARS, VEN. EN MERC. VINDING

T'GHEGHEVEN. Om met Saturnus te beginnen soo laet den tijt sijn van een dach. T'BEGEERDE. Men wil daer op vinden den loop van Saturnus in sijn wech, met stelling eens roerenden Eertcloots.

T W E R C K.

Het is te weten dat Saturnus eyghentlick wesende in sijn wech tot sulcken plaets als daermen met stelling eens vasten Eertcloots het inronts middelpunt teyckent, soo en heeft hy eyghentlick gheen ander loop, doende daghelicx als int 18 voorstel des 1 boucx o tr. 2. o. 33. 31. 28. 51. Angaende de voorofachtringhen welcke men hem siet hebben, die en commen niet van wegghen een inront datmē hem deur stelling eens vasten Eertcloots toeschrijft, maer deur den Eertcloots loop, sulcx dat hy met soodanich daghelicx roerfel eenvoudelick in sijn wech draeyt: Ende desghelijcx is oock te verstaen van Iupiter en Mars.

T' B E S L V Y T. Wy hebben dan beschreven den loop der drie bovenste Dwaelders Saturnus, Iupiter en Mars in haer wegghen, op een ghegheven tijt met stelling eens roerenden Eertcloots, na den eyfch.

V E R T O O C H. 15 V O O R S T E L.

De drie bovenste Dwaelders Saturnus, Iupiter en Mars, ontfanghen met stelling eens roerenden Eertcloots, de selve schijnbaer duyfteraerlangde en verheyte vanden Eertclood, diefe hebbē met stelling eens vasten Eertcloots.

Om dit voorstel oirdentlick te verclaren, ick salt in ses leden verdeelen, waer af int eerste sal sijn de teyckening van een der drie bovenste als Marsloop met stelling eens vasten Eertcloots.

Int tweede de teyckening van Marsloop met stelling eens roerenden Eertcloots.

Int derde t'bewijs dat Mars in d'een en d'ander stelling een selve schijnbaer duyfteraerlangde heeft, en de selve verheyte vanden Eertclood als hy is in sijn inronts verstepunt, en t'inronts middelpunt an sijn wechs verstepunt.

Daer na om te bewijfen sulcx alsoo overal te gheschien, soo sal tot bereyding van dien int vierde lidt verelaert worden, hoe dat de halfmiddellijn van Saturnuswechs middelpunt tot des inronts middelpunt, altijt ewewijdeghe is mette halfmiddellijn van des inrontwechs middelpunt, tot des inronts middelpunt: En des Eertcloodwechs halfmiddellijn van haer middelpunt totten Eertclood, altijt ewewijdeghe mette halfmiddellijn des inronts vant middelpunt tot Mars.

Int vijfde lidt, dat Mars in d'een en d'ander stelling tot allen plaetsen een selve schijnbaer duyfteraerlangde heeft, en de selve verheyte vanden Eertclood.

Int sesste vant verschil datter valt tusschen de werckingen van d'een en d'ander stelling int berekenen der schijnbaer duyfteraerlangde des Dwaelders.

1 L I D T *inhoudende de teyckening van Marsloop met stelling eens vasten Eertcloots.*

Laet voor eerste stelling ghenomen worden t'punt A een vasten Eertclood te beteyckenē, en van A tot B sy des Sonwechs uytmiddelpunticheytlijjn doende na

SUPPOSITION. To begin with Saturn, let the time be one day. **WHAT IS REQUIRED.** It is required to find in that time the motion of Saturn in its orbit, on the theory of a moving Earth.

PROCEDURE.

It is to be noted that since Saturn actually is in its orbit in a place where on the theory of a fixed Earth the epicycle's centre is drawn, in reality it has no other motion, and moves daily, as in the 18th proposition of the 1st book ¹⁾, $0^{\circ} 2,0,33,31,28,51$. As to the advance-or-lag ²⁾ that it is seen to have, this is not caused by an epicycle ascribed to it on the theory of a fixed Earth, but by the motion of the Earth, so that with this daily motion it simply revolves in its orbit. And the same is also to be understood of Jupiter and Mars.

CONCLUSION. We have thus described the motion of the three upper Planets Saturn, Jupiter, and Mars in their orbits in a given time, on the theory of a moving Earth; as required.

THEOREM.

15th PROPOSITION.

The three upper Planets Saturn, Jupiter, and Mars on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

In order to explain this proposition properly, I will divide it into six sections, the first of which is to comprise the drawing of one of the three upper Planets, namely Mars' motion, on the theory of a fixed Earth.

The second, the drawing of Mars' motion on the theory of a moving Earth.

The third, the proof that on either theory Mars has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the apogee of its orbit.

Thereafter, in order to prove that this happens thus always, in preparation of this it will be explained in the fourth section how the semi-diameter from the centre of Mars' orbit ³⁾ to the planet itself ⁴⁾ is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre, and the semi-diameter of the Earth's orbit from its centre to the Earth always parallel to the semi-diameter of the epicycle from its centre to Mars.

The fifth section, that in all places Mars has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

The sixth, of the difference between the operations on either theory in the computation of the apparent ecliptical longitude of the Planet.

1st SECTION, comprising the drawing of Mars' motion on the theory of a fixed Earth.

Let it first be assumed that the point *A* denotes a fixed Earth, and let the line from *A* to *B* be the line of eccentricity of the Sun's orbit, according to *Ptolemy's*

¹⁾ See Stevin's "2nd Method" in the 19th proposition of the 1st book, p. 97.

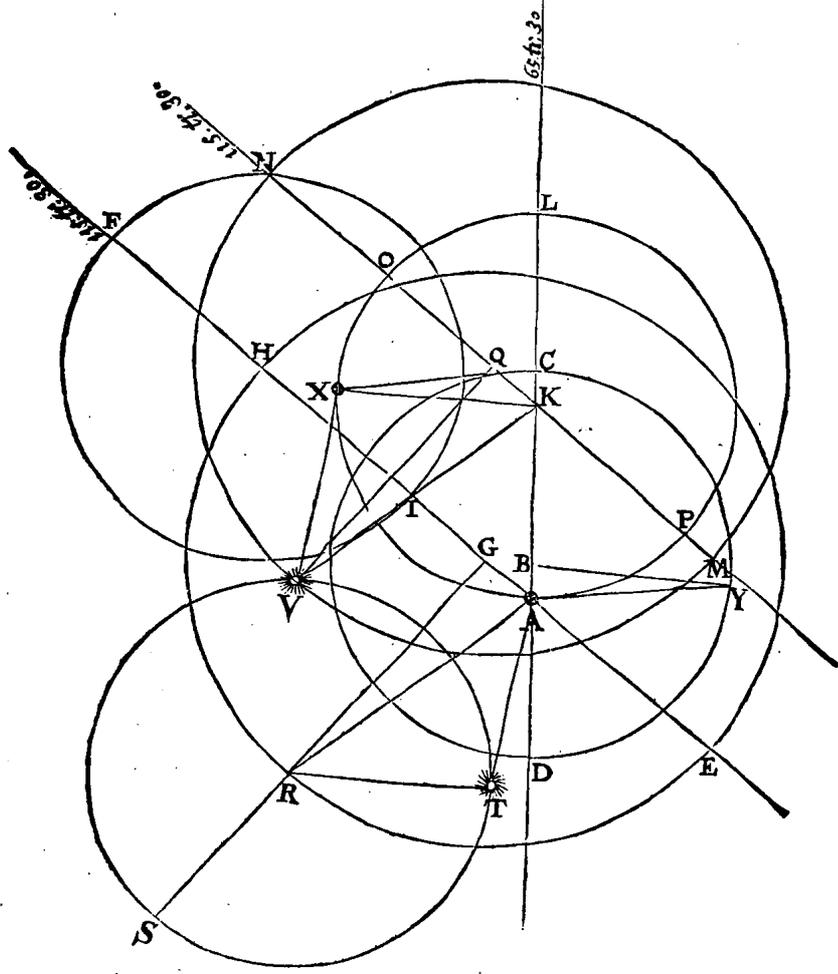
²⁾ Here advance-or-lag is used, as it was in the 18th Definition of the First Book, for the total deviation, by the epicycle as well as by the eccentricity.

³⁾ For *Saturnuswechs* in the Dutch text read *Marswechs*.

⁴⁾ For *des inronts middelpunt* in the Dutch text read an expression equivalent to "the planet itself".

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de na *Ptolemæus* rekening r'ijnder tijt sulcke 417, alser des Sonwechs halfmiddellijn die B C sy 10000 doet deur het 13 voorstel deses 3 boucx, mette selve B C sy op B als middelpunt beschreven de Sonwech C D, waer in C A voortghetrocken tot D, soo is D t'naestepunt, C t'verstepunt wefende tot *Ptolemæus* tijt onder des duystraers 65 tr. 30 ☉. Om hier op nu te doen de teyckening des loops van eenige der drie boveschreven Dwaelders, ick neem tot voorbeeld daer toe als boven gheseyt is Mars, wiens inrontwechs verstepunt ten tijde van



Ptolemæus gheweest hebbende onder des duystraers 115 tr. 30 ☉ deur het 13 voorstel deses 3 boucx, ick treck deur den vasten Eertclood A de lini E F van A tot F na des duystraers boveschreven 115 tr. 30 ☉, te weten soo dat den houck C A F doe 50 tr. dieder sijn vandē 65 tr. 30 ☉ daer C onder is, tottē 115 tr 30 ☉ daer F onder is. Ick stel daer na in A F t'punt G, soo dat A G sy Mars inrontwechs uytmiddelpunticheytlijn, doende deur het 13 voorstel sulcke 1519, alser des Sonwechs halfmiddellijn 10000 doet: Ick teycken daer na inde lini G F t'punt

computation making at his time 417 such parts as the semi-diameter of the Sun's orbit, which shall be BC , has 10,000, by the 13th proposition of this 3rd book; with the said BC let there be described about B as centre the Sun's orbit CD , in which, when CA is produced to D , D is the perigee, C the apogee, which in *Ptolemy's* time was at $65^{\circ}30'$ of the ecliptic. In order to base on this the drawing of the motion of one of the three aforesaid Planets, I take as example, as has been said above, Mars; the apogee of the latter's deferent having been in *Ptolemy's* time at $115^{\circ}30'$ of the ecliptic, by the 13th proposition of this 3rd book, I draw through the fixed Earth A the line EF from A to F towards the point at $115^{\circ}30'$ of the ecliptic described above, to wit, in such a way that the angle CAF be the 50° which are from the $65^{\circ}30'$ at which C is situated to the $115^{\circ}30'$ at which F is situated. I then take on AF the point G such that AG be the line of eccentricity of Mars' deferent, making by the 13th proposition 1,519 such parts as the semi-diameter of the Sun's orbit has 10,000. I then mark on the line GF the

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r punt H, alsoo dat G H doet 15190 voor Mars inrontwechs halfmiddellijn, welke deur het 13 voorstel van dier langde is in suleke deelen alffer des Sonwechs halfmiddellijn B C 10000 doet: Ick beschrijf daer na op G als middelpunt, mette halfmiddellijn G H den inrontwech H E, sniende A F in H als haer verstepunt, en E is t'naestepunt: Daer na beschrijf ick tot eenige plaets Mars inront, latet ten eersten sijn op t' verstepunt H als middelpunt, en dat mette halfmiddellijn H F even an B C (want ghelijck B C of H I 10000, tot G H, alsoo *Ptolemeus* gevonden reden van des inronts halfmiddellijn tot haer wechs halfmiddellijn, te weten van 39 deel 30 \odot , tot 60 deel deur het 11 voorstel) welck inront sy F I, diens verstepunt F, naestepunt I, waer me Mars voorghenomen teyckening met stelling eens vasten Eertcloots A voldoen is.

2 L I D T inhoudende de teyckening van Marsloop met stelling eens roerenden Eertcloots.

Om nu te commen totte teyckening met een roerenden Eertcloot, soo neem ick den Eertcloot A nu te loopen, en de Son an C vast te slaen, en teycken in C A t'punt K, alsoo dat de uytmiddelpunticheytlijn C K even sy an A B, en beschrijf op K als middelpunt, mette halfmiddellijn K A die even is met B C het rondt A L als Eertclootwech, diens naestepunt L, verstepunt A: Om nu hier op te doen de teyckening van Marsloop met stelling eens roerenden Eertcloots, ick trek deur t'punt K de lini M N ewewijdeghe met E F, en sal daerom K N oock strecken na des duystraers 115 tr. 30 \odot ghelijck A H; Ick stel daer na in K N t'punt Q, alsoo dat K Q even sy ande uytmiddelpunticheytlijn A G, en teycken t'punt N, soo dat Q N even sy an Mars inrontwechs halfmiddellijn G H, en beschrijf daer me Marswech N M, sniende de lini K N in N als verstepunt, en in M als naestepunt.

3 L I D T inhoudende bevvijst dat Mars in d'een en d'ander stelling een selve schijnbaer duyster aerlangde heeft, en de selve verheyte vanden Eertcloot als hy is in sijn inronts verstepunt, en t'inronts middelpunt an sijn wechs verstepunt.

Ghenomen dat Mars inronts middelpunt H met stelling eens vasten Eertcloots A sy an sijn wechs verstepunt H, en Mars ant inronts verstepunt F, soo salt van A tot F sijn inde grootste verheyte die Mars vanden vasten Eertcloot wesen can, ende dien volghens soo sal Mars met stelling eens roerenden Eertcloots moeten wesen an sijn wechs verstepunt N, en den roerendē Eertcloot an P, want daer me ist vanden roerenden Eertcloot P tot Mars an N, even soo verre als vanden Eertcloot A, tot Mars int inront an F, uyt oirfaeck dat Marswechs halfmiddellijn Q N mette uytmiddelpunticheytlijn Q K, even sijn an des inrontwechs halfmiddellijn G H mette uytmiddelpunticheytlijn G A, en boven dien K P halfmiddellijn des Eertclootwechs, even met H F halfmiddellijn des inronts. Voort want P N en A F ewewijdeghe sijn, soo wort Mars in d'een en d'ander stelling schijnbaerlick tot een selve plaets des duystraers ghesien: Ende om d'ergelijcke redenen ist openbaer, dat de aldercoriste verheyte P M met stelling eens roerenden Eertcloots even moet sijn an d' aldercoriste met stelling des vasten Eertcloots, t'welck soude sijn de lini van A tot des inronts naestepunt by aldient op E als middelpunt beschreven waer, maer ongheteyckent ghelaten is om deur veel liniē ghen duysterheyte te versorsaken.

4 L I D T

point H such that GH makes 15,190 for the semi-diameter of Mars' deferent, which by the 13th proposition is of that length in such parts as the semi-diameter of the Sun's orbit BC has 10,000. I then describe about G as centre, with the semi-diameter GH , the deferent HE , intersecting AF in H as its apogee, and E is the perigee. Thereafter I describe in some place Mars' epicycle: let it first be about the apogee H as centre, such with the semi-diameter HF equal to BC (for as BC or HI 10,000 is to GH , so is the ratio of the epicycle's semi-diameter to the semi-diameter of its orbit found by *Ptolemy*, to wit, of $39^{\circ}30'$ to 60° , by the 11th proposition), which epicycle shall be FI , whose apogee is F and perigee I ; with which the proposed drawing of Mars on the theory of a fixed Earth A has been completed.

2nd SECTION, comprising the drawing of Mars' motion on the theory of a moving Earth.

In order to come to the drawing on the theory of a moving Earth, I now take the Earth A to move and the Sun to be fixed at C , and I mark on CA the point K such that the line of eccentricity CK be equal to AB , and I describe about K as centre, with the semi-diameter KA , which is equal to BC , the circle AL as the Earth's orbit, whose perihelion is L and aphelion A . In order to base on this the drawing of Mars' motion on the theory of a moving Earth, I draw through the point K the line MN parallel to EF , and therefore KN will also tend towards $115^{\circ}30'$ of the ecliptic, like AH . I then take on KN the point Q such that KQ be equal to the line of eccentricity AG , and I mark the point N such that QN be equal to the semi-diameter of Mars' deferent GH , and with this I describe Mars' orbit NM , intersecting the line KN in N as apogee and in M as perigee.

3rd SECTION, comprising the proof that on either theory Mars has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the apogee of its orbit.

Assuming that the centre H of Mars' epicycle, on the theory of a fixed Earth A , be at the apogee H of its orbit, and Mars at the epicycle's apogee F , the distance from A to F will be the greatest distance at which Mars can be from the fixed Earth, and consequently on the theory of a moving Earth Mars will have to be at the apogee N of its orbit and the moving Earth at P , for thus it is as far from the moving Earth P to Mars at N as from the Earth A to Mars in the epicycle at F , because the semi-diameter of Mars' orbit QN , with the line of eccentricity QK , is equal to the deferent's semi-diameter GH with the line of eccentricity GA , while moreover KP , semi-diameter of the Earth's orbit, is equal to HF , semi-diameter of the epicycle. Further, since PN and AF are parallel lines, on either theory Mars is seen apparently in the same place of the ecliptic. And for similar reasons it is evident that the shortest distance PM on the theory of a moving Earth must be equal to the shortest distance on the theory of a fixed Earth, which would be the line from A to the epicycle's perigee, if it were described about E as centre, but it has not been drawn, so as not to cause obscurity on account of a multitude of lines.

4 *LIDT dat de halfmiddellijn van Mars vechs middelpunt tot des inronts middelpunt, altijd ewewijdeghe is mette halfmiddellijn van des inrontsvrechts middelpunt tot des inronts middelpunt: En des Eertclootvrechts halfmiddellijn van haer middelpunt totten Eertclood, altijd ewewijdeghe mette halfmiddellijn des inronts want middelpunt tot Mars.*

Laet Mars inronts middelpunt ghecommen sijn van H tot R, deur welcke R getrocken de rechte G R S, soo sy S middelverstepunt, van t'welck daerentuschen Mars met stelling eens vasten Eertcloots ghecommen sy tot T: En hier om sal Mars met stelling eens roerenden Eertcloots op dien tijt ghecommen sijn van N tot V, soo dat de booch N V even is ande booch H R, en ghetrocken Q V sy is even en ewewijdeghe met G R.

Oock sal den roerenden Eertclood die doen was an P, van daer gedaen hebben den loop P L X, even ande boveschreven twee als P L even neem ick an S T, en L X ghelijck met N V, of den houck L K X even mettē houck N Q V, het welck soo wesen moet deur het 33 voorstel des 1 boucx, alwaer bewesen is Mars verstepuntloop met sijn inrontloop t'samen even te wesen mette Sonloop, t'welck hier oock is metten Eertcloodloop.

Nu dan den roerenden Eertclood gesien uyt K, sal sulcken loop gedaen hebben in schijnbaer duysteraerlangde als de booch P L X mebrengt: En dergelijcke loop in schijnbaer duysteraerlangde sal oock gedaen hebben Mars met stelling eens vasten Eertcloots. Dit soo wefende, den roerenden Eertclood gesien uyt sijn wechs middelpunt K, sal sulcken loop ghedaen hebben in schijnbaer duysteraerlangde, als Mars met stelling eens vasten Eertcloots ghesien uyt sijn inronts middelpunt, en daerom moeten hun twee halfmiddellijnen K X, R T, ewewijdeghe sijn ghelijckse waren int begin des loops, te weten H F met K P, en ghelijck de strecking vant middelpunt K totten Eertclood P, doen was na de tegenoversijde der strecking vant middelpunt H tot Saturnus F, also is nu oock de strecking vant middelpunt K totten Eertclood X, na de teghenoversijde der strecking vant middelpunt R tot Saturnus T.

5 *LIDT dat Mars in d'een en d'ander stelling tot allen plaetsen een selve schijnbaer duysteraerlangde heeft, en de selve verheyt vanden Eertclood.*

Om tottet bewijs te commen ick treck de vier linien A R, A T, K V, V X, en segh daer me aldus: Angesien des driehoucx K V Q sijde K Q, evē en ewewijdeghe is met des driehoucx A R G sijde A G, gelijcx Q V even en ewewijdeghe met G R deur het 4 lidt, soo moet de derde sijde K V even en ewewijdeghe sijn mette derde A R: Voort segh ick dat anghesien des driehoucx K V X sijde K V, even en ewewijdeghe is met A R, en K X even en ewewijdeghe met R T, deur het 4 lidt, soo moet de derde sijde X V even en ewewijdeghe sijn mette derde A T, en daerom sietmen Mars an V uyt den roerenden Eertclood X, schijnbaerlick totte selve plaets des duysteraers daermē hem an T siet uyt den vasten Eertclood A, en is soo verre van V tot X, als van T tot A.

6 *LIDT*

4th SECTION, that the semi-diameter from the centre of Mars' orbit to Mars itself (QV)¹⁾ is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre. And the semi-diameter of the Earth's orbit from its centre to the Earth always parallel to the semi-diameter of the epicycle from its centre to Mars.

Let the centre of Mars' epicycle have travelled from H to R , and if through this R is drawn the straight line GRS , let S be the mean apogee, from which meanwhile let Mars on the theory of a fixed Earth have travelled as far as T . And for this reason, on the theory of a moving Earth Mars will in that time have travelled from N to V ²⁾, so that the arc NV is equal to the arc HR , and when QV is drawn, it is equal and parallel to GR .

The moving Earth, which then was at P , will also have performed thence the motion PLX , equal to the above-mentioned two, namely PL , which I take to be equal to ST , and LX equal to NV , or the angle LKX equal to the angle NQV , which must be so by the 33rd proposition of the 1st book, where it has been proved that the motion of Mars' apogee together with the motion in its epicycle is equal to the Sun's motion, which is here also equal to the Earth's motion.

Now the moving Earth, seen from K , will have performed the same motion in apparent ecliptical longitude as the arc PLX amounts to. And a similar motion in apparent ecliptical longitude will also have been performed by Mars on the theory of a fixed Earth. This being so, the moving Earth, seen from the centre of its orbit K , will have performed the same motion in apparent ecliptical longitude as Mars on the theory of a fixed Earth, seen from the centre of its epicycle, and therefore their two semi-diameters KX , RT must be parallel, as they were at the beginning of the motion, to wit, HF to KP , and just as the direction from the centre K to the Earth P then was opposite to the direction from the centre H to Mars³⁾ F , so the direction from the centre K to the Earth X is now opposite to the direction from the centre R to Mars⁴⁾ T .

5th SECTION, that in all places Mars has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

In order to come to the proof, I draw the four lines AR , AT , KV , VX , and then say as follows: Since the side KQ of the triangle KVQ is equal and parallel to the side AG of the triangle ARG , and likewise QV equal and parallel to GR , by the 4th section, the third side KV must be equal and parallel to the third side AR . Further I say that since the side KV of the triangle KVX is equal and parallel to AR , and KX equal and parallel to RT , by the 4th section, the third side XV must be equal and parallel to the third side AT , and therefore Mars is seen at V from the moving Earth X , apparently in the same place of the ecliptic where it is seen at T from the fixed Earth A , and it is as far from V to X as from T to A .

¹⁾ For *des inronts middelpunt* in the Dutch text read the equivalent of "Mars itself". Here the parallelism of QV to GR is demonstrated: QV belongs to the theory of the moving Earth, in which no epicycles occur, V is the position of Mars.

²⁾ Note that V is not really situated on the epicycle ST , but only on the orbit MN .

³⁾ For *Saturnus* in the Dutch text read *Mars*.

⁴⁾ For *Saturnus* in the Dutch text read *Mars*.

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6 LIDT van t'verschil datter valt tusschen de overkinghen van d'eenen d'ander stelling, int berekenen der schijnbaer duysteraerlangde der Dwaelders.

Met stelling een vasten eertcloots ontmoett ons int rekenen der soucking van Mars schijnbaer duysteraerlangde defes voorbeelts den gemeenen vierden houck A G R T, met vijf bekende palen, te weten drie sijden A G, G R, R T, alijt van een selve bekende langde: Voort den houck A G R als halffrontschil des bekenden houcx H G R, middelloop van des inronts middelpunt, en den houck G R T als halffrontschil des bekenden houcx S R T middelloop van Mars int inront, waer me deur het 6 voorstel inde Byvough der platte veelhoucken ghevonden sijnde den onbekenden houck G A T, en die verough totte bekende duysteraerlangde daer A G henē strect, dats na den 115 tr. 30 ①, men heeft t'begheerde.

Maer met stelling eens roerenden Eertcloots ontmoet ons hier den cruysvierhouck K Q V X, met vijf bekende palen, te weten drie sijden K Q, Q V, K X, alijt van een selve bekende langde, voort den houck K Q V, als halffrontschil des bekenden houcx N Q V middelloop van Saturnus in sijn wech, en dē houck Q K X, wesende des Eertcloots middellangde L K X, min den houck L K O van 50 tr. waer me deur het 6 voorstel inde Byvough der platte veelhoucken ghevonden sijnde den onbekenden houck K X V, en die ghetrocken vande bekende duysteraerlangde daer X K henen streckt, t'welck is de schijnbaer duysteraerlangde der Middelfon K, men heeft t'begheerde, en moet nootfakelick t'felve besluyt voortbrengghen datmen deur d'eerste wercking heeft.

Sulcx als hier is gheweest het bewijs van Mars, soo salt oock sijn van Saturnus en Iupiter.

T B E S L V Y T. De drie bovenste Dwaelders dan Saturnus, Iupiter en Mars ontfangghen met stelling eens roerenden Eertcloots de selve schijnbaer duysteraerlangde, en verheynt vanden Eertclood, diese hebben met stelling eens vasten Eertcloots, t'welck wy bewijsen moesten.

1 M E R C K.

Hier machmen nu sien d'oirfaeck hoet inde oneyghen stelling eens vasten Eertcloots by comt, dat des inronts middelpunts loop in sijn wech, en des Dwaelders loop in sijn inront, t'famen even vallen ande Sonloop, en niet int wesen te bestaen dat die Dwaelders totte Son een opsicht nemen als tot haer Coninck, ghelijck int 6 voorstel defes 3 boucx gheseyt is dat sommige hemlien daer in verwonderen, want inde wesentlicke sake en isser niet dan den Dwaelder, en den Eertclood, elck met sijn eygen roersel in sijn wech, waer me de oneyghen versierde stelling des vasten Eertcloots gheraect sulcken overeencomminghen te krijghen.

2 M E R C K.

Om in dit voorbeelt noch te sien de overeencomming vande Son metten Eertclood en Saturnus, in d'een en d'ander stelling, ick treck de lini X C vandē roerenden Eertclood X totte vaste Son C: Maer want in die stelling den roerenden Eertclood is ghecommen van A over P en L tot X, soo sal op den selven tijt met stelling eens vasten Eertcloots, de roerende Son moeten ghecommen sijn van C over D tot Y, sulcx dat de booch C D Y, even sy mette booch A P L X:

6th SECTION, of the difference between the operations on either theory in the computation of the apparent ecliptical longitude of the Planets.

On the theory of a fixed Earth we meet, in the computation of the finding of Mars' apparent ecliptical longitude in this example, with the ordinary quadrilateral $AGRT$, with five known terms, to wit: three sides AG , GR , RT , always of the same known length; further the angle AGR as supplement of the known angle HGR , mean motion of the epicycle's centre, and the angle GRT as supplement of the known angle SRT , mean motion of Mars in the epicycle; and thus, the unknown angle GAT being found, by the 6th proposition in the Supplement of Plane Polygons ¹⁾, and added to the known ecliptical longitude towards which AG tends, *i.e.* $115^{\circ}30'$, the value required is obtained.

But on the theory of a moving Earth we meet here with the crossed quadrilateral $KQVX$, with five known terms, to wit: three sides KQ , QV , KX , always of the same known length, further the angle KQV as supplement of the known angle NQV , mean motion of Mars ²⁾ in its orbit, and the angle QKX , being the Earth's mean longitude LKX minus the angle LKO of 50° ; and thus, the unknown angle KXV being found, by the 6th proposition in the Supplement of plane polygons ¹⁾, and subtracted from the known ecliptical longitude towards which XK tends — which is the apparent ecliptical longitude of the Mean Sun K — the value required is obtained, and this operation must necessarily lead to the same conclusion as that obtained by the first operation.

As the proof for Mars has been here, such it will also be for Saturn and Jupiter.

CONCLUSION. The three upper Planets Saturn, Jupiter, and Mars therefore on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth; which we had to prove.

1st NOTE.

Here we can see the cause why it is that according to the untrue theory of a fixed Earth the motion of the epicycle's centre in its orbit and the Planet's motion on its epicycle together are equal to the Sun's motion, and that in reality those Planets do not take their guidance from the Sun as from their King, as has been said in the 6th proposition of this 3rd book that some people wonder about it, for in reality there is nothing but the Planet and the Earth, each with its own movement in its orbit, as a result of which the untrue, fictitious theory of the fixed Earth obtains such correspondences.

2nd NOTE.

In order to see in this example the correspondence of the Sun with the Earth and Mars ³⁾, on either theory, I draw the line XC from the moving Earth X to the fixed Sun C . But since on this theory the moving Earth has travelled from A via P and L to X , the moving Sun in the same time, on the theory of a fixed

¹⁾ Stevin's *Trigonometry*, Work XI; i, 12, Appendix on Plane Polygons. See Vol. II B, p. 755. In the 6th proposition he considers a quadrangle and shows how all 8 angles and sides may be found, if 5 independent ones are given.

²⁾ For *Saturnus* in the Dutch text read *Mars*.

³⁾ For *Saturnus* in the Dutch text read *Mars*.

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APLX: T'welckfoo wesende AY moet even en ewewijdeghe sijn met XC, en vervolghens de vaste Son C, wort uyt den roerenden Eertclood X ghesien onder de selve duyfteraerlangde als de roerende Son an Y, uyt den vasten Eertclood A. Voort want XC ewewijdeghe is met AY, en XV met AT, soo is den houck VXC, even metten houck TAY, waer deur oock de schijnbaer verheyte van Saturnus V totte vaste Son C, ghesien uyt den roerenden Eertclood X, even is ande schijnbaer verheyte van Saturnus T totte roerende Son Y ghesien uyt den vasten Eertclood A.

3 M E R C K

Anghesien de twee stellinghen des eersten en tweeden lids, d'een met een vasten Eertclood, d'ander met een roerenden, t'samen in een form staen, deur dien t'bewijs sulcx daer vereyschte, en dattet tot meerder claarheyte can dienen, de selve elck besonderlick te sien, soo heb ickse hier van malcander ghescheyden als blijkt.

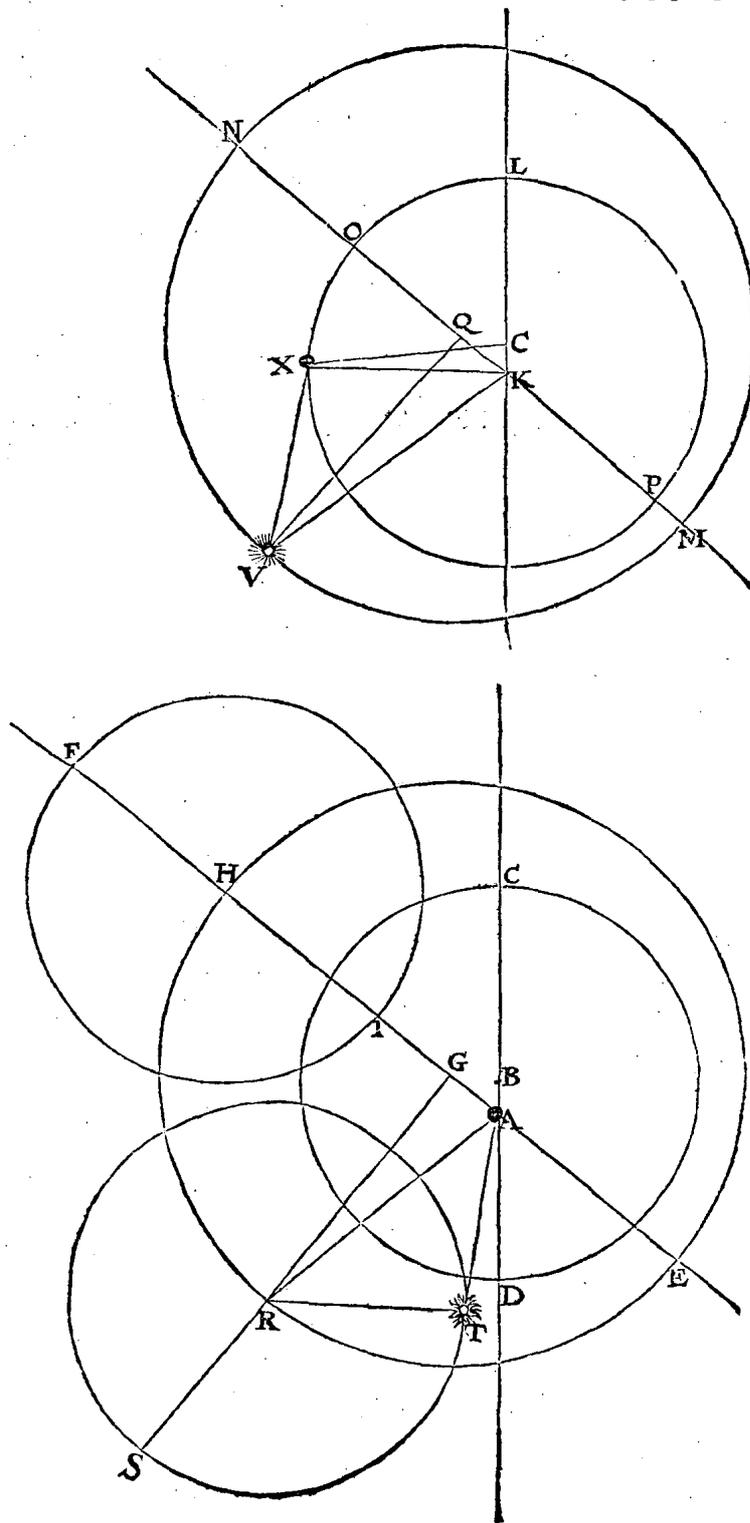
Bb 16 VOOR-

Earth, must have travelled from C via D to Y , so that the arc CDY shall be equal to the arc $APLX$. This being so, AY must be equal and parallel to XC , and consequently the fixed Sun C is seen from the moving Earth X at the same ecliptical longitude as the moving Sun at Y from the fixed Earth A . Further, since XC is parallel to AY , and XV to AT , the angle VXC is equal to the angle TAY , as a result of which also the apparent distance from Saturn V to the fixed Sun C , seen from the moving Earth X , is equal to the apparent distance from Mars ¹⁾ T to the moving Sun Y , seen from the fixed Earth A .

3rd NOTE.

Since the two theories of the first and the second section, one with a fixed and the other with a moving Earth, have been given together in one figure, because the demonstration required this, and since it may serve to make things clearer when each is seen separately, I have here separated them, as appears hereafter.

¹⁾ For *Saturnus* in the Dutch text read *Mars*.



16 VOORSTEL.

Te beschrijven den loop der twee onderste Dwaelders Venus en Mercurius in haer vveghen, op een ghegeven tijt, met stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Om met Venus te beginnen, soo laet den tijt sijn van een dach. T'BEGHEERDE. Men wil daer op vinden de loop in haer wech met stelling eens roerenden Eertcloots.

T W E R C K.

Totten loop van haer inronts middelpunt met stelling eens vasten Eertcloots doende op een dach deur

het 36 voorstel des 1 boucx otr. 59. 8. 17. 13. 12. 31.

Vergaert haer inronts loop eens dachs, doende deur

het 41 voorstel des 1 boucx otr. 36. 59. 25. 53. 11. 28.

Comt t'samen voor den begheerden loop eens dachs

van Venus met stelling eens roerenden Eertcloots 1 tr. 36. 7. 43. 6. 23. 59.

Ghemerckt het volghende * vertooch weseñde het 17 voorstel tot bewijs *Theorema.* deses * werckstucx dient, soo sullen wy dat daer voor laten verstrecken. Ende *Problema.* sghelijcx sal oock sijn den voortganck met Mercurius. T'BESLVYT. Wy hebben dan beschrevē den loop der twee onderste Dwaelders Venus en Mercurius in haer wegheñ, op een ghegheven tijt, met stelling eens roerenden Eertcloots, na den eysch.

VERTOPOCH. 17 VOORSTEL.

De twee onderste Dwaelders Venus en Mercurius, ontfangen met stelling eens roerenden Eertcloots de selve schijnbaer duyfteraerlangde, en verheyt vanden Eertclood, diese hebben met stelling eens vasten Eertcloots.

Om dit voorstel oirdentlick te verclaren, ick salt in sulcke ses leden verdeden, als int 15 voorstel mette drie bovenste ghedaen is, ghebruyckende Venus tot voorbeelt.

1 LIDT inhoudende de teyckening van Venus loop met stelling eens vasten Eertcloots.

Laet voor eerste stelling A ghenomen worden een vasten Eertclood te be- teyckenen, en van A tot B sy des Sonwechs uytmiddelpunticheytlijn, doende na *Prolemus* rekening t'sijnder tijt sulcke 417 alffer des Sonwechs halfmiddel- lijn die B C sy 10000 doet deur het 13 voorstel deses 3 boucx, mette selve B C sy op B als middelpunt beschreven de Sonwech C D, waer in C A voortghetroc- ken tot D, soo is D t'naestepunt, C t'verstepunt, weseñde tot *Prolemus* tijt on- der des duyfteraers 65 tr. 30 \odot . Om hier op nu te doē de teyckening des loops van eenige der twee boveschreven Dwaelders, ick neem tot voorbeelt daer toe als boven gheseyt is Venus, wiens inrontwechs verstepunt ten tijde van *Prole-*

Bb 2 *meus*

16th PROPOSITION.

To describe the motion of the two lower Planets Venus and Mercury in their orbits, in a given time, on the theory of a moving Earth.

SUPPOSITION. To begin with Venus, let the time be one day. WHAT IS REQUIRED. It is desired to find its motion in that time in its orbit on the theory of a moving Earth.

PROCEDURE.

When to the motion of its epicycle's centre on the theory of a fixed Earth, being in one day, by the 36th proposition of the 1st book 0° 59, 8,17,13,12,31 there is added its epicyclic motion in one day, being by the 41st proposition of the 1st book 0° 36,59,25,53,11,28 this makes together, for the required motion of Venus in one day on the theory of a moving Earth 1° 36, 7,43, 6,23,59

Considering that the subsequent theorem, being the 17th proposition, serves as proof of this problem, we shall use it as such. And the procedure with Mercury will be similar.

CONCLUSION. We have thus described the motion of the two lower Planets Venus and Mercury in their orbits, in a given time, on the theory of a moving Earth; as required.

THEOREM.

17th PROPOSITION.

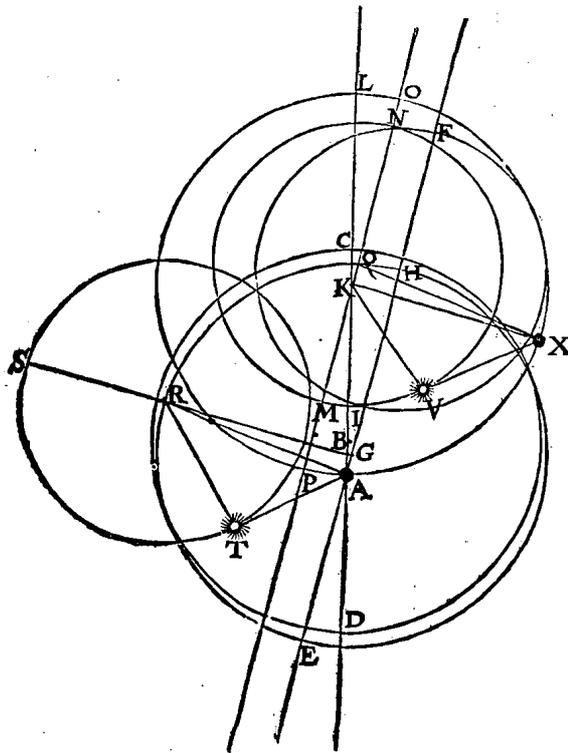
The two lower Planets Venus and Mercury on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth.

In order to explain this proposition properly, I shall divide it into six sections, as has been done in the 15th proposition for the three upper Planets, taking Venus as example 1).

1st SECTION, comprising the drawing of Venus' motion on the theory of a fixed Earth.

First let *A* be assumed to denote a fixed Earth, and let the line from *A* to *B* be the line of eccentricity of the Sun's orbit, according to *Ptolemy's* computation making at his time 417 such parts as the semi-diameter of the Sun's orbit, which shall be *BC*, has 10,000, by the 13th proposition of this 3rd book. With the said *BC*, about *B* as centre, let there be described the Sun's orbit *CD*, in which, when *CA* is produced to *D*, *D* is the perigee; *C* the apogee, which in *Ptolemy's* time was at 65°30' of the ecliptic. In order to base on this the drawing of the motion of either of the two aforesaid Planets, I take as example, as has been said above,

1) For this theory, see the explanatory diagrams and text in the Introduction, p. 16—17.



mens gheweest heb-
bende onder des duy-
steraers 55 tr. deur het
13 voorstel deses 3
boucx, ick treck deur
den vasten Eertcloot
A, de lini EF van A
tot F, na des duyste-
raers boveschreven
55 tr. te wetē soo dat
dē houck CAF doe
10 tr. 30 ①, dieder
sijn vanden 55 tr.
daer Fonder is, tottē
65 tr. 30 ①, daer C
onder is: Ick stel daer
nain AF t'punt G, so
dat AG sy Venus in
rontwechs uytmid-
delpunticheytlijn,
doende deur het 13
voorstel sulcke 208,
alser des Sonwechs
halfmiddellijn BC
10000 doet: Maer
want Venus inronts
wech even is metten

Eertclootwech, ghelijck ick gheseyt heb int 13 voorstel, en den selven Eert-
clootwech even mette Sonwech, soo beschrijf ick op t'punt G als middelpunt,
mette halfmiddellijn GA evē an des Sonwechs halfmiddellijn BC, dē inront-
wech HE, sniende HF in Hals haer verstepunt, en E is t'naestepunt: Ick be-
schrijf daer na tot eenige plaets Venus inrontwech, latet ten eersten sijn opt ver-
stepunt H als middelpunt, en dat mette halfmiddellijn HF, doende deur t'ghe-
ne verclaert is int 13 voorstel 7194, welck inront sy FI, waer me Venus teyck-
kening met stelling eens vasten Eertcloots A voldaan is.

2 LIDT inhoudende de teyckening van Venusloop met stelling
eens roerenden Eertcloots.

Om nu te commen totte teyckening met een roerendē Eertcloot, soo neem
ick den Eertcloot A nu te loopen, en de Son an C vast te staen, en teycken in
CA t'punt K, alsoo dat de uytmiddelpunticheytlijn CK even sy an AB, en be-
schrijf op K als middelpunt, mette halfmiddellijn KA die even is met BC, het
inront AL als Eertclootwech, diens naestepunt L, verstepunt A: Om hier op
te doen de teyckening van Venusloop met stelling eens roerenden Eertcloots,
ick treck deur t'punt K de lini MN ewwijdege met EF, sniende den Eertcloot-
wech in O en P, en sal daerom KN oock streckē na des duysteraers 55 tr. gelijk
AH: Ick stel daer na in KN t'punt Q, alsoo dat KQ even sy ande uytmid-
delpunticheytlijn AG, en teyckē t'punt N, soo dat QN even sy an Venus inronts
halfmiddellijn HF, en beschrijf daer me Venuswech NM, sniende de lini KN
in N als verstepunt, en in M als naestepunt.

3 LIDT

Venus; the apogee of the latter's deferent having been in *Ptolemy's* time at 55° of the ecliptic, by the 13th proposition of this 3rd book, I draw through the fixed Earth *A* the line *EF* from *A* to *F* towards the aforesaid point at 55° of the ecliptic, to wit, in such a way that the angle *CAF* be the $10^\circ 30'$ which are from the 55° at which *F* is situated to the $65^\circ 30'$ at which *C* is situated. I then take on *AF* the point *G* such that *AG* be the line of eccentricity of Venus' deferent, making by the 13th proposition 208 such parts as the semi-diameter of the Sun's orbit *BC* has 10,000. But because Venus' deferent is equal to the Earth's orbit, as I have said in the 13th proposition, and the said Earth's orbit is equal to the Sun's orbit, I describe about the point *G* as centre, with the semi-diameter *GH* ¹⁾ equal to the semi-diameter of the Sun's orbit *BC*, the deferent *HE* intersecting *HF* at *H* as its apogee, and *E* is the perigee. Thereafter I describe in some place Venus' epicycle ²⁾, let it be firstly about the apogee *H* as centre, such with the semi-diameter *HF*, making — by what has been explained in the 13th proposition — 7,194, which epicycle shall be *FI*, with which the drawing of Venus on the theory of a fixed Earth *A* is completed.

2nd SECTION, comprising the drawing of Venus' motion on the theory of a moving Earth.

In order to come to the drawing on the theory of a moving Earth, I now take the Earth *A* to move and the Sun to be fixed at *C*, and I mark on *CA* the point *K* such that the line of eccentricity *CK* be equal to *AB*, and I describe about *K* as centre, with the semi-diameter *KA* which is equal to *BC*, the circle ³⁾ *AL* as the Earth's orbit, whose perihelion is *L* and aphelion *A*. In order to base on this the drawing of Venus' motion on the theory of a moving Earth, I draw through the point *K* the line *MN* parallel to *EF*, intersecting the Earth's orbit in *O* and *P*, and therefore *KN* will also tend towards 55° of the ecliptic, like *AH*. Thereafter I take on *KN* the point *Q* such that *KQ* be equal to the line of eccentricity *AG*, and I mark the point *N* such that *QN* be equal to the semi-diameter of Venus' epicycle *HF*, and with this I describe Venus' orbit *NM*, intersecting the line *KN* in *N* as aphelion and in *M* as perihelion.

¹⁾ For *GA* in the Dutch text read *GH*.

²⁾ For *inrontswech* in the Dutch text read *inront*.

³⁾ For *inront* in the Dutch text read *ront*.

3 *L I D T* inhoudende bewijs dat Venus in d'een en d'ander stelling een selve schijnbaer duyfteraerlangde heeft, en de selve verheyt vanden Eertcloon als sy is in haer inronts verstepunt, en t'inronts middelpunt an sijn wechs verstepunt.

Ghenomen dat Venus inronts middelpunt H met stelling eens vasten Eertcloots A, sy in sijn wechs verstepunt H, en Venus ant inronts verstepunt F, soo valt van A tot F sijn inde grootste verheyt die Venus vanden vasten Eertcloon wesen can, en dien volghens soo sal Venus met stelling eens roerenden Eertcloots moeten wesen an haer wechs verstepunt N, en den roerenden Eertcloon an P, want daer me ist vanden roerendē Eertcloon P tot Venus an N, even soo verre als vandē vasten Eertcloon A, tot Venus int inront an F, uyt oirsaeck dat des Eertcloonwechs halfmiddellijn K P, mette uytmiddelpunticheytlijn Q K, even sijn an des inrontwechs halfmiddellijn G H mette uytmiddelpunticheytlijn G A, en boven dien Q N halfmiddellijn van Venuswech, even met H F halfmiddellijn des inronts: Voort want P N en A F ewewijdeghe sijn, soo wort Venus in d'een en d'ander stelling schijnbaerlick tot een selve punt des duyfteraers ghesien: En om dergheleijcke redenen ist openbaer dat de aldercortste verheyt N O met stelling eens roerendē Eertcloots, even moet sijn an d'andercortste met stelling des vasten Eertcloots, t'welck soude sijn de lini van A tot des inronts naestepunt, by aldient op E als middelpunt beschreven waer, maer onghereyckent ghelaten is om deur veel linien gheen duyfterheyt te veroirsaken.

4 *L I D T* dat de halfmiddellijn van des Eertcloonwechs middelpunt totten Eertcloon, altyt ewewijdeghe is mette halfmiddellijn van des inrontwechs middelpunt, tot des inronts middelpunt: En Venuswechs halfmiddellijn want middelpunt tot Venus, altyt ewewijdeghe mette halfmiddellijn des inronts want middelpunt tot Venus.

Laet Venus inronts middelpunt ghecommen sijn van H tot R, deur welcke R ghetrocken de rechte lini G R S, soo sy S middelverstepunt, van t'welck daerentusschen Venus met stelling eens vasten Eertcloots ghecommen sy tot T: Ende want den loop van Venus in haer wech even is ande twee loopen, d'een vant inronts middelpunt dats hier den houck H G R, d'ander, van Venus int inront, dats hier den houck S R T, soo moet Venus inde wech van N af daerentusschen gedaen hebben een loop even an die twee voorschreven houcken, welcke sy den verkeerden houck N Q V, ende ghetrocken Q V, sy moet even en ewewijdeghe sijn met R T, om t'bewijs dat op dergheleijcke ghedaen is van Mars int 4 lidt des 13 voorstels deses 3 boucx: Oock sal den roerenden Eertcloon die doen was an P, van daer ghedaen hebben den loop P X, even an des inronts middelpunts loop H R, waer deur oock de twee halfmiddellijnen K X, G R, dier twee even ronden even en ewewijdeghe moeten sijn.

5 *L I D T* dat Venus in d'een en d'ander stelling tot allen plaetsen een selve schijnbaer duyfteraerlangde heeft, en de selve verheyt vanden Eertcloon.

Bb 3 Om

3rd SECTION, comprising the proof that Venus on either theory has the same apparent ecliptical longitude and the same distance from the Earth when it is at its epicycle's apogee and when the epicycle's centre is at the orbit's apogee.

When it is assumed that on the theory of a fixed Earth A the centre of Venus' epicycle H is in its orbit's apogee H , and Venus at the epicycle's apogee F , from A to F will be the greatest distance at which Venus can be from the fixed Earth, and consequently, on the theory of a moving Earth, Venus will have to be at its orbit's apogee N , and the moving Earth at P , for thus it is just as far from the moving Earth P to Venus at N as from the fixed Earth A to Venus on the epicycle at F , because the semi-diameter of the Earth's orbit KP , and the line of eccentricity QK , are equal to the semi-diameter of the deferent GH and the line of eccentricity GA , while moreover QN , the semi-diameter of Venus' orbit, is equal to HF , the semi-diameter of the epicycle. Further, since PN and AF are parallel lines, Venus on either theory is apparently seen at the same point of the ecliptic. And for similar reasons it is evident that the very shortest distance NO on the theory of a moving Earth must be equal to the very shortest ¹⁾ distance on the theory of a fixed Earth, which would be the line from A to the epicycle's perigee, if it had been described about E as centre; but it has not been drawn, in order not to obscure the drawing by a multitude of lines.

4th SECTION, that the semi-diameter from the centre of the Earth's orbit to the Earth is always parallel to the semi-diameter from the deferent's centre to the epicycle's centre; and the semi-diameter of Venus' orbit from the centre to Venus always parallel to the semi-diameter of the epicycle from the centre to Venus.

Let Venus' epicycle's centre have travelled from H to R , and when through this R is drawn the straight line GRS , S shall be the mean apogee, from which meanwhile Venus, on the theory of a fixed Earth, shall have arrived at T . And because the motion of Venus in its orbit is equal to the two motions, one of the epicycle's centre, that is here the angle HGR , the other of Venus on the epicycle, that is here the angle SRT , Venus must meanwhile have performed in the orbit from N a motion equal to the two aforesaid angles, which shall be the opposite angle NQV ; and when QV is drawn, it must be equal and parallel to RT , because of the proof furnished in a similar case for Mars in the 4th section of the 13th proposition of this 3rd book. The moving Earth, which then was at P , will also have performed from there the motion PX , equal to the motion of the epicycle's centre HR , in consequence of which the two semi-diameters KX and GR of those two equal circles must also be equal and parallel.

5th SECTION, that in all places Venus has on either theory the same apparent ecliptical longitude and the same distance from the Earth.

¹⁾ For *andercortste* in the Dutch text read *aldercortste*.

290 SAT. IVP. MARS, VEN. EN MERC. VINDING

Om tottet bewijs te commen ick treck de vier linien AR, AT, QV, VX, en segh daer me aldus: Anghesien des driehoucx KXQ sijde KQ, even en ewewijdeghe is met des driehoucx ARG sijde AG, sghelijcx QX even en ewewijdeghe met GR, soo moet de derde sijde KX, even en ewewijdeghe sijn mette derde AR: Voort segh ick dat anghesien des driehoucx QVX sijde QX, even en ewewijdeghe is met AR, en QV even en ewewijdeghe met RT deur het 4 lidt, so moet de derde sijde XV, even en ewewijdege sijn mette derde AT: En daerom sietmen Venus an V uyt den roerendē Eertclood X, schijnbaerlick totte selve plaats des duyfteraers datmē Venus siet uyt den vasten Eertclood A, en is soo verre van V tot X, als van T tot A.

6 LIDT van t'verschil datter valt tusscheu de werckingen van d'een en d'ander stelling, int berekenen der schijnbaer duyfteraerlangde der Dwaelders.

Met stelling eens vasten Eertcloots ontmoet ons int rekenen der soucking van Venus schijnbaer duyfteraerlangde deses voorbeelts den ghemeenen vierhouck AGR T, met vijf bekende palen, te weten de drie sijden AG, GR, RT. altijd van een selve bekende langde: Voort den houck AGR, als halfrontschil des bekenden houcx HGR, middelloop van des inronts middelpunt, en den houck GRT, als halfrontschil des bekendē houcx SRT middelloop van Venus int inront, waer me deur het 6 voorstel inde byvough der platte veelhoucken ghevonden sijnde den onbekenden houck GAT, en die vervough totte bekende duyfteraerlangde daer AG henen streckt, dats na den 55 tr. men heeft het begheerde.

Maer met stelling eens roerendē Eertcloots ontmoet ons hier dē cruysvierhouck KQVX met vijf bekende palen, te weten drie sijden KQ, QV, KX, altijd van een selve bekende langde, voort den houck KQV, als halfrontschil des bekendē houcx NQV rontschil des middelloops van Venus in haer wech en dē houck QKX, wefende rontschil van des Eertcloots middellangde, waer me deur het 6 voorstel inde byvough der platte veelhoucken, ghevonden sijnde den onbekendē houck KXV, en die vergaert totte bekende duyfteraerlangde daer XK henen streckt, t'welck is de schijnbaer duyfteraerlangde der middelso K men heeft t'begeerde, en moet nootsakelick het eerste beslyt voortbrengghen datmen deur d'eerste wercking heeft.

Sulcx alshier is gheweest het bewijs van Venus, soo salt oock sijn van Mercurius.

T'BE SLVYT. De twee onderste Dwaelders dan Venus en Mercurius, ontfangghen met stelling eens roerenden Eertcloots de selve schijnbaer duyfteraerlangde en verheynt vanden Eertclood, diese hebben met stelling eens vasten Eertcloots, t'welck wy bewijssen moesten.

M E R C K T.

T'is kennelick datmen de twee stellinghen des eersten en tweeden lidts van Venus t'samen in een form ghemengt sijnde, soude meughen scheidē, gheelijck van Mars ghedaen wiert int 3 merck des 15 voorstels.

18 V O O R S T E L.

Te verclaren de reden vvaerom ick inde 8. 12. 15. en 17
VOOR-

In order to come to the proof, I draw the four lines AR , AT , QV , VX , and then say as follows: Since the side KQ of the triangle KXQ is equal and parallel to the side AG of the triangle ARG , and likewise QX equal and parallel to GR , the third side KX must be equal and parallel to the third side AR . Further I say that since the side QX of the triangle QVX is equal and parallel to AR , and QV equal and parallel to RT by the 4th section, the third side XV must be equal and parallel to the third side AT . And for this reason Venus is seen at V from the moving Earth X , apparently in the same place of the ecliptic where Venus is seen from the fixed Earth A ; and it is as far from V to X as from T to A .

6th SECTION, of the difference between the operations on either theory, in the computation of the apparent ecliptical longitudes of the Planets.

On the theory of a fixed Earth we meet, in the computation of the finding of Venus' apparent ecliptical longitude in this example, with the ordinary quadrilateral $AGRT$, with five known terms, to wit: the three sides AG , GR , RT , always of the same known length; further the angle AGR , as supplement of the known angle HGR , mean motion of the epicycle's centre, and the angle GRT , as supplement of the known angle SRT , mean motion of Venus on the epicycle. And thus, the unknown angle GAT being found, by the 6th proposition in the Supplement of Plane Polygons ¹⁾, and added to the known ecliptical longitude towards which AG tends, *i.e.* the point at 55° , the value required is obtained.

But on the theory of a moving Earth we here meet with the crossed quadrilateral $KQVX$, with five known terms, to wit: three sides KQ , QV , KX , always of the same known length; further the angle KQV , as supplement of the known angle NQV , supplementing to 360° the mean motion of Venus in its orbit, and the angle QKX , supplementing to 360° the mean longitude of the Earth. And thus, the unknown angle KXV being found, by the 6th proposition in the Supplement of Plane Polygons, and added to the known ecliptical longitude towards which XK tends, which is the apparent ecliptical longitude of the mean sun K , the value required is obtained, and this operation must necessarily lead to the first result, obtained by the first operation.

As the proof for Venus has been here, such is also that for Mercury.

CONCLUSION. The two lower Planets Venus and Mercury therefore on the theory of a moving Earth acquire the same apparent ecliptical longitudes and distances from the Earth that they have on the theory of a fixed Earth; which we had to prove.

NOTE.

It is obvious that the two theories of the first and the second section of Venus, which are combined in one figure, might be separated, as was done for Mars in the 3rd Note of the 15th proposition.

18th PROPOSITION.

To explain the reason why in the 8th, 12th, 15th, and 17th propositions I have proved that the Planets on the theory of a moving Earth are found in the same

¹⁾ See note ¹⁾ on p. 189.

MET EEN ROERENDEN EERTCLOOT. 291

voorstellen bevvesen hebbe de Dvvaelders deur stelling eens roerenden Eertcloots, bevonden te vvorden totte selve schijnbaer plaetsen, en verheden van malcander, diemense met stelling eens vasten Eertcloots bevint, mette omstandighen van dien.

Copernicus neemt wel, soot schijnt, dat de tweede stellinghen, d'een met een vasten d'ander met een roerenden Eertcloon (welverstaende als men uyt d'een treckt *Ptolemens* byvoughsels der onbekende oneventheden, en uyt d'ander de sijne) een selve bestuyt voortbrengken van der Dwaelders plaetsen en verheden van malcander, maer hy en doetter geen bewijs af, achtende soot schijnt, de sake soo claer datse t'bewijs niet en behouft, t'welck my nochtans tot grondelicker kennis der oirsaken dochte meerder versekerheyt te vereyssen, om verscheyden dinghen die my daer af int gedachte te vooren quamen: Ten eersten soo was ick een tijt lanck van meyning (ghelijcker meer sijn onder de gene die hun inde stelling eens roerendē Eertcloots oeffenen) datmen mette roerende Son deur t'ghedacht vast te stellen ter plaets des vasten Eertcloots, en daer teghen den Eertcloon roerende int rondt der Son, datmen daer me soude hebben t'begin der spiegheling des roerenden Eertcloots: Maer want de voorstellen op sulcken gront ghebout ghebreckich vielen, soo bevant ick de boveschreven verstelling des Eertcloots int rondt t'welck te vooren de Son wech gheweest hadde, niet te moeten gheschieden, maer dat om de vaste Son een ander rondt als Eertcloon wech behoort beschreven te worden. Ende want my dese dinghen soo kennelick niet en dochten datter gheen verclaring af en soude behouven, soo beschreef ick van dies het 8 voorstel deses 3 boucx: Ick bevant oock dat Saturnus met d'ander Dwaelders ghesien uyt de vaste Son, niet de selve duyfteraerlangde en hadden die des inronts middelpunt heeft ghesien uyt den vasten Eertcloon: Als by ghelijckenis int 15 voorstel deses 3 boucx, de lini A R niet even en ewewijdeghe te wesen met C V by aldiense ghetrocken waer, maer wel met K V. Voort sach ick dat met stelling eens roerenden Eertcloots haer wechs verstepunt valt op de teghenoversijde des verstepunts vande Son wech met stelling eens vasten Eertcloots, t'welck my oock met een opsicht soo kennelick niet en docht dattet gheen bewijs en soude behouven, sulcx dat ick t'selve verclaerde int 9 voorstel des 3 boucx.

Boven dien hoewel de Sonnens schijnbaer duyfteraerlangde gesien uyt den Eertcloon het teghenoverpunt is van des Eertcloots schijnbaer duyfteraerlangde gesien uyt de Son, waer deurmen lichtelick soude meynen, dat wanneer de Son met d'een stelling waer in des wechs eerste halffront hebbende achtring, dat op den selven tijt den Eertcloon met d'ander stelling soude sijn in des wechs tweede halffront hebbende voordering niet evegroot wesende mette Sonnens achtring: Nochtans sijn de Son en den Eertcloon in haer roerende stelling op een selve tijt elck inde wechs eerste halffront met achtring, of elck inde wechs tweede halffront met voordering, en beyde evegroot, welke dinghen my oock dochten haer bewijs te vereyssen, en daerom heb ick gedaen int 10 voorstel.

Ten tweeden so is my indē handel des Maenloops dit bejgent: T'is kennelick dat den Eertcloon in een uytmiddelpuntige wech draeyende, de selve onevenheyt of voorofachtring crijcht diemen met stelling eens vasten Eertcloots de Son toeschrijft, maer den Hemel der Maen den Eertcloon omvanghende en t'samen een cloot wesende, soo ist daer voor te houden, dat dien grooter cloot,

Bb 4 de sel

apparent places and distances from each other that they are found on the theory of a fixed Earth, with the circumstances relating thereto.

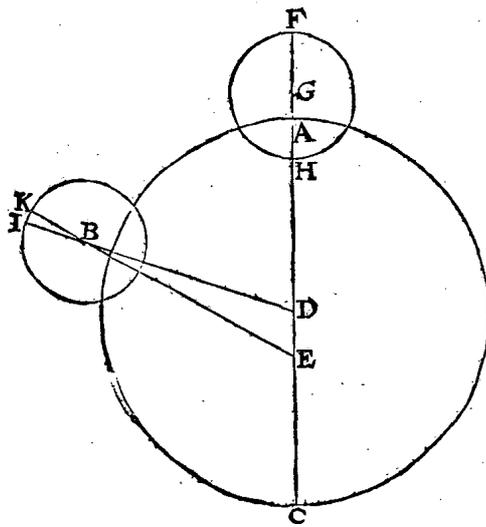
Copernicus indeed assumes, as it seems, that the two theories, one with a fixed and the other with a moving Earth (that is to say, if from the one are removed *Ptolemy's* additions of the unknown inequalities, and from the other, his own), lead to the same result with regard to the Planets' places and mutual distances, but he does not prove it, apparently considering the matter so clear that it does not call for any proof; but it appeared to me, with a view to the more thorough knowledge of the causes, to require greater certainty, on account of various things that occurred to me in this connection. In the first place, for some time I held the view (as there are more among those who study the theory of a moving Earth) that by imagining the moving Sun to be in the place of the fixed Earth, and on the other hand the Earth as moving in the circle of the Sun, one would have the beginning of the theory of the moving Earth. But since the propositions based on such a foundation turned out defective, I found that the aforesaid displacement of the Earth into the circle which had previously been the Sun's orbit should not be effected, but that about the fixed Sun another circle ought to be described as the Earth's orbit. And since these things did not appear to me so obvious as not to need an explanation, I described of this the 8th proposition of this 3rd book. I also found that Saturn and the other Planets, when seen from the fixed Sun, did not have the same ecliptical longitude that the epicycle's centre has, when seen from the fixed Earth; for example, in the 15th proposition of this 3rd book the line *AR* would not be equal and parallel to *CV*, if this line were drawn, but is equal and parallel to *KV*. Further I saw that on the theory of a moving Earth the aphelion of its orbit falls on the opposite side to the apogee of the Sun's orbit on the theory of a fixed Earth, which also did not appear to me so obvious at simple view as not to require any proof, so that I have explained it in the 9th proposition of the 3rd book.

Moreover, though the Sun's apparent ecliptical longitude, when seen from the Earth, is the opposite to the Earth's apparent ecliptical longitude, when seen from the Sun — in consequence of which one might easily think that if the Sun on the one theory were in the orbit's first semi-circle, being in lag, in the same time the Earth on the other theory would be in the orbit's second semi-circle, being in advance for an amount not equal to the Sun's lag — yet the Sun and the Earth, when assumed to be moving, are in the same time each in the orbit's first semi-circle with lag, or each in the orbit's second semi-circle with advance, and both equal in amount, which things also appeared to me to require a proof, and therefore I have given it in the 10th proposition.

Secondly, I met with the following in dealing with the Moon's motion. It is obvious that the Earth, when it revolves in an eccentric orbit, shows the same inequality or advance-or-lag that is ascribed to the Sun on the theory of a fixed Earth; but since the Heaven of the Moon contains the Earth and they are together one sphere, it is to be assumed that this greater sphere shows the

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de selve voorschreven onevenheyt crijcht, die de cleender den Eertcloot heeft waer uyt men niet onbillich en schijnt te besluyten dat haer verstepunt daer af oock de selve onevenheyt crijcht, t'welck my eerst dede twijffelen of dit geen oirsaeck vande tweede onbekende onevenheyt en mocht wesen die *Prolemus* meent bemerct te hebbē. Maer om hier in vaster te gaen, ick heb de saeck vorder overdocht, en daer toe een form gheteyckent op dese wijze: Laet A B C den Eertclootwech beteyckenen, diens middelpunt D, de Son E, deur welke twee punten D, E, ghetrocken de middellijn A D E C, soo beteyckent A t' verstepunt, C t'naestepunt, en den Eertcloot sy an A. Daer na verlangt D A tot E, en tusschen A en F ghestelt sijnde t'punt G, ick beschrijf daer op als middel-



punt de Maenwech F H, diēs verstepunt F: Maer want het roersel des seldē verstepunts tottet gene wy verclaren willen gheen verandering by en brengt, om dattet sonder onevenheyt is, soo neem ick de Maenwech geen eygen roersel te hebben, maer het middelverstepunt als F alijt te blijven inde lini ghetrocken uyt het middelpunt D deur den Eertcloot. Dit soo wesende, laet den Eertcloot met haer Maenwech gecommen sijn van A tot B, en sal haer middelverstepunt volghens t'voorgaende ghestelde, sijn in de voortghetrocken lini

D B, het welck sy in I, ick treck daer na van E deur B tot inde Maenwech de lini E B K, soo dat K t' verstepunt bediet, en den houck E B D of I B K de achtring des Eertcloots B, en daerom oock des heelen Maenwechs, en vervolghens des middelverstepunts I, t'welck tot een ander plaets gesien wort uyt E dan uyt D: Nu is de vraech of t' selve verstepunt I, mette comst des Eertcloots van A tot B, gheen onevenheyt ghecreghen en heeft van wegghen des Eertcloots uytmiddelpuntighe wech? Maer alles wel overdocht sijnde, soo valter onderscheyt te maken tusschen de plaetsen des sienders, want soomen het ooght stelt ande Son E als duystraers middelpunt, t' verstepunt I sal van daer ghesien een onevenheyt ghecreghen hebben: Maer want de dadelicke siening daer t'gheschil af is, niet en ghebeurt uyt E, maer uyt den Eertcloot B, soo moet men se daer nemen, t'welck doende t'punt K wijst an des Eertcloots schijnbaer duystraerlangde, als commende de lini E B K uyt des duystraers middelpunt E, sulcx dat by aldiē t'middelverstepunt daer gesien wierde, het soude mette comst des Eertcloots van A tot B de onevenheyt ghecreghen hebben des houck I B K: Maer t'wort an I ghesien, soo veel voorwaert als den houck K B I bedraecht, en daerom salt mette comst des Eertcloots van A tot B, schijnbaerlick inden duystraer soo veel ghelooopen hebben als den booch A B of den houck A D B bedraecht, dat is effen soo veel als den Eertcloot eyghentlick gelooopen heeft, sonder eenige onevenheyt: Inder vougen dat ick doen sach, dat hoewel den Eertcloot en oock den heelen Maenwech de selve onevenheyt heeft diemen met stelling

same aforesaid inequality that the smaller — the Earth — has, from which it seems to be concluded not unjustly that their apogee also receives from this the same inequality; which first caused me to doubt whether this could not be the cause of the second unknown inequality which *Ptolemy* thinks he has perceived. But to be more certain in this, I considered the matter further, and for this purpose drew a figure in the following manner: Let ABC denote the Earth's orbit, its centre D , the Sun E , and when through these two points D, E is drawn the diameter $ADEC$, A denotes the aphelion, C the perihelion, and the Earth shall be at A . When thereafter DA is produced to F , while between A and F is marked the point G , I describe about this as centre the Moon's orbit FH , whose apogee is F . But since the motion of this apogee does not make any difference to what we wish to explain, because it is without inequality, I assume that the Moon's orbit has no proper motion, but that the mean apogee, namely F , always remains on the line drawn from the centre D through the Earth. This being so, let the Earth with the Moon's orbit have travelled from A to B , then its mean apogee according to the foregoing supposition will be on the line DB produced, which shall be in I ; thereafter I draw from E through B into the Moon's orbit the line EBK , so that K denotes the apogee, and the angle EBD or IBK the lag of the Earth B , and therefore also of the whole Moon's orbit, and consequently of the mean apogee I , which is seen from E in another place than it is from D . Now the question is whether this apogee I has not, upon the passage of the Earth from A to B , acquired an inequality, on account of the Earth's eccentric orbit? But when everything has been well considered, a distinction has to be made between the places of the observer, for if the eye is put at the Sun's place E as centre of the ecliptic, the apogee I , when seen from there, will have acquired an inequality. But since the practical observation, with which the question is concerned, does not take place from E , but from the Earth B , it must be taken there; and when this is done, the point K indicates the Earth's apparent ecliptical longitude, since the line EBK comes from the ecliptic's centre E , so that, if the mean apogee were seen there, upon the passage of the Earth from A to B it would have acquired the inequality of the angle IBK . But it is seen at I , so much forwards as the angle KBI amounts to, and therefore upon the passage of the Earth from A to B it will apparently have moved in the ecliptic as much as the arc AB or the angle ADB amounts to, that is exactly as much as the Earth has really moved, without any inequality. In such a manner that I then saw that, though the Earth and also the whole Moon's orbit has the same inequality that is ascribed

Stelling eens vasten Eertcloots de Son toeschrijft, dat nochtans des Maenwechs verstepunt uyt den Eertcloon ghesien die onevenheyt niet en crijcht, twelck my uyt de boveschreven twijffeling brocht: Ick bevant oock datter gheen verandering en viel deur het stellen van des verstepunts eyghen loop, sdaechs op 52 ① 27 ② tegen t'vervolgh der trappē als int 11 voorstel deses 3 boucx, dat met stelling eens vasten Eertcloots is van 6 ① 41 ② na t'vervolgh der trappen deur het 11 voorstel des 1 boucx: Sghelijcx oock datter gheen verandering en viel deur het stellen des duysteringsnees eyghen loop sdaechs van 1 tr. 2. 19 tegen t'vervolgh der trappen als int selve 11 voorstel deses 3 boucx, dat met stelling eens vasten Eertcloots is van 3 ① 11 ② tegen t'vervolgh der trappen deur het 11 voorstel des 1 boucx.

Maer hoewel my dese voorschreven dinghen alsoo bekent wierden, dat en vertoonde sich niet soo openbaerlick, dat icker uyt soude willen besluuten de overeencommingen des Maenloops deur stelling eens roerenden Eertcloots, mette Maenloop deur stelling eens vasten Eertcloots gheen verclaring te behouven, sulcx dat ick daer af beschreef het 12 voorstel.

Ten derden soo is my int overlegghen des loops van d'ander Dwaelders, te weten Saturnus, Iupiter, Mars, Venus en Mercurius, dit te voeren ghecommen: T'is kennelick datmen tottet berekenen der Dwaelderplaetsen met stelling eens vasten Eertcloots, verscheyden linien treckt uyt des Sonwechs uytmiddelpuntichpunt, dats den selven vasten Eertcloon, tot d'ander noodighe punten, als inde form des 15 voorstels de linien A R, A T, ghetrocken uyt den vasten Eertcloon A. Ende dien volghens men soude lichtelick meynen dat tottet berekenen der Dwaelderplaetsen met stelling eens roerenden Eertcloots, sulcke linien als K X, K V, behooren ghetrocken te worden niet uyt K, maer uyt des Eertcloonwechs uytmiddelpuntichpunt C, dats de vaste Son, gemerckt die plaets sulcx wat is inde stelling eens roerenden Eertcloots, gelijk de plaets des vasten Eertcloots inde stelling der roerende Son: Inder voughen dat my dit oock docht bewijs te vereyschen.

Ten vierden machmen hier noch by voughen degroote verandering die der valt, deur datmen in des roerenden Eertcloots stelling het inrondt wech doet, den Dwaelder seght an des inronts middelpunt te wesen, en weerom daer teghen datmen den Eertcloon die vast ghestelt wiert, sulcken loop gheeft als ghesien is.

Al dit en dochten my gheen dinghen om an te nemen voor eyntlick een selve besluut uyt te brengghen, sonder daer af de reden te weten en bewijs te doen: T'is oock daer voor te houden dat sy die tot kennis dier gelijkheyt eerst gherochten, seker middelen tot inleyding hadden waer deur sy sulcx voor gewis hielden. Maer anghesien *Copernicus* die niet beschreven en heeft, nochians al t'ghene hy van dies stelt vast gaet (als de Maen met stelling eens roerenden Eertcloots soo te berekenen ghelijck met stelling eens vasten Eertcloots, sonder van wegghen de stelling des Eertcloonloops verandering te crijghen, voort in d'ander Dwaelders de linien als de boveschreven niet te trecken uyt de Son als weerelts middelpunt, maer uyt des Eertcloonwechs middelpunt) t'gheeft billichlick vermoeden een van tweeën, te weten of dat hem d'oirsaeck deser ghelijckheyt anders bejehent is dan my, en t'selve met sulcken lichticheyt dat hem onnoodich docht verclaring daer af te doen, of dies niet, dat hy by ghevalle ghecreghen heeft eenighe oude boucken vande beschrijving des loopenden Eertcloots, sonder sulcke redenen daer in vervatet te wesen, en dat hy daerom sulcx alsoo mach uytghegheven hebben.

Maer,

to the Sun on the theory of a fixed Earth, nevertheless the apogee of the Moon's orbit, when seen from the Earth, does not acquire that inequality, which removed my aforesaid doubts. I also found that no change was caused by putting the true motion of the apogee at $52'27''$ a day, contrary to the order of the degrees, as in the 11th proposition of this 3rd book, which on the theory of a fixed Earth is $6'41''$ in the order of the degrees, by the 11th proposition of the 1st book. Likewise that no change was caused by putting the proper motion of the line of nodes at $1^{\circ}2'19''$ a day contrary to the order of the degrees, as in the said 11th proposition of this 3rd book, which on the theory of a fixed Earth is $3'11''$ contrary to the order of the degrees, by the 11th proposition of the 1st book.

But though these aforesaid things thus became known to me, this was not manifested so clearly that I should be inclined to conclude from it that the correspondences of the Moon's motion on the theory of a moving Earth with the Moon's motion on the theory of a fixed Earth do not require any explanation; so that I described thereof the 12th proposition.

Thirdly, the following occurred to me in considering the motion of the other Planets, to wit, Saturn, Jupiter, Mars, Venus, and Mercury. It is evident that for the computation of the places of the Planets on the theory of a fixed Earth various lines are drawn from the point of eccentricity of the Sun's orbit, *i.e.* the said fixed Earth, to the other necessary points, namely in the figure of the 15th proposition the lines *AR*, *AT*, drawn from the fixed Earth *A*. And consequently one might easily think that for the computation of the places of the Planets on the theory of a moving Earth such lines, as *KX*, *KV*, ought to be drawn, not from *K*, but from the point of eccentricity of the Earth's orbit *C*, *i.e.* the fixed Sun, observing that this place is the same in the theory of a moving Earth as the place of the fixed Earth in the theory of the moving Sun; so that I thought that this, too, required to be proved.

Fourthly, there may be added the great change which is caused by the fact that in the theory of the moving Earth the epicycle is taken away, the Planet is said to be at the epicycle's centre, and again, on the other hand, that the Earth, which was assumed to be fixed, is given such a motion as has been seen.

All these appeared to me not to be things that could be taken to lead ultimately to the same result, without knowing the reason thereof and giving a proof. It is also to be assumed that those who first attained to knowledge of this equality had secure aids to guide them, in consequence of which they considered this certain. But since *Copernicus* has not described these, while yet all that he states about this is firmly established (such as computing the Moon on the theory of a moving Earth in the same manner as on the theory of a fixed Earth, without its undergoing any change because of the assumption of the Earth's motion; further, with the other Planets, not to draw the lines described above from the Sun as the world's centre, but from the centre of the Earth's orbit), it gives a just surmise of either one thing or the other, to wit, either that the cause of this equality appeared different to him from what it does to me, and this with such easiness that he thought it unnecessary to give an explanation of it, or otherwise that he had perhaps access to some old books describing the moving Earth, which did not contain such reasons, and that he may therefore have edited them like this.

Maer, mocht nu ymant segghen, laet een van beyden wesen, wat nut is uyt foodanich verhael te trecken? Dit: Byaldien d'oirsaek der gelijkheyt en overcomming deser twee stellinghen eens roerenden en vasten Eertcloots, voor *Copernicus* soo claer gheweest heeft, datse hem docht gheen uytlegging te behouven, t'fal anderen meughen beweghen daer op te letten, en naerder wech te beschrijven, om niet deur een langhet te doen dat deur een corter can ghedaen worden. Maer soo hy sulcx ghecreghen heeft uyt eenighe oude boucken hem ter handt ghecommen, t'foude hope gheven datter vande wetenschappen des wijfentijts herwaerts of derwaerts in d'een of d'ander * bouckcasse noch meer overblijffels meughen sijn dan die, en oirsaek gheven van vlietelicker daer na te vernemen dan men ghedaen heeft. T' B E S L V Y T. Hier is dan verclaert de reden waerom ick inde 8. 12. 15. en 17 voorstellen bewefen hebbe de Dwaelers deur stelling eens roerenden Eertcloots bevonden te worden totte selve schijnbaer plaetsen en verheden van malcander, diemense met stelling eens vasten Eertcloots bevint, mette omstandighen van dien, na den eyfch.

Bibliotheca.

19 V O O R S T E L.

Te verclaren op vvelcke stelling, te vveten de oneygen met een vasten Eertclood, of de eyghen met een roerende, oirboirft schijnt de rekeninghen te maken vande langde-loop der Dwaelers.

Int voorgaende 1 bouck beschreven sijnde den loop der Dwaelers deur ervarings dachtafels, en int 2 deur wisconftighe wercking beyde ghegront op de oneyghen stelling eens vasten Eertcloots, en dat ick volghens mijn voorneemen daer na in dit 3 bouck beschreven heb dien loop ghegront op de eyghen stelling des waren roerenden Eertcloots, soo mocht ymant dencken, dat nadien wy voor t'beginnen deses handels wisten die stelling oneyghen te wesen, en dese warachtich, oft niet beter en waer gheweest dien oneygen onbeschreven te laten, en den tijt mettet ondersoucken der selfde niet deur te brengen, maer in die plaets ten eersten ant warachtighe te vallen: Hier op antwoorde ick int ghemeen, dat ick de kennis van d'een en d'ander seer noodich acht, en om daer afint besonder breeder te spreken, segh aldus: Dese twee stellingen sulcke ghelijckheyt hebbende als int voorgaende blijktt, soo acht ick onnoodich te beschrijven nieuwe werckstucken vande vinding der Dwaelderlangden, effening der daghen, saminghen, teghestanden, duyfteringhen, en meer anderen, ghegront op stelling eens roerenden Eertcloots: la mijn meyning is dat de voorvallende rekeninghen, deurgaens bequamelicker souden gedaen worden deur vervouging des ghedachts op de oneyghen stelling eens vasten Eertcloots, en op ghetyckende formen na den eyfch van dien, dan op de eyghen des roerenden Eertcloots, al waren sy oock volcommelick beschreven.

Om van t'welck by voorbeelt breeder redenen te verclaren, ghenomen dat ymant in een varende schip bevale een pack thien ghemeten voeten achterwaert te legghen: Die sulck bevel na wil commen, stelt uyt sijn ghedacht het roersel des schips, die thien voeten daer in metende even al oft stil laghe, want anders verstaen sijnde, tusschen den tijt dattet bevel gheschiede, en t'werck gedaen wient, mach t'schip 1000 voet voort ghevaren sijn, inder voughen datmen het pack in plaets van 10 voeten, soude moeten 1010 voeten achterwaert legghen,

But, someone might now say, let it be one of the two; what profit is to be gained from such a story? This: If the cause of the equality and correspondence of these two theories of a moving and a fixed Earth was so clear to *Copernicus* that he thought it did not require any explanation, it may induce others to pay heed to this and describe a shorter course, in order not to do by a longer procedure what can be done by a shorter. But if he has learned this from some old books that came into his hands, this would raise the hope there may be more remains than these of the knowledge of the Age of the Sages, here or there in some library or other, and would cause us to inquire thereafter more diligently than has been done.

CONCLUSION. Here the reason has thus been explained why I have proved in the 8th, 12th, 15th, and 17th propositions that the Planets on the theory of a moving Earth are found in the same apparent places and distances from each other that they are found on the theory of a fixed Earth, with the circumstances relating thereto; as required.

19th PROPOSITION.

To explain on which theory, to wit, the untrue theory with a fixed Earth or the true theory with a moving Earth, it seems most suitable to base the computations of the motion in longitude of the Planets.

Since in the foregoing 1st book the motion of the Planets has been described by means of empirical ephemerides, and in the 2nd by mathematical operations, both based on the untrue theory of a fixed Earth, and since according to my intention I have thereafter described in this 3rd book the said motion according to the true theory of the moving Earth, it might be thought (since before we began this discussion, we knew the former theory to be untrue and the latter true) whether it would not have been better to leave the untrue one undescribed and not to waste time in studying it, but instead to begin at once with the true theory. To this I reply in general that I consider the knowledge of one theory and the other as highly necessary, and to speak of it specially in more detail, I say as follows: Since these two theories have such equality as appears in the foregoing, I deem it unnecessary to describe new problems about the finding of the Planets' longitudes, equality of time, conjunctions, oppositions, eclipses, and other things, based on the theory of a moving Earth. Nay, I am of opinion that the computations needed would generally be made more easily by directing our thought to the untrue theory of a fixed Earth and to figures drawn in accordance with its requirements than to the true theory of the moving Earth, even if they were described perfectly.

In order to explain this more fully by an example, assume that a man in a sailing vessel should order a parcel to be put ten paced feet backwards. He who wants to obey this order, puts out of his mind the movement of the ship, since those ten feet have exactly the same length as they would if it were lying still; for if it were understood differently: between the time when the order was given and that when the work was performed the ship may have sailed on 1,000 feet, so that, instead of 10 feet, the parcel would have to be put 1,010 feet

ghen, dat waer buyten t'schip misschien int water: T welck de meyning niet wesende, soo ist in sulcken ghevalle beter, her spreken en t'doen te voughen na het schijnbaerlick, dats na de schijnbaer stilstandt der inwendighe stoffen des schips, dan na het eyghen. Maer sooder een ander gheschil waer, niet van een pack te verleggen, maer van een pael, neem ick, int water te moeten slaen, thien voeten achterwaert van een seker plaets ant schip, daer verstaemen de saeck eyghentlick, te weten thien voeten achterwaert, van die plaets (t'schip mach daerentusschen ghevaren hebben hoet wil) daer den bevcelder af sprack. Sulcx dat in smenschen handelingen tweederley wijze van spreken en doen valt, d'ee- ne ghegront opt schijnbaer, d'ander opt eyghen, waer af men altijd die behoort te verkiesen, deur welcke men t'voornemen best can verstaen en uytrechten. Dit soo toeghelaten, t'schijnt oirboir in dit Hemelloopschrift t'ghedacht een gront te gheven oock op tweederley stelling, d'ee- ne schijnbaer, d'ander eyghen elck na den eyfch van t'voornemen daer de saeck lichtelicx deur can verstaen worden. Te weten opt schijnbaer, int leeren der beginfelen, en in t'maken der boveschreven rekeninghen, om datmen de woorden dier stof op desen loopen- den Eertcloot ghebruyckt al offe stil laghe: Als wannemen spreeckt van der weereltlichten opganck boven den sichteinder, onderganck onder den sicht- einder, comste tottet middachront, en veel dierghelijcke, t'welck eyghentlick heel verkeert sichteinders onderganck, en opganck is, en comste des middach- ronts totte lichten, welcke woorden duyster souden vallen: Ia en sijn by *Coper- nicus* self niet ghebesicht, hoewel sijn voornaemste wit was vanden roerenden Eertcloot te schrijven. Voort ghelijckt int varende schip nutter was sijn roer- sel uyt het ghedacht te stellen, en te houden al offe stil stonde, en t'schijnbaer te nemen al offe eygen waer, alsoo ist in desen ghevalle bequamer totte leering, het roersel des Eertcloots uyt het ghedacht te stellen, en te houden al offe stil stonde, het schijnbaer nemende al offe eyghen waer. Maer wesende gheschil vande breedeloop der Dwaelders Saturnus, Iupiter, Mars, Venus, en Mercu- rius, van welcke int volghende gheseyt sal worden, daer ist reden (ghelijck vant slaen der pael int water gheseyt is) sijn rekeningen eerst te gronden op formen gheteyckent na den eyfch der eygen stelling des roerenden Eertcloots, uyt oir- faeck dat wy daer deur beter connen gheraken tot oirfakelicke kennis des bree- deloops, en dat de wercking ghegront op de vierde stelling eens vasten Eert- cloots daer uyt ghetrocken wort op de wijze als ick hier na beschrijven sal.

Dits nu soo mijn ghevoelen, en op sulcx heb ick dese beschrijving geformt; doch soo ymant ander wichtigher redenen hadde my onbekent, deur welcke hy oirboirder bevonde anders te doen, hy soude die meughen volghen.

Tot hier toe vande langdeloop der Dwaelders gheseyt wesende, ick sal nu commen tottet beschrijven vande breedeloop.

VANDE

backwards, which would be outside the ship, perhaps in the water. This not being what was intended, in such a case it is better to speak and act according to appearance, *i.e.* the apparent rest of the internal materials of the ship, than according to the truth. But if there were another dispute, not of moving a parcel, but, for example, of having to strike a pole in the water, ten feet behind a given place in the ship, there the matter is taken literally, to wit, ten feet behind that place (the ship may meanwhile have sailed in any desired direction) of which the commander spoke. So that in men's actions there are two ways of speaking and doing, one based on appearance and the other on the truth, of which we should always choose that one by means of which the intention can best be understood and carried into effect. If this is admitted, it seems suitable in this book on the Heavenly Motions to base our thoughts also on two theories, one apparent, the other true, each according to the requirement of what is intended, by which the matter can be understood most easily. To wit, on the apparent theory in the learning of the elements and in the making of the aforesaid computations, because the words of this subject matter are applied to this moving Earth as if it were lying still. For example, when we speak of the luminaries rising above the horizon, setting beneath the horizon, their arrival in the meridian, and many similar things, which in reality, quite the reverse, is the setting and rise of the horizon, and arrival of the meridian at the luminaries, which words would be obscure. Nay, they have not been employed by *Copernicus* himself, though his chief object was to write about the moving Earth. Further, just as in the sailing ship it was more useful to put its movement out of one's mind and to assume that it lay still, and to take appearance as if it were the truth, thus in this case it is better for didactic purposes to put the motion of the Earth out of one's mind, and to assume that it stands still, taking appearance as if it were the truth. But when there is a dispute about the motion in latitude of the Planets Saturn, Jupiter, Mars, Venus, and Mercury, which will be discussed in what follows, there is good reason (as it has been said of the striking of a pole in the water) for first basing one's computations on figures drawn in accordance with the requirements of the true theory of the moving Earth, because thus we can better attain to knowledge of the causes of the motion in latitude, and because the operation based on the fictitious theory of a fixed Earth is inferred from it in the manner I shall describe hereinafter.

This is my opinion, and on this I have based this description, but if anyone had other, weightier reasons, unknown to me, for which he should find it more suitable to proceed otherwise, he may follow them up.

The motion in longitude of the Planets having been discussed up to this point, I shall now come to the description of the motion in latitude.

VANDE BREE- DE LOOP.

VYFDE ONDRSCHEYT DES DERDEN BOVCX VAN-

de breedloop der vijf Dvvaelders

Saturnus, Iupiter, Mars, Venus,

en Mercurius, met stelling eens

roerenden Eertcloots.

CORT BEGRYP DESES VYFDEN ONDRSCHEYTS.



*A*ghezien den roerenden Eertclood en de stilstaende Son, deur t'ghestelde alijt inden duysteraer sijn sonder afvrieking of breedde, en dat de Manens breedloop beschreven int 2 bouck beginnende ant 34 voorstel met stelling eens vasten Eertcloots, geen verschil en heeft dat verclaring behouft mette breedloop der stelling eens roerenden Eertcloots, soo en valt van hemlien breedde hier niet te segghen, maer alleenelick van die der vijf ander als volgt:

Ghelijck de Ouden in de beschrijving vande Dvvaelders langdeloop billichlick beginnen met dadelicke ervaringen van yders loop op bekenden tijt, om daer uyt ghemeene regelen te trecken dienende tottet vinden haers loops, in toecommende tijden, alsoo vereyscht oock sulcx de natuerlicke oirden in de beschrijving des breedeloops, inder vougen dat ick daer me in elck Dvvaelders beschrijving beginnen sal: Ende want de reghel van eenen Dvvaelder voor allen dient, soo sal ick alleenelick vanden eersten of oppersten Saturnus gheformde voorstellen beschrijven, en van d'ander sulck vermaen doen als breeder verclaring vereyscht. Hier af sal ick ses voorstellen beschrijven.

Het eerste vvesende in d'oirdē het 20, is beschrijving van Ptolemeus dadelicke ervaringen van Saturnus schijnbaer duysteraerbreedde, dienende om daer uyt ghemeene regel te trecken van sijn breedeloops eyghenschappen.

Het 2 vvesende in d'oirden het 21, om te vinden de vvechlangden der
vvee

OF THE MOTION IN LATITUDE

FIFTH CHAPTER

OF THE THIRD BOOK

Of the Motion in Latitude of the Five Planets
Saturn, Jupiter, Mars, Venus, and Mercury,
on the Theory of a Moving Earth

SUMMARY OF THIS FIFTH CHAPTER.

Since the moving Earth and the fixed Sun by supposition are always in the ecliptic without any deviation or latitude, and since the motion in latitude of the Moon (described in the 2nd book, starting with the 34th proposition) on the theory of a fixed Earth has no such difference from its motion in latitude on the theory of a moving Earth as would require an explanation, it is not necessary here to speak of their latitude, but only of that of the five others, as follows.

Just as the Ancients in describing the Planets' motion in longitude rightly start with practical experiences of the motion of each in a known time, in order to derive therefrom common rules serving to find their motion in future times, the same is also required by the natural order in the description of the motion in latitude, so that I will start therewith in the description of each Planet. And because the rule of one Planet applies to all, I will describe propositions with figures only of the first or upper Planet Saturn, and for the others I will give such description as is necessary for a fuller explanation. I will describe hereof six propositions.

The first, which is the 20th in the sequence, is a description of *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitude, serving to derive therefrom a common rule for the properties of its motion in latitude.

twee punten van Saturnusvechs schijnbaer grootste afwijckingen van den duyfteraer, metsgaders de cortste verheden vanden Eertclootvech totte selve twee punten: Oock me de langde der heele lini vant eenpunt dier grootste afwijcking tottet ander, in sulcke deelen alser des Eertclootvechs halfmiddellijn 10000 doet, deur wvifconftighe wvercking ghegront op stelling eens roerenden Eertcloots.

Het 3 wvesende in d'oirden het 22, om te vinden Saturnusvechs afwijcking vandē duyfteraer: Metsgaders hoe verre de duyfteraersne van des Eertclootvechs middelpunt valt, in sulcke deelen alser des Eertclootvechs halfmiddellijn 10000 doet, deur wvifconftige wvercking gegront op stelling eens roerenden Eertcloots.

Het 4 wvesende in d'oirden het 23, om te vinden de wechlangde der twee uyterste punten vande duyfteraersne, en vande twee uyterste punten der afwijcking in Saturnusvech: Oock der lini die van Saturnusvechs middelpunt op de duyfteraersne rechthouckich valt, in sulcke deelen alser des Eertclootvechs halfmiddellijn 10000 doet, deur wvifconftighe wvercking ghegront op stelling eens roerenden Eertcloots.

Het 5 wvesende in d'oirden het 24, om te vinden de langde der lini die van een gegevē punt in Saturnusvech rechthouckich valt opt plat des duyfteraers, in sulcke deelē alser des Eertclootvechs halfmiddellijn 10000 doet, deur wvifconftige wvercking gegront op stelling eens roerenden Eertcloots.

Het 6 wvesende in d'oirden het 25, om te vinden Saturnus schijnbaer duyfteraerbreedte op een ghegeven tijt, deur wvifconftighe wvercking ghegront op stelling eens roerenden Eertcloots.

Daer na sal volgen het bovēschreven vermaen van d'ander Dwaelders, sonder daer af gheformde voorstellen te maken.

E E R S T V A N S A T U R N U S
B R E E D E L O O P.

20 V O O R S T E L.

Te beschrijven *Ptolemeus* dadelicke ervarighen van Saturnus schijnbaer duyfteraerbreedten, dienende om daer uyt gemeene reghel te trecken van sijn breedeloops eyghenschappen.

I L I D T.

Want mijn gevoelen is datter by de menschen eertijts een grondelicker ervarentheyt gheweest heeft, vande Dwaelders langdeloop met stelling eens roerenden Eertcloots, soo vermoede ick daer uyt by hemlien oock kennils geweest

Cc te sijne

The 2nd, which is the 21st in the sequence, to find the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 3rd, which is the 22nd in the sequence, to find the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes is from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 4th, which is the 23rd in the sequence, to find the orbital longitudes of the two extremities of the line of nodes and of the two extremities of the deviation in Saturn's orbit; also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 5th, which is the 24th in the sequence, to find the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth.

The 6th, which is the 25th in the sequence, to find Saturn's apparent ecliptical latitude at a given time, by mathematical operation based on the theory of a moving Earth.

This is to be followed by the above-mentioned description of the other Planets, without making thereof illustrated propositions.

FIRST OF SATURN'S MOTION IN LATITUDE.

20th PROPOSITION.

To describe *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitudes, serving to derive therefrom a common rule for the properties of its motion in latitude.

1st SECTION.

Because it is my opinion that in earlier times people had more thorough experience of the Planets' motion in longitude on the theory of a moving Earth,

te sijne van der selve breedeloop, om datse, alsmender wat op let ghenouchsaem heur selven daer in openbaert, sulcx dat ick meen *Ptolemus* gagheslaghen ervaringhen van dies niet d'eerste te wesen, maer datmender voor hem al dapperlick op ghelet, en spiegheling daer af gheformt heeft, doch de selve ter handt van hem noch niemant anders gecommen sijnde, datmen weet, soo sullen wy danckbaerlick annemen den vlietighen arbeyt by hem hier in ghedaen, sonder welcke het nu swaerlick soude by comen tot bescheyt deses handels te gaken, ghemerckt men te weynich gaslaghers vindt. Om dan totte sake te comen ick segh aldus: Ghelijckmen tottet soucken der ghedaente vande Sonnens schijnbaer evenaerbreede, en vande Manens schijnbaer duystraerbreede, ten eersten tracht dadelick te vinden haer meeste afwijckinghen na het Zuyden en Noorden, om daer deur te comen tot kennis der plaats vande duystraersne, en de rest dies angaende, alsoo heeft *Ptolemus* int dadelick ondersoucken der ghedaente van Saturnus schijnbaer duystraerbreede, ten eersten ghetracht na sijn meeste afwijckingen, en die bevonden op de Noortsijde van 3 tr. 2 \odot (soo staets in sijn tafel) ghebeurende altijts als sijn inronts middelpunt schijnbaerlick was 50 tr. voor sijn wechs verstepunt, te wetē onder des duystraers 183 tr. en Saturnus an des inronts naestepunt: Maer buyten het naestepunt wesende, soo was sijn Noorderfche afwijcking voor datmael cleender, ende ten minsten doen hy alsoo ant verstepunt was, want hoewel hy dat dadelick niet sien en conde, deur dien Saturnus doen by de Son was, soo merckte hijt nochtans by giffing deur de daghelicksche minderinghen die hy in sijn uyterte verschijnighen gade slouch.

2 L I D T.

Op de Zuytsijde bevant hy de meeste afwijcking van 3 tr. 5 \odot , ghebeurende altijt als sijn inronts middelpunt was onder het reghenoverpunt des boveschreven 183 tr. dats onder des duystraers 3 tr. en Saturnus an sijn inronts naestepunt: Maer buyten het naestepunt wesende, soo was sijn Zuydersche afwijcking voor datmael cleender, en ten minsten doe men hem alsoo an het verstepunt vandt.

3 L I D T.

Tot hier toe is gheseyt van Saturnus eyghenscappen wesende het inronts middelpunt an sijn wechs grootste afwijckingen, maer wesende tusschen beyden in een der duystraersneen twee uytersten, soo bevant hy Saturnus altijt int plat des duystraers sonder breede, tot wat plaats des inronts hy oock mocht wesen.

4 L I D T.

Het schijnbaerlickste dat *Ptolemus* bedencken conde van d'oirsaek deses seltsaem roersels, om daer op een spiegheling te gronden, deur welcke men Saturnus breede op alle toecommende tijden berekenen en van te vooren weten mocht, heeft hy beschreven int 3 hoofstlick sijns 13 boucx, waer af den sin dusdanich is.

Laet AB't plat des duystraers beteyckenen overcant ghesien, diens middelpunt dats den vasten Eertclood C, waer deur ghetrocken is de wech DE, en streckende CD op de Noortsijde na des duystraers 183 tr. als int 1 lidt, sulcx dat

I suppose from this that they also were acquainted with their motion in latitude, because, if some heed is paid thereto, it is sufficiently revealed therein, so that I think that *Ptolemy's* observations were not the first in this respect, but that before his day it had already been attentively observed and theories had been framed on it; but since these have not been handed down to him or anyone else as far as is known, we will gratefully accept the diligent work he has done in this respect, without which it would now be difficult to attain to knowledge of this matter, seeing that too few observers are to be found. To come to the matter, I say as follows. Just as, in order to find the nature of the Sun's apparent equatorial latitude and of the Moon's apparent ecliptical latitude, it is first attempted to find directly their greatest deviations towards the South and the North, in order thus to know the place of the line of nodes and the rest relating thereto; thus *Ptolemy* in practically investigating the character of Saturn's apparent ecliptical latitude first inquired into its greatest deviations and found them on the North side to be $3^{\circ}2'$ (thus it appears in his table 1), which always occurred when its epicycle's centre was apparently at 50° ahead of its orbit's apogee, to wit at 183° of the ecliptic, and Saturn at the epicycle's perigee. But when it was outside the perigee, its Northerly deviation was for that case smaller, and smallest when it was thus at the apogee; for though he could not see this actually, because Saturn then was near the Sun, yet he found this by guessing, from the daily decrements he observed in its extreme appearances.

2nd SECTION.

On the South side he found the greatest deviation to be $3^{\circ}5'$, which always occurred when the epicycle's centre was at the point opposite the above-mentioned 183° , *i.e.* at 3° of the ecliptic, and Saturn at its epicycle's perigee. But when it was outside the perigee, its Southerly deviation was for that case smaller, and smallest when it was thus found at the apogee.

3rd SECTION.

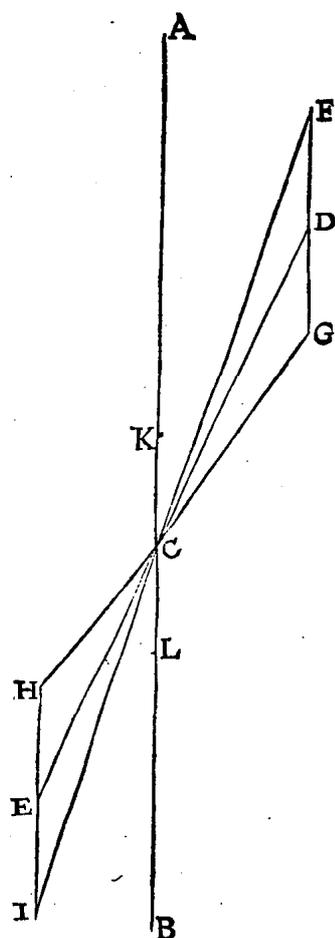
Up to this point Saturn's properties have been described when the epicycle's centre was at its orbit's greatest deviations, but when it was between the two at one of the two extremities of the line of nodes, he always found Saturn in the plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

4th SECTION.

The most plausible explanation that *Ptolemy* could think of as the cause of this curious movement, to base thereon a theory by means of which one might calculate and know in advance Saturn's latitude at all future times, he has described in the 3rd chapter of his 13th book, the meaning of which is as follows.

Let *AB* denote the plane of the ecliptic, seen transversely, whose centre is the fixed Earth *C*, through which has been drawn the orbit *DE*, *CD* tending on the North side towards 183° of the ecliptic, as in the 1st section, so that the two

¹⁾ *Syntaxis* XIII, 6 (Manitius II, p. 376), where also the tables for the other planets are found.



dat de twee houckē ACD, BCE des wechs afwijcking bediē, en op sijn uyerste punt D na het Noordē als middelpunt is beschrevē inront FG bycans ewewijdich mettē duyfteraer AB , diēs naestepunt G , verstepunt F , vā welcke getrockē sijn de twee liniē GC, EC . Ick heb hier geseyt bycans ewewijdich, deur dien t'verschil seer cleē is, alleenlick vā 2 tr. 4 \textcircled{D} , want dē houck ACD berekēde hy op 2 tr. 26 \textcircled{D} , en CDG op 4 tr. 30 \textcircled{D} , welcke twee houckē om volcommē ewewijdicheyt te hebben evengroot soudē moetē sijn. Dese boveschrevē rondē wordē altemael gelijk vanden duyfteraer geseyt is verstaē overcant gesiē te sijn. Voort wesende Saturnus ant inronts naestepunt G , so wiert sijn schijnbaer afwijcking vandē duyfteraer, dats dē houck ACG , ten grootstē bevonden van 3 tr. 2 \textcircled{D} daer af gheseyt is int 1 lidt. Maer buyten het naestepūt sijnde, als neem ick voor D , of an F , so was sijn Noordersche afwijcking voor datmael cleender, en ten minsten ant verstepunt F wesende, want cleender is dē houck ACF dā ACD . Maer om dergelijke verclaring oock te doen op de Zuytsijde, so laet op des wechs uyerste punt E als middelpūt, beschreven sijn het inront HI , oock bycans ewewijdich mettē duyfteraer AB , diens naestepunt H , verstepunt I , van welcke getrockē sijn de twee liniē HC, IC . Voort wesende Saturnus ant inronts naestepunt H , so wiert sijn afwijcking vanden duyfteraer, dats den houck BCH , tē grootstē bevonden vā 3 tr.

5 \textcircled{D} , daer af geseyt is int 2 lidt, maer buyten het naestepunt sijnde, als neem ick voor E of in I , soo was sijn Zuydersche afwijcking voor datmael cleender, en tē minsten ant verstepunt wesende, want cleender is den houck BCI dan BCE .

Dese twee eyghenschappen der afwijcking als het inront is ande uyersten DE aldus beschreven sijnde, wy sullen nu comen totte verclaring van d'oirsaek der eyghenschappen verhaelt int 3 lidt, te weten alst tusschen beyden is. *Ptolemēus* ghevoelt dat commende het inront van D na de duyfteraersne, het blijft daerentusschen altijt bycans ewewijdich metten duyfteraer, te weten soo gematicht, dat wesende het inront FG voor C , sijn plat is dan altijt teenemael int plat des duyfteraers, als ter plaets van KL , en alsdan soo moetmē gelijk int 3 lidt gheseyt is, Saturnus altijt bevinden inden duyfteraer sonder breede, tot wat plaets des inronts hy oock wesen mocht: Ende op sulcke spiegheling heeft *Ptolemēus* sijn voorstellen, tafelen en rekeninghen ghemaect, welcke ick na mijn stijl beschreven hadde, op dattet anderen tot hulp mochte strecken die na beter spiegheling trachten wilden, maer daer na ghevonden hebbende t'gene ick meen uyt kennis der oirsaken te comen, ghelijck ick dat int volgende beschrijven sal, heb sulcx onghedruckt ghelaten, ghenouch achtende de eyghenschap-

angles ACD , BCE denote the deviations of the orbit. And about its most northerly point D as centre has been described the epicycle FG , almost parallel to the ecliptic AB , whose perigee is G , apogee F , from which have been drawn the two lines GC , FC ¹⁾. I have here said "almost parallel", because the difference is very small, only $2^{\circ}4'$, for he calculated the angle ACD to be $2^{\circ}26'$, and CDG to be $4^{\circ}30'$, which two angles would have to be equal for perfect parallelism. The above-mentioned circles are all understood to be seen transversely, as has been said of the ecliptic. Further, when Saturn was at the epicycle's perigee G , its apparent deviation from the ecliptic, *i.e.* the angle ACG , was found at most to be $3^{\circ}2'$, as has been said in the 1st section. But when it was outside the perigee, I assume in front of D or at F ; its Northerly deviation was for that case smaller, and smallest when it was at the apogee F , for the angle ACF is smaller than ACD . But to give a similar explanation also for the South side, let there be described about the orbit's extremity E as centre the epicycle HI , also almost parallel to the ecliptic AB , whose perigee is H , apogee I , from which have been drawn the two lines HC , IC . Further, when Saturn was at the epicycle's perigee H , its deviation from the ecliptic, *i.e.* the angle BCH , was found to be at most $3^{\circ}5'$, as has been said in the 2nd section, but when it was outside the perigee, I assume in front of E or at I , its Southerly deviation was for that case smaller, and smallest when it was at the apogee, for the angle BCI is smaller than BCE .

These two properties of the deviation when the epicycle is at the extremities D , E thus having been described, we now come to the explanation of the cause of the properties described in the 3rd section, to wit, when it is between the two. *Ptolemy* is of opinion that when the epicycle moves from D to the nodes, meanwhile it always remains almost parallel to the ecliptic, to wit, so moderately that when the epicycle FG is in front of C , its plane is always altogether in the plane of the ecliptic, as at KL , and in that case, as has been said in the 3rd section, Saturn must always be found in the ecliptic without any latitude, no matter in what place of the epicycle it may be. And on this theory *Ptolemy* has based his propositions, tables, and calculations, which I had described in my own manner, in order that it might help others, who wanted to find a better theory. But since after that I found what in my opinion results from knowledge of the causes, as I shall describe in the following pages, I have left it unprinted, considering it sufficient that the properties of the first experiences, observed by *Ptolemy*

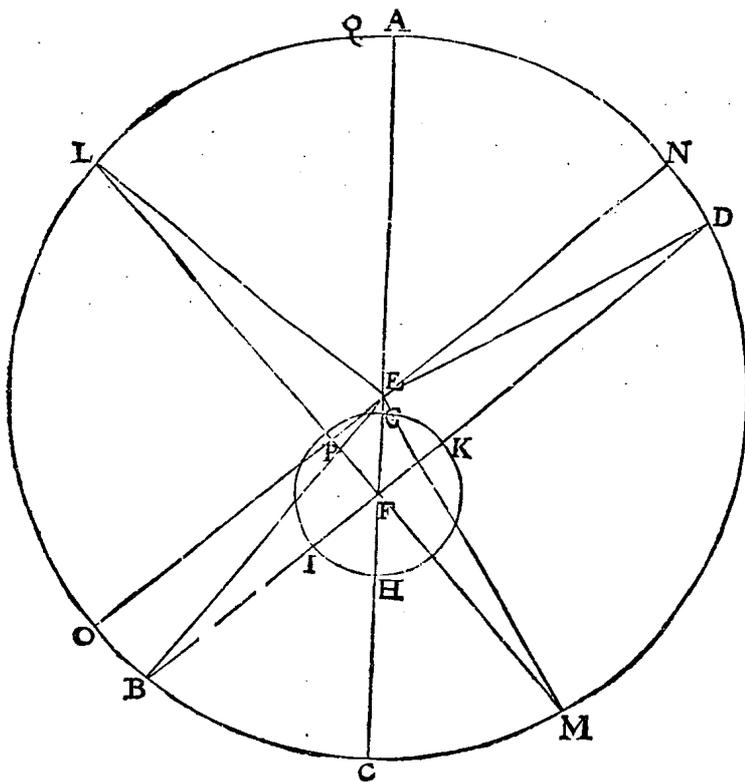
¹⁾ For EC in the Dutch text read FC .

schappen der eerste ervaringhen deur *Ptolemeus* mer seltsaem yver en neersticheyt gagheslagen, int gemeen aldus verclaert te wesen, en daer me te commen totte voorghenomen spiegheling ghegront op stelling eens roerenden Eertcloots. T' B E S L V Y T. Wy hebben dan beschreven *Ptolemeus* dadelicke ervaringhen van Saturnus schijnbaer duyfteraerbreden, dienende om daer uy: gemeene reghel te trecken van sijn breedeloops eyghenschappen, na den eyfch.

21 VOORSTEL.

Te vinden de vvechlangden der tyveepunten van Saturnusvvechs schijnbaer grootste afvvijskingen vanden duyfteraer, metsgaders de cortste verheden vanden Eertcloodvvech totte selve tyveepunten. Oock me de langde der heele lini vant een punt dier grootste afvvijsking totter ander, in sulcke deelē aliser des Eertcloodvvechs halfmiddellijn 10000 doet, deur vvisconftighe vvercking ghegront op stelling eens roerenden Eertcloots.

T' G H E G H E V E N. Laet *A B C D* Saturnuswech beteyckenen, diens middelpunt *E*, des eertcloodvvechs middelpunt *F*, waer op beschreven is den Eert-



clood.

with exceptional zeal and diligence, had thus been explained generally and that thus I had come to the intended theory based on the theory of a moving Earth. CONCLUSION. We have thus described *Ptolemy's* practical experiences of Saturn's apparent ecliptical latitudes, serving to derive therefrom a common rule for the properties of its motion in latitude; as required.

21st PROPOSITION.

To find the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let *ABCD* denote Saturn's orbit, whose centre is *E*, the centre

clootwech GH, en ghetrocken deur de twee punten F E Saturnuswechs middellijn AC, soo is A t'verstepunt, wesende onder des duyfteraers 233 tr. deur het 13 voorstel defes 3 boucx, ende want deur het 1 lidt des 20 voorstels, Saturnus grootste Noordersche afwijcking gebeurde als hy schijnbaerlick was 50 tr. voor des duyfteraers 233 tr. soo trek ick deur t'punt F de lini EFD, sniende den Eertclootwech in I en K, soo dat den houck AFD doe de selve 50 tr. en sullen daer me D en B sijn de twee punten van Saturnuswech daer hy schijnbaerlick in sijn grootste afwijckinghen vanden duyfteraer can gesien worden, endat uyt de punten K en I.

T'BEGHEERDE. Wy moeten vinden de wechlangde der twee punten B en D, dat sijn de twee boghen AB en ABD, voort de cortste twee verheden KD, IB, vanden Eertclootwech totte selve twee punten, oock me de langde der heele lini vant een punt dier grootste afwijcking tottet ander, in sulcke deelen alser des Eertclootwechs halfmiddellijn 10000 doer.

T'BEREYTSSEL. Laet ghetrocken sijn EB, ED.

T W E R C K.

De driehouck EFD heeft drie bekende palē, te wetē de uytmiddelpunticheytlijn EF 5256, des wechs halfmiddellijn ED 92308 deur de Byeenvouging van het 13 voorstel defes 4 boucx, en den houck EFD 50 tr. deur t'gegevē: Hier me gefocht de houck EDF, wort bevonden deur het 5 voorstel der platte driehoucken van 2 tr. 30.
 Welcke vergaert totten houck AFD 50 tr. deur t'ghegheven, comt voor den houck AED of booch AD 52 tr. 30.
 Maer t'verstepunt A is t'begin des wechs, daerom AD 52 tr. 30 ① ghetrocken vant heel ront 360 tr. blijft voor de begeerde wechlangde van t'punt der schijnbaer grootste uysterste Noordersche afwijcking D, dats den booch ABD 307 tr. 30.
 Ende de sijde FD wort bevonden van 95610, waer af getrocken des Eertclootwechs halfmiddellijn FK 10000, blijft voor de begeerde cortste verheyte KD vanden Eertclootwech tottet punt der schijnbaer grootste Noordersche afwijcking D 85610.
 Ende sghelijcx gedaen metten driehouck EFB, soo wort den houck EBF bevonden van 2 tr. 30.
 Welcke ghetrocken vanden houck BFC 50 tr. (als even sijnde met haer teghenoverhouck AFD) blijft voor den houck CEB 47 tr. 30.
 De selve ghetrocken vant halffront ABC, blijft voor de begheerde wechlangde vā t'punt der schijnbaer grootste Zuydersche afwijcking B, dats de booch AB 132 tr. 30.
 Ende de sijde FB wort bevonden van 88849, waer af ghetrocken des Eertclootwechs halfmiddellijn FI 10000, blijft voor de begeerde cortste verheyte IB, vanden Eertclootwech tottet punt der schijnbaer grootste Zuydersche afwijcking B 78849.
 En om te hebben de langde der heele lini BD, t'is kennelick dat die deur de drie deelen BI 78849, IK 20000, en KD 85610, t'samen bevonden wort voor t'begheerde van 184459.
 Waer af t'bewijs deur t'werck openbaer is. T'BESLYT. Wy hebben dan ghevonden de wechlangden der twee punten van Saturnuswechs schijnbaer grootste afwijckinghen vandē duyfteraer, metsgaders de cortste verhedē vanden Eertclootwech totte selve twee punten, oock me de langde der heele lini vant

Cc 3 cca

of the Earth's orbit F , about which has been described the Earth's orbit GH ; and when through the two points F, E is drawn the diameter of Saturn's orbit AC , A is the apogee, which is at 233° of the ecliptic by the 13th proposition of this 3rd book. And because by the 1st section of the 20th proposition Saturn's greatest Northerly deviation occurred when it was apparently at 50° before 233° of the ecliptic, I draw through the point F the line BFD , intersecting the Earth's orbit in I and K , so that the angle AFD shall make the said 50° ; then D and B will be the two points of Saturn's orbit where it can apparently be seen in its greatest deviations from the ecliptic, such from the points K and I .

WHAT IS REQUIRED. We have to find the orbital longitudes of the two points B and D , *i.e.* the two arcs AB and ABD , further the two shortest distances KD, IB from the Earth's orbit to these two points; also the length of the whole line from the one point of that greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000.

PRELIMINARY. Let there be drawn EB, ED .

PROCEDURE.

The triangle efd has three known terms, to wit, the line of eccentricity $EF = 5,256$, the orbit's semi-diameter $ED = 92,308$ by the Compilation of the 13th proposition of this 3rd book ¹⁾, and the angle $efd = 50^\circ$ by the supposition. When the angle edf is sought therewith, this is found, by the 5th proposition of plane triangles ²⁾, to be

$2^\circ 30'$

When this is added to the angle $afd = 50^\circ$ by the supposition, the angle aed or arc ad becomes

$52^\circ 30'$

But the apogee A is the beginning of the orbit, therefore when $AD = 52^\circ 30'$ is subtracted from the whole circle = 360° , there is left for the required orbital longitude of the point of the apparently greatest extreme Northerly deviation D , *i.e.* the arc ABD ,

$307^\circ 30'$

And the side FD is found to be 95,610, and when from this is subtracted the semi-diameter of the Earth's orbit $FK = 10,000$, there is left for the required shortest distance KD from the Earth's orbit to the point of the apparently greatest Northerly deviation D

85,610

And when the same is done with the triangle efb , the angle ebf is found to be

$2^\circ 30'$

When this is subtracted from the angle $bfc = 50^\circ$ (as being equal to its opposite angle afd), there is left for the angle ceb

$47^\circ 30'$

When this is subtracted from the semi-circle abc , there is left for the required orbital longitude of the point of the apparently greatest Southerly deviation B , *i.e.* the arc AB ,

$132^\circ 30'$

And the side fb is found to be 88,849, and when from this is subtracted the semi-diameter of the Earth's orbit $fi = 10,000$, there is left for the required shortest distance ib , from the Earth's orbit to the point of the apparently greatest Southerly deviation B ,

78,849

And in order to have the length of the whole line BD , it is obvious that by the addition of the three parts $BI = 78,849$, $IK = 20,000$, and $KD = 85,610$, this required value is found to be

184,459

¹⁾ For *dese 4 boucx* in the Dutch text read *dese 3 boucx*.

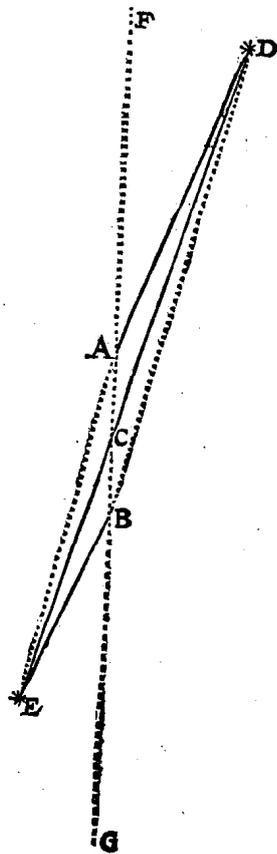
²⁾ Stevin's *Trigonometry* (Work XI; i, 12), p. 152. — See Vol. II B, p. 755.

een punt dier grootste afwijking tottet ander, in sulcke deelen alffer des Eertcloodwechs halfmiddellijn 10000 doet, deur wilconftighe wercking ghegront op stelling eens roerenden Eertcloods, na den eysch.

22 VOORSTEL.

Te vinden Saturnusvechs afvvijsking vandē duysteraer. Metfgaders hoe verre de duysteraersne van des Eertcloodvechs middelpunt valt, in sulcke deelen alffer des Eertcloodvechs halfmiddellijn 10000 doet, deur vvisconftighe vvercking ghegront op stelling eens roerenden Eertcloods.

T'GHEGHEVEN. Laet de lini A B den Eertcloodwech bedien overcant ghesien, die ick even teycken an des Eertcloodwechs middellijn I K inde form des 21 voorstels, C sy t'middelpunt, daer na treck ick dese D E, even met die D B inde selve form des 21 voorstels, doēde 1844959, en dese C D evē met die F D, dese C E even met die F B, en sniende dese twee linien makcander in of ontrent C, sulcx dat dē houck der afwijking dier twee platten is A C D, en A D sy de lini vanden Eertclood an A, tot Saturnuswechs uysterste noortsche punt der awijking doēde deur het 21 voorstel 85610, weseude daer de lini K D: Sgelijcx sy B E de lini vanden Eertclood an B, tot Saturnuswechs uysterste zuysche punt der afwijking E, doende deur het selve 21 voorstel 78849, welke daer beteykent is met F B, daer na A B an weder sijden verlangt weseude tot F en G, soo sijn A F en B G int plat des duysteraers, en Saturnusmeeste noordersche breede sy dē houck F A D, doende deur het 3 lidt des 20 voorstels 3 tr. 2 ①, maer sijn meeste zuyersche breede dē houck G B E, doende deur het 2 lidt des 20 voorstels 3 tr. 3 ①.



T'BEGERDE. Wy moeten vinden Saturnuswechs afwijking vanden duysteraer, dats dē houck A C D, metfgaders hoe verre de duysteraersne van des Eertcloodwechs A B middelpunt C valt.

T W E R C K.

De cruyfviethouck A B D E heeft vijf bekende palen, te weten de sijn A B middellijn des Eertcloodwechs 20000 deur t'ghefelde: Boven dien soo doet deur t'ghegheven A D 85610, B E 78849, den houck D A B 176 tr. 58 ①, want soo veel blijfter almen treckt F A D 3 tr. 2 ①, vant halfront, en den houck E B A 176 tr. 55 ①, want soo veel blijfter

The proof of which is evident from the procedure. **CONCLUSION.** We have thus found the orbital longitudes of the two points of the apparently greatest deviations of Saturn's orbit from the ecliptic, as well as the shortest distances from the Earth's orbit to these two points; also the length of the whole line from the one point of this greatest deviation to the other, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth; as required.

22nd PROPOSITION.

To find the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes is from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let the line AB denote the Earth's orbit, seen transversely, which I draw equal to the diameter of the Earth's orbit IK in the figure of the 21st proposition; let C be the centre. Thereafter I draw this DE equal to that DB in the said figure of the 21st proposition, making 1,844,959, and this CD equal to that FD , this CE equal to that FB , and these two lines intersecting each other in or about C , so that the angle of the deviation between those two planes is ACD , then AD shall be the line from the Earth at A to the extreme northerly point of the deviation of Saturn's orbit, making by the 21st proposition 85,610, which there is the line KD . In the same way let BE be the line from the Earth at B to the extreme southerly point of the deviation of Saturn's orbit E , making by the said 21st proposition 78,849, which is there denoted by FB . Thereafter, AB being produced on either side to F and G , AF and BG are in the plane of the ecliptic, and Saturn's greatest northerly latitude shall be the angle FAD , making by the 1st ¹⁾ section of the 20th proposition $3^{\circ}2'$, but its greatest southerly latitude the angle GBE , making by the 2nd section of the 20th proposition $3^{\circ}5'$.

WHAT IS REQUIRED. We have to find the deviation of Saturn's orbit from the ecliptic, *i.e.* the angle ACD , as well as how far the line of nodes is from the centre C of the Earth's orbit AB .

PROCEDURE.

The crossed quadrilateral $ABDE$ has five known terms, to wit, the side AB (the diameter of the Earth's orbit) = 20,000 by the supposition; moreover, by the supposition AD makes 85,610, BE 78,849, the angle DAB $176^{\circ}58'$, for that is what is left when $FAD = 3^{\circ}2'$ is subtracted from the semi-circle, and the angle $EBA = 176^{\circ}55'$, for that is what is left when $GBE = 3^{\circ}5'$ is subtracted

¹⁾ For 5 in the original read 1.

blijfter als men treckt GBE 3 tr. 5 ① vant half front: Merct noch dat benevens de boveschreven vijf bekende palen, tot meerder gerief bekennt sijn drie ander, te weten DE lini tusschen de twee punten der uysterste breedten van Saturnus, doende deur het 21 voorstel deses 3 boucx 1844959, de lini AE 98849, als evē ghenouch sijnde met AB 20000, en BE 78849 t'samen: Voort de lini BD 105610, als even genouch sijnde met AB 20000, en AD 85610 t'samen: Hier me ghesocht den houck ACD , wort bevonden deur het 6 voorstel inde Byvough der platte veelhoucken voor Saturnuswechs begeerde afwijcking vanden duyfteraer 2 tr. 43 ①, waer voor *Ptolemus* int 3 Hoofstlick sijns 13 boucx al tastende vandt 2 tr. 26 ①.

Angaende de verheynt der duyfteraersne van des Eertcloodwechs middelpunt C , sy wort ghenouchsaem bevonden daer in te vallen, want onder verscheyden neminghen der palen daer t'werck deur gedaen can worden, soo viel my deur de ghene die ick nam den houck ADC van 19 ①, waer me des driehouck DAC drie bekende palen, te weten de twee houcken ADC , DCA , en de sijde AD my uytbrochten de lini AC van 10294, t'welck te veel is 294, want om volcommen te wesen soudet sijn 10000, maer t'schilt soo weynich dat nemende voor den houck ADC 18 ①, in plaets van 19 ① als vooren, soo comt dan voor AC 9732, t'welck 268 weynigher is dan de volcommenheit vereyscht, sulcx datment daer voor houden mach de duyfteraersne te vallen deur des Eertcloodwechs middelpunt C , overeencommende mette rekening die *Ptolemus* op sijn spiegheling ghemaect heeft, aldaer vallende deur den vasten Eertclood, want de selve in die stelling sulcx wat is als des Eertcloodwechs middelpunt inde hare, ghelijck ick int 18 voorstel deses 3 boucx daer af wat verclaring ghedaen heb.

Merckt noch dat ander manier van wercking dan de voorgaende mach gedaen worden, met eerst te vinden de lini CD (dat is van des Eertcloodwechs middelpunt tot Saturnus ter ghegheven plaets sijns wechs) deur het 6 lidt des 15 voorstels deses 3 boucx, want die bekennt sijnde, soo heeft den driehouck ADC drie bekende palen, te weten benevens die CD noch de lini AD , en de houck ACD , waer me d'ander onbekende palen ghevonden worden.

T B E S L V Y T. Wy hebben dan gevonden Saturnuswechs afwijcking vanden duyfteraer: Metsgaders hoe verre de duyfteraersne van des Eertcloodwechs middelpunt valt, in sulcke deelen alffer des Eertcloodwechs halfmiddellijn 10000 doet, deur wisconftighe wercking ghegront op stelling eens roerenden Eertcloods, naden eysch.

MERCK INHOVDENDE VER- claring dat Saturnus breedeloopsker ghetuychnis gheeft want roersel des Eertcloods.

Als men met stelling eens vasten Eertcloods seght Saturnus inronts middelpunt te wesen ande duyfteraersne, soo bevint men hem metter daet alijt int plat des duyfteraers sonder breedte, tot wat plaets hy oock int inront sijn mocht deur het 3 lidt des 20 voorstels, t'welck voor een wonder gehouden sijnde, soo heeft *Ptolemus* daer toe verdocht sulcke spiegheling als beschreven is int selve 20 voorstel, maer met dese stelling eens roerendē Eertcloods siemē alles noot-sakelick te moeten volghen uyt de eenvoudighe draeying van Saturnus hemel op haer as, sonder yet nieus of vreemts daer by te moeten versieren, en dat men sich niet verwonderen mocht, sooment anders bevonde, want commende by

C c 4 de by

from the semi-circle. Note also that besides the above-mentioned five known terms for greater convenience three more are known, to wit, DE , the line between the two points of the extreme latitudes of Saturn, making by the 21st proposition of this 3rd book 1,844,959, the line $AE = 98,849$, as being sufficiently equal to $AB = 20,000$ and $BE = 78,849$ together. Further the line $BD = 105,610$ as being sufficiently equal to $AB = 20,000$ and $AD = 85,610$ together. When the angle ACD is sought therewith, by the 6th proposition in the Supplement of plane polygons¹⁾ there is found for [it, *i.e.* for] the required deviation of Saturn's orbit from the ecliptic $2^{\circ}43'$, for which *Ptolemy* in the 3rd Chapter of his 13th book tentatively found $2^{\circ}26'$.

As to the distance of the line of nodes from the centre of the Earth's orbit C , it is found substantially to fall therein, because among several assumptions of the terms by means of which the operation can be performed I found among those I took the angle ADC equal to $19'$, by means of which the three known terms of the triangle DAC , to wit, the two angles ADC , DCA and the side AD , yielded me the line $AC = 10,294$, which is 294 too much, for in order to be perfect it would have to be 10,000; but the difference is so small that, taking the angle ADC equal to $18'$ instead of $19'$, as above, I get $AC = 9,732$, which is 268 less than perfection requires; so that it may be assumed that the line of nodes falls through the centre of the Earth's orbit C , which corresponds to the calculation performed by *Ptolemy* on the basis of his theory, where it falls through the fixed Earth, for the latter in that theory is the same as the centre of the Earth's orbit is in this theory (of the moving Earth), as I have explained in the 18th proposition of this 3rd book.

Note also that another method of operation than the preceding one may be used by first finding the line CD (*i.e.* from the centre of the Earth's orbit to Saturn at the given point of its orbit) by the 6th section of the 15th proposition of this 3rd book, for when that is known, the triangle ADC has three known terms, to wit, besides the said CD also the line AD and the angle ACD , by means of which the other unknown terms are found.

CONCLUSION. We have thus found the deviation of Saturn's orbit from the ecliptic, as well as how far the line of nodes falls from the centre of the Earth's orbit, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth; as required.

NOTE

consisting in a statement that Saturn's motion in latitude furnishes certain evidence of the motion of the Earth.

When it is said on the theory of a fixed Earth that the centre of Saturn's epicycle is in the line of nodes, it is indeed always found in the plane of the ecliptic without any latitude, no matter in what place it may be on the epicycle, by the 3rd section of the 20th proposition; and because this was considered a wonder, *Ptolemy* devised for it the theory that has been described in the said 20th proposition; but with the present theory of a moving Earth it is seen that everything must necessarily follow from the simple rotation of Saturn's heaven about its axis, without anything new or strange having to be invented; and that one would

¹⁾ See p. 189, note 1.

de by voorbeeld gheseyt, Saturnus van E tot dat hy is ande duyfteraersne voor C, soo is hy dan int plat des duyfteraers, en wanter den Eertcloot nummermeer buyten en loopt, soo volght daer uyt dat tot wat plaets haers wechs den selven Eertcloot dan is, soo en can Saturnus van daer niet dan inden duyfteraer ghesien worden: En is onder anderen hier me het roersel des Eertcloots soo openbaer, dat ment by de ghene diet ontkennen, voorgebreck van ervarentheyt houden mach.

23 VOORSTEL.

Te vinden de vvechlangde der tvvee uysterste punten vande duyfteraersne, en vande tvvee uysterste punten der afvvijsking in Saturnusvvech, oock der lini die vā Saturnusvvechs middelpunt op de duyfteraersne rechthouckich valt in sulcke deelen alser des Eertclootvvechs halfmiddellijn 10000 doet, deur vvisconstige vvercking ghegront op stelling eens roerenden Eertcloots.

T G H E G H E V E N. Anghesien ick om mijn voornemen wel te verclaren hier soude moeten vertheycken de form des 21 voorstels, om daer noch by te vervougē r'gene in dit voorstel vereyscht is, soo sal ick de voorschrevē form des 21 voorstels self daer toe gebruyckē, tot welcken einde ick aldus segh: Nadien de duyfteraersne deur des Eertclootwechs middelpunt streckt, volgens t'inhout des 22 voorstels, soo treck ick inde form vā 21 voorstel deur des Eertclootwechs middelpunt F de lini L M, als duyfteraersne rechthouckich op D B, van wiens twee uysterste punten L, M, ick treck de twee linien L E en M E: Treck daer na deur t'punt E Saturnuswechs middellijn N O ewwijdeghe met D B, en sniende L M in P. T B E G H E E R D E. Wy moeten vinden de wechlangden vā L, M, wesen de twee uysterste punten vande duyfteraersne L M; Sghelijcx de wechlangde vā N O, wesen de uysterste punten der afvvijsking in Saturnuswech, ende E P langde der lini die vā Saturnuswechs middelpunt E op de duyfteraersne L M rechthouckich comt.

T W E R C K.

Vanden rechthouck L F D doende 90 tr. ghetrocken den houck A F D, doende deur het 1 lidt des 20 voorstels 50 tr. blijft voor den houck L F E	40 tr.
De driehouck L F E heeft drie bekende palen, te weten L E als halfmiddellijn des wechs 92308, de uytmiddelpunticheytlijn E F 5256 deur het Byeeuvoughsel vā 13 voorstel deses 3 boucx, en den houck L F E 40 tr. eerste in d'oiden: Hier me ghesocht den houck F I E, wort bevonden deur het 5 voorstel der platte driehoucken vā	2 tr. 6.
Welcke vergaert tōne 40 tr. des houcx L F E, comt voor den houck A E I, of booch A L, als begheerde wechlangde vā een punt der duyfteraersne L	42 tr. 6.
Maer om te vindē de wechlangde vā ander punt M, t'is kennelick den houck F M E, even te wesen metten houck F I E, en te doen als die	

wonder with reason if one found it otherwise, for when *e.g.* Saturn moves from *E* until it is in the line of nodes in front of *C*, it is in the plane of the ecliptic, and because the Earth never moves outside it, it follows that no matter in what place of its orbit the said Earth then is, Saturn cannot thence be seen anywhere but in the ecliptic. And from this, among other reasons, the motion of the Earth is so evident that those who deny it are to be considered lacking in experience.

23rd PROPOSITION.

To find the orbital longitudes of the two extremities of the line of nodes, and of the two extremities of the deviation in Saturn's orbit, also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Since in order to explain my intention properly I should have to draw anew the figure of the 21st proposition and add thereto what is required in the present proposition, I will use the aforesaid figure of the 21st proposition itself, to which end I say as follows: Since the line of nodes passes through the centre of the Earth's orbit, according to the contents of the 22nd proposition, I draw in the figure of the 21st proposition, through the centre of the Earth's orbit *F*, the line *LM* as the line of nodes at right angles to *DB*, from whose two extremities *L*, *M* I draw the two lines *LE* and *ME*. I then draw through the point *E* the diameter of Saturn's orbit *NO* parallel to *DB* and intersecting *LM* in *P*. WHAT IS REQUIRED. We have to find the orbital longitudes of *L*, *M*, which are the two extremities of the line of nodes *LM*; also the orbital longitudes of *N*, *O*, which are the extremities of the deviation in Saturn's orbit, and *EP*, the length of the line which from the centre of Saturn's orbit *E* is dropped perpendicular to the line of nodes *LM*.

PROCEDURE.

When from the right angle *LFD*, making 90° , is subtracted the angle *AFD*, making by the 1st section of the 20th proposition 50° , there is left for the angle *LFE*

40°

The triangle *LFE* has three known terms, to wit, *LE* (as the semi-diameter of the orbit) = 92,308, the line of eccentricity *EF* = 5,256 by the Compilation of the 13th proposition of this 3rd book, and the angle *LFE* = 40° (the first in the present list); when the angle *FLE* is sought therewith, it is found by the 5th proposition of plane triangles ¹⁾ to be

$2^\circ 6'$

When this is added to the 40° of the angle *LFE*, I get for the angle *AEL* or arc *AL*, as the required orbital longitude of the one point of the line of nodes *L*,

$42^\circ 6'$

But in order to find the orbital longitude of the other point *M*, it is obvious that the angle *FME* is equal to the angle *FLE* and, like the latter, makes $2^\circ 6'$; when this is subtracted from the angle *CFM* which, being the opposite angle to *LFE* = 40° (the first in the present list), has the same value, there is left for the angle *FEM*, that is also *CEM*, or for the arc *CM*, $37^\circ 54'$. When this is added to the semi-circle *ABC* = 180° , the required orbital longitude of the point *M* is

$217^\circ 54'$

¹⁾ See p. 223, note 2.

als die 2 tr. 6 ③, de selve ghetrocken vanden houck CFM, die als teghenoverhouck van LFE 40 tr. eerste in d'oiden oock soo veel doet, blijft voor den houck FEM, dats oock CEM, of voor de booch CM 37 tr. 54 ④: De selve vergaert totter halfrontd ABC 180 tr, comt voor begheerde wechlangde des punts M 217 tr. 54.

Maer om nu te vinden de wechlangde vande twee uysterste punten der afwijking in Saturnuswech C, (t'welck om bekende redenen niet en sijn de twee puntē B, D, hoewel Saturnus daer uyt dē Eertclood inde aldergrootste schijnbaer duyfteraerbreede can ghesien wordē, maer N, O) ick segh aldus: Angesien dē houck EDF doet deur het 21 voorstel 2 tr. 30 ①, en dat dē houck NEDevē is an EDF, om dat ED is tusschen de twee ewwijdegen EN, FD, soo doet dē houck END, of de booch DN oock 2 tr. 30 ①, die vergaert totte wechlangde des punts D, doende deur het 21 voorstel 307 tr. 30 ②, comt voor begheerde wechlangde des punts N 310 tr.

Ende O teghenoverpunt van dien moet sijn in des wechs 101 tr.

Om nu te vinden de lini EP, soo heeft daer toe den driehouck ELP drie bekende palen, te weten de halfmiddellijn EL 92308 deur de Byenvougingh van het 13 voorstel deses 3 boucx, den houck ELP 2 tr. 6 ① tweede in d'oiden, en den houck EPL recht: Hier me ghesocht de sijde EP, wort bevonden deur het 4 voorstel der platte driehoucken voor begheerde lini van 3378.

Waer af t'bewijs deur t'werck openbaer is. T'BE SLVYT. Wy hebben dan ghevonden de wechlangde der twee uysterste punten vande duyfteraerfne, en vande twee uysterste punten der afwijking in Saturnuswech, oock der lini die vā Saturnuswechs middelpunt op de duyfteraerfne rechthouckich valt, in sulcke deelen alffer des Eertcloodwechs halfmiddellijn 10000 doet, deur wilconftighe wercking ghegront op stelling eens roerenden Eertcloots, na den eyfch.

VERVOLGH.

T'is kennelick dat Saturnus in sijn wech vanden 42 tr. 6 ③ derde in d'oiden, dats inde form des 21 voorstels van L over B tot M, alijt Zuyderlick moet wesen, maer vanden 217 tr. 54 ④ vierde in d'oiden, totten 42 tr. 6 ③, dats van M over A tot L alijt Noorderlick: Om t'welck noch opentlicker te verclaren, merckt dat de voorschreven booch van L over B tot M doende 217 tr. 54 ④, inde form des 22 voorstels anghewesen is deur de lini CF, waer me t'selve rontfdeel LBM beteyckent wort overcant ghesien te wesen: T'welck soo sijnde, Saturnus en can in CE der form des 22 voorstels tot sulcken plaets niet wesen, dat hy ghesien uyt den Eertclood tot wat plaets die oock in haer wech AB mocht wesen, anders verschijne dan op de Zuytsijde des duyfteraers FG: En sghelijcx en can hy int deel CD der form des 22 voorstels (beteyckenende het boveschreven rontfdeel MAL der form des 21 voorstels) tot sulckē plaets niet wesen, dat hy ghesien uyt den Eertclood, tot wat plaets die oock in haer wech AB mocht wesen, anders verschijne dan op de Noortsijde des duyfteraers FG.

24 VOORSTEL.

Te vinden de langde der lini die van een gegeven punt in Saturnusvech rechthouckich valt opt plat des duyfteraers

But in order now to find the orbital longitudes of the two extremities of the deviation in Saturn's orbit C (which for known reasons are not the two points B, D , though Saturn can there be seen from the Earth at the greatest apparent ecliptical latitude, but N, O), I say as follows: Since the angle EDF by the 21st proposition makes $2^{\circ}30'$, and the angle NED is equal to EDF , because ED is situated between the two parallel lines EN, FD , the angle NED 1) or the arc DN also makes $2^{\circ}30'$. When this is added to the orbital longitude of the point D , which by the 21st proposition makes $307^{\circ}30'$, the required orbital longitude of the point N becomes

310°
 130°

And O , the opposite point thereto, must be in the orbit at

In order now to find the line EP , the triangle ELP has three known terms, to wit, the semi-diameter $EL = 92,308$ by the Compilation of the 13th proposition of this 3rd book, the angle $ELP = 2^{\circ}6'$ (the second in the present list), and the angle EPL right. When the side EP is sought therewith, the required line is found, by the 4th proposition of plane triangles 2), to be

3,378

The proof of which is evident from the procedure. CONCLUSION. We have thus found the orbital longitudes of the two extremities of the line of nodes, and of the two extremities of the deviation in Saturn's orbit, also of the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth; as required.

SEQUEL.

It is obvious that Saturn in its orbit from longitude $42^{\circ}6'$ (the third in the present list), that is in the figure of the 21st proposition from L via B to M (*i.e.* to $217^{\circ}54'$), must always be on the South side, but from longitude $217^{\circ}54'$ (the fourth in the present list) to longitude $42^{\circ}6'$, that is from M via A to L , always on the North side. In order to explain this even more plainly, note that the aforesaid arc from L via B to M , making $175^{\circ}48'$ 3), in the figure of the 22nd proposition is designated by the line CE 4), by which this part of the circle LBM is denoted to be seen transversely. This being so, Saturn cannot in CE of the figure of the 22nd proposition be in such a place that, when seen from the Earth — no matter in what place the latter may be in its orbit AB — it appears anywhere but on the South side of the ecliptic FG . And in the same way, in the part CD of the figure of the 22nd proposition (denoting the above-mentioned part of the circle MAL of the figure of the 21st proposition) it cannot be in such a place that, when seen from the Earth — no matter in what place the latter may be in its orbit AB — it appears anywhere but on the North side of the ecliptic FG .

24th PROPOSITION.

To find the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-

1) For END in the Dutch text read NED .

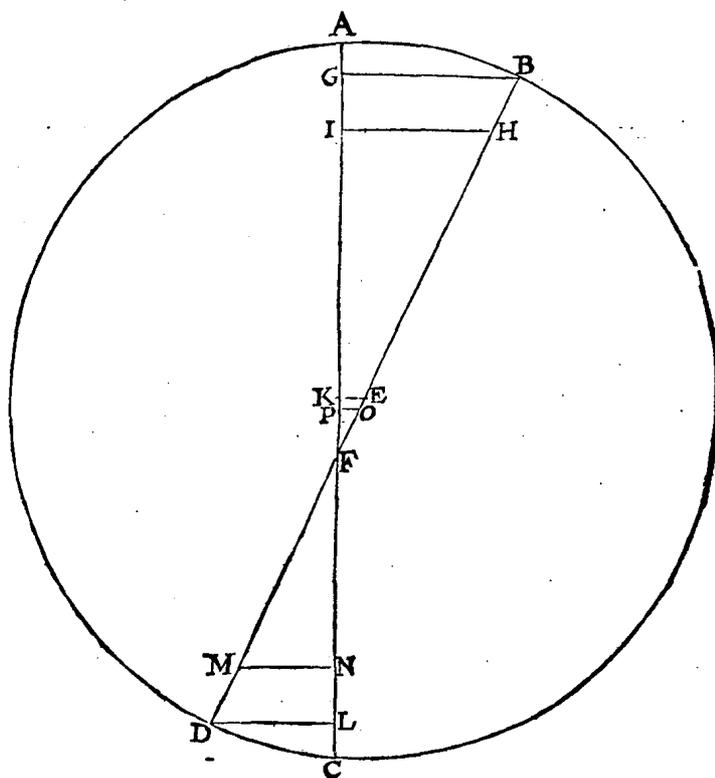
2) Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

3) For $217^{\circ}54'$ in the Dutch text read $175^{\circ}48'$.

4) For CF in the Dutch text read CE .

steraers, in sulcke deelen alser des Eertclootvechs halfmiddellijn 10000 doct, deur vvisconstighe vvercking gegront op stelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet $A B C D$ Saturnus hemel sijn, diens middelpunt E , waer deur ghetrocken is de rechte $B E D$ (van gedaente als $N E O$ inde form des 21 voorstels) beteyckenende Saturnuswech overcant gesien, daer in t'punt



F sy de duyfteraersne oock overcant ghesien, soo dat $E F$ (even an $E P$ int 21 voorstel) is de lini die van Saturnuswechs middelpunt op de duyfteraersne rechthouckich comt, doende deur het 23 voorstel 3378, daer na ghetrocken deur t'punt F de rechte $A F C$, bediende t'plat des duyfteraers overcant ghesien, ende den houck $B F A$, wefende de afwijcking des wechs van t'plat des duyfteraers doet deur het 22 voorstel deses 3 boucx 2 tr. 43 \odot . T'WERCK.

1 VOORBEELT.

Ghenomen ten eersten dat hier in ghevonden moet sijn de langde der lini $B G$, die van des wechs meeste afwijcking B , rechthouckich valt opt plat des duyfteraers $A C$. Om daer toe te comen ick segh des wechs halfmiddellijn $E B$ te doen 92308 deur het Byeenvoughsel van 13 voorstel deses 3 boucx, en $F E$ 3378 deur t'ghegheven, welke vergaert maken t'samen 95686 voor de lini $F B$, waer me de driehouck $B F G$ drie bekende palen heeft, te weten de sijde

$F B$

diameter of the Earth's orbit has 10,000, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let $ABCD$ be Saturn's Heaven, whose centre is E , through which has been drawn the straight line BED (of the same character as NEO in the figure of the 21st proposition), designating Saturn's orbit, seen transversely, wherein the point F shall be the line of nodes, also seen transversely, so that EF (equal to EP in the 21st proposition) is the line which from the centre of Saturn's orbit is dropped perpendicular to the line of nodes, by the 23rd proposition making 3,378. Thereafter through the point F is drawn the straight line AFC , denoting the plane of the ecliptic seen transversely, and the angle BFA , which is the deviation of the orbit from the plane of the ecliptic, by the 22nd proposition of this 3rd book makes $2^{\circ}43'$.

PROCEDURE.

1st EXAMPLE.

Let it firstly be assumed that in this has to be found the length of the line BG , which from the greatest deviation of the orbit B is dropped perpendicular to the plane of the ecliptic AC . In order to come to this, I say that the semi-diameter of the orbit EB makes 92,308 by the Compilation of the 13th proposition of this 3rd book, and FE 3,378 by the supposition, which when added together make 95,686 for the line FB , so that the triangle BFG has three known terms, to wit, the side $FB = 95,686$, the angle $BFG = 2^{\circ}43'$, and

FB 95686, den houck BFG 2 tr. 43 ①, en den houck BGFrecht deur t'ghegheven: Hier me ghesocht de sijde B G, wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 4535.

2 V O O R B E E L T.

Ghenomen ten tweeden dat gevonden moet sijn de lini HI, commende uyt des wechs 30 trap, van t'punt der uysterste afwijcking B ghetelt, maer want die lini even is ande lini dieder valt uyt des wechs halfmiddellijn EB, van t'punt H als uysterste des houckmaet pijs BH, van 30 tr. soo laet ons E B ansien voor wechs halfmiddellijn, waer in EH sal sijn houckmaet van 60 tr, dats van t'vierendeelrontschil der ghegeven 30 tr. doende die houckmaet 8660. Dir soo sijn- de ick segh aldus: Doende de halfmiddellijn EB 10000, soo doet EH 8660, wat sal EH doen wesende EB ghestelt op 92308? comt alsdan voor EH 79939, daer toe vergaert EF doende deur t'ghegeven 3378, comt voor FH 83317, waer me de driehouck HFI drie bekende palen heeft, te weten de sijde FH 83317, den houck HFI 2 tr. 43 ①, en den houck HIF recht: Hier me ghesocht de sijde HI, wort bevonden deur het 4 voorstel der platte driehoucken voor het begheerde van 3949. En is kennelick HI te commen uyt Saturnuswechs 340 tr. want dese B wort inde form des 21 voorstels beteyckent met N, wesende in des wechs 310 tr. deur het 23 voorstel, waer toe de 30 tr. comt als vooren 340 tr.

Merckt nu dat sulcx als hier gheweest is de manier van t'vinden der lini HI tusschen E en B, ofte int halfont EB, alsoo salse oock sijn tot allen plaetsen int selve halfont.

3 V O O R B E E L T.

Ghenomen ten derden dat gevonden moet sijn de lini EK, commende uyt des wechs 90 tr. van t'punt der uysterste afwijcking D ghetelt. Om daer toe te commen, ick segh dat anghesien EF doet 3378 deur t'ghegeven, soo heeft den driehouck EFK drie bekende palen, te weten FE 3378, den houck EFK 2 tr. 43 ①, en den houck EKF recht: Hier me ghesocht de sijde EK wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 160. Dufdanich dan gheweest hebbende den voortganck int halfont EB, wy sullen nu derghelijcke doen int ander halfont ED.

4 V O O R B E E L T.

Ghenomen ten 4 dat ghevonden moet sijn de langde der lini DL, die van des wechs meeste Zuydersche afwijcking D, rechthouckich valt opt plat des duystraers AC: Om daer toe te commen, ick segh des wechs halfmiddellijn ED te doen 92308 deur het Byenvougsel vant 13 voorstel deses 3 boucx, en FE 3378 deur t'ghegeven, welke ghetrocken vande 92308, blijft 88930, voor de lini FD, waer me de driehouck DFL drie bekende palen heeft, te weten de sijde FD 88930, den houck DFL 2 tr. 43 ①, en den houck DLF recht deur t'ghegeven: Hier me ghesocht de sijde DL, wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 4215.

5 V O O R B E E L T.

Ghenomen ten vijften dat ghevonden moet sijn de lini MN, commende uyt des wechs 30 trap van t'punt der uysterste afwijcking D ghetelt, maer want
die lini

the angle BGF right by the supposition. When the side BG is sought therewith, the required value is found by the 4th proposition of plane triangles to be 4,535.

2nd EXAMPLE.

Let it secondly be assumed that the line HI has to be found, which proceeds from 30° of the orbit, taken from the point of the greatest deviation B ; but because this line is equal to the line which extends from the semi-diameter of the orbit EB , from the point H as extremity of the versed sine BH of 30° , let us regard EB as the semi-diameter of the orbit, in which EH shall be the sine of 60° , *i.e.* the complement of the given 30° , said sine making 8,660. This being so, I say as follows: When the semi-diameter EB makes 10,000, EH makes 8,660. What will EH be when EB is taken 92,308? Then EH becomes 79,939. When to this is added EF , making by the supposition 3,378, FH becomes 83,317, so that the triangle HFI has three known terms, to wit, the side $FH = 83,317$, the angle $HFI = 2^\circ 43'$, and the angle HIF right. When the side HI is sought therewith, the required value is found by the 4th proposition of plane triangles ¹⁾ to be 3,949. Then it is obvious that HI proceeds from 340° of Saturn's orbit, for this B in the figure of the 21st proposition is denoted by N , which is situated at 310° of the orbit by the 23rd proposition; when to this is added the 30° , the required value is 340° , as above.

Now note that such as here has been the method of finding the line HI between E and B , or in the semi-circle EB , the same it will also be in any place in this semi-circle.

3rd EXAMPLE.

Let it thirdly be assumed that the line EK has to be found, which proceeds from 90° of the orbit, taken from the point of the greatest deviation D . In order to come to this, I say that since EF makes 3,378 by the supposition, the triangle EFK has three known terms, to wit, $FE = 3,378$, the angle $EFK = 2^\circ 43'$, and the angle EKF right. When the side EK is sought therewith, the required value is found by the 4th proposition of plane triangles to be 160. The procedure in the semi-circle EB having been thus, we shall now do the same in the other semi-circle ED .

4th EXAMPLE.

Let it fourthly be assumed that the length of the line DL has to be found, which from the greatest Southerly deviation of the orbit D is dropped perpendicular to the plane of the ecliptic AC . In order to come to this, I say that the semi-diameter of the orbit ED makes 92,308 by the Compilation of the 13th proposition of this 3rd book, and FE 3,378 by the supposition. When the latter is subtracted from the 92,308, there is left 88,930 for the line FD , so that the triangle DFL has three known terms, to wit, the side $FD = 88,930$, the angle $DFL = 2^\circ 43'$, and the angle DLF right by the supposition. When the side DL is sought therewith, the required value is found by the 4th proposition of plane triangles to be 4,215.

5th EXAMPLE.

Let it fifthly be assumed that the line MN has to be found, which proceeds from 30° of the orbit, taken from the point of the greatest deviation D ,

¹⁾ Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

die lini even is ande lini dieder valt uyt des wechs halfmiddellijn, van t'punt M als uysterste des houckmaetpijls D M van 30 tr. soo laet ons E D an sien voor wechs halfmiddellijn, waer in E M sal sijn houckmaet van 60 tr. dats van t'viendeelrontschil der ghegeven 30 tr. doende die houckmaet 8660: Dit soo sijnde ick segh aldus: Doende de halfmiddellijn E D 10000, soo doet E M 8660, wat sal E M doen wefende E D ghestelt op 92308? comt alsdan voor E M 79939, daer afghetrocken E F doende deur t'ghegeven 3378, blijft voor F M 76561, waer me de driehouck M F N drie bekende palen heeft, te weten de sijde F M 76561, den houck M F N 2 tr. 43 \odot , en den houck M N F recht: Hier me ghesocht de sijde M N, wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 3629.

Merckt nu dat sulcx als hiergheweest heeft de manier van t'vinden der lini M N tusschen F en D, alsoo salse oock sijn tot allen plaetsen tusschen F en D.

6 V O O R B E E L T.

Ghenomen ten seften dat ghevonden moet sijn de lini O P, commende uyt des wechs 89 tr. van t'punt der uysterste afwijcking D ghetelt: Maer want die lini even is ande lini dieder valt uyt des wechs halfmiddellijn E D van t'punt O als uysterste des houckmaetpijls D O van 89 tr. soo laet ons E D an sien voor wechs halfmiddellijn, waer in E O sal sijn houckmaet van 1 tr. dats van t'viendeelrontschil der ghegeven 89 tr. wefende die houckmaet van 175. Dit soo sijnde ick segh doende de halfmiddellijn E D 10000, soo doet E O 175, wat sal E O doen wefende E D ghestelt op 92308? comt alsdan voor E O 1615, die getrocken van E F doende deur t'ghegeven 3378, blijft voor O F 1763, waer me de driehouck O P F drie bekende palen heeft, te weten de sijde F O 1763, den houck O F P 2 tr. 43 \odot , en den houck O P F recht: Hier me ghesocht de sijde O P wort bevonden deur het 4 voorstel der platte driehoucken voor t'begheerde van 84.

Merckt noch dat sulcx als hier gheweest is de manier van t'vinden der lini O P tusschen E en F, alsoo salse oock sijn tot allen plaetsen tusschen E en F.

T' B E S L V Y T. Wy hebben dan gevonden de langde der lini die van een geve punt in Saturnuswech rechthouckich valt opt plat des duystraers, in sulcke deelen alser des Eertcloorwechs halfmiddellijn 10000 doet, deur wisconstighe wercking ghegrontop stelling eens roerenden Eertcloots, na den eyfch.

V E R V O L G H.

T'is kennelick hoe datmen om de begheerde lini van Saturnuswech rechthouckich opt plat des duystraers met lichticheyt te vinden, sal meughen maken een tafel dier linien van trap tot trap, vant verstepunt beginnende, als by voorbeelt om te hebbē de lini vallende alsoo van Saturnuswechs eersten trap, ick neem voor my de form des 21 voorstels, daer in teykenende t'punt Q, alsoo dat van des wechs verstepunt A tot Q 1 tr. bereyckent, waer uyt volght N Q te doē 51 tr. (want de wechlangde van N doet deur het 23 voorstel 310 tr. wiens rontschil voor N A 50 tr. daer toe A Q 1 tr. comt als boven voor N Q 51 tr.) daerom als men deur dit 24 voorstel vindt de hanghende vanden 51 tr. van Bafghetelt (ghelijck vooren mette hangende H I van 30 tr. gedaen wiert) men heeft t'begheerde, om dat die lini vallen sal uyt Saturnuswechs 1 tr. en alsoo met alle ander.

Merckt noch dat wanneer dese dinghen niet aldus voorbeeltische wijze ghesdaen en wordē, maer met ernst om op een toecommende tijt dadelick de breedte te

but because this line is equal to the line which extends from the semi-diameter of the orbit, from the point M as extremity of the versed sine DM of 30° , let us regard ED as the semi-diameter of the orbit, in which EM shall be the sine of 60° , that is of the complement of the said 30° , said sine making 8,660. This being so, I say as follows: When the semi-diameter ED makes 10,000, EM makes 8,660. What will EM be when ED is taken 92,308? EM then becomes 79,939. When from this is subtracted EF , making by the supposition 3,378, there is left for FM 76,561, so that the triangle MFN has three known terms, to wit, the side $FM = 76,561$, the angle $MFN = 2^\circ 43'$, and the angle MNF right. When the side MN is sought therewith, the required value is found by the 4th proposition of plane triangles to be 3,629.

Now note that such as here has been the method of finding the line MN between F and D , the same it will also be in any place between F and D .

6th EXAMPLE.

Let it sixthly be assumed that the line OP has to be found, which proceeds from 89° of the orbit, taken from the point of the greatest deviation D . But because this line is equal to the line which extends from the semi-diameter of the orbit ED , from the point O as extremity of the versed sine DO of 89° , let us regard ED as the semi-diameter of the orbit, in which EO shall be the sine of 1° , that is of the complement of the given 89° , said sine making 175. This being so, I say: When the semi-diameter ED makes 10,000, EO makes 175. What will EO be when ED is taken 92,308? EO then becomes 1,615. When this is subtracted from EF , making by the supposition 3,378, there is left for OF 1,763, so that the triangle OPF has three known terms, to wit, the side $FO = 1,763$, the angle $OPF = 2^\circ 43'$, and the angle OPF right. When the side OP is sought therewith, the required value is found by the 4th proposition of plane triangles to be 84.

Note also that such as here has been the method of finding the line OP between E and F , the same it will also be in any place between E and F .

CONCLUSION. We have thus found the length of the line which from a given point in Saturn's orbit is dropped perpendicular to the plane of the ecliptic, in such parts as the semi-diameter of the Earth's orbit has 10,000, by mathematical operation based on the theory of a moving Earth; as required.

SEQUEL.

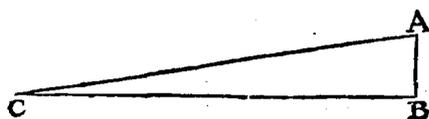
It is obvious that in order easily to find the required line from Saturn's orbit perpendicular to the plane of the ecliptic, it is possible to make a table of those lines from degree to degree, starting at the apogee. For example, in order to have the line thus extending from the first degree of Saturn's orbit, I take before me the figure of the 21st proposition and mark therein the point Q such that the distance from the orbit's apogee A to Q denotes 1° , from which it follows that NQ makes 51° (for the orbital longitude of N by the 23rd proposition makes 310° , whose difference from 360° , NA , makes 50° ; when to this is added $AQ = 1^\circ$, NQ becomes, as above, 51°). If therefore by this 24th proposition the perpendicular at 51° is found, taken from B (as has been done above for the perpendicular HI at 30°), the required value is obtained, because that line

de te vinden, dattet noodich soude sijn deur dadelicke ervaring eerst te vinden waer nu des wechs verstepunts schijnbaer duyfteraerlangde is: Ten anderē hoe de duyfteraersne verlopen mach sijn sichtent *Ptolemeus* tijt (diens ervaringen ick voorbeeltſche wijſe ghenomen hebbe om de redenen t'haerder plaats verclaert) want de Beschrijvers der dachtafels nemen al of den loop van Saturnus duyfteraersne, even waer met des wechs verstepuntsloop altijt 50 tr. van malcander blijvende, maer daer soude nieuwe dadelicke gagheslaghen ervaring af behooren te wesen, want by al dien den loop des verstepunts en des duyfteraersnees van Saturnus verscheyden sijn, ghelijck vande Maen ghebeurt, soo mocht sulcx oirfaeck wesen waer deur sijn breedten niet dadelick soo bevonden en wierden, ghelijck rekeninghen der dachtafels mebrenghen.

25 VOORSTEL.

Te vinden Saturnus schijnbaer duyfteraerbreede op een ghegeven tijt, deur wifconſtighe vvercking gegront op ſtelling eens roerenden Eertcloots.

T'GHEGHEVEN. Laet Saturnus op den ghegeven tijt wesen in sijn wechs 340 tr. alwaer de lini van hem rechthouckich opt plat des duyfteraers



doe sal 3949 deur het 2 voorbeelt des 24 voorſtels defes 3 boucx: En de verheyte van hem totten Eertclood die gevonden wort na de manier verclaert int 6 lidt des 15 voorſtels defes 3 boucx, wefende

daer de lini XV, sy neem ick, van 80000. T'BEGHEERDE. Wy moeten hier me vinden Saturnus schijnbaer duyfteraerbreede. T'BEREYTSSEL. Ick teycken de rechthouckige driehouck ABC recht an B, doende de ſijde AB de ghegeven 3949, en AC 80000.

T'WERCK.

De driehouck ABC heeft drie bekende palen, te weten de ſijde AB 3949, de ſijde AC 80000, en den houck B recht: Hier me ghesocht den houck ACB, wort bevonden deur het 5 voorſtel der platte driehoucken voor de begheerde breede van 2 tr. 50 ①. Maer om nu te weten of die Zuydelick of Noordelick is, dat wijft sijn wechlangde, want deur t'vervolgh des 23 voorſtels blijktt, dat die wefende tuſſchen den 44 tr. 12 ①, ende den 215 tr. 48 ①, ſy is Zuydelick, maer inde reſt des wechs (daer defen 340 tr. in valt) Noordelick. Waer af t'bewijs openbaer is. T'BESLVT. Wy hebben dan gevonden Saturnus schijnbaer duyfteraerbreede op een ghegeven tijt, deur wifconſtighe wercking ghegront op ſtelling eens roerenden Eertcloots, na den eyſch.

MERCKT.

Angheſien de reghel der ſes voorſtellen tot hier toe van Saturnus breede beſchreven, gemeen is over de breedten van d'ander vier Dwaelders Jupiter, Mars, Venus en Mercurius, soo en sal ick daer op gheen gheformde voorſtellen maken, ghelijck oock verhaelt is int Cortbegrijp defes 5 Onderscheyts, maer be-

D d schrij.

will extend from the first degree of Saturn's orbit, and the same with all the others.

Note also that when these things are not done thus by way of example, but in earnest, in order to find practically the latitude at future times, it would be necessary first to find by practical experience where the apparent ecliptical longitude of the orbit's apogee is now situated. Secondly, how the line of nodes may have shifted since the days of *Ptolemy* (whose experiences I have taken by way of example for the reasons explained in their place), for the Describers of the ephemerides all assume the motion of Saturn's line of nodes to be equal to that of the orbit's apogee, always remaining 50° apart, but this ought to be ascertained by new direct observational experience, for if the motion of the apogee and that of Saturn's line of nodes are different, as is the case with the Moon, this might be the cause why its latitudes were not found in practice to be such as calculations of the ephemerides imply ¹⁾).

25th PROPOSITION.

To find Saturn's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a moving Earth.

SUPPOSITION. Let Saturn be at the given time at 340° of its orbit, where the line dropped perpendicular from it to the plane of the ecliptic shall be 3,949 by the 2nd example of the 24th proposition of this 3rd book. And the distance from it to the Earth, which is found in the manner explained in the 6th section of the 15th proposition of this 3rd book, which there is the line *XV*, I assume to be 80,000. WHAT IS REQUIRED. We have to find herewith Saturn's apparent ecliptical latitude. PRELIMINARY. I draw the right-angled triangle *ABC*, right-angled in *B*, the side *AB* making the said 3,949 and *AC* = 80,000.

PROCEDURE.

The triangle *ABC* has three known terms, to wit, the side *AB* = 3,949, the side *AC* = 80,000, and the angle *B* right. When the angle *ACB* is sought therewith, the required latitude is found by the 5th proposition of plane triangles to be $2^\circ 50'$. But in order to know whether this is Southerly or Northerly, this is indicated by its orbital longitude, for by the sequel of the 23rd proposition it appears that when it is between $44^\circ 12'$ and $215^\circ 48' 2)$, it is Southerly, but in the rest of the orbit (in which falls this 340°) it is Northerly. The proof of which is evident. CONCLUSION. We have thus found Saturn's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a moving Earth; as required.

NOTE.

Since the rule of the six propositions described so far for Saturn's latitude is common to the latitudes of the other four Planets Jupiter, Mars, Venus, and Mercury, I will not make any illustrated propositions thereon, as has also been

¹⁾ Cf. footnote ³⁾ to p. 279.

²⁾ The provenance of the longitudes $44^\circ 12'$ and $215^\circ 48'$ is not clear. The values are obtained when *twice* the angle $2^\circ 6'$ is added to 40° and subtracted from 220° .

schrijven alleenelick by manier van vermaen t'gene breeder verclaring schijnt te vereyffchen, beginnende met Iupiter als volght.

NV VAN IVPITERS

B R E E D E.

Ptolemeus heeft deur dadelicke gagheslaghen ervaringhen de gedaenten van Iupiters breedeloop alsins bevonden gelijk van Saturnus int voorgaende geseyt is, welcke int besonder dusdanich sijn :

Ten eersten soo was sijn meeste breedte op de Noortsijde van 2 tr. 4 \odot , gebeurende altijd als sijn inronts middelpunt schijnbaerlick was 20 tr. voor sijn wechs verstepunt, te weten onder des duyfteraers 141 tr. en Iupiter an des inronts naestepunt, maer buyten het naestepunt wefende, soo was sijn Noordersche breedte voor datmael cleender, en ten minsten doen hy alsooan het verstepunt was.

Ten tweeden soo bevant hy op de Zuytsijde de meeste breedte van 2 tr. 8 \odot , gebeurende altijd als sijn inronts middelpunt was onder het teghenoverpunt des boveschrevē 141 tr. dats onder des duyfteraers 321 tr. en Iupiter an sijn inronts naestepunt : Maer buyten het naestepunt wefende, soo was sijn Zuydersche breedte voor datmael cleender, en ten minsten doenmen hem an het verstepunt vandr.

Ten derde wefende het inronts middelpunt in een der duyfteraersnees twee uysterfē, so bevant hy Iupiter, gelijk oock van Saturnus geseyt is, altijd int plat des duyfteraers sonder breedte, tot wat plaets des inronts hy oock wesen mocht.

De spiegeling by *Ptolemeus* hier op met stelling eens vasten Eertcloots veroirdent, is van ghedaente teenemael gheweest als die van Saturnus, sulcx dattet onnoodich schijnt de selve hier te verhalen: Ende want de spiegeling met stelling eens roerenden Eertcloots aude voorgaende van Saturnus oock ghelijck is, en daer deur verstaen wort, soo en beschrifick die niet int langhe.

NV VAN MARS

B R E E D E.

Ptolemeus heeft deur dadelicke gagheslaghen ervaringhen de gedaenten van Mars breedeloop alsins bevonden ghelijck van Saturnus int voorgaende geseyt is.

Ten eersten soo was sijn meeste breedte op de Noortsijde van 4 tr. 21 \odot , gebeurende altijd als sijn inronts middelpunt was an sijn wechs verstepunt (het welcke deur de Byenvougingh des 13 voorstels is onder des duyfteraers 115 tr. 30 \odot) en Mars in des inronts naestepunt, maer buyten het naestepunt wefende, soo was sijn Noordersche breedte voor datmael cleender, en ten minstē doen hy alsoo ant verstepunt was.

Ten tweeden soo bevant hy op de Zuytsijde de meeste breedte van 7 tr. 7 \odot , gebeurende altijd als sijn inronts middelpunt was in sijn wechs naestepunt onder het teghenoverpunt des boveschreven 115 tr. 30 \odot , dats onder des duyfteraers 295 tr. 30 \odot , en Mars an sijn inronts naestepunt: Maer buyten het naestepunt wefende, soo was sijn Zuydersche breedte voor datmael cleender, en ten minsten doemen hem al soo ant verstepunt vandr.

Ten derde wefende het inronts middelpunt an een der duyfteraersnees twee uysterfē, soo bevant hy Mars, ghelijck oock van Saturnus geseyt is, altijd int plat

said in the Summary of this 5th Chapter, but I will merely mention those things which seem to call for a fuller explanation, starting with Jupiter, as follows.

NOW OF JUPITER'S LATITUDE.

Ptolemy through direct observational experiences has found the character of Jupiter's motion in latitude to be in every respect as has been said of Saturn in the foregoing, which in particular is as follows.

Firstly, its greatest latitude on the North side was $2^{\circ}4'$, which always occurred when its epicycle's centre was apparently at 20° ahead of its orbit's apogee, to wit, at 141° of the ecliptic, and Jupiter at the epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when it was thus at the apogee.

Secondly, he found on the South side the greatest latitude of $2^{\circ}8'$, which always occurred when its epicycle's centre was in the point opposite to the above-mentioned 141° , *i.e.* at 321° of the ecliptic, and Jupiter was at its epicycle's perigee. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

Thirdly, when the epicycle's centre was in one of the two extremities of the line of nodes, he always found Jupiter, as has also been said of Saturn, in the plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

The theory that *Ptolemy* based on this, on the assumption of a fixed Earth, was entirely of the same character as that of Saturn, so that it seems unnecessary to relate it here. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn, and is understood therefrom, I shall not describe it in full.

NOW OF MARS' LATITUDE.

Ptolemy through direct observational experiences has found the character of Mars' motion in latitude to be in every respect as has been said of Saturn in the foregoing.

Firstly, its greatest latitude on the North side was $4^{\circ}21'$, which always occurred when its epicycle's centre was at its orbit's apogee (which by the Compilation of the 13th proposition is at $115^{\circ}30'$ of the ecliptic), and Mars in the epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

Secondly, he found on the South side the greatest latitude of $7^{\circ}7'$, which always occurred when its epicycle's centre was at its orbit's perigee, the point opposite to the above-mentioned $115^{\circ}30'$, *i.e.* at $295^{\circ}30'$ of the ecliptic, and Mars at its epicycle's perigee. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

Thirdly, when the epicycle's centre was at one of the two extremities of the line of nodes, he always found Mars, as has also been said of Saturn, in the

plat des duystraers sonder breede, tot wat plaets des inronts hy oock mocht wesen.

De spiegheling by *Ptolemus* hier op met stelling eens vasten Eertcloots veroirdent, is van ghedaente teenmael gheweest als die van Saturnus, sulcx datter onnoodich schijnt de selve hier te verhalen: Ende want de spiegeling met stelling eens roerenden Eertcloots, ande voorgaende van Saturnus oock ghelijck is, en daer deur verstaen wort, soo en beschrijfick die niet int langhe: Doch sal ick hier verhalen dat in dit werck wat cortheit valt anders als in dat, deur dien Mars uysterle breeden an sijn wechs verstepunt en naestpunt ghebeuren, als gheseyt is, want anghesien de lini tusschen de selve twee punten beken is deur de Byeenvouging des 13 voorstels deses 3 boucx, alwaer Marswechs halfmiddellijn staet op 15190, welcke tweemaal ghenomen comt voor de selve lini 30380, soo en behouftmen die niet te soucken na de manier des 21 voorstels, noch oock als int 23 voorstel de wechlangde vande twee uysterle punten der afwijking in Marswech, anghesien de selve het verstepunt en naestpunt sijn.

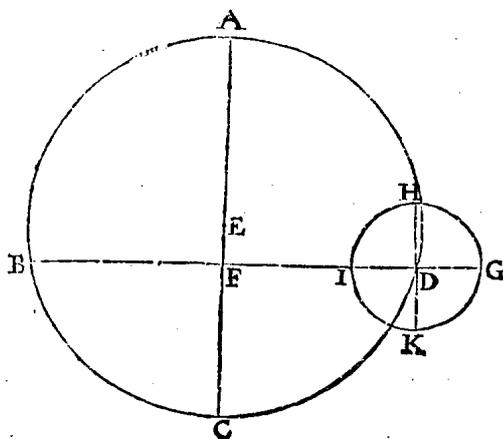
N V V A N M E R C V R I V S

B R E E D E.

I L I D T.

Inhoudende *Ptolemus* dadelicke gagheslaghen ervaringhen van Mercuriusloop.

Om beuqamelicker te verclarē *Ptolemus* dadelicke gagheslagē ervaringhen der gedaēte vā Mercurius breedeloop mette somme vā sijn spiegeling daer op veroirdent, so laet *A B C D E*



so laet *A B C D E* inrontwech beteyckenen, diens middelpunt is *E*, dē vasten Eertcloot *F*, deur welke en oock deur *E* ghetrocken sy de lini *C F E A*, soo dat *A* t' verstepunt is, *C* t' naestpunt, en deur *F* ghetrocken wesende de lini *B F D* rechthouckich op *A C*, so bedien de twee punten *B, D*, des wechs 90 tr. en 270 tr. en hoewel dat so veel verschilt als de uymiddelpunticheit *E F* veroirsaeckt, nochtans angesien *Pto-*

lemus de selve twee punten den 90 en 270 noemt, wy sullen ser voor nemen, daer na sy opt punt *D* als middelpunt beschreven het inront *G H I K*, diens verstepunt *G*, naestpunt *I*, en middelverheden *H, K*, tusschen welke vier punten gheteyckent sijn de twee middellijnen *I G, H K*.

Dit aldus wesende, soo heeft *Ptolemus* deur dadclieke ervaring bevonden dat Mercurius meeste breede op de zuysijde was van 4 tr. 5 ⊙, ghebeurende altijt *D d 2* als

plane of the ecliptic without any latitude, no matter in what place of the epicycle it might be.

The theory that *Ptolemy* based on this on the assumption of a fixed Earth was entirely of the same character as that of Saturn, so that it seems unnecessary to describe it here. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn, and is understood therefrom, I shall not describe it in full. But I will relate that in this procedure some abridgement can be made different from that, because Mars' greatest latitudes occur at its orbit's apogee and perigee, as has been said, for since the line between these two points is known by the Compilation of the 13th proposition of this 3rd book, where the semi-diameter of Mars' orbit is given as 15,190, which, when taken twice, gives 30,380 for the said line, it need not be sought after the manner of the 21st proposition, nor, as in the 23rd proposition, the orbital longitudes of the two extremities of the deviation in Mars' orbit, since these are the apogee and the perigee.

NOW OF MERCURY'S LATITUDE.

1st SECTION.

Consisting in *Ptolemy's* practical observational experiences of Mercury's motion.

In order to explain more easily *Ptolemy's* practical observational experiences of the character of Mercury's motion in latitude, together with the summary of his theory based thereon, let *ABC* denote the deferent, whose centre is *E*, the fixed Earth *F*, through which and also through *E* let there be drawn the line *CFEA*, so that *A* is the apogee, *C* the perigee, and when through *F* is drawn the line *BFD* perpendicular to *AC*, the two points *B, D* denote 90° and 270° of the orbit, and though this differs as much as amounts to the eccentricity *EF*, yet since *Ptolemy* calls these two points 90° and 270° , we shall take them thus. Thereafter let there be described about the point *D* as centre the epicycle *GHIK*, whose apogee is *G* and perigee *I*, and points of medium distance *H, K*, between which four points are drawn the two diameters *IG, HK*.

This being so, *Ptolemy* found by direct experience that Mercury's greatest latitude on the South side was $4^\circ 5'$, which always occurred when its epicycle's

als sijn inronts middelpunt was 90 tr. van sijn wechs verstepunt, soo wel ter eender als ter ander sijde, dat is soo wel inden 270 tr. an D, als inden 90 an B (welcke 90 tr. is onder des duystraers 280 tr. gemerckt deur de Byeenvouging des 13 voorstels deses 3 boucx, het verstepunt onder des duystraers 190 tr. is) en Mercurius in des inronts naestepunt als an I: Maer buyten het naestepunt wefende, soo was sijn zuyderfche breede voor datmael cleender, en ten minsten doen hy also ant verstepunt G was. D'oirfacck waerom de boveschreven breedten even vallē als het inronts middelpunt is an D of B, blijkt inde form, om dat de lini vanden Eertclood F tot D, even is ande lini van F tot B: Want daer uyt volght dattet inronts middelpunt an B wefende, sijn verstepunt en naestepunt in sulcken gheslalt uyt den Eertclood F ghesien worden, als wanneert an D is.

Ten tweedē so bevant hy op de noortsijde de meeste breede van 1 tr. 45 \odot , ghebeurende altijd als sijn inronts middelpunt was 90 tr. vā sijn wechs verstepunt, soo wel ter eender als ter ander sijde, en Mercurius an sijn inronts naestepunt, maer buyten het naestepunt wefende, soo was sijn noorderfche breede voor datmael cleender, en ten minsten doemē hem alsoo ant verstepunt vandt.

2 L I D T.

Inhoudende *Ptolemæus* spiegheling ghetrocken uyt de voorgaende dadelicke ervaringhen des 1 lidts.

Anghesien *Ptolemæus* voorgaende ervaringhen, noch breeder connen verclaert worden deur sijn spieghelinghen die hy daer op verdocht heeft, soo sal ick de somme van dien hier by voughen als volght.

Nadien des inrontwechs twee punten die uyt den vasten Eertclood F inde meeste breede ghesien worden sijn B en D, soo soude daer uyt volghen dat de ghemeene sne van die wech en den duystraer, rechthouckich moet commen op B D, ende want *Ptolemæus* sich in aller Dwaelders breedeloopen voorstelde die te gaen deur den Eertclood F, soo soude volgens die oirden de lini A F C de duystraersne moeten sijn, en daerom het inronts middelpunt wefende an A of C, en Mercurius ant inronts verste of naestepunt, soo soude hy uyt den Eertclood F moeten gesien worden int plat des duystraers sonder breede: Maer de ervaring wees hem anders, want hy doē sijn breede altijd bevant vā 45 \odot na t'Zuyden: Sulcx dat hy daerom (benevens noch een ander redē daerinden Byvough des breedeloops met stelling eens vasten Eertcloots af geseyt sal worden) Mercurius spiegheling anders veroirdende als vande drie bovenste Dwaelders, t'welck aldus toeginck: Hy heeft gheseyt de lini B D streckende deur de twee punten der uysterste afwijking, te wesen de ghemeene sne des inrontwechs en duystraers, teghen de natuerlicke reghel: Maer weerom ter ander sijde gaf den inrontwech en het inront seker drie seer versierlicke waggelende roersels, die onderscheyts halven ghenoecht worden *afweging, *afwijking, en *afkeering, waer me hy tot sijn voornemen gherocht, waer af ick oock van elck int besonder verhael sal doen, dat treckende uyt de bepalinghen die *Purbachius* tot dese plaets van dit roersel verstandelick doet.

Devatio.
Declinatio.
Reflexio.

Devatio.

Om dan mette *afweging te beginnen, soo is te weten dat *Ptolemæus* den inrontwech gheseyt heeft op B D als as een waggelende roersel te hebben overentweer gaende of waggelende als de balckeens waeghs, ghenoecht afweging, sulcx dattet inronts middelpunt wefende an B of D, soo en isser gheen afweging,

centre was at 90° from its orbit's apogee, both on one side and on the other, *i.e.* both at 270° in *D* and at 90° in *B* (which 90° is at 280° of the ecliptic, seeing that by the Compilation of the 13th proposition of this 3rd book the apogee is at 190° of the ecliptic), and Mercury was at the epicycle's perigee, namely at *I*. But when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee *G*. The cause why the above-mentioned latitudes are equal when the epicycle's centre is at *D* or *B* appears from the figure, because the line from the Earth *F* to *D* is equal to the line from *F* to *B*. For from this it follows that when the epicycle's centre is at *B*, its apogee and perigee are seen from the Earth *F* in the same manner as when it is at *D*.

Secondly, he found on the North side the greatest latitude of $1^\circ 45'$, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, and Mercury at its epicycle's perigee; but when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

2nd SECTION.

Consisting in *Ptolemy's* theory derived from the foregoing practical experiences of the 1st section.

Since *Ptolemy's* foregoing experiences can be explained even more fully by means of the theories he has conceived thereon, I will here add the summary thereof, as follows.

Since the two points of the deferent which from the fixed Earth *F* are seen at the greatest latitude are *B* and *D*, it would follow therefrom that the intersection of that deferent and the ecliptic must be perpendicular to *BD*, and because *Ptolemy*, in the case of all the Planets' motions in latitude, imagined them to pass through the Earth *F*, according to this rule the line *AFC* would have to be the line of nodes, and therefore, if the epicycle's centre were at *A* or *C*, and Mercury at the epicycle's apogee or perigee, it would have to be seen from the Earth *F* in the plane of the ecliptic without any latitude. But experience taught him differently, for he then always found its latitude to be $45'$ towards the South, so that he therefore (apart from another reason, which is to be dealt with in the Supplement of the motion in latitude on the assumption of a fixed Earth) framed Mercury's theory differently from that of the three upper Planets, in the following way. He said that the line *BD* passing through the two points of the greatest deviation was the intersection of the deferent and the ecliptic, contrary to the natural rule. But on the other hand again he gave the deferent and the epicycle three highly artificial oscillatory motions, which for the sake of distinction are called deviation, declination, and deflection¹⁾, with which he achieved his intention, of which I will also give an account for each in particular, taking

¹⁾ The term "deviation" is used here in a more restricted sense than previously, *e.g.* page 75, line 5 and further on, where it was the translation of *afwycking* in general. The name for the second oscillatory motion of Mercury, *afwycking*, has been translated "declination", in accordance with the Latin term in the margin, though this word has here quite a different meaning from that of the later equatorial co-ordinate, which in Dutch books is often denoted by the same word. For the corresponding verb, *afneyghen* is sometimes used; this is translated here by "to diverge". For the Latin *reflectio*, here indicating bending back, we have taken "deflection" as the best English equivalent.

ging, maer is des wechs heel plat int plat des duyfteraers: Daer na het inronts middelpunt van B of D vertreckende, soo begint den wech af te wegghen, of te neyghen altyt na het Zuyden, en vermeerdert dese afweging gheduerlick tot dattet inronts middelpunt ghecommen is ant verstepunt A, of naestepunt C, en dan is de afweging ten grootsten vande boveschreven 45 O ; Welck daer na weerom vermindert, tot dattet inronts middelpunt ghecommen is an B, of D, alwaer dan weerom gheen afweging en is. En blijkt hier uyt dattet roertel t'welck het inronts middelpunt vande afweging ontfangt nummermeer op de noortsijde en gheschiet.

Maer om nu vande *afwijcking te segghen, soo is te weten dattet plat des inronts deur een waggelende roersel afneyght vant plat sijns wechs op tweeder- *Declinatio-*
ley wijze: Ten eersten met afwijcking op de middellijn H K, streckende deur de *ne.*
middelverhedē H, K: Met dit roersel gebeuret dat de middellijns I G verstepunt G, op d'een sijde des wechs afwijckt, en t'naestepunt I op d'ander. Dese afwijcking houdt dusdanige reghe: Wanneer des inronts middelpunt als D, is an des wechs verstepunt A, de voorschrevē middellijn I G is in t'plat des wechs: Maer het inronts middelpunt van t' verstepunt A vertreckende, so begint des inronts verstepunt af te wijcken na het Zuyden, ende het naestepunt na het Noorden, welke afwijcking geduerlick vermeerdert tot dat des inronts middelpunt ande middelverhey D gecommen is, en alsdan gebeurt de middellijns als I G meeste afwijcking, welke daer na gheduerlick vermindert, tot dattet middelpunt des inronts is an des wechs naestepunt C, alwaer de voorschrevē middellijn als I G wederom geen afwijcking en heeft, maer het inronts middelpunt van C vertreckende na de middelverhey D, het verstepunt als G begint af te wijcken van t'plat des wechs na het Noorden, en het naestepunt gelijk I na het Zuyden, en vermeerdert die afwijcking geduerlick tot dat des inronts middelpunt ant uytterste D gecommen is, alwaer die afwijcking weerom ten grootsten wort. Van daer voort vermindertse tot dattet inronts middelpunt an t' verstepunt A is, alwaer gelijk int begin de voorschrevē middellijn als I G, weerom int plat des wechs is: En daer na volgt weerom d'eerste gestalt. Vyt t'gene geseyt is blijft dat des wechs afweging tē grootstē sijnde, so heeft het inront geen afwijcking, maer de wech sonder afweging wesende, dat dā des inronts afwijcking tē grootstē is.

Nu restet noch de *afkeering te verclaren welke dusdanich is: Des inronts *Reflexio.*
plat heeft boven de voorschreven waggeling op den as H K, noch een ander op den as I G tusschen des inronts verstepunt en naestepunt (ghenouchsaem toe- gaende gelijk de tweederley waggeling die het zeecompas ontfangt, met twee assen op malcander rechthouckich commende) deur welck roersel gebeurt dat de middellijn H K het plat des wechs deursnijt inde lini I G als gemeene sine, so dat des inronts sliicker helft op d'een sijde des inrontwechs, de rechter helft op d'ander sijde afkeert: Dese middellijns afkeering houdt dusdanige regel: Wanneer des inronts middelpunt is ande middelverhey D, soo en heeft de voorschreven middellijn H K geen afkeering vande wech A B C D, dan is int plat der selve: Maer het inronts middelpunt van D vertreckende na het verstepunt A, soo begint de middellijns sliicker helft H D af te keeren na het Zuyden, de ander helft na het Noorden, welke afkeering geduerlick vermeerdert tot dattet inronts middelpunt ant verstepunt A ghecommen is, alsdan ten grootsten wesende. Daer na voortgaende na het ander punt B soo vermindertse weerom tot datse an gecommen is, alwaer weerom geen afkeering en gheschiet. Maer het inronts middelpunt van die plaets vertreckende na het naestepunt C, so begint de voorschreven sliicker helft ghelijck H D weerom af te keeren na het

Dd 3 Noor.

this from the description which *Purbachius* gives of this motion in a comprehensible manner in this place.

To begin with the deviation, it is to be noted that *Ptolemy* has said that the deferent has about BD as axis an oscillatory motion, going to and fro or oscillating like the beam of a balance, called deviation, so that when the epicycle's centre is at B or D , there is no deviation, but the entire plane of the orbit is in the plane of the ecliptic. Thereafter, when the epicycle's centre starts from B or D , the orbit begins to deviate or incline always to the South, and this deviation increases continuously until the epicycle's centre has arrived at the apogee A or perigee C , and then the deviation is greatest: the above-mentioned $45'$. Thereupon it decreases again until the epicycle's centre has arrived at B or D , where then again there is no deviation. And from this it is apparent that the motion which the epicycle's centre receives from the deviation never occurs on the North side.

But to speak now of the declination, it is to be noted that the plane of the epicycle through an oscillatory motion diverges from the plane of its orbit in two ways. Firstly, with declination about the diameter HK , passing through the points of medium distance H , K . With this motion the apogee G of the diameter IG diverges on one side of the orbit and the perigee I on the other side. This declination observes the following rule. When the epicycle's centre (as in the present case D) is at the orbit's apogee A , the aforesaid diameter IG is in the plane of the orbit. But when the epicycle's centre starts from the apogee A , the epicycle's apogee begins to decline towards the South and the perigee towards the North, which declination constantly increases until the epicycle's centre has arrived at the point of medium distance B , and then there is the greatest declination of the diameter (IG), which thereupon decreases continuously until the centre of the epicycle is at the orbit's perigee C , where the aforesaid diameter (IG) again has no declination; but when the epicycle's centre starts from C towards the point of medium distance D , the apogee (G) begins to decline from the plane of the orbit towards the North, and the perigee (I) towards the South, and this declination increases continuously until the epicycle's centre has arrived at the extremity D , where this declination again becomes greatest. From this point onwards it decreases until the epicycle's centre is at the apogee A , where, just as in the beginning, the aforesaid diameter (IG) is again in the plane of the orbit. And this is followed again by the first situation. From what has been said it appears that when the orbit's deviation is greatest, the epicycle has no declination, but when the orbit is without deviation, then the epicycle's declination is greatest.

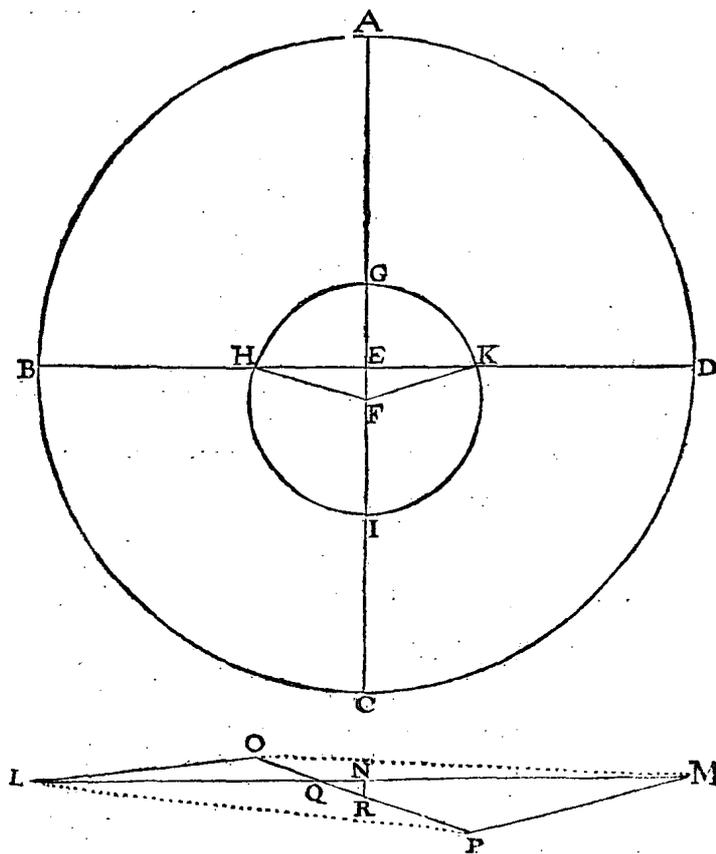
Now it still remains to explain the deflection, which is as follows. The epicycle's plane has, in addition to the aforesaid oscillation about the axis HK , yet another oscillation about the axis IG between the epicycle's apogee and perigee (which proceeds substantially like the two kinds of oscillation which the mariner's compass receives, with two axes at right angles to one another), in consequence of which motion the diameter HK intersects the plane of the orbit in the line IG as intersection, so that the epicycle's left half deflects on one side of the deferent, the right half on the other side. This deflection of the diameter observes the following rule. When the epicycle's centre is at the point of medium distance D , the aforesaid diameter HK has no other deflection from the orbit $ABCD$ than that which is in the plane of the latter.

Noorden, en vermeerdt alsoo tot datse ant naestpunt C is, alwaer men se dan weerom ten grootsten bevint : Van daer vermindertse geduerlick tot dattet inronts middelpunt comt an d'ander middelverhey D, alwaer weerom geen afkeering en is, en alsdan begint weerom de voorgaende gestalt. Hier uyt is kenelick dat ter plaets des inrontwechs daer het inront geen afwijking en heeft, sijn meeste afkeering te ghebeuren.

3 L I D T.

Inhoudende verclaring vā Mercurius breedeloop met stelling eens roerenden Eertcloots.

Hier vooren beschreven sijnde het 1 lidt inhoudende *Ptolemeeus* dadelicke ervaringen (waer by tot breeder verclaring noch vervought wiert het 2 lidt van sijn spiegeling) men soude deur vervouging der selve ervaring op stelling eens roerenden Eertcloots volgens de voorgaende gemcene regel totter begheerde commen, nochtans angesien de form vande onderste Dwaelders binnen den Eertclootwech loopende, wat anders valt als vande bovenste, en dat Mercurius wech niet deur des Eertclootwechs middelpunt en strect, t'welck voor de sommige verclaring mocht vereytschen, soo sal icker wat af seggen. Laet A B C D



den

But when the epicycle's centre starts from D towards the apogee A , the left half of the diameter HD begins to deflect towards the South, the other half towards the North, which deflection increases continuously until the epicycle's centre has arrived at the apogee A , when it is greatest. Thereupon proceeding towards the other point B , it decreases again until it has arrived there, where again there is no deflection. But when the epicycle's centre starts from that place towards the perigee C , the aforesaid left half (HD) begins to deflect towards the North again, and thus increases until it is at the perigee C , where it is then again found greatest. From there onwards it decreases continuously until the epicycle's centre arrives at the other point of medium distance D , where again there is no deflection, and then the preceding situation occurs again. From this it is obvious that at the place of the deferent where the epicycle has no declination its greatest deflection occurs.

3rd SECTION.

Consisting in the explanation of Mercury's motion in latitude on the assumption of a moving Earth.

Since above has been described the 1st section consisting of *Ptolemy's* practical experiences (to which, for a fuller explanation, was also added the 2nd section of his theory), by applying this experience to the assumption of a moving Earth one might according to the preceding common rule arrive at what is required. Nevertheless, since the figure of the orbits of the lower Planets within the Earth's orbit is somewhat different from that of the upper Planets, and since Mercury's orbit does not pass through the centre of the Earth's orbit, which for some might require an explanation, I will say something about it. Let $ABCD$ denote the Earth's orbit, whose centre is E , the centre of Mercury's orbit F , about which has been described its orbit $GHIK$; when thereafter through E and

den Eertclootwech beteycken en, diens middelpunt E, Mercuriuswechs middelpunt F, waer op beschreven is sijn wech G H I K, daer na deur E en F ghetrocken de middellijn A C, en deur t'punt E de lini B D rechthouckich op A C, soo is I t'punt verft van des Eertclootwechs middelpunt E, en G het naeste punt: Voort anghesien *Ptolemaeus* bevonden heeft dat Mercurius meefte breedden al tijt gebeurden als het inronts middelpunt was by de middelverheden, so volghet daer uyt met stelling eens roetendē Eertcloots, dat de selve meefte breedē al tijt gesien worden als Mercurius is an sijn wechs middelverheden H en K, en den Eertcloot an B of D: Nu dan H en K wescende de twee puntē welcke in Mercuriuswech de grootste afwijcking krijgē diemē uyt den Eertclootwech sien can, soo seggen wy dier twee platten gemene sine te moeten commen rechthouckich op B D, dats in G I, of ewewijdich, mette selve, en niet in H K gelijk *Ptolemaeus* die stelt: Maer om nu te vindē waer de selve duyfteraersne valt, metsgaders Mercuriuswechs afwijcking van t'plat des duyfteraers, ghelijck van Saturnus int 21 en 22 voorstelghedaen wiert, ick souck voor al de langde der lini H K, tot dien eynde aldus segghende: De driehouck E K F heeft drie bekende palen, te wetē Mercuriuswechs halfmiddellijn F K 3572, en de uytmiddelpunticheytlijn E F 947 deur de Byenvouging des 13 voorstels deses 3 boucx, en dē houck K E F recht: Hier me gefocht de sijde E K, wort bevonden deur het

5 voorstel der platte driehoucken van

3444.

Daer toe noch soo veel voor E K, comt voor de begheerde H K

6888.

En van E D 10000, ghetrocken E K 3444 eerste in d'oirden, blijft voor de lini K D, oock me voor H B

6556.

Dit aldus bekent sijnde ick teycken een ander form als gedaen wiert met Saturnus int 22 voorstel, treckende ten eersten L M als Eertclootwech overcant ghesien, even an B D 20000, en stel int middel van L M t'punt N als Eertclootwechs middelpunt, ick treck daer na de lini O P van 6888 even an H K tweede in d'oirden, en sniende L M in Q, daer na L O evē met H B 6556 derde in d'oirden, sgelijcx M P even met K D, dats oock doende 6556, en segghen houck O L Q te doen 1 tr. 45 ①, en P M Q 4 tr. 5 ① volghens d'ervaring int 1 lidt: Dit soo wescende de cruyfvierhouck L O M P heeft vijf bekende palen, te wetē L M 20000, L O 6556, M P 6556, den houck O L Q 1 tr. 45 ①, en den houck P M Q 4 tr. 5 ① deur t'ghegheven. Merckt noch dat benevens de boveschrevē vijf palen tot meerder gerief bekent sijn drie ander, te wetē O P 6888, P L 13444 als even genouch sijnde met O P 6888 en O L 6556 t'samen, voort O M doende oock soo veel: Hier me ghesocht den houck O Q L, wort bevonden deur het 6 voorstel inde Byvough der platte veelhoucken voor Mercuriuswechs begheerde afwijcking vanden duyfteraer

5 tr. 32 ①.

En de lini Q M van 11364, waer af ghetrocken M N 10000, blijft voor de lini vande duyfteraersne Q totten Eertclootwechs middelpunt N.

1364.

Tot hier toe is beschreven t'ghene ick van Mercurius int besonder verclaren wilde. Belanghende ander byvallen als inde ses voorstellen van Saturnus breedde beschreven sijn, die houden wy als voor ghemeene reghel over Mercurius en d'ander Dwaelders te verstrecken.

Angaende overeencomminghen en verschil deser stelling van Mercurius

D d 4 bree-

F is drawn the diameter *AC*, and through the point *E* the line *BD* perpendicular to *AC*, *I* is the point furthest from the centre of the Earth's orbit *E*, and *G* is the nearest point. Further, since *Ptolemy* found that Mercury's greatest latitudes always occurred when the epicycle's centre was at the points of medium distance, it follows therefrom on the assumption of a moving Earth that these greatest latitudes are always seen when Mercury is at its orbit's points of medium distance *H* and *K*, and the Earth at *B* or *D*. *H* and *K* now being the two points which in Mercury's orbit receive the greatest deviation that can be seen from the Earth's orbit, we say that the intersection of these two planes must be perpendicular to *BD*, *i.e.* in *GI*, or parallel thereto, and not in *HK*, as *Ptolemy* assumes. But in order now to find where this line of nodes falls, as also the deviation of Mercury's orbit from the plane of the ecliptic, as was done with Saturn in the 21st and 22nd propositions, I seek first of all the length of the line *HK*, saying to this end as follows. The triangle *EKF* has three known terms, to wit, the semi-diameter of Mercury's orbit $FK = 3,572$, and the line of eccentricity $EF = 947$ by the Compilation of the 13th proposition of this 3rd book, and the angle *KEF* right; when the side *EK* is sought therewith, this is found by the 5th proposition of plane triangles to be

3,444

When to this is added the same value for *EK*, the required *HK* becomes

6,888

And when from $ED = 10,000$ is subtracted $EK = 3,444$ (the first in the present list), there is left for the line *KD*, and also for *HB*,

6,556

This being known, I draw another figure, as was done with Saturn in the 22nd proposition, first drawing *LM* as the Earth's orbit, seen transversely, equal to $BD = 20,000$, and I mark in the middle of *LM* the point *N*, as centre of the Earth's orbit; I then draw the line *OP* of 6,888 equal to *HK* (the second in the list) and intersecting *LM* in *Q*, then *LO* equal to $HB = 6,556$ (the third in the list), also *MP* equal to *KD*, *i.e.* also making 6,556, and I say that the angle *OLQ* makes $1^{\circ}45'$, and *PMQ* $4^{\circ}5'$, according to the experience in the 1st section. This being so, the crossed quadrilateral *LOMP* has five known terms, to wit, $LM = 20,000$, $LO = 6,556$, $MP = 6,556$, the angle $OLQ = 1^{\circ}45'$, and the angle $PMQ = 4^{\circ}5'$ by the supposition. Note also that besides the above-mentioned five terms for greater convenience three more are known, to wit, $OP = 6,888$, $PL = 13,444$ as being substantially equal to $OP = 6,888$ and $OL = 6,556$ together, further *OM* having the same value. When the angle *OQL* is sought therewith, by the 6th proposition in the Supplement of plane polygons ¹⁾ the required deviation of Mercury's orbit from the ecliptic is found to be

5°32'

And when from the line *QM* of 11,364 is subtracted $MN = 10,000$, there is left for the line from the line of nodes *Q* to the centre of the Earth's orbit *N*

1,364

Up to this point has been described what I wished to explain in particular for Mercury. As to other cases, such as have been described in the six propositions

¹⁾ See p. 189, note 1.

316 VANDE BREEDELOOP DER DVVAELD.
breedeloop met die van *Ptolemus*, daer af sal gheseyt worden inden volghenden
Byvough des breedeloops met stelling eens vasten Eertcloots.

N V V A N V E N V S
B R E E D E.

Ptolemus heeft deur dadelicke gagheslaghen ervaringhen de ghedaenten van Venus breedeloop alsins bevonden ghelijck van Mercurius int voorgaende gheseyt is, welcke int besonder dusdanich sijn.

Ten eersten soo was haer meeste breede op de noortsijde van 6 tr. 22 $\text{\textcircled{D}}$, gebeurende altijd als haer inronts middelpunt was 90 tr. van sijn wechs verstepunt soo wel ter eender als ter ander sijde, dat is soo wel inden 270 tr. als inden 90 (welcke 90 tr. is onder des duyfteraers 145 tr. ghemerckt deur de Byeenvonging des 13 voorstels deses 3 boucx het verstepunt onder des duyfteraers 35 tr. is) en Venus in des inronts naestepunt: Maer buyten het naestepunt wesende, soo was heur noorderfche breede voor datmael cleender, en ten minsten doen sy alsoo ant verstepunt was.

Ten tweeden soo bevant hy op de zuytsijde de meeste breede van 1 tr. 2 $\text{\textcircled{D}}$, gebeurende altijd als heur inronts middelpunt was 90 tr. van sijn wechs verstepunt soo wel ter eender als ter ander sijde, en Venus an haer inronts naestepunt, maer buyten het naestepunt wesende, soo was haer zuyderfche breede voor datmael cleender, en ten minsten doenmense alsoo ant verstepunt vandt.

De spiegheling by *Ptolemus* hier op met stelling eens vasten Eertcloots veroirdent is met haer afweging, afwijcking, afkeering, en ander ghedaente, te nemaal gheweest als die van Mercurius, sulcx dattet onnoodich schijnt de selve alhier te verhalen: Ende want de spiegeling met stelling eens roerenden Eertcloots ande voorgaende van Saturnus en Mercurius oock ghelijck is, en daer deur verstaen wort, soo en beschrijf ick die niet int langhe.

Angaende overeencomminghen en verschil deser stelling van Venus breedeloop met die van *Ptolemus*, daer af sal gheseyt worden inden volghenden Byvough des breedeloops met stelling eens vasten Eertcloots.

of Saturn's latitude, we assume that these apply as a common rule to Mercury and the other Planets.

As regards the correspondences and difference of this theory of Mercury's motion in latitude and that of *Ptolemy*, these are to be discussed in the subsequent Supplement of the motion in latitude on the assumption of a fixed Earth.

NOW OF VENUS' LATITUDE.

Ptolemy through practical observational experiences has found the character of Venus' motion in latitude to be in every respect as has been said of Mercury in the foregoing, which in particular is as follows.

Firstly, its greatest latitude on the North side was $6^{\circ}22'$, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, *i.e.* both at 270° and at 90° (which 90° is at 145° of the ecliptic, seeing that by the Compilation of the 13th proposition of this 3rd book the apogee is at 55° of the ecliptic), and Venus was at the epicycle's perigee. But when it was outside the perigee, its Northerly latitude was for that case smaller, and smallest when at the apogee.

Secondly, he found on the South side the greatest latitude to be $1^{\circ}2'$, which always occurred when its epicycle's centre was at 90° from its orbit's apogee, both on one side and on the other, and Venus was at its epicycle's perigee; but when it was outside the perigee, its Southerly latitude was for that case smaller, and smallest when at the apogee.

The theory that *Ptolemy* on the assumption of a fixed Earth based thereon was, with its deviation, declination, deflection, and other properties, altogether the same as that of Mercury, so that it seems unnecessary here to describe it. And because the theory on the assumption of a moving Earth is also equal to the preceding one of Saturn and Mercury, and is understood therefrom, I will not describe it in full.

As regards the correspondences and difference of this theory of Venus' motion in latitude and that of *Ptolemy*, these are to be discussed in the subsequent Supplement of the motion in latitude on the assumption of a fixed Earth.

BYVOUGH

DES BREEDELOOPS

DER VYF DWAELDERS

Saturnus, Iupiter, Mars, Venus, en

Mercurius, ghegront op stelling

eens vasten Eertcloots.

SUPPLEMENT
OF THE MOTION IN LATITUDE
OF THE FIVE PLANETS

Saturn, Jupiter, Mars, Venus, and Mercury,
Based on the Theory of a Fixed Earth

C O R T B E G R Y P D E S E S B Y V O V G H S.

Net 19 voorstel des 3 boucx is gheseyt, gherievigher te sijn re-
keninghen vande langdeloop der Dvvaelders te maken op de
oneyghen stelling eens vasten Eertcloots, dan op de eygen eens
roerenden, en dien volghens soudemen om de selve reden me-
ghen segghen dergelijcke met stelling eens vasten Eertcloots oockoirboirder
te sijn vande breedeloop, en dat daerom de natuerlicke oir den soude
vereysschen desen Byvough niet hier, maer int tweede bouck met stelling
eens vasten Eertcloots behooren vervought te wesen, ghelijck mette Ma-
nens breedeloop daer ghedaen is. Om hier op te antwoorden, soo is te w-
eten dat nadien grondelicke kennis deses handels ghetrocken wort uyt de
breedeloop met stelling eens roerenden Eertcloots, welcke alsdoen noch niet
beschreven en was, noch volghens mijn voorghenomen oir den beschreven
en moest wesen, soo en conde dit daer niet bequamelick commen: Maer de
selve breedeloop nu int derde bouck verclaert sijnde, soo connen wy daer
uyt trecken t'ghene de breedeloop met stelling eens vasten Eertcloots ver-
eyscht, en daerom heb ick die hier beschreven.

Theoremata. De selve sal ses voorstellen hebben, wesen de eerste vier * vertoo-
Problemata. ghen, de laetste twee * verckstucken, te weten:

Het 1 voorstel, dat de ronden der twee onderste Dvvaelders Venus en
Mercurius die byde stelders eens vasten Eertcloots inront dragers genoemt
wordē, inronde sijn, en t'gene sy inronde heeten, inront dragers te wesen.

Het 2, dattet plat des inronts der vijf Dvvaelders Saturnus, Jupiter,
Mars, Venus en Mercurius met stelling eens vasten Eertcloots, alijt e-
vrijdich is met tet plat des duysteraers.

Het 3, dat wesen de twee even en evervijdege ronden het een hooger alst
ander, de lini tusschen het middelpunt v ant leeghste, en een punt inden om-
treck v ant hooghste, even en evervijdege te sijn met te lini tusschen sijn lij-
standich tegenoverpunt int leeghste, en het middelpunt v ant hooghste.

Het 4, dat de Dvvaelders met stelling eens vasten Eertcloots de selve
schijnbaer duysteraerbree de ontfangen, dese hebben met stelling eens roeren-
den Eertcloots.

Het 5, wesen de gegeven eens Dvvaelders meeste noordersche en zuy-
dersche bree de, te vindē sijn wechs afwijcking vanden duysteraer: Oock
mede hoe verre de duysteraersne vanden Eertcloon valt, deur wvifcon-
stige vercking ghegront op stelling eens vasten Eertcloots.

Het 6, om te vindē eens Dvvaelders schijnbaer duysteraerbree de op een
gegeve tijt, deur wvifconstige vercking gegrot op stelling eens vasten Eertcloots.

SUMMARY OF THIS SUPPLEMENT.

In the 19th proposition of the 3rd book it has been said that it is more convenient to make calculations of the motion in longitude of the Planets on the untrue theory of a fixed Earth than on the true theory of a moving Earth; consequently it might be said for the same reason that similar calculations on the theory of a fixed Earth are also more suitable for the motion in latitude, and that therefore the natural order would require that this Supplement should not be added here, but in the second book, based on the theory of a fixed Earth, as has been done there with the motion in latitude of the Moon. In order to answer this argument, it is to be noted that since thorough knowledge of this subject is derived from the motion in latitude on the theory of a moving Earth, which had not yet been described at that moment, nor should have been described according to the sequence planned by me, this could not be properly inserted there. But now that this motion in latitude has been explained in the third book, we can derive therefrom what is required for the motion in latitude on the theory of a fixed Earth, and that is why I have described it here.

The Supplement is to comprise six propositions, the first four being theorems and the last two being practical problems, to wit:

The 1st proposition, that the circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are epicycles, and what they call epicycles are deferents.

The 2nd, that the planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are always parallel to the plane of the ecliptic.

The 3rd, that when there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one.

The 4th, that on the theory of a fixed Earth the Planets acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth.

The 5th, given a Planet's greatest northerly and southerly latitudes, to find its deferent's deviation from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth.

The 6th, to find a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth.

VERTOOCH. I VOORSTEL.

De ronden der tvvee onderfte Dwaelders Venus en Mercurius die by de stelders eens vasten Eertcloots ghe-noemt vvorden * inrontdragers, sijn inronden: En t'ge-^{Deferens} ne sy inronden heeten, sijn inrontdragers. ^{epicyclorum.}

T'is deur kennis der voorgaende stelling eens roerenden Eertcloots openbaer, dat de inronden der drie bovenste Dwaelders met stelling eens vasten Eertcloots niet int wesen en bestaen, dan versiert worden even te sijn anden Eertcloonwech, maer t'gaet anders toe mette twee onderste, want t'ghene men Venus of Mercurius inront noemt, en is niet versiert noch even anden Eertcloonwech, dan om eyghentlick te spreken het is hun wech self daer sy dadelick in loopen, en in stelling eens vasten Eertcloots, volghens d'oiden der drie bovenste, inrontwech behoort te heeten: En daerom als men de voorgaende regel der langdloop vande bovenste met stelling eens vasten Eertcloots, soude willen ghemeen hebben over de twee onderste, men soude daer moeten op de selve wijze spreken, te weten inrontwech noemen het cleenste dat eyghentlick inrontwech is, en inront het grootste dat versiert wort even anden Eertcloonwech, want daer me dan ghedaen als mette bovenste, de reghel sal ghemeen sijn. Maer om van sulcke stelling noch claerder te spreken ick seggh aldus: Angesien yder Dwaelders inront even is anden Eertcloonwech, soo moeten se met malcander al evegroot sijn, waeruyt wijder volghet, dat de wegghen vande leegher Dwaelders teghen haer inronden verleken, minder fullen sijn dan de wegghen vande hoogher Dwaelders.

Laet tot voorbeelt in dese eerste volghende form op A als middelpunt, beschrevē worden het rondt B C als Saturnus inrontswech, diens halfmiddellijn A B doet deur de Byeenvouging des 13 voorstels vant 3 bouck 92308, en op B als middelpunt, het inrontt diens halfmiddellijn D B even an den Eertcloonwech diens halfmiddellijn doet 10000.

Ten tweeden sy op A als middelpunt, beschreven het rondt E F als Iupiters inrontswech, diens halfmiddellijn A E doet deur de Byeenvouging des 13 voorstels 52174, en op E als middelpunt het inrontt diens halfmiddellijn E G 10000.

Ten derden sy op A als middelpunt beschreven het rondt H I als Mars inrontswech, diens halfmiddellijn A H doet deur de Byeenvouging des 13 voorstels 15190, en op H als middelpunt het inrontt diens halfmiddellijn H K 10000.

Ten vierden sy op L als middelpunt, beschreven het rondt M N als Venus inrontswech, diens halfmiddellijn M L doet deur de Byeenvouging des 13 voorstels 7194, en op M als middelpunt het inrontt, diens halfmiddellijn M O 10000, welck inrontt in hem heeft des inrontwechs middelpunt L, anders dan een der drie voorgaende formen: Doch is kennelick dat de reghel des langdloops voor alle vier ghemeen moet sijn.

Ten vijften sy op P als middelpunt beschreven het rondt Q R als Mercurius inrontswech, diens halfmiddellijn P Q doet deur de Byeenvouging des 13 voorstels 3572, en op Q als middelpunt het inrontt, diens halfmiddellijn Q S 10000, welck inrontt in hem heeft den heelen inrontwech Q R, anders dan in een der vier voorgaende formen, doch is kennelick dat de reghel des langdloops

THEOREM

1st PROPOSITION.

The circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are epicycles; and what they call epicycles are deferents.

It is evident from a knowledge of the foregoing theory of a moving Earth that the epicycles of the three upper Planets on the theory of a fixed Earth do not really exist, but are imagined to be equal to the Earth's orbit; but matters are different with the two lower Planets, for what is called Venus' or Mercury's epicycle is neither imagined nor really equal to the Earth's orbit; but, accurately speaking, it is their orbits themselves in which they move in actual practice, and which, on the theory of a fixed Earth, according to the arrangement of the three upper Planets, ought to be called deferents. Therefore, if the foregoing rule about the motion in longitude of the upper Planets on the theory of a fixed Earth should be desired to apply also to the two lower Planets, it would be necessary to speak there in the same way, to wit, call deferent the smallest, which really is a deferent, and epicycle the largest, which is imagined to be equal to the Earth's orbit, for if these are then dealt with in the same way as the upper Planets, the rule will apply generally. But to speak even more clearly of this theory, I say as follows: Since the epicycle of each Planet is equal to the Earth's orbit, they must all be equal to one another, from which it follows further that the orbits of the lower Planets, when compared with their epicycles, will be smaller than the orbits of the upper Planets.

By way of example, in this first subsequent figure let there be described about A as centre the circle BC as Saturn's deferent, whose semi-diameter AB by the Compilation of the 13th proposition of the 3rd book ¹⁾ makes 92,308, and about B as centre the epicycle, whose semi-diameter DB , equal to the semi-diameter of the Earth's orbit, makes 10,000.

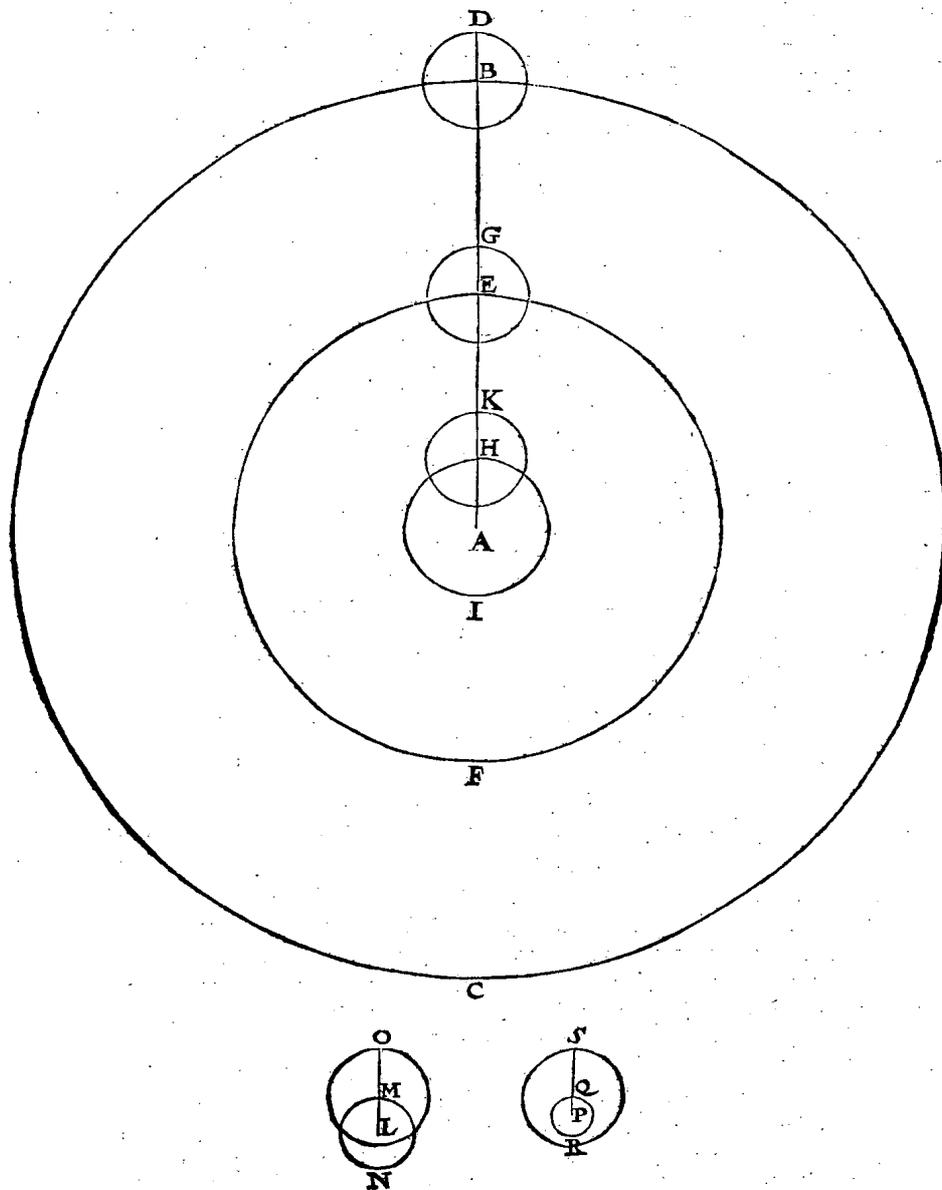
Secondly, let there be described about A as centre the circle EF as Jupiter's deferent, whose semi-diameter AE by the Compilation of the 13th proposition makes 52,174, and about E as centre the epicycle, whose semi-diameter EG makes 10,000.

Thirdly, let there be described about A as centre the circle HI as Mars' deferent, whose semi-diameter AH by the Compilation of the 13th proposition makes 15,190, and about H as centre the epicycle, whose semi-diameter HK makes 10,000.

Fourthly, let there be described about L as centre the circle MN as Venus' deferent, whose semi-diameter ML by the Compilation of the 13th proposition makes 7,194, and about M as centre the epicycle, whose semi-diameter MO makes 10,000, which epicycle has within it the deferent's centre L , unlike anyone of the three foregoing figures. It is obvious that the rule of the motion in longitude must be the same for all four Planets in common.

Fifthly, let there be described about P as centre the circle QR as Mercury's deferent, whose semi-diameter PQ by the Compilation of the 13th proposition makes 3,572, and about Q as centre the epicycle, whose semi-diameter QS makes 10,000, which epicycle has within it the entire deferent QR , unlike anyone of the four foregoing figures. It is obvious that the rule of the motion in longitude must be

¹⁾ See p. 173.



loops voor alle vijf gemeen moet sijn, sonder de verkeerde haspeling te vallen diemen ontmoet anders doende. Dit oude misbruyck heeft sijn bekende oir-faack, want tewijle d'eerste ondersouckers des Hemelloops gheen kennis en hadden vande ghedaente des roerenden Eertcloots, soo en conden sy niet be-ter schrijven dan van t'ghene voor hemlien uytlick schein te wesen. Noch is oock te anmercken dat de regel des breedeloops (soo wel als des langdeloops) met stelling eens vasten Eertcloots, hier deur over allen ghemeen is ghelijck
int vol.

the same for all five Planets in common, without the occurrence of the tangle, due to inversion, which we meet with if we do otherwise. This ancient abuse has a well-known cause, for since the first investigators of the Heavenly Motions were not acquainted with the character of the moving Earth, they could merely describe things as they appeared to them. It is also to be noted that the rule of the motion in latitude (as well as of the motion in longitude) on the theory of a fixed Earth is thus common to all, as will become apparent in the sequel.

int volghende blijcken sal. T' B E S L V Y T. De rondan dan der twee onderste Dwaelers Venus en Mercurius die by de stelders eens vasten Eertcloots ghe-noemt worden inrontdragers, sijn inronden : En t'ghene sy inronden heeten, sijn inrontdragers, t'welck wy bewijfen moesten.

V E R T O O C H. 2 V O O R S T E L

Het plat des inronts der vijf Dwaelers Saturnus, Iupiter, Mars, Venus en Mercurius, is met stelling eens vasten Eertcloots altijt euewijdich mettet plat des duysteraers.

Voor al anghesien het inront versiert is euegroot metten Eertcloodwech, en dat den Dwaeler daer in ghenomen wort een langdeloop te hebben even en ghelijck mette langdeloop des Eertcloots in haer wech, soo volghet daer ghenouchsaem uyt deser twee rondan euewijdicheyt behooren toegelaten te worden, ghemerckt datter anders gheen volcommen ghelijcke langdeloop sijn en soude, nochtans anghesien de selve euewijdicheyt uyt het voorgaende can bewesen worden, soo sal ick die beschrijven als volghet: Ick verkies hier toe de form des 15 voorstels vant 3 bouck, waer me ick aldus segh: Ten eersten soo is deur de ghemeene reghel beschreven int 22 voorstel des 3 boucx, bekennt hoe groot dat moet sijn de breedte van Mars ter plaets des punts N ghesien uyt den roerenden Eertclood O, maer soo groot die daer wesen moet, even soogroot moet Mars breedte oock sijn ter plaets van I met stelling eens vasten Eertcloots, dats ghesien uyt A: Ende want A I even is met O N, soo volghet daer uyt dat de lini van I rechthouckich opt plat des duysteraers even moet sijn ande lini van N rechthouckich opt plat des duysteraers. Ten tweedē segh ick, dat soo lanck als is de lini van N tot opt plat des duysteraers gesien uyt den roerenden Eertclood an P, soo lanck moet oock sijn de lini van F tot opt plat des duysteraers ghesien uyt den vasten Eertclood A, om dat A F even is met P N.

Maer elck der twee linien van F en N rechthouckich opt plat des duysteraers, aldus even sijnde ande lini van N tot opt plat des duysteraers, soo moeten die selve twee linien van F en N rechthouckich op t'plat des duysteraers met malcander even sijn, en vervolgens soo is des inronts heele halfmiddellijn I F euewijdich mettet plat des duysteraers. Ende op de selve voet ist openbaer te connen behoot worden de middellijn deur H rechthouckich op F I soose ghetrocken waer, oock euewijdich te wesen mettet plat des duysteraers, want soo men deur t'punt K treckt een middellijn rechthouckich op O P, en datmen naem den roerenden Eertclood te wesen ande uystersten van dien (het welck cortheitshalven ghelaten wort) t'bewijs soude daer me sijn als vooren: Nu dan de middellijn F I, met d'ander middellijn deur H, soose als gheseyt is getrocken waer, beyde euewijdich sijnde metten duysteraer, soo volghet uyt het heel plat des inronts F I euewijdich te wesen mettet plat des duysteraers. Ende sulcx als dit bewijs is wesen de het inront met sijn middelpunt a H, alsoo sal derghelijcke bewesen worden tot alle plaetsen. Voort gelijck dit bewijs is geweest mette form des 15 voorstels, dienende voor de drie opperste Dwaelers, alsoo salt oock sijn mette form des 16 voorstels dienende voor de onderste.

T' B E S L V Y T. Het plat dan des inronts der vijf Dwaelers Saturnus, Iupiter, Mars, Venus en Mercurius, is met stelling eens vasten Eertcloots altijt euewijdich mettet plat des duysteraers, t'welck wy bewijfen moesten.

E e V E R-

CONCLUSION. The circles of the two lower Planets Venus and Mercury, which are called deferents by those who hold the theory of a fixed Earth, are therefore epicycles; and what they call epicycles are deferents; which we had to prove.

THEOREM.

2nd PROPOSITION.

The planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are always parallel to the plane of the ecliptic.

First of all, since the epicycle is imagined to be equal to the Earth's orbit and since the Planet is assumed to have therein a motion in longitude equal and similar to the motion in longitude of the Earth in its orbit, it follows sufficiently that the parallelism of these two circles should be admitted, since otherwise there would be no perfectly similar motion in longitude; nevertheless, since this parallelism can be proved from the foregoing, I will describe it as follows. I choose for this the figure of the 15th proposition of the 3rd book, with regard to which I say as follows: Firstly, by the common rule described in the 22nd proposition of the 3rd book it is known how great must be the latitude of Mars at the point N , when seen from the moving Earth O ; but as great as it must be there, so great must also be Mars' latitude at I on the theory of a fixed Earth, *i.e.* when seen from A . And because AI is equal to ON , it follows that the line from I perpendicular to the plane of the ecliptic must be equal to the line from N perpendicular to the plane of the ecliptic. Secondly, I say that as long as is the line from N to the plane of the ecliptic, when seen from the moving Earth at P , so long must also be the line from F to the plane of the ecliptic, when seen from the fixed Earth A , because AF is equal to PN .

But each of the two lines from F and I ¹⁾ perpendicular to the plane of the ecliptic thus being equal to the line from N to the plane of the ecliptic, those two lines from F and I ¹⁾ perpendicular to the plane of the ecliptic must be equal to one another, and consequently the epicycle's entire semi-diameter IF is parallel to the plane of the ecliptic. And on the same grounds it is evident that it can be proved that the diameter through H perpendicular to FI , if it were drawn, would also be parallel to the plane of the ecliptic, for if through the point K is drawn a diameter perpendicular to OP , and if the moving Earth were assumed to be at the extremities thereof (which is omitted for brevity's sake), the proof would be the same as above. The diameter FI , with the other diameter through H , if — as has been said — it were drawn, thus both being parallel to the ecliptic, it follows that the entire plane of the epicycle FI is parallel to the plane of the ecliptic. And such as is this proof when the epicycle is with its centre at H ; a similar proof can be given for any place. Further, as this proof has been with the figure of the 15th proposition, serving for the three upper Planets, so will it also be with the figure of the 16th proposition, serving for the lower Planets.

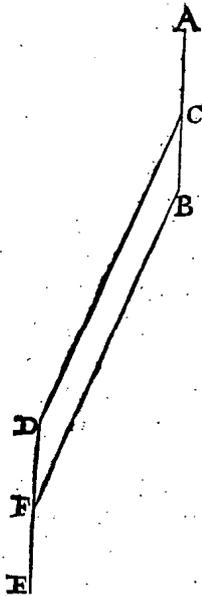
CONCLUSION. The planes of the epicycles of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury on the theory of a fixed Earth are thus always parallel to the plane of the ecliptic; which we had to prove.

¹⁾ For N in the Dutch text read I .

VERTOORCH. 3 VOORSTEL.

Wesende twee even en ewevijdeghe ronden het een hooger als t'ander, de lini tusschen het middelpunt vant leeghste, en een punt inden omtreck vant hooghste, is even en ewevijdege mette lini tusschen sijn lijckstandich teghenoverpunt int leeghste, en het middelpunt van het hooghste.

T'GHEGHEVEN. Laet AB het hooghste rondt sijn overcant ghesien diens middelpunt C , opperste punt A , onderste punt B , en DE sy het leeghste rondt, even en ewevijdege met AB , diens middelpunt F , opperste punt D , onderste punt E , en FB sy de lini tusschen het middelpunt F vant leeghste rondt, en eenich punt inden omtreck vant hooghste rondt, daer ick hier toe neem het onderste punt B : Sghelijcx sy DC de lini tusschen sijn lijckstandich teghenoverpunt int leeghste, dat is het opperste punt D , en het middelpunt C vant hooghste rondt. Ende is te weten dat ick D ghenoeemt hebbe lijckstandich teghenoverpunt van B , uyt oirsaeck dat E een lijckstandich punt van B is (als wesende in sijn rondt soo het onderste ghelijck D int sijne) en D teghenoverpunt van E .



T'BEGHEERDE. Wy moeten bewijfen dat FB even en ewevijdege is met DC . T'BEWYS. Anghesien CB en DF even en ewevijdege halfmiddellijnen sijn deur t'ghegheven, soo sijn FB en DC twee linien tusschen de uytersten van twee even en ewevijdege, en daerom sijnse oock self even en ewevijdege: En sulcx als hier bewesen is mettet lijckstandich teghenoverpunt vant onderste punt B , alsoo is derghelijcke openbaer mettet lijckstandich teghenoverpunt van alle voorghestelt punt des omtreex.

T'BESLUYT. Wesende dan twee even en ewevijdege ronden het een hooger als t'ander, de lini tusschen het middelpunt vant leeghste, en een punt inden omtreck vant hooghste is even en ewevijdege mette lini tusschen sijn lijckstandich teghenoverpunt int leeghste, en het middelpunt vant hooghste, t'welck wy bewijfen moesten.

VERTOORCH. 4 VOORSTEL.

De Dvvaelders ontfanghen met stelling eens vasten Eertcloots de selve schijnbaer duyfteraerbreede, diese hebben met stelling eens roerenden Eertcloots.

Anghesien de Son of Eertcloot nummermeer breede en hebben, soo en valter niet af te segghen. Angaende de breede der Maen, het is int voorgaende ghe-

THEOREM.

3rd PROPOSITION

When there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one.

SUPPOSITION. Let AB be the higher circle seen transversely, its centre being C , the uppermost point A , the lowermost point B , and let DE be the lower circle, equal and parallel to AB , its centre being F , the uppermost point D , the lowermost point E , and let FB be the line between the centre F of the lower circle and some point on the circumference of the higher circle, for which I here take the lowermost point B . Similarly, let DC be the line between its homologous opposite point in the lower circle, *i.e.* the uppermost point D , and the centre C of the higher circle. And it is to be noted that I have called D homologous opposite point to B because E is a point homologous to B (as being in its circle the lowermost point just as D is in its circle) and D is the point opposite E .

WHAT IS REQUIRED. We have to prove that FB is equal and parallel to DC .
PROOF. Since CB and DF are equal and parallel semi-diameters by the supposition, FB and DC are two lines between the extremities of two equal and parallel lines, and for this reason they are also equal and parallel themselves. And such as has here been proved for the homologous point opposite the lowermost point B , so the same is evident for the homologous point opposite any given point of the circumference.

CONCLUSION. When there are two equal and parallel circles, one higher than the other, the line between the centre of the lower one and a point on the circumference of the higher one is thus equal and parallel to the line between its homologous opposite point in the lower one and the centre of the higher one; which we had to prove.

THEOREM.

4th PROPOSITION.

On the theory of a fixed Earth the Planets acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth.

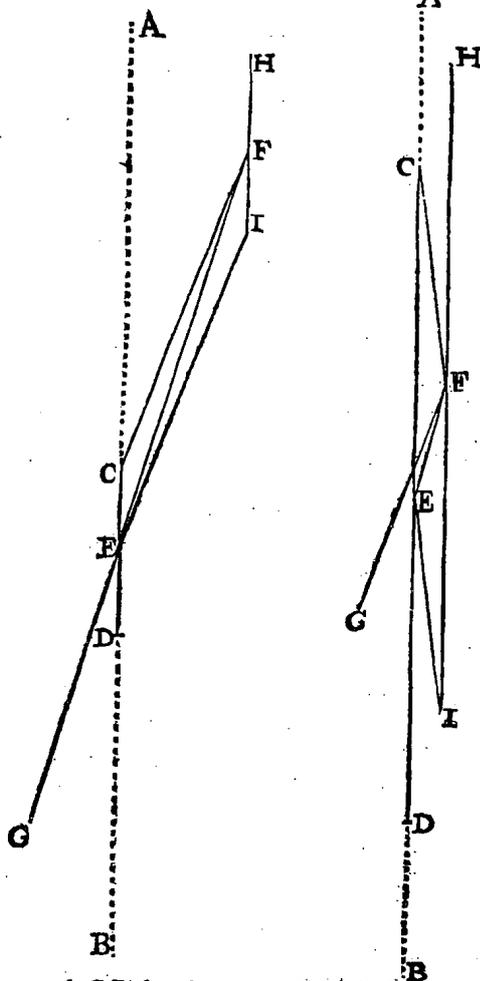
Since the Sun or Earth never have any latitude, there is nothing to be said about them. As to the latitude of the Moon, it has appeared in the foregoing

de ghebleken die in één en d'ander stelling een selve te wesen, sulcx datter alleenelick vande vijf ander verclaring behouft.

T'GHEGHEVEN. Laet AB den duyfteraer bedien overcant ghesien ghe-lijckal d'ander rondon, CD sy den Eertcloodwech, diens middelpunt E, waer

1 FORM.

2 FORM.



deur als inde eerste form dienēde voor de drie bo-
vēste Dwaelers, of waer
buytē als inde 2 form we
sende Mercurius teycke-
ning dienende voor de
twee onderste, getrockē
den Dwaelderwech FG,
in welke ick neem den
Dwaelder te wesen ant
opperste punt F, en den
Eertclood an haer wechs
opperste pūt C. Tot hier
toe de teyckening gedaē
sijnde vande stelling eēs
roerendē Eertcloots, ick
sal nu die eens vasten be-
ginnen, tot welckē eyn-
de ick des Eertclood-
wechs middelpūt E voor
vasten Eertclood neem,
want dat in die stelling
daer tegen sulcke beteyc-
kening heeft deur het 15
voorstel des 3 boucx, en
FG die eerst was Dwael-
derwech, neem ick nu
voor inrontwech: Voort
anghesien den Dwael-
der ghenomen is te we-
sen an des wechs opper-
ste punt F, soo moet het
inront t'welck HI sy,
met sijn middelpunt we-
sen an F, oock even en
evewijdich mettē Eert-

cloodwech CD deur het 2 voorstel deses Byvoughs. Voort want den roeren-
den Eertclood ghenomen wiert an C, soo moet den Dwaelder ghenomen sijn
an I als lijkstandich tegenoverpunt van C, waer af de reden blijktt int 15 voor-
stel des 3 boucx. T'BEGHEERDE. Wy moeten bewijsen dat den Dwaelder
an I, met stelling des vasten Eertcloots an E, de selve schijnbaer duyfteraer bree-
de ontfangt, die hy heeft an F met stelling des roerenden Eertcloots an C.

T'BEREYTSSEL. Laet getrockē wordē de twee linien EI, CF. T'BEWYS.
Anghesien CD even en evewijdighe is met HI, soo sijn haer helften EC, FI
oock even en evewijdighe, en de twee linien daer tusschen EI, CF moeten
oock evewijdighe sijn, en daerom oock den houck AEI, wefende des Dwael-

Ee 2 ders

that it is the same in one theory as well as the other, so that an explanation is required only for the five other Planets.

SUPPOSITION. Let AB denote the ecliptic, seen transversely like all the other circles; let CD be the Earth's orbit, its centre being E , through which — as in the first figure serving for the three upper Planets — or alongside of which — as in the 2nd figure, which is a drawing of Mercury, serving for the two lower Planets — is drawn the Planet's orbit FG , in which I take the Planet to be at the uppermost point F , and the Earth at its orbit's uppermost point C . The drawing so far having been made for the theory of a moving Earth, I will now start on one for the theory of a fixed Earth, to which end I take the centre of the Earth's orbit E for the fixed Earth, for in this theory this has the said significance by the 15th proposition of the 3rd book, and FG , which first was the Planet's orbit, I now take for the deferent. Further, since the Planet is assumed to be at the orbit's uppermost point F , the epicycle, which shall be HI , must be with its centre at F , also equal and parallel to the Earth's orbit CD by the 2nd proposition of this Supplement. Further, because the moving Earth was taken at C , the Planet must be taken at I as homologous point opposite C , the reason of which appears from the 15th proposition of the 3rd book. **WHAT IS REQUIRED.** We have to prove that the Planet at I , on the theory of the fixed Earth at E , acquires the same apparent ecliptical latitude that it has at F on the theory of the moving Earth at C .

PRELIMINARY. Let the two lines EI , CF be drawn. **PROOF.** Since CD is equal and parallel to HI , their halves EC , FI are also equal and parallel, and the two lines in between, EI and CF , must also be parallel, and therefore also the

ders schijnbaer duyfteraerbreedeghesien uyt den vasten Eertcloot E, is even anden houck A C F wefende des Dwaelders schijnbaer duyfteraerbreedeghesien uyt den roerenden Eertcloot C: Maer sulcx als hier bethoont is wefende den roerenden Eertcloot en den Dwaelder tot dier rondē hooghste en leeghste punt, sal oock alsoo blijcken tot alle plaetsen, om dat ghelijck in dese twee ewewijdege rondē CD, HI, de lini EI tusschen het middelpunt E vant leeghste rondt, en het punt I inden omtreck vant hooghste, even en ewewijdeghe is mette lini CF, tusschen sijn lijckstandich teghenoverpunt C int leeghste, en het middelpunt F vant hooghste, soo sijn alle sulcke linien overal ewewijdeghe deur het 3 voorstel deses Byvoughs. T' B E S L V Y T. De Dwaelders dan ontfanghen met stelling eens vasten Eertcloots de selve schijnbaer duyfteraerbreedē diefe hebben met stelling eens roerenden Eertcloots, t'welck wy bewijsen moesten.

5 V O O R S T E L.

Wefende ghegheven eens Dwaelders meeste noordersche en zuydersche breedē, te vinden sijn vvechs afvvijsking vanden duyfteraer: Oock mede hoe verre de duyfteraersne vanden Eertcloot valt, deur vvijsconstighe vvercking ghegront op stelling eens vasten Eertcloots.

Anghen den inrontwech der drie bovenste Dwaelders deur den Eertcloot streckt, of ghenomen wort voor na ghenouch daer deur te strecken, maer vande twee onderste daer buyten, soo sal ick daer af twee voorbeelden beschrijven.

1 Voorbeelt vande drie bovenste Dwaelders.

T' G H E G H E V E N. Laet A B den duyfteraer beteyckenē overcant ghesien, diens middelpunt dats den vasten eertcloot C, waer deur ghetrocken is Saturnus inrontswech D E, streckende C D na de noortsijde, C E na de zuytsijde, sulcx dat de twee houcken A C D, B C E, des inrontwechs afwijcking bedien, en by sijn uyterste punt D na t'Noorden als middelpunt is beschreven het inront F G, ewewijdich metten duyfteraer A B, diens verstepunt F, naestpunt G, van t'welck getrocken sy de lini G C, en den houck A C G, doet als bevonden wiert inde ervaring des 20 voorstels des 3 boucx 3 tr. 2 ①: Sghelijcx is opt uyterste punt E na het Zuyden als middelpunt, beschreven het inront H I, ewewijdich metten duyfteraer A B, diens verstepunt I, naestpunt H, van t'welck getrocken sy de lini H C, en den houck B C H doet als bevonden wiert inde ervaring des voorschreven 20 voorstels 3 tr 5 ②. T' B E G H E E R D E. Wy moeten vinden des inrontwechs D E afwijcking vanden duyfteraer, dats den houck A C D.

T' W E R C K.

De driehouck C D G heeft drie bekende palen, te weten D G 10000 deur rgestelde, C G soo veel als ten tijde der ervaring dede de lini vanden Eertcloot tottet inronts naestpunt, t'welck deur t'vervolgh van het 13 voorstel des 3 boucx int ghemēen bekend wort, en besonderlick bevonden is int 21 voorstel des 3 boucx van 88718, of anders om gherieviger wercking machmen nemen de lini

angle AEI , being the Planet's apparent ecliptical latitude when seen from the fixed Earth E , is equal to the angle ACF , being the Planet's apparent ecliptical latitude when seen from the moving Earth C . But such as has here been proved when the moving Earth and the Planet are at their circles' uppermost and lowermost points, the same will also be apparent in any place, because just as in these two parallel circles CD , HI the line EI between the centre E of the lower circle and the point I on the circumference of the higher circle is equal and parallel to the line CF between its homologous opposite point C in the lower and the centre F of the higher circle, thus all such lines are always parallel by the 3rd proposition of this Supplement. CONCLUSION. On the theory of a fixed Earth the Planets thus acquire the same apparent ecliptical latitude that they have on the theory of a moving Earth; which we had to prove.

5th PROPOSITION.

Given a Planet's greatest northerly and southerly latitudes, to find its deferent's deviation from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth.

Since the deferent for the three upper Planets passes through the Earth, or is assumed to pass through it approximately, but for the two lower Planets outside it, I will describe two examples thereof.

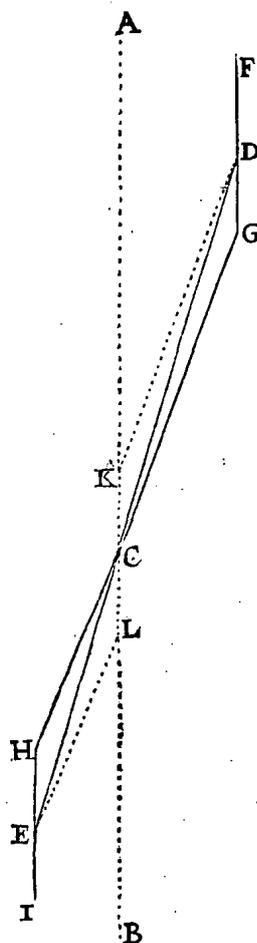
1st Example of the Three Upper Planets.

SUPPOSITION. Let AB denote the ecliptic, seen transversely, whose centre is the fixed Earth C , through which is drawn Saturn's deferent DE , CD tending towards the north side, CE towards the south side, so that the two angles ACD , BCE denote the deferent's deviation, and about its extremity D towards the North as centre has been described the epicycle FG , parallel to the ecliptic AB , its apogee being F , its perigee G , from which let there be drawn the line GC ; then the angle ACG , as was found in the experience of the 20th proposition of the 3rd book, makes $3^{\circ}2'$. Similarly, about the extremity E towards the South as centre has been described the epicycle HI , parallel to the ecliptic AB , its apogee being I , its perigee H , from which let there be drawn the line HC ; then the angle BCH , as was found in the experience of the aforesaid 20th proposition, makes $3^{\circ}5'$. WHAT IS REQUIRED. We have to find the deviation of the deferent DE from the ecliptic, *i.e.* the angle ACD .

PROCEDURE.

The triangle CDG has three known terms, to wit, $DG = 10,000$ by the supposition, CG as much as the line from the Earth to the epicycle's perigee was at the time of the experience, which by the sequel of the 13th proposition of the 3rd book becomes known generally and has been found in particular in the 21st proposition of the 3rd book to be 88,718 ¹⁾, or otherwise, with a view

¹⁾ This value is not to be found in Proposition 21 of the Third Book.



de lini CD, doende na ghenouch 10000 meer, dat 98718, en den houck DGC 176 tr. 58 ①, als even sijnde metten houck GCB, die halfrontschil is der bekende grootste breedte ACG 3 tr. 2 ①: Hier me gefocht den houck CDG, wort bevonden deur het 5 voorstel der platte driehoucken van 2 tr. 43 ①, twelck oock is voor den houck ACD begheerde afwijcking des wechs vanden duyfteraer, overeencommende mette 2 tr. 43 ① die int 22 voorstel des 3 boucx ghevonden wierden. Ick doe daer na dergelijcke wercking over d'ander sijde, fouckende den houck BCE, tot dien eynde aldus segghende: De driehouck CHE heeft drie bekende palen, te weten EH 10000 deur t'ghestelde, CH 75210 deur t'vervolgh van het 13 voorstel des 3 boucx, en den houck CHE 176 tr. 55 ①, als even sijnde mettē houck ACH, die halfrontschil is vande bekende grootste breedte 3 tr. 5 ①: Hier me ghesocht den houck HEC, wort bevonden deur het 6 voorstel der platte driehouckē van 2 tr. 43 ①, t'welck oock is voor den houck BCE, even vallende mettē boveschreven houck ACD oock van 2 tr. 43 ①, sulcx dat volghens dese rekening GH deur den vasten Eertcloot C streckt. Maer by aldien foodanighe evenheyt niet ghecommen en waer, ghelijckt mette twee onderste Dwaelders ghebeurt, men soude dan volghen de wijze des nabeschreven 2 voorbeelts.

I M E R C K

Noch is te weten dat Saturnus inrontwechs afwijcking vanden duyfteraer met stelling eens vasten Eertcloots can gevonden worden opeen ander wijze, welke alsoose noch opentlicker verclaert de ghemeenschap der twee stellinghen eens

vasten en roerenden Eertcloots, soo sal ick die met een beschrijven. Laet als by manier van bereytsel ghetrocken worden DK, even en ewewijdeghe met GC: Sghelijcx EL, even en ewewijdeghe met HC, t'welck soo sijnde de cruyf vierhouck KDEL, is even en ghelijck metten cruyf vierhouck ADEB des 22 voorstels vant 3 bouck, daerom hier me ghesocht des inrontwechs CD afwijcking vanden duyfteraer na de manier des selven 22 voorstels, t'ghene daer uyt comt is t'begeerde, en moet nootsakelick even sijn mettē besluyt vande voorgaende eerste wercking. En blijkt hier me sichtbaerlick hoe de form der stelling eens roerenden Eertcloots, te weten den cruyf vierhouck KDEL, metten Eertcloodwech KL, een selve besluyt voortbrenghet als de form der stelling eens vastē Eertcloots C, en haer inront ter eender sijde GDF, ter ander HEI, want den driehouck CDG is even en gelijk metten driehouck DCK, alsoo oock is CEH met ECL, waer uyt volghet dat alsulcken afwijcking als van CD bevonden wort deur de bekende palen des driehoucx DCK, foodanighe moeder oock bevonden worden deur de bekende palen des driehoucx CDG. Oock is

E c 3 s ghe-

to a more convenient procedure, the line CD may be taken, which is approximately 10,000 more, *i.e.* 98,718; and the angle $DGC = 176^{\circ}58'$ as being equal to the angle GCB , which is the supplement of the known greatest latitude $ACG = 3^{\circ}2'$. When the angle CDG is sought therewith, it is found by the 5th proposition of plane triangles to be $2^{\circ}43'$, which is also the value of the angle ACD , the required deviation of the deferent from the ecliptic, corresponding to the $2^{\circ}43'$ found in the 22nd proposition of the 3rd book. I then perform a similar operation on the other side, seeking the angle BCE , saying to this end as follows. The triangle CHE has three known terms, to wit, $EH = 10,000$ by the supposition, $CH = 75,210$ by the sequel of the 13th proposition of the 3rd book, and the angle $CHE = 176^{\circ}55'$, as being equal to the angle ACH , which is the supplement of the known greatest latitude $3^{\circ}5'$. When the angle HEC is sought therewith, it is found by the 6th proposition of plane triangles¹⁾ to be $2^{\circ}43'$, which is also the value of the angle BCE , which is equal to the above-mentioned angle ACD , also $2^{\circ}43'$, so that by this calculation GH passes through the fixed Earth C . But if such equality had not resulted, as is the case with the two lower Planets, the method of the following (2nd) example would have to be used.

NOTE.

It is also to be noted that the deviation of Saturn's deferent from the ecliptic on the theory of a fixed Earth can be found in a different manner, and since this makes the similarity of the two theories of a fixed and a moving Earth even clearer, I will describe it at the same time. By way of preliminary let there be drawn DK , equal and parallel to GC . Similarly EL , equal and parallel to HC . This being so, the crossed quadrilateral $KDEL$ is equal and similar to the crossed quadrilateral $ADEB$ of the 22nd proposition of the 3rd book. Hence, when the deviation of the deferent CD from the ecliptic is sought therewith in the manner of the said 22nd proposition, the result is the required value and must needs be identical with the result of the foregoing first procedure. And thus it is clearly evident that the figure of the theory of a moving Earth, to wit, the crossed quadrilateral $KDEL$, with the Earth's orbit KL , leads to the same result as the figure of the theory of a fixed Earth C , and its epicycle on one side GDF , on the other side HEI , for the triangle CDG is equal and similar to the triangle DCK , and so is CEH to ECL , from which it follows that the same deviation that is found for CD , from the known terms of the triangle DCK , must also be found

¹⁾ Stevin's *Trigonometry* (Work XI; i, 12) p. 156.

from the known terms of the triangle CDG . The same is also to be understood for the other two triangles. And such as has here been the example with Saturn, the same will it obviously also be with the other two upper Planets Jupiter and Mars.

2nd Example of the Two Lower Planets.

Although all the letters in the figure of this 2nd example have the same denotation as those of the first example, and it may be considered that the meaning of this example is sufficiently clear from the first, nevertheless, recognizing the difference of outward appearance, to wit, that here the deferent is much smaller than the epicycle, whereas in the 1st example the deferent was much larger, while moreover this deferent does not pass through the Earth, I will also describe this 2nd example.

SUPPOSITION. Let the parts of the following figure $ABCDEFGHIKL$ be of Mercury, with the same denotation as the parts of the figure of the first example of Saturn, but differing in that this deferent DE , which shall have been found in the manner of the procedure described in the 1st example, does not pass through the Earth C , but intersects the ecliptic in the point M . Secondly, whereas the epicycle of Saturn was much smaller than its deferent, it is much larger here instead, to wit, FG in the same ratio to DE as in the figure of the 1st proposition of this Supplement the line PQ to QR . Further the angle BCG makes $1^{\circ}45'$ and ACH $4^{\circ}5'$, as was found in the experience of Mercury's latitude, after the 25th proposition of the 3rd book. WHAT IS REQUIRED. We have to find the deviation of the deferent DE from the ecliptic, *i.e.* the angle AMD .

PROCEDURE.

From the three known terms of the triangle DCG is found, in the same manner as in the 1st example, the angle

And the angle

When to this is added the angle

We get the angle

When this is subtracted from 180° , there is left the angle

When to this is added the given angle

We get the angle

From the three known terms of the triangle ECH there is found, in the manner of the 1st example, the angle

When this is added to DCH (the seventh in this list), we get the angle

The triangle DCE has three known terms, to wit, that angle DCE (the ninth in the list) and the two lines CD , CE , from the Earth to the

CDG .

DCG .

BCG ,

BCD .

DCK .

KCH ,

BCD .

HCE .

DCE .

deur t'vervolgh van het 13 voorstel des 3 boucx: Hier me wort ghevonden den houck

CDE.

Daer toe vergaert dē houck CD G eerste in d'oirden, comt dē houck E DG, dats oock dē houck MD G, welcke evē sijnde met A MD, om dat MD is tusschen de twee ewewijdege KM, DG, soo is daer bekenet de begheerde afwijcking

AMD.

T'is oock kennelick hoe ghevonden sal worden de langde der lini MC, dat is soo verre de duyfteraersne vanden Eertcloodt valt, want den driehouck DCM heeft drie bekende palē, te weten dē houck CDM, als wesende den houck CDE achtste in d'oirden, voort den houck MCD, als even sijnde metten houck CDG eerste in d'oirden, om dat CD is tusschen de twee ewewijdeghe KC, DG, ten derden de sijde CD, met welcke drie palen als gheseyt is bekenet wort de lini

MC.

Noch is te weten dat Mercurius inrontwechs afwijcking vanden duyfteraer met stelling eens vasten Eertcloodts can gevonden worden op een ander wijse, welcke alsoofse noch opentlicker verclaert de gemeenschap der twee stellinghen eens vasten en roerenden Eertcloodts, soo sal ick die met een beschrijven. Laet als by manier van bereytsel ghetrocken worden DK, even en ewewijdeghe met GC: Sghelijcx EL even en ewewijdeghe met HC, t'welck soo sijnde, de cruyfvierhouck KDE L, is even en ghelijck metten cruyfvierhouck LOMP int 3 lidt van Mercurius breede achter het 25 voorstel des 3 boucx, daerom hier me ghesocht des inrontwechs MD afwijcking vanden duyfteraer na de manier des selven 3 lidts, t'ghene daer uyt comt is t'begheerde, en moet nootfakelick even sijn metten besluyt vande voorgaende eerste wercking. En blijkt hier me sichtbaerlick hoe de form der stelling eens roerenden Eertcloodts, een selve besluyt voortbrengt als de form der stelling eens vasten Eertcloodts, also van dergelijcke wat breeder gheseyt is achter het 1 voorbeelt.

En sulcx als hier gheweest is het voorbeelt met Mercurius, alsoo ist kennelick te sullen sijn met dander onderste Dwaelder Venus. T' BESLUYT. Wesende dan ghegheven eens Dwaelders meeste noordersche en zuydersche breedde, wy hebben gevonden sijn wechs afwijcking vanden duyfteraer: Oock mede hoe verre de duyfteraersne vanden Eertcloodt valt, deur wisconstighe wercking ghegront op stelling eens vasten Eertcloodts, na den eysch.

6 VOORSTEL.

Tevinden eens Dwaelders schijnbaer duyfteraerbreedte op een ghegheven tijt, deur vvifconstighe vvercking gegront op stelling eens vasten Eertcloodts.

Om t'begheerde te krijghen, wy sullen eerst vinden de lini vanden Dwaelder int inront rechthouckich op den duyfteraer. Als by gelijckenis inde form des 5 voorstels deses Byvoughs, de verdochte lini van Frechthouckich opt plat des duyfteraers AB: Maer sulcke lini is tot allen plaetsen des inronts, even an de lini van des inronts middelpunt als D, rechthouckich op AB, deur diē DF ewewijdeghe is met AB, en daerom salmen dese altijt vinden in plaets van die, t'welck doende soo en sal de manier van t'vinden der selve geen verschil hebben, mette vinding van dergelijcke lini in stelling eens roerenden Eertcloodts

Ee 4. beschre-

epicycle's centre, which become known from the sequel of the 13th proposition of the 3rd book.

Herewith is found the angle

CDE.

When to this is added the angle CDG (the first in the list), we get the angle EDG , *i.e.* also the angle MDG , and since this is equal to AMD , because MD is between the two parallel lines KM , DG , there is known the required deviation

AMD.

It is also obvious how the length of the line MC has to be found, *i.e.* as far as the line of nodes is from the Earth, for the triangle DCM has three known terms, to wit, the angle CDM , which is also the angle CDE (the eighth in the list); further the angle MCD , which is equal to the angle CDG (the first in the list) because CD is between the two parallel lines KC , DG ; thirdly, the side CD , by means of which three terms, as has been said, is found the line

MC.

It is also to be noted that the deviation of Mercury's deferent from the ecliptic on the theory of a fixed Earth can be found in a different manner, and since this makes the similarity of the two theories of a fixed and a moving Earth even clearer, I will describe it at the same time. By way of preliminary let there be drawn DK , equal and parallel to GC . Similarly EL equal and parallel to HC . This being so, the crossed quadrilateral $KDEL$ is equal and similar to the crossed quadrilateral $LOMP$ in the 3rd section of Mercury's latitude, after the 25th proposition of the 3rd book¹⁾. Hence, when the deviation of the deferent MD from the ecliptic is sought therewith after the manner of the said 3rd section, the result is the required value and must needs be identical with the result of the foregoing first procedure. And from this it is clearly evident that the figure of the theory of a moving Earth leads to the same result as the figure of the theory of a fixed Earth, as has been said somewhat more fully in a similar case after the 1st example.

And such as has here been the example with Mercury, the same will it obviously be with the other lower Planet Venus. CONCLUSION. Given a Planet's most northerly and southerly latitudes, we have thus found the deviation of its deferent from the ecliptic; also how far the line of nodes is from the Earth, by mathematical operations based on the theory of a fixed Earth; as required.

6th PROPOSITION.

To find a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth.

In order to get the required value, we first have to find the line from the Planet in the epicycle perpendicular to the ecliptic; for example, in the figure of the 5th proposition of this Supplement, the imagined line from F perpendicular to the plane of the ecliptic AB . But such a line is in any place of the epicycle equal to the line from the epicycle's centre (D) perpendicular to AB , because DF is parallel to AB , and therefore the one will always be found instead of the other. When doing so, the method of finding it will not be different from the finding of a similar line, described in the theory of a moving Earth in the 24th proposition of the 3rd book. The cause of this similarity is even more

¹⁾ In the figure on page 248.

beschreven int 24 voorstel des 3 boucx: De oirfaeck deser ghelijckheyt is daer deur noch kennelicker, dat den Dwaelder met stelling eens roerenden Eertcloots, comt ter plaets van des inronts middelpunt met stelling eens vasten Eertcloots. Nu dan sulcke lini alsoo bekent wordende, en daer benevens deur t'vervolgh vant 13 voorstel des 3 boucx de lini vanden vasten Eertclood totten Dwaelder, soo heeft den rechthouckighen driehouck begrepen onder de selve *Hypothensse* twee linien, en de derde * schoensche drie bekende palen, waer me opbaerlick ghevonden wort de begheerde breedte als int 25 voorstel des 3 boucx met stelling eens roerenden Eertcloots, sulcx dat my onnoodich ghedocht heeft het selve hier andermael int langhe te beschrijven. T B E S L V Y T. Wy hebben dan ghevonden eens Dwaelders schijnbaer duyfteraerbreede op een ghegheven tijt, deur wisconfighe wercking ghegront op stelling eens vasten Eertcloots, na den eyfch.

S A M I N G

Van ettelicke overeencommingen en verschillen tusschen de beschrijving des breedeloops deses Byvoughs, mette breedeloop van *Ptolemeus*.

Alsoo my inde voorgaende beschrijving deser stof des breedeloops, somwijlen voortvielen eenighe verscheydenheden tusschen de selve en de breedeloop by *Ptolemeus* beschreven, soo heeft my bequamer ghedocht die hier by den anderen te versamen, dan inde voorgaende leering te vermenghen, sulcx dat ick daer af stellen sal de volghende leden.

I L I D T.

Anghesien d'oirfaeck des breedeloops der Dwaelders ghetrocken wort nyt stelling eens roerenden Eertcloots die tot *Ptolemeus* tijt onbekent schein, soo heeft hy nochtans int veroirdenen der spiegheling vande drie bovenste, de sake al seer na ghecommen, stellende het inront te wijle het loopt altijt bycans ewewijdich vanden duyfteraer te blijven, want soo hy dat gelijk de sake vereyscht, en int 2 voorstel des Byvoughs bewesen is te moeten sijn, voor heel ewewijdich ghenomen hadde (t'schilde als blijkt int 3 Hoofstlick sijns 13 boucx in Saturnusloop alleeneclck 2 tr. 4 ⊕), dat den houck gemaect vant inront mette wech, ten alderhoochsten grooter was dan den houck gemaect vande wech metten duyfteraer: In Iupiter was sulck verschil alleeneclck van 1 tr. 6 ⊕, in Mars van 1 tr. 15 ⊕, ja het inront anden duyfteraer sijnde, t'waffer volgens sijn stelling teene mael in sonder snyen, ghelijck na de volcommen spiegheling sijn moet) soo en waer de tastende onseker wercking int soucken vande afwijcking der inrontswegen beschreven int selve 3 Hoofstlick niet noodich gheweest, en souden d'ander rekeninghen van der Dwaelders breeden, dan deurgaens overeenghecommen hebben met dese, sonder daer af inde rest soo te verschillen als int volghende 2 lidt gheseyt sal worden.

2 L I D T.

Ptolemeus tafel van Saturnus breedeloop int 5 voorstel sijns 13 boucx, en schijnt geen genouchsaem overeencomming te hebben met sijn spiegheling. Om van t'welck breeder verclaring te doen, soo is te weten dat hy int 4 voorstel vant

obvious from the fact that on the theory of a moving Earth the Planet comes at the place of the epicycle's centre on the theory of a fixed Earth. Hence, this line thus becoming known, and in addition, from the sequel of the 13th proposition of the 3rd book, also the line from the fixed Earth to the Planet, the right-angled triangle contained between these two lines and the third (the hypotenuse) has three known terms, with which the required latitude is evidently found, as in the 25th proposition of the 3rd book on the theory of a moving Earth, so that I deemed it unnecessary to describe this here once more at length. CONCLUSION. We have thus found a Planet's apparent ecliptical latitude at a given time, by mathematical operations based on the theory of a fixed Earth; as required.

COLLECTION.

Of some similarities and differences between the description of the motion in latitude according to this Supplement and the motion in latitude according to *Ptolemy*.

Since in the foregoing description of this subject matter of the motion in latitude I sometimes came across some differences between it and the motion in latitude described by *Ptolemy*, it seemed more suitable to me to collect them here with the others than to mix them with the foregoing subjects, so that I will give thereof the following sections.

1st SECTION.

Since the cause of the motion in latitude of the Planets is derived from the theory of a moving Earth, which seemed unknown in *Ptolemy's* time, nevertheless in framing the theory of the three upper Planets he came very near to the matter, assuming the epicycle, while moving, to remain always nearly parallel to the ecliptic; for if — as the matter requires and has been proved to be true in the 2nd proposition of the Supplement — he had taken it to be perfectly parallel (as appears in the 3rd Chapter of his 13th book, the difference for Saturn's motion was only $2^{\circ}4'$ by which amount the angle made by the epicycle with the deferent was greater at most than the angle made by the deferent with the ecliptic; with Jupiter this difference was only $1^{\circ}6'$, with Mars $1^{\circ}15'$, nay, when the epicycle was in the ecliptic, according to his assumption it fell altogether in it, without intersecting it, as it must be according to the perfect theory), the tentative, uncertain procedure in the seeking of the deviation of the deferents, described in the said 3rd Chapter, would not have been necessary, and the other calculations of the Planets' latitudes would generally have agreed with the present calculations, without further differing therefrom as much as is to be mentioned in the following (2nd) section.

2nd SECTION.

Ptolemy's table of Saturn's motion in latitude in the 5th chapter 1) of his 13th book seems not to be sufficiently in agreement with his theory. To explain this more fully, it is to be noted that in the 4th chapter of the said 13th book he takes the deferent's deviation from

1) Stevin designates *Ptolemy's* chapters by the same name of *Voorstel* which he also uses for the propositions.

- vant selfde 13 bouck, des inrontwechs afwijking vanden duyfteraer, die inde eerste form vant 5 voorstel deses Byvoughs beteykent sy metten houck A C D neemt op 2 tr. 26 ①.
- Den houck C D G op 4 tr. 30 ①.
- Het halfrontschil van dien doet voor den houck C D F 175 tr. 30 ①.
- By aldien nu vanden Eertcloot C, tot des inronts verstepunt F, wese sende mettet middelpunt an sijn wechs verstepunt D, getrocken waer de lini C F, sy soude doen deur de Byeenvouging des 13 voorstels vant 3 bouck 108718.
- Waer me sulcke driehouck C D F drie bekende palen soude hebbē, te weten dē houck C D F 175 tr. 30, derde in d'oirden, D F 10000, en C F 108718 vierde in d'oirden: Hier me ghefocht den houck D C F, wort bevondē deur het 5 voorstel der platte driehouckē vā 25 ①.
- Die ghetrocken vanden houck A C D 2 tr. 26 ① eerste in d'oirden, blijft voor den houck A C F 2 tr. 1 ①.
- En soo veel moet volghens *Ptolemews* spiegeling wesen Saturnus breede als hy is an des inronts verstepunt F, met des inronts middelpunt an sijn wechs verstepunt D. Maer volgens t'gebruyck der boveschrevē tafels, so is hy dan sonder breede, waer deur die tafels soo veel van haer spiegheling verschillen.
- En doēde dergelijcke op d'ander sijde als Saturnus is ant inrōts verstepunt I, men sal sijn breede bevinden van 1 tr. 38 ①, in welcker plaets hy volghens de tafels sonder breede bevonden wort. Inder vougen dat, volghens t'ghene ick voorgenomen hadde te verclarē, *Ptolemews* tafel van Saturnus breedeloop geē genouchsaem overeencomming en schijnt te hebbē met sijn spiegelingen. En dergelijcke is oock te verstaen vande tafels en spiegelingē der Dwaelders Iupiter en Mars. Maer hoe groot soodanighe breedten vallen met stelling eens roerenden Eertcloots, dats openbaer deur het 25 voorstel des 3 boucx.

3 L I D T.

Int 1 lidt is geseyt dat *Ptolemews* niet verre vande rechte spiegheling en was, stellende de inronden vande drie bovenste Dwaelders altijt bycans ewewijdich vanden duyfteraer te blijven, maer sulcx en is mette twee onderste soo na niet geluckt, waer af hy uyt sijn gageslagen ervaringen niet sulcken wijze van spiegeling trecken en conde, als hy uyt de drie bovenste gedaen hadde: De voornamelicste oorfaeck die hem verhinderde schijnt tweederley, d'eerste dusdanich: T'gene hy Mercurius inront noemide en daer voor gebruyste en wast niet deur het 1 voorstel deses Byvoughs, maer veel cleender dan na t'behooren, sulcx dat als hy t'selve al prouvende annam voor altijt bycans ewewijdich vanden duyfteraer te blijven, gelijk hy mette inronden der drie bovenste dede, dat en hadde mette ervaringen geen gemeenschap, want het was t'ander groot ront datmen daer toe nemen moest. D'ander oorfaeck te weten die hem de waggeling des inrontwechs dede besluyten, en de duyfteractie op een verkeerde sijde nemen, schijnt dar dusdanich was: Hy heeft sich in gebeelt alle gemeene sneen der inrontwegen en des duyfteraers deur den Eertcloot te strecken: Als by voorbeeld dē inrontwech D E, die inde form des 5 voorstels deses Byvoughs dē duyfteraer eygentlick in Mdeursnijt, heeft hy ghemeent te strecken deur den vasten Eertcloot C, waer op hy sijn rekeningē makende, soo isser uyt gevolght dat doen hy meende het inronts middelpunt gecommē te wesen van D tot C, en alsdan behooren inden duyfteraer A B te wesen sonder breede, soo bevant hij metter dact

the ecliptic, which in the first figure of the 5th proposition of this Supplement ¹⁾ is denoted by the angle ACD , to be

2°26'

The angle CDG to be

4°30'

The supplement thereof (the angle CDF) makes

175°30'

Hence, if from the Earth C to the epicycle's apogee F , being with the centre at its deferent's apogee D , the line CF were drawn, by the Compilation of the 13th proposition of the 3rd book it would make ²⁾

108,718

Thus, this triangle CDF would have three known terms, to wit, the angle $CDF = 175°30'$ (the third in the present list), $DF = 10,000$, and $CF = 108,718$ (the fourth in the list). When the angle DCF is sought therewith, by the 5th proposition of plane triangles it is found to be

25'

When this is subtracted from the angle $ACD = 2°26'$ (the first in the list), there is left for the angle ACF

2°1'.

And this, according to *Ptolemy's* theory, must be the amount of Saturn's latitude when it is at the epicycle's apogee F , with the epicycle's centre at its deferent's apogee D . But according to the use of the above-mentioned tables it is then without latitude ³⁾, owing to which these tables differ so much from the theory.

And when we do the same on the other side, when Saturn is at the epicycle's apogee I , its latitude will be found to be $1°58'$, whereas according to the tables it is found without latitude, so that, in accordance with what I had intended to explain, *Ptolemy's* table of Saturn's motion in latitude seems not to be sufficiently in agreement with his theories ⁴⁾. And the same is also to be understood for the tables and theories concerning the Planets Jupiter and Mars. But the amounts of these latitudes on the theory of a moving Earth are evident from the 25th proposition of the 3rd book.

3rd SECTION.

In the 1st section it has been said that *Ptolemy* was not far from the true theory in assuming the epicycles of the three upper Planets to remain always nearly parallel to the ecliptic, but he did not succeed as well with the two lower Planets, for which he could not deduce from his observational experiences the same kind of theory as he had done for the three upper Planets. The chief cause that prevented him from doing so seems to be of a twofold nature, the first being as follows. What he called Mercury's epicycle (and used as such) was not so by the 1st proposition of the present Supplement, but much smaller than it ought to be, so that when, testing it, he assumed this to remain always nearly parallel to the ecliptic, as he did with the epicycles of the three upper Planets, this did not agree with the experiences, because it was the other greater circle that ought to be taken for it. The other cause, to wit, the one that made him deduce the oscillation of the deferent and take the line of nodes on the wrong side, seems to have been as follows. He imagined all the intersections of the deferents and the ecliptic to pass through the Earth. For example, he thought

¹⁾ Page 270.

²⁾ This value is not found in the 13th proposition. Compare footnote to page 269.

³⁾ ⁴⁾ Stevin is wrong here, because he overlooks *Ptolemy's* remark that before entering the table of the sixtieths the argument has to be increased by $50°$ (for Saturn; decreased by $20°$ for Jupiter).

330 BYVOUGH DES BREEDELOOPS.

daet daer buyten an N (welverstaende dat DN even geteyckent is an DC) met soo veel breede als veroirsaecht wort deur de verhey van C tot N, welke misgrijping hem heeft doē des inrontwechs waggeling versieren, die beschreven is int 2 lidt van Mercurius breede achter het 25 voorstel des 3 boucx.

4 L I D T.

Maer angesien *Ptolemus* de voorschreven waggeling des Dwaelders van C tot N, seyde deur ervaring in Mercurius bevondē te hebben van 45 ①, soo sulle wy nu onderfouckē hoe sulcx hier me overeencomt: Tot dien einde segh ick dat de verhey van C tot N, anghewesen wort mette lini NR inde form des 3 lidts van Mercurius loop achter het 25 voorstel des 3 boucx, diens langde ick aldus vinde: De driehouck QNR heeft drie bekende palē, te weten dē houck NQR 5 tr. 32 ①, als even sijnde met OQL 5 tr. 32 ① deur des voorschrevē 3 lidts vierde in d'oiden, QN 1364 deur des selfden lidts vijfde in d'oiden, en den houck QNR recht: Hier me ghesocht de lini NR, wort bevonden deur het 4 voorstel der platte driehoucken van 132.

By aldien my *Ptolemus* verclaert hadde, tot wat plaets de Son was ten tijde van sijn dadelicke ervaringē doen hy de afweging van 45 ① vandt, waer deur ons met stelling eens roerenden Eertcloots bekend soude sijn des selven Eertcloots plaets, soo mochten wy na die overeencommingen met meerder sekerhey trachten, maer dat niet gheschiet sijnde, wy sullen de twee uyterste afwijkinghen soucken om te sien ofter de 45 ① oock tusschen vallen, als volght:

Doende de lini NR als vooren 132, en datmen daer an vervought na de manier des 25 voorstels int 3 bouck de meeste verhey van Mercurius totten Eertclood, doende deur de Byenvouging des 13 voorstels vant 3 bouck 14519, soo wort deur de gemeene regel des selfden 25 voorstels Mercurius breede daer me bevonden van 31 ①: Maer van 1 tr. 21 ① op de minste verhey 5481, tusschen welke 1 tr. 21 ① en 31 ①, de boveschreven 45 ① der ervaring sijn. Soo nu bekend waer de lini vanden Eertclood tot Mercurius ten tijde der ervaring, mē soude als geseyt is van dese overeencomming noch eygentlicker connē spreke.

Angaende Venus afweging welke *Ptolemus* stelt op 10 ①, die soude na de voorgaende wijze van Mercurius ten cleensten bevonden worden van 20 ①: Watter eyghentlick af is, daer soudemen deur nieuwe dadelicke gaflaginghen nauwer af connen oordeelen.

B E S L V Y T D E S B R E E D E L O O P S.

Tot hier toe is vande breedeloop der vijf Dwaelders Saturnus, Inpiter, Mars, Venus en Mercurius, beschreven t'ghene ick voor my ghenomen hadde, waer me ick oock meyne de selve niet meer voor soo een onbekent roersel behouren ghenomen te worden ghelijckmen ghedaen heeft, als blijkende deur des Eertcloots loop sulcke verscheydenheden nootfakelick te moeten vallen, en elck Dwaelders hemel eenvoudelick te dracyen op haer as, sonder datmen behoufs eenich onnauwerlick roersel daer by te versieren, en daimen al die voorgaende tijtslijtighe hapseling sal moghen verlaten, oock met oirfakelicker kennis sich voortatn daer in oefenen.

A N-

that the deferent DE , which in the figure of the 5th proposition of the present Supplement really intersects the ecliptic in M , passes through the fixed Earth C , and when he based his calculations thereon, it followed that when he thought the epicycle's centre had moved from D to C , and then ought to be in the ecliptic AB , without any latitude, in actual fact he found it outside, at N (it being understood that DN has been drawn to be equal to DC), with so much latitude as is caused by the distance from C to N ; this misconception made him imagine the deferent's oscillation, described in the 2nd section of Mercury's latitude, after the 25th proposition of the 3rd book.

4th SECTION.

But since *Ptolemy* said that for Mercury he had found the aforesaid oscillation of the Planet from C to N by experience to be $45'$, we will now examine how this agrees therewith. To this end I say that the distance from C to N is denoted by the line NR in the figure of the 3rd section of Mercury's motion, after the 25th proposition of the 3rd book, whose length I find as follows. The triangle QNR has three known terms, to wit, the angle $NQR = 5^{\circ}32'$, as being equal to $OQL = 5^{\circ}32'$, by the fourth in the list of the aforesaid 3rd section, $QN = 1,364$ by the fifth in the list of the said section, and the angle QNR being a right angle. When the line NR is sought therewith, by the 4th proposition of plane triangles it is found to be 132.

If *Ptolemy* had told me at what place the Sun was at the time of his practical experiences, when he found the deviation of $45'$, from which on the theory of a moving Earth we should know the place of the said Earth, we might try with greater certainty to find those correspondences, but since this has not happened, we have to seek the two extreme deviations, to see whether the $45'$ fall in between, as follows: When the line NR makes, as above, 132 and to this is added, after the manner of the 25th proposition in the 3rd book, the greatest distance from Mercury to the Earth, which by the Compilation of the 13th proposition of the 3rd book makes 14,519, by the common rule of the said 25th proposition Mercury's latitude is therewith found to be $31'$. But this would be found to be $1^{\circ}21'$ at the smallest distance 5,481, and between this $1^{\circ}21'$ and $31'$ lie the aforesaid $45'$ of the experience. If now the line from the Earth to Mercury at the time of the experience were known, it would be possible, as has been said, to speak more truly of this correspondence.

As to Venus' deviation, which *Ptolemy* takes to be $10'$, by the foregoing method for Mercury this would be found to be at least $20'$. The true state of affairs might be judged more accurately by means of new practical experiences.

CONCLUSION OF THE MOTION IN LATITUDE.

Up to this point that part of the motion in latitude of the five Planets Saturn, Jupiter, Mars, Venus, and Mercury which I had intended has been described, and therefore I am also of opinion that this should no longer be considered such an unknown motion as it has been done, since it appears that from the Earth's motion such differences are inevitable, and that the Heaven of each Planet simply turns about its axis, without our having to invent any unnatural motion, and that all the foregoing time-devouring complications can be abandoned, and the matter can henceforth be practised with better knowledge of the causes.

A N H A N G H

DES HEMELLOOPS

V A N D E R D W A E L D E R S

onbekende roersels by *Ptolemeus* gage-

slagen : En vande spieghelingen by

hem en *Copernicus* daer uyt

beschreven.

C O R T B E G R Y P D E - S E S A N H A N G S .

Nadien *Ptolemeus* ter hande ghecommen was het Hemel-
loopchrift met stelling eens vasten Eertcloots, eenvoudich ge-
lijckt int voorgaende verclaert is, soo heeft by der *Dwael-*
ders plaetsen en loopen seer neerstelicken gageslagen en onder-
socht, om te sien hoe se daer me overeenquamē. De beschrijving des loops der
Son (daer *Hypparchus* an *ruiffelde*) docht hem recht, maer niet van
d'ander ses *Dwaelders*, want hoer vel hemlien middelloop op seer langhe
zijt effen genouch uyt quam, soo oordeelde hy nocht ans inde besonder keeren
verscheydenheit te wesen, sulcx dat hy daer af sijn spiegheling beschreef,
en die vermengde onder de voornoemde eenvoudige loop mette stelling eens
vasten Eertcloots. En dergelijcke vermenging heeft daer na *Copernicus*
van sijn spiegeling oock gedaen mette stelling eens roerendē Eertcloots. Maer
want de plaetsen der *Dwaelders* daer na gageslagen, niet bevonden en
worden met die regels overeen te commen, en dat daerom die roersels als
noch onbekent schijnen, soo heb ick se int voorgaende in d' een en d' ander stel-
ling weerom uyt be bekende ghescheyden : Maer op dat men eyghentlicker
weten soude hoedanich die uytghecheyden byvooging was, soo is om de-
se redenen en noch ander die int voorgaende breeder verclaert sijn, daer
af desen Anhang gemaeft, die derthien voorstellen sal hebben, wesen de
seven van *Ptolemeus* bygevoughde spiegheling, te weten d' eerste vijf
vande Maen. Het sesste van *Saturnus*, *Jupiter*, *Mars* en *Venus*. Het se-
vende van *Mercurius*. D' ander vijf sijn van *Copernicus* bygevough-
de spiegeling te weten het 8 vande Maen. Het 9 van *Saturnus*, *Jupiter* en
Mars. Het 10 van *Venus*. Het 11 en 12 van *Mercurius*. Het 13 een ver-
bael op der sterren onbekende loop, en des duysteraers onbekende afwijking
vanden evenaer.

E E R S T

APPENDIX

TO THE HEAVENLY MOTIONS

OF THE PLANETS' UNKNOWN MOTIONS

observed by Ptolemy; and the Theories
derived therefrom by him and Copernicus

SUMMARY OF THIS APPENDIX

After the description of the Heavenly Motions on the assumption of a fixed Earth had come into the hands of *Ptolemy*, in the simple form in which it has been set forth in the foregoing, he very industriously observed and investigated the Planets' positions and motions, so as to see in how far they agreed therewith. The description of the motion of the Sun (which *Hipparchus* doubted) appeared to him to be correct, but not that of the other six Planets, for though their mean motion accorded well enough in a very long time, yet he was of opinion that there was some difference in the separate revolutions, so that he described his theory about this and combined it with the aforesaid simple motion on the assumption of a fixed Earth. And a similar combination was also made thereafter by *Copernicus* for his theory on the assumption of a moving Earth. But because the positions of the Planets observed thereafter are not found to agree with those rules, so that those motions still seem unknown, I have separated them again from the known motions in the foregoing, on one as well as the other assumption. But in order that the reader might know more truly of what nature those separated additions were, for this reason as well as for others, which have been set forth more fully in the foregoing, this Appendix has been made about them. It is to comprise thirteen propositions, seven of them relating to the theory added by *Ptolemy*, to wit, the first five of the Moon, the sixth of Saturn, Jupiter, Mars, and Venus, the seventh of Mercury. The other five relate to the theory added by *Copernicus*, to wit, the 8th of the Moon, the 9th of Saturn, Jupiter, and Mars. The 10th of Venus. The 11th and 12th of Mercury. The 13th is an account of the stars' unknown motions and the ecliptic's unknown deviation from the equator.

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EERST PTOLEMEVS BYGHE-
voughde spiegeling des Dvvaelders met
stelling eens vasten Eertcloots.

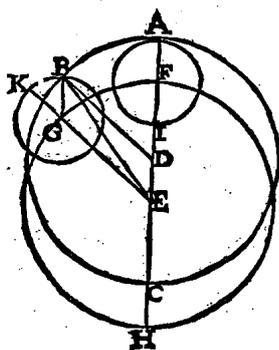
I VOORSTEL.

Wesende een Dvvaelder gestelt in een uytmiddelpuntichront, of anders in een inront diës halfmiddellijn even is an dese uytmiddelpuntichronts uytmiddelpunticheytlijn, en dat inront in een middelpuntichront, even in grootheyt en loopant uytmiddelpuntich, en des Dvvaelders loop int inront ghelijck metten loop des inronts int middelpuntichront, doch op een verkeerde sijde: Die twee stellinghen gheven den Dvvaelder een selve plaets.

Anghesien de stelling der Maen in een uytmiddelpuntighe wech na de manier inde voorgaende twee boucken beschreven, en volghens t'ghene inde natuur schijnt te bestaen, heur de selve plaets gheeft diese crijcht met een middelpuntich inront op de wijze deses voorstels, die *Ptolemeus* vercoos om sijn ghevonden tweede oneventheden bequamelicker te verclaren, en dat ick de selve oneventheden hier beschrijven wil, soo heeft my voughelick ghedocht eerst te bewijfen (ghelijck *Ptolemeus* oock gedaen heeft int 5 Hoofstuck sijns 4 boucx) sulcke twee stellingen alsoo te overcommen, t'welck d'oirsaek is der beschrijvinghen van dit voorstel tot dese plaets.

T'GHEGHEVEN. Laet voor eerste stelling het uytmiddelpuntichront *ABC* de Dwaelderwech beteyckenen, diens middelpunt *D*, en *E* den vasten Eertcloot, t'punt *A* sy den Dwaelder ten eersten ant verstepunt, welke daer na ghedaen hebbe een loop van *A* tot *B*, dats oock den houck *ADB*.

Laet nu voor tweede stelling op *E* als middelpunt, beschreven worden het middelpuntichront *FGH*, even an *ABC*, en op *F* mette halfmiddellijn *FA*



(die even moet vallen met *ED*) het inront *AI*, diës verstepunt *A*, daer na sy het inronts *AI* middelpunt *F* ghecommen an *G*, sulck dat sijn loop *FG*, of houck *FE G*, even sy anden loop der eerster stelling *ADB*, en het inront beschrevē opt middelpunt *G*, sy *BK*, waer in van *E* deur *G* ghetrocken sy *E G K*, soo dat *K* des inronts verstepunt beteyckent, van t'welck den Dwaelder daerentusschen ghelooopen heb na *B* (tegen d'eerste loop van *F* na *G*) een booch ghelijck met *FG*, of anders geseyt sulcx dat den houck begrepen tusschen de lini *K G*, en de lini van *G* na *B* tot inden omtreck des inronts, welke lini men

crijcht treckende van *G* een ewewijdeghe met *EF*. T'BEGHEERDE. Wy moeten bewijfen dat dē Dwaelder in dese tweede stelling int selve punt *B* valt, daer

FIRST THE THEORY OF THE PLANETS,
ADDED BY PTOLEMY, ON THE ASSUMPTION
OF A FIXED EARTH

1st PROPOSITION.

When a Planet is placed on an eccentric circle or otherwise on an epicycle whose semi-diameter is equal to the line of eccentricity of this eccentric circle, while that epicycle is on a centric ¹⁾ circle, equal in magnitude and motion to the eccentric circle, and the Planet's motion on the epicycle is equal to the motion of the epicycle on the centric circle, but in an opposite direction, those two locations give the Planet the same place.

Since the location of the Moon in an eccentric orbit, after the manner described in the foregoing two books and in accordance with what seems to exist in nature, gives it the same position it gets with a centric epicycle ²⁾ in the manner of the present proposition, which *Ptolemy* chose in order to explain more conveniently the second inequalities found by him, and since I here wish to describe these inequalities, it seemed appropriate to me first to prove (as *Ptolemy* has also done in the 5th Chapter of his 4th book) that these two locations agree, which is the cause of the descriptions of this proposition in this place.

SUPPOSITION. For the first location let the eccentric circle ABC denote the Planet's orbit, its centre being D , and E the fixed Earth; let the point A be the Planet first at the apogee, which thereafter shall have moved from A to B , that is also the angle ADB .

Now for the second location let there be described about E as centre the centric circle FGH , equal to ABC , and about F with the semi-diameter FA (which has to be equal to ED) the epicycle AI , its apogee being A . Thereafter let the centre F of the epicycle AI have reached G , so that its motion FG , or the angle FEG , be equal to the motion of ADB in the first location. And let the epicycle described about the centre G be BK , in which let there be drawn from E through G the line EGK , so that K denotes the epicycle's apogee, from which let the Planet meanwhile have moved to B (contrary to the first motion from F to G) through an arc equal to FG , or, in other words, equal to the angle contained between the line KG and the line from G to B as far as the circumference of the epicycle, which line is obtained by drawing from G a line parallel to EF . WHAT IS REQUIRED. We have to prove that in this second location the Planet falls

¹⁾ As contrasted with an eccentric circle.

²⁾ Meaning an epicycle whose centre describes a centric circle.

daer hy in d'eerste stelling was. T'BEWYS. Anghefien DB, EG twee even en ewewijdeghe halfmiddellijnen sijn, tusschen welke ED comt, en dat de lini van G na B even en ewewijdeghe met ED is, deur t'gegeven, soo moet de form begrepen tusschen de vier linien BD, DE, EG, en de lini van G na B ewewijdeghe met ED, dats de form BDEG een * ewewijdich vierhouck sijn, en vervolghens t'punt B, te weten den Dwaelder, is soo wel uysterste der lini GB int inront KB na d'eerste stelling, als uysterste der lini DB int uytmiddelpuntichront AB na d'eerste stelling, en vervolgens den Dwaelder gesien vanden Eertcloodt E an B int inront sonder uytmiddelpuntichront, of an B int uytmiddelpuntichront sonder inront, het is hem al tot een selve plaets ghesien. T'BE-SLVYT. Wefende dan een Dwaelder ghestelt &c.

M E R C K T.

Hoewel dese manier de Manens ware plaets oock anwijst, soo schijnt noch-rans datmense niet en behoort te ghebruycken, eensdeels om dat haer duyfter plecken die alijt na den Eertcloodt ghekeert staen, betuyghen datse in geen inront en draeyt, ten anderen om datmen met meer hapseling twee ronden stelt daer deur een can ghedaen worden: En daerom is by *Ptolemeus* int stellen des Sonloops, en by *Copernicus* des Eertcloodtloops, met reden het uytmiddelpuntichront vercoren, t'welck *Ptolemeus* int stellen des Maenloops oock soude genomen hebben, ten waer dat, soo hy seght int 3 Hoofstuck sijns 4 boucx, de stelling des inronts hem bequamcr viel om daer deur sijn voornemen der na-volghende tweede oneventheden te verclaren.

2 V O O R S T E L.

Te verclaren d'oirfaeck die *Ptolemeus* beveeghde tottet ondersoucken van sijn tvveede oneventheden der Maen, metsgaders de ghedaente van sijn byghevoughde spiegheling int ghemeen.

Alfoo *Ptolemeus* gheduerlick en seer ernstelick gaslouch de Manens schijnbaer duyfteraerlangden, heeft dickwils bevonden die te verschillen mette regelen hier vooren beschreven, en hem van sijn voorganghers naghelaten, welck hy d'eerste oneventheden noemt, sulcx dat hy daer af na sijn goetduncken verbetering ghedaen heeft, ende een tweede onevenheyt daer by vervought. Om hier af sijn meyning te verclaren ick seggh aldus: Hy heeft bevonden dat haer uysterste voorofachtringhen in saminghen en tegestanden der Son alijt waren van 5 tr. ghelijck int 30 voorstel des 2 boucx d'eerste onevenheyt mebrengt, maer daer buyten vielder verandering, en dat ten grootsten in vierdeschijn, alwaerse condé vallen van 7 tr. 40 ①, dats 2 tr. 40 ① meer als d'ander, en soo veel mochte int oordeel der toecommende Maenplaetsen feyl vallen, alsmē int rekenen alleenelick sach na d'eerste onevenheyt. Om nu te verclaren d'oirfaeck die hy stelt van dit verschil, soo is vooral te weten dat hy toite bequaemste uytlegging sijns voornemens (ghelijck hy seght int 5 Hoofstuck sijns 4 boucx) eerstelick ghenomen heeft de stelling der Maen niet in een uytmiddelpuntighe wech, ghelijck wy die hier vooren beschreven hebben, maer in een * middel-puntichront, dat is te loopen in een inront, en t'selve in een middelpuntighe wech, na de manier beschreven int 1 voorstel deses Anhangs.

E f L a e t

in the same point B where it was in the first location. PROOF. Since DB and EG are two equal and parallel semi-diameters, between which ED comes, and since the line from G to B is equal and parallel to ED , by the supposition, the figure contained between the four lines BD , DE , EG , and the line from G to B parallel to ED , *i.e.* the figure $BDEG$, must be a parallelogram, and consequently the point B , to wit, the Planet, is the extremity of the line GB on the epicycle KB according to the second¹⁾ location as well as the extremity of the line DB on the eccentric circle AB according to the first location, and consequently whether the Planet is seen from the Earth E at B on the epicycle without eccentric circle or at B on the eccentric circle without epicycle, it is always seen in the same place. CONCLUSION. When therefore a Planet is placed, etc.

NOTE.

Although this method also indicates the Moon's true position, nevertheless it seems it ought not to be used, in the first place because the Moon's obscure portions, which are always turned towards the Earth, show that it does not revolve in an epicycle, secondly because it is a greater complication to assume two circles if it can be done with one. And for this reason, when *Ptolemy* framed the theory of the Sun's motion and *Copernicus* that of the Earth's motion, the eccentric circle was rightly chosen, which *Ptolemy* would also have taken when framing the theory of the Moon's motion if it were not for the fact, which he mentions in the 5th Chapter of his 4th book, that the assumption of the epicycle was more convenient to him for explaining by this means his intention with regard to the subsequent second inequalities.

[The propositions 2 - 5, not reproduced here, concern the Moon's motion]

¹⁾ For *eerste* in the Dutch text read *tweede*.

De driehouck EBF heeft drie bekende palen, te weten EB 48 deel 31 ① eerste in d'oirden, EF 10 deel 19 ① deur het 3 voorstel des Anhangs, en de houck BEF 89 tr. 30 ①, als halfrontschil des gegeven houck DEB 90 tr. 30 ①: Hier me ghesocht den houck EBF, wort bevonden deur het 6 voorstel der platte driehoucken van

12 tr. 1.

Vande booch HIGK doende deur t bereytsel 333 tr. 12 ①, getrocken de booch HG 180 tr. blijft de booch GK, of houck GBK 153 tr. 12.

Daer toe den houck EBF 12 tr. 1 ① tweede in d'oirden, comt voor den houck EBK

165 tr. 13.

De driehouck EBK heeft drie bekende palen, te weten den houck EBK 165 tr. 13 ① vierde in d'oirdē, EB 48 deel 31 ① eerste in d'oirden, en des inronts halfmiddellijn BK 5 deelen 15 ①: Hier me ghesocht den begheerden houck der voordering BEK, wort bevonden deur het 6 voorstel der platte driehoucken van 1 tr. 25 ①, latet sijn

1 tr. 26.

T B E S L V Y T. Wy hebben dan ghevonden op een ghegeven tijt de Manens voorofachtring, deur wercking gegront op stelling van *Ptolemus* bygevoughde spiegheling, na den eyfch.

VERVOLGH.

Deur dit vinden der voorofachtring is openbaer hoe bekend sal worden de Manens schijnbaer duyfteraerlangde op een ghegeven tijt, want totte Middelmanens duyfteraerlangde vervoughde de voorofachtring men heeft t begerde. Merckt noch dat deur *Ptolemus* en anderen verscheyden tafels ghemaect sijn, om met lichicheyt te vinden t'ghene hier boven deur rekeninghen der platte driehoucken met meerder moeyte ghevonden wort, welcke tafels wy hier onbeschreven laten, als totte kennis vande gedaente deser tweede oneventheden of bygevoughde spiegheling der Maen van *Ptolemus* niet voordelick. Dit dan t'ghene sijnde t'welck wy vande selve tweede oneventheden voorghenomen hadden te segghen, sullen nu commen tot sijn onbekende oneventheden van d'ander Dwaelders.

6 VOORSTEL.

Te verclaren de somme van *Ptolemus* bygevoughde spiegheling des langdeloops van Saturnus, Iupiter, Mars en Venus.

Anghemerckt wy deser Dwaelders eenvoudighe loop ghelijckse *Ptolemus* ter handt quam voor bekend nemen, als int eerste en tweede bouck beschreven sijnde, so sal t'verclaren van sijn byvougung hier cort en licht vallen.

Voor al iste weten dat hy deser vier Dwaelders schijnbaer duyfteraerlangden, in saminghen en teghestanden der Middelfon als des inronts middelpunt wasan sijn wechs verstepunt of naestepunt, altijd bevant ghelijck me brocht de rekening des eenvoudighen loops soose hem eerst ter handt ghecommen was, want hy in yder Dwaelder op sulcke stelling vant en berekende de langde der nymiddelpunticheytlijn, en des inronts halfmiddellijn, maer daer buyten viel der verandering.

Om dan

6th PROPOSITION.

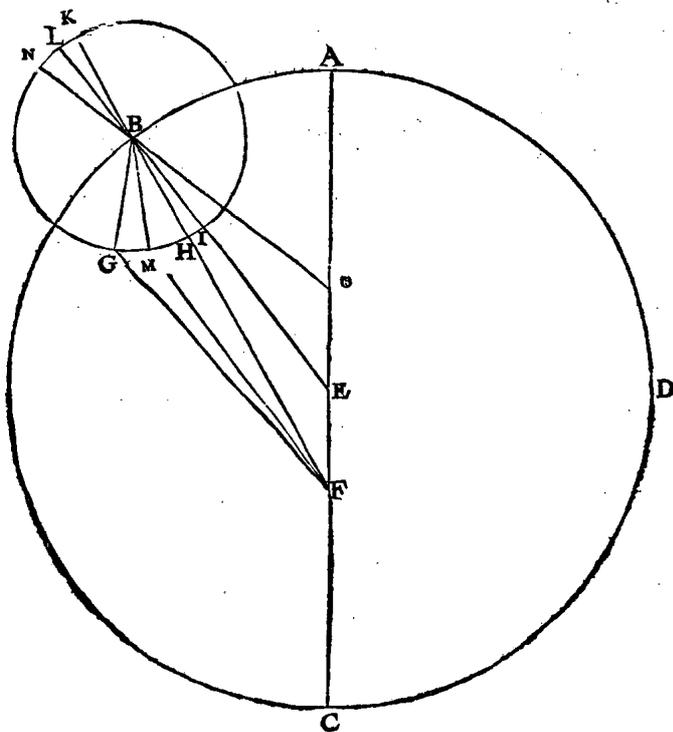
To expound the sum of the theory of the motion in longitude of Saturn, Jupiter, Mars, and Venus, added by *Ptolemy*.

Since we assume the simple motion of these Planets, as it was handed down to *Ptolemy*, to be known, as having been described in the first and the second book, the explanation of his addition will here be short and easy.

First of all it is to be noted that he always found the apparent ecliptical longitudes of these four planets, in conjunctions with and oppositions to the Mean Sun, when the epicycle's centre was at its deferent's apogee or perigee, to be as resulted from the calculation of the simple motion as it had first been handed down to him; for he found and calculated for each Planet, in such a location, the length of the line of eccentricity and of the epicycle's semi-diameter; but for other points he found differences.

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Om dan totte faecke te commen laet $ABCD$ den inrontswech fijn, diens middelpunt E , verstepunt A , uytmiddelpuntichpunt of den Eertcloot F , des inronts middelpunt B , waer op befchreven is het inront $GHIK$, daer na van E en F deur B ghetrocken sijnde de twee rechte linien EL , FK , fy snyen het inront in H en I , soo dat H is t'naestpunt, K t'verstepunt, I middelnaestpunt, L middelverstepunt: Voort neem ick dat den Dwælder na uytwijfen der rekening des tweeden boucx, dat is volghens den eenvoudigen loop soofc *Ptolemeu* eerst



ter handt quam, moeft fijn an G , soo dat fijn inronts middellangde LG doet 150 tr. en fijn fchijnbaer verheyte van t'verstepunt A ghesien uyt den Eertcloot F , soude hebben moeten fijn den houck AFG : Maer hy bevant fulckē houck metter daet cleender, als neem ick AFM , inder voughen dat den Dwælder int inront wesen tlick was an M , en niet volghens de rekening an G .

Hier uyt heeft hy aldus gedocht, nadien vanden Dwælder tottet middelverstepunt moet wesen 150 tr. gelijk eerst gestelt wiert de booch LG , en dat den Dwælder int inront soo veel voorders is dan G , als van G tot M , soo moet het middelverstepunt oock even soo veel voorders fijn van L , twelck sy an N , en hier me is NM van 150 tr. ghelijck LG .

Nu dan N ghenomen sijnde als voor middelverstepunt, van daermen des Dwælders middelloop begint te tellen, hy heeft van N deur B ghetrocken een rechte lini snyende de middellijn AC in O : Heeft daer na ghefocht hoe lanck de lini EO viel, en want alle noodige palen hem daer toe bekennt waren, heeftse even bevonden met EF , en dat niet alleen in dit voorbeeld, maer in allen ande-

ren

Therefore, to come to the matter, let $ABCD$ be the deferent, its centre being E , its apogee A , the point of eccentricity or the Earth F , the epicycle's centre B , about which has been described the epicycle $GHIK$. If thereafter from E and F through B there are drawn the two straight lines EL , FK , they intersect the epicycle in H and I , so that H is the perigee, K the apogee, I the mean perigee, L the mean apogee. Further I assume that, as shown by the calculation of the second book, *i.e.* according to the simple motion as it was first handed down to *Ptolemy*, the Planet should be at G , so that its epicycle's mean longitude LG makes 150° , while its apparent distance from the apogee A , as seen from the Earth F , ought to have been the angle AFG . But he found this angle in reality to be smaller — I take AFM — so that the Planet on the epicycle was in reality at M and not, as according to the calculation, at G .

From this he concluded as follows: Since from the Planet to the mean apogee there must be 150° , as the arc LG was first assumed to be, and since the Planet on the epicycle is as much in advance of G as the distance from G to M , the mean apogee must also be as much in advance of L ; let this be N , then herewith NM is 150° , like LG .

If therefore N is assumed to be the mean apogee, from which we begin to count the Planet's mean motion, *Ptolemy* drew from N through B a straight line intersecting the diameter AC in O . Thereafter he sought the length of the line EO , and because all the necessary terms were known to him, he found it to be equal to EF , such not only in the present example, but in all others, in whatever

ren tot wat plaets het inronts middelpunt in sijn wech, en den Dwaelder int inront wesen mochten: Sulcx dat hy hier af willende sijn spiegheling beschrijven, heeft O genoemt het * onevenheys punt, an t'welck genomen sijnde het oogh gestelt te wesen, men siet het inronts middelpunt B inden inrontswech A B C D, en den Dwaelder an G oirdentlick draeyen, waer uyt volght de voornomde twee punten B en M oirdentlick te draeyen ghesien uyt des inrontwechs middelpunt E, of anders geseyt dattet inronts middelpunt B in sijn wech, en den Dwaelder M int inront, d'een tijt rassicher als d'ander moetē loopen, teghen de ghemeene natuerlicke oirden, en volghens sulcke stelling gevonden wesende der Dwaelders voorofachtringen, en schijnbaer duystraerlangde, soo bestaet hier in *Ptolemæus* voorschreven vondt en spiegeling die hy vermengt heeft by den loop hem van sijn voorgangers ter handt gecommen.

T B E S L V Y T. Wy hebbē dan verclaert de somme van *Ptolemæus* bygevoughde spiegheling des langdeloops van Saturnus, Iupiter, Mars en Venus, na den eyfch.

*Punctum
inequalitatis.
Purbachius
centram
equantia.*

7 VOORSTEL.

Te verclaren de somme van *Ptolemæus* bygevoughde spiegheling des langdeloops van Mercurius.

Alfoo *Ptolemæus* dickwils gaflouch Mercurius plaetsen, en daer benevens noch acht nam op de gaflaginghen sijnder voorganghers, heeft bevonden dat hy in elcken keer tweemaal ten naesten by den Eertclood quam (ghelijck de Maen, die in elck Maenschijn na sijn segghen tweemaal ten naesten comt) het welcke t'elckens ghebeurde doen sijn inronts middelpunt schijnbaerlick was 120 tr. over weder sijden vant schijnbaer verstepunt, dat hy bevant in des duystraers 190 tr. D'oirsaek hier af stelde hy te wesen dattet inronts middelpunt totte twee voorschreven plaetsen altijt den Eertclood ten naesten was. Dit int ghemeen gheseyt sijnde, wy sullen nu totte besonder verclaring van sijn meyning commen.

Laet in de volgende eerste form A B C D dē wech des inronts sijn, diēs middelpunt is E, verstepunt A, uytmiddelpunticheytpunt of den Eertclood F, des inronts middelpunt A, waer op beschrevē is het inront G H I K, diens verstepunt G, naestepunt I, voort sy Mercurius ant verstepunt G, en dit alles ghenomen na den eenvoudigen loop soose *Ptolemæus* ter handt quam, en beschreven is int 2 bouck, met welcke ghestalt *Ptolemæus* gheen feyl en vandt, want hier me heeft hy int 8 Hoofstlick sijns 9 boucx ghesocht de reden der uytmiddelpunticheytlijn, en des inronts halfmiddellijn tot des inrontwechs halfmiddellijn: Maer Mercurius buyten des inronts verstepunt of naestepunt wesende, of des inronts middelpunt buyten sijn wechs verstepunt of naestepunt, soo vielder verschiel, t'welck *Ptolemæus* na verscheyden tastingen en onderfouckingen int 9 Hoofstlick sijns 9 boucx verbeterde, en in form van spiegheling brocht als volght: Hy heeft opt middelpunt E des wechs in dese gestalt sijnde, beschreven een rondecen L M N, snyende A C in L en N, wesende des selfden rondecens halfmiddellijn E N den helft der uytmiddelpunticheytlijn E F, hier in heeft hy t'punt L anghesien als voor middelpunt des inrontwechs, hoe wel het in dees form eyghentlick E is, en t'selve middelpunt L gheseyt te draeyen int rondecen van L tegē het vervolgh der trappē na M, doende daer in een

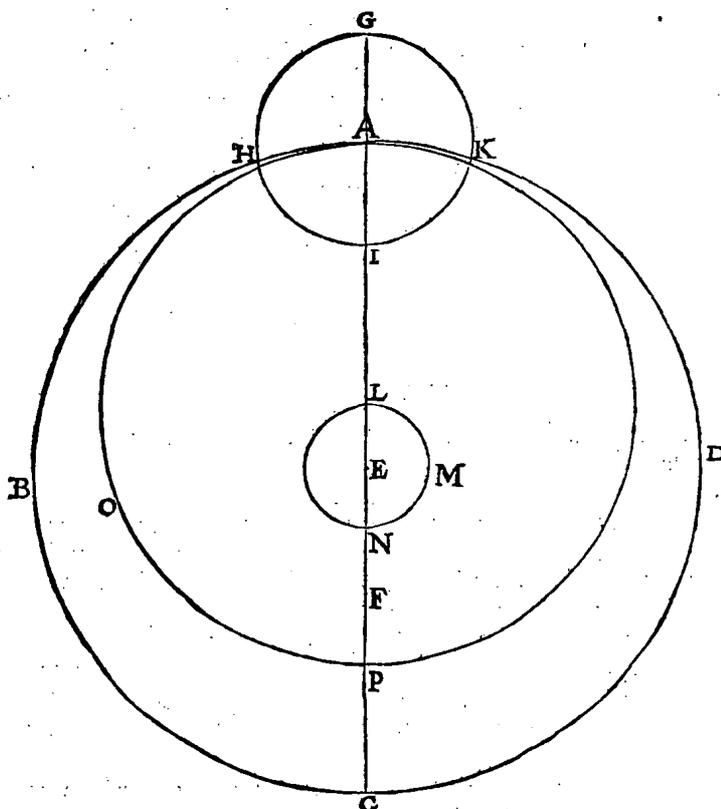
place the epicycle's centre might be on its deferent and the Planet on the epicycle. Thus, wishing to describe hereof his theory, he called O the point of inequality; and if the eye is taken to be situated in this point, the epicycle's centre B is seen to move regularly on the deferent $ABCD$ and the Planet to move regularly at G , from which it follows that the aforesaid two points B and M are seen from the deferent's centre E to move irregularly or, in other words, that the epicycle's centre B on its deferent and the Planet M on the epicycle must move faster at one time than at another, contrary to the common natural order of things; and when according to this assumption the Planets' advance-or-lag and apparent ecliptical longitudes are found, this constitutes *Ptolemy's* aforesaid discovery and theory, which he combined with the motion that was handed down to him by his predecessors. CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Saturn, Jupiter, Mars, and Venus, added by *Ptolemy*; as required.

7th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Mercury, added by *Ptolemy*.

Since *Ptolemy* frequently observed Mercury's positions, and in addition also noted the observations of his predecessors, he found that twice in each revolution it came as near as possible to the Earth (just as the Moon, which according to him comes as near as possible twice in each lunation), which happened whenever its epicycle's centre was apparently at 120° on either side of the apparent apogee, which he found to be at 190° of the ecliptic. He assumed that the cause of this was that the epicycle's centre was always as near as possible to the Earth in the two aforesaid places. This having been said in a general way, we shall now come to the separate exposition of his view.

In the subsequent first figure let $ABCD$ be the deferent, whose centre is E , its apogee A , the point of eccentricity or the Earth F , the epicycle's centre A , about which has been described the epicycle $GHIK$, whose apogee shall be G , its perigee I . Further let Mercury be at the apogee G , all this being taken according to the simple motion as it was handed down to *Ptolemy* and has been described in the 2nd book. With this disposition *Ptolemy* found no fault, for herewith he sought in the 8th Chapter of his 9th book the ratio of the line of eccentricity, and of the epicycle's semi-diameter, to the deferent's semi-diameter. But when Mercury was outside the epicycle's apogee or perigee, or the epicycle's centre was outside its deferent's apogee or perigee, there was a difference, which *Ptolemy* after several tentative efforts and investigations corrected in the 9th Chapter of his 9th book and cast into the form of a theory, as follows. About the centre E of the deferent in this disposition he described a small circle LMN , intersecting AC in L and N , the semi-diameter of this small circle EN being one half of the line of eccentricity EF ; in this he took the point L for the centre of the deferent, though in this figure it is really E , and said that this centre L moved on the small circle from L , against the order of the degrees, to M , performing therein one



een keer, op den selven tijt dattet inronts middelpunt in sijn wech een keer doet, op d'ander sijde na t'vervolgh der trappen, dats van A na B, en alsoo met hem draghende den heelen inrontwech.

Maer anghesien A L hier genomen is voor wechs halfmiddellijn defer bygevoughde spiegeling van *Ptolemew*, soo moet des selven wechs omtreck cleender sijn dan A B C D na stelling des eenvoudighen loops soose *Ptolemew* ter handt quam, daerom laet ons op L als middelpunt beschrijven den inrontwech der spiegeling van *Ptolemew* A O P: En hoe wel die nu cleender is dan d'cerste, soo blijft hier nochtans de verheyte vandē Eertcloot F tot A de selve, en vervolghens soo blijft oock des inronts grijphouck de selve: En als des wechs middelpunt L ghecommen sal sijn van L over M tot N, soo sal volghens dit gestelde des inronts middelpunt A ghecommen sijn over d'ander sijde an C, en sal aldan de verheyte vanden Eertcloot F, tot des inronts middelpunt, en vervolghens des inronts grijphouck oock de selve sijn als na de eenvoudige loop soose *Ptolemew* ter handt quam: Sulcx dat dese spiegeling van *Ptolemew* geen verandering en gheeft int voorschreven soucken der reden vande uytmiddelpunticheytlijn, en inronts halfmiddellijn, totte lini tusschen den Eertcloot en t'verstepunt.

Dese ghedaente aldus verclaert sijnde, wefende des inronts middelpunt an t'punt

revolution in the same time in which the epicycle's centre performs one revolution on its deferent — to the other side, in the order of the degrees, *i.e.* from *A* to *B* — while carrying with it the whole deferent.

But since *AL* has here been taken for the deferent's semi-diameter according to this additional theory of *Ptolemy*, this deferent's circumference must be smaller than *ABCD* (which followed from the simple motion as it was handed down to *Ptolemy*); therefore let us describe about *L* as centre the deferent of the theory of *Ptolemy* *AOP*. And though this is now smaller than the first, nevertheless the distance from the Earth *F* to *A* remains the same here, and consequently the epicycle's angular diameter also remains the same. And when the deferent's centre *L* has moved from *L* via *M* to *N*, according to this supposition the epicycle's centre *A* will have moved on the other side to *C*, and then the distance from the Earth *F* to the epicycle's centre, and consequently the epicycle's angular diameter, will also be the same as according to the simple motion as it was handed down to *Ptolemy*, so that this theory of *Ptolemy* does not bring about any change in the aforesaid seeking of the ratio of the line of eccentricity and the epicycle's semi-diameter to the line between the Earth and the apogee.

This disposition thus having been explained with the epicycle's centre at the

point A , furthest away from the Earth F , and also at its opposite point C , we will now also give an example with the epicycle's centre at another place — I assume at 60° of the mean motion from the apogee A — and Mercury at a longitude in the epicycle of 100° . To this end, in this second figure let the line AB pass through the apogee A (which *Ptolemy* found at 190° of the ecliptic), and let its opposite point be B , the Earth C , the centre of the small circle D , about which has been described this small circle $FGEH$ (instead of the small circle LMN of the first figure), the apogee thereof being F , the perigee E , and the semi-diameter DE being equal to EC . Further let there be marked the point G such that the arc from F to G , against the order of the degrees, makes the given 60° , and about G as centre let there be described the deferent IKL . Thereafter I draw DK so that the angle ADK is equal to $FDG = 60^\circ$, it being understood that from I to K it is, in the order of the degrees, just as it is from F to G against this order, all this as required by the above supposition. I describe thereafter about K as centre the epicycle $MNOP$; further I draw from G through K the line GKM , so that M denotes the epicycle's mean apogee. Then I take N to be Mercury's position and the arc MN to make the given 100° , and I draw KN , NC , CK , intersecting the epicycle in O , thereafter EK , EG , DK , DG , and I take the epicycle to be intersected by GK in P as the mean perigee.

Now it is also to be noted that *Ptolemy* assumes the epicycle's centre K to move regularly when seen from E , and for that reason it moves irregularly when

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middelpunt G, dat is inde selve wech d'een tijt rasscher als d'ander. Maer he oogh ghestelt an des wechs middelpunt G, soo neemt hy Mercurius, van daer ghesien oirdentlick te loopen in sijn inront, en daerom loopt hy self int inront oock oirdentlick al tijt even ras. En volgens sulcke stelling gevonden wesende Mercurius voorofachtring en schijnbaer duysteracrlangde, soo bestaet hier in *Ptolemus* spiegheling. T B E S L V Y T. Wy hebben dan verclaert de somme van *Ptolemus* byghevoughde spiegheling des langdeloops van Mercurius, na den eyfch.

Tot hier toe is gheseyt van *Ptolemus* byghevoughde spiegheling des langdeloops die hy vermengt heeft by den eenvoudighen loop hem van sijn voorgangers ter handt gecommen. Angaende sijn spiegheling des breedeloops, de somme daer afschijnt inde voorgaende breedeloop ghenouch verclaert.

N V V A N C O P E R N I C V S B Y G H E - voughde spiegheling der Dvaelders met stelling eens roerenden Eertcloots.

8 V O O R S T E L.

Te verclaren de somme van *Copernicus* byghevoughde spiegheling der Maen.

Nemende *Copernicus* dat *Ptolemus* ervaringhen vande Maens gageflagen schijnbaer plaetsen recht waren, om daer op als vaste gront een spiegheling te stichten, en siende dat hy in sijn byghevoughde spiegheling haer loop int inront, en des inronts loop in sijn wech oneven stelde, d'een tijt rasscher als d'ander, ghelijck vooren geseyt is in deses Anhangs 2 voorstel, sulcx heeft hem ongheschickt ghedocht, alsoo oock dede het stellen van haer groote naerdering totten Eertclood, strijdende teghen d'ervaring, deur welcke men bevint haer grijphouck de verandering niet te crijghen dieder uyt soude moeten volghen, inder vougen dat hy in die plaets een ander wijze beschreef: Om welcke te verclaren, soo is te weten dat hy lettende op de 5 tr. grootste voorofachtring der Maen in middelteghstant en middelfaming, maer van 7 tr. 40 \odot in ettelicke middelvierdeschijnen door *Ptolemus* gagheslaghen, als vooren gheseyt is, heeft ghenomen die regel valt te gaen, en d'oirsaeck ghestelt dusdanich te wesen: Laet A B den inrontwech der Maen beteyckenen, diens middelpunt dats den Eertclood C, en D E F het inront daer de Maen in loopt, volghens de eenvoudighe stelling soose *Ptolemus* eerst ter handt quam, diens middelpunt A, en getrocken de rechte D A F C, oock E C gerakende het inront an E, soo doet den houck A C E de boveschreven 5 tr. daer na sy ghetrocken de lini C G, soo dat den houck A C G doe de 7 tr. 40 \odot achtering, en op H als middelpunt sy beschreven het rondeken E G, gherakende de linien C E, C G, daer na sy getrocken van A deur t'punt des boveschreven raeckfels E de lini A E H G: En heeft de Maen in dit rondeken E G een loop dobbel ande middelmaenwinft, en des rondekens middelpunt H heeft sijn loop ghelijck mette loop die de Maen gheseyt wort te hebben int inront, volghens de eenvoudighe stelling ghelijckse
Ptole-

seen from the deferent's centre *G*, that is in the same orbit faster at one time than at another. But when the eye is situated at the deferent's centre *G*, he assumes Mercury, when seen from there, to move regularly on its epicycle, and for that reason it also moves regularly on the epicycle, always with the same speed. And when according to this supposition Mercury's advance-or-lag and apparent ecliptical longitude have been found, this constitutes *Ptolemy's* theory. CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Mercury, added by *Ptolemy*; as required.

Up to this point the theory of the motion in longitude added by *Ptolemy*, which he combined with the simple motion handed down to him by his predecessors, has been discussed. As to his theory of the motion in latitude, the summary of this seems to have been explained sufficiently in the foregoing motion in latitude.

NOW OF THE THEORY OF THE PLANETS,
ADDED BY COPERNICUS, ON THE ASSUMPTION
OF A MOVING EARTH

[The 8th proposition, concerning the motion of the Moon, has not been reproduced here].

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ghesocht des inronts halfmiddellijn A E, wort bevonden deur het 4 voorstel der platte driehoucken van 872, latet sijn soo *Copernicus* stelt 860.

De driehouck E C G heeft drie bekende palen, te weten den houck E C G 2 tr. 40 ①, de sijde E C 10000, als even genouch sijnde met A C, en den houck G E C recht: Hier me ghesocht des rondekens middellijn E G, wort bevondē deur het 4 voorstel der platte driehoucken van 466, latet sijn soo *Copernicus* stelt 474.

Diens helft voor de halfmiddellijn E H 237.

Tot C A 10000, vergaert A D 860, als even sijnde met A E eerste in d'oirdē, en daer toe noch D K 474, als even sijnde met E G tweede in d'oirden, comt voor C K meeste verheyte vande Maen tottē Eertcloot 11334.

Van C A 10000, getrocken A F 860, als even sijnde met A E eerste in d'oirden, en noch 474 voor de middellijn vant rondeken alst an F is, blijft voor de minste verheyte vande Maen totten Eertcloot 8666.

Sulcx dat de meeste verheyte tot de minste in sulckē reden is als 11334, tot 8666, t'welck soo groot verskil der Manens grijphoucken niet en geeft als *Ptolemus* palen int 3 voorstel deses Anhangs van 65 deelē 13 ① tot 34 deelen 9 ①, want segghende 34 deelen 9 ①, geven 65 deelen 13 ①, wat de Manens minste grijphouck 30 ①? Comt deur het 6 voorstel des 2 boucx de Manens meeste grijphouck na *Ptolemus* byghevoughde spiegheling 57 ①, maer na *Copernicus* stelling alleenelick 39 ①, want segghende 8666, gheven 11334, wat de Manens minste grijphouck 30 ①? comt als vooren 39 ①: Doch ist 3 ① meer dan na de eenvoudighe stelling soose *Ptolemus* eerst ter handt quam, welke dede 36 ① (overeencommende soo men seght mette dadelicke ervaringhen) want als gefeyt is inde Byeenvouging vant 13 voorstel des 3 boucx, de meeste verheyte doet 531, de minste 445, daerom seggende 445 gheeft 531, wat de Manens minste grijphouck 30 ①? comt als vooren 36 ①, sulcx dat *Copernicus* de sake daer in naerder ghecommen is dan *Ptolemus*.

Ten anderen soo en heeft *Copernicus* mettēt rondeken gheen ongheregelde loop d'een mael rasscher als d'ander, ghelijck *Ptolemus* stelling inhoudt.

Angaende d'overeencomminghen mette dadelicke ervaringhen, *Copernicus* en *Ptolemus* stellinghen gheven een selve besluyt in alle middelsaminghen en middeltegestanden, en oock in alle middelvierdeschijnen in welke de Maen by des inronts middelverheden is, want alsdan de voorofachtring na d'een en d'ander wijze 7 tr. 40 ① doet, maer buyten die plaetsen valter tusschen d'een en d'ander stelling vershil. T' B E S L V Y T. Wy hebben dan verclaert de somme van *Copernicus* byghevoughde spiegheling der Maen, na den cysch.

9 VOORSTEL.

Te verclaren de somme van *Copernicus* byghevoughde spiegeling des langdeloops van Saturnus, Iupiter en Mars.

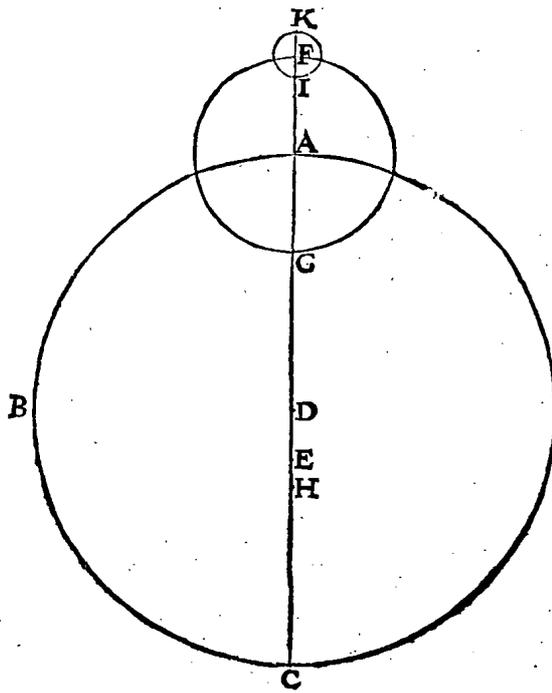
Nemende *Copernicus* dat *Ptolemus* ervaringhen van Saturnus, Iupiters en Mars gaghelaghen plaetsen recht waren, om daer op als vaste gront een spiegheling te sichten, en siende dat hy in sijn byghevoughde spieghelingen haer loop int inront, en des inronts loop in sijn wech oneven stelde, d'een tijt rasscher

9th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Saturn, Jupiter, and Mars, added by *Copernicus*.

When *Copernicus* assumed that *Ptolemy's* experiences of the observed positions of Saturn, Jupiter, and Mars were right, to base thereon a theory as a firm foundation, and he saw that in his additional theories he assumed their motion on the epicycle and the epicycle's motion on its deferent to be unequal, at one

scherals d'ander, ghelijck vooren gheseyt is in deses Anhangs 6 voorstel, sulcx heeit hem ongheschickt ghedocht, inder voughen dat hy in die plaets een ander wijze beschreef aldus toe gaende: Laet A B C den inrontwech beteyckenen van een der drie bovenste Dwaelders, ick neem Saturnus, diens middelpunt D, den vasten Eertcloot E, deur welcke ghetrocken is de rechte lini A C, soo dat A t'verstapunt bediet; C t'naestepunt, D E de uytmiddelpunticheytlijn, doende na *Ptolemus* rekening 3 deelen 25 ①, sulcke alster D A 60 doet, daer na sy op A als middelpunt beschreven het inront F G, diens verstapunt F, naestepunt G, waer in den Dwaelder sy ant verstapunt F. Dit is tot hier toe de form der eenvoudighe stelling met een vasten Eertcloot, soose *Ptolemus* eerst ter handt quam, waer op hy sijn boveschreven spiegheling veroidende: Maer de selve an *Copernicus* als gheseyt is niet bevallende, heeft gedocht of men sijn versierde roersels niet bequamelicker en soude te wege brengen, met den Dwaelder te doen loopen in een rondecen beschreven int inroni: En na verscheyden tastingen quam tot dusdanich besluyt: Hy nam voor sich de lini tusschen den Eertcloot en het onevenheypunt by *Ptolemus* in sijn byghevoughde spiegheling int 5 Hoofstlick sijns 11 boucx bevonden van 6 deelen 50 ②, sulcke alster des inrontwechs halfmiddellijn 60 doet, waer voor *Copernicus* int 5 Hoofstlick sijns 5 boucx ghebruyckt 1139 en 10000: Van dese 1139 der lini tusschen den Eertcloot en het onevenheypunt, trock hy het vierēdeel doende 285, en bleef 854, die heeft hy ghereyckent van D tot H, nemende t'selve punt H nu voor Eertcloot, die eerst deur E hadde, beteyckent geweest: Nam daer na het derden-deel van D H 854, dats E H doende 285, en teyckende die langde van F tot I, beschreef met F I als halfmiddellijn het rondecen I K, nemende den Dwaelder

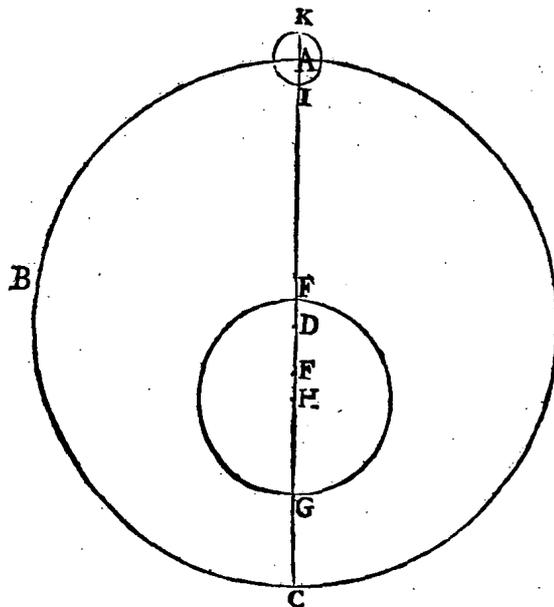


ten tijde van sijn saming mette middelson te wesen an het naestepunt I, hebbende eē loop mettet vervolgh der trappen van I na K, even metten loop des inronds middelpunt A na B: Maer wesende den Dwaelder in dese middelsaming alsoo an het rondecens naestepunt I, t'is kennelick dat hy in alle middelsaminghen daer an moet wesen, in alle middelteghestant an het verstapunt K, en in alle middelvierdeschijn an een middelverheyt, en quamen hier me Gg 3 over-

time faster than at another — as has been said previously in the 6th proposition of this Appendix — this appeared objectionable to him, so that instead of it he described another method, as follows. Let ABC denote the deferent of one of the three upper Planets — I assume Saturn — its centre being D , the fixed Earth E , through which has been drawn the straight line AC such that A denotes the apogee, C the perigee, DE the line of eccentricity, making according to *Ptolemy's* calculation $3\frac{25}{60}$ parts such as DA makes 60. Thereafter let there be described about A as centre the epicycle FG , its apogee being F , its perigee G , on which let the Planet be at the apogee F . This is, up to this point, the figure of the simple assumption with a fixed Earth, as it was first handed down to *Ptolemy*, on which he framed his above-mentioned theory. But since, as has been said, this did not satisfy *Copernicus*, he considered whether his imagined motions could not be brought about more conveniently by having the Planet move on a small circle described upon the epicycle. And after several tentative efforts he arrived at the following conclusion. He took before him the line between the Earth and the point of inequality, found by *Ptolemy* in his additional theory in the 5th Chapter of his 11th book to be $6\frac{50}{60}$ parts such as the deferent's semi-diameter makes 60, for which *Copernicus* in the 5th Chapter of his 5th book uses 1,139 and 10,000. From these 1,139 of the line between the Earth and the point of inequality he subtracted one fourth, making 285, upon which there were left 854, which he drew from D to H , now taking this point H for the Earth, which had first been denoted by E . Thereafter he took one third of $DH = 854$, i.e. EH , making 285, and drew this length from F to I , described with FI as semi-diameter the small circle IK , assuming the Planet at the time of its conjunction with the mean sun to be at the perigee I , moving in the order of the degrees from I to K , equally to the motion of the epicycle's centre A to B . But if the Planet is in this mean conjunction at the perigee of the small circle I , it is obvious that it must be there in any mean conjunction, in any mean opposition at the apogee K , and in any mean quadrature at a point of medium

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overeen Saturnus dadelicke langden na sijn meyning. Doch want sijn voor- nemen niet en was dese beschrijving aldus te doen met stelling eens vasten Eertcloots, maer met stelling eens roerenden, hy heeft die form verandert, beschrijvende op t'punt ghelijck H als middelpunt, een Eertcloodwech even ant boveschreven inrondt, en opt verstepunt des Dwaelderwechs een rondeken even an t'ander, nemende den Dwaelder daer in te loopen. Laet tot open- licker verclaring vandien A B C andermael het uytmiddelpuntichront betey- kenen, diens middel- punt D, en t'punt E sy hier in sulckē ver- heyt van D, alst in de eerste form van D was, daer na sy be- schreven den Eert- cloodwech F G op t'punt H als middel- punt, wesende t'selve punt H in sulckē ver- heyt van D als t'punt H in d'eerste form vā D was, te weten 854 deelen, en des Eert- cloodwechs halfmid- dellijn F G, doe soo veel als daer des in- ronts halfmiddellijn E G 1083, daer na sy mette halfmiddellijn evē an H E, beschre- ven op A als middelpunt het rondeken I K, even ant rondeken I K der eerste form, waer in Saturnus sy ant naestepunt I, met een loop als in d'eerste form vā I na K: Dese form met stelling eens roerenden Eertcloots, gheeft den Dwael- der al de selve schijnbaer duysteraerlangden die uyt d'eerste form met stelling eens vasten Eertcloots volgen, waer af ick eerst besonder bewijs gedaen hadde, maer denckende daer na de sake verstanelick ghenouch te connen sijn deur het ghene van dergelijcke vertoont is int 15 voorstel des 3 boucx, ten anderen let- tende op de onsekerheyt des gronts daer dese stelling op gebout schijnt, ick heb dat bewijs cortheytshalven onbeschreven ghelaten.



kenen, diens middel- punt D, en t'punt E sy hier in sulckē ver- heyt van D, alst in de eerste form van D was, daer na sy be- schreven den Eert- cloodwech F G op t'punt H als middel- punt, wesende t'selve punt H in sulckē ver- heyt van D als t'punt H in d'eerste form vā D was, te weten 854 deelen, en des Eert- cloodwechs halfmid- dellijn F G, doe soo veel als daer des in- ronts halfmiddellijn E G 1083, daer na sy mette halfmiddellijn evē an H E, beschre-

ven op A als middelpunt het rondeken I K, even ant rondeken I K der eerste form, waer in Saturnus sy ant naestepunt I, met een loop als in d'eerste form vā I na K: Dese form met stelling eens roerenden Eertcloots, gheeft den Dwael- der al de selve schijnbaer duysteraerlangden die uyt d'eerste form met stelling eens vasten Eertcloots volgen, waer af ick eerst besonder bewijs gedaen hadde, maer denckende daer na de sake verstanelick ghenouch te connen sijn deur het ghene van dergelijcke vertoont is int 15 voorstel des 3 boucx, ten anderen let- tende op de onsekerheyt des gronts daer dese stelling op gebout schijnt, ick heb dat bewijs cortheytshalven onbeschreven ghelaten.

Merck noch hier vooren geseyt te wesen dat *Copernicus* het rondeken eerst verdocht int inront met stelling eens vasten Eertcloots, ende hoewel sulcx in sijn schriften niet en blijkt, ick hebt nochtans soo gheseyt om mijn mey- ning int corte beter te verclaren, en dattet metter daet soo schijnt toeghegaen te hebben.

Deur dese voorgaende spiegheling seght *Copernicus* den Dwaelder alijt tot sijn behoerlicke plaets gevonden te worden.

Merckt noch int boveschreven te blijcken, dat *Copernicus* byghevoughde spiegeling een vermenging is van sijn vondt met wat uyt *Ptolemews* byvougning ghetrocken, ghemerckt de lini daer hy de drievierendeel afneemt by *Ptolemews* ghebruyckt wiert.

Angaende het soucken van des Dwaelders schijnbaer duysteraerlangde vol- ghens

distance, and in his view Saturn's practical longitudes agreed with this. But because it was not his intention to make this description thus on the assumption of a fixed Earth, but on the assumption of a moving Earth, he changed that figure, describing about the point H as centre an Earth's orbit equal to the above epicycle, and about the apogee of the Planet's deferent a small circle equal to the other, assuming the Planet to move thereon. For a fuller explanation of this, let ABC again denote the eccentric circle, its centre being D , and let the point E be here at the same distance from D as it was from D in the first figure. Thereafter let there be described the Earth's orbit FG about the point H as centre, this point H being at the same distance from D as the point H was from D in the first figure, to wit, 854 parts, and let the semi-diameter of the Earth's orbit FG make the same as there the epicycle's semi-diameter $EG = 1,083$. Thereafter let there be described, with the semi-diameter equal to HE , about A as centre the small circle IK , equal to the small circle IK of the first figure, on which let Saturn be at the perigee I , with a motion as in the first figure from I to K . This figure on the assumption of a moving Earth gives the Planet quite the same apparent ecliptical longitudes as follow from the first figure on the assumption of a fixed Earth, of which I had first given a separate proof, but considering afterwards that the matter could be understood easily enough from what has been shown for a similar case in the 15th proposition of the 3rd book, and secondly taking into account the uncertainty of the foundation on which this assumption seems to be built, for brevity's sake I have left this proof unwritten.

Note also that it has been said above that *Copernicus* first imagined the small circle on the epicycle on the assumption of a fixed Earth, and though this does not appear from his writings, I have nevertheless said it so in order better to explain my view in brief, and because it seems really to have taken place like this.

Copernicus says that by the foregoing theory the Planet is always found at its appropriate place.

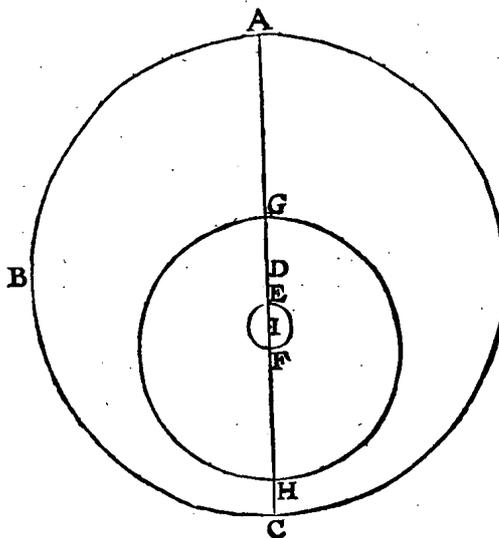
Note also that it appears from the above that *Copernicus'* additional theory is a combination of what he found and what was deduced from *Ptolemy's* addition, since the line of which he takes three fourths was used by *Ptolemy*.

gens dese spiegeling, t'is kennelick datmen inde tweede form sal vindē des ron-
dekens middelpunts schijnbaer duyfteraerlangde op de wijze ghelijck int 6 lidt
vant 15 voorstel des 3 boucx ghevonden wiert des Dwaelders schijnbaer duy-
fteraerlangde, en daer noch by voughen de voorofachtring die den Dwaelder
deur het rondeken crijcht, want datter uyt comt is t'begeerde. Deurt'ghene wy
tot hier toe van Saturnus gefeyt hebben, is dergheijcke te verstaen van Iupiter
en Mars. T B E S L V Y T. Wy hebben dan verclaert de somme van *Coperni-
cus* bygh evoughde spiegheling van Saturnus, Iupiter en Mars, na den eysch.

10 VOORSTEL.

Te verclaren de somme van *Copernicus* byghevoughde
spiegheling des langdeloops van Venus.

Angefen Venus met stelling eens roerenden Eertcloots alsoo loopt in haer
wech die binnen den eertclootwech is, ghelijck de drie opperste Dwaelders in
haer weghen loopen die boven den Eertclootwech sijn, soo soude mijns be-
dunckens *Copernicus* meyning uyt het voorgaende bekent sijn, als men het ron-
deken beschreef op Venuswech binnen den Eertclootwech: Doch te wijle hy
achte de sake claerder te wesen, met Venuswechs middelpunt te doen draeyen
in een rondeken even an t'ghene men anders op de wech als gheseyt is mocht
beschrijven, soo sal ick dat verclaren. Laet A B C dē Eertclootwech sijn, diens



middelpunt D, halfmiddellijn D A doende 10000, en E is de plaets daer Venus
wechs middelpunt bevonden wiert volghens de rekening gemaectt op de een-
voudighe stelling soose *Ptolemeus* eerst ter handt quam, alwaer de uytmiddel-
punticheytlijn D E dede 208: Maer na *Copernicus* spiegeling sy geteyckent het
punt F, soo dat E F even is met E D, beduydende t' selve punt F Venuswechs
middelpunt, waer op mette halfmiddellijn F G doende 7194, beschrevē is Ve-
nuswech G H in dese ghestalt: Daer na sy van E F ghenomen het middel I, en

G g 4 daer

As to the computation of the Planet's apparent ecliptical longitude according to this theory, it is obvious that in the second figure the apparent ecliptical longitude of the small circle's centre has to be found in the manner in which in the 6th section of the 15th proposition of the 3rd book the Planet's apparent ecliptical longitude was found, to which has to be added the advance-or-lag which the Planet gets from the small circle, for the result is the required value. From what we have hitherto said about Saturn the same is to be understood for Jupiter and Mars. CONCLUSION. We have thus expounded the sum of the theory of Saturn, Jupiter, and Mars, added by *Copernicus*; as required.

10th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Venus, added by *Copernicus*.

Since on the assumption of a moving Earth Venus moves in its orbit, which is within the Earth's orbit, just as the three upper Planets move in their orbits, which are above the Earth's orbit, in my opinion *Copernicus*' view would be known from the foregoing if the small circle were described on Venus' orbit within the Earth's orbit. But since he deemed the matter to be clearer if the centre of Venus' orbit were made to move on a small circle equal to the one that might otherwise — as has been said — be described on the orbit, I will explain this. Let ABC be the Earth's orbit, its centre being D , its semi-diameter DA , making 10,000; and E is the place where the centre of Venus' orbit was found according to the calculation made on the simple assumption, as it was first handed down to *Ptolemy*, where the line of eccentricity DE made 208. But according to *Copernicus*' theory let there be marked the point F such that EF is equal to ED , this point F denoting the centre of Venus' orbit, about which, with the semi-diameter FG making 7,194, has been described Venus' orbit GH in this disposition. Thereafter let there be taken of EF the middle point I , and

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daerop als middelpunt beschreven het roncken EF, daer des wechs GH middelpunt in draeyt, tweemaal so ras als den Eertcloot, en oock na t'vervolgh der trappen, sulcx dat wanneer den Eertcloot is an t'verstapunt A of naestapunt C, soo is t'middelpunt des wechs GH altijd an E, maer den Eertcloot an een der middelverhedē sijnde, als B, soo is dat middelpunt altijd an F. En hier me seght *Copernicus* dat Venus altijd tot haer behoerlicke langde ghevonden wort.

T B E S L V Y T. Wy hebben dan verclaert de somme van *Copernicus* byghevoughde spiegheling des langdeloops van Venus, na den eyfch.

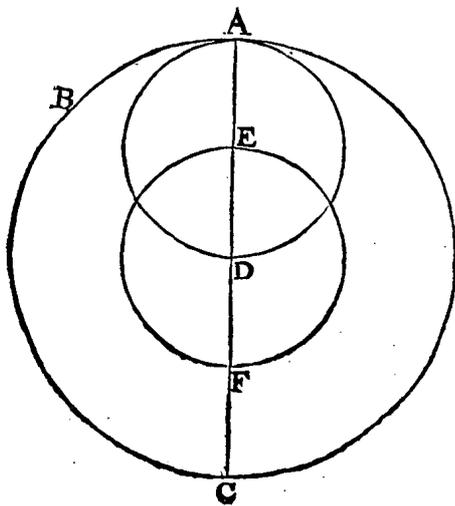
M E R C K T.

Alsoo *Copernicus* int 25 Hoofstuck des 5 boucx, tot sijn spiegeling noodich verstont Mercurius te doen overentweer loopē in een rechte lini, en nochtans niet willende toelaten eenighe ongheschickte loop van rondē d'eenmael rascher als d'ander, soo heeft hy int 4 Hoofstuck sijns 3 boucx beschrevē een vertooch waer deur sulcx met even roersel van rondē te weghe can ghebrocht worden t'selve vertooch (daer oock afghehandelt wort deur *Proclus* uytlegger der beginselen van *Euclides*) sal ick hier by setten als 11 voorstel, om te dienen tot bewijs des volghenden 12 voorstels.

V E R T O O C H. II V O O R S T E L.

Wesende opt middelpunt eens eerste ronts beschreven een tvveede, diens halfmiddellijn even is ande helft der halfmiddellijn van t'eerste, en opeen punt indē omtreck vant tvveede beschrevē een derde, even ant tvveede, hebbende een loop dobbel anden loop vant tvveede, en op een ander sijde. Yder punt des omtrexx vant derde beschrijft een rechte halfmiddellijn vant eerste.

I F O R M.



T G H E G E V E N. Laet in dees eerste form ABC het eerste rontd sijn, diens middelpunt D, halfmiddellijn AD, en op D sy beschreven een tweede rontd EF, diens halfmiddellijn DE even is anden helft vā AD des eerste ronts ABC, daer na sy op een punt als E indē omtreck vant tweederont EF, beschreven een derde AD even ant tweede EF, en hebbende een loop van A na de rechter sijde, dobbel anden loop vant tweede-rontd EF na de sijncker: Laet voort A eenich

about this as centre let there be described the small circle EF , on which the centre of the orbit GH moves, twice as fast as the Earth, and also in the order of the degrees, so that when the Earth is at the apogee A or the perigee C , the centre of the orbit GH is always at E , but when the Earth is at one of the points of medium distance such as B , that centre is always at F . And *Copernicus* says that herewith Venus is always found at its appropriate longitude.

CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Venus, added by *Copernicus*; as required.

NOTE.

Since *Copernicus* in the 25th Chapter of the 5th book deemed it necessary for his theory to make Mercury move to and fro in a straight line and yet was not prepared to admit any objectionable motion of circles, at one time faster than at another, in the 4th Chapter of his 3rd book he gave a demonstration by means of which this can be brought about with a uniform motion of circles; I will here add this demonstration (which is also dealt with by *Proclus*, the commentator of the elements of *Euclid*) as the 11th proposition, to serve as a proof of the subsequent 12th proposition.

THEOREM.

11th PROPOSITION.

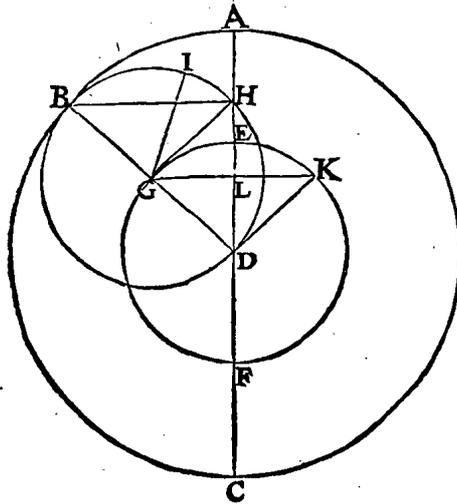
When about the centre of a first circle a second is described, whose semi-diameter is equal to one half of the semi-diameter of the first, and when about a point on the circumference of the second a third is described, equal to the second, having a motion that is twice the motion of the second and in an opposite direction, every point of the circumference of the third describes a straight semi-diameter of the first.

SUPPOSITION. In this first figure let ABC be the first circle, its centre being D , its semi-diameter AD , and about D let there be described a second circle EF , whose semi-diameter DE is equal to one half of AD of the first circle ABC . Thereafter in a point such as E on the circumference of the second circle EF let there be described a third circle AD , equal to the second EF and having a motion from A to the right that is twice the motion of the second circle EF to

eenich genomē punt sijn des omtreex vant derde rondt A D. T' BEGEERDE. Wy moeten hier me bewijfen dattet punt A des omtreex A D vant derde ront, een rechte middellijn A C beschrijft des eersten rondts A B C.

T'BEREYTSSEL. Latet punt E des tweede rondts E F inde eerste form gecommen sijn na de sijncker sijde tot G als in dees tweede gelooopen hebbende dē booch E G, of houck A D B, sulcx dattet verstepunt A des derde ronts A D inde eerste form moet gecommē sijn vā A tot B in dees tweede: Hierentusschē

2 FORM.



moetter genomen roerende punt in het derde rondt gecommen sijn volgende t'gestelde van B na H, te weten na de rechter sijde, sulcx dat den booch BH dobbel is an E G, of den houck B G H, dobbel anden houck B D H, latet sijn tot H. Om nu op dese derde form het begeerde noch opentlicker te verclaren, wy moeten bewijfen dat H valt inde lini A C. T'BEWY S. Angesien den houck B G H opt middelpunt G int rondt B H D dobbel is anden houck B D H opt punt des omtreex D int selve ront B H D deur t'bereytsel, en ooc deur het 20 voorstel des

3 boucx van *Euclides*, so segh ick hierom t'punt H nootfakelick te vallen in A C, want by aldient daer buyten viel, ick neem an I, soo en soude dan den houck B G I niet her dobbel connen sijn des houcx E D G, het welck teghen t'gestelde waer. T' B E S L V Y T. Wefende dan opt middelpunt eens rondts, beschreven &c.

V E R V O L G H.

De lini B H valt rechthouckich op A C om dese reden : Laet gheteyckent worden inden omtreck E F het punt K, soo dat de booch E K even is met E G, en daerom oock de booch B H met G K, daer na sy ghetrocken de lini D K en G K : Twelck soo wefende A D snijt de rechte G K in haer middel L, sulcx datse op malcander rechthouckich commen : Maer den driehouck B G H is even en ghelijck metten driehouck G D K, en de sijde B G light met G D in een rechte lini, waer deur B H ewewijdeghe is met G K, en comt ghelijck de selve G K oock rechthouckich op A C. Dit soo sijnde ick segh aldus : Want op een gegeven tijt bekent is de booch A B, soo wort ghevonden de houckmaetpijl H A des houckmaets H B, of houcx A D B, in sulcke deelen als men A D toeschrijft deur het 14 voorstel vant Houtmaetmaecksel, maer mettet punt H wort Mercurius beteyckent, waer uyt blijkt sijn plaets inde middellijn A C bekent te worden.

the left. Further let A be any point taken on the circumference of the third circle AD . WHAT IS REQUIRED. We have to prove herewith that the point A of the circumference AD of the third circle describes a straight diameter AC of the first circle ABC .

PRELIMINARY. Let the point E of the second circle EF in the first figure have moved to the left to G , as having moved on this second circle through the arc EG , or angle ADB , so that the apogee A of the third circle AD in the first figure must have moved from A to B on this second circle. In the mean time the moving point taken in the third circle must, according to the supposition, have moved from B to H , to wit, to the right, so that the arc BH is twice EG , or the angle BGH twice the angle BDH , let it be as far as H . In order now to explain in this third figure more clearly what is required, we have to prove that H falls in the line AC .

PROOF. Since the angle BGH at the centre G in the circle BHD is twice the angle BDH at the point of the circumference D in the said circle BHD , by the preliminary and also by the 20th proposition of the 3rd book of *Euclid*, I say that for this reason the point H necessarily falls on AC , for if it fell outside it — I assume at I — the angle BGI could not be twice the angle EDG , which would be contrary to the supposition. CONCLUSION. When thus about the centre of a circle is described, etc.

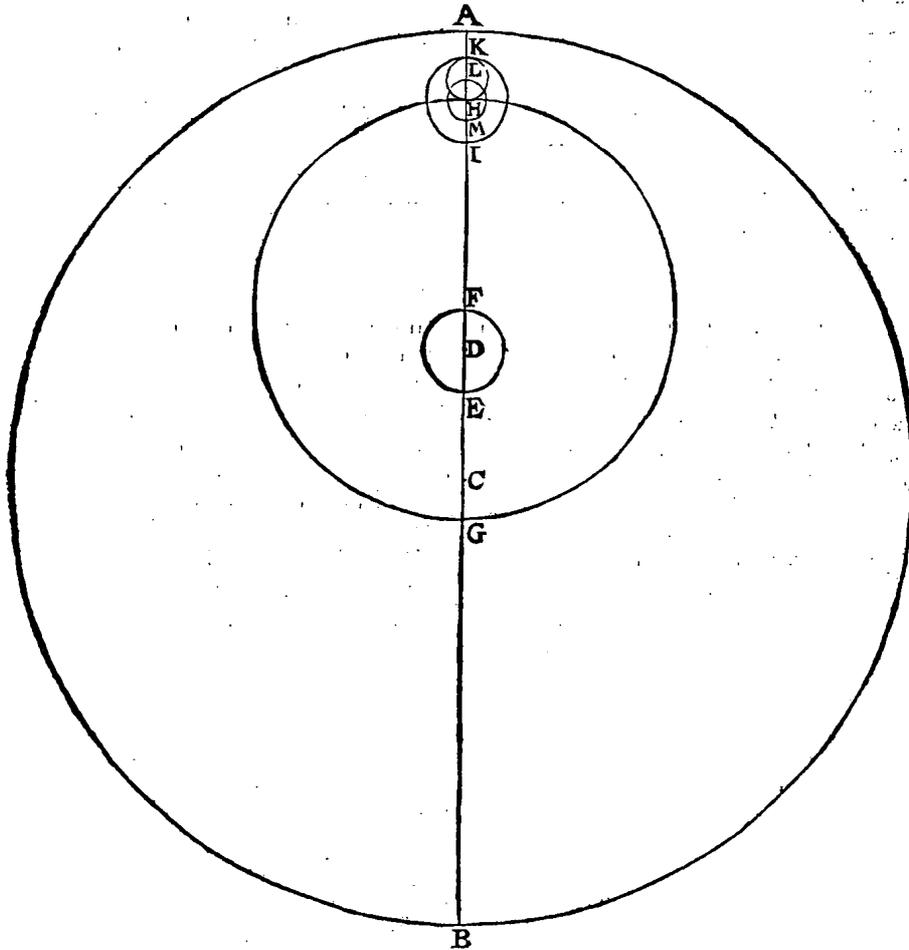
SEQUEL.

The line BH is perpendicular to AC , for the following reason: Let there be marked on the circumference EF the point K such that the arc EK is equal to EG , and consequently also the arc BH to GK . Thereafter let there be drawn the lines DK and GK . This being so, AD intersects the straight line GK in its middle point L , so that they come perpendicular to one another. But the triangle BGH is equal and similar to the triangle GDK , and the side BG lies with GD in a straight line, owing to which BH is parallel to GK , and like this GK also comes perpendicular to AC . This being so, I say as follows: Because at a given time the arc AB is known, the versed sine HA of the sine HB or of angle ADB is found, in such parts as are ascribed to AD by the 14th proposition of Trigonometry ¹⁾, but by the point H Mercury is denoted, from which its position in the diameter AC appears to become known.

¹⁾ For *Houtmaetmaecksel* in the Dutch text read *Houckmaetmaecksel*. See Work XI; i, 11, p. 60.

Te verclaren de somme van *Copernicus* byghevoughde spiegheling des langdeloops van Mercurius.

Laet *A B* den Eertclootwech beteyckenen, diens middelpunt *C*, middellijn *A B*, uyt middelpunticheytlijn *D C*, ghevonden na de wijze der eenvoudighe stelling soose *Ptolemeus* eerst ter handt quam van 947 deelen, sulcke alffer des Eertclootwechs halfmiddellijn *A C* 10000 doet, ende op *D* als middelpunt, en



mettet derdendeel van *DC*, t'welck sy *DE*, is beschreven een roncken *EF*, sulcx dat *F* t'verstapunt beteyckent vā *C*, en *E* t'naestepunt, daer na op *F* als middelpunt sy beschrevē den inrontwech *GH*, op wiens verstapunt *H* als middelpunt ick het inront *IK* teycken, en opt selve middelpunt *H* noch een cleender rondt *LM*, diens halfmiddellijn *HM*, even is an den helst der halfmiddellijn *HI*, en

12th PROPOSITION.

To expound the sum of the theory of the motion in longitude of Mercury, added by *Copernicus*.

Let AB denote the Earth's orbit, its centre being C , its diameter AB , the line of eccentricity DC , found by the method of the simple assumption, as it was first handed down to *Ptolemy*, to be 947 parts, such as the semi-diameter of the Earth's orbit AC makes 10,000, and about D as centre and with one third of DC , which shall be DE , there is described a small circle EF such that F denotes the point furthest from C (apogee) and E the nearest point (perigee). Thereafter about F as centre let there be described the deferent GH , about whose apogee H as centre I draw the epicycle IK , and about this centre H also a smaller circle LM , whose semi-diameter HM is equal to one half of the semi-diameter HI and intersects AB in L and M ; thereafter about L as centre the small circle KH , whose semi-diameter LH is the same as that of the circle described about HL ; in this small circle KH Mercury is carried. This figure thus being completed, we will explain its significance. F , the centre of the circle HG , performs two revolutions a year, one which it receives from the Earth's orbit, the other being its proper motion, and both in the order of the degrees. The centre L of the circle KH is fixed on the circumference of the circle LM and is carried therein against the order of the degrees, performing one revolution a year. The motion of Mercury on the circle KH is in the order of the degrees, equal to the above-mentioned motion of F , to wit, two revolutions a year, from which it follows that Mercury always moves to and fro on the diameter KI without leaving it, by the 11th proposition of this Appendix. Further it follows that when the Earth is at A or B , the centre of the circle HG must always be at F , that is the point of the circle EF furthest from C . But when the Earth is at the points of medium distance, *i.e.* 90° from A or B , the aforesaid centre of the circle HG must always be at E , that is the point of the circle EF nearest to C , which takes place in the opposite way to what happened with Venus. Further it follows that if, as has been said, Mercury moves to and fro on the diameter IK , the nearest point will always be at I when the Earth is at A or B , but at the furthest point K when the Earth is at the points of medium distance. In this way it occurs that the centre of HG (F) on the circle EF , and Mercury on the diameter IK , each perform two revolutions a year. And meanwhile the proper motion of the epicycle IK or the line FH in the circle HG is uniformly about 88 days for one revolution. And herewith *Copernicus* says that Mercury is found at its appropriate place.

CONCLUSION. We have thus expounded the sum of the theory of the motion in longitude of Mercury, added by *Copernicus*; as required.

HI, en sniende AB in L en M, daer na op L als middelpunt het rōndekē KH, diens halfmiddellijn LH de selve is des ronts beschreven op HL, in dit rōndeken KH wort Mercurius ghedreghen. Dese form aldus voldoen sijnde, soo fullen wy haer beteyckening verclaren : F middelpunt des ronds HG doet des jaers twee keeren, d'een diet vanden Eertclootwech ontfangt, d'ander sijn eyghen, en beyde na t'vervolgh der trappen : T middelpunt L des ronds KH, is vast inden omtreck des ronds I. M, en wort daer in ghedreghen teghen het vervolgh der trappen doende des jaers een keer : Den loop van Mercurius int rōndt KH, is na t'vervolgh der trappen, even anden boveschreven loop van F, te weten des jaers twee keeren, waer uyt volghet dat Mercurius altijt overentweert loopt inde middellijn KI sonder daer uyt te commen, deur het 11 voorstel deses Anhangs. Wijder volghet dat wesende den Eertcloot an A of B, soo moet het middelpunt des ronds HG dan altijt sijn an F, dats des ronds EF verstepunt van C : Maer den Eertcloot ande middelverheden sijnde, dats 90 tr. van A of B, soo moet het voorschreven middelpunt des ronds HG dan altijt sijn an E, dats des ronds EF naestepunt van C, het welck op verkeerde wijze toegaet van t'ghene met Venus ghebeurde. Wijder volghet dat Mercurius de middellijn I K overentweert loopende soo gheseyt is, altijt het naestepunt an I sal sijn wesende den Eertcloot an A of B, maer an t'verstepunt K den Eertcloot ande middelverheden sijnde : Hier me ghebeuret dat het middelpunt van HG als F, int rōndt EF, en Mercurius inde middellijn IK des jaers elck twee keeren doen : En hierentusschen is den eyghen loop des ronds IK, of de lini FH int rōndt HG eenvaerdelick ontrent de 88 daghen over een keer. En hier me segt *Copernicus* dat Mercurius tot sijn behoerlicke plaets ghevonden wort.

T B E S L V Y T. Wy hebben dan verclaert de somme van *Copernicus* bygevoude spiegheling des langdeloops van Mercurius, na den eyfch.

13 VOORSTEL.

Verhael op der sterren onbekende loop: En des duyteraers onbekende afsvijcking vanden * evenaer.

Aequator.

De vaste sterren worden gheseyt een onbekende loop te hebben d'een tijt raffcher als d'ander, want hoewelmen se altijt eeverre van malcander bevint, nochtans heeft den heelen Hemelscloot een roersel van Westen int Oosten, sulcx dat haer duyteraerlangde die ande lentsne begint, gheduerlick grooter wort, en van *Ptolemeus* tijt tot nu toe over de 21 \odot vermeerderd is : Maer volghens de ervaringhen sedert gheschiet, soo wort dit roersel seer onghereghelt gheacht, d'eenmael slapper als d'ander, ja soo eenighe meynen somwijlen ruggheling te loopen. Hier afsijn by ettelicke als *Thebis*, de *Alfonsinen*, *Purbachius*, *Copernicus*, *Ioannes Venerus*, verscheyden * spieghelinghen verdocht, elck na sijn goetduncken. Maer om van dit onbekent roersel mijs ghevoelen te segghen, het is te weten dat der sterren * wanschacuwing grooter bevonden wort in landen na den * afpunt dan na den evenaer, waer af wy seer merckelick voorbeelt hebben, deur de ervaring gheschiet op de vermaerde seylage van Willem Barentsen metten sijnen in Nova Zembla, wesende daer des afpunts verheffing van 76 tr. alwaer hemlien de Son eerst onder den * sichteinder ginck den vierden November 1596, diese op den eersten behoorden verlooren te hebben, sulcx dat se hoogher scheen dan se eyghentlick was, of wanschacuw-

Theoria.

Refraatio.

Polus.

Horizonte.

13th PROPOSITION.

An account of the stars' unknown motion; and the ecliptic's unknown deviation from the equator.

The fixed stars are said to have an unknown motion, at one time faster than at the other, for though they are always found at the same distances from one another, nevertheless the whole Heavenly Sphere has a motion from West to East, so that their ecliptical longitude, which starts at the vernal equinox, becomes continuously greater, and from *Ptolemy's* time to the present day has increased by more than 21° ¹⁾. But according to the experiences that have been gained since, this motion is considered to be very irregular, at one time slower than at another, nay, as some think, it is sometimes a backward motion. About this many authors, such as *Thebit*, the *Alphonsines*, *Purbachius*, *Copernicus*, *Johannes Vernerus* ²⁾, have invented different theories, each according as he thought fit. But to give my opinion about this unknown motion, it is to be noted that the refraction of the stars is found greater in countries towards the pole than towards the equator, of which we have a very notable example through the experience gained during the famous voyage of *Willem Barentsen* and his men to *Nova Zembla* ³⁾ where the elevation of the pole was 76° and where the Sun did not go down beneath the horizon until the fourth of November 1596, though they should have lost it on the first, so that it seemed to be higher than it really was, or had

¹⁾ For 21 ① in the Dutch text read 21 tr.

²⁾ Abū-l-Ḥasan Thābit ibn Qurra ibn Marwān al-Harrānī (Harran 826—Bagdad 901), mathematician and astronomer at the court of the Caliph Almustadid. His theory of „trepidation” became known mainly through al-Zarkali and the tables of Toledo; a treatise of Thābit was translated into Latin and printed at Leipzig (1503).

The Alphonsine Tables were constructed about 1250—1300 under the patronage of King Alphonso X of Castile. The original version is unknown, but modified tables bearing the same name existed in Paris about 1320; they came into general use and were repeatedly printed between 1483 and 1649: *Alphonsi, regis Castellae, caelestium motuum tabulae* (Venice 1483).

Georg von Peurbach or Purbach (Peurbach in Austria 1423—Vienna 1461), professor in Vienna. His treatise, *Theoricae novae planetarum*, passed through many editions from 1507 onwards, during the whole of the 16th century.

Johannes Werner (Nuremberg 1468—*ibid.* 1528), astronomer, mathematician and cartographer, discussed the precession in his treatises: *De motu octavae sphaerae*; *Summaria enarratio Theoriae motus octavae Sphaerae* (Nuremberg 1522). — For *Venerus* in the Dutch text read *Vernerus*.

³⁾ *Novaya Zemlya*. — Towards the end of the sixteenth century Dutch navigators tried to discover a passage towards the Indies through the arctic seas. These pioneers were obliged to winter on N.Z., where relics of their equipment have been found. See the account of the voyage by one of the participants, G. de Veer: *Reizen van Willem Barents, Jacob van Heemskerck, Jan Cornelisz. Rijk en anderen naar het Noorden (1594-1597)*. Edited by S. P. L'Honoré Naber (Linschoten Vereniging XIV, 1917); translated by the Hakluyt Society (1876).

The *Novaya Zemlya* phenomenon has been discussed by several modern authors, of whom we mention especially S. W. Visser (Proc. Acad. Amsterdam, Ser. B, vol. 59 (1956), p. 375). The exceptional refraction of $4^{\circ}17'$ may be explained by a pronounced ground inversion, extending over a large distance. A similar observation was made by Shackleton in 1915, when the refraction reached 2° .

schaeuwing had ghelijck elck berekenen mach, van ontrent 1 tr. 9 \odot : Maer 81 daghen daer na, te weten den 24 Ianuarius 1597, soo heeft den randt der Son haer weerom begonnen te openbaren, welcke sy sooder geen wanschaeuwing gheweest en had, op den 9 Februarius eerst behoorden gesien te hebben, inder voughen datse hoogher scheen danse eyghentlick was, of wanschaeuwing had by de 5 tr. welcke in dese laetste ervaring veel meerder bevonden wiert als in d'eerste, waer af d'oirsaeck bekent gheworden is an *Philippus Lans-*

Observator. bergius vlietich * Gaslagher en vermaert Wisconstenauer, die daer te voeren in sijn ervaringhen deur oneven wanschaeuwinghen langhe in twijffel gheweest had, want hoewel de Son tot lijckstandighe plaetsen vant * Winters keerpunt was, als neem ick 50 of 56 tr. daer voor, en 56 tr. daer na, soo en heeft hysc nochtans in d'een en d'ander niet met een selve hooghde boven den sichteinder bevonden, maer meerder inde laetste dan in d'eerste plaets, achtende nochtans aldoen dit seyl van wegghen wanschaeuwing niet te connen commen, om dat, als gheseyt is, de Son in d'een en d'ander ervaring eueverre van het winters keerpunt was: Maer her boveschreven voorbeelt van Nova Zembla t'sijnder kennis ghecommen sijnde, heeft d'oirsaeck besloten te wesen, en mijns bedunckens met goede reden, dat de coude winterseche vochtigheden in Februarius dicker en waterachtigher sijn, dan inde warmer maent October eynde des voorganghen Somers. Nu dan de wanschaeuwing in Nova Zembla soo uytnemende groot wesende, en tot ander plaetsen daer de ervaringhen ghebeurt sijn, als Pruyssen, Duytslandt, Spaeyne, Italic, Egipten, soogroot als yders Noorderlickheynt mebrengt, boven dien tot een selve plaets op d'een tijt des jaers grooter als op d'ander, sonder nochtans by de Beschrijvers der voornoemde spieghelinghen daer op acht ghenomen te wesen, soo valt daer uyt te besluyten, dat de onevenheden by hemlien bevonden, niet noofsake-lick en sijn van wegghen onevenheynt des loops der sterren; want al waer die gantsch even, soo souder om de verscheydenheynt der wanschaeuwing moeten schijnen onevenheynt te wesen: Oft anders gheseyt, soo by hemlien volcommen evenheynt bevonden waer, t'soude teycken sijn van onevenheynt int wesen. Tot breeder bevesting van het voorseyde valt noch te anmercken, ten eersten dat uyt de ervaringhen in Egipten gheschiet, even loop bevonden wiert, te hondert Iaren van 1 tr. totten tijt van 432 Iaren toe: Ten anderen dat de onevenheynt met rasscher loop, gheoirdeelt wort uyt ervaringhen die daer na in verscheyden Noorderlicker landen gageslaghen sijn, t'welck met reden vermoeden gheeft de onberekende wanschaeuwinghen daer af oirsaeck te meughen wesen, of volcommelick, of ten deele, maer welck van beyden men neemt, de voorschreven spieghelinghen vallen ongheront, en schijnt dat soo de Schrijvers van dien dese verscheyden grootheyden der wanschaeuwinghen ghenouch bedocht hadden, datse de selve spieghelinghen soo niet en souden veroirdent hebben, en daerom laet ick die onbeschreven, achtende datter voor al behooren te sijn ervaringhen van ghenouch saem sekerheynt, eermen totter veroirdenen en besluyten van sulcke spieghelinghen comt. Hier toe soudet connen voordêrlick sijn, datmen tot Alexandria in Egipten, daer de voorschreven eerste ervaringhen ghebeurden, dede gaslaen de Sonnens inganck der lentse en erfisne, met haer meeste afwijcking vanden evenaer, en datmen daer na saghe hoe die overcommen met dergelijcke ons Noorderlicker ervaringhen op den selven tijt gheschiet.

Tot hier toe is gheseyt vande onbekende roersels der vaste sterren: Angaende die der Dwaelers, van welcke in desen Anhang ghehandelt is, ick acht *Pro-*
lemeus

a refraction, as anyone may calculate, of about $1^{\circ}9'$. But 81 days afterwards, to wit, on the 24th of January 1597, the rim of the Sun began to show again, though, if there had been no refraction, they ought not to have seen it until the 9th of February, so that it seemed higher than it really was or had a refraction of about 5° , which in this last experience was found much greater than in the first. The cause of this was revealed to *Philippus Lansbergius* ¹⁾, industrious Observer and famous Mathematician, who previously had long been in doubt in his experiences owing to unequal refractions, for though the Sun was in homologous places in relation to the winter solstice — I assume 50° or 56° in advance of it and 56° behind it — yet he did not find it at the same height above the horizon in one place and the other, but at a greater height in the last place than in the first; yet he then thought that this error could not be due to refraction, because, as has been said, the Sun was at equal distances from the winter solstice in one experience and the other. But when the aforesaid example of Nova Zembla had come to his knowledge, he concluded the cause to be — in my opinion with good reason — that the cold, humid winter atmosphere in February is denser and contains more water than in the warmer month of October, the end of the preceding summer. Since therefore the refraction in Nova Zembla was so extraordinarily great, and in other places where the experiences were gained, such as Prussia, Germany, Spain, Italy, Egypt, as great as was conditioned by the Northerliness of each, while moreover in the same place it was greater at one time of the year than at another, but without the Describers of the aforesaid theories having paid heed to it, it may be concluded that the inequalities found by them need not necessarily be due to inequality of the motion of the stars, for even if this were altogether equal, there must seem to be inequality owing to the variation of the refraction. Or in other words: if they had found perfect equality, this would be a sign of real inequality. For a fuller confirmation of the above it is also to be noted that from the experiences gained in Egypt there was found equal motion, of 1° in a hundred years, up to 432 years. Secondly, that the inequality with faster motion is judged from experiences observed thereafter in different more Northerly countries, which justly raises the suspicion that the uncalculated refractions may be the cause of this, either altogether or in part. But no matter which of the two is taken, the above-mentioned theories are unfounded, and it seems that if the Authors thereof had taken sufficient account of these different amounts of the refraction, they would not have framed the said theories as they did; and for that reason I leave them undescribed, being of opinion that before all there should be experiences of sufficient certainty before one proceeds to frame and derive such theories. To this end it might be advantageous if at Alexandria in Egypt, where the aforesaid first experiences were gained, the Sun's passage through the vernal and the autumnal equinox were made to be observed, with its greatest deviation from the equator, and that it were thereafter seen how they correspond with similar more Northerly experiences gained at the same time.

Up to this point the unknown motions of the fixed stars have been discussed. As to those of the Planets, which have been dealt with in this Appendix, I also

¹⁾ Philippus van Laensbergh or Lansbergen (Ghent 1561—Middelburg 1632), surgeon and clergyman, studied the motion of the planets and advocated the theory of Copernicus from 1619 onwards. His *Opera Omnia* were published at Middelburg, 1663. In his printed works, no considerations about abnormal refractions are found.

Ptolemus ervaringhen daerſe op ghegront ſijn oock voor onſeker, want hoewel hy daer in groote loofficken aerbeyt ghedaen heeft, nochtans ghemerckt dat toeginck met een cleene cooper* Hemelloopſtuych, van verſcheyden ringhen ghemaect, elcke op haer as draeyende, ſulcx datmen des tuychs duyste-<sup>Inſtrumento
Aſtronomi-
co.</sup> raer altyt ewewijdich creech metten hemelſchen: Voort dat hy de ſterren deur ſichtgaetkens der wijslijnen ſach, met meerder plaets dan de grootheyd der ſterre, onſekerder toegaende dan na het gebruyck van Tuiſcho Brahe, ſoo is hemlien die met ſulcke reetſchappen ommegeen, kennelick ghenouch de onſekerheden dieder in vallen, hoe ſorchvuldich die oock ghemaect ſijn, ghelijck *Ptolemus* ſelf daer af ſomwijlen vermaen doet. Maer der ſterren hoochden of verheden van malcander met groote reetſchappen ghemeten, en de reſt deur rekeninghen der platte en clooſche driehoucken ghewrocht, daer machmen vaſtelicker op te wercke gaen.

Noch vielder in *Ptolemus* ervaringhen eenige onſekerheyd van der Dwaelders ſchijnbaer duysteeraerlangden, om dat de vaſte ſterren ſelf daer de dadelicke meting op ghegront is, tot die tijt rouwelick beſchreven waren met 10 ① voor cleenſte maet, als in ſijn tafelen blijktt. Voorts boven ſtelling van oneven onnatuerlicke loopen, ſoo betuyghen noch de dadelicke ghemeten grijphoucken der Dwaelders, voornamelick van Mars en de Maen, datſe niet in ſulcke verheyd en naerheyd des Eertcloots en commen als de ſpiegeling mebringt, gelijk wy elders breeder verclaert hebben.

Benevens t'ghene tot hier toe ghefeyt is noch vervought t'ghetuychnis van verſcheyden Gaſſaghers, als onder anderen *Regiomontanus*, *Bernhardus Waltheri*, en *Purbachius*, in druck uytgaende, ſoo en worden der Dwaelders plaetſen niet bevonden t'overkommen mette ſpieghelinghen: Al t'welck anghemerckt, ten ſchijnt niet ſeker ghenouch ofter der Dwaelders onbekende roerſels ſijn, of wefende, dat wy van haer ghedaente gheen ghenouchſaem beſcheyt en hebben, daerom meyn ick dat ſoo ymant voor hem naem een nieuwe ſpiegheling van dien te beſchrijven, dattet oirboir waer ſich voor al van ſoo veel ghewiſſe ervaringhen te voorsien, datſe voor gront mochten verſtrecken om op te bouwen. Doch hier mede mijn teghenwoordich ghevoelen verclaert ſijnde, laet daerentuffchen elck ſijn goerduncken volgen.

T'BEſLVYT. Wy hebben dan ghedaen een verhael op der ſterren onbekende loop, en des duysteeraers onbekende afwijking vanden Evenaer, na den eyſch.

DES DERDEN BOVCX
EY N D E.



consider *Ptolemy's* experiences, on which they are founded, to be uncertain, for though he has performed a great deal of praiseworthy work in this respect, nevertheless, considering that this took place with a small astronomical instrument of copper, made of different rings, each revolving about its axis, so that the ecliptic of the instrument was always obtained parallel to the celestial ecliptic, and since he saw the stars through small peepholes in the diopter rings, wider than the size of the star, which was a more uncertain procedure than by the practice of Tycho Brahe, it will be sufficiently obvious to those who handle such instruments to what uncertainties they give rise, however carefully they may have been made, as *Ptolemy* himself sometimes warns his readers. But when the altitudes of the stars or their distances from each other are measured with large instruments and the rest is achieved by calculations of plane and spherical triangles, this forms a firmer foundation on which to proceed.

There was also some uncertainty in *Ptolemy's* experiences concerning the Planets' apparent ecliptical longitudes, because the fixed stars themselves, on which practical measurement is based, up to that time had been roughly described with 10' as the smallest measure, as appears from his tables. Further, apart from the supposition of unequal unnatural motions, the practically measured angular diameters of the Planets, chiefly of Mars and the Moon, also show that they do not come as far from and near to the Earth as the theory implies, as we have explained more fully elsewhere.

When to the statements made up to this point there is further added the testimony of different Observers, such as, among others, *Regiomontanus*, *Bernhardus Waltheri*, and *Purbachius*¹⁾, which has been printed, the Planet's positions are not found to agree with the theories. Taking all this into consideration, it does not seem to be certain enough whether there are unknown motions of the Planets or whether we do not have sufficient information about their character. I am therefore of opinion that if anyone intended to describe a new theory about this matter, it would be proper first of all to procure so many certain experiences that they might serve as a foundation on which to build such a theory. But since herewith my present view has been set forth, let everyone meanwhile use his own discretion.

CONCLUSION. We have thus given an account of the stars' unknown motions and the ecliptic's unknown deviation from the Equator; as required.

END OF THE THIRD BOOK.

¹⁾ Johannes Müller (Unfind near Königsberg 1436—Rome 1476), celebrated astronomer, pupil and collaborator of Peurbach, worked in Vienna, in Italy, Hungaria, and Nuremberg.

Bernhard Walther (Nuremberg 1430—*ibid.* 1504), a wealthy citizen, gave financial aid to Regiomontanus and continued his observations. Only parts of his manuscripts have been preserved.

Georg von Peurbach, see note p. 315.

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VAN DE SPIEGHELING DER
EBBENVLOET

OF THE THEORY OF
EBB AND FLOW

From the *Wisconstighe Ghedachtenissen* (Work XI; i, 26)

INTRODUCTION

HISTORICAL REMARKS ¹⁾

In Antiquity many authors described tidal phenomena and indicated their connection with the moon, though often also a comparison with the breathing of an animal was made. Pytheas of Massilia is said to have been the first to indicate the connection of the half-monthly variation of the tide with the phases of the moon. Strabo in his *Geography* ²⁾ calls attention to the tidal currents in narrow straits, which are connected with the risings, settings, and meridian passages (upper and lower) of the moon. Pliny the Elder ³⁾ is acquainted with the facts that the tide for any place has a constant time-interval with the moon, that the height of the tide varies with the phase of the moon, and also that near the equinoxes the tides are higher than at the solstices, and that on the coasts of the Atlantic they are higher than along the shores of the Mediterranean.

The revival of learning in the sixteenth century, which first had to resuscitate and assimilate the science of Antiquity, did not show much progress as to the knowledge of the Tides. The great navigations of those days indeed made known a wealth of new facts on the movements of the waters. But in the reports of the navigators the westward flow in the oceans was attributed to the westward diurnal motion of the heavens. In a work by Julius Caesar Scaliger ⁴⁾ (the father of the renowned mathematician and chronologist Josephus Justus), published in 1557, the "desire of the moon" is mentioned as the cause of the tides; the alternation of ebb and flow is explained by the repulsion exerted by the long stretch of the Western Continent lying in the way of the westward stream.

Francis Bacon ⁵⁾ also speaks of the general westward motion of the sea, and he believes that the short period of half a lunar day is due to obstruction by the shores. In his zeal to refute old superstitions he refuses to assume here an effect of the moon. "Yet it will not immediately follow . . . that things which correspond in the course and periods of time, or even in the manner of carriage, are in their nature subordinate, and the cause one of the other. For I do not go so

¹⁾ An extensive historical survey of the investigations on the tides is found in: Rollin A. Harris, *Manual of Tides* (Appendix 8 to U.S. Coast and Geodetic Survey, 1897). — Pytheas wrote about 325 B.C.

²⁾ Strabo (c. 54 B.C.—c. 24 A.D.), *Geography*, I.3.11-12. *cap.* 54 & 55; 3.5.8. *cap.* 173-174.

³⁾ Plinius (23-79 A.D.), *Natural History*, II. 212-222 (Loeb Classical Library, 1938, ed. Rackham).

⁴⁾ J. C. Scaliger, scientist and physician (Padua 1484—Agen 1558), *Exotericarum Exercitationum ad Cardanum* (Exerc. LII).

⁵⁾ Francis Bacon 1561-1626, *De fluxu et refluxu maris* (Ed. Spedding, Vol. V, p. 441).

far as to assert that the motions of the moon or sun are set down as the causes of the inferior motions which are analogous to them; or that the sun and moon (as is commonly said) have dominion over these motions of the sea; . . . indeed in that very half-monthly motion (if rightly observed) it would be a very strange and novel kind of obedience, for the tides at the new and full moon to be affected in the same way, while the moon is affected in opposite ways . . ." 6)

Johannes Kepler in a letter to J. G. Herwartus (1598) says that the seas might adhere to the moon 7). In his *Astronomia Nova* (1609) he writes: "If the earth should cease to attract its waters, all marine waters would be elevated and would flow into the body of the moon" 8). Yet at other times he compares the tide to the breathing of an animal.

It is well known that Galileo Galilei in 1616 framed a different theory 9). Ascribing the tides to a periodical irregularity in the velocity of the terrestrial surface, due to a combination of the diurnal rotation and the annual revolution, he makes them an important argument in favour of the Copernican world-system. This effect indeed must be present, but it is too small to be observed and is lost among the irregularities of the small tidal phenomena in the Mediterranean.

The treatise of Stevin on the Tides is part of his *Wisconstighe Ghedachtenissen* (*Mathematical Memoirs*), published at Leyden in 1608 (Work XI; i, 26); it was translated into Latin by Snellius (Leyden 1608, Work XI b) and into French by Girard (Leyden 1634, Work XIII; ii, 2). In these Memoirs, it is Book VI of the *Eertclootschrift* (*Geography*).

6) Francis Bacon, *Works, I.c.*, p. 448.

7) Joh. Kepler, *Gesammelte Werke*, Vol. XIII, p. 193 (München, 1945).

8) Joh. Kepler, *Astronomia Nova* (*Gesammelte Werke*, Vol. III, p. 26. Ed. Max Caspar, München 1937).

9) Galileo Galilei, *Discorso sopra il flusso e refluxo del mare* (Ediz. Nazion. V, 371); also: *Dialogo* (4th dialogue) (Ediz. Nazion. VII, 442).

SUMMARY OF THE WORK.

The Tides are treated by Stevin as a chapter of geography. His knowledge of this phenomenon was determined by the conditions on the coasts of Western Europe, especially the North Sea. To him the tides were a phenomenon to be explained, as they were by Kepler, by an attraction of the terrestrial oceans by the moon. In this attraction there is nothing of the later Newtonian theory; it is simply an expression of direct experience. The same experience taught him a second fundamental fact: the waters were attracted not only towards the moon but also towards the opposite side. The third fundamental fact, also taken simply from experience, was the alternation of spring tide at the times of the full and the new moon, and neap tide at the times of the quarters. There is no trace of an attempt to account for this alternation by a combination of solar and lunar attraction; the tides to him were a purely lunar phenomenon.

Starting from this basis, Stevin derives the phenomena of the tides first by means of what in later years was called the static theory. The earth is supposed to be entirely covered with water, the surface of which takes the form of an elongated ellipsoid with the high tops exactly below the moon and at its opposite point, and the places of lowest level forming a circle on the globe at a distance of 90° between the tops. Projected upon the celestial sphere, they are the place of the moon in the sky and its opposite point, and a celestial circle at a distance of 90° from the moon. The observer's zenith, also projected upon the sphere, owing to the diurnal rotation describes a parallel circle with a declination equal to his terrestrial latitude. Thus it is a problem of spherical trigonometry: when the observer's zenith passes the moon (or its opposite) at the shortest distance — *i.e.* when it has the same right ascension — there is high tide; when it intersects the 90° circle of lowest level, the observer witnesses low tide.

These conditions are worked out for a number of special cases. When the moon is in the vernal or autumnal equinox, the low-level circle is the colure through the poles, at 90° right ascension; high tide and low tide alternate with a six-hour interval (neglecting the moon's interim motion in longitude). However, when the moon at 90° or 270° right ascension reaches its maximum declination (neglecting its latitude), the low-level circle passes through the equinoxes and the poles of the ecliptic; the parallel circle described by the observer's zenith intersects it at a point nearer to the pole of the ecliptic, at a right ascension that has to be computed by spherical trigonometry. Then the intervals between the moments of high and low tide are unequal; the rise of the water is slow, its fall is rapid. Since the height of the high tide depends on the distance from the zenith to the moon's position, it will — for medium latitudes — if the moon's declination is large, be different for the two high tides of one day. This difference increases with the observer's latitude, and beyond 61° (determined by the maximum declination $28^\circ.6$ of the moon) one of them disappears, so that instead of a 12-hour, a 24-hour period in the oceanic level appears. At the poles there is always an interval of 7 days between high tide and low tide.

Thus Stevin deduces the phenomena of the tides from simple static theory. But he is well aware that the real phenomena of the tides deviate from this theory and are far more intricate. In the next chapters he therefore gives an explanation, first of the fact that high tide occurs many hours (different for different coasts) after the moon's culmination; this is due to the resistance which the coasts of the continents offer to the progress of the tidal wave. The fact that in the seas of Western Europe the high tide proceeds in the wrong direction, from West to East, he explains by stating that small and inland seas are only imperceptibly attracted by the moon. An exact treatment of these phenomena was not yet possible at that time (a century before Newton). Stevin tries to give a demonstration in his third proposition; comparing a small and a large vessel with water, he shows that in the latter a greater weight of fluid can be sustained by a smaller sideways pressure. Thus the small attraction by the moon is more effective for the oceans than for minor waters, such as the North Sea; these receive their motion from the progress of the tide in the oceans. Hence, the tidal wave slowly runs up from the western sea eastward into the broad river estuaries.

Stevin emphasizes the fact that the tidal phenomena are complicated, and that their theory may be highly defective. Because the opinion had been expressed that the motion of the waters was due not to an attraction but to a repulsion exerted by the moon, he examines the consequences of such a force. They can be described by interchanging the expressions high tide and low tide in the preceding results, so that at medium latitudes the rise of the water is rapid and the fall is slow; they are at variance with the phenomena observed. And he concludes his treatise with the wish that all over the earth the tides should be carefully studied, in order that a satisfactory theory might be based on the observations.

S E S T E
B O V C K D E S
E E R T C L O O T S C H R I F T S,
V A N D E
S P I E G H E L I N G D E R *Theorie.*
E B B E N V L O E T.

C O R T B E G R Y P.


 Ngefen ervaringsen de sekerste gront sijn, als int voor-
 gaende breeder gheseyt is, daermen ghemeene reghelen
 uyt treckt, om tot kennis aer saken te commen, en dat
 ons deur deser landen groote seylagen, bequamer middel
 ont moet d'ander te vooren gheveest is, om te geraken
 tot veel ghevijste ervaringsen der eyghenschappen
 van ebbe en vloet: Soo heeft my tottet bevoorderen van sulcx, oirboir ghe-
 docht van dese stof een * Spiegheleing te beschrijven, ghegront ten deele
 op ervaringsen diemen nu heeft, ten deele op stelling die de natuerlicke reden
 lijckformich schijnt, dienerde als begin, om by manier van beschreven const
 hier af te handelen, en deur breeder ervaringsen diemen namaels crijghen
 mocht, oir aentlick na grondelicker kennis te trachten.

Theoriam.

Angaende ymant dencken mocht, dattet van my voor t'uytgeren van
 desen vougheucker waer gheveest, sulcke dinghen eerst sekerlick onder-
 socht te hebben, of doen ondersoucken: Hier op seghick dat sulcx niet een of
 veynich menschens verck vvesende, soo heeft my dit de bequaemste
 vvech ghedocht, om op corten tijt veel bescheyt en sekerbeyt te crijgen, vvant
 veel menschen totte borveschreven gaslagingshen vermaent sijnde, t'can ghe-
 beuren datter hun tot verscheyden plaetsen meer toe sullen begeren, dan deur
 mijn besonder voor dering an besonder menschen meughelick soude vvesen.

I BE-

OF THE THEORY OF EBB AND FLOW

SUMMARY.

Since experience is the surest ground, as has been said more fully in the foregoing, from which to draw general rules in order to gain knowledge of things, and since thanks to the great navigations of these countries we have better means than before for obtaining many sure experiences of the properties of ebb and flow, it seemed suitable to me, in order to promote this, to describe a theory of this subject matter, based in part on experiences now available and in part on suppositions which seem to be in accordance with natural reason, which description may serve as a starting point, in order to deal with this in the manner of a textbook and properly strive to gain fuller knowledge by means of ampler experience that may be obtained later.

If anyone should be of opinion that it would have been more fitting if, before publishing this treatise, I had first examined these things with certainty or caused them to be so examined, I say to this that since that is not the work of one man or a small number of men, this seemed to me the best method for getting much information and certainty in a short time, for when many people have been admonished to make the above-mentioned observations, it may happen that in different places more people will proceed to do so than would be possible by my private admonition to some private persons.

BEGHEERTEN.

1 BEGHEERTE.

Postulatur.

Wy begheeren toegelaten te vvorden, dat de Maen en haer teghepunt het vvater des Eertcloots gheduerlick na hun suyghen.

VERCLARING.

MEN bevint deur daghelicksche gheduerighe ervaringhen, dat ebbe en vloet vande Maen gheregiert worden, oock den vloet ten hoochsten te commen in volle en nieu Maen, diemen dan sprinckvloet noemt, maer ten leeghsten in * vierdeschijn: Waer *Quadratur.* af sulcken gheneenen kennis sijnde, datmen met ghenouchsaem sekerheyt van te vooren weet de uyr van toecommende ghetijden, tot groot voordeel der zeevaert, soo en ist niet noodich daer af toelating te begheeren. Maer want met elcken Maenkeer om den Eertclood (die ontrent alle 25 uyren eens ghebeurt) twee vloeden en oock twee ebben commen, soo wort by ettelicke vermoet de Maen en haer teghepunt een eyghenschap te hebben, datse het water na hun suyghen inde hooghde: Doch ist onseker oft inde natuer soo toegaet, want deur haer perfsing (t welck t verkeerde van suyging is) souden oock dagelick twee ebben en vloeden commen. Maer welck van beyden, of wat ander derde natuerlicke eyghenschap daer af d'oirsaeck is, meyn ick onbekent te sijn deur ghebreck van ervaring. Ick heb eenighe ons Indivaerders ondervraeght na de ghedaente van waterghetyen tot seker plaetsen, maer niet connen vinden t'ghene ick socht: Doch anghesien t' vermoen van ettelicke is, t'selve deur de voorschreven suyging te geschien, so ist dat wy begheeren sulcx toegelaten te worden, om also ons voorghenomen spiegheling een gront te gheven, welcke beschreven sijnde, sullen eyntlick int naest laetste voorstel segghen wat verandering uyt perfsing volghen soude.

2 BEGHEERTE.

Den Eertclood heel met vvater bedeckt te sijn, sonder vvint of yet dat an ebbe en vloet verhindernis gheeft.

VERCLARING.

De natuerlicke oirden van ebbe en vloet, wort deur winden, oock deur landen boven t'water uytstekende, verhindert, sulcx dat tot alle plaetsen geen hoochste water en is, wesende de maen of haer teghepunt int middachront, ghelijck de ghemeene reghel vereyscht volghende d'eerste begheerte, maer mach dan ten leeghsten sijn; of min verschillen: Ten anderen dat de vloet niet ancommen en sal uyt den oosten na westen, maer uyt den westen of eenigen anderen oirt. Ten derden datmen tot sommighe plaetsen diens toppunt verrevande Maen is, den daghelickschen vloet hoogher bevint dan tot ander plaetsen over diens toppunt de Maen gheweest heeft, daerse volghende t'gestelde ten hoochsten soude sijn.

Maer

POSTULATES.

1st POSTULATE.

We postulate that the Moon and its opposite continually suck the water of the Earth towards them.

EXPLANATION.

It is found by daily continual experiences that ebb and flow are governed by the Moon, and also that the flood-tide is highest at full and new Moon, which is then called spring-tide, but lowest at quadrature. This being such common knowledge that the hour of future tides is known beforehand with sufficient certainty, to the great advantage of navigation, it is not necessary to postulate this. But because with every revolution of the Moon about the Earth (which takes place once in about every 25 hours) there are two flood-tides and also two ebb-tides, it is suspected by several people that the Moon and its opposite have the property of sucking the water up towards them. It is, however, uncertain whether it happens like this in nature, for in consequence of their pressure (which is the opposite of suction) there would also be two flood-tides and two ebb-tides every day. But I think that for lack of experience it is unknown which of the two or what other (third) natural property is the cause of this. I have inquired with some of our mariners sailing to the Indies about the nature of the tides in certain places, but have not been able to find what I sought. But since several people suppose that it is due to the aforesaid suction, we postulate this in order thus to give our intended theory a basis. And after this has been described, we will finally say in the last proposition but one what change would result from pressure.

2nd POSTULATE.

That the Earth is covered entirely with water, without the wind or anything else impeding ebb and flow.

EXPLANATION.

The natural order of ebb and flow is impeded by winds, also by lands sticking out above the water, so that not in all places is the tide highest when the moon or its opposite is in the meridian, as the general rule requires according to the first postulate, but it may be lowest or differ less. Secondly, the flood may come not from the east to the west, but from the west or some other direction. Thirdly, in some places, whose zenith is far from the Moon, the daily flood-tide is found to be higher than in other places, in whose zenith the Moon has been, where according to the supposition it ought to be highest.

180 6 BOVCK DES EERTCLOOTSCHRIFTS,

Theoria Geometrie.
Praxis Geometrie.

Maer op dat alle dese ongheregheltheden, ons niet en verhinderen om te begrijpen de groote ghemecene eyghenschap van ebbe en vloed, die wy spiegelingfche wijze voornemen te beschrijven, soo begheeren wy hier boven toegelaten te worden, den Eertclood heel met water bedeckt te sijn, sonder wint of yet dat an ebbe en vloed hinder gheeft, om daer na vande ghedaente der beletfelen onderscheydelicker te meughen spreken: Want gelijk * spiegeling der Meetconst voordelick is * totte meetdaet, hoe wel daer in nochtans platten en rechte linien der meetbaer velden en lichamen, niet de volcommenheyt en hebben die de bepalinghen der Spiegelinghen mebrengghen, alsoo can dese spiegeling oock voordelick sijn totte daet, voornamelick int stuck der zeevaert, hoe wel nochtans de formen der zeen, niet de volcommenheyt en hebben die dese begheerte en de volghende bepalinghen inhouden.

BEPALINGHEN.

BEPALING.

Wesende ghetrocken een rechte lini van des Eertcloots middelpunt totte Maen: Het punt daerse des vvaters oppervlack gheraecht noemen vvy Manens vloettop: En t punt daer teghenover Teghepunts vloettop.

VERCLARING.

Latet rondt ABCD den Eertclood beteycken en, diens middelpunt E, welke teenemael bedeckt sy met water, totten omtreck FGH I toe, en dat sonder storm, wint, of eenich beletfel, na luyt der boveschreven 2 begheerte: Voort sy het rondt KL de Maenwech, waer in K de Maen bedict, en L haer teghepunt: Welcke twee deur haer trecking niet toe en laten, het water om den Eertclood sijnde, hem totte rontheyt te begheven, maer een eyfche form doen hebben: Ick treck daer na de rechte lini KEL, snyende des waters oppervlack in F en H: Voort treck ick IE Grechthouckich op KL. Dit so sijnde, F is des vloets hoochste punt, of top na de Maen toe, t welck ick daerom noem Manens vloettop. En om dergheelicke redenen H haer teghepunts vloettop.

2 BEPALING.

Het rondt op den Eertclood diens plat de rechte lini tusschen beyde de vloettoppen int middel deursnijt, en daer op rechthouckich is, noemen vvy Ebront.

VERCLARING.

Als het rondt IFG overcant ghesien, heet Ebront, om dat daer in altijt ebbe of leeghste water gheschiet. En valt daer in ten alderleeghsten om dat EI de cortste lini is, diemen van des Eertcloots middelpunt E, tot des waters oppervlack trecken can.

NV

But in order that all these irregularities may not prevent us from understanding the great general property of ebb and flow, which we intend to describe in theory, we postulated above that the Earth is covered entirely with water, without the wind or anything else impeding ebb and flow, after which we may discuss more particularly the character of the impediments. For just as Theoretical Geometry forms a suitable preliminary to practical geometry, although in the latter the planes and straight lines of the fields and bodies to be measured do not have the perfection which the definitions of the Theories imply, the present theory may also form a suitable preliminary to practice, especially in the matter of navigation, although nevertheless the forms of the seas do not have the perfection which the present postulate and the following definitions imply.

DEFINITIONS

1st DEFINITION.

When a straight line is drawn from the centre of the Earth to the Moon, we call the point where it touches the surface of the water the Moon's Flood Top, and the point opposite it the Opposite's Flood Top.

EXPLANATION.

Let the circle $ABCD$ denote the Earth, whose centre is E , and let it be covered entirely with water, up to the circumference $FGHI$, such without storm, wind or any impediment, according to the above-mentioned 2nd postulate. Further let the circle KL be the Moon's orbit, in which K denotes the Moon and L its opposite. Owing to their attraction these two do not permit the water surrounding the Earth to take a spherical shape, but cause it to have the shape of an egg. I then draw the straight line KEL , intersecting the water's surface in F and H . Further I draw IEG at right angles to KL . This being so, F is the highest point of flood, or the top directed towards the Moon, which I therefore call the Moon's flood top. And for the same reasons I call H its opposite's flood top.

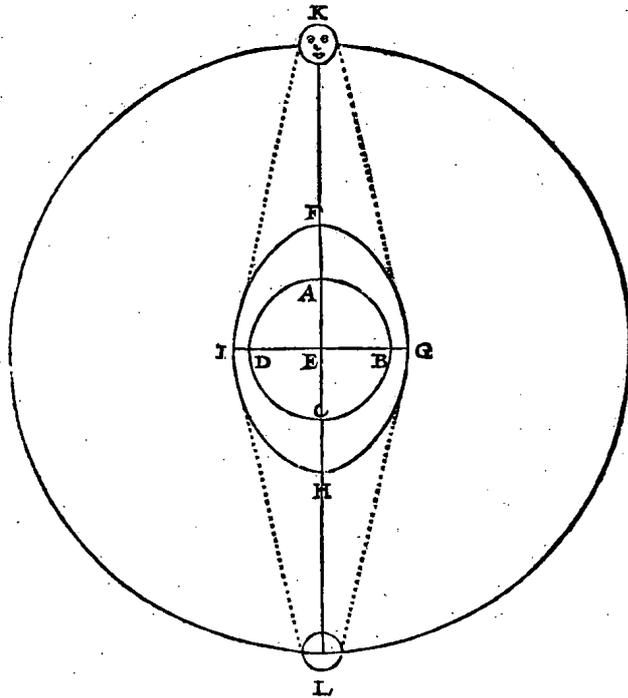
2nd DEFINITION.

We call the circle on the Earth, whose plane intersects the straight line between the two flood tops in the middle and is at right angles thereto, Ebb Circle.

EXPLANATION.

Thus the circle IEG ¹⁾, seen transversely, is called Ebb Circle, because ebb or lowest tide always occurs in this. And it is lowest of all in this, because EI is the shortest line that can be drawn from the centre of the Earth E to the water's surface.

¹⁾ IFG is a printer's or clerical error in the Dutch text.



N V D E
V O O R S T E L L E N .

I V O O R S T E L .

T'onderfoucken de ghemeene ghedaente van ebbe en vloet.

Om lijckformicheyt te hebben der gheftalt van vloettoppen en ebront tot alle plaetsen des Eertcloots, soo ist voorderlick te doen maken een ebbenvloetruych welcke op een Eertcloodt gheleyt, en daer op verschoven sijnde na ons wille, alijt de selve twee vloettoppen en ebront anwijfe: Hier toe heeft sijn VORSTELICKE GHENADE hem in dees spiegheling oeffenende, doen bereyden seker ronden van stijf papier, daer me sulcx te weghe ghebrocht wiert, t'welck yder tot derghelijcke lust hebbende, oock so sou meughen doen. Doch want op de hemelclooten der vaste sterren, seker twee ronden en vier punten ghereyckent worden, die ons verftrecken meugen voor ebronden en vloettoppen van eenighe befonder plaetsen, soo sullen wy die voorbeeldsche wijfe daer toe ghebruycken, want den sin daer me verftaen sijnde, soo falfe metten boveschreven ebbenvloetruych openbaer wesen int gemeen overal.

Q I Voor.

NOW THE PROPOSITIONS

1st PROPOSITION.

To examine the general figure of ebb and flow.

In order to have uniformity in the form of flood tops and ebb circle in all places of the Earth, it is efficacious to have a tidal instrument constructed, which, when laid on a Globe and displaced thereon as desired, shall always show the two flood tops and ebb circle. To this end his PRINCELY GRACE, when he practised the present theory, had certain circles of stiff paper prepared, with which this was brought about, a thing which anyone having a mind to it might also do in this way. But because on the celestial globe of the fixed stars are drawn two circles and four points which may serve as ebb circles and flood tops of some particular places, we will use them for this purpose by way of example, for if the meaning has been understood in this way, it will be clear in general with the above-mentioned tidal instrument.

1 Voorbeeld gaende de Maen onder de lensne, dats boven t' middelbront.

*Sethonem
vernalen &
autumnalem.* In handen hebbende een Hemelclood, ick stel my voor * lensne en herbsne de twee vloettoppen te sijn, wese de Maen onder t' begin des duyfteraers, en sal alsdan ebront sijn het rondt deur den 90 tr. en de aspunten des Eertcloots. Dit aldus wese de, soo lang de Maen onder de lensne loopt, salt tot elke plaets opt middelrondt daer de Maen boven is hoochwater sijn: En 6 uyren daer na (meer van weghen haer eyghen loop ontrent $\frac{1}{4}$ dats $6\frac{1}{4}$ uyren) sal het ebront tot die plaets ghecommen sijn, en daerom alsdan t' water ten leeghsten: En binnen ander $6\frac{1}{4}$ uyr daer nae weerom hooghe vloet, en soo overhant gheduerlick mette volghende ebbe en vloet.

2 Voorbeeld gaende de Maen onder des duyfteraers 90 tr.

Ten tweeden stel ick my op den Hemelclood voor, des duyfteraers 90 tr. en haer teghenoverpunt den 270 tr. de twee vloettoppen te sijn, wese de Maen onder den 90 tr. en sal alsdan t'ront deur de lensne, en des duyfteraers aspunten ebront sijn. Hier me en sullen de tijden tusschen ebbe en vloet niet eve groot vallen als int eerste voorbeeld, maer verschillen, en dat tot d'een plaets meer als d'ander na t'verschil haerder breeden. Om nu t'selve te vinden tot een ghegeven breedte, als van 50 tr. ick verhef den aspunt op sulcken hooghe boven den sichteinder, en breng den 90 tr. des duyfteraers onder t' middachront, keer daer na den clood tot dattet ebront deur snijt het middachront inden voorschreven 50 tr. der breedte, dats int toppunt: En sie hoe veel trappen des evenaers daerentusschen verlopen sijn, bevinde neem ick 121 tr. 13 ①, die, 15 tr. op de uyr gherekent, bedraghen 8 uyr 5 ①, voor den tijt van t'hoochste water totter leeghste: En van daer tot d'ander vloet sal sijn 3 uyr 55 ①, te weten t'verschil tusschen 8 uyr 5 ① en 12 uyren; welverstaende dat hier tot elck noch soude moeten vergaert worden t'ghene de Manens eyghenloop veroirsaect, t'welck ick cortheys halven achterlaet.

M E R C K T.

Sijn VORSTELICKE GHENADE dit boveschreven 2 voorbeeld metten Hemelclood onderfouckende, en voorder lettende opt vervolgh van verscheyden ebben en vloeden achter malcander, heeft daer in de volghende oirden bemerckt.

Den tijt van ebbe na de boveschreven vloet, sal sijn van 3 uyr 55 ①: En de volghende vloet weerom 8 uyr 5 ①. Sulcx dat by aldien de Maen gheduerlick siep in des duyfteraers 90 tr. (t'welck om de leerings wil soo mach ghestelt worden) d'oirden van ebbe en vloet soude dusdanich sijn.

Ebbe	8 uyr 5 ①
Vloet	3 uyr 55 ①
Ebbe	3 uyr 55 ①
Vloet	8 uyr 5 ①
Ebbe	8 uyr 5 ①
Vloet	3 uyr 55 ①
Ebbe	3 uyr 55 ①
Vloet	8 uyr 5 ①
Ebbe	8 uyr 5 ①

En

1st Example, the Moon moving under the vernal equinox, i.e. above the Equator.

Having in my hands a celestial globe, I imagine the vernal and the autumnal equinox to be the two flood tops when the Moon is at the beginning of the ecliptic; then the circle through 90° and the poles of the Earth will be the ebb circle. This being so, as long as the Moon is moving under the vernal equinox, it will be high tide in every place on the equator where the Moon is in the zenith. And 6 hours later (about $\frac{1}{4}$ hour more, on account of its own motion, *i.e.* $6\frac{1}{4}$ hours) the ebb circle will have reached that place, and therefore the water will then be lowest. And another $6\frac{1}{4}$ hours later it will be high flood-tide again, and so on continually with the next ebb and flow.

2nd Example, the Moon moving under the point at 90° of the Ecliptic.

Secondly I imagine on the Celestial Globe that the point at 90° and its opposite at 270° [of ecliptical longitude] are the two flood tops, the Moon being under 90° , then the circle through the vernal equinox and the poles of the ecliptic will be the ebb circle. Herewith the intervals between ebb and flow will not be equally long, as they were in the first example, but they will be different, such in one place more than in another, according to the difference of their latitudes. In order to find it at a given latitude, *e.g.* of 50° , I raise the pole to that height above the horizon and bring the 90° of the ecliptic under the meridian, then turn the globe until the ebb circle intersects the meridian at the aforesaid 50° of latitude, *i.e.* in the zenith, and I ascertain how many degrees of the equator have passed between the two positions. I find *e.g.* $121^\circ 13'$, which — taking 15° to the hour — is equivalent to 8 hours 5 minutes for the interval from the highest to the lowest tide. And from there to the next high tide will be 3 hours 55 minutes, to wit, the difference between 8 hours 5 minutes and 12 hours, it being understood that to each there would still have to be added what the Moon's own motion causes, which I omit for brevity's sake.

NOTE.

When his PRINCELY GRACE examined the above-mentioned 2nd example with the Celestial Globe and further observed the sequence of many successive tides, he noted in this the following regularity.

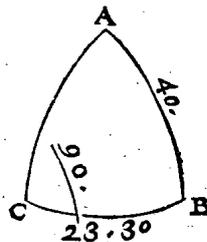
The time from the above flood-tide to the next ebb-tide will be 3 hours 55 minutes, and then to the next flood-tide again 8 hours 5 minutes, so that if the Moon were continually at 90° of the ecliptic (which may thus be assumed for didactic purposes), the order of the tides would be as follows:

Ebb-tide	after 8 hours	5 minutes
Flood-tide	„	3 hours 55 minutes
Ebb-tide	„	3 hours 55 minutes
Flood-tide	„	8 hours 5 minutes
Ebb-tide	„	8 hours 5 minutes
Flood-tide	„	3 hours 55 minutes
Ebb-tide	„	3 hours 55 minutes
Flood-tide	„	8 hours 5 minutes
Ebb-tide	„	8 hours 5 minutes

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En soo gheduerlick voort. Doch valt hier daghelicx noch sulcke verandering by, als de Maenloop veroirfaect.

Maer om de voorschreven vinding des tijts deur rekening der clootsche driehoucken af te veerdigen, ick aensie int eynde der voorgaende wercking op den Hemelcloot, de gestalt des driehoucx begrepen tusschen de drie punten als Eertcloots aspunt, Duysteraers aspunt, metter toppunt, en merck den houck an des duysteraers aspunt recht te wesen: De sijde tusschen beyde de aspunten te doen 23 tr. 30 ①: En de sijde van des Eertcloots aspunt tottet toppunt 40 tr. te weten schilbooch vande gegevē breede 50 tr. Sulckē driehouc teycken ic hier als ABC, alwaer A het toppunt bediet, B des Eertcloots aspunt, C des duysteraers aspunt, wefende den houck ande selve C recht, de sijde AB van 40 tr. en BC 23 tr. 30 ①. Sulcx dat den driehouck drie bekende palen heeft, waer me ghesocht den houck B, wort deur het 32 voorstel der clootsche driehoucken bevonden van 58 tr. 47 ①: Die ghetrocken van 180 tr. blijft 121 tr. 13 ①, en foogrootis den verlopen evenaerbooch die doet voor t'begheerde 8 uyr 5 ①.



T B E W Y S.

B A, B C, voortghetrocken totten evenaer, begrijpen daer des evenaers booch als grootheyt des houcx B, dats 58 tr. 47 ①: Maer de voortghetrocken BC valt in des duysteraers 270 tr. tusschen welke en des evenaers 90 tr. sijn 180 tr. die ten tijde des hooghen waters int middachront was, daerom vanden tijt des hoogsten waters totten tijt des leeghsten dattet ebront deursneet het middachront inden 50 tr. der breede, sijn verlopen 180 tr. min den houck B 58 tr. 47 ①, dats 121 tr. 13 ①, als int werck.

1 V E R V O L G H.

T water is op de Eertcloots aspunten ten leeghsten, loopende de Maen onder den evenaer als gheseyt is int 1 voorbeelt; En ten hoogsten wefende in haer uyerste noortsche en zuysche breede, waer uyt volght datmen daer met elck Maenschijn maer tweemaal hooch water en heeft, en tweemaal leeghwater: En dattet ten leeghsten sijnde daer na ontrent 7 daghen lanck waft, en weerom ontrent 7 daghen daelt, en so overhant voort. Doch crijcht dese oirdentlicke ebbe en vloed soo daer als overal eenighe verandering deur dien de vloettoppen hoogher rijzen, in saming en teghestant van Son en Maen, dan alsse een vierendeelronts schijnbaerlick van malcander sijn.

2 V E R V O L G H.

Tis kennelick dat ebbe en vloed opt middelront elck altijt ghedueren ontrent $6\frac{1}{4}$ uyren, tot wat plaets de Maen oock is. Lactse by voorbeelt sijn in des Duysteraers 90 tr. Dit soo ghenomen, het ront deur le lentse en weerelts aspunten streckende is dan ebront, dat openbaerlick 6 uyren in evenaerloop verschilt vanden selven 90 tr. des duysteraers: En derghelelike bevint sich oock alsoo tot alle plaetsen daer de Maen is, ghelijckmen lichtelicker siet deur t'behulpeens papieren ebbenvloettuychs daer int begin des 1 voorstels afgheseyt is.

Q 3

3 VER-

And thus continually on. But to this, daily changes have to be added as caused by the Moon.

However, in order to effect the aforesaid finding of the time by means of spherical triangles, at the end of the foregoing operation on the Celestial Globe I note the form of the triangle contained between the following three points: the pole of the Earth, the pole of the Ecliptic, and the zenith, and I note that the angle at the pole of the ecliptic is right, that the side between the two poles is $23^{\circ}30'$, and that the side from the pole of the Earth to the zenith is 40° , to wit, the complement of the given latitude of 50° . I here draw this triangle, namely ABC , where A denotes the zenith, B the pole of the Earth, C the pole of the ecliptic, the angle at the said C being right, the side AB 40° , and BC $23^{\circ}30'$, so that the triangle has three known terms. When by these means the angle B is sought, by the 32nd proposition of spherical triangles ¹⁾ it is found to be $58^{\circ}47'$. When this is subtracted from 180° , there remains $121^{\circ}13'$, and this is the arc of the equator passed through, which is equivalent to the required 8 hours 5 minutes.

PROOF.

When BA , BC are produced to the equator, they there contain the equator's arc which is the magnitude of the angle B , *i.e.* $58^{\circ}47'$. But BC produced passes through 270° of the ecliptic, and between this and 90° of the equator there are 180° , which were in the meridian at the time of high tide; therefore from the time of the highest tide to the time of the lowest, when the ebb circle intersected the meridian at 50° of latitude, 180° have passed *minus* the angle B of $58^{\circ}47'$, *i.e.* $121^{\circ}13'$, as in the trigonometrical operation.

1st SEQUEL.

The water is lowest at the poles of the Earth when the Moon moves under the equator, as has been said in the 1st example, and highest when it is at its extreme northerly and southerly latitudes, from which it follows that there they have twice high tide and twice low tide in every lunation, and that when it is lowest, thereafter it rises for about 7 days and ebbs again for about 7 days, and thus continually. But this regular ebb and flow will undergo some change there as well as everywhere because the flood tops rise higher when Sun and Moon are in conjunction and opposition than when they are apparently a quarter circle apart.

2nd SEQUEL.

It is evident that on the equator low and high tide each always lasts about $6\frac{1}{4}$ hours, no matter in what place the Moon is. Let it be, for example, at 90° of the Ecliptic. This being assumed, the circle passing through the vernal equinox and the poles of the ecliptic ²⁾ is then the ebb circle, which evidently differs 6 hours in equatorial motion from the said 90° of the ecliptic. And the same is also found in all places where the Moon is, as is seen more easily by means of a tidal instrument of paper as referred to at the beginning of the 1st proposition.

¹⁾ Stevin's *Spherical Trigonometry* (Work XI; i, 13), p. 234.

²⁾ For *weerelks aspunten* in the Dutch text read *duysteraers aspunten*.

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3 V E R V O L G H.

Het blijkt op den Eertclood, dat plaetsen naerder den aspunt, dan de Maen den evenaer, aldan vant ebront niet gherocht en worden: Sulcx datse op dien tijt soo leeghen ebbe niet en crijghen, als wanneer het ebront daer over comt.

M E R C K T.

Alfoo sijn VORSTELICKE GHENADE het inhoud des voorgaende 3 vervolghs deur een Hemelclood ondersocht, heeft met een willen weten, wat tijt datter tot sulcke plaets verloopt, tusschen het leeghste der ebbe en hoochste des vloets; Maer bevant (soo yghelick sal die derghelijcke doet) al-tijt 12 uyren, besloot daer uyt dat sulck geval somwijlen soude moeten over-commen alle plaetsen diens breedte over de 61 tr. 30 ① streckt, want soo veel blijfter alsmen de Manens grootste evenaerbreede 28 tr. 30 ①, treckt van 90 tr.

Nu ghedenckt myeenich zeevarent volck te hebben hooren bevestighen, datse tot plaetsen gheweest hadden daert 12 uyren ebde, en 12 uyren vloeyde, doch dat sulcx niet lang en duerde, dan dat daer na weerom een onghereghelde ebbe en vloet volghde. Maer of dit om dese oirsaeck gheschiet, daer soudemen bescheyt af connen weten, alsmen de ervaringhen te werck stelde daer int 9 voorstel af gheseyt sal worden.

4 V E R V O L G H.

Tis kennelick dattet een der twee vloettoppen t'welck een gestelde plaets naeft comt, aldaer hoogher vloet veroirsaeckt als rander. Waer uyt volght dat wesende de Maen over de noortsijde, soo sullen des Eertcloots plaetsen over de noortsijde ghelegghen hoogher vloet crijgen vande Manens vloettop, als van haer teghepunts vloettop. Maer wesende het teghepunt over de noortsijde, dat aldan t'verkeerde ghebeuren sal.

M E R C K T.

Uyt het voorgaende valt te besluyten, datter op den Eertclood vier merkelicke verscheydenheden van ebben en vloedden sijn angaende de ghedue-richeyt.

Ten eersten opt middelront alijt van ontrent $6\frac{1}{4}$ uyren, als int 1 voorbeelt en 2 vervolgh.

Ten tweeden buyten t'middelront, tot op ontrent den 61 tr. 30 ① der bree-de, alwaerse verschillen connen na t'inhout des 2 voorbeelts.

Ten derden vanden 61 tr. 30 ① tot byden aspunt, daerse tot sommige tijden elck van 12 uyren sijn, als int Merck des 3 vervolghs.

Ten vierden onder den aspunt, daer ebbe en vloet alijt elck ontrent 7 daghen duiren als int 1 vervolgh.

5 V E R V O L G H.

Tis kennelick dat deur de boveschreven twee voorbeelden en vervolghen, in welcke vloettoppen en ebronden op den Hemelclood gheteyckent waren, lichtelick can verstaen worden de ghemeene regel van ander voorbeelden, diens vloettoppen en ebront daer op niet en sijn, doch daer op beteyckent connen worden metten ebbenvloettuych daer int begin deses voorstels af geseyt

3rd SEQUEL.

It appears on the Earth that places nearer to the pole than the Moon is to the equator are not reached by the ebb circle, so that at that time they do not get as low an ebb-tide as when the ebb circle passes through them.

NOTE.

When his PRINCELY GRACE examined the contents of the foregoing 3rd sequel by means of a Celestial Globe, he wanted to know at the same time how much time elapses in that place between the lowest point of the ebb-tide and the highest point of the flood-tide. But he found (as will anyone who does so) this was always 12 hours. From this he concluded that this case would sometimes have to occur for all those places whose latitude is more than $61^{\circ}30'$, for this is what remains when the Moon's greatest equatorial latitude of $28^{\circ}30'$ is subtracted from 90° .

Now I remember that I have heard certain mariners assert that they had been in places where the water ebb'd during 12 hours and flow'd during 12 hours, but that this did not last long and was followed again by an irregular ebb and flow. But whether this is due to this cause might become known if the experiences to be dealt with in the 9th proposition were put to use.

4th SEQUEL.

It is evident that the one of the two flood tops which comes nearest to a given place causes a higher flood-tide there than the other. From this it follows that when the Moon is on the north side, the places on the Earth lying on the north side will get a higher flood-tide from the Moon's flood top than from the flood top of its opposite. But when the opposite is on the north side, the contrary will happen.

NOTE.

From the foregoing it is to be concluded that on the Earth there are four notable diversities between the tides as to their alternating duration.

Firstly, on the equator always about $6\frac{1}{4}$ hours, as in the 1st example and the 2nd sequel.

Secondly, outside the equator, up to about $61^{\circ}30'$ of latitude, where they may differ, according to the 2nd example.

Thirdly, from $61^{\circ}30'$ to near the pole, where at some times they both last 12 hours, as in the Note to the 3rd sequel.

Fourthly, at the pole, where ebb- and flood-tide each always lasts about 7 days, as in the 1st sequel.

5th SEQUEL.

It is evident that from the above two examples and sequels, in which flood tops and ebb circles were drawn on the Celestial Globe, the general rule for other examples can easily be understood, for which the flood tops and ebb circle are not present thereon, but can be drawn thereon with the tidal instrument

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 gefeyt is. T B E S L V Y T. Wy hebben dan onderfocht de gemeene gedaente
 van ebbe en vloet, na den eyfch.

2 VOORSTEL.

Te verclaren d'oirfaeck vvaerom na de hooghe sprinck-
 vloeden leegher ebben volghen dan na leeghe vloeden.

T'wort dadelick bevonden dat de ebbe die nae sprinckvloet volght, leegher
 comt dan de ebbe na vloet ontrent vierdeschijn: Sulcx datmen in zeevercken
 die op diepe drooghe gront moeten ghemaect worden, als onder anderen nu
 ter tijt inde belegherde Stadt Ostende, na sprinckvloet wacht, om het strant lee-
 ghe drooch te hebben. D'oirfaeck hier af is openbaer om dese reden: Anghe-
 sien inde form van d'eerste bepaling t'water met sprinckvloet, die in saming en
 teghestant ghebeurt, in meerder menichte hoogher ghetrocken wort an F en H,
 dan ten tijde van vierdeschijn, soo moetet aldan opt ebront I G meer ghebren-
 ken, en daer minder en leegher wesen dan ten tijde van vierdeschijn.

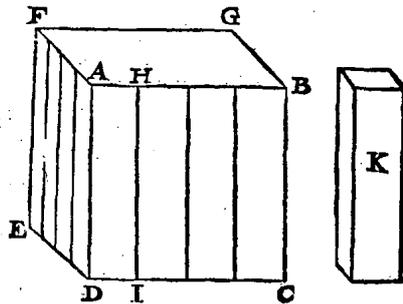
T B E S L V Y T. Wy hebben dan verclaert d'oirfaeck waerom na de hooghe
 sprinckvloeden leegher ebben volghen dan na leeghe vloeden, na den eyfch.

Tot hier toe sijn de voorstellen gheweest vande gemeene eyghenschappen
 van ebbe en vloet: De volghende sullen van besonder wesen, te weten van
 de oirfaken waerom op besonder plaetsen des Eertcloots, de voorgaende ghe-
 meene reghels gheen plaets en houden.

3 VOORSTEL.

Te verclaren de reden, vvaerom cleene vvateten vande
 Maen en haer teghepunt, soo hoog niet ghetrocken en
 vvorden als groote.

Men merckt niet dat cleene waterkens, als binnelantsche meerkens, grachten,
 water in een glas of ander vat, vande Maen opghetrocken worden: Nochtans
 mocht ymant dencken, so de eyghenschap der Maen waer t'water na heur inde
 hooghde te trecken, dat soude so wel over sulcke cleene behooren te geschien,
 als over ander der groote zee: Ia datse de cleyne noch hoogher behoort te trec-
 ken, deur diense lichter sijn dan de groote. Om hier af de reden te verclaren, laet
 A B C D E F G een vat vol waters sijn, elcke sijde als A B C D een rechthouck
 wefende 4 voet lanck, en 4 voet breed, en sal tegen de selve sijde A B C D perssen
 een ghewicht even an t'ghewicht
 van 32 voeten waters deur het
 15 voorstel vande beginselen des
 waterwichts, en t'geheel water sal
 begripen 64 voeten. Ghenomen
 voort dat elcke sijde die wy gestelt
 hebben op 4 voet lanck en breed,
 ghemaect sy van 4 plancken elck
 lanck 4 voet, en breed 1 voet, als de
 planck A H I D met dierghelijcke,
 en sal teghen elcke planck ancom-
 men een ghewicht even an t'ghe-



Q 3

wicht

mentioned at the beginning of this proposition. CONCLUSION. We have thus examined the general figure of ebb and flow; as required.

2nd PROPOSITION.

To explain the reason why the high spring-tides are followed by lower ebb-tides than are low flood-tides.

It is found in practice that the ebb-tide following spring-tide is lower than the ebb-tide after flood-tide at quadrature, so that builders of coastal defences, which have to be made on deep, dry ground, such as *e.g.* at the present time in the besieged City of Ostend, wait until after spring-tide, to have the beach low and dry¹). The cause of this is evident for the following reason: Since in the figure of the first definition the water at spring-tide, which occurs at conjunction and opposition, is raised higher in larger quantities at *F* and *H* than at quadrature, on the ebb circle *IG* there must be a greater shortage and there it must be less and lower than at the time of quadrature.

CONCLUSION. We have thus explained the reason why the high spring-tides are followed by lower ebb-tides than are low flood-tides; as required.

Up to this point the propositions related to the general properties of ebb and flow. The following are to relate to particular properties, to wit, to the reasons why in particular places of the Earth the foregoing general rules do not apply.

3rd PROPOSITION.

To explain the reason why small waters are not raised as high as big waters by the Moon and its opposite.

Small waters, such as inland lakes, canals, water in a glass or some other vessel are not perceived to be raised by the Moon. And yet a man might think that if the property of the Moon were to raise the water up to it, this would have to happen with such small waters just as well as with others, of the big seas; nay, that it even ought to raise the smaller ones higher still, because they are lighter than the big ones. In order to explain the reason of this, let *ABCDEFGH* be a vessel full of water, each side (*e.g.* *ABCD*) being a rectangle 4 feet long and 4 feet broad; against this side *ABCD* there will press a weight equal to the weight of 32 [cubic] feet of water, by the 15th proposition of the elements of hydrostatics, and the whole of the water will be 64 [cubic] feet. Assuming further that each side, which we have said to be 4 feet long and broad, be made of 4 boards, each 4 feet long and 1 foot broad, such as the board *AHID* and the like, then the weight pressing against each board will be equal to the weight

¹) This passage gives rather precise information on the year when Stevin's *Ebb and Flow* was written. Ostend was besieged from 1601 to 1604 by the Archduke Albert and the Spanish general Spinola; finally it was taken and entirely destroyed.

wicht van 8 voeten waters, als vierendeel vande 32 voeten, en sullender in als sijn 16 sulcke plancken. Ghenomen voort dat elcke planck om in die standt ghehouden te worden, een teghendrucking moet hebben van buyten, even so stijf als t'water van binnen, dat is t'ghewicht ghelijck voor gheseyt is van 8 voet waters: Hier toe neem ick te wesen 64 versheyden crachten, of om by claerder voorbeeld te spreken 64 mannen, te wesen soo veel alser voeten waters int vat sijn, en sal commen tegen elcke planck 4 mannen. Laet nu K een ander cleender vat vol waters sijn, ghemaectt alleenlick van 4 plancken, even en ghelijck mette voorgaende, en sal dat vat begrijpen 4 voet waters, staende 4 voet hooch. Dit soo sijnde, dat ymant nu aldus seyde: Anghesien 64 mannen ant groot vat connen houden 64 voeten waters 4 voet hooch, soo connen 4 mannen ant cleen vat houden 4 voet waters 4 voet hooch, en wilde over sulcx teghen elcke der vier plancken des cleen vats K, stellen alleenlick een man, daer tegen yder planck des groot vats ghesfelt sijn 4 mannen, dat waer ghemist, om dat teghen elcke planck des cleen vats, even soo veel ghewicht perft als teghen elcke planck des groot vats deur het 11 voorstel vande beginselen des waterwichts. Sulcx dat teghen de 4 plancken des cleen vats, niet en souden moeten ghesfelt sijn 4 mannen, om t'water in die standt te houden, maer 16 mannen: En vervolghens 4 mannen en sullen 4 voeten waters int cleen vat soo hooch niet connen opperffen, als 64 mannen sullen connen opperffen 64 voeten waters int groot vat. Waer uyt wijder volght, dat soo een bevelhebber over sulck volck, tot een cleen water een menichte van mannen veroirdende, in sulcken reden totte mannen teghen een groot water, als het cleen water tottet groot, dat hy daer mede cleene wateren soo hooch niet perffen en sal als de groote: En alsoo is der ghelijcke te verstaen mette Maen en haer teghepunt, welcke deur haer trecking over eve veel waters eve veel ghewelts doende, en trecken de cleene wateren soo hooch niet als de groote: En vervolghens hoe wel sy heur treckende werck soo stetck doen op cleene waterkens als op groote, soo en cant nochtans op de cleene om de bovenschreven oirsaken niet bemerckt worden.

T B E S L V Y T. Wy hebben dan verclaert de reden waerom cleene wateren vande Maen en haer teghepunt, so hooch niet ghetrocken en worden als groote, na den eysch.

4 V O O R S T E L.

Te verclaren de reden vvaerom de vloet tot veel plaetsen niet an en comt van oosten na vvesten, ghelijck de gemeene reghel der spiegheling mebrengt.

Het blijkt deur het 2 voorstel, dat cleene waters soo hooch niet ghetrocken en worden als groote: Hier uyt volght dat rivieren cleene waters sijnde, in haer selven gheen vloet en hebben, en dat de vloet diemender in siet, niet uyt de rivier, maer uyt de zee comt, waer uyt wijder volght, dat alst vande mont der rivier opwaert, of na de lantsijde, van westen na oosten streckt, dat dan de vloet daer in moet loopen, teghen de ghemeene reghel van westen na oosten, en die strecking der rivier anders sijnde, so comt den vloet oock anders in. Als de vloet commende uyt zee in Schelde loopt van Berghen op Zoom na Antwerpen zuytwaert van daer na Baestroot ghelijck haer de form der rivier leyt. Tgene wy hier gheseyt hebben vande zee en een rivier, verstaet hem oock alsoo met een
grootte

of 8 [cubic] feet of water (being one fourth of the 32 [cubic] feet), and there will be 16 such boards in all. It is further assumed that each board, to be kept in that position, must be subjected to a counter-pressure from the outside as great as that of the water on the inside, *i.e.* the weight of 8 [cubic] feet of water, referred to before. For this I assume there are 64 distinct forces or, to give a clearer example, 64 men, to wit, as many as there are [cubic] feet of water in the vessel; then against each board will press 4 men. Now let *K* be another — smaller — vessel full of water, made of only 4 boards, equal and similar to the foregoing; then that vessel will contain 4 [cubic] feet of water, standing 4 feet high. This being so, if a man now said as follows: Since at the big vessel 64 men can retain 64 feet of water, 4 feet high, at the small vessel 4 men can retain 4 feet of water, 4 feet high, and because of this he wanted to put only one man against each of the four boards of the small vessel *K*, whereas against each board of the big vessel are placed 4 men, this would be wrong, because the same weight presses against each board of the small vessel as against each board of the big vessel, by the 11th proposition of the elements of hydrostatics ¹⁾; so that against [each of] the 4 boards of the small vessel there would not have to be placed 4 men to keep the water in that position, but 16 men. And consequently 4 men will not be able to raise 4 feet of water in the small vessel to the same height as 64 men will be able to raise 64 feet of water in the big vessel. From this it follows further that if a commander of such men were to order to a small water a number of men having the same ratio to the men he ordered to a big water as the small water itself has to the big water, he will not thus raise the small as high as the big waters. And the same is to be understood for the Moon and its opposite, which, by their attraction exercising the same force on the same quantity of water, do not raise the small as high as the big waters. And consequently, though they exercise their attraction as much on small as on big waters, nevertheless it cannot be perceived on the small ones, for the above-mentioned reasons.

CONCLUSION. We have thus explained the reason why small waters are not raised as high as big waters by the Moon and its opposite; as required.

4th PROPOSITION.

To explain the reason why in many places the flood does not come from east to west, as would be in accordance with the general rule of the theory.

It appears from the 2nd proposition that small waters are not raised as high as big waters. From this it follows that rivers, being small waters, do not in themselves have a flood, and that the flood seen in them does not come from the river, but from the sea; from which it follows further that if the direction from the mouth of the river upwards, or to the side of the land, is from west to east, the flood must enter, against the general rule, from west to east, and if the direction of the river is different, the flood also enters differently. Thus the flood coming from the sea to the Scheldt passes from Berghen op Zoom to Antwerp southwards, from there to Baestroot according as the form of the river

¹⁾ This has been demonstrated in *The Elements of Hydrostatics*, Proposition XI. See Vol. I, p. 421.

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grootte zee en een cleene: Als by voorbeeld, men siet langs de Fransche, Vlaemsche en Hollantsche stranden, den vloet incommen uyt westen na oosten, en dat deur dien de grootte wijde Spaensche zee streckende tot America toe, ghescheyden wort met Engelant en Schotland, vande cleene Duytsche zee, in welke sulcke verheffing niet wesende als in d'ander, soo comt de grootte vloet uyt de grootte zee tusschen Enghelant en Vranckrijck daer invallen van westen na oosten, ghelijck wy vooren van t'vallen des vloets in een rivier gheseyt hebben; T welck in veel zeen schijnende teghen de ghemeene reghel der spiegheling te wesen, nochtans om bekende oirsaken soo wesen moet.

T B E S L V Y T. Wy hebben dan verclaert de reden waerom de vloet tot veel plaetsen niet an en comt van oosten na westen, ghelijck de ghemeene reghel der spiegheling mebrengt, na den eysch.

5 VOORSTEL.

Te verclaren d'oirsaek, vvaerom dattet tot veel plaetsen gheen hoochste vwater en is, vvesende de Maen of haer teghenoverpunt int middachront, ghelijck de ghemeene reghel der spiegheling mebrengt.

Angesien de vloet der cleene zeen, veroirsaekt wort deur de vloet der grootte ghelijck int 4 voorstel gheseyt is, soo volght daer uyt dat de plaetsen der cleene zee naest de grootte ghelegen, eer hooch water moeten hebben als ander verder plaetsen der cleene zee: Daerom alwaert inde grootte zee altijd hooch water wesende de Maen of haer teghepunt int middachront, soo en cant op den selven tijt tot ander plaetsen verre ghenouch inde cleene zee gheen hoochste water sijn: Ghelijckmen siet in ons cleene Duytsche zee, alwaer de oostlicker plaetsen later vloet hebben dan de westlicker. **T B E S L V Y T.** Wy hebben dan verclaert d'oirsaek waerom dattet tot veel plaetsen gheen hoochste water en is wesende de Maen of haer teghenoverpunt int middachront, ghelijck de ghemeene reghel der spiegheling mebrengt, na den eysch.

6 VOORSTEL.

Te verclaren d'oirsaek vvaerom sprinckvloet tot somighe plaetsen deurgaens eenighe dagen later comt dan met volle of nieu Maen.

Int 5 voorstel is gheseyt, dattet hoochste water tot ettelicke plaetsen eenighe uyren gheschiet na de comst der Maen of haer teghenoverpunts int middachront, waer uyt niet vreemt en is dat de sprinckvloeden die altijd metten hoogen vloet commen, aldaer oock soo veel uyren later vallen dan na de ghemeene reghel: Maer ettelicke daghen van volle en nieu Maen te verschillen ghelijck dadelick bevonden wort (want voor Hollandt, soo ick van zeevolck verstaec, verschillet over de twee daghen, westwaert min, oostwaert meer) dat mocht ymant bedencking gheven. Om hier af d'oirsaek te verclaren, ick seggh voor al kennelick te wesen deur t'ghene gheseyt is int 4 voorstel, dat sprinckvloet der rivieren en cleene zeen, niet en commen uyt haer selven, maer veroirsaekt sijn deur de sprinckvloet der grootte zee, waer uyt volght dattet verschil des tijts tusschen de

Q 4

sprinck-

directs it ¹⁾). What we have here said about the sea and a river applies in the same way to a big and a small sea. For example, along the French, Flemish, and Dutch shores the flood is seen to come from west to east, such because the big, wide Spanish sea, extending to America, is separated by England and Scotland from the small Dutch sea, and since in the latter there is not the same elevation as in the other, the big flood enters from the big sea between England and France from west to east, as we have said before about the entrance of the flood into a river; and though this seems to be against the general rule of the theory in many seas, this must be so for known reasons.

CONCLUSION. We have thus explained the reason why in many places the flood does not come from east to west, in accordance with the general rule of the theory; as required.

5th PROPOSITION.

To explain the reason why in many places the tide is not highest when the Moon or its opposite is in the meridian, as would be in accordance with the general rule of the theory.

Since the flood of the small seas is caused by the flood of the big seas, as has been said in the 4th proposition, it follows that those places of the small sea which are situated nearest to the big sea must have high tide sooner than other remoter places of the small sea. Therefore, even if it were always high tide in the big sea when the Moon or its opposite is in the meridian, at the same time the tide may not be highest in other places far enough away in the small sea, as is seen in our small Dutch sea, where the more eastward places have the flood-tide later than the more westward places. CONCLUSION. We have thus explained the reason why in many places the tide is not highest when the Moon or its opposite is in the meridian, as would be in accordance with the general rule of the theory; as required.

6th PROPOSITION.

To explain the reason why in some places spring-tide usually occurs a few days later than full or new Moon.

In the 5th proposition it has been said that in several places the highest tide occurs a few hours after the Moon or its opposite is in the meridian, so that it is not strange that the spring-tides always occurring at the high flood-tide also occur there so many hours later than according to the general rule. But that they should differ several days from full and new Moon, as is found in practice (for in Holland, as I understand from mariners, the difference is more than two days, to the west less and to the east more), this might give rise to objections. To explain the cause of this, I say first of all that it is evident from what has been said in the 4th proposition that spring-tides of rivers and small seas are not due to themselves, but are caused by the spring-tide of the big sea, from which it follows that the difference in time between the spring-tides of a big and a small

¹⁾ See the map on pag. 358.

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springvloeden van een groote en cleene zee, grooter can sijn dan t'verschil des tijts tusschen de springvloet der groote zee en een rivier, uyt oirsaeck dat de rivierens springvloet om het afcommende water dat achter veel hoogher is, niet soo verre loopen en can als de springvloet der cleene zee, die van achter soo veel niet hoogher en is. Om hier af deur een form breeder verclaring te doen, laet A B C D E F G het oppervlack der cleene zee bereykenen, waer af de verheventheyt A, sy ande mont der groote zee beduydende den hooghen springvloet, welke binnen ses uyren daer na ten leeghsten sijnde, en ses uyren daer na weerom ten hooghsten, soo wort die cleene zee in gheduerlicke beweging ghehouden, met verscheyden hooghe springvloeden, en ebben tusschen beyden, d'een achter d'ander, al veroirsaect uyt den eersten hooghen springvloet A, wordende allenx cleender en cleender, sulcx datmen ten laetsten geen



ebbe en vloet meer en merckt. Maer wanttet eenighe daghen can anloopen eer den eersten hooghen springvloet A, comt totten laetsten hooghen springvloet G, en dat de hooghe vloeden in alle nieu en volle Maen oock den spring me brenghen als ghefeyt is, soo en can den grootsten vloet diemen spring noemt an A ten grootsten wesende, haer aldergrootsten vloet te weten spring an G, niet veroirsaken dan soo veel tijts na nieu of volle Maen, als den eersten spring an A, behouft om over al de springvloeden B, C, D, E, F, tot G te commen, t'welck na dattet verre is eitelicke daghen an can loopen: Als by voorbeeld: Angesien het na luyt der Almenacken, met volle en nieu Maen hooch water is tot Calis ten 10 uyren: Tot Nieupoort ten 11 uyren: Tot Ostende ten 11½ uyren. Tot Blanckeberge te 1 uyr: Tot Vlissinghen ten 2 uyren: Tot Bergen op Zoom ten 4 uyren: Tot Antwerpen ten 6 uyren: Tot Baestroo ten 8 uyren: Soo is de vloet 1 uyr doende met te commen van Calis tot Nienpoort: ½ uyr van Nieupoort tot Ostende: 2 uyren van Ostende tot Blanckeberge: ½ uyr van Blanckeberge tot Vlissing: 2 uyren van Vlissing tot Berghen op Zoom: 2 uyren van Berghen op Zoom tot Antwerpen: 2 uyren van Antwerpe tot Baestroode: Maeckt t'samen om den vloet te commen van Calis tot Baestroo 10 uyren. Waer uyt blijkt dat wanneert te Calis springvloet is, so moetet te Baestroo 10 uyren daer na eerst springvloet wesen. En die dergelijcke berekende op een plaets van Baestroo veel verder westwaert dan Calis, soude also in plaets van 10 uyren meughen vinden verschil van eenighe daghen: Daerom al ist dat springvloet ter plaets daer geen hinder en is, met volle Maen comt, volghende de ghemeene regel der spiegheling, soo moet nochtans om t'belet der uytstekende lande, tot sommighe plaetsen de springvloet eenighe daghen nae volle en nieu Maen commen. **T B E S L V Y T.** Wy hebben dan verclaert d'oirsaeck, waerom springvloet tot sommighe plaetsen deurgaens eenighe daghen later comt dan met volle of nieu Maen, na den eyfch.

7 VOORSTEL.

Te verclaren d'oirsaeck vvaerom tot sommighe plaetsen verder vanden vloettop ghelegghen als ander, nochtans hoogher vloet comt.

Na

sea may be greater than the difference in time between the spring-tide of the big sea and a river, because the spring-tide of a river, on account of the water flowing downward, which is much higher behind, cannot proceed as far as the spring-tide of the small sea, which is not so much higher behind. In order to explain this more fully by means of a figure, let *ABCDEFG* denote the surface of the small sea, of which let the elevation *A* be at the mouth of the big sea, denoting the high spring-tide, and since this is lowest six hours afterwards and six hours later highest again, this small sea is kept in continual movement, with several high spring-tides and ebb-tides between them, one after the other, all caused by the first high spring-tide *A*, which gradually become smaller and smaller, so that finally no ebb and flow is perceived any more. But because it may be some days before the first high spring-tide *A* reaches the last high spring-tide *G* and the high floods at every new and full Moon also bring on the spring-tide, as has been said, the greatest flood-tide, which is called spring-tide at *A*, when at its greatest, can only cause its greatest flood of all, to wit, the spring-tide at *G*, as much time after new or full Moon as the first spring-tide at *A* requires to reach *G* *via* all the spring-tides *B, C, D, E, F*, which may be several days, according to the distance. For example 1): according to the Almanacs at full and new Moon it is high tide at Calais at 10 o'clock, at Nieuwpoort at 11, at Ostend at 11.30, at Blankenberge at 1, at Flushing at 2, at Bergen op Zoom at 4, at Antwerp at 6, at Baestrou at 8. The flood thus takes 1 hour to come from Calais to Nieuwpoort, $\frac{1}{2}$ hour from Nieuwpoort to Ostend, 2 hours from Ostend to Blankenberge, $\frac{1}{2}$ hour 2) from Blankenberge to Flushing, 2 hours from Flushing to Berghen op Zoom, 2 hours from Berghen op Zoom to Antwerp, 2 hours from Antwerp to Baestroode. This makes together 10 hours for the flood to come from Calais to Baestrou. From this it appears that when it is spring-tide at Calais, it must be spring-tide at Baestrou as much as 10 hours later. And if anyone were to calculate this for a place much further to the west of Baestrou than Calais, he might find instead of 10 hours a difference of some days. Therefore, though spring-tide comes at full Moon in a place where there is no impediment, according to the general rule of the theory, nevertheless in some places, on account of the impediment formed by lands sticking out, the spring-tide must come some days after full and new Moon. CONCLUSION. We have thus explained the reason why in some places spring-tide usually occurs a few days later than full or new Moon; as required.

7th PROPOSITION.

To explain the reason why in some places situated further away from the flood top than others the flood-tide may yet be higher.

1) See the map on p. 358.

2) These numbers fit only if it were high tide at Blankenberge at 1.30 p.m.

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Na de ghemeene reghel der spiegheling, so behoort den vloet ten hoochsten te wesen anden vloettop, en da plaetsen die den vloettop naerder sijn, behooren den vloet oock hoogher te hebben als ander verder daer af ghelegen; nochtans sietmen dadelick tot veel plaetsen t'verkeerde ghebeuren. Om hier af d'oirfaeck te verklaren, soo seggh ick eerst by voorbeelt aldus: Men siet dat een bare waters commende teghen een hooft in zee uytstekende, datse daer vooren verhoocht, ja daer overloopt veel hoogher als ander baren in zee: De reden daer af is dese: Soo t'voorste der baer teghen t'hooft steutende alleen waer sonder vervolgh van water, het soude als kennelick is strax te rugghe keeren sonder sulcke hooge verheffing te doen: Maer de rest der sware groote baer op haer ganck weseude, en can op soo corten tijt niet keeren, dan het voorste water wort opghehouden van het volghende, en dat volghende oock van ander volghende, sulcx dat t'een t'ander onderhoudende en voortdringhende, het gheraecht tot die voorschreven verheffing, hoogher dan t'ghemeen water der zee. Dit verstaen sijnde, laet ons liet heel verheven water eens vloets nemen voor een baer, welke ettelicke mijlen lanck sijnde, ten is gheen wonder dat daer af tot sommighe plaetsen daerse recht tegen an comt, hoogher verheffing gheschiet dan in zee, als by voorbeelt de vloet der groote Spaensche zee, treckende na de leeghe cleyne Duytsche zee als boven geseyt is, en vallende teghen de landen des inhams van Bretagne, crijcht een verheffing boven de ebbe van 10 of 11 vamen, dats hoogher dan de vloet tot ander plaetsen die den vloettop naerder sijn, ja hoogher dan onder den vloettop self, en dat om bekende reden. **T B E S L V Y T.** Wy hebben dan verclaert d'oirfaeck; waerom tot sommighe plaetsen verder vanden vloettop ghelegen als ander, nochtans hoogher vloet comt, na den eyfch.

8 VOORSTEL.

Te verklaren d'eygenschappen dieder van ebbe en vloet sijn fouden, soose gheschiede deur persing der Maen en haer teghepunt.

Wy hebben tot hier toe de faeck ghenomen, al of de Maen en haer teghepunt het zeewater na hun trock of foghen, volghende t'inhoudt van d'eerste begheerte: Maer sooder een verkeerde eyghenschap van persing in waer, soo fouden eenighe reghelen verkeert vallen. Om van t'welck by voorbeelt te spreken; laet inde volghende form de letteren van beteyckehing wesen als inde form der 1 bepaling, uytghenomen dat de Maen K, en haer teghepunt L, nu t'water niet en trecken als daer, maer perssen als hier, sulcx dat de lini FH nu corter sy dan I G. T'welck soo ghenomen, F en H sullen ebtoppen sijn, I G vloetront, en veel gedaenten van ebbe en vloet sullen op verkeerde wijze commen vande voorgaende des 1 en 2 voorbeelts, oock des 1, 2, 3, en 4 vervolghs vant 1 voorstel.

Ten eersten, soo sal ter plaetsen daer de Maen boven is leegh water sijn, teghen d'eerste stelling.

Ten tweeden, gaende de Maen onder des duystraers 90 tr. soo sal t'ghene int 2 voorbeelt des 1 voorstels berekent wort op 8 uyren 5 ①, sijn van 3 uyr 55 ①; En weerom verkeert dat daer berekent wiert op 3 uyr 55 ①, sal sijn van 8 uyr 5 ①.

Ten

According to the general rule of the theory the flood-tide ought to be highest at the flood top, and the places which are nearer to the flood top also ought to have the flood-tide higher than others which are further away from it; nevertheless, in practice the contrary is seen to occur in many places. In order to explain the cause of this, I first say as follows, by way of example: It is seen that a billow of water striking on a mole sticking out into the sea is raised in front of it, nay, overflows it, much higher than other waves in the sea. The reason of this is the following: If the foremost part of the billow striking on the mole were by itself, without any water following it, it would — as is evident — at once turn back without rising to such an elevation. But the rest of the heavy billow, being already on its way, cannot turn back in so short a time; then the foremost water is retained by the next, and this next also by more that follows, so that, one retaining and urging on the other, the aforesaid elevation, higher than out at sea, is reached. This being understood, let us take the whole of the elevated water of a flood-tide for a wave; then since this is many miles long, it is no wonder that in some places against which it strikes directly it gives rise to a higher elevation than occurs out at sea; thus, for example, the flood-tide of the big Spanish sea, flowing to the low, small Dutch sea, as said above, and striking against the lands of the bay of Brittany, gets an elevation of 10 to 11 fathoms above ebb-tide, *i.e.* higher than the flood-tide in other places which are nearer to the flood top, nay, higher than at the flood top itself, such for known reasons. **CONCLUSION.** We have thus explained the reason why in some places situated further away from the flood top than others the flood-tide may yet be higher; as required.

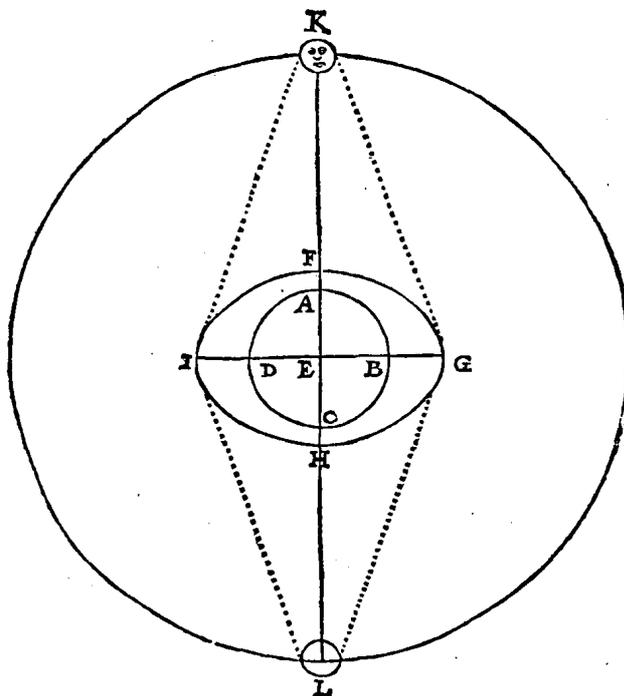
8th PROPOSITION.

To set forth the properties which ebb and flow would have if they were occasioned by pressure from the Moon and its opposite.

Up to this point we assumed that the Moon and its opposite attracted or sucked the sea-water towards them, according to the contents of the first postulate. But if it were due to the opposite property, namely pressure, some rules would become reversed. In order to speak of this by way of example, in the following figure let the reference letters be the same as in the figure of the 1st definition, except that the Moon *K* and its opposite *L* do not now attract the water as there, but press it as here, so that the line *FH* be now shorter than *IG*. When this is so taken, *F* and *H* will be ebb tops, *IG* the flood circle, and many qualities of ebb and flow will be the converse of the foregoing of the 1st and 2nd examples, as also of the 1st, 2nd, 3rd, and 4th sequels to the 1st proposition.

Firstly, in those places where the Moon is in the zenith it will be low tide, contrary to the first theory.

Secondly, when the Moon is under 90° of the ecliptic, the interval which in the 2nd example of the 1st proposition is calculated to be 8 hours 5 minutes will be 3 hours 55 minutes; and conversely, what was calculated there to be 3 hours 55 minutes, will now be 8 hours 5 minutes.



Ten derden, sal t'water op des Eertcloots afpunten ten hoogsten sijn, loopende de Maen onder den evenaer, en ten laagsten wesende in haer uiterste noortsche en zuysche breedte, teghen de reghel des 1 vervolghs vant 1 voorstel.

Ten vierden, een der twee ebtoppen dat een ghesfelde plaets naect comt, veroirsaeckt aldaer leegher ebbe als t'ander: T welck verschil heeft vant 4 vervolgh des 1 voorstels.

Ten vijften, blijktet dat plaetsen naerder den aspunt, dan de Maen den Evenaer, alsdan vant vloetront niet gherocht worden, sulcx datse op dien tijt so hooghen vloet niet en krijghen, als plaetsen daer het vloetront over comt, tegen de reghel vant 3 vervolgh des 1 voorstels.

Nu welke stelling van beyden recht is, of wat ander derde int natuerlick wesen mach sijn, daer af acht ick ons dadelicke ervaringhen te ghebreken; maer hoemen daer toe soude meughen commen, van dies sal ick mijn ghevoelen int volghende voorstel segghen. **T B E S L V Y T.** Wy hebben dan verclaert d'eygenschappen dieder van ebbe en vloet sijn souden, soose deur persing der Maen en haer teghepunt gheschiede.

9 VOORSTEL.

Te verclaren hoet schijnt datmen de saeck an soude meugen legghen, om te gheraken tot grondelicker kennis van ebbenvloet dander nu af is.

Als

Thirdly, the water will be highest at the poles of the Earth when the Moon moves under the equator, and lowest when it is at its extreme northerly and southerly latitudes, contrary to the rule of the 1st sequel to the 1st proposition.

Fourthly, the one of the two ebb tops which comes nearest to a given place causes there a lower ebb-tide than the other; which differs from the 4th sequel to the 1st proposition.

Fifthly, it appears that places nearer to the pole than the Moon is to the Equator are not then reached by the flood circle, so that at that time they do not get as high a flood-tide as places through which the flood circle does pass, contrary to the rule of the 3rd sequel to the 1st proposition.

Now I consider that we lack practical experience to decide which of the two theories is the right one, or what other (third) theory may be in accordance with nature; but in the following proposition I will give my opinion about the way in which such experience might be obtained. CONCLUSION. We have thus explained the properties which ebb and flow would have if they were occasioned by pressure from the Moon and its opposite.

9th PROPOSITION.

To explain how it seems we might proceed to gain fuller knowledge of ebb and flow than we now have.

When ebb-tide, flood-tide, and spring-tide are not in accordance as to time and magnitude with the general calculation of the tides to which mariners

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Als ebbe, vloet en sprinck, niet en overcommen in tijt en groothey, mette gemeene rekening der ghetijden daer de Schippers hun na ghevougen, als wat vrougher of later ghebeurde, of datter leeghe vloet comt, met wint die nochtans hooghe vloet veroirsacckt, of verkeert hooghe vloet met wint die cleene vloet mebrengt, men segt dan ghemeenelick daer moet ander wint in zee sijn, of gheweest hebben, daert nochtans misschien sal meugen ghebeurt sijn om redenen der ghemeene reghel die wy boven beschreven hebben, of by anderen daer af beschrijvelick sijn. Doch want hier uyt somwijlen volcht verlies van schip lijf en goet, soo en schijnet niet buyten reden datmen tracht na sekerhey en kennis der oirsaken deser stof. Maer want uyt veel ghewisse ervaringhen de sekerste reghels gemacckt worden, so soudet seer voorderlick sijn dat veel menschen, tot allen plaetsen des Eertcloots daert te pas can commen, daghelick ghesloughen en opschreven al t'ghene sy dadelick daer af sien gebeuren, als tot wat uyt ebbe, tot wat uyt vloet comt, metende oock de hooghe van yder vloet, en de leeghe van yder ebbe, daer by opteyckenende de winden of stilten die der dan sijn, sulcke schriften daer na int ghemeen, en (soo den Gaslagher self gheen spieghelaer en is) ter hant der spieghelaers commende, sy souden sien hoe alles mette ghemeene spiegheling overquaem, acht nemende op des ghemeene reghels belet van landen en winden. 'Tis oock te weten dat d'ondersoucking der ghemeene ghedaente van ebbe en vloet daer int 2 voorstel af gheseyt is, bequamelixt soude meughen gheschien op cleene Eylandkens in een groote zee gegelghen, en (om geen hinder der ebbe en vloet te hebben) seer verre van lant, als Sint Helena en dierghelijcke. Hier uyt soudemen meughen mercken, ten eersten ofter recht onder de Maen en haer tegepunt twee vloettoppen loopen met een ebront, volghende t'eerste ghestelde: Of twee ebtoppen met een vloetront, volghende het tweede ghestelde des 8 voorstels.

Ten tweeden, als de Maen onder den evenaer loopt, of dan ebbe en vloet $6\frac{1}{4}$ uyren achter malcander volghen na t'inhout der spiegheling verclaert in des 1 voorstels 1 voorbeelt.

Ten derden, als de Maen met groote afwijcking vanden evenaer loopt, of dan sulcke tijden tuschen ebbe en vloet, verschillen na t'inhout der spiegheling verclaert in des 1 voorstels 2 voorbeelt; of nae t'inhoudt des 2 lidts vant 8 voorstel, want ons dat oock versekeren soude van suyging of perfsing.

Ten vierden, oft water ontrent des eertcloots aspunten ten leeghsten is loopende de Maen onder den evenaer, volghende t'eerste ghestelde, als int 1 vervolgh des 1 voorstels, of ten hoochsten volghende het tweede ghestelde, als int 8 voorstels derde lidt, t'welck ons van suyging of perfsing oock soude wetenschap gheven.

Ten vijfden, of alsdan dien vloet hoogst comt, wiens treckende cracht te weten der Maen of haer teghepunts, naest des * Doenders toppunt is, na t'inhout der spiegheling verclaert in des 1 voorstels 2 voorbeelt; Of anders dat de vloeden even hooch commen, maer datter sulck verschil inde ebben valt, t'welck uyt perfsing volghen soude. Alle welcke dinghen soose bevonden wierden te overcommen met een der twee voorgaende spieghelinghen, t'soude verstrecken tot sekerhey des handels: Maer verschillende men soude deur dat ghevonden meughen trachten na verbetering. **T B E S L V Y T.** Wy hebben dan verclaert hoet schijnt datmen de saeck an soude meughen legghen, om te geraken tot grondelicker kennis van ebbe en vloet d'ander nu af is, na den cysch.

*Efficientis
punctum
verticale.*

E B B E N V L O E T S
E Y N D E.

conform, namely their setting in a little earlier or later, or that the flood-tide is low with a wind which nevertheless occasions a high flood-tide, or conversely the flood-tide is high with a wind which causes a low flood-tide, it is usually said that there must be or must have been a different wind at sea, thought it may perhaps have happened on account of the general rule we have described above, or according to others still to be described. But because loss of ship, life, and property sometimes results from this, it does not seem inopportune to try to get certainty and knowledge of the causes of this matter. But because the surest rules are made from many certain experiences, it would be very propitious if in all places of the Earth where this may be appropriate many people were to observe and write down daily what they see happening in actual fact, *e.g.* at what time there is ebb-tide, at what hour flood-tide, measuring also the height of every flood-tide and the depth of every ebb-tide, and also noting the wind or calm at that moment; if such writings thereafter become public and (if the Observer himself is no theoretician) fall into the hands of the theoreticians, these will have to find out how far everything was in accordance with the general theory, taking into account the impediment to the general rule caused by lands and winds. It is also to be noted that the examination of the general qualities of ebb and flow, as referred to in the 2nd proposition, might be effected most efficaciously in small islands situated in a big sea and (in order not to have ebb and flow impeded) very far from the land, *e.g.* St. Helena and the like. From this, one might note: firstly, whether directly under the Moon and its opposite there are two flood tops with one ebb circle, according to the first theory, or two ebb tops with one flood circle, according to the second theory of the 8th proposition.

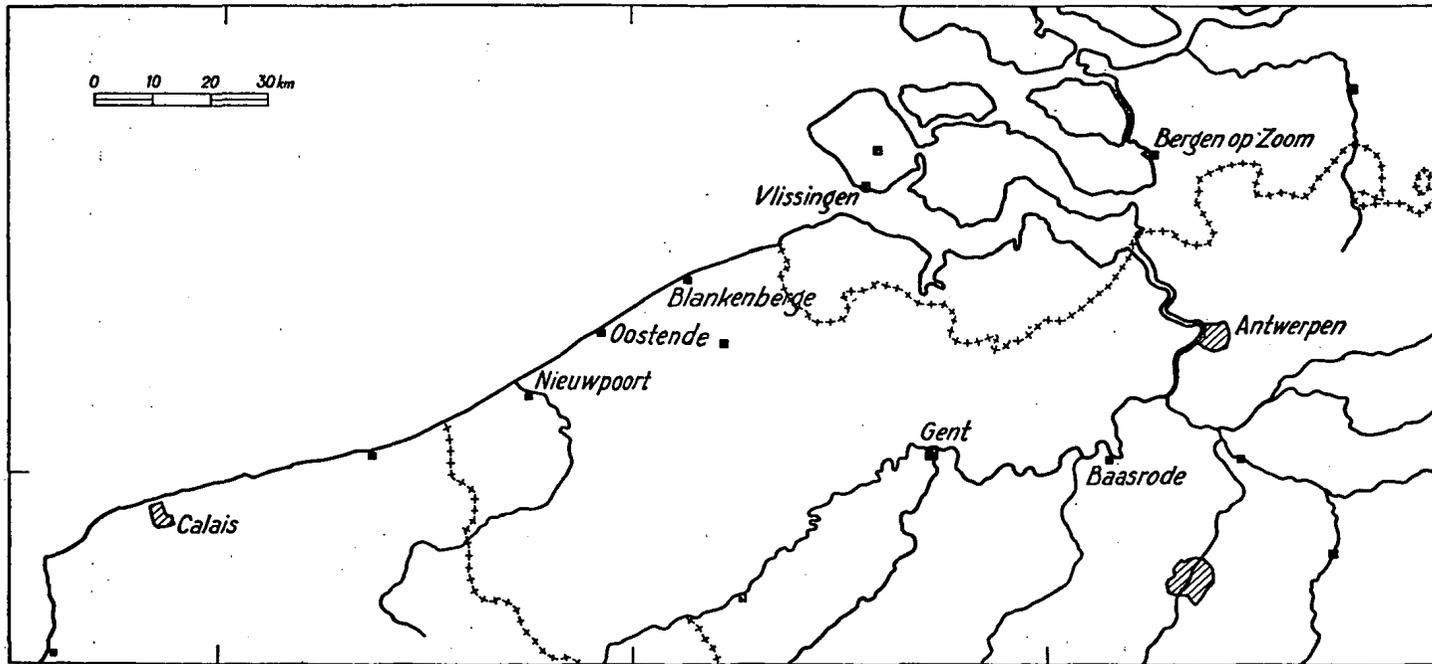
Secondly, whether when the Moon moves under the equator, the tides succeed one another after $6\frac{1}{4}$ hours, according to the contents of the theory set forth in the 1st example of the 1st proposition.

Thirdly, whether when the Moon has great deviation from the equator, these intervals between ebb-tide and flood-tide differ according to the contents of the theory set forth in the 2nd example of the 1st proposition, or according to the second section of the 8th proposition, for this would also give us certainty whether it is a matter of suction or of pressure.

Fourthly, whether the water is lowest about the poles of the earth when the Moon moves under the equator, according to the 1st theory, *e.g.* in the 1st sequel to the 1st proposition, or highest according to the second theory, *e.g.* in the third section of the 8th proposition, which would also furnish us with knowledge as to suction or pressure.

Fifthly, whether then that flood-tide will be highest for which the attraction, to wit, that of the Moon or its opposite, is nearest to the zenith of the Observer, according to the wording of the theory set forth in the 2nd example of the 1st proposition; or otherwise whether the flood-tides will be the same height, but there is such a difference between the ebb-tides, which would result from pressure. And if all these things were found to be in agreement with one of the two foregoing theories, this would provide certainty in the practical work. But if no agreement were found, one might strive for correction by means of the data found. CONCLUSION. We have thus explained how it seems we might proceed to gain fuller knowledge of ebb and flow than we now have; as required.

END OF EBB AND FLOW.



The estuary of the Scheldt and the adjoining seacoast.

THE NAUTICAL WORKS

OF

SIMON STEVIN

DE HAVENVINDING

THE HAVEN-FINDING ART

INTRODUCTION

§ 1

INTRODUCTION AND GENERAL REMARKS

The main rules followed in Stevin's day — and even for a long time after — in conducting a ship safely across the ocean to her destination were:

- a. conduct the ship to the latitude of the parallel through the place she is bound for;
- b. sail east or west along this parallel.

A place situated on the seaboard of the continent was certain to be reached in this way, but the time of arrival could not be predicted. The rules equally applied if the land to be made was an island, but this involved considerable risk, as was proved all too frequently in actual fact. Indeed, a navigator would sometimes guess he was west of an island, whereas actually he was east of it. If in such a case he headed east, he went in the wrong direction, an error which was not detected until much later and after much doubt. Stevin refers to such a case (p. 427), where a ship sought for St. Helena for several weeks and the navigator "sailed several times around it before he got there" ¹⁾.

The resultant prolongation of the voyage and the complete uncertainty as to the time at which a landfall would be made formed two out of the numerous and very great dangers attendant on navigation in those days. In fact, prolongation of the voyage entailed longer exposition to all sorts of wind and weather, a greater risk of water shortage, illness, and loss of lives. Such uncertainty involved the possibility that in the hours of the night one might suddenly approach land, land which was usually a lee shore and not lit by warning lighthouses. Under such circumstances good seamanship demanded that navigators should keep a very sharp look-out and exercise the greatest caution, while the ship must make no headway during the night, when dangers could not be sighted. Further, the lead had to be kept going. But in spite of all this, in many cases when land suddenly loomed ahead, there was neither time nor opportunity to turn the ship round and avoid peril of death. Indeed, how often a sailing-vessel is driven on by the wind at haphazard, thus meeting its doom, generally with disastrous consequences!

If the determination of longitude by land and by sea had been possible, the seaman would have found coasts and islands in the right place on the chart. He would have been able to mark the position of his ship on the chart and thus

¹⁾ When, more than sixty years later, in his *Kort Onderwijs aengaende het gebruyck der Horologien tot het vinden der lenghten van Oost en West* (see: *Oeuvres Complètes de Chr. Huygens*, Vol. XVII, La Haye 1932, pp. 191-237) Christiaan Huygens explains the use at sea of his pendulum clock, he cites a perfectly similar instance of a considerable misconception and great uncertainty. This concerned a fleet which in 1664 wanted to make land at Fogo, one of the Cape Verde Islands, on account of water shortage.

would have known its position relative to the coast, in consequence of which the uncertainty would have been removed. But this possibility did not exist. From the early sixteenth century the basic idea of the method of determining longitude with the aid of the chronometer and by means of lunar distances indeed had been known. But the practical application of both these methods was not arrived at until the second half of the eighteenth century. Aboard they were not commonly used until the nineteenth century.

During the sixteenth, seventeenth, and eighteenth centuries a solution of the great problem was diligently and passionately sought in countries like Portugal, Spain, France, England, and the Netherlands, where the seafaring trade flourished and the art of navigation was practised and developed scientifically. The problem was studied in those countries both by sailors, who daily felt the lack of knowledge, and by scholars. The latter made proposals on a great many points. But many more ideas — fantastic ideas in our eyes — were advanced by seafaring and non-seafaring men, who thought they could make useful suggestions and thus hoped to receive the high pecuniary rewards offered for the solution of the problem. The thought and endeavours of all these people form the subject-matter of an important and fascinating chapter of the history of navigation, a chapter which bears the heading "The Problem of Longitude".

A self-contained and completed section of this chapter must be considered to be formed by the efforts which sought the solution in the deviation of the magnetic needle from the astronomical meridian²⁾. The title of this part should

²⁾ The vertical plane through the vector representing the geomagnetic force bears the name of the plane of the magnetic meridian, and the intersection of this plane with the horizontal plane is called the magnetic meridian, direction of the magnetic north, or magnetic north and south line. It is the direction in which a magnetic needle pivoting on a vertical pin aligns itself under the influence of terrestrial magnetism. In general this direction does not coincide with the astronomical meridian, which is also called the true north and south line. The magnetic meridian may deviate to the east or to the west of the astronomical meridian. The angle between these two meridians is called declination. The merchant navy speaks of *variation* of the compass, a name which will be used henceforth. In the navigable parts of the world the variation is an acute angle. (see the figure on page 368)

For a given place on the earth the amount of the variation is not constant. It is liable to a very slow increase or decrease (secular variation), further to a small diurnal fluctuation, and sometimes to sudden irregularities.

At the present day maps exist on which lines have been drawn joining the places where the variation has the same value, expressed in degrees; these lines are called isogonics. The lines joining the places at which the variation is nil are called agonics. These maps mention the secular variation. They make it possible to find the amount of the variation for a given point at sea and for a given year. It is clear that the diurnal fluctuation and the above-mentioned irregularities cannot be taken into account.

When a magnetic needle is not directed exclusively under the influence of earth magnetism, but mounted on board a ship where disturbing influences due to surrounding iron occur, the magnetic needle will deviate from the magnetic north and south line. This is called *deviation*.

In those days ships were made of wood. Deviation thus could not be due to the ship's iron, but it *could* be caused by the nearness of guns, objects of iron in the neighbourhood of the compass, knives in the pockets of the helmsman, etc. It is known that a warning was sounded against such influences as far back as the seventeenth century. It may therefore be assumed that navigators were on their guard with regard to this point and that the compasses referred to in the present introduction showed no deviation, so that this deviation will not be taken into account and no more reference will be made to its existence.

be: "the determination of longitude at sea by means of the variation of the compass" ³⁾. The matter played a part around 1500 to the second half of the eighteenth century.

A good many people in Holland as well as elsewhere thought and wrote about this subject. In the sixteenth century opinions about the possibility of using the variation for the determination of longitude differed widely, between absolute rejection and hopes of complete success. Stevin was among the convinced champions. His *Haven-Finding Art*, published in 1599 ⁴⁾, is a lucid scientific treatise, in which the author, making use of existing observational data, states his view of the magnetism of the earth. In an intelligible idiom he sets forth a method which is to enable the seaman, on the strength of the variation of the compass needle observed by him, to conduct his ship unerringly to her destination, without being kept in uncertainty and without having to know either the geographical longitude of the place for which the ship is bound or the longitude of the ship at sea. In Stevin's time, however, the data about the magnetism of the earth were extremely scanty and quite insufficient to base thereon such a method of determining longitude. It was not until the eighteenth century, when more information about the variation of the magnetic needle on the oceans had been collected, that the method became of practical value and, for want of anything better, was used and appreciated by mariners. The basic idea of Stevin's system then proved to be correct. At the present day, when the variation has been measured everywhere and charts show its value for any given place in any given year, a rough determination of longitude by this method would be possible. But the need of this solution is no longer felt. Nowadays ships are steered across the ocean by astronomical navigation, while the chronometer, checked by means of radio time-signals, forms the backbone of this system. It is hardly necessary to refer to up-to-date electronic aids to navigation in this place.

However, all this does not alter the fact that Stevin has supplied the material for a particularly interesting page out of the history of navigation. His treatise may be called a very remarkable book, remarkable for its place in this history as well as for its contents.

§ 2

THE PLACE OF *THE HAVEN-FINDING ART* AMONG SIXTEENTH-CENTURY TEXTBOOKS ON NAVIGATION

The question may be asked whether *The Haven-Finding Art* is one among the early publications on navigation in Holland.

The navigator of the second half of the sixteenth century, sailing along the seaboard of western Europe and to the Baltic was in a position to consult short elementary treatises on navigation, which were included in some "rutters" and atlases destined for this trade. Such books gave a description of the route as well as some information about the compass, about the calculation of the hours of high and low water, about plane charts, etc., the latter with a view to the

³⁾ In § 5 this subject is to be briefly discussed under this title.

⁴⁾ *Works*, X.

compilation of maps⁵⁾. But in consequence of the rapid development of deep sea navigation in Holland there was a growing need of wider nautical knowledge, required for carrying on this trade. It is natural that people looked to the country which had long been in possession of knowledge and experience of deep sea navigation to supply this want. It was Spain which provided the required instruction, both directly and also indirectly *via* England. This development may be briefly outlined here.

In 1580 a Dutch translation of Medina's *Arte de navegar*⁶⁾ appeared at Antwerp⁷⁾. Although the book had been written by one who was no sailor, it was looked upon as a standard work, and it had become very famous and widely diffused. Of the original work, which had appeared 35 years earlier, Italian, French, and English translations had long been in existence. To the Dutch edition the Antwerp mathematician and nautical expert Michiel Coignet added an excellent commentary, *Nieuwe Onderwijsinghe*⁸⁾. This latter work, which was better adapted to practice than was Medina's *Arte*, argues the great nautical knowledge of the expert author. The said translation of Medina, combined with Coignet's appendix, was reprinted several times at Amsterdam, invariably by Cornelis Claeszoon, publisher of a great many books on nautical subjects. In 1598 he already published the fourth edition, which points to a rapid diffusion even in Holland. It appears that the book, in its combination with Coignet's treatise, played a useful part in the advancement of nautical knowledge.

The textbook of Zamorano⁹⁾, which was greatly valued in Spain and passed through four reprints there, appeared in a Dutch translation at Amsterdam in 1598¹⁰⁾. The book of William Bourne¹¹⁾, based on Spanish sources, which was highly popular in England and was repeatedly reprinted there, appeared in a Dutch translation at Amsterdam in 1594 and once again in 1599¹²⁾.

The fact that the enumeration of the textbooks available in Holland in 1599 is thus exhausted shows that Coignet in the southern and Stevin in the northern Netherlands were foremost among those who devoted treatises to scientific navigation, although mention must also be made of Adriaan Veen, who in the same period and during the early years of the seventeenth century tried to serve ocean navigation with his charts in the form of a spherical cap¹³⁾. The sailors, who traditionally learned to sail and find their way at sea in practice, through instruction by old salts and their own watchfulness, and who regarded this as

⁵⁾ See: Cornelis Anthoniszoon, *Onderwijsinghe vander zee, om stuurmanschap te leeren, derdewerf nu ghedruckt, neerstelijck ghecorregeerd ende verbeteri*. Anno 1558. Described by J. Keuning, *Tijdschrift Kon. Ned. Aardrijkskundig Genootschap*, 1950, p. 687. See further the works of Lucas Jansz. Waghenaer and others.

⁶⁾ Pedro de Medina, *Arte de navegar*. Valladolid 1545.

⁷⁾ Pedro de Medina, *De zeevaert oft conste van ter zee te varen. In onse nederduytsche tale overgeset ende met annotatiën verciert bij M. Merten Everaert Brug*. Antwerp 1580.

⁸⁾ Michiel Coignet, *Nieuwe onderwijsinghe op de principaelste punten der zee-vaert*. Antwerp 1580.

⁹⁾ Rodrigo Zamorano, *Compendio de la arte de navegar*. Seville 1581.

¹⁰⁾ Rodrigo Zamorano, *Cort onderwijs van de Conste der zeevaert*. Amsterdam 1598.

¹¹⁾ William Bourne, *A Regiment for the Sea*. London 1574.

¹²⁾ William Bourne, *De const der zee-vaert*. Amsterdam 1594.

¹³⁾ Contained in his book: *Tractaet vant Zee-bouck houden op de ronde gebulte pas-kaert*, Amsterdam 1597, which forms part of his *Napasser*, Amsterdam 1597. Adriaan Veen tried to overcome the defects inherent in the sea charts of those days by teaching his readers to work out the sailing problems on segments of the globe.

sufficient, were as a rule reluctant to learn from arm-chair theorists, unacquainted with day-to-day practice. Stevin was of course aware of this reluctance, but it did not deter him, sound scientist as he was, from putting his work and his discoveries at the disposal of seamen. Stevin, being convinced that he had something valuable to offer, made an attempt to improve safety at sea and, if possible, to solve a burning question, a problem which the navigational world was yearning to solve. His work roused the interest of Prince Maurice, for he gratefully relates (p. 431) how the Stadtholder had acquainted himself with the subject and had become convinced of the possibility thus opened for greater safety in navigation. The Prince gave orders for navigators henceforth to determine carefully, with the aid of suitable instruments, the variation of the magnetic needle in the places at which they touched, and to submit the results of their observations to the Admiralty. The latter was to publish the data thus collected. In this way the Stadtholder backed up Stevin. His measures advanced the research on terrestrial magnetism, the result of which was expected to benefit navigation.

Finally it may be observed that the "Privilege" with which *The Haven-Finding Art* opens shows that the States General of the United Netherlands by letters patent of 18th March 1599 granted to Christoffel van Raphelingen, printer at Leiden and a grandson of the famous Christoffel Plantijn (Christophe Plantin) at Antwerp, for a period of six years, the sole right of printing, publishing, and selling this book. We also read there that Van Raphelingen intended to publish the treatise not only in Dutch but in Latin, French, and other versions as well.

Van Raphelingen brought out Latin and French editions almost simultaneously, but he did not after all publish the treatise in other translations. The English translation, which we owe to Edward Wright, was printed and published in London, also in 1599¹⁴).

The name of the writer does not occur in the original Dutch book. Stevin's authorship appears from the Latin translation, which is due to Grotius. In the "dedication", dated 1st April 1599, Stevin is mentioned as such (p. 4), and the same is the case in Wright's preface (page B-2 verso) to his translation, the title of which is: *The Haven-finding Art*. Reference may further be made to Snellius, who says in his *Tiphys Batavus*¹⁵): "But this subject has been treated by others and it induced our Stevin (*Stevino nostro*) to write his *Haven-Finding Art* (*suae Limeneuretices*). You will find it in the work *Wisconstighe Ghedachtenissen* of H.R.H. Prince Maurice, which was translated into Latin and published by us previously."

§ 3

THE CONTENTS OF *THE HAVEN-FINDING ART* OF 1599

a. *STEVIN'S "CONJECTURE" ABOUT TERRESTRIAL MAGNETISM*

It is known, says Stevin, that for a long time past, especially after the great voyages of exploration to the Indies and America, people have sought for means to determine longitude at sea, so as to make it possible to reach one's destination. However, all these attempts failed. Some investigators had hoped that the vari-

¹⁴) *Works*, X.

¹⁵) Willebrord Snellius, *Tiphys Batavus sive histiodromice de navium cursibus et re navali*. Leiden 1624, p. 67.

ation of the compass might furnish the basis of a method for the determination of longitude, and they had assumed a magnetic pole, by which they understood a point on the earth towards which any magnetic needle, no matter where it was mounted, would point. But experience had taught that the variation had nothing to do with such a pole. This idea therefore had been found to be unsound. By emphatically rejecting such a magnetic pole, Stevin reveals one of the foundations of his standpoint, which was diametrically opposed to that of his great contemporary Petrus Plancius, the famous cartographer and scholar who prepared the ground scientifically for the important first nautical expeditions of the Dutch and taught the navigators who were sent out on these voyages.

Stevin goes on to say that investigations had indeed furnished a method which made it possible to head for the desired harbour unerringly, even if neither the longitude of the place of the ship nor that of the harbour in question was known. The only requirement was that one had to know the amount of the variation for the place of destination and had to be able to determine this for the place of the ship by observation. One then had to follow the parallel of the port of destination to a point where the variation was identical with that holding for that place. By observing whether the variation measured was increasing or decreasing the navigator learned whether he was sailing in the right direction. This method appeared so sound to Stevin that in his opinion the position found by dead reckoning had to be considered less reliable than the result obtained in this way. He regards

the variation of a given place as constant.

Stevin dismisses the objection that several places with the same variation might be situated on the same parallel. This need not cause errors, since such places were very far apart.

There was yet another reason why knowledge of the variation was necessary. The navigator of an ocean-going ship had to know it if he was to be able to deduce the true course from the course steered by the compass, in order to learn how the ship was moving on the surface of the earth in relation to the meridians ¹⁶⁾. As a rule the seaman assumed that his compass —

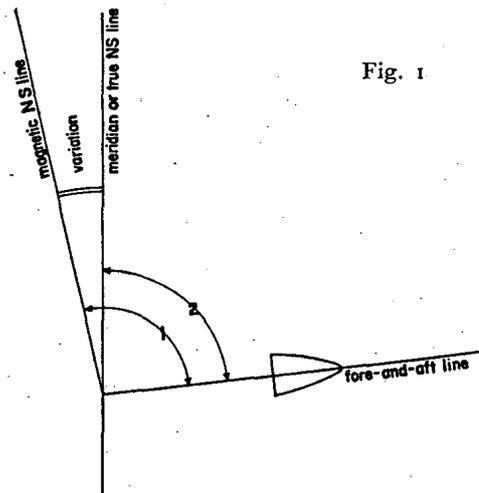


Fig. 1

¹⁶⁾ The course which is steered and kept while sailing by the compass is the angle between the compass needle and the fore-and-aft line. It is read into degrees or points on the division at the circumference of the compass card. Since the present discussion does not take account of deviation due to iron in the vicinity, the direction pointed by the compass needle implies the direction of the magnetic north and south line. In this case therefore the course steered is the angle between the magnetic north and south line and the fore-and-aft line. Because this course is measured by reference to the magnetic north and south line it is called the magnetic course (1).

The direction of the ship may also be measured by reference to the meridian. This (Continuation on next page).

constructed in such a way as to allow for the variation found at his port of departure — continued to point to the true north throughout the voyage. Most sailors had no idea that the variation changed as they moved across the earth. In consequence errors arose in the data about directions, or in the positions of places, which data they submitted upon coming home, with the object of marking them on charts and on globes. Large-scale inquiries and collection of data in the matter of measurement of the variation were desirable to avoid such inaccuracies. As has been said, the Stadtholder urged home this desirability by his measures and orders.

It is in this connection that Stevin recommends the use of a compass with an adjustable needle, a type which was in use side by side with other compasses in Holland as well as elsewhere. With such instruments it was possible to set the needle for the value of the variation at the place in question, so that the north and south line invariably indicated the true north or the direction of the astronomical meridian and the courses steered by such a compass were true courses. Compasses of this type continued to be made and used by the Dutch until far into the nineteenth century.

For the benefit of those who wanted to study this matter the book contains a table of the values of the variations in a number of places, in so far as they had already been observed, which values Plancius had "collected by protracted labour and not without great expense from different corners of the earth, both far and near". It was to Plancius, writes Stevin, that credit would be chiefly due if by this system it were to be possible for the seaman to reach different lands and harbours. Stevin feels obliged to express his gratitude, for the numerical data were taken from Plancius, and it must have been quite a job indeed to collect, elaborate, and arrange them. But in his theoretical discussions Stevin does not follow the train of thought of Plancius. He adopts a course of his own, which — as has been said — was of importance for the practice of navigation during a period long after his day, whereas the theories of Plancius were soon exploded.

Before publishing the table and proceeding to discuss it in detail, Stevin, being a scientist who hopes to make a contribution to the solution of the problem of determining longitude at sea, addresses the seaman who will have to supply the further data. He provides him with a golden rule aiming at the advancement of both theory and practice. Even if further observations of variation, latitude, and longitude should furnish values different from those listed in the table and if consequently different explanations should have to be framed, this was not to deter the seaman from the inquiry; on the contrary, he was to do everything that might help to collect more information about the method, with the ultimate object of jointly finding its true character. In his concluding sentence (p. 459) Stevin says once more that if the results of later inquiries should be different from the outlook on the

(Continuation of note 16)

course is called the true course; it is the angle between the meridian (or true north and south line) and the fore-and-aft line (2).

The course steered, which is read on the compass—in this case the magnetic course—is reduced to the true course with the aid of the variation. The amount of the variation in degrees therefore has to be added or subtracted. Once the true course is known, the direction of the path followed by the ship can be drawn on the chart in relation to the meridian.

variation based on the table of Plancius, it will be necessary to modify the conclusions.

It was actually in this way that the theory was developed. The orders of the Stadtholder were obeyed and Stevin's wish was fulfilled, though much later than he may have hoped. Indeed, we know that the gaining of an understanding of terrestrial magnetism was a process which required several centuries. The seaman may consider himself fortunate in having advanced the art of navigation by making ample contributions to the knowledge concerning the indications of the magnetic needle on the earth (see § 5). Even in the present century he made some valuable contributions. The scientific investigation of terrestrial magnetism is still far from being complete even to-day, but it is no longer a subject of study for the seamen.

For 24 places on the northern and for 19 places on the southern hemisphere the table gives the variation, the latitude, and the longitude, the latter being reckoned by reference to the meridian through Corvo, one of the westernmost islands of the Azores. The easternmost places given are situated in longitude 160° E. They are the mouth of Canton River in China in latitude 23° N. and "Bunam, 46 German miles to the east from the eastern end of Java" in latitude south 17° . No data are supplied for the part of the world to the west of Corvo and to the east of Canton, *i.e.* a part comprising 200 degrees of longitude. The observations which Plancius had received about that part of the world from Spanish, English, and Dutch sailors were not in conformity with one another, "because they had been taken without suitable instruments and sufficient knowledge". Plancius was looking forward to receiving new and more accurate data.

The variation is nil in the island of Corvo. Going eastward on the northern hemisphere, we find easterly variation which increases to $13^{\circ} 24'$ at Plymouth; upon which it decreases to zero at the island of Hjelmsöy, at about 30 miles west of the North Cape, and according to the table in longitude 60° E. On the southern hemisphere, east of the meridian through Corvo, the variation is also easterly, increasing from zero to the maximum of 19° in the vicinity of Tristan da Cunha and then decreasing again to zero in longitude 60° E. It appears that the two maximum values lie in longitude 30° E., *i.e.* on the mid-meridian of the lune contained between longitude 0° and 60° E.; this lune is called a "perck" (segment) by Stevin.

Stevin draws special attention to the regularity he seems to detect in these figures, and on this ground he arrives at a bold conclusion. In fact, he "concludes" that the value zero of the variation holds for the whole meridian through Corvo — from one pole to the other — and equally for the meridian through Hjelmsöy. The maximum values fall on the mid-meridian of longitude 30° E. He assumes that the increase of the easterly variation holds for the whole lune

¹⁷⁾ 46 German miles stand for a distance of 184 nautical miles, because 15 German miles are reckoned for one degree of latitude of 60 nautical miles. Bunam or Bima, on the north coast of "Java Menor", is to be found on the map of China, etc. by Van Langeren 1595 (reproduced in Vol. XLIII of the Linschoten Ver., Map V) and on the map of Java (in the *Caert-thresoor*, 1598, p. 61). It is referred to as Buma on the page of the 2nd edition of the map of the world of Plancius, reproduced in Wieder, *Monumenta Cartografica*, Vol. II, 1926, p. 40 III. The place Bima and Bima Bay are situated on the north coast of the island of Sumbawa.

between longitudes 0° and 30° E., and the decrease for that between longitudes 30° and 60° E.

The phenomena in the lune between longitudes 60° and 160° E. confirm this picture. Here we have westerly variation, increasing from zero to the maximum value of 33° near Novaya Zemlya in the northern, and to 22° at the island of St. Brendan's in the southern hemisphere, upon which it decreases to zero in the meridian of longitude 160° E. again in both hemispheres. The places in which the two maxima occur again lie on the mid-meridian of the lune, *viz.* in longitude 110° E.

The conclusion now is obvious. The meridian of longitude 160° E. is a line joining points where the variation is nil, a line which is now called an agonic. The meridians of longitude 0° and 60° E. too are agonics. The increasing variation holds for the whole lune between longitudes 60° and 110° E., the decreasing variation for the lune between longitudes 110° and 160° E.

When in his attempt to give a picture of terrestrial magnetism Stevin here assigns a particular property to all places situated on the same meridian because in one or two points of that meridian either a value zero or a maximum value is found for the variation, he takes a view held by others before him. This point is to be discussed more fully later on. Stevin leaves it an open question how one is to conceive the difference of the variation on the mid-meridian, considering that unequal values are found in the northern and the southern hemisphere. To him the main thing is whether during a movement either to the east or to the west an increasing or a decreasing variation is observed.

For the part of the world comprising 200 degrees of longitude for which Plancius possessed no observational material, but for which he was expecting fuller and more reliable data, Stevin merely states how he "somewhat suspects" the magnetic needle will point there. On the same lines he now continues as follows.

The property of the magnetic needle of pointing to the north on the meridian through Corvo and through those of longitude 60° and 160° E. is also assumed by him for those of longitude 180° , 240° , and 340° E., *i.e.* the meridians in the longitudes 180° away from longitudes 0° , 60° , and 160° . All these lines are agonics. In this way the surface of the earth is divided into six lunes, with alternate easterly and westerly variation, each subdivided into one half with increasing and one with decreasing variation. In each of these lunes the maximum falls on the mid-meridian, *i.e.* the 30th, the 110th, the 170th, the 210th, the 290th, and the 350th meridian.

In a remark at the end of his theoretical discussion Stevin cautiously calls his picture a "conjecture", which may not be confirmed by observation. He regards the whole picture as descriptive of the division of the earth's surface in connection with the behaviour of the magnetic needle. However, he has great confidence in the core of his idea, *viz.* the possibility of reaching a place on the earth with the aid of the latitude and the value of the variation. This confidence appears from the instance given on pp. 456 - 458 of a voyage from Amsterdam to Brazil. Here again he considers the results more reliable than the longitude deduced from the course and the distance sailed. Thus he is able to conclude his discussion with the hopeful expectation: "If therefore the needle-pointing and the latitude are duly observed in all corners of the world and made known to everybody, it will be possible to sail the world in another way than hitherto".

Stevin's method also enables ships — which have been scattered by a storm, for instance — to assemble at an appointed place at sea.

We have thus seen how the learned author ventures to draw conclusions from a small number of observations, holding only for one sixth of the earth's surface. The regularity he has found is met with again in the lune between longitudes 60° and 160° E., upon which he utters the "surmise" that it will also hold for the rest of the earth's surface. The picture here outlined was to help the seaman to find his destination as long as the great problem of the determination of longitude at sea had not yet been solved.

Stevin makes no attempt to account for all these things in his treatise. He explicitly rejected a magnetic pole, a point on which he was right as long as this pole was considered the centre at which the magnetic needle had to point. In his refusal to assume a magnetic pole Stevin took up a standpoint diametrically opposed to that of Mercator as well as Plancius.

In our view his "conjectures" and "surmises" appear extremely speculative. Many contemporaries, hankering as they were after the possibility of determining longitude at sea, held a different view of the matter from ours.

b. *THE MEASUREMENT OF THE VARIATION OF THE COMPASS ACCORDING TO STEVIN*

As was to be expected, Stevin's treatise expounds in what way the variation of the compass can be determined on board a moving ship, even though — according to the author — many people were acquainted with this subject.

The instructions found on pp. 461 - 465 do not call for much comment. They state that if a mariner's compass is used, this should be one in which the magnetic needle coincides with the north and south line of the compass card. In taking a bearing one has to hold a vertically suspended cord near the compass in such a way that the shadow cast by it passes through the centre of the graduation. The place of the shadow on the graduation is read before and after noon at equal altitudes of the sun. The point situated midway between the two observations indicates the direction of the meridian. Since the needle coincides with the north and south line, the amount and the direction of the variation are thus found.

The observation could also be taken by means of a compass-needle pivotally mounted in a box, against the inner wall of which the graduation had been marked. During the observation the box had to be held in such a way that the needle coincided with the zero line of the graduation. Otherwise the method remained unchanged.

There were some navigators who used a compass fitted with an adjustable azimuth circle, which is described on p. 467. On p. 468 it is illustrated, unfortunately not completely, for though the drawing does show the bowl, the needle, and the azimuth circle, it does not include the system of suspension. The latter it said to have been an invention due to Reynier Pietersz, who had suspended the instrument "on two different pins, in the manner of the mariner's compass". For steering compasses this suspension system was quite familiar at the time. It is usually called an invention of Cardanus (1501-1576), an Italian mathematician and physicist, astronomer, and professor of medicine at Pavia. But with regard

to an azimuth compass it was indeed justifiable to speak of an invention, as will appear in § 4 b.

Along the graduated circumference of the quadrant an alidade, equipped with sights, was adapted to pivot. This made possible a rough measurement of the altitude of the sun or another heavenly body. The azimuth quadrant turned on a pivot mounted in the centre of the glass cover. To ensure its remaining perpendicular to the glass cover of the bowl as it turned, it was provided with two lateral supports, as mentioned in the text. The bowl was weighted on the underside so as to ensure the horizontal position of the cover. Along the circumference of the bowl as well as on its inner wall, graduations had been provided, in such a way that the zero points of the two coincided. On the former the direction of the quadrant could be read. The needle was set to the zero point of the latter.

The observation took place as follows. The adjustment of the compass needle was brought about by the rotation of the compass bowl within its gimal ring. Owing to the deviations from the course to left and right due to the movement of the ship as she sailed, this adjustment was undone. Accordingly, for the neutralization of this so-called yawing the bowl had to be repeatedly turned to the right and to the left by hand. At the same time the quadrant had to be directed towards the sun. Once the quadrant had been given the right direction, the turning and readjustment of the bowl alone sufficed to maintain the proper adjustment of the needle as well as that of the quadrant. When this had been achieved, the angle between the quadrant — or, which comes to the same thing, the azimuthal direction of the sun — and the compass needle was read on the graduated circumference.

Later in the day a second bearing was taken at an altitude of the sun equal to that of the morning observation, after which the amount of the deviation of the needle from the meridian appeared from the readings of the morning and the afternoon, i.e. the deviation was equal to half the difference between these readings.

In the same way it was possible to determine the deviation by night at equal altitudes of the same star. With the aid of the moon this was impossible — as Stevin writes — in view of its rapid proper motion and its large parallax, due to its short distance from the earth ¹⁸⁾.

The allegation that the rapidity of the moon's motion gives rise to errors is already to be found in William Bourne, *A Regiment for the Sea*, in the chapter on "How to find the true meridian" (in the edition of 1577 on p. 28). About the taking of bearings it is stated: "you may do the like by night by any of the starres that you perfectly do know, doing as you do by the sun in all points, but you cannot do it so wel and truly by the moone, by the meanes of the swiftnesse of the moones motion in the Zodiack". There exist Dutch translations of Bourne's book, published at Amsterdam in 1594 and 1599. It seems fairly probable that this work was available to Stevin.

¹⁸⁾ These words will have to be interpreted as a not untimely warning: do not take bearings of the moon. The reason why this should not be done is not expressed very felicitously. It is especially the rapid change in the declination between the instants of observation which may give rise to errors, for an increase or decrease of 2° is quite conceivable. Other inaccuracies fall within the limits of the errors of observation. Perhaps Stevin considered it superfluous to give an exact explanation.

At the end of the chapter there is yet another warning. When the ship is in a region where the variation changes rapidly as one sails eastward and westward, so that the variation is not the same for the morning and the afternoon position, this appears from the fact that the deviation found by combining the first forenoon and the last afternoon observation does not agree with that obtained by the combination of the last forenoon and the first afternoon observation. According to Stevin this need not point to inaccuracy on the part of the navigator. The phenomenon might even serve to gain some idea of the speed with which the variation changed during a known number of sailing hours ¹⁹⁾.

Let us devote some attention to this remark of Stevin's.

It is only if the change of the variation due to the movement of the ship is considerable and moreover is not equal during equal movements — *i.e.* if the change is irregular — that the phenomenon referred to can be revealed. In that case high demands as regards accuracy of measurement are made on the compass. It is beyond doubt that the compasses of those days could not yet meet these demands.

But Stevin, who knew no more about the amount of the variation in various points on the earth than he mentions in *The Haven-Finding Art*, can have had no idea of the speed with which the variation changed in consequence of the said movement of the ship, nor can he have known whether this change was regular or irregular. He is not likely to have possessed observational data about this. If this had been the case, he would no doubt have mentioned it in this context.

In answer to the question what may have induced Stevin to make the above remark two suppositions may be suggested. Either we have to regard it as an entirely theoretical one, which arose in the mind of a mathematician who had reflected profoundly about his problem, or — what is more likely — the source must be sought in England again. In fact, in this connection attention may be drawn to an interesting booklet, *viz.* Robert Norman's *The Neue Attractive*, London 1581 ²⁰⁾.

In Chapter 9 Norman ²¹⁾ disputes the view of those who assume that the change of the variation from one place to another is regular; he explains quite clearly that he feels justified in speaking of regularity if with an equal move-

¹⁹⁾ Stevin has omitted to draw attention to an influence due to the change in the latitude through the movement of the ship and one in the declination of the sun between the forenoon and the afternoon observation. Either he forgot to mention it or he considered these influences to be of minor importance for practical purposes. The change in the declination, which can be no more than a few minutes of arc, indeed does not have to be taken into account. The change in the latitude will seldom have amounted to more than one degree.

²⁰⁾ Robert Norman, *The Neue Attractive: containing a short discourse of the magnes or loadstone, and amongst other his vertues of a new discovered secret and subtile propertie, concerning the declining of the needle, touched therewith under the plaine of the Horizon*. London 1581. Reprinted in 1585, in 1596, and again in 1614.

²¹⁾ Norman († 1596), who in the 18 or 20 years of his seafaring life had diligently collected data about the behaviour of the compass-needle and who, after having settled on shore, became known as "Norman, the compass-maker", was the discoverer of the inclination of the magnetic needle, which phenomenon he describes in his booklet. "A matter never before found or written by any", thus William Borough calls this phenomenon in the preface to his book *A Discourse of the Variation of the Cumpas or Magneticall Needle*, London 1581, which was intended as an "annexe" to Norman's treatise.

ment of the ship is associated an equal change of the variation. Martin Cortes, too, had assumed this regularity. But Norman calls the assumption an error. According to him no such regularity exists. In some regions the change is rapid and sudden, in others it is slow. For the same place on the earth, however, the deviation remains constant.

Norman expresses the wish that sailors on their voyages may observe the variation with accurate instruments, a practice which will benefit them at a later date, in particular in those regions where the variation changes rapidly over short distances. Norman does not state how the phenomenon is revealed by observation, as Stevin does.

It may be considered probable that Stevin was acquainted with Norman's view. Further it is quite conceivable that the wish of Norman just mentioned was the origin of Stevin's similar wish, as well as of Prince Maurice's order of 1599 about the collecting of observational data.

c. THE LATIN TRANSLATION

The jurist Hugo Grotius (1583—1645), a figure famed in world history, at the age of 16 years had translated *The Haven-Finding Art* faithfully into elegant Latin. The title of the translation is:

Limenheuretica, sive portuum investigandorum ratio.

Metaphraste Hug. Grotio Batavo.

Ex officina Plantiniana apud Christophorum Raphelengium.

Academiae Lugduno-Batavae Typographum. 1599. 22)

Grotius wrote for the booklet a dedicatory epistle, addressed to the Doge, the Senate, and the people of Venice and dated Delft, 1st April 1599. This date shows that the translation appeared almost simultaneously with the original Dutch version. It is not merely on account of the courtesy of the wording that this dedication is worth reading. It also contains some personal impressions of the author. It drives Stevin's meaning home to the reader more clearly than he himself had done and it throws full light on the importance attached to Stevin's work by the leader of the country, Lieutenant-Admiral Prince Maurice.

Grotius relates that he had met the Venetian ambassador while accompanying the Dutch embassy sent to Paris. After making a polite comparison between Venice and the Republic he states he had resolved to dedicate a work to the Venetians. The favourable occasion which was worthy of them and which enabled him to add a contribution of his own — a reference to his dedicatory epistle — had now arisen. He was able to offer and recommend a booklet containing instructions given by the Prince to the commanders of the navy and to their boards, to be followed by them. The Lieutenant-Admiral himself had previously studied the subject.

After a circumstantial discussion of the development of ancient navigation and the knowledge of the compass, Grotius recalls how on voyages from east to west the compass-needle had been found to deviate gradually and not inconsiderably

²²⁾ *Works, X.*

from the true north, which had caused great doubt and uncertainty among seamen. Thanks to prolonged observation of the magnetic declination at different times and places it had been found by the most learned mathematicians — as one of whom he considers Prince Maurice — that this was no mere accident, but that in nature a certain regularity (*ratio et norma*) existed according to which the pointings of the needle varied. The Prince had now presented these instructions, written about the matter by his mathematician Stevin, to those in authority in maritime affairs, in order that, if there should be found to exist disagreement between theory and personal observation, every effort might be made to deduce a rule from different experiments.

In order that as many data as possible might be collected, the Prince had decided to present the booklet to the Doge, so that the Venetian navigators might take similar observations, which would make for greater certainty in the finding of any destination.

Grotius concludes his dedicatory epistle with a general recommendation of the method and with the wish that "this small present" might be sympathetically received, "which will be of benefit to both parties and to the whole of the human race".

The high expectations that were entertained — by the Prince in particular — of the fruits of Stevin's work could hardly be expressed more eloquently.

d. *THE ENGLISH TRANSLATION* ²³⁾

Not long after the appearance of the Dutch edition of *The Haven-Finding Art* an English translation was published, prepared by the mathematician and nautical expert Edward Wright (1558—1615), famed in the history of navigation. He added a solemnly worded dedicatory epistle to Charles, Earl of Nottingham, Lord High Admiral of England, dated 23rd August 1599. Wright very urgently solicits the admiral's interest in the subject, which was considered by Prince Maurice to be of exceptional value for deep-sea navigation.

Wright had undertaken the translation in order to spread the knowledge of this method among all English seafarers, although he was aware that there were English navigators who had found the position of their ship by means of latitude and variation "more then ten yeres since". However, his countrymen were not to be behind the Dutch, and that was why he recommended this useful booklet — he calls it "this Dutch Pilot" — to the admiral, requesting him to induce English seamen to test the method and to take observations all over the world. "Proofo already made by some of our skilfullest English navigators" had already raised good hopes of success, a success which would benefit not only seamen, but also "the whole body of the Christian commonwealth".

After this, in an introduction Wright wishes "Richard Poulter, the maister and brotherhood of Trinitie House and all English mariners and sea-men in generall that love the perfection of their owne profession, health and happiness". Wright here uses the words in which Grotius had addressed the Doge and the Venetians, and he expatiates on the wide importance of the knowledge of the variation for the improvement of navigation.

The two introductions are followed by the translation of the body of *The*

²³⁾ *Works*, X.

Haven-Finding Art, with the exception of the Appendix at the end. The figures are reproduced extremely faithfully.

The name of Edward Wright lives on in the history of navigation on account of his famous book *Certain Errors in Navigation Detected and Corrected*, of which three editions appeared, viz. in 1599, in 1610, and the last in 1657, published posthumously and edited by Joseph Moxon. When Wright revised the work for the second edition and addressed Henry, Prince of Wales, in a dedicatory epistle, in which he described in glowing colours the perfect state of shipping and navigation at that time, he evinced such great confidence in the system of haven-finding that this nautical writer, who was one of the greatest experts of his country and his age, wrote: "this variation performeth almost so much in effect as the invention of the longitude".

Since the translation of *The Haven-Finding Art* already existed, Wright did not consider it necessary in 1610 to include this treatise in his book, but he did add *An Addition Touching the Variation of the Compasse*, an essay of only three pages. There was a definite reason for his writing this addition. The opening paragraph at once gives the explanation of it. In fact, it is stated that "some have been of opinion that there be two magneticall poles". The author mentions no names, but the identity of his adversaries is revealed by Moxon, in his preface to the reader in the third edition of *Certain Errors* referred to above. It was Anthony Linton and his followers who propagated this theory, which resembles that of Plancius. Linton published a small book²⁴⁾ containing an enumeration of all sorts of improvements to be made in navigation. One of them concerned the possibility of determining the longitude from the variation. The starting-point was that although "a great learned man and his followers absolutely denie that there is any fixed pole magneticall", there must be two fixed magnetic poles, which furnished the basis for the possibility of determining a ship's position. Both magnetic poles were situated on the surface of the earth, within the arctic circles, one in the Arctic and the other in the Antarctic regions. They were altogether different from the geographical poles and were unlike anything written up to that time about magnetic poles. Linton imagined these two poles to be joined by a magnetic axis passing through the centre of the earth. A magnetic equator with parallels and meridians could be imagined at right angles to the line joining these poles. The meridian through the two magnetic and the two geographical poles had to be regarded as the prime meridian. If the variation and the latitude of a place were known, it was assumed to be possible to determine the longitude of this place by "arithmetically calculation". Conversely, if the latitude and the longitude of a given place were known, the variation for this place could of course also be calculated.

In his *Addition* Wright refutes this opinion. In accordance with his views, always assuming that the magnetic pole must be the central point towards which the magnetic needle points, wherever it is placed on the earth, logically there would have to be as many pairs of magnetic poles as agonic were known. All those poles cannot be reduced to two. Wright draws a brief conclusion: Linton's

²⁴⁾ *Newes of the complement of the art of navigation and of the mightie Empire of Cataia together with the Straits of Anian.* by A. L., London 1609.

The author mentions his name, Anthonie Lynton, in the dedicatory epistle on page 1.

supposition "is absurd and therefore no such magneticall poles". In other words: away with them!

Wright finds the impossibility of such magnetic poles confirmed in a series of values of the variation of the compass which follows the *Addition*. The observational data had partly been supplied by the author himself, but he had obtained the majority from English and foreign mariners. The table runs to twelve pages and contains 283 values for places widely dispersed over the earth, a very considerable extension indeed of the original material of Plancius of 1599, published by Stevin at the time.

It thus appears that Wright made no alterations in Stevin's train of thought. There *can* be no magnetic pole, at least none of the kind imagined at that time. The variations of the compass are enumerated and listed such as they had been observed. No attempt is made to give an explanation of terrestrial magnetism.

And there the matter rested. As already said, Moxon published the third edition of Wright's book, because he considered it undesirable for the English nation "that this so usefull a book should sleep it self to death". The *Addition* of 1610 was printed unchanged at the end, along with the list of the observed variations. As an appendix he included *The Haven-Finding Art*, the text being that of the translation of 1599. It is curious that the same book thus contains both the list of the variations of 1599 and the enlarged list of 1610, now published again after almost fifty years. Although this seems to indicate stagnation, progress did exist in the matter of the knowledge of the variation; this will be discussed in § 5.

e. THE FRENCH TRANSLATION

The French translation ²⁵⁾ bears the following title:

Le Trouve-Port, traduit d'Alleman en François à Leyde en l'Imprimerie de Plantin, par Christoffle de Ravelengien, Imprimeur juré de l'Université de Leyde, 1599, avec privilège. ²⁶⁾

The vignette on the title-page is the same in the Dutch and the French edition. The unknown translator has closely followed the original text. The booklet contains no dedicatory epistle, like the two other translations, nor any preface to the reader or the like, so that no data are present giving special information about this version. On the last page the *Privilege* is reproduced. It is given in Dutch and is perfectly identical with that in the original edition. It is probably the same type.

f. STEVIN'S VIEW OF THE SYSTEM OF "THE HAVEN-FINDING ART" IN 1608

Nine years after the publication of *The Haven-Finding Art* the great collective work *Wisconstighe Ghedachtenissen, beschreven door Simon Stevin, Leiden 1608* ²⁷⁾ appeared, in two volumes.

²⁵⁾ The French translation of *The Haven-Finding Art* is not present in any Dutch library. Two copies of it are to be found in the Bibliothèque Nationale de Paris.

²⁶⁾ *Works*, X.

²⁷⁾ *Works*, XI.

Volume I includes, as the "Fifth Book of Geography", the *Haven-Finding Art* (pp. 163—175)²⁸). A comparison of the text with that of 1599 reveals points of resemblance as well as differences.

Long passages were taken over literally, such as the description of the finding of the variation of the compass at sea and that of the azimuth compass of Reynier Pietersz. The table with the values of the variations, originating from Plancius, has been reprinted and once more forms the starting-point of the discourse. The expression of gratitude to Plancius too is present. Just as he did in 1599, Stevin also emphatically rejects the magnetic pole in the text of 1608. In both cases he utters this central idea quite at the beginning.

A sentence has been added in which the author says emphatically that he is going to speak of "haven-finding", not of "longitude-finding, which would be a wider subject and would be of greater worth". The work has been abridged by the omission of several passages. The order of other passages has been changed.

From the omissions a changed view may be inferred.

Whereas in 1599 the statement that the meridians through Corvo and Hjelmsjö were agonic and that a maximum value for the variation occurred on the mid-meridian was for Stevin a "conclusion" based on the data, in 1608 he writes: "from this it is suspected that these properties can be assumed". Stevin therefore has grown more cautious in his wording.

The text concerning the agonic in longitude 160° E. and the meridian of longitude 110° E., with the maximum value of the variation, is taken over. But the parts east of longitude 160° E. and west of Corvo are no longer spoken of. The figure illustrating the six agonics and the text concerning the six lunes, each with an increasing and a decreasing variation, have been dropped. Apparently he now considered his earlier "conjecture" too speculative to maintain it.

In 1599 Stevin wrote that Plancius was expecting "any day" to receive new observations, taken on board ships which had been away for more than fourteen months. In 1608 they do not seem to have reached him yet, for he now writes of Plancius: "but he expects to receive more accurate information about it". Although therefore the lack of observations from remote regions is the reason why the list of the variations is neither enlarged nor corrected after nine years, one strongly suspects that acquaintance with newer data induced Stevin to make the above-mentioned alterations in the next and to be more cautious in his wording.

Like the starting-point of his theory, his confidence in his system of finding a destination through knowledge of the latitude and the variation of the compass is unaltered. The text about the deviation of the compass in the vicinity of Amsterdam, the result obtained during the voyage to Brazil, the preference of the position obtained to that by dead reckoning, the story of the ship that missed St. Helena, to be found on pages 4 to 6 in the edition of 1599, all have been taken over literally in the edition of 1608. Stevin concludes with the following words: "Thus far the appearance of the variations following from the data of the table has been described. If other, more exact observations should prove different in the future, other conclusions will have to be drawn from them, and in navigation the best must always be followed".

The *Wisconstighe Ghedachtenissen* were translated into Latin and published in two volumes under the title:

²⁸) *Works*, XI, i, 25.

Hypomnemata mathematica. Leiden 1608. 29)

Volume I contains:

*Liber quintus geographiae de Limenheuretica,
metaphraste Hugo Grotio Batavo,*

with in the margin the words: *Portuum investigandorum ratione.*

From this title it may be seen that Grotius again prepared the translation, for which naturally he now followed the text of 1608. No further remarks therefore have to be made about this, with the exception of one.

When in 1599 Stevin had written: "From this it is concluded", Grotius had translated: *ex hisce concludere volumus*. But when in the edition of 1608 Stevin used a weaker expression and wrote: "from this it is suspected", Grotius still wrote: *ex his concludere volumus*. Since the translator thus continued to speak of "concluding", he failed in this respect to render Stevin's meaning accurately.

Finally it has to be mentioned in what form *The Haven-Finding Art* was included in

Les Oeuvres mathématiques de Simon Stevin . . . Le tout revue . . . par Albert Girard, 1634. 30).

In Volume II, p. 171, we find:

Cinquiesme livre de la Géographie. Du Trouve-Port ou la manière de trouver les Havres.

This again is the text of 1608. Here the change in Stevin's words has been taken into account. Whereas the French translation of 1599 reads: "*de ceci on veut conclure*", in that of 1634 we find: "*de cecy on a opinion*".

There are no indications of any other editions or versions of *The Haven-Finding Art* having appeared since then.

§ 4

THE MEASUREMENT OF THE VARIATION OF THE COMPASS

a. THE MEASUREMENT OF THE VARIATION OF THE COMPASS BEFORE STEVIN'S DAY.

It has been said before that Stevin, when proceeding to discuss the determination of the variation of the compass, begins with the remark that this subject was "known to many people". This was no doubt true. In fact, sixteenth-century navigators had examined special phenomena observed by them at sea and they had the good sense to publish their measurements. Writers of nautical textbooks dealt with the subject. In the latter part of the sixteenth century Dutch seamen too on their ocean voyages determined the amount of the variation and recorded it in their log-books. But in practice most sailors did not concern themselves about the variation. They knew by heart by what courses they could reach their destination and sailed "by sight". The art of finding their way they had "mostly learned by their own experience and by the instruction of old and experienced pilots", as Coignet wrote in his *Nieuwe Onderwijsinghe* of 1580 and in its later editions. Since for the application of the system of "haven-finding" it was necessary to know the variation of the magnetic needle, it is obvious that for the sake

29) *Works*, XI, b.

30) *Works*, XIII.

of completeness Stevin devoted some space to the subject. What he writes about it is based on foreign works available at the moment, and further — as far as Holland was concerned — on the work of Reynier Pietersz. Stevin was not original on this point, as will appear from the following discussion.

It is known that Columbus on his first voyage across the Atlantic took bearings of Polaris³¹⁾ in the early and the latter part of the nights of 13th, 17th, and 30th September 1492. We do not know whether he did or did not take these observations with the aid of a special instrument. A bearing instrument is first described in a Portuguese manuscript of 1514³²⁾, but unfortunately this description is not very clear, so that no reliable idea can be formed of it.

A perfectly clear description is given by Francisco Faleiro, a Portuguese in the service of Spain, in his textbook *Arte del Marear* of 1535³³⁾. In Chapter VIII, in which the northeasting and the northwesting of the compass-needle are discussed, the author describes the construction of an instrument for determining the variation of the needle. It is a round, flat disc, in the centre of which a magnetic needle has been mounted. About the compass pivot as centre a graduation, consisting of four times 90°, has been marked on the disc. Coinciding with the line 0°-180° and perpendicular to the plane of the disc is a small rectangular plate, which serves as a shadow pin. In the figure illustrating the text the latter is not shown.

The observation is taken at noon. The instrument is held in the hand in such a way that it is horizontal and the plate casts no shadow. The line 0°-180° then lies in the meridian and the needle indicates the variation on the graduation. It is also possible to take the observation in the forenoon and the afternoon at equal altitudes of the sun and to read the two indications of the needle. During the second observation the mean of the readings indicates the direction of the meridian, while the angle which the needle now makes with this direction is the variation sought. Faleiro prefers the latter method of taking the observation because it can be performed many times a day. The observation at the sun's greatest altitude on the other hand might be inaccurate because the moment of noon could not be determined exactly. Finally observations at sunrise and sunset, of which in the same way the mean is taken, afford a third method for obtaining the desired result. Although nowadays this instrument appears to us to be little suited for use on board, because, while holding it horizontal and at the same time directing it, one has to compensate the rolling, pitching, and yawing movements and changes of the heeling of the ship by movements of the hand, from Faleiro's text it is quite evident that it was destined for use at sea³⁴⁾.

³¹⁾ "Taking a bearing of Polaris" or of the sun is the process of measuring the angle between the compass needle on board and the azimuthal direction of Polaris or the sun, as the case may be.

³²⁾ This manuscript is discussed in § 5, p. 393

³³⁾ Francisco Faleiro, *Tratado del esfera y del arte del marear*. Seville 1535. Facsimile reprint edited by J. Bensaude, *Histoire de la science nautique portugaise*. Vol. 4, Berne/Munich 1915. Chapter VIII, Del nordestear de las agujas, p. 79.

³⁴⁾ Alonso de Santa Cruz—an inhabitant of Seville, who died there in 1572—in 1536 was appointed *cosmografo real* and was a renowned cartographer. He also made contributions to the development of the art of navigation, in particular to the problem of the determination of longitude. Following his attendance of a *junta* of cosmographers, astronomers, and scholars he wrote a treatise entitled:

(Continuation on next page).

In discussing the compass and its properties, Medina ³⁵⁾ draws attention to the taking of bearings during navigation, saying: "the Pilot should adjust his needle to Polaris" ³⁶⁾, waiting for the moment at which the "guardians" or "wheels" (β and γ *Ursae Minoris*) are northeast and southwest of Polaris, because the latter is then in the meridian. Chapter VI describes how the direction of the needle can be tested by taking two bearings of the sun at equal altitudes, the shadow cast on the card by a style being read before and after noon, when the shadow has the same length. However, this is a method for use on land, to find whether the instrument has been properly made, not for determining the variation of the compass at sea. Medina did not believe in the existence of variation, and thus he made no contribution to the development of this nautical subject.

Zamorano ³⁷⁾ (Chapter XVIII) also teaches the observation of the Pole Star, but he calls the method "uncertain" (*ocasionada a error*, says the Spanish text). In addition his book contains the above-mentioned testing method. But he presents a new method for the measurement of the variation of the compass at sea and on land, *viz.* taking a bearing of the sun at sunrise and at sunset. At the end of his book he depicts a simple instrument, by means of which the point of sunrise or sunset in the horizon is found graphically, the declination and the latitude being known. This instrument therefore furnished the solution of the problem, which is essentially one of spherical trigonometry. With one instead of two observations the desired objective was now reached. It was the Englishman Thomas Harriot (1560-1621) ³⁸⁾ who afterwards compiled for this purpose a table of ampli-

(Continuation of note 34)

Libro de las longitudes y manera que hasta agora se ha tenido en el arte de navegar. This treatise, which he dedicated to Philip II, has been preserved in manuscript at Madrid. The year of its appearance is not recorded, but this cannot be prior to 1536, since he calls himself *cosmografo mayor* of the king. In 1921 it was published.

He devotes a good deal of space to one of the numerous methods for the determination of longitude discussed by him, *viz.* to that by means of the northeasting and north-westing of the compass needle. In this context the same instrument as that of Faleiro is described, with the only difference that here a central shadow pin is mentioned. The meridian and the magnetic declination are determined by observation of the sun in the forenoon and the afternoon, at equal altitudes. The inventor of this instrument is reported to be Felipe Guillen, dispenser and inhabitant of Seville, an intelligent and inventive man, who enlisted in the Portuguese navy in 1525. In the treatise the instrument is called "*muy comun en Portugal*". Although Guillen is looked upon as the inventor, by Al. de Santa Cruz, it is much more probable that it is of Portuguese origin.

A description of the construction and the use of the instrument, equipped with a vertical central shadow pin, is also to be found in Nunes, *Tratado da Esphera*, 1537 (p. 141) (See the facsimile edition of this book by J. Bensaude, Berne/Munich 1915). Also in: Hellmann, *Rara magnetica*, and in: *Terrestrial Magnetism and Atmospheric Electricity*, 1943-1945. Harradon, *Some early contributions to the history of geomagnetism*.

³⁵⁾ Pedro de Medina, *Arte de navegar*. Valladolid 1545. Book 6, Chapter III, p. 83 and Chapter VI, p. 86.

³⁶⁾ "*El piloto para marcar sus agujas mira el estrella del norte para las marcar por ella. . .*" (*marcar* = to adjust).

³⁷⁾ Rodrigo Zamorano, *Compendio de la arte de navegar*. Seville 1581. Reprints in 1582, 1586, 1588, 1591, 1596. Dutch translation in 1598. English translation by Edward Wright, included in his work *Certain Errors in Navigation*, 2nd edition of 1610, and again in the 3rd edition of 1657. The numerous editions and translations indicate the wide diffusion of this book.

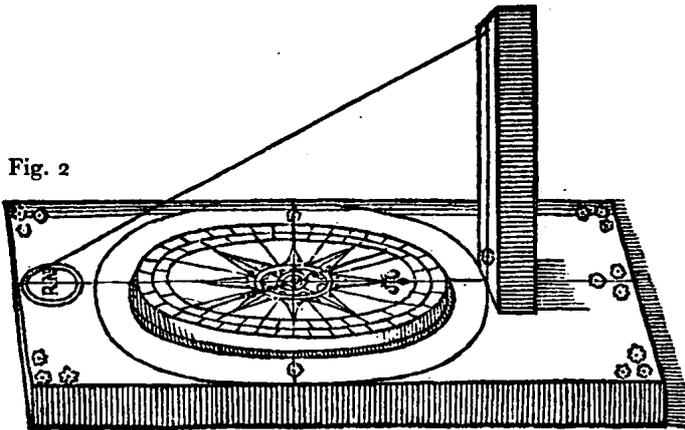
³⁸⁾ E. G. R. Taylor, *The Mathematical Practitioners of Tudor and Stuart England*. Cambridge 1954, pp. 182 and 333.

tudes for every degree of declination of the sun and for every degree of latitude up to $54^{\circ} 39'$).

A book widely used and valued in England in the last decades of the sixteenth century was the textbook of William Bourne entitled *A Regiment for the Sea*, which first appeared in 1574⁴⁰⁾. A Dutch translation, under the title of *De Const der zee-vaerdt*, was published at Amsterdam in 1594, and again in 1599. For the determination of the variation of the compass at sea the author gives five methods, *viz.* observation in the horizon at sunrise and sunset and taking the mean of the readings, taking a bearing of the sun before and after noon at equal altitudes, ditto at noon, when it is in the meridian, two observations of a star, and finally taking a bearing of Polaris. Observation of the moon is stated to be inadvisable.

With the exception of Faleiro, the authors hitherto cited do not say anything about the use of an instrument specially fitted for taking observations on board. It thus has to be assumed that the observation was taken with an ordinary mariner's compass of the type that was being used to steer by, and that the shadow cast on the card by a vertically suspended string or a vertical pin in the middle of the glass cover indicated the azimuthal direction of the sun.

An instrument specially constructed for the purpose, *i.e.* a true azimuth compass, is first found in William Borough (1537-1598), who during his seafaring years studied the variation of the compass-needle in order to make possible accurate navigation. About this subject he wrote an important treatise⁴¹⁾, which he



Instrument for the measurement of the variation after William Borough
(See note 41)

³⁹⁾ Such tables were widely used in the eighteenth century. Tables of the amplitude of the sun are also to be found in: Joao Baptista Lavanha, *Regimento Nautico*. Lisbon 1595, 2nd ed. 1606.

⁴⁰⁾ Reprints in 1576, 1577, and 1580. After the author's death in 1588 the book was republished by Thomas Hood in 1592, 1596, and 1601. The numerous reprints and the translation indicate that the book was widely diffused and well known in nautical circles.

⁴¹⁾ William Borough, *A Discourse of the Variation of the Cumpas or Magneticall Needle, wherein is mathematicallie shewed the manner of the observation, effects and application thereof, made by W. B.* London 1581. Reprints in 1585, 1596, and 1614. The title-page has the statement: "and is to be annexed to *The New Attractive of R.N.*"

regarded as a supplement to the work of Robert Norman. In the present context attention is drawn only to two very simple instruments with the aid of which it was possible to take a bearing and which are illustrated and described in the treatise.

As shown by Fig. 2, the first consists of a piece of board, on which has been fixed a round box, on the bottom of which a wind-rose is marked. Mounted in the centre of the latter is the pivot on which the magnetic needle turns. On this piece of board has been mounted a vertical style on which a line has been drawn. A second line has been made on the piece of board, from the foot of the first line and through the centre of the card. This is the centre line of the instrument. A string has been stretched from the top of the style to the end of the line on the piece of board. In order to enable the observation to be taken, the piece of board was put on a horizontal table and turned until the shadow of the string fell on the line through the centre of the card and at the same time covered the line on the vertical style. On the card the position of the needle is read, the angle between the needle and the azimuthal direction of the sun thus having been measured.

Because in Borough's opinion "imperfections" still attached to this instrument, he designed another of improved construction (Fig. 3). The needle is contained

A new Instrument for the Variation.

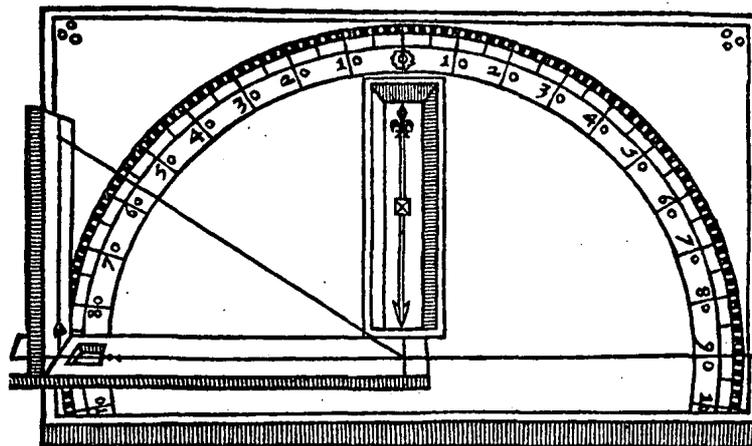
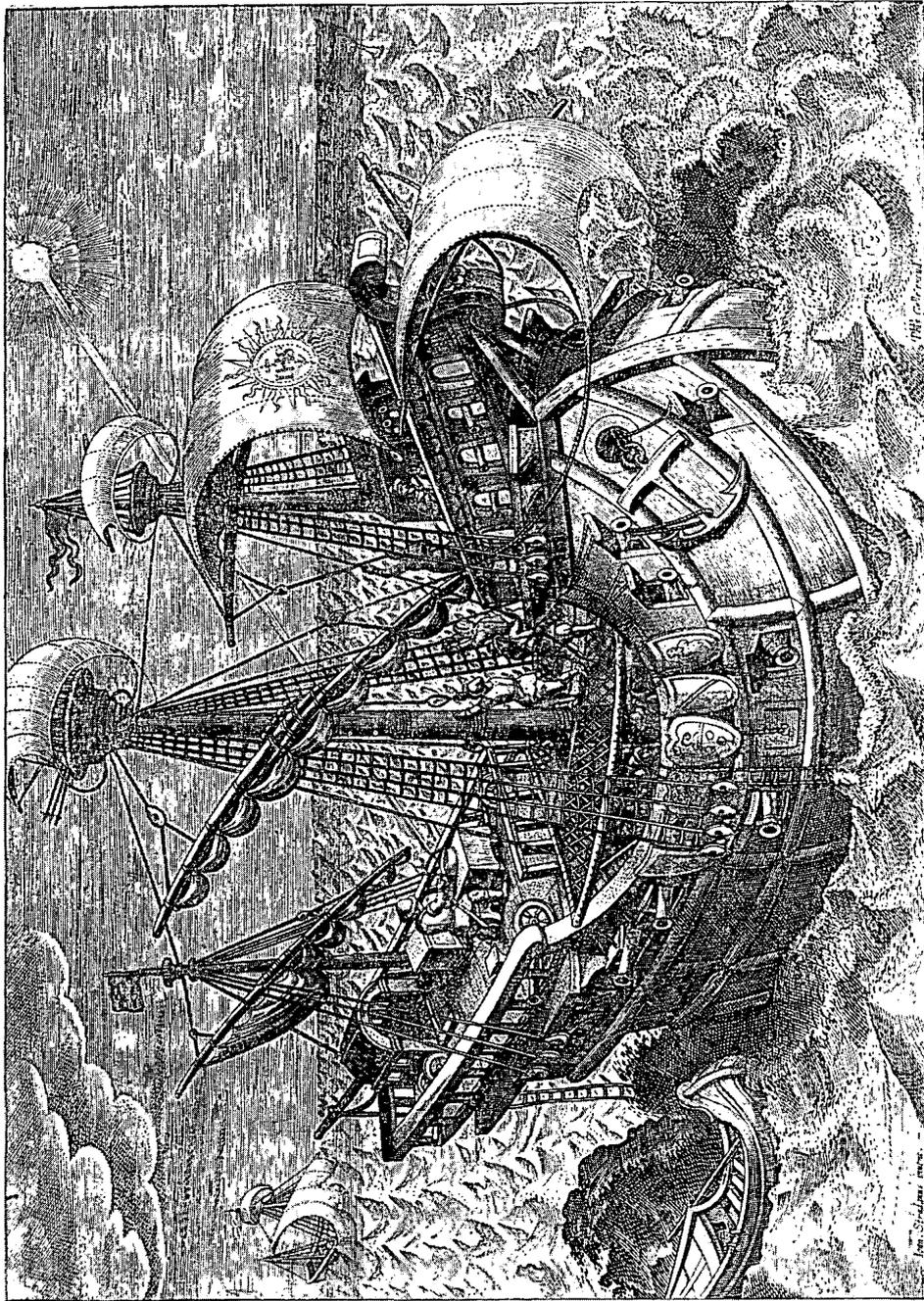


Fig. 3

Improved version for Borough's Instrument (see note 41)

in a narrow box, in consequence of which only small deflections are possible. The instrument always has to be turned until the needle coincides with the zero line of the graduation. In the illustration the instrument has been folded out into the plane of the drawing. It can be seen that it consisted of a horizontal lath (alidade), mounted to turn on a pivot in the centre of the graduation, and of a vertical lath fastened on the first. On both a mark had been made and the ends were joined by a string. In the horizontal lath, above the marks of the graduation, there was an opening, through which the position of the lath could be read.



*Magnete paulum virtutis saepe deua
Dat inuenire portum ubique Plancius.
ORBIS LONGITVDINES REPERIRE È MAGNETIS À POLO DECLINATIONE.*

Navigator, taking a bearing of the sun (Stradanus, about 1600, see note 42)

This adjustable azimuth compass may no doubt be called a considerable improvement and an invention.

When the observation was to be taken, the instrument was put on a horizontal surface or held horizontal in the hand and was turned in the right direction by the setting of the needle to the zero line of the graduation. The alidade was then turned until the shadow of the string fell on the lines of the two laths, upon which the position of the alidade was read on the graduation, and thus the angle between the needle and the azimuthal direction of the sun had been measured. It is to be noted that when the instrument was used on board a ship moving in all sorts of directions, by a movement of the hand the piece of board had to be given the position in which the prescribed adjustment of the needle was maintained. The observer thus was faced with the far from easy task of seeking two adjustments at the same time. In this respect the new azimuth compass of Borough was inferior to the first-mentioned construction, for in that case only the base of the instrument had to be given the right direction.

It is a fortunate circumstance that a fine print has been preserved, which illustrates a navigator who is taking a bearing of the sun. It occurs in a book of prints entitled *Nova reperta*, published at Antwerp about 1600⁴²). The latest inventions known at the time are illustrated. This print, reproduced on the opposite page concerns the system of determining longitude from the declination of the magnetic needle. The useful application is ascribed to Plancius. A particularly heavy ship is shown. Four relatively small, bulging sails are spread. They seem to be drawing well, but curiously enough the sheets are not taut and have the same elegant curve as the sails. Considering the appearance of the sails the ship must be running, whereas the direction in which the flags are fluttering shows that the wind is approximately ahead instead of aft. The anchor, the flukes and the stock of which lie in one and the same plane instead of being at right angles to one another, is unfit for use in this way. The artist of the time therefore appears to have proceeded very freely indeed. We have to interpret his print with the greatest caution.

On the raised quarterdeck there is a small table, covered with a cloth, at which the navigator is sitting on a seat. The compass is in front of him on the table. The bowl is a round box. Fixed on its edge is the azimuth quadrant, which consists of a small vertical style and a quarter circle. It appears not to be adjustable, nor has it got an alidade, but perhaps a sight could move along the circumference of the quarter circle, by which means it may have been possible to determine roughly the altitude of the sun. The quadrant is directed towards the sun, a position which the observer must have obtained by turning the box. If we rely upon the print in this respect, we may therefore conclude that this instrument was directed and manipulated like the first azimuth compass of Borough, but that the non-adjustable quarter circle, probably with a sight, is a new addition.

An extremely important stride forward in the system of "taking a bearing of a heavenly body" is due to the Rev. William Barlow (1544-1625), a son of the bishop of Chichester, who was greatly interested in navigation and engaged in investigations on magnetism. His book *The Navigator's Supply* (1597) contains an illustration of the "compasse of variation" designed by him and constructed by the instrument-maker Charles Whitwell. It may be described as follows.

⁴²) Joan Stradanus, *Nova reperta*. Engraved by Joan Galle. Antwerp, n.d. (about 1600).

In a square wooden box the round metal compass bowl is suspended in gimbals. The card is divided into full points and provided with a beautiful fleur-de-lis for the north point, as was the custom in those days. An azimuth instrument similar to the present-day cross bar sight is adapted to turn on a pivot in the centre of the glass cover. Hinges are shown at the foot of both sights. Both could thus be placed in the horizontal position when the box had to be closed with the sliding lid. Against the inside of the bowl there was a graduation. The position of the azimuth instrument as well as that of the north point of the compass card had to be read on this graduation if the angle between the needle and the azimuthal direction of the heavenly body observed was to be found. Although the use of gimbals and of a "thread and sights" may be looked upon as two important strides forward in the development of the azimuth compass, the fact that two readings had to be made simultaneously on a moving ship — with the card pivoting relative to the graduation and the azimuth instrument too, because this was moved by hand — formed a great inconvenience still attaching to this instrument.

In France Jean de Séville, dit le Soucy, a medical practitioner and mathematician at Rouen, in his *Compost manuel*⁴³), a book written in particular with a view to the determination of longitude, gave an unfortunately obscure description of a compass with which the variation could be measured. There is no indication that this book may have been known to the Dutch at that time, nor that any other French book may have affected their knowledge concerning the observation of the variation of the compass.

Let us now see what information was available in Holland at that time.

Since 1580 Coignet's *Nieuwe Onderwijsinghe*, appended to the Dutch translation of Medina, was no doubt the best source a seaman could use if he wanted to learn more about deep-sea navigation. It was a useful textbook for him, to be preferred to Medina in view of its simplicity, the clear language in which it was written, and above all things because it was adapted entirely to the practice of navigation. Coignet avoids long digressions about the variation of the compass, "for this would appear useless to mariners", and he is brief and to the point in giving his opinion, which agrees with that of experienced seamen. This opinion is the following. It is necessary to use on board a compass the needle of which was set under the same angle with the north and south line of the card as was the case in the compass used by the maker of the chart which the navigator consulted at sea. "Thus you will not err". Experience had proved the necessity of this. It was of greater value than the enunciations of science, "since the matter at sea cannot so far be easily stated in a general rule".

We thus find that the Dutch textbook does mention the deviation of the compass-needle as a fact, but that deliberately little is said about this matter and the determination of the amount of the variation is not discussed. Of course the observations of the variation taken during the first voyage to the Indies, the data collected on the northern voyages of Heemskerck and Barents — to mention a few important sources — bear witness to the fact that there were pioneers who had full command of the subject. With regard to the rank and file of mariners this could not be said, since most of them had gained practical experience at sea

⁴³) Jean de Séville, dit le Soucy, *Le compost manuel calendrier et almanach perpetuel, principalement pour la longitude de l'est et ouest*. Rouen 1586. Reprint 1595.

and had not learned the art of navigation from a book. The views held by many of them are revealed by Jan Huychen van Linschoten ⁴⁴), when he writes "that some masters think it is not necessary to know how much the northeasterly or northwesterly variation of the compass amounts to, giving as their reason that our ancestors were unacquainted with the compass and nevertheless drew charts of the coasts". Upon which the author gives as his opinion "that this may indeed be excused in some navigations, but in most long voyages it is highly necessary to know the northeasting and northwesting".

Considering the available information and the prevailing views in Holland, Stevin in compiling his *Haven-Finding Art* must have been convinced of the necessity of including the method of determining the variation in addition to his own theory. Just as in each case, when entering a special field of knowledge, he acquainted himself with the progress of the science concerned, he must here — not finding the source in his own country — have turned his eyes towards other countries. It is beyond doubt that he knew the works of Zamorano and Bourne, either in the original or in the Dutch translation. And although no translation exists of Borough's *Discourse of the Variation of the Cumpas*, this treatise, which was printed three times before 1599, must have been so widely known to those who studied the magnetism of the earth that neither its existence nor its contents can have escaped Stevin's notice. Nor does he mention in his text anything in this field beyond what was known from the said sixteenth-century authors.

But when he describes the azimuth compass of Reynier Pietersz to his readers, he gives something else than Borough and Barlow had found and described. This is an invention due to a compatriot of his, and we see how Stevin makes use of an instrument which at that time possessed the best properties with a view to its use in practice ⁴⁵).

⁴⁴) Jan Huychen van Linschoten, *Reijs-Gheschrift*, 1595, p. 19. *Beschrijvinghe van de Coursen nae Oost-Indien*. Printed in Vol. XLIII of *Werken der Linschoten Vereeniging*, pp. 34, 35.

⁴⁵) In the Spanish language there exists a booklet which, as the title reveals, is entirely devoted to such an instrument and which in connection with our subject is a very important treatise because it deals with the measurement of the variation, the importance of the variation for the determination of longitude, etc. It is:

Andres del Rio Riaño, *Tratado de un instrumento por el qual se conocera la nordesteacion o noroesteacion de la aguja de marear, navegando por la mayor altura del sol o de otra estrella o por dos alturas yguales y de la utilidad que del se á de seguir*. Seville, n.d.

One copy of this treatise is present in the library of the Museo Naval at Madrid. From an exhaustive search by the director of this museum it appears that it is not to be found in any other library. The copy in question is thus unique.

The year of its publication is mentioned neither on the titlepage nor anywhere else in the book. It was known at the time to Navarrete, who mentions it in his *Biblioteca marítima Española*, 1851, Vol. I, p. 97. He relates that it appeared subsequent to another work of Riaño, which was published in 1585. Further it is cited in the *Ensayo de bibliografía marítima Española*, Barcelona 1943, under No 220. In a note it is said that Navarrete puts it after 1585 and that Mendez Bejarano assumes it appeared in 1589. In the Museum it is put at 1585.

If this were true, it might just be possible that one specially interested in the subject, like Stevin, might have been acquainted with its contents. Further inquiry, however, proves it to be impossible.

The reader will encounter many statements implying an indication of time, e.g. "the

(Continuation on next page).

b. REYNIER PIETERSZ AND HIS "GOLDEN COMPASS".

Reynier Pietersz, as Stevin calls him, or Reynier Pieter van Twisch⁴⁶⁾, as his name appears in various resolutions, is known to have followed the sea for many years, in the service of shipowners at Hoorn, among others. In 1595 he is referred to as "pilot, living at Hoorn", in 1598 as "engineer". At this town he died in 1613. The Chronicle of Hoorn⁴⁷⁾ speaks, more than a century after his death, in the highest terms of this citizen, "a man greatly skilled in navigation". In the entry for 1598 the reason of his fame is given. In fact, he had "practised an exceptional skill in measuring the longitude, as simply as the southern and the northern latitude are commonly done". According to these words therefore he made the long-sought determination of longitude as simple as the determination of latitude, which had been known for several centuries. The solution was effected with the aid of an instrument which the inventor called the "Golden Compass". The brief description of it states "that it floated⁴⁸⁾ and thus always placed itself horizontal, in order to be able to use it in ships which list badly at sea or are tossed in other ways". The chronicler describes the process of taking the observation as follows: "the reading took place by means of the light of the sun, which fell through a certain sight on to the graduation marked around, and this indicated exactly, by means of the deviation of the needle from the true meridian, how far one was to the east or to the west".

From the fact that his inventions are mentioned in various resolutions it appears that the determination of the variation of the compass occupied his mind very intensively. We will now give some details.

On 12th July 1595 the States of Holland and West Frisia granted him letters patent on an invention of nautical instruments, which, "being useful to general navigation", he wanted "to benefit others as well"⁴⁹⁾. The reference

(Continuation of note 45)

fact that the whole fleet was lost in this year". Only three such statements will be cited here.

On page 5 *verso* a problem involving the determination of longitude by means of the moon is discussed, which is put at May 1, 1607. Who was likely to do this in 1585?

On page 3 *verso* of the "address to the reader" reference is made to the 8th chapter of the book which "el padre Clavio" wrote about sundials. Chr. Clavius, a well-known Jesuit father, who died at Rome in 1612, wrote:

Gnomonices libri octo, in quibus non solum horologiorum solarium. . .", which appeared at Rome in 1602.

There are several references to a book by Joan Garcia de Cespedes, *Regimiento de marear*. This name is not known among those of nautical writers. A name that is known is that of Andres Garcia de Cespedes, whose book *Regimiento de navegacion* appeared in 1606. No doubt it is the latter book which is meant. It can safely be said that 1585 is not the year in which Riaño's booklet appeared. It will probably have to be dated in 1607.

It is quite certain that the booklet has to be dated after the invention of the azimuth compass by Reynier Pietersz. It need not therefore be cited in connection with the development of the azimuth compass before Stevin's day.

⁴⁶⁾ The name is also spelled Van 't Wisch, Twisck, and Twisk. Twisk is the name of a village in the neighbourhood of Medemblik, in the province of North Holland.

⁴⁷⁾ Th. Velius, *Chronijk van Hoorn*, 4th edition, annotated by S. Centen, 1740, p. 501.

⁴⁸⁾ *I.e.* it was suspended horizontally.

⁴⁹⁾ G. Doorman, *Octrooien 1940*, p. 281. *Resolutiën Staten van Holland en West-Friesland*. 12th July 1595, fol. 278.

is to the determination of latitude, of longitude, and of the variation of the compass in remote regions, "hitherto never found or revealed so fully by anyone".

The same inventions are spoken of in somewhat clearer language in the letters patent granted to the inventor by the States General on 8th March 1597 for a period of twelve years⁵⁰). This document speaks of "two new instruments, invented" and already made by Reynier Pietersz, "being highly useful aids and very essential to navigation, the one to be used in order to learn, by measurement of the sun, the deviation of the needle and also to measure the longitude, east and west, *i.e.* how far one is removed from every meridian". The other must have been an instrument for measuring the altitudes of the sun or a star, but not with the aid of the visible horizon, as takes place in a fog or in observations on land.

In the document it is stated that a drawing accompanied it. Unfortunately this drawing has not been preserved, so that we lack further data about the appearance and the construction of these instruments, which are explicitly called "new inventions".

A year later he addressed to the States of Holland and West Frisia a request for a subvention in connection with the expense incurred in the construction of the two instruments just mentioned, upon which on 13th March 1598⁵¹) the States requested Jos. Scaliger, Snellius, Ludolf van Ceulen, and Stevin, together with the deputies of Amsterdam, Rotterdam, Hoorn, and Enkhuizen, to examine them and to report on them. The fact that the judgment of these learned and still famous men was sought shows very clearly that the inventions of Reynier Pietersz were considered important and that great expectations were entertained of them. Perhaps it was due to this that Stevin became acquainted with Reynier Pietersz and his invention.

In 1611 the latter applied once more to the States, asking them for a subvention⁵²).

It is not known on what lines the learned committee made their report. The Chronicle of Hoorn does contain a judgment based on a practical examination of the "Golden Compass", which reads as follows: it was "thus put to the test by several experienced pilots in many places, both to the east and to the west, even on the coasts of Guinea and the East and West Indies, and was everywhere found correct and without error, to the surprise of those who undertook the tests".

Although the authentic drawing, which might have provided a conclusive confirmation, is lacking, it is beyond doubt that the "Golden Compass" of Reynier Pietersz as described in the Chronicle of Hoorn and the cited resolutions is the same as that which Stevin illustrates, describes, and recommends in *The Haven-Finding Art*. On a great many points this invention of his compatriot brought improvements of the foreign instruments with which he may have been acquainted.

⁵⁰) G. Doorman, *Octrooien* 1940, p. 97. Also printed in De Jonge, *Opkomst*, Vol. I, p. 176. Original: Public Record Office, Records of the States General, item 3328 of the inventory, fol. 243.

⁵¹) De Jonge, *Opkomst*, Vol. I, p. 176. *Resolutie Staten van Holland*, 13th March 1598. De Jonge here adds a note reading: This instrument is illustrated in the *Haven-Finding Art* of Simon Stevin.

⁵²) G. Doorman, *Octrooien* 1940, p. 288. *Staten van Holland en West-Friesland*, 1st September 1611, p. 174.

Indeed, Borough's azimuth compass consisted of a small piece of board which had to be put horizontally on a table. It was thus suitable for use on land, but on a moving and rolling sailing-vessel no instrument can be mounted so as to be permanently horizontal, nor can such an adjustment be ensured by holding it in the hand. For the "Golden Compass" cardanic suspension was used. For a steering compass this feature was not novel. What *was* novel was its application in an azimuth compass, so that the horizontal position was ensured. Stevin was thus justified in speaking of an invention.

A second feature, which appears neither from the cited letters patent nor from *The Haven-Finding Art*, was the following. The graduation was marked on the inside of the bowl. It was to the zero point of this graduation that the needle had to be set. In consequence it was necessary during the taking of the observation to move the bowl in the direction contrary to that of the ship's movement. At the same time the quadrant had to be directed towards the sun. But as soon as the right direction had been given to the quadrant, the position of the needle as well as of the quadrant was maintained by the mere movement and readjustment of the box. It can readily be understood that this method of making the cardanically suspended bowl rotatable produced more accurate results than those that could be obtained with Borough's instrument.

The revolving quadrant, furnished with an alidade, may also be called an improvement, but it is especially the method of adjustment to which this remark applies. This method appears from the description found in the Chronicle of Hoorn, from which it may be understood that the sunlight falling through the sights cast a spot of light on the graduation along the circumference of the bowl. One observer could thus manipulate the instrument and bring about the two desired adjustments simultaneously. For this he merely had to look from above at the circumference of the bowl and at the needle.

Briefly, with the „Golden Compass” of Reynier Pietersz Stevin no doubt presented the most efficient instrument which was to be found in those days and which moreover satisfied the demands made by his system of „haven-finding”. And yet in practice its success seems to have been scanty, at least if we are to believe the Hoorn chronicler, who continues his favourable report on the trial of the instrument with the words: "I really do not know what is the cause that it is not used any further, but I think the age and the lack of means of the inventor are not among the least of the impediments and obstructions".

These suppositions on the part of Velius were probably wrong. The reason will rather have to be sought in the technical sector. Indeed, although the instrument may have been suitable for use as an azimuth compass, it naturally failed and disappointed as a means for the determination of longitude at sea. There is another point as well. The necessity of constantly moving and readjusting the whole bowl was a drawback of which the observer must have been aware. Improvement on this point was quite possible. In fact, such movement is superfluous with an azimuth compass equipped not only with a single magnetic needle but also with an ordinary compass card, while the position given by the bearing-instrument is read on a graduation along the circumference of the card. In that case it is only the azimuth quadrant which is constantly directed towards the sun by being turned against the movement of the ship. The reading on the card does not change in consequence of the movements of the ship. A compass of this type, which also amounted to a considerable improvement on that of Barlow

again, must have supplanted the "Golden Compass" of Reynier Pietersz as well as that of Barlow very readily indeed.

Finally the reader should realize what it must have been like to work with such compasses, in those days when the "compass-maker" was an artisan rather than an instrument-maker, hardly any knowledge existed as to the most suitable materials, and the requirements which a properly working mariner's compass was to satisfy had not yet been defined, *viz.* steadiness, sensitivity, light weight, and powerful directive force, while finally science had not yet prescribed the means which led to the satisfaction of these requirements. At that time the compass card was heavy. The cap in the card was imperfectly constructed. The pivot on which the card turned was not made of the most suitable material, nor was its point sufficiently sharp. The friction between pivot and cap accordingly was considerable. The needle, which had been touched with loadstone, was weakly magnetized, so that the magnetic moment was small. Owing to these causes the violent movements to which a sailing-vessel is liable dragged along the card and made it unsteady. In spite of the warning to be found in the textbooks, the fact that the bowl was not air-tight and the glass cover was insufficiently sealed resulted in air being blown in and pressure being exerted on the card and the needle, which caused deviations. The card oscillated and did not easily return to the original position. This applied all the more in high latitudes, where the directive force is less than in low latitudes. Steering on such a compass as well as reading a bearing must have been a difficult task, apt to involve considerable inaccuracy. Under difficult conditions it may even have become impossible ⁵³).

Here again it is evident why the task of the navigator in those days was arduous and extremely dangerous. Small wonder therefore that many of them, in spite of eminent seamanship, fell victims to the primitive level of navigation. Its development, both theoretical and technical, was ardently wished for, but it was not realized until the second half of the eighteenth century, when great possibilities were opened up in the matter of the determination of longitude and latitude at sea. The octant, the chronometer, the determination of longitude by lunar distances, the nautical almanac, the method of determining latitude invented by Cornelis Douwes, increased mathematical knowledge, etc., brought the solution. Those who paved the way for all this were good servants of their country.

⁵³) The unsteadiness of the compass was so familiar a thing that it had even become known outside nautical circles. The great seventeenth-century Dutch poet Joost van den Vondel, who celebrated navigation in his work and who borrowed numerous expressions from its idiom in his poetry, was also acquainted with the phenomenon. In his *Gijsbrecht van Aemstel* (1637) he alludes to the oscillation of the compass-needle and the unreliability of its indications. In the Prologue he says that the armorial bearings of the city of Amsterdam will in future be a reliable guide for the seaman and, leading him on like

. . . . a celestial symbol
On high, unsoiled by fog or earthly mist,
Give heart to the heroic helmsman
Where, frightened by the needle's swinging, he
Drifts in the Arctic Sea.

(transl. James S. Holmes)

THE DETERMINATION OF LONGITUDE AT SEA BY MEANS OF THE
VARIATION OF THE COMPASS

BRIEF SUMMARY

a. *THE EARLIEST VIEWS.*

As an introduction to this chapter, it may be recalled without a detailed discussion that the Chinese literature of the first centuries of the Christian era refers to the attraction of an iron needle by a loadstone and to the directive force of a magnetized needle. From a Chinese work dating from between 1111 and 1117 Balmer⁵⁴) cites the mention of the fact that the magnetic needle does not point exactly in the direction north-south, but always shows a small declination to the west. From this it is evident that the variation had been discovered at that time. It was attributed to errors in workmanship and in the manipulation of the magnet⁵⁵).

It has been proved that even before 1269 the variation was recognized as a real phenomenon in Europe. Its amount was measured, as is evident from Italian portolani. On a sun-dial made at Nuremberg in 1451, which is equipped with a compass, the magnetic north and south line is shown⁵⁶).

What was the state of affairs at sea? It must be considered impossible that the Portuguese mariners on their voyages of exploration of the fifteenth century should not have watched the behaviour of their compass intently. A mariner pioneering in unfamiliar waters, who has to consider the direction in which he is sailing — if only because at some future time he must be able to return to his home port — cannot be expected to do otherwise. What means of verification was at his disposal? It was nature herself which furnished him with it, as one among the numerous valuable clues for which the seaman of those days used to keep a sharp lookout⁵⁷) and which he interpreted with much greater ease than the present-day navigator, who has become alienated from those signs because so many technical aids are at his disposal. The seaman of those days was able to compare the uncertain direction shown by the needle with the certain direction which the Pole Star furnished. Thus he was induced to take his bearings from the Pole Star, a method which was recorded by Medina in his *Arte de navegar* of

⁵⁴) H. Balmer, *Beiträge zur Geschichte der Erkenntnis des Erdmagnetismus*. Aarau 1956, p. 36.

⁵⁵) For further details reference may be made to: E. O. von Lippmann, *Die Geschichte der Magnetnadel bis zur Erfindung des Kompasses (gegen 1300)*. Quellen und Studien zur Geschichte der Naturwissenschaften und der Medizin, Band 3, Heft 1 (Berlin 1932).

⁵⁶) *Comptes rendus du Congrès Int. de Géographie*. Amsterdam 1938, Tome II (Géographie historique), p. 55; H. Winter, *Die Erkenntnis der magnetischen Missweisung und ihr Einfluss auf die Kartographie*.

⁵⁷) These valuable clues include: the discoloration of the sea and the change in appearance of the waves as the ship approaches shallows, the change in the temperature of the sea-water, plants or weeds floating about the ship, clouds and lightning above remote land, the scent of vegetation, which can be detected at a great distance, the booming of the surf on the shore, which can be heard from afar, birds of particular species and in certain numbers flying round the ship, etc.

1545⁵⁸). During such operations the fact that there was a constant declination to one side — *i.e.* the variation — must undoubtedly have struck these mariners.

The first to record such an observation in his logbook was Columbus. The fact that he took bearings from the Pole Star on his first outward voyage on 13th, 17th, and 30th September 1492 in the early and the latter part of the night is famous in the history of navigation and has been repeatedly referred to. Let us follow the conclusions reached by Franco after an exhaustive study of these observations and of the later reports of Columbus in this field⁵⁹).

The three observations convinced the admiral that the observed difference in azimuth pointed to a movement of the Pole Star about the pole. The observations do not create the impression of being a nautical discovery. Columbus on his voyage saw the variation change in value and he was the first to observe the passage from easterly to westerly variation. He knew that in different places of the world the declination of the magnetic needle had certain values and he was convinced that these values were constant for each place. He had a dim suspicion of a relation between the change of longitude and the change in the variation of the compass.

The fact that seamen exchanged their observations and findings gave rise to a manuscript written by Joao de Lisboa and dating back to 1514. This is now the oldest known document that contains a treatise about the declination of the magnetic needle from the meridian and about the determination of longitude by means of this declination. The manuscript in question formed part of the library of the Marquis De Castello Melhor. Because it was reprinted in 1903 it became accessible and known. It was included in the *Livro de Marinharia de Joao de Lisboa*⁶⁰). The chapter bears the title: "Here begins the treatise about the compass-needle made by Joao de Lisboa in the year 1514, through which one can know from the declination of the magnetic needle — wherever one may be — how far one is away from the true meridian".

The treatise is subdivided into ten short sections and opens with a paragraph in which the Genoese and French compasses are discussed, the needles of which were attached to the card at an angle equal to the local declination relative to its north and south line.

The first section describes how a compass has to be made if it is to be suitable for the purpose. The needle must be as large as possible and must be attached to the card in such a way that it coincides with the north and south line, not deviating from it. The card must be large and the space between the card and the inside of the bowl must be narrow, in order to make accurate readings possible. The second section describes the bowl, on the inside of which a division into 32 points was marked. The third section reveals that a bearing instrument was fixed on the bowl and warns the navigator that the bowl has to be horizontal when he takes a bearing. The description is not sufficiently clear to give an idea of the bearing instrument.

The fourth section teaches in a clear way how the true north is found by an observation of Polaris. The observation has to be taken when the "guardians" —

⁵⁸) This method was already cited in § 4, p. 382

⁵⁹) Salv. Garcia Franco, *Historia del arte y ciencia de navegar*. Madrid 1947, Tomo I, p. 49.

⁶⁰) *Livro de Marinharia, tratado da agulha de marear, de Joao de Lisboa, copiado e coordenado por J. I. de Brito Rebello*. Lisbon 1903.

i.e. the stars β and γ of the Little Bear - are northeast or southwest of the Pole Star. In the former case the Pole Star is in the meridian, beneath the celestial pole, in the latter it is in the meridian $3\frac{1}{2}^\circ$ above the celestial pole. The position of the compass-needle relative to the bearing instrument is the declination of the needle. This observation could also be taken with other stars besides the "guardians".

South of the equator the observation could be taken in a similar way by reference to the Southern Cross, as is taught in the fifth section. This constellation was shown in a star map made by the author.

The next section contains some directions concerning the accurate taking of the observation. The seventh section is a very important one. It says that the meridian on which the needle points exactly towards the north pole of the world, *i.e.* the one on which the variation of the compass is nil, passes through the island of Santa Maria and towards the island of San Miguel of the Azores, thus dividing the world into two equal parts. The meridian passes *via* the Cape Verde Islands to the island of St. Vincent, said meridian lying between the Cape of Good Hope and Cape Frio. The author had invariably found that on this meridian the needle pointed in the direction of the terrestrial pole. He had never met with any other meridian on which the same phenomenon occurred.

Finally the tenth section discusses the relation between the declination of the compass-needle and terrestrial longitude. This passage is not easy to understand, but its purport is as follows. When one moves between latitudes 30° and 45° round the earth, starting from the prime meridian, for every 250 *leguas* the variation will be found to increase by one point, to a maximum of four points to the east or to the west, in longitude 90° E. and W. It then decreases again to zero in longitude 180° . In this way it was possible to compute the longitude of any place from the variation observed in that place. "With the greatest ease" are the words used elsewhere.

Again, during the first circumnavigation of the world (1519-1522), a Spanish undertaking under the command of the Portuguese Magellan, certain particulars were noted in the direction indicated by the needle in various places, which the Italian Ant. Pigafetta mentions in the journal he kept during the voyage. Pigafetta was of the opinion that the variation of the compass furnished a convenient means of finding one's distance from the prime meridian. He stated that the method had already been proved by experience ⁶¹).

In his textbook Faleiro (see note 33) devotes a chapter to the variation of the

⁶¹) *Premier voyage autour du monde par le Chevalier Ant. Pigafetta sur l'escadre de Magellan 1519-1522*, Paris, an IX (1801), p. 281, quoted in Linschoten Vereeniging, *Tweede Schipvaart*, Vol. XLIV, p. XXXII.

J. A. Robertson, *Magellan's Voyage Around the World by Ant. Pigafetta*. Cleveland 1906. Vol. I, p. 89 and p. 253, Note 229.

Linschoten Vereeniging, *Reizen van Barents, Heemskerck, Rijk, en anderen*. Vol. XV, p. XXI, Note 2: reference to *Relazione di Ant. Pigafetta sul primo viaggio*, Rome 1894.

J. Bensaude (*Histoire de la science nautique portugaise*, Résumé, Geneva 1917, pp. 12-22) is of the opinion that Pigafetta's remarks are based on the work of the Portuguese Ruy Faleiro—brother to Francisco—who studied the problem of the determination of longitude. He had proposed three methods, *viz.* two with the aid of the moon and the third by means of the variation of the compass. The latter method is identical with that of Joao de Lisboa, 1514, according to which the longitude is proportional to the amount of the variation.

compass-needle (Chapter VIII, p. 79). He states the problem clearly and writes: "The north-easting of the needles causes navigators many doubts, from which they may be freed by knowing precisely how much the needles northeast or northwest. In addition to this, other advantages will follow, such as knowing exactly in what direction they are sailing. Knowing this, they will follow exactly their courses without error or wandering and also it will help much to a knowledge of the longitude in which they are navigating". He describes the relation between longitude and variation as follows.

On the meridian through Corvo the variation is nil, apart from small discrepancies to be accounted for as being due to differences between the compasses. When the ship sails westward, the needle will get westerly variation; when she sails eastward, the variation will be easterly, while the variation increases as the ship moves further away. Maximum values are reached at a difference of longitude of 90° . After this the variation decreases again to the meridian of 180° from Corvo, where the variation is nil again. The principle accordingly is the same as that described by Joao de Lisboa in 1514. It thus appears that this knowledge, gained at sea, mentioned by a practical man, subsequently elucidated in a scientific manuscript, now benefits navigation thanks to a nautical textbook. This process took more than fifty years.

Let us now proceed — remaining in the field of Portuguese science — to discuss the learned mathematician and astronomer Pedro Nunez (1492-1577), royal cosmographer and professor in the university of Coimbra. His work includes a treatise entitled

Tratado em defensam da carta de marear, 1537⁶²), which he dedicated to the Infante Dom Luys, who was his pupil. The determination of the magnetic declination with the aid of the instrument known to us is clearly explained (p. 141). The method of taking observations at two equal altitudes of the sun is followed. Nunez writes (p. 140) that he is convinced of the existence of the variation of the compass, but wonders what is its amount. He had no confidence in the navigators, who gave different values for the same place, sometimes greater, sometimes smaller. In the absence of data he was unable to study the possible relation between the variation of the compass and longitude. He aspired to carry out an inquiry that would benefit the development of the nautical chart and of navigation. This learned theoretician and mathematician, whose investigations and writings were of fundamental importance for the evolution of navigation, calls in the aid of practice. This aid was to be given presently by one of his pupils, Joao de Castro (1500-1548), who in 1538 set sail for Goa in India as master of the ship Gryfo in a fleet under the command of his uncle Garcia de Noronha. His training and scientific bent induced Castro to observe the variation of the compass very carefully on many occasions. In the modern edition of his logbook⁶³) a list is given of 43 variations obtained during the voyage from Lisbon to Goa, along the coast of India and in the Red Sea. On these data he bases his own conclusion, *viz.* that it is certain from these observations that there is no relation between the variation of the compass and the longitude of a place. The fact that

⁶²) It forms part of: Pedro Nunez, *Tratado da sphaera*, 1537, an extremely rare book, which was published in facsimile by J. Bensaude, *Histoire de la science nautique portugaise*, Vol. 5, Berne/Munich 1915.

⁶³) Joao de Castro, *Roteiro de Lisboa a Goa*. Annotado por Joao de Andrade Corvo. Lisbon 1882.

the variation could be different for places situated on the same meridian puzzles him, "from which we are justified in supposing that such variations are caused by particular and inherent mysteries concealed by mighty Nature in her vast and secret workshops". It was due to Castro's conclusion that in Portugal an important phase in the development of scientific navigation was closed.

In other countries the problem continued to occupy the minds of scholars as well as seamen. Naturally the question also used to be asked as to what caused the directive force of the needle. Towards what point did it point? How did the declination arise?

The point towards which the needle pointed was sought outside the earth by some scholars. We are not going into this question now, for it falls outside the scope of this introduction.

It was long thought that the needle ought to point towards the geographical pole, and that, if it did not, this was due to poor workmanship. Wishing to remedy this, Bartolomeo Crescentio ⁶⁴⁾ tried to give the needle a special shape, and Fournier ⁶⁵⁾ asserts that he had put a needle of this type to the test at Paris and La Rochelle, in order to verify whether it really pointed to the pole.

The suspicion about the existence of a magnetic mountain on the earth dates far back to antiquity and seems to have arisen wherever the mysterious tendency of the magnetic needle to point in a particular direction was discovered. In Europe as well as among the Arabs (Arabian Nights' Entertainments) this mountain struck terror in the hearts of seamen, and the loss of innumerable ships was attributed to it. Balmer ⁶⁶⁾ quotes a thirteenth-century Italian poet who spoke about a magnetic mountain ⁶⁷⁾. The map of the world of Andreas Walsperger, of 1448, bears a legend in the north to the effect that no navigation takes place in that big sea because of the magnets. A warning against the great danger of magnetic attraction is sounded in a legend on the map of the world of Ruysch (Rome, 1508), north of Iceland: "ships having iron in them cannot return" ⁶⁸⁾. The existence of such a mountain in the extreme north was also assumed by the great geographer and cartographer from the Southern Netherlands, Gerhard Mercator (1512-1594). In his chapter devoted to magnetism and marine compasses Coignet ⁶⁹⁾ clearly says that Mercator at 16° 30' from the north pole "puts a very prodigious rock and mine of loadstone, towards which all other loadstones in the whole world are attracted". There is no longer any reference to its being a danger to navigation. It is now only the loadstones and the compasses which are attracted by this rock.

⁶⁴⁾ Bartolomeo Crescentio, *Nautica Mediterranea*. Rome 1607.

⁶⁵⁾ Georges Fournier, *Hydrographie*. Paris 1643, p. 550.

⁶⁶⁾ Balmer, *Beiträge zur Geschichte der Erkenntnis des Erdmagnetismus*. Aarau 1956, p. 533.

⁶⁷⁾ *ibid.* *Die Sage vom Magnetberg*, p. 651. Hennig, *Terrae Incognitae*, Vol. III, Chapter 149: *Die Seefahrt eines Oxforder Geistlichen in den Atlantischen Norden und die Frage des Magnetbergs*. 1360.

⁶⁸⁾ The map of Ruysch is to be found in the edition of Ptolemy's atlas of 1508. The map is discussed and reproduced by Nordenskjöld, *Facsimile Atlas*, p. 65, and map XXXII. The legend reads: *hic incipit Mare Sugenum. Hic compassus navium non tenet, nec naves quae ferrum tenent revertere valent.*

Nordenskjöld utters the suspicion that these words might perhaps point to the experience that the compass becomes useless in the extreme north, *i.e.* in the neighbourhood of the magnetic pole.

⁶⁹⁾ Michiel Coignet, *Nieuwe onderwijsinghe op de principaelste punten der Zee-vaert*. Antwerp 1580, p. 5 verso.

b. *MERCATOR (1512-1594)*

Mercator, being convinced that the magnetic needle - irrespective of the place on the earth where it was put up - was drawn in the direction of the magnetic pole, thus regarded the magnetic meridian of a place as a great circle passing through that pole. This pole was the point of intersection of all magnetic meridians, and its position on the earth might be found if for two points the positions of which were known the variation had been determined by measurements. In a letter of 1546 to Ant. Perrenot, Bishop of Arras, Mercator writes that near Walcheren the variation was 9° east and at Danzig presumably 14° east ⁷⁰). For the point of intersection of the magnetic meridians he had found a point in latitude 79° N. The prime meridian had thus at the same time become known. With the aid of further data he afterwards obtained improved results. The system was to the effect that for any place on the earth the variation could be calculated and that conversely the longitude could be found from the amount of the variation.

Mercator's famous chart of the world, destined for navigation and dating from 1569 ⁷¹), illustrates his views at that moment. The magnetic pole is indicated in the top lefthand corner in the form of a steep rock with a mark to show its exact position. It lies on the meridian of longitude 180° and in latitude $73^{\circ}30'$ N. This meridian passes through the Strait of Anian, which was assumed to connect the Arctic Ocean and the Pacific between Asia and America. Because the chart covers more than 360 degrees of longitude, the 180th degree of longitude appears a second time on the righthand side. On this meridian and in the above-mentioned latitude the same magnetic pole is shown once more, with the following legend: "from sure calculations it is here that lies the magnetic pole and the very perfect magnet, which draws to itself all others, it being assumed that the prime meridian be where I have placed it". Not far from this rock a kind of island is shown, again with a point marked in the middle, the position of which is latitude 77° N. and longitude 174° E. A legend states that the magnetic pole will be found in this place if the meridian through the island of Corvo is taken as the magnetic prime meridian.

In an elaborate legend, printed near these poles, Mercator accounts for his procedure, saying that the very capable seaman François van Dieppe states that the magnetic needle points due north on the Cape Verde Islands Sal, Boa Vista, and Mayo. The legend continues: "This is closely supported by those who state that this occurs at Terceira or S. Maria (which are isles of the Azores). Some believe that this is the case at the most westerly of these islands, which is called Corvo. Now, since it is necessary that longitudes of places should, for good reasons, have as origin the meridian which is common to the magnet and the World, in accordance with a great number of testimonies I have drawn the prime meridian through the said Isles of Cape Verde; and as the magnet deviates elsewhere more or less from the pole, there must be a special pole towards which magnets turn in all parts of the world, therefore I have ascertained that this is in reality at the spot where I have placed it by taking account of the magnetic

⁷⁰) In 1539 Rheticus had found 13° at Danzig. See: Balmer, p. 103. Letter reproduced in Balmer, p. 313.

⁷¹) *Drei Karten von Gerhard Mercator. Europa, Britische Inseln, Weltkarte. Facsimile-Lichtdruck nach den Originalen der Stadtbibliothek zu Breslau. Herausgegeben von der Ges. für Erdkunde zu Berlin.* 1891.

declination observed at Ratisbon. But I have likewise calculated the position of this pole with reference to the Isle of Corvo in order that note be taken of the extreme positions between which, according to the extreme positions of the prime meridian, this pole must lie until the observations made by seamen have provided more certain information" ⁷²).

In the bottom lefthand corner of the chart of the world there is an inset, *viz.* of the arctic regions, which could not be shown in the big chart, since the latter was one on Mercator's projection, in which the pole cannot be present. On the north pole of the earth a steep black rock has been marked. The rock on the magnetic pole lying on the meridian through the Cape Verde Islands and the magnetic pole without a rock in longitude 174° E. on the meridian through Corvo are both shown.

In the chart of the world in two hemispheres, published in 1587 ⁷³) by Rumoldus Mercator, the latter followed his father's example. On the 180th meridian, being the prime meridian through the Cape Verde Islands, and in latitude 73° 30' N. the magnetic pole is to be found.

After a new edition of the above-mentioned chart of the arctic regions had appeared in 1595, Rumoldus prepared a new edition of it, brought up to date, which is easily recognized because it includes, among other things, "*het behouden buis*", — a name calling to mind the wintering of 1596/7 — and names given by Barents to capes on the coast of Novaya Zemlya ⁷⁴). Rumoldus maintained the four polar islands which his father had imagined, and the two possible positions of the magnetic pole in latitudes 73° 30' and 77° N. He only altered the longitude of both. For the meridian through the Cape Verde Islands he now gives longitude 178° E. and for the meridian through Corvo 172° E., thus a displacement of 2° ⁷⁵).

In the chart of the arctic regions made by Willem Barents ⁷⁶) and published in 1598 — the year after his death — the polar islands no longer appear, which amounts to an important correction. However, the author still stuck to the "*polus magnetis*", shown as a tall rock in latitude 74° N. and lying on the meridian corresponding to that through the Cape Verde Islands in the arctic chart of Mercator.

c. *PLANCIUS (1552-1622)*

Plancius entirely followed Mercator in this matter. In the map of the world in two hemispheres which he appended to the Dutch bible published by Laurens Jacobsz he marked the magnetic pole in latitude 74° N. And when he compiled his big map of the world, dating from 1592 ⁷⁷), it was again the map of his

⁷²) This text has been taken from: *The Hydrographic Review*, Vol. IX, No 2, Nov. 1932, p. 7. Text and translation of the legends of the original chart of the world by Gerhard Mercator, issued in 1569.

⁷³) It occurs in: *Galliae tabulae geographicae per Ger. Mercatorem*. Duisburg, 1585.

⁷⁴) It appears on p. 73 of: Mercator, *Atlas sive cosmographicae meditationes de fabrica mundi et fabricati figura*. Ed. decima. Henr. Hondius. Amsterdam 1628.

⁷⁵) It is curious that the atlas mentioned in Note 74 has on p. 65 a map of Asia, in which the magnetic pole has been maintained on the 180th meridian, which clearly proves the uncertainty prevailing with respect to the position of the magnetic mountains.

⁷⁶) Reproduced in Vol. XV of *Werken uitgegeven door de Linschoten-Vereeniging: Reizen van Willem Barents, Jacob van Heemskerck*.

⁷⁷) See: F. C. Wieder, *Monumenta Cartographica, reproductions*. Vol. II. 1926. Text and map pages 27, 30, and 35.

learned predecessor which formed his principal source and starting point, though Plancius did not adopt the *rete* employed by Mercator and made ample use of Portuguese data wherever he found them to be more up to date. Owing to his critical method Plancius gave the best representation of land and sea that was possible at the time.

Just as in Mercator's map, in the top righthand corner the small sketch is to be found of the big, steep rock in the Strait of Anian, which indicates the magnetic pole and which lies in latitude $73^{\circ} 30' N.$ and longitude 180° . A legend is added to the effect that it lies on the meridian through the Cape Verde Islands. The prime meridian appears to fall through the island of San Thiago. The other possible position of the magnetic pole, referred to the meridian through Corvo, is to be found in latitude $77^{\circ} N.$ and longitude $173^{\circ} E.$ The legend occurring on Mercator's map of 1569 as to the position of the two poles is repeated verbally, the source being mentioned at the end in the form of the name Ger. Merc. Analogously to Mercator's map we again find in the top lefthand corner the former of these two poles, and in the bottom lefthand corner the inset of the arctic regions.

Finally attention may be drawn to the map of the world by Plancius, engraved by Josua van den Ende and dating from 1604⁷⁸⁾. The two magnetic poles are to be found in the corresponding places, and so is Mercator's legend of 1569, with the initials. On the globe of Jacob Florentius van Langren, dating from 1589, the two magnetic poles are marked, with the legend stating that they refer to the meridians through Corvo and the Cape Verde Islands respectively. On the later specimen of this globe, of 1612, the fact that the same legends are used shows that the view concerning the poles was maintained by the author.

The geographers of the early seventeenth century begin to think differently of it. The tide had turned. The map of the world in two hemispheres of Willem Jansz. Blaeu of 1605⁷⁹⁾ no longer contains a magnetic mountain, nor does the map of the world on Mercator's projection by Jodocus Hondius of 1608⁸⁰⁾. The same applies to the latter's map of the world of 1611⁸¹⁾. The prime meridian in this map falls through the Cape Verde Islands.

This changed view is accounted for on the wonderful globe of 1613 by Jodocus Hondius Jr. and Adriaan Veen. In the place where the poles used to be marked, the following legend in Latin is to be found⁸²⁾:

"Gerardus Mercator, and others who followed him in this, had put in this place two magnetic poles, the one referred to the Cape Verde Islands, the other to the islands of Corvo and Flores. However, because there is no cer-

⁷⁸⁾ It was described and reproduced by Marcel Destombes. *La mappemonde de Petrus Plancius, gravée par Josua van den Ende, 1604, d'après l'unique exemplaire de la Bibliothèque Nationale de Paris.* 1944. Publications de la Société de Géographie de Hanoi.

⁷⁹⁾ *World Map 1605 Willem Jansz. Blaeu, facsimile of the unique copy belonging to the Hispanic Soc. of America,* text Edw. L. Stevenson, New York 1914.

⁸⁰⁾ *The Map of the World on Mercator's Projection by Jodocus Hondius,* Amsterdam 1608. From the unique copy in the collection of the Royal Geographical Society with a memoir by Edw. Heawood. London 1927.

⁸¹⁾ *World Map by Jodocus Hondius, 1611.* Ed. by Edw. L. Stevenson and J. Fisher, New York 1907.

⁸²⁾ This globe is present in the Netherlands Historical Maritime Museum at Amsterdam. On another, undated specimen, also present there, the same legend is to be found.

tainty at all about this and daily experience teaches us otherwise about the declination of the compass needle, we have omitted both."

The theoretical conception of Mercator had been unable to hold its own against the results obtained in practice.

As long as one magnetic pole, lying in line with the meridian through Corvo, continued to be assumed, this involved the consequence that easterly variation was bound to exist east of this meridian up to longitude 180° , and westerly variation in the western hemisphere. Maximum values for the variation had to be found in longitudes 90° E. and W. This picture had already been sketched in the *Livro de Marinharía* and later by Faleiro, but with them there was no question in these places of a magnetic pole with the property of attracting the needle. Later on Coignet, in his *Nieuwe Onderwijsinghe*, expounds very lucidly that all these phenomena must be due to the presence of one magnetic pole. Moreover he shows by means of a drawing that when two places in the northern hemisphere lie on the same meridian, the variation must be greater for the more northerly one than for the other. In order to demonstrate that the whole conception is correct, he concludes by stating that according to "Mercator's rule" the variation at Antwerp will have to be 9° to the east, and that he had found this value "by experience" ⁸³).

Plancius, who diligently tried to solve the mystery of terrestrial magnetism, must soon have found that the data about the variation collected by him did not fit in with this simple scheme. What can have been the source of these data?

Plancius himself provides the answer to this question in a manuscript written by him, which is now kept in the Public Record Office at The Hague. ⁸⁴) This manuscript is of the utmost importance to us for three reasons. In the first place because it mentions the author's sources. Further it helps us to understand Plancius' theory concerning the possibility of determining longitude at sea. Finally it appears to be the source of the list of values of the variation, to be found in *The Haven-Finding Art*, a fact which accounts for Stevin's gratitude towards Plancius.

The manuscript in question contains three treatises, which are entitled:

1. *Van de graden der lancte ende het affmeten der selver door het Noordoosten ende Noordwesteren der naelde*
(Of the degrees of longitude and the measurement thereof by means of the northeasting and northwesting of the needle).
2. *Van de Oost-Indische zeevaart ende haren eygenschappen ende aenmerkingen*
(Of the navigation to the East Indies and its character, with notes).
3. *Naerder verclaringe van de Oost-Indische zeevaart*
(Further comment on the navigation to the East Indies).

The treatises sub 1 and 2 were drawn up after the return of De Houtman about

⁸³) Thus to be found in the first edition of 1580, published at Antwerp, as well as in the fourth edition, Amsterdam 1598.

⁸⁴) Public Record Office, The Hague. Loketkas Admiraliteit No 10.

1598, while that sub 3 is of a later date and must have been written after the return of the fleet of Van Neck, in 1599 or 1600 ⁸⁵).

Before restricting our attention to the contents of the treatise sub 1 — the most important for the present purpose — we may remind the reader that an obvious reaction to the question concerning the origin of Plancius' data would have been to think of Jan Huychen van Linschoten. In fact, in his *Reijsgheschrift van de navigatiën der Portugaloyzers* ⁸⁶), the guide published at Amsterdam in 1595, which was of inestimable value to all contemporary explorers and seafarers, this author included hundreds of values of the variation. In addition he published a series of values for a number of specified places lying on the route from Portugal to the East Indies ⁸⁷). This list is more than a mere enumeration of values for particular places. Indeed, with its aid an observed variation gave an indication of the longitude in which the ship was at that moment. Thus we read: if the compass has a northeasterly variation of half a point, "you may know that you are near Cabo de bona Esperança", and if it has a northwesterly variation of $\frac{3}{4}$ point, "beware of the Isle of Sant Lourenço, for you will see it presently". Although in our view the way in which the place is indicated is very vague, yet some value has to be attached to this list in the above sense, as appears from the two instances here given as well as from others. Considering the provenance of these data, it is evident that no mention is made of any rule as to the relation between variation and longitude, for the Portuguese had already rejected such a relation some fifty years before.

That Plancius read and knew the writings of Van Linschoten is evident from the criticism uttered by him in his *Naerder verclaringe*, but he does not mention his name in the treatise written after the return of De Houtman: *Van de Oost-Indische zeevaert*. It is thus not certain whether Plancius did or did not make use of the work of Van Linschoten, with whom he sometimes disagreed considerably.

Although a description of the contents of the last-mentioned treatise falls beyond the scope of the present introduction, however valuable it may be in view of the fact that the experiences of Dutch, French, English, and Portuguese seafarers are here set forth for the benefit and instruction of others, one remark has to be made. This treatise contains a considerable number of statements about the amount of the variation in various places, which — just as in the writings of Van Linschoten — were intended by Plancius as aids to navigation. Thus we find the instruction to cross the equator on the outward voyage "at $8^{\circ}30'$, 9° , or $9^{\circ}30'$ of increasing northeasting", followed by the advice to pass Cape St. Augustine on the coast of Brazil in a place where the variation was 6 or 7 degrees of increasing northeasting, "then you will be on the right track to sail successfully with God's aid past the shoals of Abrolho". If one acted on this advice, one would be far enough from the coast to be safe with respect to this notorious point. We find the warning: "who pays no attention to the variation of the needle and to the currents will get further from his course than he thinks, as has befallen many people". From these examples it will be sufficiently evident in what respect Plancius considered it important to know these values, and thus also what importance he

⁸⁵) The treatises mentioned sub 2 and 3 are included in: De Jonge, *De opkomst van het Nederlandsch gezag in Oost-Indië*, Vol. I, 1862, pp. 184-194 and 194-200.

⁸⁶) See: *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. XLIII, *Het Itinerario van Jan Huygen van Linschoten*, 1579-1592, Vols IV and V, 1939.

⁸⁷) *Idem*, Vol. V, pp. 367-370.

attached to the observation of the variation on board ⁸⁸). No reference is made in this passage to any special relation between these values and the longitude at sea.

We will confine ourselves to the treatise mentioned above sub 1, *viz.* "of the degrees of longitude", which forms the source from which we may learn to understand the theory of Plancius concerning the relation between the geographical longitude and latitude of a place on the earth and the amount of the variation in that place. His theory forms a unique and interesting page of the chapter in the book of navigation with which we are here concerned, and it has to be discussed before Stevin's work can be assigned its place. It was published under the title of *Memorie van Plancius* in Vol. XXXII of *Werken der Linschoten-Vereeniging* ⁸⁹), but with the omission of three pages at the end. It is precisely these pages whose contents are of the utmost importance for our purpose. The main title mentioned above is followed by the words:

"Extract from the writings of Frederick Houtman, from which it is understood how the ships of Amsterdam on their first voyage to the East Indies took the deviation of the needle of the compass by means of the rising and the setting sun, which can also be measured at all the points and the degrees of the compass and at all hours and minutes of the day."

We will follow the treatise sub 1, devoted to "the degrees of longitude".

Whilst formerly, on his chart of the world, Plancius had taken over from Mercator the two magnetic poles lying close together and the prime meridians, he has now made his choice and dropped the prime meridian through the Cape Verde Islands. The prime meridian, from which the degrees of longitude are reckoned, is the one through the islands of Flores and Corvo. On this meridian "the needle of the compass points due south and north without any deviation". Plancius now assumes that there are four such meridians on which the variation is nil and which might thus be called agonics. They are those of 0, 60, 160, and 260 degrees of longitude — reckoned in the eastward direction — which divide the surface of the earth into four lunes, of which one therefore covers 60 and the other three 100 degrees of longitude each. In the lune from 0 to 60° the variation is easterly, in the second it is westerly, and thus alternately. In each lune the value increases from zero to a maximum lying on the median meridian of the lune. The maxima thus lie at 30°, 110°, 210°, and 310°. We read that a point at 17 German miles — *i.e.* 68 nautical miles — east of Cape Agulhas lies on the meridian of 60°E.L. and that the island of Hjelmsöy at the North Cape too lies on this meridian. On the meridian of 160°E.L. — "or very close to it" — lies Canton in China, on that of 260° the westernmost part of California, on that of 310° "Nombre de Dios, a famous port in America". This name is still found on the northern coast of Panama, slightly to the east of Colón. Considering the difference of longitude from Corvo, this place may be meant.

⁸⁸) In this context reference may be made to the very rare book of William Barlow, *The Navigator's Supply*, London 1597. The author speaks about a voyage made by Sir Francis Drake from the western tip of Cuba to Virginia: "and found his destination only through the navigator's knowledge of the variation of the compass".

⁸⁹) *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. XXXII. *De eerste schipvaart der Nederlanders naar Oost-Indië onder Cornelis de Houtman*, 1595-1597. Part III, 1929, pp. 411-432.

After this outline of the magnetism of the earth Plancius states his sources. This is what he writes:

"The foregoing foundations and grounds are found certain and true by frequent experiences and observations, made with great diligence and skill by many learned men, intelligent masters, and good navigators of Spain, Portugal, France, England, The Netherlands, and other nations in many countries and places of Europe, Asia, Africa, Peruana and Mexicana ⁹⁰), both south and north of the equator. But of the deviation of the needle in the countries and seas lying between China and Mexico I have not so far been able to get accurate information, confirmed by experience." ⁹¹)

These words are followed by the instructions teaching how the longitude is derived from the known amount of the variation in a place whose latitude is given. Although this derivation is obscure to the uninitiated reader, it appears at once that this is a problem of spherical trigonometry. Just as Mercator had done, Plancius assumes a mathematical relation between these quantities, which admits of being expressed by means of this auxiliary science. It is not through calculation, but through measurements in a plane surface that the problem is solved. For this purpose two instruments are employed. The first is the "general astrolabe" (*astrolabium catholicum*), an instrument which had long been known and widely used to solve problems of spherical trigonometry, and the second is a "longitude-finder" specially designed for this method ⁹²). Only in this way was it possible for Plancius to make the subject accessible and manageable for the seaman of around 1600. In fact, the latter was unable to perform calculations and it was not until the end of the eighteenth century that his accomplishments included a knowledge of spherical trigonometry and its applications.

Mercator formerly had placed the magnetic pole on the continuation of the prime meridian in a latitude determined by calculation, and Plancius had imitated him. Since the latter now, on the ground of observational data, assumed four meridians on which the variation was nil, he could not mark four magnetic poles in a corresponding way, for in that case no variation zero could have occurred on any of the agonic. The term magnetic pole has disappeared from this treatise, although the concept has not been rejected (in so many words).

The treatise shows that on the ground of the observational data at his disposal — and undoubtedly after endless patient trials — Plancius arrived at a thesis reading as follows:

⁹⁰) Names used at that time for North and South America: *Mexicanae* and *Peruvianae Pars*.

⁹¹) Balmer, p. 129, points out that the learned Spanish Jesuit Acosta, basing himself on the reports of Portuguese seafarers, mentions four lines of no variation. The reference is to his *Historia natural y moral de las Indias*, Book I, Chapter 17.

José de Acosta (1539-1600) was a missionary in Peru for many years. His aforesaid book, which appeared at Seville in 1590, was very soon translated into six languages, the Dutch version, *Historie naturael ende morael van de Westersche Indiën*, being published at Enkhuizen in 1598. The translator was Jan Huychen van Linschoten, who laid hands on the book as early as 1594 and who thought so highly of its contents that he was no longer satisfied with his own work.

In the passage in question (p. 37) it is said that "an experienced Portuguese navigator" had furnished him with the information. He did not recall the names of the "places" (*sic*) having no variation, except the "vicinity" of Corvo.

⁹²) For the construction and the use of these two instruments, see the Appendix.

the point of intersection of the magnetic meridian through a given place with the nearest agonic falls on the same parallel for all places.

The longitude of a place is calculated in a spherical triangle, the known elements of which are: the complement of the latitude of the parallel just referred to, the complement of the latitude of the place, and the variation in that place ⁹³). The latitude determined by Plancius for that parallel must have been approximately $65^{\circ}30'$ ⁹⁴).

He then starts to apply his rule. On the ground of a Portuguese observation of the variation at Cape St. Augustine in Brazil, amounting to 3° northeasting, he calculates that the longitude of this cape, lying in latitude $8^{\circ}30'$ south, must be $5^{\circ}50'$. In the same way he calculates dozens of longitudes, and he does this also for the observational data brought by the ships which completed the first voyage to the East Indies. He had found these observations to be in agreement with those of the Portuguese seafarers. We need not consider the results of the calculations, because the determinations of longitude are based on a relation which does not exist. However, we find that out of the large body of material a number of places, with an appended statement of the pertinent latitude, longitude, and variation, have been collected in two almost identical lists accompanying the treatise. They form the contents of the three pages referred to on p. 401, and these lists are again identical — except for a few minor differences — with the lists which Stevin gives in his *Haven-Finding Art* (pp. 438 - 440).

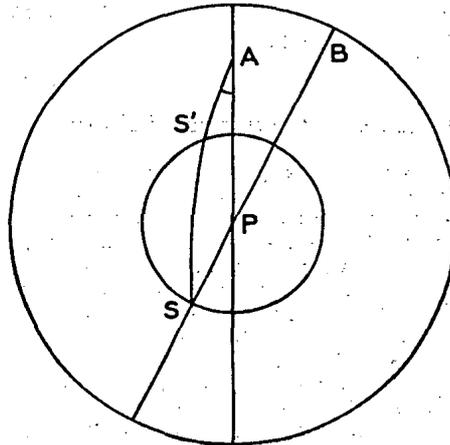
d. STEVIN

Since the reader has become acquainted with the contents of *The Haven-Finding Art* in § 3, while the present section has thrown light on the conceptions of Plancius, Mercator, and others, it is now possible to see Stevin's ideas against the background formed by the work of his predecessors.

The fact that Stevin absolutely rejects a magnetic pole — which impresses the reader all the more because he does so in the opening pages of his treatise —

Fig. 5

⁹³) Figure:
 Outer circle: equator; inner circle: parallel at latitude $65^{\circ}30'$
 PB: agonic; PA: meridian through A
 AP: 90° - latitude of A
 \angle PAS: known variation at A
 S: point of intersection of the magnetic north and south line in A with the parallel SS'
 \angle BPA: longitude of A in relation to the nearest agonic BPS
⁹⁴) See Appendix, especially fig. 6 and fig. 8 on p. 414 and 417.



implies a definite attitude and a radical change of opinion as compared with his predecessors. On the ground of observations of the variation which had become available — it should be noted that they were taken precisely from the work of Plancius — the impossibility of the existence of a magnetic pole had become established, at least in the sense in which it was taken at that time.

A second and extremely important point of difference from Plancius and Mercator was the fact that no use was made of spherical trigonometry, an auxiliary science of which Stevin, in contrast to Plancius, had perfect command. Stevin rejects the mathematical relation between the amount of the variation and the longitude: a perfectly fundamental difference indeed.

In his *Memorie* Plancius had pointed out briefly that, passing to the east from the meridian of Corvo, the easterly variation increased in the northern as well as the southern hemisphere to a maximum in longitude 30° , after which it decreased again to 0° in longitude 60° . Without any further comment, and as a fully established fact, he mentions the existence of the four agonics, and the increase and decrease of the variation in the three lunes of 100° difference of longitude. He states the latter although he admits he has no data at his disposal about the variation between China and Mexico.

Stevin interprets the material, which he obtained from Plancius and which he presents in *The Haven-Finding Art* (pp. 438 - 440) in his own way. His description of the increase and decrease of the variation in the lunes from 0° to 60° and from 60° to 160° accords with Plancius, but from that point on he is more cautious than the latter. About the part from longitude 160° to 360° , for which no data are available, he says how "it is suspected to some extent" that the variation will appear to be. Instead of four he assumes six agonics, *viz.* the meridians of longitude 0° , 60° , 160° , 180° , 240° , and 340° , and he calls his system a "conjecture". It has been stated above that in 1608 he expressed himself more cautiously than he had done in the first edition of *The Haven-Finding Art*.

Plancius thought it possible to derive the longitude mathematically from the variation observed. He thus offered a method for the actual determination of longitude.

On the available material Stevin was unable to find anything more than a practical method of "haven-finding", a method which was serviceable and was to prove useful, but which yet was not equivalent to a method for the determination of longitude. In fact, in 1608 he said that the latter had a wider scope and was "worthier". In this way he himself points out the difference.

Summarizing, one may conclude that the allegation that Stevin recorded and put into print the ideas of Plancius with respect to the determination of longitude is by no means true. On the contrary, Stevin held views of his own and he published his own conceptions under an appropriate title. Relying completely on what appeared certain to him, he furnished seamen with two rules, *viz.*:

- a. in seeking your destination and sailing along the parallel, note the increase or decrease of the variation of the compass.
- b. take plenty of observations, which will make it possible to improve upon the theory and to add to the knowledge of terrestrial magnetism, which will benefit your successors.

Whereas the system of Plancius after some time was called "imperfect and

quite frivolous" on the ground of practical experience ⁹⁵), so that it soon fell into oblivion, the gist of Stevin's rules proved serviceable and was put into practice.

In view of these considerations Stevin ranks high among the men who attempted to determine longitude on the ground of the variation of the compass. Although the appeal to navigators to take observations was heard abroad as well, he may be considered to have been the first to advance navigation by giving a satisfactory rule and at the same time recommending a suitable instrument for taking the necessary observations. An increased knowledge of terrestrial magnetism resulted from his work.

In *The Haven-Finding Art* Stevin does not account in any way for the curious phenomenon of the deviation of the compass-needle, or for terrestrial magnetism in general. The reader's attention is drawn to the fact that in his *Wisconstighe Ghedachtenissen* of 1605—1608 he discusses terrestrial magnetism in the 2nd Proposition of Book III, Ch. 1 of *The Heavenly Motions* (pp. 253—255) under the heading: "To expound the motion of the earth in its place, and its magnetic rest." In that context he refers to the work of William Gilbert, *De Magnete* ⁹⁶ — which appeared in 1600 and is to be mentioned once more at the end of the present section (p. 410) — and says the following about the magnetic condition of the earth: "in the earth there is found such a large amount of loadstone and other substances with magnetic force that, like a big loadstone, it has in itself the properties that are found in small spheres made of loadstone." The earth itself is a magnet. It is this view which was introduced by the above-mentioned English scholar.

Stevin also adopts Gilbert's explanation of the fact that the amount of the variation is not the same in all places of the world, a phenomenon about which, as Stevin says, "many people have wondered and puzzled so long". He compares the earth to a spherical magnet, in which a deep and wide groove has been made. The phenomenon is attributed to the influence of the land, not of the sea. This is to be learned from the following curious and simple explanation: "Since the earth is a sphere of loadstone with deep pits, viz. the seas, in whose moving water the above-mentioned magnetic character cannot be present, straight northward pointing indeed is found about the middle of the great seas, such as in the ocean between America and Europe, but when we get to the east, towards Europe, the needle deviates towards the east, and when we get to the west, towards America, it deviates towards the west."

In Gilbert's great work, Book IV, Chapter VII, p. 163, this thesis is set forth in detail. We quote: "It is not easy to determine by any general method how great the arc of variation is in all places and how many degrees and minutes it subtends on the horizon, since it becomes greater or less from diverse causes. For both the strength of true verticity of the place and of the elevated regions, as well as their distances from the given place and from the poles of the world, must be considered and compared; which indeed cannot be done exactly. Nevertheless by our method the variation becomes so known that no grave error will

⁹⁵) Testimony to be found in the *Journal gehouden bij schipper Jan Cornelisz May, schipper op de Vos*, 3rd July 1611. See: *Werken uitgegeven door de Linschoten-Vereeniging*, Vol. I, p. 16.

⁹⁶) A book devoted to Gilbert and to his main work, its influence, etc., is: Duane H. D. Roller, *The De Magnete of William Gilbert*. Amsterdam 1959.

perturb the course at sea. If the positions of the lands were uniform and straight along meridians and not defective and rugged, the variations near lands would be simple" . . . "but the inequalities of the maritime parts of the habitable earth, the enormous promontories, the very wide gulfs, the mountainous and more elevated regions, render the variations more unequal or sudden or more obscure and moreover less certain and more inconstant in the higher latitude." Although Stevin only took over the core of this reasoning, he must have been acquainted with the whole text of Gilbert, who deals extensively with the subject of variation in his Fourth Book 97).

In the year after the publication of *The Haven-Finding Art* a booklet appeared under the title *Een corte onderrichtinge* 98); it had been written by a practical seafaring man, viz. Albert Haeyen, known as the compiler of a useful book containing charts and sailing directions for the North Sea and its coast from Nieuwpoort to Jutland. As the title says, it is intended to refute the errors daily increasing in the finding of ports "owing to incapable teachers". It is an attack in sailors' jargon. The writer inveighs against haven-finders — by which are to be understood both Plancius and Stevin — who are landlubbers, not versed in nautical terminology, and who in the writer's opinion are not entitled to write about practical nautical matters and to instruct sailors. The system was "built on a straw". The results were poor, a statement which he proves by means of instances taken from the practical experience of the first voyage to the Indies and voyages to the North (p. 21). In one case, when the result was particularly poor, the crew wanted to throw the pilot overboard. The writer's attack is mainly directed against the difficulty of measuring the variation of the compass with sufficient accuracy. The instrument recommended in *The Haven-Finding Art* was no good. "That the taking of a bearing with the compass is a ticklish matter becomes quite clear because several seafaring men, when standing behind a compass, will not agree to half a point in taking a bearing, nay — which is more — not even to a full point, as we have found at sea as well as on the land" (p. 10). It is alleged that navigators, when returning from the first voyage to the Indies, refused to surrender their observational data, obtained with such great risks and difficulties, and that others had faked them, because they were unwilling to put their knowledge and experience, which represented their living, "their field and their plough", readily at the disposal of the scholars on shore. It was in this way that the data for the table of the variation were said to have been collected. We are not going to discuss the further contents of the booklet, which was written with some pretence of learning. Haeyen did not prove himself a man of good education. The disinclination to adopt innovations is after all a familiar phenomenon. When towards the end of the seventeenth century Christiaan Huygens wanted to put his pendulum clock to the test on board, the two navigators who took the observations in accordance with the instructions met with opposition and were heaped with ridicule.

97) The quotations have been taken from the English translation of *De Magnete* by Silvanus P. Thompson, London 1900.

98) *Een corte onderrichtinge belanghende die kunst van der zeevaart, waer in gehandelt wordt hoe men die selfde sullen moghen verbeteren ende oock met een wederleijt, die abuysen die daghelicks door onbequame leer-meesters om havens te vinden, wassen ende toenemen, allen die ter zee handelen tot een ghetrouwe waerschouwinghe door Aelbert Hendricksz, anders Aelbert Haeyen. Amsterdam 1600.*

Another writer who was sceptical of the accuracy of the indications of the compass was Adriaan Metius (1571—1635), mathematical professor in Franeker University. In his *Nieuwe geographische onderwijsinge* ⁹⁹⁾ he devotes a chapter to the defectiveness of navigation, in which he shows how in an experiment on shore the compass-needle, after being brought out of its position of equilibrium, frequently fails to return to its original position (pp. 58 and 59). "What can be accomplished on the moving ship by those who pretend that by an observation of the deviation of the needle the longitude for sailing to the east and the west can be found accurately enough?"

A valuable book was that devoted by Keteltas to the azimuth compass, its construction, and its use ¹⁰⁰⁾. In the dedicatory epistle to Prince Maurice the writer expresses his expectation that the mysterious properties of the magnetic needle will be useful for the determination of longitude. He therefore believes in a relation between them. Many navigators, when sailing to the Indies, had intently observed the variation of the compass after having been instructed by Plancius, and they had found the division of terrestrial magnetism by the four agonics, in accordance with "the thesis of the aforesaid master", to be confirmed in actual practice. Attention is drawn to the number four in connection with the "thesis". If it were possible to measure the variation accurately — the writer speaks of an accuracy to within the nearest minute of arc — the problem of longitude would be solved, and for this reason he wrote a treatise about the construction of a number of instruments by means of which the variation could be measured, about the method of checking such instruments and the means for correcting them, in order that by a thorough understanding of the subject one might be able to rely upon one's measurements. His work is comprehensive, clear, and valuable. It includes a fine illustration of an azimuth compass, suitable for use on board and fitted with an azimuth-circle with a cross-bar sight, "an instrument which is in daily common use among navigators".

A repudiating and almost scornful statement is to be found in the book of Robert Hues, translated and annotated by Jodocus Hondius ¹⁰¹⁾. "There are some people who pretended to derive some measure or certain rule about the variation, as if it occurred in a measurable and proportional way, but it is all in vain. Experience shows that there is no system in it." And somewhat further on he says: "they had better keep silent who think one might make calculations from these deviations in order to find the longitudes of places, which were to be wished, and it would actually occur in this way if the needle pointed always to one and the same point" (Part IV, Chapter 15). It is obvious that this criticism is levelled at Plancius. In fact, at the time it was uttered, *The Haven-Finding Art* had not yet appeared. But Stevin's system was condemned by it too.

The famous Willem Jansz. Blaeu (1571—1638), a nautical expert and a competent judge, also joins the ranks of the opponents, as is evident from the epistle to the reader in the first part of his *Het Licht der Zeevaart* (Light of Navigation) of 1608. He opens this work with *Eene korte onderwijsinghe in de Const*

⁹⁹⁾ Adr. Metius. *Nieuwe geographische onderwijsinge*. Amsterdam 1621.

¹⁰⁰⁾ Barent Ev. Keteltas, *Het ghebruyck der naeld-wijsinge tot dienste der zee-vaert beschreven*. Amsterdam 1609.

¹⁰¹⁾ Robert Hues, *Tractaet ofte handelinge van het gebruyck der hemelscher ende aertscher globe, in 't Latijn beschreven door Robert Hues en nu in Nederduytsch over-gheset ende met Annotatien vermeerdert door Judocum Hondium*. Amsterdam 1623. First edition 1597.

der Zee-vaert (Brief instruction in the art of navigation), in which everything a seaman needed had been included, but from which learned astronomical matters were omitted. He continues: "we have equally omitted to write somewhat about the finding of longitude, which is commonly called east and west, about which some people claim to have found great things. Nay, even to the extent that one can find one's course towards the east and the west just as certainly as towards the south and the north. But all that has so far been published about this is not only useless, but (if one were to rely thereon) also prejudicial and deceptive, about which we intend to write more fully in the fourth part of this book as also what profit a navigator may gain from the deviation of the needle or variation of the compass, on which these new discoveries have been baselessly founded." We do not know what Blaeu intended to write in support of his criticism and about this "profit", for no copy of Part IV of *Het Licht der Zeevaart* is known. However, his condemnation was clear enough, and Plancius had good reason to be embittered against Blaeu because of his destructive criticism: useless, prejudicial, and deceptive. These terms must have been directed no less against Stevin's work.

We may learn more about Blaeu's adverse judgment from a legend in Latin occurring on a globe made by him, which is preserved in the Antwerp Library and which is dated 1620¹⁰²). Here the author explains his choice of the place of the prime meridian, and in this connection he refers to the "contemporaries" who wanted to follow an indication given by nature and choose for the prime meridian the meridian on which the magnetic needle shows no declination. But "they err". The needle cannot serve to find the origin of longitude, "since it shows different declinations on the same meridian, according as it is nearer to this or that continent". He thus says that the variation may have different values on the same meridian and that its amount depends on the proximity of land. These are ideas which were no doubt borrowed from Gilbert's work.

Thus in Holland the controversy had not yet been decided. Plancius continued to propagate his doctrine and he induced navigators to take his instruments with them on their voyages. Stevin had *The Haven-Finding Art* reprinted and included in his *Wisconstighe Ghedachtenissen*, in a Dutch and in a Latin edition. Steering the same course as regards this doctrine was Keteltas, but it was condemned by Hondius, Alb. Haeyen, Blaeu, and Metius. It was the data collected at sea which led to a better understanding. Stevin was among those who insisted that frequent observations be carried out. This advice was hammered into navigators both in Holland and elsewhere, and to this day "taking an azimuth" is among the actions performed by them several times a day. The object is no longer to determine the variation, but to compute the declination of the compass-needle from the magnetic meridian — the deviation, which is due to the ship's iron — *i.e.* still in order to be accurately informed about the direction in which the ship is moving.

Though we will not mention further Portuguese, Spanish, French, and English authors who did not believe in a relation between variation and longitude, or who hoped to find it¹⁰³) or believed that it did exist — such an enumeration would

¹⁰²) A. M. Dermul, *De aard- en hemelgloben van W. Jansz Blaeu in de Antwerpsche Bibliotheek*. Antwerp 1940.

¹⁰³) William Borough, *A Discourse of the Variation of the Cumpas or Magneticall Needle*. 1585. Also in an edition of 1596, Ch. 10.

fall outside the scope of this introduction — we wish to draw attention to two important works, which we cannot pass by in the context of this section.

The first was written by Guillaume de Nautonier, Sieur de Castelfranc en Languedoc, and is entitled *Mécometrie de Leymant* ¹⁰⁴); it is a very extensive work. His system is similar to that of Mercator. He assumes two magnetic poles, one in each hemisphere, in latitude 67° . On the meridian which passes through these two as well as the geographic poles and on which also the island of Ferro is situated, there is no variation of the compass. The tables of the variation given by the author were questioned by some of his contemporaries. Like this author, Kepler long continued to cherish the hope that there would be a simple correlation between the network of meridians and parallels on the earth and the magnetic network.

Fournier, in his voluminous book *Hydrographie* ¹⁰⁵), devotes a long chapter to the determination of longitude, *i.e.* including the method relying upon the variation of the compass (*Livre XII, chap. XXXIV*, pp. 606—608). After having uttered his criticism, he concludes: "*c'est donc une folie de s'amuser à chercher les longitudes par telles voyes*". The matter is thus dismissed with good-natured mockery.

The second of the books referred to above was published in England.

The work in question is that about "the magnet, magnetick bodies also and on the great magnet the earth", by William Gilbert ¹⁰⁶) (born at Colchester in 1544; died 1603), a book that has gained fame in the history of science. It appeared in 1600, *i.e.* very shortly after Stevin's *Haven-Finding Art*. The author was a medical practitioner and a physicist. He engaged for many years in empirical scientific investigations, and he took for granted only those things which had been confirmed by reliable tests. Edward Wright wrote an encomiastic preface for the book, in which he styles the author the "father of magnetick philosophy". It is not surprising that this book contains a chapter entitled "whether the terrestrial longitude can be found from the variation" (Book IV, Chapter IX), and that the name of Stevin and his *Haven-Finding Art* are mentioned in it. Whilst Gilbert calls the values of the variation for the first "segment" from longitude 0 to 60° "in some part true", he disputes the view that the variation is zero all over the meridians through Corvo and Hjelmstöy, or $13^\circ 24'$ over the whole length of that through Plymouth. The value zero does hold at the island of Corvo, but not in other latitudes. He concludes his discussion with the following passage:

"Consequently the limits of variation are not conveniently determined by means of great circles and meridians, and much less are the ratios of the increment or decrement toward any part of the heavens properly investigated by them. Wherefore the rules of the abatement or augmentation of north-easting or northwesting or of increasing or decreasing of the magnetick deviation, can by no means be discovered by such an artifice. The rules which

¹⁰⁴) Guillaume de Nautonier, Sieur de Castelfranc en Languedoc, *Mécometrie de Leymant, c'est à dire la manière de mesurer les longitudes par le moyen de l'eymant . . . aussi facilement comme la latitude. Imprimé à Venes chez l'auteur. 1603.*

¹⁰⁵) Georges Fournier, *Hydrographie, contenant la théorie et la pratique de toutes les parties de la navigation. Paris 1643.*

¹⁰⁶) Guilielmi Gilberti Colcestrensis, medici Londinensis, *De magnete, magneticisque corporibus et de magno magnete tellure; Physiologia nova, plurimis & argumentis & experimentis demonstrata. London 1600.*

follow later for variation in southern parts of the earth investigated by the same method are altogether vain and absurd. They were put forth by certain Portuguese mariners, but they do not agree with the observations and the observations themselves are admitted to be bad." (Translation by S. P. Thompson)

Once again it appears that the values of the variation as determined in actual practice overthrew the theory. The passage ends with the following curious conclusion:

"But the method of haven-finding in long and distant voyages by carefully observed variation (such as was invented by Stevinus and mentioned by Grotius) is of great moment, if only proper instruments are in readiness, by which the magnetick deviation can be ascertained with certainty at sea."

The honourable place assigned to Stevin in the preceding pages is thus confirmed by his contemporary.

e. THE EVOLUTION OF THE SUBJECT IN THE 17th AND 18th CENTURIES.

It was to be expected that those who collected and studied the values of the variation as observed by mariners on their voyages should indicate the figures on a chart, in the places where such observations had been taken. This was the method by which a convenient survey could be obtained. The oldest of such "variation charts" is now said to be that of Alonso de Santa Cruz, referred to by him in his *Libro de las Longitudes*. It must therefore date from about 1535 or 1540. Its value was shaken by the findings of De Castro.

Athanasius Kircher (1601—1680) in his great work on magnetism ¹⁰⁷) mentions a "mappa geographico-magnetica", made by Christophorus Burrus or Cristoforo Borri, an Italian Jesuit priest, who lived in Lisbon and in the Portuguese colonies.

Sir Robert Dudley (1573—1649) in his marine atlas *Arcano del mare* ¹⁰⁸) shows the variation in many places in his charts. Five such figures are found on the chart of the North Sea, from Dover and Nieuwpoort to the mouth of the river Weser.

Further a magnificent large chart on Mercator's projection is to be found in the third edition of Wright's *Certaine Errors in Navigation*, published in 1657. It bears the title "A plat of all the world, projected according to the truest rules", and it had been revised and corrected, as had the book, by Jos. Moxon. It is dated 1655. In a cartouche we read the following words:

"The numbers scatteringly dispersed here and there in this sea chart signifie the variation of the compasse. The letters E and W shewe whether it be East or West. The other letters following signifie the observers names, as D Davis, K Kendal, H Hall, L Lynschot, C Candish, CA John de Castro, etc."

On the west coast of Novaya Zemlya is to be found an observation marked WB, which means of course: Willem Barents.

¹⁰⁷) Athanasius Kircher, *Magnes sive de arte magnetica*. Rome 1641 (as well as Cologne 1643).

¹⁰⁸) Robert Dudley, *Arcano del mare*. Florence. - 1st ed. 1646-1647, present in the University Library, Leiden. - 2nd ed. 1661, present in the Netherlands Historical Maritime Museum, Amsterdam.

An unprecedented increase of knowledge was due to the mathematician and astronomer Edmund Halley (1656—1742), who during long ocean voyages carried out observations and published their results — as he had found them in 1700 — in a large map of the world on Mercator's projection ¹⁰⁹). The map was destined for navigators. In the seas it showed the isogonics, with the exception of those in the Pacific. For the benefit of the user, Halley added an elaborate explanation to the map. To navigators a knowledge of the variation is of the utmost importance if they are to be able to determine their course accurately at moments when weather conditions do not permit them to determine the variation for themselves by observation. With respect to the possibility of determining longitude by means of the chart, the explanation says as follows:

"A further use is in many cases to estimate the Longitude at sea thereby: for where the curves run nearly North and South, and are thick together, as about C. Bonne Esperance, it gives a very good indication of the distance of the land, to ships to come from far: for there the variation alters a degree to each two degrees of longitude nearly, as may be seen in the Chart. But in this Western Ocean, between Europe and the North America the curves lying nearly East and West, cannot be serviceable for this purpose."

In view of the slow change to which the variation is subject, which necessitates revision of the chart, Halley requests the cooperation of navigators. Observations will be gratefully acknowledged by him.

The chart had a wide diffusion. In Holland it was included in the *Atlas van Zeevaart en Koophandel* by Louis Renard, Amsterdam 1745. In France it was also known. In England the investigations were continued, and variation charts were compiled by other authors as well. Such charts were also to be found in Holland. Nautical handbooks pointed out the usefulness of these charts.

How Halley's advice about navigation in the vicinity of the southern point of Africa was applied in practice can be learned, as far as Holland is concerned, from the *Zeilage-Ordre, om ten allen tijde van Straat Sunda over Kaap de Goede Hoop naar Nederland te zeilen* (Sailing direction, for sailing at any time from Sunda Strait via the Cape of Good Hope to the Netherlands), issued by the East India Company to their ships, which instructions were approved in 1783. It is stated there:

"From the longitude and latitude just mentioned, head west by south so as to pass at about 30 miles beyond the shoals lying at the southern end of Madagascar, up to longitude 61° to 62° , where there is now no greater variation than 23° to 24° north-westing.

From there again steer west-south-west, in order to catch sight of the coast of Africa, in the neighbourhood of Punto de Fontes or Algoa Bay. In this channel between Madagascar and Punto de Fontes the greatest variation is found to be between 26° and 27° , and as one approaches the land of Africa or the aforesaid Punto de Fontes, it will decrease to $23^{\circ}30'$ to 23° north-westing. Subsequently by repeated sounding try to find the Reef of Agulhas, so as to call at the Cape of Good Hope or Bay False, as you shall have been ordered in the Instructions."

¹⁰⁹) *Nova et accuratissima totius terrarum orbis tabula nautica variationum magneticarum index. Juxta observationes anno 1700 habitas constructa per Edm. Halley.*

When in 1754 Cornelis Douwes (1712—1773) published in a treatise¹¹⁰) his method of determination of latitude, which became internationally known, he spoke with some reservation about this method of determination of longitude. It was only "in some regions" that by means of the variation one "had roughly some certainty whether one was east or west of a known place", when after a long voyage the estimated longitude had become very unreliable. This was contrasted with the view of Pybo Steenstra, lecturer of mathematics, navigation, and astronomy in the "Atheneum Illustre" of Amsterdam. When in 1770 the latter published his lessons on the finding of longitude at sea, he referred to the method based on the variation of the compass, among the other possibilities, as "so far the best and readiest means of finding the true longitude at sea", provided it be applied in a place where the declination varied rapidly with the ship's movement¹¹¹). None of the existing methods for the determination of longitude produced a reliable result in those days. The development of this problem was very backward as compared with the accuracy of the determination of latitude, which had been enormously advanced by Hadley's invention of the octant in 1731. The result was now reliable to within a few minutes of arc. We can thus sympathize with Steenstra when on 21st November 1763, upon assuming his office as a lecturer, he exclaimed in his inaugural address:

"Would that it were possible to find the true longitude at all times with the same certainty and ease; then the art of navigation would attain to a degree of perfection which it now with good reason despairs of ever reaching."

The speaker was taking too gloomy a view. At that very time the marine chronometer was being constructed and the results obtained with it were promising. The method of determining longitude by means of lunar distances was being developed. The "Nautical Almanac" needed for the computation was soon to appear. The new instrument for measuring angles permitted a reasonable measurement of distances. The mathematical knowledge of the sailor was brought to a higher level with a view to all this.

For the determination of longitude on the basis of the variation of the compass the knell had sounded. This subject, at which scholars as well as practicians had worked for nearly 300 years, had become a thing of the past.

§ 6

APPENDIX

a. *THE CONSTRUCTION AND THE USE OF THE ASTROLABIUM CATHOLICUM.*

Round about 1600 spherical trigonometry was a subject far above the heads of navigators. And yet they had to do with problems of spherical trigonometry, such as the computation of longitude according to Plancius and the determination of the distance on a great circle between two points on the earth. The ways in which

¹¹⁰) *Verhandeling om buiten den middag op zee de ware middagsbreedte te vinden.* Verhandelingen Hollandsche Maatschappij der Wetenschappen, Haarlem 1754, p. 146.

¹¹¹) Pybo Steenstra, *Openbaare lessen over het vinden der lengte op zee.* Amsterdam 1770, p. 45.

they were enabled to solve the latter problem appear very clearly from a legend on the big map of the world of Jodocus Hondius of 1611 ¹¹²).

There were three methods:

1. by projection and construction of plane triangles
2. with the aid of the *astrolabium catholicum*
3. by measurements on the globe.

Directions for all three methods are given, and a fine and clear illustration of

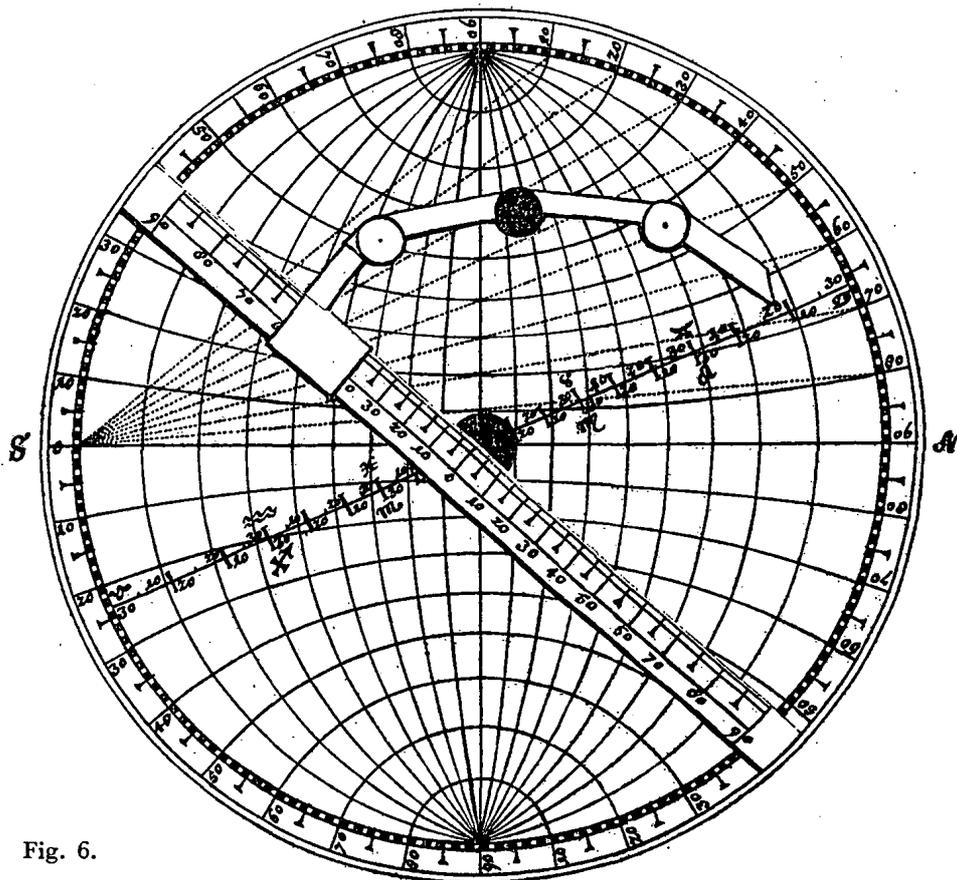


Fig. 6.

Illustration of the network of meridians, parallels, etc., on the astrolabium catholicum, taken from the manuscript of the surveyor Jan Nanninghsoon at Broek-in-Waterland, 1647, present at the State Record Office of North Holland, Haarlem.

¹¹²) *World Map by Jodocus Hondius 1611*. Edited by Edw. L. Stevenson and J. Fisher. New York 1907.

the *astrolabium catholicum* is included. The instrument generally serves to solve problems of spherical trigonometry, which is done by measurements in a plane. It may be looked upon as a variant of the astronomical astrolabe, which had been known for many centuries past.

The instrument consisted of a flat disk on which was drawn a circular network of lines, being the network of meridians and parallels in equatorio-stereographic projection. Because the centre of projection has been chosen in the equator, the projection of the latter is a straight line, at the same time the diameter in the figure. The meridians meeting in the two poles — at a distance of 90° from the equator — are parts of arcs of a circle, and so are the parallels. In fact, we here have to do with the peculiar property of this projection, that circles on the globe become circles in the projection. Its other special feature is that an angle between curved lines on the globe is equal to the angle between their stereographic projections (Fig. 6).

Adapted to turn about a spindle in the centre of the figure was a rule, the edge of which could be made to coincide with the equator. It carried a division which was identical with that on the equator produced by its points of intersection with the meridians. Along the rule could be moved a small block, which was equipped with a metal point consisting of a number of relatively movable links. By means of this construction it was possible to place the end of this point over any given point of the *rete*, which was necessary in the manipulation.

No specimens of the *astrolabium catholicum* are known. We merely find illustrations of it in various places, such as the title-page of Willem Janszoon Blaeu's atlas *Licht der Zeevaart* of 1608 and the title-page of the marine atlas *De groote lichtende ofte vijfjarige colom* (The Lighting Colonne or Sea-Mirroure) of Jacob Aertsz. Colom, Amsterdam 1661. The latter has an illustration of a navigation lesson given in a church, where the teacher from the pulpit — it may be assumed with good reason that it is Plancius who is shown here — hands the instrument to one of his pupils standing at the foot of the pulpit. At the Rijksmuseum, Amsterdam, among the relics of the wintering of Willem Barents in Novaya Zemlya (1596—1597) there is to be found a specimen of the above-mentioned block with the point consisting of links. This proves that an *astrolabium catholicum* formed part of the nautical equipment of the expedition.

For an explanation of the way the instrument was used, further particulars, and illustrations the reader is referred to the present author's article in the maritime review *De Zee*, 1916, p. 180:

„Het gebruik van het *Astrolabium Catholicum*”, and further to the following three works edited by the Linschoten Vereeniging:

- Vol. XV *Reizen van Willem Barents*, etc. 1917, pp. XXI ff.
- Vol. XXXII *De eerste schipvaart*, etc. 1929, pp. 433 ff.
- Vol. XLIV *De tweede schipvaart*, etc. 1940, p. XXXIV.

b. THE CONSTRUCTION AND THE USE OF THE LONGITUDE-FINDER OF PLANCIUS

Only one specimen of this instrument has been preserved. It is also present at the Rijksmuseum among the relics of the wintering of Willem Barents in Novaya Zemlya (Fig. 7).

It is further illustrated in the above-mentioned Works of the Linschoten

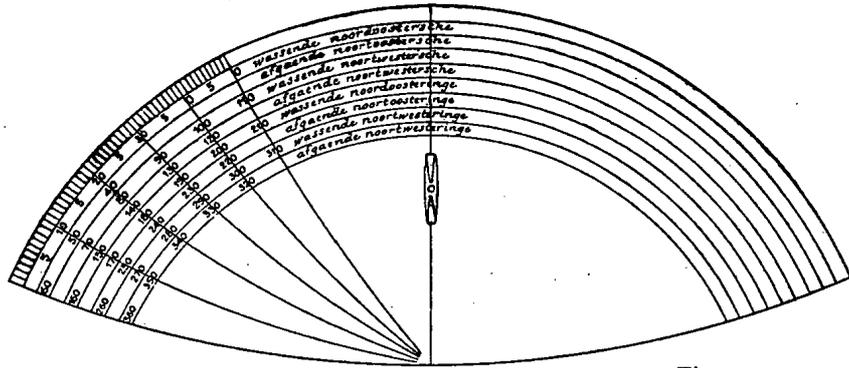


Fig. 7

Longitude-finder of Peter Plancius (Rijksmuseum Amsterdam).

Vereeniging, Vol. XV, p. XXV and Vol. XXXII, p. 436. It is nothing but a flat copper plate, 21.5 cm long, bounded by two arcs of a circle, and it is provided with a number of lines engraved in it. The radius of the circular edge of the

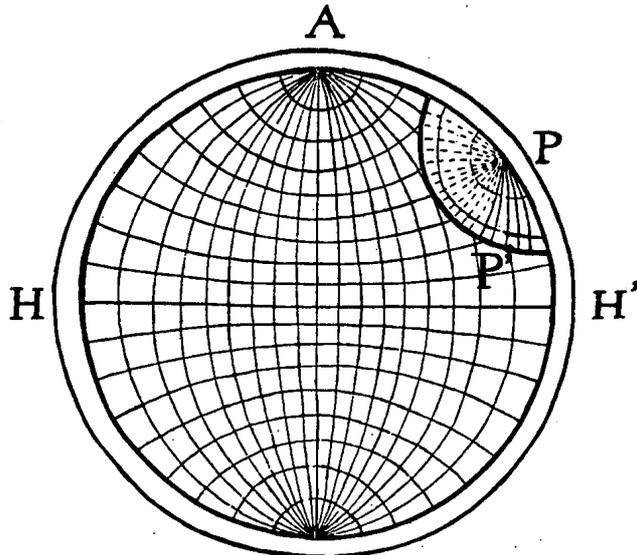


Fig. 8.

This figure shows how the longitude-finder of Plancius has to be used in conjunction with the astrolabium catholicum. *A* is the place on the earth for which the variation and the latitude are known, while the longitude has to be found. The centre of the division on the longitude-finder is the North Pole. Moreover

$$AP = 90^\circ - \text{latitude } A.$$

On the division of the longitude-finder the point *P'*, where the magnetic meridian through *A* intersects the edge of the longitude-finder, indicates the difference of longitude between *A* and the nearest agonic.

plate through construction is found to be 263 mm. The plate was intended to be placed against the edge of the *astrolabium catholicum* and to fit against it (Fig. 8).

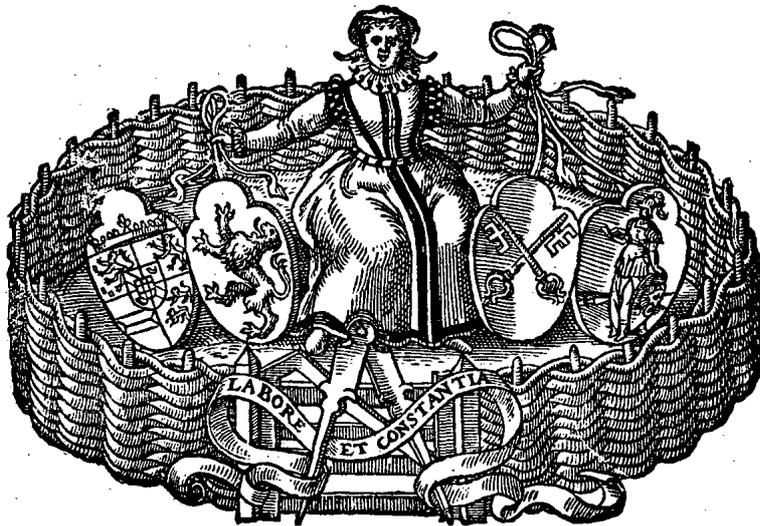
The two instruments accordingly had the same radius. The longitude-finder represents the polar cap of the astrolabe and the engraved curved lines are merely parts of meridians on the *rete* of the astrolabe. The second arc-shaped edge of the plate is a parallel of latitude, *viz.* the one on which Plancius assumed all points of intersection of agonic and magnetic meridians to fall. The terrestrial latitude of this locus was an important feature in his theory. Through construction it appears that this latitude must have been slightly less than $65^{\circ}30'$. This value can be confirmed as follows.

The objects at the Rijksmuseum referred to above include a sheet of paper on which a circular *rete* of the same kind as that on the astrolabe is printed. This sheet of paper lay for nearly 300 years in the ramshackle hut in Novaya Zemlya; it was brought to the Netherlands and smoothed as well as possible, and is now preserved between cardboard and a glass plate. It is obvious that the paper has been subject to warping. Nor is the *rete* complete. In spite of all this, in spite of folds and tears, the edge of the longitude-finder still fits tolerably well against the edge of the *rete* on the paper. The other edge falls on a parallel of 65° to $65^{\circ}30'$. How did Plancius arrive at this figure? Did he choose it because the results of his work were thus most satisfactory, or did he take it perhaps from William Borough, who in his *A Discourse of the Variation of the Cumpas or Magneticall Needle*, London 1596, in the 8th chapter speaks about the magnetic pole being situated at a distance of $25^{\circ}44'$ from the geographic pole?

When L'Honoré Naber, editor of Vol. XV of the Works of the Linschoten Vereeniging, verified the calculations of Plancius, he assumed that the latter had taken the polar circle for this parallel. As a locus, the polar circle in his opinion was "in a sence cosmically hallowed". Although Naber's results agreed satisfactorily with those of Plancius, the dimensions of the plate show that a small correction has to be made. This does not of course affect the existing view of the theory advocated by Plancius.

Plancius was not the only one to teach the use of these two instruments and to equip navigators with them in order that they might use them on board. Yet another teacher of navigation may be mentioned on whose programme the same subject figures. We are referring to "Jan van den Brouck, a professional navigator in the famous commercial town of Rotterdam", writer of *Instructie der Zee-Vaert*, a small book published in 1610, which was entirely adapted to the needs of the simple would-be navigator. He devotes a good deal of attention to the *astrolabium catholicum*, the practical instrument "by means of which one can do anything that one could do with a celestial or terrestrial globe". By way of example he works out a great many problems, in all sorts of fields. "In this way one will soon be a master in handling the astrolabe." He also works out examples with the longitude-finder, under the heading: "an example of how by means of the plate one can find the longitude according to the method of D.P. Plancius."

DE
HAVEN-
VINDING.



TOT LEYDEN,
IN DE DRUCKERYE VAN PLANTIIN,
BY CHRISTOFFEL VAN RAVELENGHIEN,
Gesworen drucker der Vniuersiteyt tot Leyden.

c 1 5 . 1 5 . 1 6 .

Met Priuilegie.

Privilegie.

Op den achtthienden Martij anno xvi^e negentnegen-
gentic, hebben die Staten generael der Vereenichde Ne-
derlanden geconsenteert, ende gheoctroyeert, Consenteren,
ende Octroyeren mits desey, Christoffel Wapfelengius
Bouckdrucker tot Leyden, alleene binney den tijdt van
zesse jaeren naestcommende, inde Gooz. Vereenichde Ne-
derlanden te mogen drucken, doen drucken, uytgeuen, en-
de Vercoopen, zeker bouck geintituleert Haven-vindingh,
Welcken hy van meyninghe is, soo int Latijn als int
Franchois, ende Suytsch, oft oyck in ander spraken
te laten uytgaen, Interdicerende, ende Verbiedende alleyn
ende een y gelyck den Gooz. bouck, int geheel oft int deel, in
eenigerhande spraken na te drucken, noch oyck buyten de
Vereenichde Nederlanden nagedruckt, inde selue uyt te ge-
uen, oft Vercoopen, sonder consent vanden Gooz. Wapfel-
engius, opte Verbeurte Gande naghedruckte Exemplaria,
ende de boete int Originael verhaelt, &c.

Sloeth.^{vr}

Ter ordonnantie vande zelue
Heeren Staten generael.

G. Aerssen.

PRIVILEGE.

On the eighteenth of March of the year 1500 and ninety-nine the States General of the United Netherlands consented and granted a patent, and they consent and grant a patent by the presents, to Christoffel Raphelengius, printer at Leyden, only within the time of the next six years, to print, have printed, publish, and sell in the aforesaid United Netherlands a certain book entitled *Haven-vindingh*, which he intends to publish both in Latin and in French and Dutch, or also in other languages, prohibiting and forbidding one and all to reprint the aforesaid book, wholly or in part, in any language, or, if reprinted outside the United Netherlands, to publish or sell it in the said country without the consent of the aforesaid Raphelengius, on pain of confiscation of the reprinted copies and the penalty mentioned in the original, etc.

Sloeth.

At the decree of the said
States General.

C. Aerssen.

D E

HAVENVINDING.



Is kennelick datmen over langhen
 tijt, voornaemlick sedert dat de groo-
 te zeevaerden op Indien en America
 begosten, middel gefocht heeft, waer
 deur den Stierman op zee mocht we-
 ten, de eertijcxlangde der plaets daer
 teghenwoordelick sijn schip is, om alsoo te commen
 totte havens daer hy begeert te wesen, sonder datmen
 alsnoch tot sulcke ghewisse kennis der langde heeft
 connen ghecommen: want sommighe verhopende
 die te vinden deur de verscheenwysing der zeylnael-
 de, hebben de selve verscheenwysing een * aspunt
 toeghescreven, die noemende † seylsteens aspunt. maer
 men bevint na wyder ervarighen, dat die afwijckin-
 ghen sich na gheen aspunt en schicken. Doch so heeft
 nochtans het foucken van dien, middel veroirsaect
 om tot een begerde hauen te gheraken, niet teghen-
 staende des havens en schips ware langden beyde on-
 bekent sijn. Om twelck eerst by voorbeelt te vercla-
 ren, en daer na te beschrijven de omstandighen desen
 handel aengaende, waer deur de ghebruyck noch ge-
 meender en sekerder sal worden, soo is voor al te we-
 ten, datmen deur ervaring bevint, de zeylnaelde tot
 verscheyden plaetsen (hoewel op gheen seylsteens
 aspunt reghel houdende ghelijck gheseyt is) seer ver-
 scheydelick te wijzen, als tot sommighe oirten recht

* *Polum.*
 † *Polum*
magnetis.

A 2 Noort,

THE HAVEN-FINDING ART.

It is known that for a long time past, principally since the great voyages to the Indies and America began, a means has been sought by which the navigator might know at sea the longitude of the place where his ship is at the moment, in order thus to get to the harbours to which he wishes to go, but that hitherto it has not been possible to arrive at such accurate knowledge of the longitude. For some people, hoping to find it through the variation of the compass, ascribed a pole to the said variation, calling it magnetic pole, but it is found upon further experience that these variations do not obey a pole. Nevertheless the search for this has furnished a means for reaching a desired harbour, even though the true longitudes of both the harbour and the ship are unknown. In order that this may first be explained by means of an example and then the circumstances of the method may be described, as a result of which the application will become even more general and certain, first of all it is to be noted that it is found by experience that the magnetic needle (though it does not obey a magnetic pole, as has been said) points very differently in different places, to wit, in some places due North, in

Noort, tot ander wyckse na t'Oosten, elders na t'Westen, welke veranderinghen als men van t'Oosten na t'Westen treckt, op kleyne weghen seer merckelick sijn, als bij voorbeelt t'Amsterdam wyckse na t'Oosten 9. * trappen 30. ①. An t'voorlant van Enghelant 11. tr. Te Lonnen 11. tr. 30. ①. By Timouth in zee 12. tr. 40. ①. en soo voorts.

* Gradus.

*Hoemen een haven of landt vindt, daer
af de breedte en naeldwycking bekennt is.*

SVLCKE naeldwycking, metsgaders de breedte der plaetsen bekennt sijnde, deur ervaring der ghene diet metter daet alsoo bevonden hebben, men can daer me sonder langde te weten de plaets vinden. Als by voorbeelt, an een Stierman bekennt sijnde, dat de breedte van Amsterdam is 52. trappen 20. ①. met naeldwycking na t'Oosten van 9. tr. 30. ①. ende dat hy hem vindt op zee inde selve breedte van 52. tr. 20. ①, mette voorscreven Oostersche naeldwycking van 9. tr. 30. ①. Hy weet dat hy ontrent Amsterdam moet wesen, t'sij mette langde van Amsterdam hoet wil. Aengaende ymant mocht seggen; datter wel noch ander plaetsen sijn vande selve breedte en naeldwycking, nochtans niet Amsterdam: Tis waer, maer sij vallen seer verre van daer, ende can uyt dander onderkent worden, deur seker omstandighen, van welke wy hier na segghen fullen. Merckt noch dat hoewel de Stierlieden Amsterdam anders connen vinden deur omligghende

others it declines towards the East, elsewhere towards the West, which differences are very noticeable for small distances in going from the East to the West; thus, for instance, at Amsterdam it declines $9^{\circ}30'$ towards the East. Off the Foreland of England 11° , at London $11^{\circ}30'$, off Tynemouth in the sea $12^{\circ}40'$, and so on.

*How a Harbour or Land is found whose
Latitude and Needle-Pointing are known.*

The needle-pointing as well as the latitude of the place being known through the experience of those who have found it so in practice, the place can be found by this means without the longitude being known. Thus, for instance, if a navigator knows that the latitude of Amsterdam is $52^{\circ}20'$, with an easterly variation of $9^{\circ}30'$, and he is at sea in the said latitude of $52^{\circ}20'$, with the aforementioned easterly variation of $9^{\circ}30'$, he knows that he must be off Amsterdam, whatever may be the longitude of Amsterdam. If anyone were to say that there are also other places having the same latitude and variation, which yet are not Amsterdam, this is true, but they are very far away from it and it can be distinguished from the others by certain circumstances, which we shall discuss hereafter. Note also that though navigators are able to find Amsterdam

ghende landen, giffing, diepten, sanden, en ander teyckens, sonder acht op naeldwyfing te nemen, nochtans hebben wy dat voorbeeld van die bekende plaets ghestelt, om daer deur te oentlicker te verclaren de gheemeenheit vande reghel op verre seylagen, daermen op langhe tijt gheen landt en fiet : Als, neem ick, een Stierman begeerende van hier te seylen tot Cabo Sant Augustijn in Brasilie, ende wetende dat de naeldwycking daer is, van (ghelyckmen segt) noort na t'oosten 3. tr. 10. ①, en de zuyderfche breede 8. tr. 30. ①. als hy derwaert varende tot sulcken naeldwycking en breede ghecommen is, hy weet hem ontrent Cabo Sant Augustijn te wesen : Ende hoewel giffing hem anders dede vermoeden, sal die verlaten, als deur oosterfche of westerfche verborghen stroomen bedroghen sijnde, of qualick ghegift hebbende : Want dat de naeldwycking die eertijts tot Cabo Sant Augustin was 3. tr. 10. ①. nu daer niet wesen. en soude, de reden en laet niet toe sich sulcx voor te stellen om daer op te werck te gaen : Of dat ymant op zee een ander naeldwijfing vindt dan de voorscreuen, die hy weet tot Cabo S. Augustin te sijn van 3. tr. 10. ①. ende nochtans willende d'ervaring der naelde verlaten, en giffing volghen, sich seyde ontrent Cabo S. Augustin te wesen, wie en verstaet niet sulcx sonder reden te sijn? als van een die sich selfs teghenspreect, seggende die naeldwijcking aldaer te sijn van 3. tr. 10. ①. ende soo niet te wesen.

Merckt wijder wel ghebeurt te sijn, dat eenen seylende na het Eylant van Sint Helena, ende gecommen wesende tot des selven Eylants breede, nochtans dat

A 3 Eylant

in other ways, from surrounding lands, by conjectural reckoning, depths, sands, and other signs, without paying heed to the needle-pointing, we have nevertheless given this instance of a known place in order thus to set forth more manifestly the general applicability of the rule during long voyages, when no land is seen for a long time. Thus, if a navigator, desiring to sail from hence to Cape St. Augustine in Brazil and knowing that the variation is there (as is said) $3^{\circ}10'$ to the east of the true north and the southern latitude $8^{\circ}30'$, in sailing in that direction has come to this variation and latitude, he knows he is off Cape St. Augustine. And even if conjectural reckoning made him suppose otherwise, he must disregard this, assuming that he has been deceived by unknown eastern or western currents or that he has guessed wrongly. For reason does not permit him to imagine that the variation which was previously $3^{\circ}10'$ off Cape St. Augustine should not have this value now ¹⁾, and to proceed accordingly. Or if a man finds at sea another variation than the above, which he knows to be $3^{\circ}10'$ off Cape St. Augustine, and nevertheless, wishing to disregard the observation of the needle and to rely on conjecture, were to say that he was off Cape St. Augustine, who does not deem this to be an unreasonable procedure, like that of a man who contradicts himself, saying that this variation is $3^{\circ}10'$ there and that it is not.

Note further that it has sometimes happened that a man, sailing to the Island of St. Helena and, having come to the latitude of this island, yet not finding this

¹⁾ Stevin here assumes that the variation holding good for a given place is invariable. However, it is now well-established that the variation at any one place is slowly changing with time.

Eylant daer niet vindende, oock niet wetende of hyder ooft of west af was, heeft al ramende ooftwaert ghesocht, dat westwaert lach, ende hoe hy verder alsoo voer, hoe hy verder vande begeerde plaets gerocht: Denckt nu eens, soo dien Stierman (die wel ettelicke weken lanck dat Eylant socht, ende ettelicke mael daer rontom voer eer hyder in gherocht) had bekent gheweest hoe de naelde op Sint Helena wees, ende daer beneffens wetenschap ghehadt vande naeldwijfing op zee te vinden, of hy moetwillichlick na een grooter naeldwijking soude ghevaren hebben, wetende dat de plaets daer hy begeerde te wesen een kleender hadde?

* Rumbi
wordense
byde Portu-
gesen ghe-
noemt.

Hierby machmen verstaen hoe noodich de kennis der naeldwijfing is: Te meer dat de gene die met wetenschap der * seylstreken wil varen (twelck den Stierman op groote seylaghen niet en behoort onbekent te sijn) over al het recht noort moet weten, welck noort op zee deur kennis der naeldwijking ghevonden wort.

Soomen hier beneffens noch insiet de onsekerheyt vande ware plaetsen der landen, die na tsegghen der Stierlieden op de eertsclooten gheteyckent worden, spruytende daer uyt, datse het wijfen der leli die elck van huys brengt, alijt voor noort houden, streckende daerenboven noch tot meerder onsekerheyt in haer seyling: Men sal verstaen het gaslaen der naeldwijfinghen daer in oock seer oirboir te wesen, wantmen deur zeecompassen daer toe bereyt, de leli al seylende alijt recht noort can doen wijfen, midts de naelde of
r'bestre-

island there nor knowing whether he was to the east or to the west of it, by conjecture sought to the east what lay to the west, and that the further he thus sailed, the further he got from the desired place. Now just consider whether this navigator (who sought this island for several weeks and sailed several times around it before he got there), if he had known how the needle pointed off St. Helena and in addition had known how to find the variation at sea, would deliberately have sailed to a place where the variation is greater, though he knew that the place to which he desired to go had a smaller one?

From this it may be understood how needful the knowledge of the variation is, especially since those who wish to be certain of the course they are following (which ought not to be unknown to the navigator during long voyages) have to know everywhere the true north, which is found at sea by knowledge of the variation.

If further the uncertainty is also recognized of the true positions of the lands which are drawn on the globes according to the information of navigators, which uncertainty results from the fact that they always think the point indicated by the fleur-de-lys which each of them brings from home to be the true north, which moreover leads to greater uncertainty in sailing, it will be understood that the observation of the needle-pointing is also very useful in this respect because it is possible by means of mariner's compasses¹⁾ prepared for this purpose to make the fleur-de-lys point always due north during the voyage, provided the

¹⁾ For the construction of such compasses, see the Introduction, § 3b, p. 369.

rbestrecken ijsen, soo veel vande leli te verdraeyen, als de saeck vereyscht.

Dit alles wel anghemerct, ende toeghelaten wesende verscheyden landen sulcke verscheyden naeldwijsinghen te hebben, gelijk deur ettelicke betuycht wort, het schijnt dat de ghene die niet toe en staen, deur r'behulp der selve naelwijsing de seylage te connen bevoordert worden, of datse de saeck niet en verstaen, of wat anders daer teghen weten dat yghelick niet bekent en is.

Nu alsoo Sijn EXCELLENTIE de voorgaende saken rijpelijk overdocht hadde, ende sich inghebeelt meughelick te sijn, de bovescreven voordering der seylage hier deur merckelick te connen geschien, heeft als Admirael vander zee, ande Admiraliteyt seker oirden ghestelt, ende onderwijs ghegheven, om te weghe te brengen dat de Stierlieden op sulcke reysen varende, hun daer na ghevougen: Namelick datse van nu voortaan tot veel plaetsen daerse commen, metter daet ende wel forchvuldelick, ondersoucken de afwijkinghen der seylnaelde vant noorden, nemende daer toe reetschap wel bequaem: Ende van haer reysen weerghekeert sijnde, daer af ghetrouwelick verwitting doen ande voorscreven Admiraliteyt, welcke de selve ervaringen sullen doen in oirden stellen, ende ten ghemeenen oirboire an yghelicken openbaer maken.

Maer op dat elck die wil noch opentlicker verstaen mach alle omstandighen dese saeck aengaende, soo sullen wy hier stellen een begin, van r'ghene men
deur

needle or the magnetized iron is turned away from the fleur-de-lys as much as is required.

All this being considered and it being admitted that different countries have such different needle-pointings, as is testified by many people, it seems that those who do not admit that navigation can be advanced with the aid of the said needle-pointing either do not understand the matter or know something else to the contrary, which is not known to everyone.

When therefore His Excellency had thoroughly considered the above matters and conceived that it was possible for the above-mentioned advancement of navigation to be appreciably effected by this means, as Lord High Admiral he gave order and instruction to the Admiralty to see to it that navigators going on such voyages should act accordingly, namely, that henceforth in many places where they come they should find out actually and very carefully the variations of the needle from the north, using very suitable instruments for this, and upon their return from their voyages should faithfully report the results to the aforesaid Admiralty, which will cause these observations to be listed and published for the use of all.

But in order that anyone who wishes may understand more clearly all the circumstances relating to this matter, we shall here set down the principle of that

deur breeder ervaringhen in wille is voorder te vervolghen, tafelwijs vervatende de naeldwijfighen die der alree gagheslaghen sijn, welck den hoochgeleerden * Eertrijcxschrijver Heer Petrus Plancius, deur langdeurighen arbeyt, en niet sonder groote cotten by een vergaert heeft, uijt verscheyden houcken des eerbodems, soo wel verre als na ghelegghen: Sulcx dat als de Stierlieden int ghemeen deur dese manier landen en havens fullen vinden, soo wel als eenighe int besonder die alree ghevonden hebben, den selven Plancius ghehouden mach worden voor een der voornaemlickste oirsaken van dien. De voornoemde tafel waer af breeder verclaring ghedaen sal worden is als volght.

Verclaring op de na-volghende Tafel.

EER wy commen totte verclaring, willen voor al segghen, dat by aldien namals deur nauwer en sekerder ervaringhen, der plaetsen naeldwijfsingen, breedten en langden, anders bevonden wierden dan inde tafel staet, ende datmen alsdan ander manier van verclaring en bepaling van woorden behoufde, dat de volghende, dat ons sulcx van t'voornemen deser ondersoucking niet en behoort af te keeren, maer veel eer daer toe te trecken, als allencx gerakende tot meerder en sekerder kennis eens handels ghefticht op sulcken gront als vooren verclaert is. Dese meyning volghende, wy fullen mettet waerschijnlickste dat ons
nu be-

which it is desired to continue further by means of wider experience, listing in a table the variations that have already been observed, which the learned geographer Mr. Petrus Plancius has collected by protracted labour and not without great expense from different corners of the earth, both far and near, so that, if navigators shall find lands and harbours generally in this way, as some in particular have already found them, the said Plancius may be considered one of the principal causes of this. The aforesaid table, a more detailed explanation of which will be given, is as follows.

Explanation of the Following Table.

Before we come to the explanation, we wish to say first of all that if afterwards, by more accurate and more exact observations, the needle-pointings, latitudes, and longitudes of the places should be found to be different from those in the table, and if in that case another way of explanation and definition of words should be required than the following, this ought not to keep us from undertaking this investigation, but rather to incite us to it, so that we may gradually attain to greater and more exact knowledge of a method based on the foundation explained above. Following this opinion, we shall proceed with the most probable knowledge

nu bekend is voortvaren , al oft warachtich waer; want elck tijnder tijt der ghelijcke doende, men sal t'warachtichste dat inde nateur daer af is, allenk naerder en naerder meughen gheraken.

Dit soo sijnde , ende om nu tot verclaring der tafel te commen , soo sietmen voor al datter sijn drie pilaren, deerste van der plaetsen naeldwijssingen, de tweede vande breedte, waer by noch ghevoucht is de derde vande gheraemde langde , op dat de plaetsen inde eertsclooten te lichtelicker ghevonden worden , oock mede om de ghedaenten der naeldwijssinghen daer deur int volghende opentlicker te verclaren. De letter N bediet inde tweede pilaer overal noordersche breedte, maer Z zuyersche breedte.

Voort wantter gheseyt wort van naeldwijcking, oostering, westering, vergrootende ende verkleenende , oock van eerste en tweede perck , welcke als eygen constwoorden haer bepalinghen vereytschen, soo is voor al kennelick , de zeylnaelde seker eyghenschap te hebben , datse op een selve plaets een selven oirt wijst, maer niet den selven oirt over al, want tot sommighe plaetsen wijstse recht noort, tot ander wijckse na t'oosten, elders na t'westen , daerom segghen wy by manier van bepaling als volghen sal:

siet het 12. blas.

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T A-

we now have as if it were true; for if everyone does the same in due time, it will be possible to come gradually nearer and nearer to that which is most true in the nature of things.

This being so, and in order to come now to the explanation of the table, it is seen first of all that there are three columns, the first of the variations of the places, the second of the latitude, to which has been added the third, of the estimated longitude, in order that the places may be found more easily on the globes, and also in order to explain the character of the variations more clearly in what follows. The letter *N* in the second column everywhere designates northern latitude, and *S* southern latitude.

Further, because mention is made of variation, easterly variation, westerly variation, increasing and decreasing, and also of first and second segments¹⁾, which as special technical terms require a definition, it is to be known first of all that the magnetic needle has the particular property that in the same place it points in the same direction, but not in the same direction everywhere, for in some places it points due north, in others it declines to the east, elsewhere to the west; for this reason by way of definition we say as follows:

¹⁾ This term will here be used to render the Dutch "perck", which stands for the part of the earth's surface that is bounded by two half-meridians.

TAFEL DER NAELD- WIISINGHEN.

		Ostering.	Breede.	Langde.	
		tr. ①.	tr. ①.	tr. ①.	
Eerste percx opde noort- zijde	Ver- grootende oostering	Een der Vlaemsche Eylanden Corvo.	0. 0.	0. 0.	
		Opt Vlaemsch Eylant Sancta Maria.	3. 20.	8. 20.	
		Neffens het Eylant Maio.	4. 55.	11. 20.	
		By t'Canarische Eylant Palma.	6. 10.	16. 20.	
		By Cabo de Roca by Lisbona.	10. 0.	24. 30.	
	Ver- cleenende oostering	Het westerlickste van Yrlandt.	11. 0.	24. 12.	
		Engelants eint.	12. 40.	28. 0.	
		Een mijl oostwaert van Pleymouth.	13. 24.	30. 0.	
		By Timouth in zee.	12. 40.	33. 0.	
		Londen in Engelant.	11. 30.	34. 6.	
Twee- de percx opde noort- zijde	Ver- grootende westering	Het voorlant van Engelant.	11. 0.	35. 40.	
		Amsterdam.	9. 30.	39. 30.	
		Westering.			
		Helmshuy by westen de Noortcaep in Finmarck.	0. 0.	N	60. 0.
		Noortcaep in Finmarken.	0. 55.	N 71. 25.	61. 30.
	Ver- cleenende westering	Noorkin.	2. 0.	N 71. 10.	63. 30.
		Sint Michiel in Ruffia genaemt Arch- angel.	12. 30.	N 64. 54.	83. 30.
		De zuyderlicke stract van Vaygats.	24. 30.	N 69. 30.	103. 0.
		Langenes in Nova Zembla.	25. 0.	N 73. 20.	100. 30.
		Willems Eylant by Nova Zembla.	33. 0.	N 75. 35.	110. 0.
Ver- cleenende westering	Yshouck in Nova Zembla.	27. 0.	N 77. 12.	120. 30.	
	Het winterhuys in Nova Zembla.	26. 0.	N 76. 0.	120. 30.	

Op

		Ostering.	Breede.	Langde.
		tr. ①.	tr. ①.	tr. ①.
Eerste percx op de zuyt- zijde	Ver- grootende oostering	Op 105. Spaensche mijlen westwaert van Cabo Sant Augustin en Brasilia.	0. 0.	0. 0.
		By Cabo S. Augustin in Brasilia.	3. 10.	Z. 8. 30. 6. 0.
		Zuyt en noort met Cabo das Almas in Guinea.	12. 15.	Z. 0. 0. 29. 0.
	Ver- cleenende oostering	Noortwest wel soo noordelick vande Eylanden van Tristan da Cunha.	19. 6.	Z. 31. 30. 30. 0.
		Noortwest wel soo westelick vande voorscreven Eylanden.	15. 0.	Z. 31. 30. 36. 0.
		Zuyt en noort met Cabo de Bona espe- rance.	2. 30.	Z. 35. 30. 57. 0.
Tweede percx op de zuyt- zijde (uyt- genom- men Goa Cochin en Canton)	Ver- grootende westering	Op 17. duytsche mijlen van Cabo das Aguillas oostwaert.	0. 0.	Z. 60. 0.
		Ontrent 5. mijlen in zee vant lant Na- tal	4. 30.	Z. 33. 0. 66. 0.
		By de Baixos da India.	11. 0.	Z. 22. 0. 79. 30.
		Mofambique.	11. 0.	Z. 14. 50. 81. 40.
		Inden inwijck van S. Augustin in Ma- dagascar.	13. 0.	Z. 23. 30. 83. 0.
		Zuyt van Cabo Sant Romain.	16. 0.	Z. 28. 0. 86. 20.
		Inden inwijck van Anton Gil in Ma- dagascar.	15. 0.	Z. 16. 20. 91. 0.
		34. Duytsche mijlen zuytooft van S. Brandaon.	22. 0.	Z. 19. 20. 110. 0.
		Goa een vermaerde coopstat in India.	15. 10.	N 15. 30. 120. 0.
		Cochin.	15. 0.	N 9. 45. 121. 0.
		25. Duytsche mijlen west ten noorden vande zuytwesthouck van Samatra.	6. 0.	Z 5. 28. 147. 0.
		Bantan een coopstadt in India.	4. 45.	Z 6. 0. 150. 0.
		Her Eylant Lubock.	2. 25.	Z 6. 10. 155. 0.
		De zuytwesthouck vant Eylant Balij.	1. 30.	Z 8. 40. 157. 0.
De mont der Rivier van Cantan in China.	0. 0.	N 23. 0. 160. 0.		
Bunam 46. Duytsche mijlen van het oostende van Iava na het oosten.	0. 0.	Z 160. 0.		

B 2

Bepa-

TABLE OF THE VARIATIONS.

		Easterly Variation deg. min.	Latitude deg. min.	Longitude deg. min.
First segment on the northern hemisphere	Increasing easterly variation	Corvo, Azores	N 37 0	0 0
		On the Island of St. Mary, Azores	N 37 0	8 20
		Off the Island of Maio ¹⁾	N 15 0	11 20
		Off the Island of Las Palmas, Canary Isl.	N 28 30	16 20
		Off Cabo da Roca, near Lisbon	N 38 55	24 30
		The westernmost part of Ireland	N 52 8	24 12
		Land's End	N 50 21	28 0
	Decreasing easterly variation	One mile eastward from Plymouth	N 50 18	30 0
		Off Tynemouth ²⁾ in the sea	N 55 0	33 0
		London in England	N 51 24	34 6
		The Foreland of England ³⁾	N 51 8	35 40
		Amsterdam	N 52 20	39 30
			Westerly Variation	
	Second segment on the northern hemisphere	Increasing westerly variation	Hjelmsöy to the west of the North Cape in Finnmark ⁴⁾	N
North Cape in Finnmark			N 71 25	61 30
Nordkinn ⁵⁾			N 71 10	63 30
St. Michael in Russia, called Archangel			N 64 54	83 30
The strait to the south of Vaygach Isl.			N 69 30	103 0
Langenes on Novaya Zemlya ⁶⁾			N 73 20	100 30
"Willems Eylant", off Novaya Zemlya ⁷⁾			N 75 35	110 0
Decreasing westerly variation		"Yshouck" on Novaya Zemlya ⁸⁾	N 77 12	120 30
		"Het winterhuys" on Novaya Zemlya ⁹⁾	N 76 0	120 30

(For notes, see p. 441).

		Easterly	Latitude	Longitude	
		Variation	deg. min.	deg. min.	
		deg. min.			
First segment on the southern hemisphere	Increasing easterly variation	At 105 Spanish miles westward from Cape St. Augustine in Brazil ¹⁰⁾	0 0	S	0 0
		Off Cape St. Augustine in Brazil	3 10	S 8 30	6 0
		To the south of Cabo das Almas in Guinea ¹¹⁾	12 15	S 0 0	29 0
	Decreasing easterly variation	Slightly more northerly than northwest from the Islands of Tristan da Cunha	19 0	S 31 30	30 0
		Slightly more westerly than northwest from the aforesaid Islands	15 0	S 31 30	36 0
		To the south of the Cape of Good Hope	2 30	S 35 30	57 0

(For notes, see p. 441)

		Westerly	Latitude	Longitude
		Variation	deg. min.	deg. min.
		deg. min.		
Second segment on the southern hemisphere (except Goa, Cochin, and Canton)	Increasing westerly variation	At 17 German miles eastward from Cape Agulhas 12)	0 0 S	60 0
		About 5 miles in the sea from the land of Natal	4 30 S 33 0	66 0
		At the Baixos da Judia 13)	11 0 S 22 0	79 30
		Mozambique	11 0 S 14 50	81 40
		In S. Augustin Bay, Madagascar 14)	13 0 S 23 30	83 0
		South of Cape St. Romain 15)	16 0 S 28 0	86 20
	Decreasing westerly variation	In Antongil Bay, Madagascar 16)	15 0 S 16 20	91 0
		34 German miles south-east from St. Brendan 17)	22 0 S 19 20	110 0
		Goa, a famous market-town in India	15 10 N 15 30	120 0
		Cochin	15 0 N 9 45	121 0
		25 German miles west by north from the southwestern corner of Sumatra	6 0 S 5 28	147 0
		Bantam, a market-town in the East Indies 18)	4 45 S 6 0	150 0
		The Island of 'Lubbock' 19)	2 25 S 6 10	155 0
		The southwestern corner of the Island of Bali	1 30 S 8 40	157 0
		The mouth of the River of Canton in China	0 0 N 23 0	160 0
		Bunam, 46 German miles to the east from the eastern end of Java 20)	0 0 S	160 0

(For notes, see p. 441).

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- 1) Cape Verde Islands.
 2) Northumberland, east coast of Great Britain.
 3) In view of the latitude given, South Foreland near Dover must be meant.
 4) The island of Hjelmsöy, to the west of the North Cape.
 5) Nordkyn, Finnmark, a cape to the east of the North Cape.
 6) Langenes (Capo de Prior), Sukhoy Nos, a cape on the west coast of Novaya Zemlya, in about latitude 73°30' N.
 7) Berg Island, one of the Gorbovi Islands, near the west coast of Novaya Zemlya.
 8) Cape Bolshaya Ledyanoi, north point of Novaya Zemlya.
 9) "Het behouden huis", where Heemskerck and Barents passed the winter of 1596/97.
 10) East coast of Brazil, near Pernambuco.
 11) What is meant is: in the meridian of Cape Palmas, coast of Liberia, in latitude 4°25' N.
 12) Cape Agulhas, south point of Africa, in latitude 34°50' S.
 13) Baixos da Judia, Mozambique Channel. This shoal is called Judia, after the ship „de Jodin" (the Jewess), which got into trouble there.
 14) West coast of Madagascar, latitude 23°30' S.
 15) Cabo Sant Roman, Cape Andavaka, south coast of Madagascar, situated southwest of Fort Dauphin.
 16) East coast of Madagascar, latitude 16° S.
 17) A mythical island, imagined east of Madagascar.
 18) Java.
 19) Lubock is the island of Bawean, situated north of Surabaya, in about latitude 6° S. The name is a corruption of the Malay word *lubuk*, which means harbour basin. The island owes this name to the big inlet Sangkapura, on the southern side of the island.
 20) For Bunam or Bima, see page 370, note 17.

1. Bepaling.

DE afwijking der naelde vant noorden na t'oosten, heet oostering, maer na t'westen, westering, ende int ghemeen naeldwijking: Maer naeldwijking en rechte noortwijking, int ghemeen naeldwijking.

AENGAENDE de woorden van vergroo-
tende en verkleenende oostering, en weste-
ring, oock van eerste en tweede perck, die vereyschen
eer wy totte bepaling commen, wat breeder vercla-
ring, tot welcken einde wy aldus segghen: Men siet
inde tafel, dat de naelde in Corvo recht noort wijft,
maer van daer oostwaert commende, datse begint te
oosteren allencx meer en meer, tot een mijl oostwaert
van Plemouth, alwaer de afwijking ten grootsten
is van 13. tr. 24. ①. Ende van daer voorder commen-
de, sij begint te vercleynen tot Helmschuy by westen
de Noortcaep van Finmarcken toe, alwaerse weerom
recht noort wijft. Voort is de langde van Corvo tot
Helmschuy van 60. tr. Twelck soo sijnde, het blijkt
dat de voorf. grootste naeldwijking van 13. tr. 24. ①.
by Plemouth wiens langde 30. tr. gheschiet int mid-
del der twee plaetsen daer de naelde recht noort
wijft, want den 30. tr. is int middel tusschen t'begin
en den 60. tr.

Tghene.

1st Definition.

The declination of the needle from the north to the east is called easterly variation, but to the west, westerly variation, and in general variation; but variation as well as north-pointing of the needle are called needle-pointing.

As regards the words increasing and decreasing easterly and westerly variation, and also first and second segment, these require some further explanation before we come to the definition, to which end we say as follows: It is seen in the table that in Corvo the needle points due north, but that, when from there one comes to the east, it gradually begins to decline to the east more and more, up to one mile eastward from Plymouth, where the variation is greatest, namely $13^{\circ}24'$. And when from there one gets further, it begins to decrease up to Hjelmsöy to the west of the North Cape of Finnmark, where the needle points due north again. Further the longitude from Corvo to Hjelmsöy is 60° . This being so, it appears that the aforesaid greatest variation of $13^{\circ}24'$ off Plymouth, whose longitude is 30° , is midway between the two places where the needle points due north, for 30° is in the middle between zero and 60° .

Tghene wy hier ghefeyt hebben, vande verandering der naeldwijfing op de noortsijde des eertrijcx, dergelijcke bevint fich deur ervaring oock opde zuyt-fijde, want op 105. Spaensche mijlen weftwaert van Cabo Sant Augustin opt begin der langde, wijft de naelde recht noort, alsoofse oock doet ter plaets inde tafel ghefeyt, op 17. Duytsche mijlen van Cabo das Aguillas, wefende op 60. tr. der langde, ende int middel tuffchen beyden, dats op den 30. tr. valt aldaer oock ghelijck opde noortsijde de grootste ooftering, dats ter plaets inde tafel ghenaeint, noortweft wel soo noorderlick vande Eylanden van Trifan da Cuncha, doende die wijcking 19. tr.

Hier uijt wilmen besluyten, dat de naelde recht noort wijft tot alle plaetsen ghelegen inde twee halfmiddachfronden deur Corvo en de Helms Huy, van deen * afpunt tot dander. Oock mede dat de naeldens * *Polo.* ooftering ten grootsten is, tot alle plaetsen int halfmiddachfront ftreckende deur de plaets ghelegen een mijle ooftwaert van Plymouth.

Inder voughen dat in sulck ansien, het eertrijcxdeel begrepen tuffchen die twee halfmiddachfronden, 60. tr. in langde van malcander, is een perck int welck de naelde over al vant noorden na t'oosten wijckt, ende inden helft van dien, dat is het eertrijcx deel begrepen tuffchen de twee halfmiddachfronden, teerste deur t'begin, het ander deur den 30. tr. soude over al fijn vergrootende ooftering: Ende in dander helft verkleenende ooftering: welverftaende als men vant werten na t'oosten trect, dats na t'vervolg vande tr. der langde.

A similar thing to that which we have here said about the change of the needle-pointing on the north side of the earth is also found by experience on the south side, for at 105 Spanish miles westward from Cape St. Augustine in longitude 0° the needle points due north, as it also does in the place given in the table, at 17 German miles from Cape Agulhas, which is in longitude 60° , and in the middle between the two, *i.e.* at 30° , as on the north side, falls the greatest easterly variation, *i.e.* in the place mentioned in the table, slightly more northerly than northwest from the Islands of Tristan da Cunha, said variation being 19° .

From this it is concluded that the needle points due north in all places situated in the two meridian semi-circles through Corvo and Hjelmsöy, from one pole to the other. Also that the easterly variation of the needle is greatest in all places in the meridian semi-circle passing through the place situated one mile eastward from Plymouth.

Thus, considering the above, the part of the earth contained between those two meridian semi-circles, 60° in longitude distant from each other, is a segment in which the needle declines everywhere from the north to the east, and in one half of it, *i.e.* the part of the earth contained between the two meridian semi-circles, the first through longitude 0° , the other through 30° , there is everywhere increasing easterly variation, and in the other half decreasing easterly variation, that is to say: when one goes from west to east, *i.e.* in the order of the degrees of longitude.

Deur tgene tot hier toe gheseyt is vant eerste perck met oostering, ende sijn twee deelen, t'een met vergrootende oostering, t'ander met verkleenende, mach men lichtelick verstaen derghelijcke ghedaenten vant tweede perck met westering, ende sijn twee deelen, t'een met vergrootende westering, t'ander met verkleenende: Want inde mont der Rivier van Cantan in China, ligghende in langde 160. tr. van Corvo, daer wijst de naelde de derdemael recht noort, daerom aldaer ghetrocken een derde halfmiddachfront, soo is het eertrijcxdeel begrepen tusschen dat tweede halfmiddachfront ende dit derde 100. tr. van malcander, een perck int welck de naelde overal vant noorden na t'westen wijckt, ende int middel van dese twee, dats int halfmiddachfront 30. tr. vant tweede, ende oock soo veel vant derde, oft andersins 110. tr. vant eerste door Corvo, daer heefmen oock de grootste afwijking der naelde, soot inde tafel tot twee plaetsen blijktt, deen op Willems Eylant by Nova Zembla, alwaer de grootste westering op die breede bevonden is van 33. tr. dander 34. Duytsche mijlen zuytoost van S. Brandaon, alwaer de grootste naeldwijck op die breede bevonden is van 22. tr. wesende de langde van elck dier twee plaetsen 110. tr. Sulcx dat inden helft van dit tweede perc, dats het eertrijcxdeel begrepen tusschen de twee halfmiddachfronten, t'eerste deur den 60. tr. t'ander deur den 110. tr. soude overal sijn vergrootende westering, in dander helft verkleenende westering.

Van dese 160. tr. der langde, twelck op 20. tr. na,
den

From what has so far been said about the first segment with easterly variation and its two parts, the one with increasing, the other with decreasing easterly variation, it can easily be understood that the second segment with westerly variation and its two parts, the one with increasing, the other with decreasing westerly variation, is of the same nature. For in the mouth of the River of Canton in China, situated in longitude 160° distant from Corvo, the needle points due north the third time; consequently, if a third meridian semi-circle is drawn there, the part of the earth contained between the second and the third meridian semi-circle, 100° distant from each other, is a segment in which the needle declines everywhere from the north to the west, and in the middle of these two, *i.e.* in the meridian semi-circle 50° distant from the second and as much from the third, or in other words 110° distant from the first through Corvo, the greatest variation of the needle is to be found, as appears in the table in two places, the one in "Willems Eylant", near Novaya Zemlya, where the greatest westerly variation in that latitude is found to be 33° , the other at 34 German miles southeast from St. Brendan, where the greatest variation in that latitude is found to be 22° , the longitude of each of those two places being 110° . Thus in the one half of this second segment, *i.e.* the part of the earth contained between the two meridian semi-circles, the first through 60° , the other through 110° ; there would be everywhere increasing westerly variation, in the other half decreasing westerly variation.

HAVENVINDING.

den helft des eertrijcx is, heeft de voorf. Plancius de naeldwijfinghen becommen ghelijcke hier vooren beschreven fijn: Maer vande rest des eertrijcx, te weten van Cantan ooftwaert, of van Corvo weftwaert, en overcommen de ervaringhen niet die hem van Spaengjaerden, Engelschen en onse zeevaerders ter handt ghecommen fijn, als ghedaen wefende sonder bequamen tuych, en ghenouchfaem wetenschap: Doch verwacht hy van daer alle daghe nieuwe zeker ervaringhen, deur schepen die meer dan veerthien maenden uijtgheweest fijn. Maer daerentuffchen fullen wy segghen tgene men van dat deel eenichfins vermoet als volght: By aldien de eyghenschap der rechtnoortwijfing, niet alleen en is inde voorf. drie boghen, diemen meent halfmiddachfronden te wesen, ghelijck wy vooren gheseyt hebben van deen aspunt tot dander, maer inde heele ronden, soo foudender opt eertrijck in als fulcke ses halffronden fijn, vervanghende ses percken.

Teerste met oostering lanck 60. tr.

Het tweede met westering lanck 100. tr.

Het derde met oostering lanck 20. tr.

Het vierde met westering lanck 60. tr.

Het vijfde met oostering lanck 100. tr.

Het fefte met westering lanck 20. tr.

Om tgene voorfeyt is deur een form noch oentlicker te verclaren, soo laet A B C D E F G H I K L M, het middelront des eertcloots beteyckenen, diens aspunt N. Voort fij N A den helft van teerste halfmiddachfront deur Corvo, N C het tweede, N E het derde, N G het

Of these 160° of longitude, which is 20° short of one half of the earth, the aforesaid Plancius obtained the variations as they have been described above. But for the rest of the earth, to wit, from Canton eastward or from Corvo westward, the observations he obtained from the Spaniards, the English, and our own navigators do not agree, because they have been made without suitable instruments and sufficient knowledge. But he expects any day to receive new and more exact observations about that part through ships that have been away for more than fourteen months. Now in the meantime we shall say what is assumed about that part, as follows. If the property of the north-pointing applied not only in the aforesaid three semi-circles, which are thought to be meridian semi-circles, as we have said above, from one pole to the other, but in the whole circles, there would on the earth be six such semi-circles in all, containing six segments.

The first with easterly variation 60° long

The second with westerly variation 100° long

The third with easterly variation 20° long

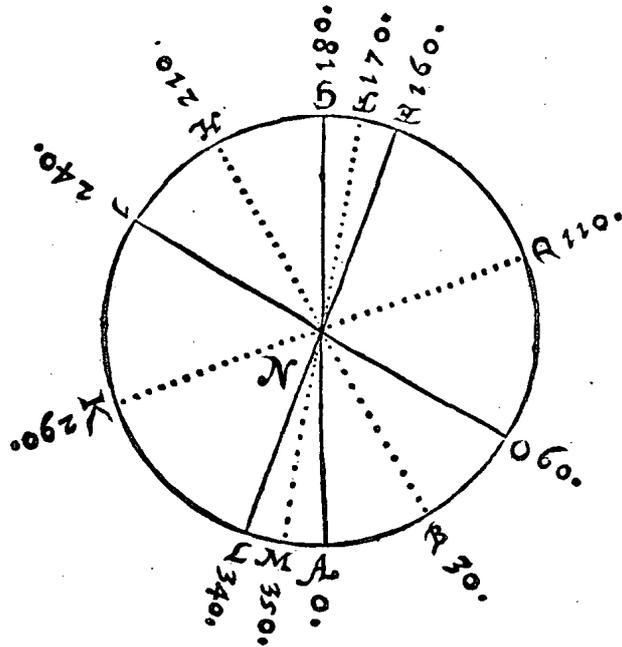
The fourth with westerly variation 60° long

The fifth with easterly variation 100° long

The sixth with westerly variation 20° long

In order to explain the above even more clearly by means of a figure, let *ABCDEFGHIKLM* designate the equinoctial circle of the earth, its pole *N*. Further let *NA* be the half of the first meridian semi-circle through Corvo, *NC*

het vierde, N I het vijfde, N L het sefte, ende alsoo dat de booch A C doe 60. tr. C E 100. tr. dats A E 160. tr. E G 20. tr. dats A G 180. tr. G I 60. tr. dat A I 240. tr. I L 100. tr. dats A L 340. tr. L A 20. tr. dats tgeheel rondt 360. tr. Voort sijn de ses punten B, D, F, H, K, M, middelen tusschen A C, C E, E G, G I, I L, L A. Dit aldus wvefende soo bediet,



A N C het 1^e perck met oostering.
 A N B des 1^e percx vergrootende oostering.
 B N C des 1^e percx verkleenende oostering.
 C N E het 2^e perck met westering.
 C N D des 2^e percx vergrootende westering.
 D N E des 2^e percx verkleenende westering.

E N G

of the second; *NE* of the third, *NG* of the fourth, *NI* of the fifth, *NL* of the sixth, and such that the arc *AC* makes 60° , *CE* 100° , and so *AE* 160° ; *EG* 20° , and so *AG* 180° ; *GI* 60° , and so *AI* 240° ; *IL* 100° , and so *AL* 340° ; *LA* 20° , and so the whole circle 360° . Further let the six points *B*, *D*, *F*, *H*, *K*, *M* be the mid-points between *A* and *C*, *C* and *E*, *E* and *G*, *G* and *I*, *I* and *L*, *L* and *A*. This being so,

- ANC* designates the 1st segment with easterly variation
- ANB* the increasing easterly variation of the 1st segment
- BNC* the decreasing easterly variation of the 1st segment
- CNE* the 2nd segment with westerly variation
- CND* the increasing westerly variation of the 2nd segment
- DNE* the decreasing westerly variation of the 2nd segment

H A V E N V I N D I N G.

17

E N G het 3^e perck met ooftering.
 E N F des 3^e percx vergrootende ooftering.
 F N G des 3^e percx verkleenende ooftering.
 G N I het 4^e perck met westering.
 G N H des 4^e percx vergrootende westering.
 H N I des 4^e percx verkleenende westering.
 I N L het 5^e perck met ooftering.
 I N K des 5^e percx vergrootende ooftering.
 K N L des 5^e percx verkleenende ooftering.
 L N A het 6^e perck met westering.
 L N M des 6^e percx vergrootende westering.
 M N A des 6^e percx verkleenende westering.

M E R C T. Hoewel het te vermoeden is, datmen de drie laetste halfronden niet vinden en sal van ghe-daente als de voorgaende giffing inhoudt, maer mis-schien in menichte meer of min, en van ander ghe-stalt; doch soo is hier me voorbeeltsche wijze ver-claert de manier hoemen de weerelt int gheheel sal meughen deylen, met sulcke halfronden alffer alsdan ghevonden sullen worden: Boven dien is deur tvoor-gaende ghenouch te verstaen, wat bedien vergrooten-de en verkleenende ooftering, en westering, oock eer-ste en tweede halfmiddachsfront, met eerste en twee-de percken: Om welcke by manier van bepaling te vervaren, men soude meughen aldus segghen:

C

2. Bepa-

ENG the 3rd segment with easterly variation
ENF the increasing easterly variation of the 3rd segment
FNG the decreasing easterly variation of the 3rd segment
GNI the 4th segment with westerly variation
GNH the increasing westerly variation of the 4th segment
HNI the decreasing westerly variation of the 4th segment
INL the 5th segment with easterly variation
INK the increasing easterly variation of the 5th segment
KNL the decreasing easterly variation of the 5th segment
LNA the 6th segment with westerly variation
LNM the increasing westerly variation of the 6th segment
MNA the decreasing westerly variation of the 6th segment

NOTE. Though it is to be expected that the three last semi-circles will not be found to be such as the preceding conjecture implies, but perhaps in a quantity either more or less and of a different form, yet it has thus been explained by way of example in what manner the whole world may be divided by such semi-circles as shall be found. Moreover it can be sufficiently understood from the foregoing what is the meaning of increasing and decreasing easterly and westerly variation, also of the first and the second meridian semi-circle, with the first and second segments. In order to summarize this in the form of definitions, it might be said as follows:

2. *Bepaling.*

VERGROOTENDE oostering of westering, is die welcke de naelde van westen na oosten voortghebrocht zijnde, vergroot: Ende verkleenende, die aldan verkleent.

3. *Bepaling.*

DE halfmiddachfronden daer de naelde recht noort in wijst, heeten wy eerste, tweede halfmiddachfront, en soo oirdentlick voort na tvervolgh vande trappen der langde soo veel alser sulcke ronden zijn, beginnēde vant halfmiddachfront deur Corvo.

4. *Bepaling.*

TVLACK begrepen tusschen eerste en tweede halfmiddachfront, noemen wy eerste perck, en dander oirdentlick vervolghende tweede, derde perck, tottet laetste.

D E

2nd Definition.

Increasing easterly or westerly variation is that which increases when the needle is carried from west to east; and decreasing variation, which then decreases.

3rd Definition.

The meridian semi-circles in which the needle points due north we call the first and the second meridian semi-circle, and so on in the order of the degrees of longitude, as many such semi-circles as there are, starting from the meridian semi-circle through Corvo.

4th Definition.

The surface contained between the first and the second meridian semi-circle we call the first segment, and the others in due order the second segment, the third, up to the last.

DE ghedaenten der naeldwijfinghen aldus beschreven sijnde, wy sullen nu deur voorbeelt verklaren, ghelijck voornemen was, dat hoewelder op een selve breede evegroote naeldwijkinghen sijn tot verscheyden plaetsen des eertrijcx, dat nochtans den Stierman can weten in welcke der selve hy is. Laet tot desen einde een schip andermael moeten varen van Amsterdam na Cabo Sant Augustin, in Brasilië, wiens breede inde tafel beschreven staet van 8. tr. 30. ①, ende de naeldwijfing vergrootende oostering des eersten percx van 3. tr. 10. ①. Tselve schip afvarende, ende commende voorby Engelandt, bevindt sijn naeldwijfing daghelicx meer en meer te oosteren, tot by Plymouth toe, alwaerse ten grootsten wesende van 13. tr. 24. ①, het verskert hem dat hy tot daer toe ghevaren heeft in verkleenende oostering des eersten percx, ende dat hy van daer voort seylt inde vergrootende oostering, welcke hy bevindende van 10. tr. opde breede van 38. tr. 55. ①. weet hem te wesen ontrent Cabo de Roca by Lisbona: Van daer af, ontrent zuytwest anvarende, sal daghelicx bevinden de breede te minderen, ende de naelde noordelicker te keeren: Oft andersins soo die daghelicksche noordering niet en blecke, maer dat de naelde een selve streeck wese, oft oostlicker keerde, salt daer voor houden dat onbemerckelicke stroomen sijn schip al varende oostwaert drijven: Om twelck te voorkomen, salt soo veel westelicker an setten, dat hy daghelicx de naeldens behoirlicke noordering krijghe. Maer soo hy quaem totte oostering van 3. tr. 10. ①. eer hy

The character of the needle-pointings thus having been described, we shall now explain by means of examples, as was our intention, that although at the same latitude there are equal variations in different places of the earth, the navigator may nevertheless know in which of these places he is. For this purpose let a ship have to sail once more from Amsterdam to Cape St. Augustine in Brazil, the latitude of which is given in the table as $8^{\circ}30'$ and the variation is increasing easterly variation of the first segment of $3^{\circ}10'$. When this ship puts off and sails past England, the variation will be found to become more and more easterly every day up to Plymouth, and since it is there at its greatest, namely $13^{\circ}24'$, this assures the navigator that he has so far sailed in the decreasing easterly variation of the first segment and that from there he will further sail in the increasing easterly variation, and when he finds this to be 10° in latitude $38^{\circ}55'$, he knows that he is off Cabo da Roca near Lisbon. When from there he sails about southwest, he will daily find the latitude decreasing and the needle returning to the north ¹). Or else, if this daily return further to the north did not become apparent, but the needle pointed in the same direction or declined further to the east, he will assume that imperceptible currents are driving his ship eastward; and in order to prevent this, he will direct it so much further westward that he may daily obtain the proper return of the needle to the north. But if he comes to

¹) The ship, starting from Amsterdam, navigates first in the section BNC ("decreasing easterly variation of the 1st segment"); then in the section ANB ("increasing easterly variation of the 1st segment"). See p. 451.

gherocht totte zuyderlicke breedte van 8. tr. 30. ①. hy fal maken soo veel hem meughelick is, die naeldwijcking int zuytwaert varen te behouden, soo veel oostelicker of westelicker seyléde als de saeck vereyscht. Ende hoewel hem na gissing docht anders te behouren, en sal die nochtans niet volghen, om redenen hier vooren breeder verclaert, want commende alsoo totte zuyersche breedte van 8. tr. 30. ①. met vergrootende oostering van 3. tr. 10. ①. hy moet (tmach mette langde dier plaets sijn soot wil) ontrent Cabo Sant Augustin wesen, ende dat met sekerheyt: daermen anders op gissing betrouwende, ettelicke hondert mijlen vande begeerde plaets gheraect, sonder te weten of mender oost of west af light, ghelijck op sulcke reysen metter daet ghenouch ghebleken heeft. Daerom tot allen houcken des weerelts de naeldwijcking en breedte wel ghenomen sijnde, ende an alle man bekent ghemaect, men sal de weerelt anders connen beseylen danmen ghedaen heeft.

Tot hier toe sijn beschreven de ghedaenten der naeldwijckinghen, volghende uijt het ghestelde des tafels: Soo ander sekerder ervaringhen in toecommen den tijt anders wesen, men sal daer uyt anders meughen besluyten, ende inde zeyling sich na t'beste altijt ghevoughen.

Hoe-

the easterly variation of $3^{\circ}10'$ before he has reached latitude $8^{\circ}30'$ South, he must do all he can to keep this variation in going to the south, sailing so much more towards the east or the west as required. And though by conjecture he thinks it ought to be otherwise, he must not proceed on this, for the reasons set forth more fully above, for if he thus comes to latitude $8^{\circ}30'$ South with increasing easterly variation of $3^{\circ}10'$, he must be (whatever may be the longitude of the place) off Cape St. Augustine, with certainty, whilst otherwise, relying on conjecture, one gets many hundreds of miles from the desired place, without knowing whether one is to the east or the west of it, as has appeared often enough in practice during such voyages. If therefore the needle-pointing and the latitude are duly observed in all corners of the world and made known to everybody, it will be possible to sail the world in another way than hitherto.

Thus far the character of the variations following from the data of the table has been described. If other, more exact observations should prove different in the future, other conclusions can be drawn from them, and in navigation the best must always be used.

*Hoemen het noortpunt en naeld-
u-wijfing vindt.*

HOE wel het vinden der naeldwijfing (daer af wy hier vooren dickwils ghefeyt hebben) an velen bekent is, nochtans fullen wy daer af fchrijven voor de ghene diet niet en weten.

Anghesien men hier begeert te vinden de afwijking der naelde vant noorden, soo souctmen eerst het noortpunt, om de naeldwijfing daer by te verlijcken. De manier der vinding vant selve noortpunt in een beweghende fchip op zee, heeft groote ghemeenfchap mette manier der vinding vant noortpunt, of vande middachflijn opt vast lant, ende mach onder anderen aldus uijtgherecht worden: Men doet int zecompass de leli recht overcommen mettet noortende vant ftael, of vande zeylnaelde daer onder ligghende: Of noch beter machmen in plaets vande leli, een naelde felf boven opt papier vast legghen, deelende r'rondt van tselve papier in fijn 360. tr. beginnende ande naeldens noortpunt als hier onder het rondt A B C D, waer in de naelde beteyckent is met A C, vastghemaect wefende opt selve papier, E is tmiddelpunt: Tgebruyck hier me is dusdanich: Ghelijck den Stierman int foucken der breede, wacht tot dat de middach ghecommen is, te weten tot dat de fchaeu van een hangfnoer of rechtsnoer, overcomt mette lini die hy in fijn compass voor de middachflijn houdt, alsoo fal hy hier doen, uijtghenomen dat hy begint 3. 4. of 5. uijren of

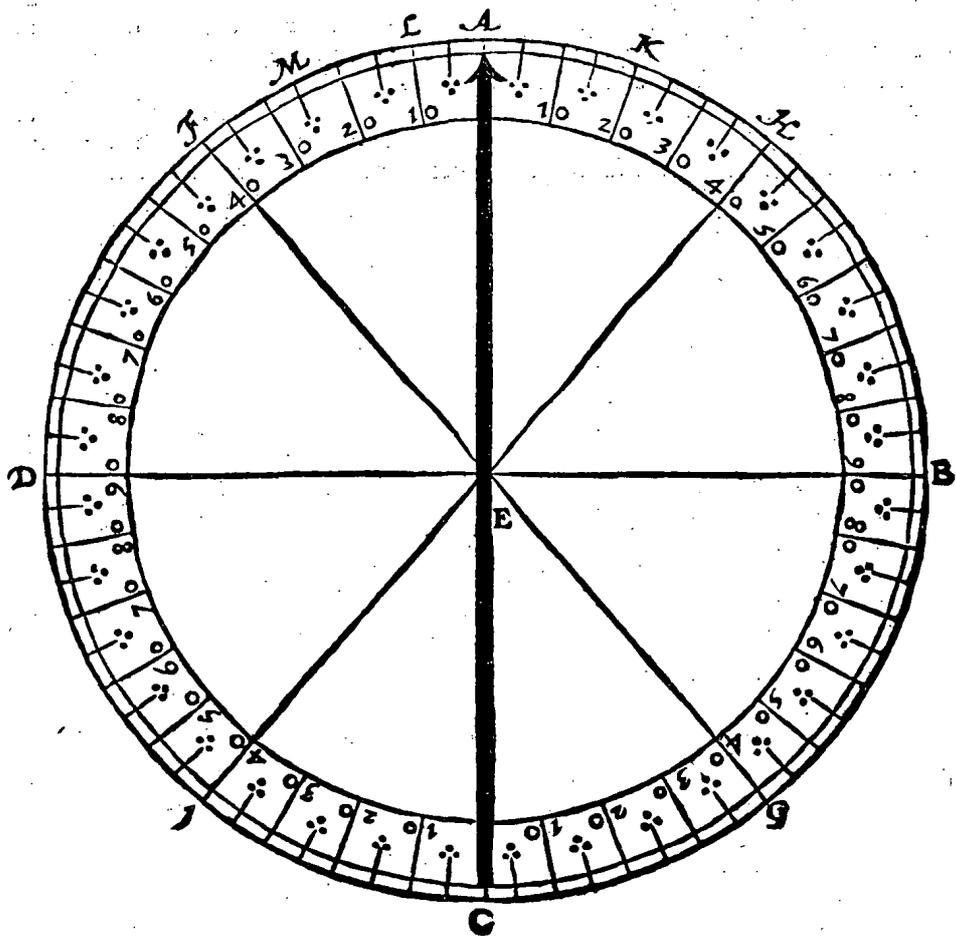
C 3 meer

How the True North and the Variation are found.

Although the finding of the variation (about which we have often spoken above) is known to many people, we shall write about it for those who do not know it.

Since it is desired to find the variation of the needle from the north, first the true north is sought, in order that the needle-pointing may be compared with it. The method of finding the said true north in a moving ship at sea greatly resembles the method of finding the true north or the meridian line on the land, and can be carried out, among other things, as follows. The fleur-de-lys in the mariner's compass is made to correspond exactly with the north end of the steel or of the magnetic needle lying underneath. Or, better still, instead of the fleur-de-lys a needle itself can be fixed on top of the paper and the circumference of the said paper can be divided into its 360°, starting at the north-point of the needle, *e.g.* the circle *ABCD* below, in which the needle is denoted by *AC*, being fixed on the said paper, *E* being the centre. The use of this is as follows. Just as the navigator, when seeking the latitude, waits until noon has come, to wit, until the shadow of a plumb-line coincides with the line which he regards as the meridian line in his compass, so he must do here, except

meer voor middach alsdan, acht nemende op wat trap en ghedeelte van dien de schaeu des hangfnoers wijft, bevint die, neem ick, op den 40. tr. gheteyckent F, sulcx dat G E F, de schaeu bediet, ende nemende alsdan de Sonnens hooghde, bevint die, by voorbeelt, van 25. tr. welcke hy, metfgaders de 40. tr. tot ghedachtnis opteyckent : Wachtende voorts soo lang na



middach,

that he begins 3, 4 or 5 hours or more before noon, noting at what degree and part of it the shadow of the plumb-line points. Let us assume that he finds this at 40° , designated by F , so that GEF denotes the shadow. Then, taking the Sun's altitude, he will find this to be *e.g.* 25° , which he notes down, together with the 40° , as an aid to memory. Then, waiting so long after noon until the

middach, tot dat de Son weerom ghedaelt is tot op de selve hooghe alsvooren van 25. tr. sal sien waer de schaeu vant hangfnoer alsdan opt papier wijft, twelck sij, neem ick, 40. tr. over dander sijde, als an H, sulcx dat I E H, de schaeu bediet. Dit soo sijnde, t'middel des boochs F H, als A, is tbegeerde noortpunt, ende want de naelde daer recht op wijft, soo en heeftse in dat voorbeelt gheen wijcking, dan wijft recht noort. Maer soo inde voorf. ervaring na middach de schaeu vant hangfnoer niet ghewesen en hadde 40. tr. over dander sijde van A, maer by voorbeelt alleene-lick 20. tr. tot K; In sulcken ghevalle deeltmen den booch F K, doende 60. tr. door tghedacht in tweeen an L, sulcx dat L F, L K, elck doen 30. tr. Twelck soo sijnde, L ist noortpunt, ende de begeerde naeldwijcking daer af is oostering van L tot A 10. tr.

Maer byaldien inde voorf. ervaring na middach, de schaeu vant hangfnoer ghewesen hadde op L, dats 30. tr. van F, soo deeltmen den booch F L, doende 30. tr. doortghedacht in tweeen an M, sulcx dat M F, M L, elck doen 15. tr. twelck soo sijnde, M is tnoortpunt, ende de begeerde naeldwijcking daer af, wesende oostering van M tot A 25. tr. ende alsoo met alle voorbeelden. Maer soo de naelde alleen draeyde, fonder an een papier ghehecht te sijn als hier vooren, ende dat de trappen op den cant vande casse gheteyckent waren, ghelijck wel ghedaen wort: Tghebruyck is daer me alsvooren, midts datmen ten tijde der ervaring, de casse keert tot dat de naelde opt begin der trappen wijft.

Ander

Sun has descended again to the same altitude as before, namely 25° , he must find where the shadow of the plumb-line then points on the paper. Let us assume this to be 40° on the other side, namely at H , so that IEH denotes the shadow. This being so, the middle of the arc FH , namely A , is the desired true north, and because the needle points straight to it, it has no variation in this example, but points due north. But if in the aforesaid observation after noon the shadow of the plumb-line is not 40° on the other side of A , but *e.g.* only 20° , at K , in such a case the arc FK , which makes 60° , is divided in two in imagination at L , so that LF and LK each make 30° . This being so, L is the true north, and the desired variation is an easterly variation from L to A of 10° .

But if in the aforesaid afternoon observation the shadow of the plumb-line is at L , *i.e.* 30° from F , the arc FL , which makes 30° , is divided in two in imagination at M , so that MF and ML each make 15° . This being so, M is the true north, and the desired variation, which is an easterly variation from M to A , is 25° , and the same applies to all examples. But if the needle alone is turned, without being fastened to a paper, as above, and the degrees are marked on the rim of the box, as is sometimes done, it is used in the same way as above, provided the box is turned at the moment of the observation until the needle points at the zero point of the graduation.

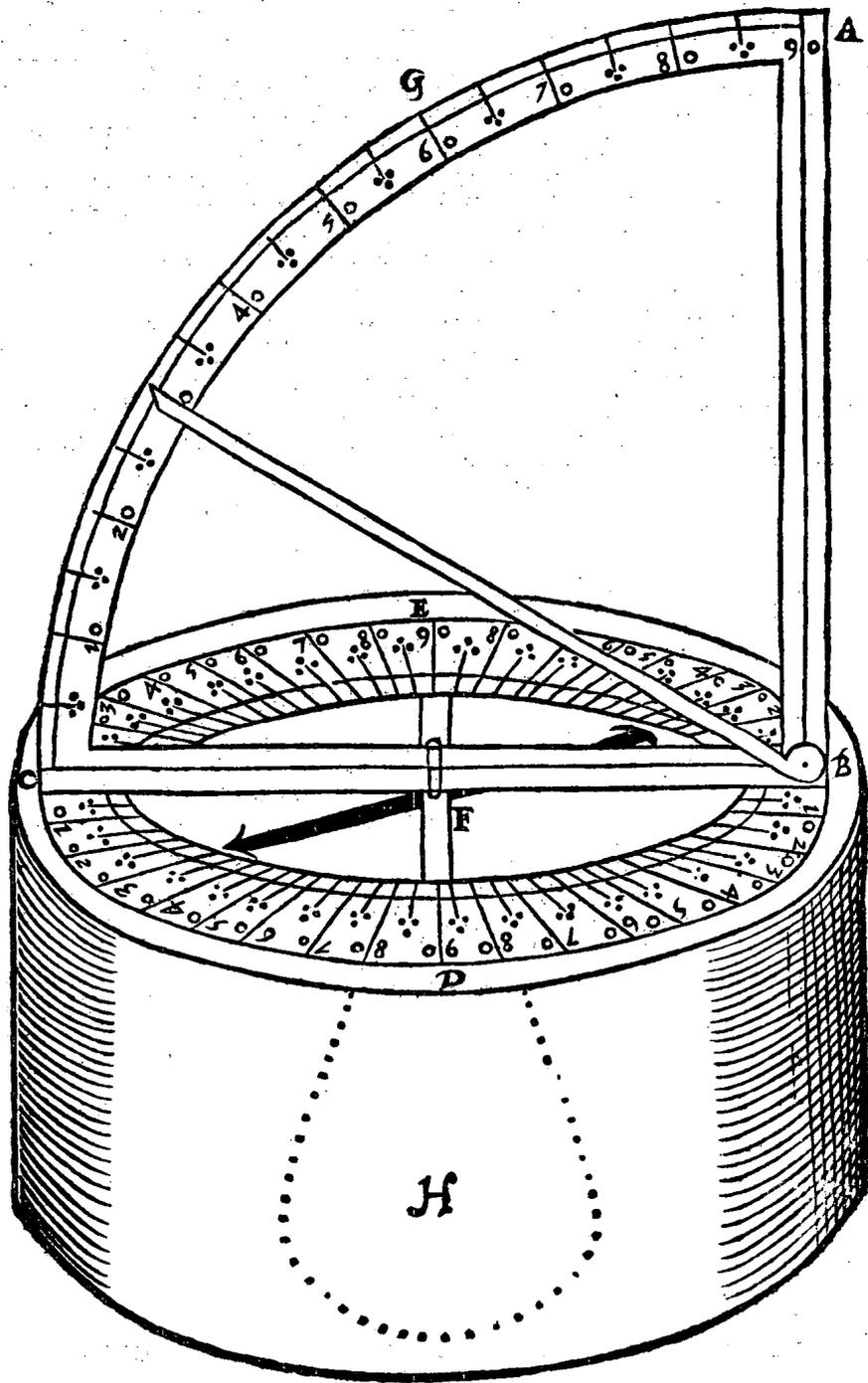
* *Quadrantem
Azimuthalium
 seu verti-
culū cuius
planū ho-
rizontale.*

Ander sijnder die nemen een * foppich vierendeel ronts, wiens sichteindersplat, niet teghenstaende de beweeghlicheyt des schips, alijt in waterpas blijft, deur sulcke manier als int volghende gheseyt sal worden. Hier me vintmen de Sonnens hooghde met haer fopbooch beyde tseffens: De form daer af mach dusdanich wesen: $A B C$ bediet een vierendeel ronts, staende rechthouckich opt rondt $B D C E$, ghedeek in sijn 360. trappen, twelck het sichteindersplat beteyckent, sijn middelpunt is F , waer op tvierendeelronts draeyen can, ende op dattet alsins rechthouckich blijft opt voorf. rondt $B D C E$, soo comt van deen en dander sijde een steunsel, als van G tot by D en E , vast ghemaect an tvoorf. vierendeelrōts, om daer me te drayen. Voort isser int rondt $B D C E$ een glas, en daer onder sijn seylnaelde, soo lanck alse ten langsten inde casse bequamelick vallen mach, ende heeft de selve casse van binnen heur 360. tr. daer de punt der naelde scherpe-lick op wijzen mach, overcominende die 360. tr. met dander 360. tr. boven opt sichteindersfront. Desen tuych is deur de vondt van Reyner Pietersz. hanghende ghemaect op twee verscheyden assen, na de manier der zeecompassen, op dat alsoo het rondt $B D C E$, inde beweginghen vant schip alijt ewewijdich vanden sichteinder blijve: Ende op dattet selve noch meerder sekerheyt hebbe, soo wortter onder een ghewicht an vervought gheteyckent H , van 25. of 30. pont, of soo veel als de grootheyt vanden tuych vereyscht.

Tis oock te ghedencken oirboir te wesen, dattet vierendeelronts tsijnder plaets recht overende staende,
over

There are others who take an azimuthal quadrant, the horizontal plane of which, notwithstanding the movement of the ship, always remains level, by the method to be described in the following. By this means the Sun's altitude is found along with its azimuth. The figure relating thereto may be as follows. *ABC* designates a quadrant of a circle, at right angles to the circle *BDCE*, divided into its 360 degrees, which denotes the horizontal plane. Its centre is *F*, about which the quadrant can turn, and in order that it may always remain at right angles to the aforesaid circle *BDCE*, a support is provided on either side, namely from *G* to *D* and *E*, fastened to the aforesaid quadrant, in order to turn along with it. Furthermore there is in the circle *BDCE* a glass, and underneath it the magnetic needle, which has the maximum length possible in the box, and on the inside of this box are marked the 360 degrees, at which the point of the needle can point accurately, these 360 degrees corresponding to the other 360 degrees on top of the horizontal circle. By the discovery of Reynier Pietersz ¹⁾ this instrument has been suspended on two different shafts, in the manner of the mariner's compass, in order that the circle *BDCE* may thus, in spite of the movements of the ship, always remain parallel to the horizon. And in order that this may be even more certain, a weight is fixed underneath it, marked *H*, namely 25 or 30 pounds or as much as the size of the instrument makes necessary.

¹⁾ See the Introduction, § 4 b.: Reynier Pietersz and his "Golden Compass".



D over

over deen en dander sijde eveswaer sij , dat is de sijde van F na C, soo swaer als van F na B, twelckmen weten can mits tvierendeelrondts af te nemen , ende te hanghen met G neerwaert an een draet, vast gemaect int middel van B C by F, ende alsdan salmen vande swaerste sijde veel af vijlen , tot dat de reghel BC in waterpas hangt.

** Alidada.* Angaende ymant mocht dencken, dat de * wijfreghel in verscheyden plaetsen hooger of leegher ghedraeyt, te groote verandering int ghewicht mocht geven, daer af en is gheen merckelick feyl te verwachten , om t'groot ghewicht van H, ende de lichticheyt der wijfreghel.

De ghebruyck daer af, om t'noortpunt en naeldwijfsing te vinden, is dusdanich: Men begint, gelijk in deerste wijze, ettelicke uijren voor middach, draeyende den tuych tot dat de naelde opt begin des ronts wijft, daer na keertmen het vierendeel ronts soo lang herwaerts en derwaerts, tot dat de Son deur de sichtgaetkens schijnt: Twelck soo sijnde, men bevint, neem ick, dat den ondersten cant of wijfer vant vierendeelronts, wijft int sichteinderflat op den 40. trap, ende de hooghde der Son, die int vierendeelronts anghewesen wort van, neem ick, 25. tr. welcke men, mitsgaders de 40. tr. tot gedachtnis opteyckent. Wachende voort soo lang na middach , tot datmen de Son deur den seluen tuych ghedaelt vindt tot opde selve hooghde alsvooren van 25. tr. men keert alsdan den stoel ter eender en ander sijde, tot dat de Son deur de sichtgaetkens schijnende , de naelde vveerom wijft
opt

It is also to be remembered that it is suitable for the quadrant which is vertical in its place to have the same weight on either side, *i.e.* the side from *F* to *C* to have the same weight as that from *F* to *B*, which can be known if the quadrant is taken off and suspended with *G* downward by a thread fastened in the middle of *BC* at *F*, and then so much must be filed away from the heavier side, until the line *BC* hangs level.

If anyone should think that the pointer might bring about too great a variation in the weight in different places according as it is turned higher or lower, no appreciable error is to be expected from this, because of the great weight of *H* and the lightness of the pointer.

The way in which this instrument is used to find the true north and the variation is as follows. The observation should be started, as in the first case, a few hours before noon, the instrument being turned until the needle points at the zero point of the graduation. Thereupon the quadrant is turned this way and the other until the Sun shines through the sights. This being so, it is found *e.g.* that the lower edge or pointer of the quadrant points in the horizontal plane at 40° , while the altitude of the Sun, which is indicated in the quadrant, is *e.g.* 25° , which is noted down, together with the 40° , as an aid to memory. Then one should wait after noon until by means of the instrument the Sun is found to have descended to the same altitude as before, namely 25° . Then the quadrant is turned this way and the other until, the Sun shining through the sights, the

opt begin des ronts: Twelck soo sijnde, t'middelste punt des boochs int sichteindersplat tusschen deerste en tweede ervaring, is tgesochte noortpunt: Ende soo veel de naelde aldan daer af wijckt, dat de begeerde naeldwijcking, gelijk int eerste voorbeelt wat breeder van sulcx gheseyt is.

Deur tghene hier boven gheseyt is vande ervaring mette Son des daechs, mach dergelijcke verstaen worden ende gheschien met yder vaste sterre des nachts, die ghebruyckende al oft de Son waer: maer niet de Maen, eensdeels om heur rassche eyghen loop, ten anderen om tgroot * verscheensicht datse heeft * Parallaxim.

Merckt noch datmen voor den middach twee drie vier of meer ervaringhen mach doen: Als by gelijknis, deerste wessende de Son boven den sichteinder 10. tr. inde tweede 15. tr. inde derde 20. tr. ende doende dergelijcke drie ervaringen op sulcke hoogden na middach, soo bevintmen hoe deen met dander overcomt, ende als men alsins een selve noortpunt crijcht, tgheeft den Stierman meerder betrouwen op sijn werck.

Seylende een Stierman van ooft na west of van west na ooft, t'can ghebeuren dat hy op den tijt van 10. of 12. uijren tusschen deerste ervaring en de laetste, een trap of meer verandering der naeldwijcking crijge, waer uyt wijder volghen can, dattet noortpunt gevonden deur deerste voormiddachsche ervaring, en de laetste namiddachsche, niet overcommen en sal met t' noortpunt gevonden deur de laetste voormiddachsche

needle again points at the zero point of the graduation. This being so, the mid-point of the arc in the horizontal plane between the first and the second observation is the desired true north. And as much as the needle there declines, that is the desired variation, as has been described somewhat more fully in the first example.

From what has been said above about the observation of the Sun in the daytime the same may be understood and done with any of the fixed stars at night, which may be used as if it were the Sun; but the Moon should not be used for this, on the one hand because of the rapidity of its proper motion, on the other hand because of the large parallax it has, because it is so near to the earth.

It is further to be noted that two, three, four or more observations may be made before noon. Thus, for instance, the first when the Sun is 10° above the horizon, in the second 15° , in the third 20° . And if three similar observations are made at the same altitudes after noon, they are found to correspond one with the other, and if the same true north is always obtained, this gives the navigator greater confidence in his work.

When a navigator sails from east to west or from west to east, it may happen that in the interval of 10 or 12 hours between the first observation and the last there is a difference of one or more degrees in the variation, from which it may follow further that the true north found from the first forenoon observation and the last afternoon observation will not agree with the true north

sche ervaring, en deerste namiddachsche, sonder nochtans dat den Stierman int werck gheseylt heeft. Dit hem soo ontmoetende, hy can daer uijt ramen hoe veel op seker uijren varens de naeldwijsing verandert, ende daer op gissing maken, om trechte noortpunt en naeldwijsing met noch meerder sekerheyt te hebben. Tselve canmen oock weten deur de naeldwijsing ghevonden op voorgaende daghen, ende die verleken mette wijsing des teghenwoordighen dachs.

B Y V O V G H.

GH E M E R C K T de ghegeven naeldwijsing en breede tsamen een seker punt anwijzen, soo wel op zee als te lande: Soo volght daer uijt meughelijk te sijn, dat schepen op een bestemde plaets in zee, verre van landt malcander vinden connen. Twelck oirboir is onder anderen, om na storm de schepen van een vlote weerom by een te gheraken. Men can daer deur oock setten een * saemplaets, om aldaer schepen van verscheyden oirten, op een bestemde tijt te doen vergaren.

* Rende-
vouw.

F I N I S.

De feylen verbeterd aldus.

Inde 11. side inde cant, voor Canton, lees Cantan. Inde 24. side inde cant, voor Azimuthalium seu verticulum, lees Azimuthalem seu verticalen. Inde 24. side, voor sichteinderplat, lees overal sichteinderplat, ende voor sichteinderfront, lees sichteinderfront. Inde 26. side inde sifste reghel, voor syvaerste sijde veel afvilen, lees syvaerste sijde soo veel afvilen.

found from the last forenoon observation and the first afternoon observation, without the navigator having made an error in the work. When he finds this, he may estimate from it how much the variation differs in a given number of hours' sailing, and from this may make a conjecture to have the true north and the variation with even greater accuracy. One can also know this when the variations found on preceding days are noted down and compared with the variation of the day in question.

APPENDIX.

Since the given variation and latitude in combination indicate a definite point, both at sea and on the land, it follows from this that it is possible for ships to find each other at a given point at sea, far from the land. This is useful, among other things, to help the ships of a fleet to reassemble after a storm. By this means it is also possible to fix a rendez-vous where ships coming from different directions may meet at a predetermined time.

FINIS.

VAN DE ZEYLSTREKEN

THE SAILINGS

From the *Wisconstighe Gedachtenissen* (Work XI, i, 24)

INTRODUCTION

§ 1

THE CONTENTS OF THE TREATISE DEVOTED TO THE "ZEYLSTREKEN"

The treatise about the *Zeylstreken*. (The Sailings) is devoted to two special tracks, entirely different in character, along which a ship can move over the earth's surface, *viz.* great circle and loxodrome. It is succinctly and lucidly written. As in *The Haven-Finding Art*, the words express Stevin's meaning in a perfectly clear way. That is why this introduction is no more than a short explanatory commentary on the original text here presented. The reader is assumed to be familiar with nautical terminology. Nevertheless it is necessary to make a few preliminary remarks, in order to point out certain differences in character between *The Haven-Finding Art* and *The Sailings*.

The Haven-Finding Art was intended for sailors, and the author hoped that the practice of navigation might at once reap benefit from the method discovered by him, for which reason it was published in a separate booklet; it included an urgent entreaty — backed by Prince Maurice — to test its value at sea. The publication of translations of the work testify to Stevin's desire that seamen of other nationalities might also profit by it. The treatise on *The Sailings*, on the other hand, is a theoretical discussion of a subject belonging to the practice of navigation indeed, but not one which formed a daily concern of seamen sailing in European waters. In this work Stevin deals with problems which had so far been studied by just a few pioneers of nautical science abroad, a study which had been induced by the fact that seamen who had made ocean-voyages had been confronted with these problems in practice and had been unable to solve them. This treatise drew the attention of the Dutch to a subject which was of fairly recent date. In 1534 Nunes had taken it up, and Mercator and Edward Wright had continued the work. With the aid of their publications — not forgetting those of Apian — Stevin had studied the subject, upon which he continued the work of his predecessors, making use of their writings and calculations.

Here another point of difference between the two treatises becomes apparent. It was possible to speak of a personal conception and an original work of Stevin in the case of the description of terrestrial magnetism given in *The Haven-Finding Art* and the profitable use that could be made of the declination of the magnetic needle. *The Sailings* on the contrary is no original work, as the labour of the pioneers forms the starting-point and the backbone of this treatise. The author, however, managed to produce a systematic, well-arranged and complete summary, of the subject, cast in a clever and instructive form. It is the presentation of the matter that we are entitled to call Stevin's own conception.

Another difference between the two works consists in the language in which Stevin addresses the reader. That of *The Haven-Finding Art* was simple, and the seaman of 1600 could easily follow the argument if he wished to. For *The Sailings* this holds only in so far as the application of spherical geometry and the

solution of problems by means of measurements on the globe are concerned. But the subject-matter was beyond the mental range of the contemporary seaman as soon as Stevin began to deal with spherical trigonometry or to solve problems mathematically. In those cases it was only the mathematically trained reader who could follow him. This is the reason why *The Sailings* was not of direct use to navigation at the time of its appearance. It was not until much later and very gradually that this knowledge reached the seaman through the textbooks of navigation and that Stevin's work became of use to practice at sea.

As the "Summary of the Sailings" states, this treatise forms part of Stevin's *Hydrography*. It comprises four definitions, followed by eleven propositions, of which two relate to the great-circle track and nine to the loxodrome. At the end there is an "Appendix" on loxodromes.

The first definition says that a "zeylstreeck" is the line which ships describe when they are sailing, *i.e.* the line a ship follows. The name is identical with the term "sailing track", which is now used to indicate the extended fore-and-aft line. In the special case of steering due east or west Stevin speaks of "*oost en west streeck*" (east and west track). Courses pertaining to other directions bear different names.

The second definition concerns the great-circle track, called "*rechte streeck*" (straight track) by Stevin and defined as the shortest distance between two points on the globe. To those who wonder that arcs are called "straight tracks", he says that one may speak of straightness because these lines do not deviate either to the right or to the left. In present-day terminology he would have said that the great circle on the sphere corresponds to a straight line in a plane, in contrast to the "*cromme streken*" (curved tracks), which are defined in the third definition.

Stevin explicitly excludes the equator and the meridians from the "*cromstreken*". Why he does so, will be explained presently. He defines the "*cromstreeck*" as the line described by a ship steering a constant course. This line is now called a loxodrome, and this term will be used henceforth. Stevin compares the loxodrome and the great circle. A ship when moving along the former follows a constant course, when sailing along the latter a variable course. Nowadays we say that the loxodrome is a line on the earth's surface which cuts all the meridians at a constant angle. This line does not lie in a plane and accordingly is a curve of double curvature. Loxodromes pass round the earth, through higher and higher latitudes; they never reach the pole, for then they would have to run towards the north, which is contrary to the definition. Stevin speaks of "*slangstreken*" and in the margin of "*spirales*". Although the equator and the meridians are curves on which the course is constant, so that by the definition they are loxodromes, yet they are great circles and do lie in a plane, which is the reason why Stevin does not include them among the "*cromstreken*". He is addressing the navigator directly when he gives the advice to become thoroughly familiar with the character of these curves and in cases of uncertainty in the position not to attribute errors too readily to the influence of unknown ocean-currents.

In the fourth definition the loxodromes N. by E., N.N.E., etc., are denominated 1 to 8. The last-mentioned one is the course east, falling along the parallel. In the four quadrants the loxodromes have the same form four and four, such as N. by E., N. by W., S. by E., and S. by W., etc. Whatever holds for one out of a group, applies equally to the corresponding loxodrome in another group.

The definitions are followed by the propositions. In the first of these, two points in a given latitude and longitude are assumed on the earth's surface. With the pole these points form a spherical triangle. Of this triangle the six elements are mentioned, while it is stated that if three of them are known, the other three can be found. As examples Stevin takes the determination of: 1) the distance along the great circle between those two points, and 2) the angles between the great-circle and the meridians through the two places, in other words: the course of the ship in the place of departure and that when she has reached her destination. The problem is solved with the aid of spherical trigonometry, for which Stevin refers to the 40th proposition on spherical triangles occurring in his *Trigonometry*.

The second proposition relates to great-circle sailing. It is taught in two different ways how the courses are determined which have to be steered to follow the great circle. The one method is "*tuychwerckelick*" (mechanical), *i.e.* by means of an instrument, in this case the globe. The other is mathematical. According to the first-mentioned method the place of departure is sought on the globe, after which this place is brought in the zenith by rotation of the globe in its stand. The desired course of departure is read on the horizon between the meridian through the place of departure and a pivoting vertical circle set over the place of destination. Use is thus made of a property of the poles of great-circles, *viz.* that the angle between two great-circles is measured by an arc of a great-circle, a pole of which is the point of intersection of these great-circles. As is evident from this, the reader is required to have some knowledge of spherical geometry. The course found is then followed a certain distance — Stevin speaks of 3 or 4 "*trappen*" (degrees), *i.e.* 180 to 240 nautical miles — upon which in the position thus reached the course is again determined in the same way. Stevin points to the change in the course which comes to light if one proceeds in this way, and to the fact that the displacements of the ship each time take place along a loxodrome. By taking the displacements small, one avoids inaccuracy. It can be checked whether the ship is still on the great-circles, as she should be, by a determination of the latitude from observation of the sun or the stars.

For the mathematical solution of the problem the reader is again referred to the treatise on spherical triangles. If the spherical triangle in question is oblique-angled, the 40th proposition mentioned above can again be applied, but the object can also be attained by making use of right-angled spherical triangles, which is less difficult. Each time, after a given displacement of the ship, the course is determined again. If the change in the course is found to be small, the displacements can be taken larger.

In the third proposition Stevin proceeds to deal with the loxodrome. First he discusses the drawing of it on the globe. As one of the aids with which this can be done, a simple instrument is mentioned, a copper model of the loxodrome fitting on the globe, the idea of which — as Stevin states — had been borrowed from the globe-makers. It is clearly described how such models are made for each track, *i.e.* seven in all, and also how the loxodromes are drawn on the globe, from degree to degree of difference of longitude, with the aid of the models. It is mentioned that in theory the loxodrome cannot reach the pole, though in drawing it seems to do so. But then, drawing does not furnish an accurate result.

With the aid of mathematically calculated tables the determination of the shapes of the loxodromes can take place more accurately than by the method described

above. To make it possible to construct a complete table of loxodromes, the latitude has to be determined of the points of intersection of seven loxodromes with the meridians, at differences of longitude of one degree.

Stevin describes two methods by which the object can be attained.

The first of these methods is the result of calculation. In small right-angled spherical triangles, the base of which is one degree of the equator or one degree of a parallel, the hypotenuse is a loxodrome, and the perpendicular side is a part of the meridian, the length of this part of the meridian is calculated each time. For this calculation Stevin again refers to his *Trigonometry*, viz. to the 36th proposition concerning spherical triangles. (The reader will be well aware of the fact that the above mentioned triangles are not spherical, as two of the sides are not great-circles.) The result of the calculations are added together. The bases of these triangles are required to be known. This is the case, for Stevin has at his disposal a table in which the length of one degree of the parallel in a given latitude is expressed in minutes of the equator. It is the table of the "*achtste cromstreeck*" (eighth loxodrome), occurring on pp. 138/9, after the "*Tafels der cromstreken*" (Tables of Loxodromes). It corresponds to the present-day table for the reduction of the difference of longitude to the departure ($\text{dep.} = \Delta L \cdot \cos b$), with the only difference that in Stevin's table the interval is 30', whilst in the modern table the interval is 1° and decreases to 10' as the latitude increases.

When at the end of the fourth proposition Stevin explains the table of the eighth loxodrome, he says he has taken it without any modifications from the *Cosmographia*¹⁾ of Peter Apian (Petrus Apianus, 1495—1552)²⁾. The table in question is already to be found in the first edition — of 1524 — of this author's widely distributed and well-known book (Book I, pp. 42-43). Apart from a few differences, to be ascribed to printer's errors on both sides, it appears to have been copied faithfully in its entirety from Apian, including the numbers indicating the decrease of the length of the degree of the parallel, from 30' to 30' difference of latitude, expressed in seconds. It is most likely that Stevin used a late sixteenth-century edition of the book for his purpose. The edition of 1524 is merely mentioned here to show that the table had existed long before this.

After thus having clearly explained how one is to proceed, Stevin says that it would have taken him too much time to construct in this way a complete table of loxodromes by calculation. He therefore merely pointed the way, without following it himself. Continuing his train of thought in the same direction, he might have pointed out that the numbers mentioned opened up the possibility of interpolation for the latitude, and that interpolation was necessary for the performance of the calculation of the table outlined by him, if accuracy was to be attained. But he omitted to do so.

On the other hand he did take over the *Table of Rumbes* of the English mathematician and nautical expert Edward Wright (1558-1615). His candid

¹⁾ *Cosmographicus liber Petri Apiani mathematici studiose collectus* (Landshut 1524).

This book was particularly widely distributed. It was translated into many languages and passed through a great many reprints in the sixteenth and the early seventeenth century.

²⁾ Peter Apian, whose real name was Benewitz or Bienewitz, born at Leisnig in Saxony, geographer and astronomer, from 1527 Professor of Mathematics at Ingolstadt. Maker of maps and instruments, and famous as an observer of astronomical phenomena.

admission of this opens his explanation of the second method by which the table could be made. His source is Wright's *Certain Errors in Navigation*³⁾, a book which had gained fame in England and had first appeared in London in 1599, the very year in which Wright had translated Stevin's *The Haven-Finding Art*, thus making it known in his country.

Stevin took over the table without any modifications, notwithstanding the fact that he had found "some imperfection" in it. He says he will recur to his objections in the "Appendix". The explanation of its arrangement is preceded, by way of introduction, by the explanation of another table, viz. that of the "*versaemde snijlijnen*" (assembled secants), also copied by him from Wright. In the latter's work this table was called "Table for the true dividing of the meridians in the sea-chart". Stevin shows that it is produced by the constant addition of the secants of angles increasing by 10', as follows:

sec 10' = 10,000,042
 sec 20' = 10,000,168; the sum is 20,000,210
 sec 30' = 10,000,381; the sum is 30,000,591

After the performance of the additions he drops — without stating his reason for it — the last five digits, so that the numbers in the table correspond to a radius $R = 100$. This explanation is followed by the table, which thus gives the sum of the secants of angles from $0^{\circ}10'$ to $89^{\circ}50'$, the interval being 10'.

Stevin says nothing about the meaning of these numbers. For the benefit of the reader it is here observed that in the Mercator chart the length of one minute of the meridian in a given latitude is equal to one minute of the equator multiplied by the secant of that latitude. That is why in that chart the distance from a given parallel to the equator is equal to one minute of the equator, multiplied by the sum of the secants of angles from 0° to that latitude, taken minute by minute. It is this sum of the secants — though at an interval of 10' — which the table furnishes. It is thus essentially identical with our present-day table of meridional parts⁴⁾. The latter is no longer calculated by addition of the secants, but from

the relation $\int \sec \varphi \, d\varphi = \text{Intg} \left(45^{\circ} + \frac{\varphi}{2} \right)$. If we compare the table published by Stevin with the modern one, the values to be found in the latter have to be multiplied by 10. If we round off the numbers to integers and disregard differences due to rounding off, etc., the tables are seen to agree up to latitude 38° . Beyond that, Stevin's — or rather: Wright's — values become greater. At 60° the difference is 0.3 minute of the equator and it increases to 2.8 minutes of the equator at 82° , where the modern table ends because the practice of navigation requires no data for higher latitudes. The table in Wright/Stevin goes as far as $89^{\circ}50'$, against which the number 226,223 is mentioned.

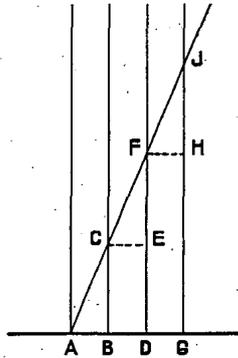
Next comes the promised explanation of the second method by which the table of the loxodromes can be constructed. The original text (p. 104, which refers to the figure on p. 95) can easily be followed. As an instance, Stevin takes the

³⁾ *Certain Errors in Navigation, arising either of the ordinarie erroneous making or using of the sea chart, compasse, crosse staffe and tables of declination of the sunne and fixed starres, detected and corrected by Edward Wright* (London 1599). Copies in the British Museum, London, and in the Bodleian Library, Oxford; not in the National Maritime Museum, Greenwich.

⁴⁾ The present-day definition is as follows: meridional parts for any latitude is the length of a meridian, expanded on a Mercator chart, between the equator and that parallel of latitude expressed in minutes of arc of the equator.

determination of the latitudes of the points of intersection of the first loxodrome with the meridians at differences of longitude of 1° , commencing at the equator. In the right-angled triangle, of which the base is an arc of one degree of the equator and the hypotenuse is the first loxodrome (N. by E.) — a triangle which on account of its smallness is regarded as a plane triangle — the latitude of the point of intersection of the loxodrome with the meridian at 1° of longitude is calculated with the aid of the 4th proposition of plane trigonometry. It "is found" that this latitude is $5^\circ 1'$, and this value is included in the table. In order to find the latitudes of the subsequent points of intersection, Stevin now proceeds as follows. He takes the table of "assembled secants", seeks the meridional parts of $5^\circ 1'$, and finds 3,014. This number is multiplied by two. Against 6,028 he finds $10^\circ 0'$, which is the required latitude for a difference of longitude of 2° . Against $3 \times 3,014$ he finds the latitude for the third point of intersection. We shall recur presently to the calculation of the first point of intersection.

The instructions given by Stevin are indeed perfectly clear, but he does not go into the essence of the matter, nor does he explain why the multiples in question have to be determined. However, this operation becomes clear if we transfer it to the Mercator chart.



In the above figure, which represents a section of a Mercator chart, AG is a part of the equator. The points A , B , D , and G are points of intersection with meridians, at intervals of one degree of longitude. The line AJ is the first loxodrome. The latitude of the point of intersection C was calculated mathematically by Stevin. The table of "assembled secants" showed him how many minutes of the equator the distance BC amounts to. Now $\triangle CEF$ is congruent with $\triangle ABC$. Consequently EF is as many minutes of the equator as BC . The distance DF in minutes of the equator is thus twice the meridional parts of BC . In the table it is looked up what is the latitude corresponding to this number. Thus the latitude of F has been determined. The number is included in the table. The congruence applies to all subsequent triangles. In each case, therefore, to find the latitude of a point of intersection, the meridional parts of BC have to be added to those of the preceding point of intersection; in other words, the meridional parts of each of the points of intersection form a multiple of the meridional parts of BC . The use of the table of "assembled secants" was a strikingly ingenious idea. Thanks to this, it was possible to reduce the extremely laborious calculation of the shapes of the seven loxodromes to the simple performance of additions and the search of numbers in that table. Although Stevin concludes his argument rather

laconically with the words "and so on with the rest of the seven loxodromes", he cannot have failed to realize that the man who originally calculated the table of loxodromes performed an enormous amount of work and made an important contribution to the development of the art of navigation. In Stevin's book the table runs into thirty pages, on each of which the latitudes of 120 points of intersection are given. Stevin was convinced that it would advance both science and cartography. In fact, he emphatically states that it has now become possible to draw the loxodromes with great accuracy on globes and to check their shapes in sea-charts (p. 140); from these words we may infer Stevin's opinion about Wright's work, *viz.* that he had achieved his object. Indeed, the latter had written (page F-2): "the special use of this table is for the true drawing of the rumbes in the globe and the chart". Wright had gone no further than the calculation of the points of intersection in question. It was Stevin who made the next move, thus bringing the matter into the nautical sphere, where science is applied in practice.

In his discussion Stevin includes the distance run by the ship. He explains how with the aid of trigonometry the distance along the loxodrome from one point of intersection to the next can be calculated. He performs this calculation only for a very few cases, *viz.* those which he requires in instances to be dealt with later on. The fact that in his opinion Wright's table was still insufficiently accurate — this is the second reference to such inaccuracy — deterred him from completing this calculation. Moreover, he states that he was hindered from doing so by other matters, just as he did before. Thus unfortunately this part of the work remained incomplete from the nautical point of view. Stevin merely pointed the way. Those having a mind and an opportunity for it might complete the table in this respect with the aid of the example.

Next the construction of the table of the eighth loxodrome is explained. In our modern, simple notation it is based on the formula: one degree of the parallel in latitude b is equal to one degree of the equator multiplied by the cosine of b (1° par. in latitude $b = 1^\circ$ eq. $\times \cos b$).

At the end of the fourth proposition Stevin refers again to the "mechanical" method. It may be convenient to have at one's disposal a number of models of loxodromes, made of copper. They could be shaped to correspond to the loxodromes drawn on a globe. Twice seven such models in all were required.

The reader may have wondered why Stevin reckoned with seven loxodromes, *i.e.* with a division of the quadrant into eight parts — corresponding to the division of the compass rose — instead of with a difference in the course of less than one point or $11^\circ 15'$. The answer lies in the field of the practice of sailing. The ships of the time, being small in size, were difficult to steer. They used to yaw violently and were very unsteady and restless at sea. To be able to steer them with an accuracy to within one point was a reasonable result. A smaller subdivision was senseless. Even small modern sailing craft and yachts, which sail much better and are more manageable than the unwieldy ships of Stevin's day, will do hardly better than this. In consequence the navigator will usually steer on full points and seldom on half points. It is obvious that if the ship is steered on full points, the true course deduced from the compass course need not fall on a full point.

We are now coming back to the passage in which Stevin states that for the point of intersection of the first loxodrome with the meridian at 1° of longitude he "has found" a latitude of $5^\circ 1'$, in which latitude he found 3,014 for the

meridional parts; he continued his explanation with this value. Stated in this way, his words produce the impression that he calculated this value for himself. This, however, is to be doubted, as may be demonstrated by the following.

What were the directions given by Wright in this respect? The following passage is to be found in his book (page F *verso*), where he is referring to his "Table of rumbes", the "Table for the true dividing of the meridians in the sea chart", and to the construction of the former of these tables:

"This table of rumbes is most easily made by addition only with helpe of the table before mentioned shewing how the meridians or degrees of latitude in the nauticall planisphere are to be divided, after this manner. Multiplie the tangens of the angle that the rumb maketh with the equinoctial by 60, the product shall be the first number at the beginning of each table of each rumb, to bee set over against one degree of longitude⁵⁾, and all the rest are found by perpetuall addition of this number, first to it selfe — for the summe is the number answerable to two degrees of longitude — then to this summe, and so forth in all the rest. These numbers being found out in the table before mentioned did shew at what minute of latitude each rumb should crosse the meridian for every degree of longitude . . . which being once found, these numbers serve to no further use."

As was the custom in those days, these directions omit to account for the rules given. For the present-day reader this forms no difficulty. He will understand the words and consider their import correct. In fact, the latitude of the first point of intersection is calculated in the Mercator chart — *i.e.* in a flat surface — and is expressed in minutes of the equator. It is of this product that the multiples are taken, upon which the latitudes corresponding to "these numbers" are sought in the Table of meridional parts; and these are the values which have to be included in the table. Still, the rule cannot be said to be formulated very clearly, and it certainly has to be called obscure for the reader of 1600, who was being confronted with a new subject. This reader was bound to expect that the operation would take place in a spherical triangle, and he therefore ought to have been told that it was carried out in a plane triangle. The words "the product shall be the first number . . ." were likely to mislead him. It is true that the last sentence of Wright's argument, which is correct, could cure him of his error; but it is justifiable to speak of a lack of clearness in the text, since for the reader it was hard to realize that "these numbers" also refers to "the first number".

When we now read the corresponding passage in Stevin on p. 535, which refers to the figure on p. 522, we find that Stevin takes as starting-point the spherical triangle XRQ , which he treats as a plane triangle, "on account of the smallness of the sides". The side QX , or the latitude of the first point of intersection, can be found with the aid of the fourth proposition of plane triangles, to the effect that the side subtending the known acute angle — in this case a part of the meridian — is equal to the base multiplied by the tangent of this known acute angle.

It appears that Stevin has not fully grasped Wright's argument. Possibly he was misled by the insufficient clearness of the text. It was a mistake for him to speak of a spherical triangle. He failed to notice that he was operating with a plane

⁵⁾ The meaning is: place this number in the table, for the latitude of the first point of intersection, against 1° of difference of longitude.

triangle. Wright's calculation yields the meridional parts of the latitude of the first point of intersection, at the same time the basis for the further operations. If Stevin had understood the directions, he would have had to cipher as follows:

$$\begin{aligned} \text{meridional parts } b &= 60 \times \text{tg } 78^{\circ}45' = 60 \times 5.02734 = \\ &301.64 \text{ min. of the equator} = \\ &3,016.4 \text{ units of the table.} \end{aligned}$$

His own directions in the form of a formula are:

- a) measured in degrees: latitude $b = 1^{\circ} \text{ eq.} \times \text{tg } 78^{\circ}45' =$
 $1^{\circ} \text{ eq.} \times 5.02734 = 5^{\circ}.02734 =$
 $5^{\circ}1'38''.$
- b) measured in units of the table: latitude $b = 60 \times \text{tg } 78^{\circ}45' = 3,016.4.$

We have seen that he made no use of the answer 3,016, nor of $5^{\circ}1'38''$, although it is certain that he knew the latter value, because we meet with it in his criticism of Wright, on p. 581 of the Appendix. The latitude of $5^{\circ}1'$, "found" by him, virtually springs from nowhere, and it has to be assumed that he took it from Wright's table (p. 537 of our edition), and looked up the corresponding number: 3,014.

Stevin's text in some places is incoherent. He understood the operation imperfectly and made the mistake of regarding as latitude the value which really stood for the meridional parts of it. In consequence his directions were incorrect. When we consider this mistake in connection with the imperfections in Wright's text, it becomes evident how difficult it was to attain to the right understanding, although in our eyes it is quite simple. And if Stevin can be said not to have clearly realized the possibilities of the Mercator chart fifty years after its appearance, it is all the less surprising that more than a century had to elapse before this chart with its queer distortion was generally accepted by sailors.

The next seven propositions concern problems which are now called sailing problems. They form the application of the previously discussed theory and an exercise for the student. In each case, a number of elements being known, certain unknowns have to be determined. For all of them Stevin gives three methods of solution, *viz.* one with the aid of the copper models of the loxodromes, one by means of the loxodromes drawn on the globe, and finally the arithmetical method making use of the tables. The text does not call for any further comment. The last-mentioned method was not yet suitable for practical application, since the table was incomplete owing to the absence of one element: the distance. Because Stevin's comprehensive and scientific book in two bulky volumes, of which the treatise on the sailings formed only a modest part, decidedly did not come under the eyes of the practical seaman, the latter derived no benefit from it. No textbook of navigation that was destined for this purpose and contained this information was yet available at that time.

It is only with regard to the problem discussed in the fifth proposition that two remarks have to be made. Here it is required to determine the distance between two places situated in latitude 24° , their difference of longitude being 30° . It is correctly calculated that the distance measured along the parallel — we should now say: the departure — must be $27^{\circ}24'$. But since the circumference of the earth was still unknown, $27^{\circ}24'$ could not be expressed in a linear measure. In this connection Stevin speaks of a variety of miles which are in use in different countries. Miles of different length give different results. In this respect Stevin could not do otherwise than follow the opinion of many people, *viz.* that a degree

was equivalent to 18 hours' walk, at 8,000 paces an hour. An hour's walk also used to be equated to 1,500 Rhineland roods, which makes a pace equivalent to $2\frac{1}{4}$ Rhinelandfeet. The author advises navigators — in the apparent belief that he will be read by them after all — to keep to the result expressed in degrees and minutes, so as to understand one another properly.

In the second place it has to be pointed out that at the end of this proposition (p. 553) Stevin mentions the possibility of the application of interpolation in using the table of the eighth loxodrome.

As announced by Stevin at the beginning, the treatise ends with an "Appendix". This consists of five chapters, in each of which a particular point is dealt with. In the main this amounts to a criticism of the works of Nunes and Wright, the very authors who had formed his sources and the contents of whose works constitute the gist of his treatise. In the Appendix he collects his remarks and objections.

In the first chapter Stevin points out that the method of numbering the loxodromes is not always the same. Thus Hues⁶⁾ reckons from the meridian, calling N. by E. the first loxodrome, whereas Wright commences at the equator, referring to E. by N. as the first loxodrome. Stevin advocates uniformity in this respect and explains why it is preferable to reckon the loxodromes from the meridian rather than from the equator. To him the first way is the natural order, with the "east and west track" or "eighth loxodrome" also classed with the loxodromes, as it ought to be. During the further development of the subject this system was found to be sound and efficient. Wright abandoned his own system.

In the second chapter of the Appendix a remark is made about Nunes (1492—1577) and his treatise on the shapes and the properties of the loxodromes. Stevin disproves a proposition given by Nunes. We shall revert to this criticism in § 2.

As appears from the title, the third chapter treats of inaccuracies in Wright's table of loxodromes. The English, who began to engage in deep-sea navigation after the Portuguese and the Spaniards, also proceeded to study the loxodromes and discovered the mistake made by Nunes. Then Wright's tables were published, which in Stevin's opinion marked a big stride forward. To check them, he made a random test. He figured out the shape of the fourth loxodrome — the fourth involved the smallest amount of figure-work for him — by the method based on spherical trigonometry, described by him in the fourth proposition. He found the point of intersection of this loxodrome with the meridian of 78° to be in latitude $61^\circ 26'$, whilst in Wright's table he found $61^\circ 14'$, a difference of "only 12''". He had reason to think that his result was slightly too high, so that he could "assume" — as he says — that the tables in question were "rather accurate".

As to this difference of 12', or slightly less, it may be observed that the correct value is $61^\circ 14' 52''$, so that the value in Wright's table appears to be

⁶⁾ Robert Hues (1553-1632) accompanied Thomas Cavendish on his voyage round the world (1586-1588). He was a mathematician and a geographer. Like Harriot and some others, he was patronized by the Earl of Northumberland; he is known in particular for his book *Tractatus de globis et eorum usu* (London 1593), of which an edition appeared at Leiden in 1594.

sufficiently accurate. The difference had arisen because the intervals taken by him in the calculation — *i.e.* 30', in the table of the eighth loxodrome — were too great. Stevin's procedure consequently was unsound, and so in our opinion was his criticism.

Again Stevin says he had no time to pursue his verification any further. He cannot yet call the table quite perfect, and to justify this statement he derives a proposition with which he had found the numbers not to be in agreement. The operation, which is given on pp. 577 - 583, can be described more simply as follows (See the figure on p. 576 of our edition).

A loxodrome EI — the first is meant — commencing at the equator, cuts some meridians at differences of longitude of one degree in the points K , L , and M . Through these points are drawn parallels, which cut the meridians in N , P , and Q . NPQ is again a first loxodrome. K lies in latitude b_1 , L in latitude b_2 . The difference of latitude between K and P is called Δb_1 , that between L and Q , Δb_2 . Because they have very "small sides", Stevin takes the triangles NKP and PLQ approximately as plane similar triangles and writes:

$$\begin{aligned}
 KP : LQ &= NK : PL, \text{ or} \\
 \Delta b_1 : \Delta b_2 &= 1^\circ \text{ eq. } \cos b_1 : 1^\circ \text{ eq. } \cos b_2 \\
 \frac{\Delta b_1}{\Delta b_2} &= \frac{\cos b_1}{\cos b_2} = \frac{\sec b_2}{\sec b_1}
 \end{aligned}$$

Now that this proportion is known and its correctness has been proved, he is going to perform the verification announced by him. He chooses the first loxodrome and will calculate the latitudes of the points of intersection with the 1st and 2nd meridian, *i.e.* FK and GL . He does so by two different methods, which are here described as methods A and B.

(A) Stevin takes the spherical triangle KFE , which to begin with he regards as a plane triangle. Next, by means of his method explained in the fourth proposition (p. 535), he calculates the latitude of the first point of intersection, finding $5^\circ 1' 38''$, a value already referred to. In accordance with the above proposition NO becomes $5^\circ 0' 28''$, and thus GL becomes the sum of these values, or $10^\circ 2' 6''$, which latitude he says he also finds when ciphering according to the first procedure of the 4th proposition, *viz.* the stepwise calculation. Incidentally it may be remarked that it involved inaccuracy because Stevin was not yet able to make sufficient allowance for the gradual decrease of the cosine of the latitude.

(B) Subsequently determining GL by making use of the "table of assembled secants", he finds a different value, in the following way. The meridional parts of $5^\circ 1' 38''$ amount to 3,020; this number, multiplied by 2, gives 6,040, which stands for the meridional parts of $10^\circ 1'$, a value which is thus found to be $1' 16''$ less than the value obtained by method A. He thus thinks he has detected an inaccuracy, and states that the difference in question will become greater as the operation proceeds.

If he regarded the triangle KFE as a spherical instead of a plane triangle, and then calculated KF , he found $5^\circ 0' 51''$ 7), or $47''$ less than the first result, which was at least slightly better. This difference again, though small at first, increases

7) see: next page.

as the calculation proceeds. Stevin maintains his criticism.

This criticism, however, was not justifiable. Again we find him making the mistake pointed out above: he took for latitude what were meridional parts. In fact, he should have used 3,016 as meridional parts; in that case he would have found the corresponding latitude to be $5^{\circ}1'.2$. This value of 3.016, multiplied by 2, makes 6,032, the corresponding latitude being $10^{\circ}0'.1$. Wright gives $5^{\circ}1'$ and $10^{\circ}0'$ respectively in his table, so that he was right.

In the fourth chapter Stevin states that the proportion referred to above may serve to construct a reliable table. The interval of longitude is discussed. If the interval of 1° is found too great, the calculation may be performed with $15'$ or even $10'$. But it is doubtful whether the gain in accuracy justifies the additional work. The table of the eighth loxodrome too would have to be calculated for an interval of $1'$. Stevin actually seems to have considered this desirability. If he had taken this work in hand himself, some passages of his treatise would have been different and he would not have reached his present conclusions.

The fifth and last chapter deals with a proposal for an improvement of the mariner's compass. This is a purely technical matter, while all the other subjects formed part of the theory of navigation. Some people wanted to replace the usual division of the rose into 32 points by one into 64 parts, although others thought that such small angular differences in the division of the rose could not be read at sea. But Prince Maurice, after having reflected about the matter of accurate steering, had proposed a division into degrees, even into fractions of degrees, provided the compass were large enough in size and were made with great care. The Prince was not aware — nor could he have been — what consequences the use of a large, and consequently heavy, needle entailed: increased friction between the pivot and the cap on which the rose is supported, resulting in unsteadiness of the rose. In the first example a compass is described the needle of which can be moved in relation to the rose, for the adjustment of the variation. A description is also given of the way in which the compass can be mounted accurately on board.

In the second place it is proposed not to make use of a compass-rose to which, as usual, a double magnet in the form of a rhomb had been attached, but to omit the rose and to equip the compass with one magnetic needle. The round bowl was then provided with a division into degrees on the inside. In such a compass the words East and West have to change places. The special difficulties which this apparatus involved in practice with regard to the indication and the reading of courses is clearly described in the text. On such a compass it was also possible to steer true courses. The compass bowl then had to be mounted pivotally on a small pin, so that the lubber line might be made to diverge as many degrees from the longitudinal axis of the ship as the variation amounted to.

⁷⁾ This value is correct and is found as the result of one of the formulas for the right-angled spherical triangle: $\operatorname{tg} b = \operatorname{tg} E \sin \Delta L$ or $\operatorname{tg} b = \operatorname{tg} 78^{\circ}45' \sin 1^{\circ}$.

§ 2

STEVIN'S PREDECESSORS:
APIAN, NUNES, MERCATOR, AND WRIGHT

In response to questions which had come up from the practice of navigation, Pedro Nunes (1492—1577)⁸⁾, the learned Portuguese astronomer, mathematician, and nautical expert, in 1534 proceeded to study the line on the earth's surface which is described by ships steering a constant course, *i.e.* the line cutting the meridians at a constant angle. By drawing such lines on a globe he succeeded in appreciating their shape. He found them to be complicated in character. A few years later, in his *Tratado em defensam da carta de marear* of 1537, he gave a definition and description of these lines, which he called "linhas dos rumos"⁹⁾. On p. 143 he states: "The rhumb lines are not circles, but irregular curves, which will make equal angles with all the meridians which they cut"¹⁰⁾. The author in this treatise brings out the difference between sailing on this line and on a great-circle. At first Nunes thought that these lines met at the pole, but later he retracted this statement. In 1566 he published a Latin version of his treatise about the sea-chart, in a more detailed and elaborated form, as part of a longer work, which appeared under the title *Opera*¹¹⁾. In this it is described — with figures to illustrate the matter — how the loxodromes are drawn on the globe by mechanical means. The aid which Nunes made use of was a flexible model (*quadrante esférico flexível*) for each of the eight "rumos", with which to determine the points of intersection with meridians, at equal differences of longitude. Although on a globe of a relatively small size this procedure was defective, so that the line drawn was liable to errors, yet it formed the starting-point for later knowledge about the loxodrome. A second Latin edition of the treatise on the sea-chart is included in the great work of Nunes: *De arte atque ratione navigandi* (Coimbra 1573). In Cap. 26, under the title *Propositum globum rumbis deliniare* (p. 116), the model is illustrated once more and its use is clearly described.

Nunes ascribed a particular property to the loxodrome, *viz.* that the sines of the polar distances of the points of intersection with meridians at equal differences of longitude form a continued proportion. It is this statement which Stevin refutes in the second chapter of the Appendix, making use of the proposition he had

⁸⁾ Nunes, from 1529 onwards cosmographer to King Emanuel of Portugal, Professor of mathematics at Coimbra University, founder of scientific navigation.

⁹⁾ This treatise, together with another treatise and the translation of the famous work of Sacrobosco, forms the *Tratado da Sphera* of Pedro Nunes, published in 1537 at Lisbon. This work was published in facsimile in 1915 by J. Bensaude.

¹⁰⁾ *Nam serem os rumos circulos, mas linhas curvas irregulares, que vam fazendo com todo los meridianos que passamos angulos iguaes.*

¹¹⁾ A. Fontoura da Costa in his *A marinharia dos descobrimentos* (Lisbon 1933) says (p. 423) that some authors assume an earlier edition of this work appeared in 1546 at Coimbra. Thus, already Rödning in his *Allgemeines Wörterbuch der Marine* (Hamburg 1794) mentions such an edition. But Luciano Pereira da Silva, who described the works of Nunes (*As Obras de Pedro Nunes. Sua cronologia bibliográfica*, Coimbra 1915), denies that such an edition ever existed.

derived in the third chapter. By calculation he shows the unsoundness of Nunes' belief. It is striking that he does so in a cautious and gentle way, and by no means in the form of a reproof.

Stevin's respect for the pioneer in this field is reflected in these words. At the same time it becomes evident that Stevin, when undertaking to study a new subject, goes back to the source. The existence of this source was previously known also to Robert Hues, who in his *Tractatus de globis coelesti et terrestri eorumque usu* (London 1594) in the chapter devoted to the drawing of loxodromes on the globe and to the use of these lines says: *Inventio haec & consideratio delineandi rumbos in globo aliquanto est antiquior. Petrus Nonius Lusitanus multa de his in duobus lib. quos de navigandi ratione conscripsit. Mercator etiam in suis Globis eas expressit* ¹²).

To Nunes is due the credit of having raised the art of finding one's way across the ocean — an art which had existed for many centuries before his day — to the rank of a science. Thanks to his work and studies, in Portugal a great height was attained, greater than anywhere else in the sixteenth century. At the same time his work concluded an era in the history of the art of navigation in Portugal. The work of the Portuguese was continued in Flanders, in the first place by Gerhard Mercator (1512—1594) ¹³).

Mercator is known to have made a terrestrial globe which carries the date of 1541 as the year in which it appeared ¹⁴). Many compass-roses divided into points appear on it. These directions have been extended so as to form a network of loxodromes. Those who studied and described the globe called this network accurate and admired Mercator's skill in drawing it.

An achievement of unprecedented importance for the art of navigation, owing to which Mercator's name will always be honourably mentioned in the history of navigation, was the compilation of his world-map destined for use at sea. It appeared in 1569. The network of meridians and parallels had been designed in such a way that the loxodromes appeared in it as straight lines. Meridians and parallels are straight lines as well. It is this chart which is known as the Mercator chart, sometimes less properly called the chart on Mercator's projection. It contains a great many legends in Latin. In one of them the requirements which the chart has to satisfy are enumerated. Mercator says: "It is for these reasons that we have progressively increased the degrees of latitude towards each pole in proportion to the lengthening of the parallels with reference to the equator. Thanks to this device we have obtained that . . . no trace will anywhere be found of any of those errors which necessarily be encountered on the ordinary charts of shipmasters, errors of all sorts, particularly in high latitudes" ¹⁵).

It is not known by what method Mercator constructed the network of his chart. He himself has not disclosed this in any posthumous work. The idea that he

¹²) "This invention and idea of drawing rhumb lines on a globe is rather old. Petrus Nonius from Portugal speaks repeatedly about it in the two books he has written on the subject of navigation. Mercator, too, has represented them on his globes."

¹³) Gerhard Mercator (or Kremer), a Flemish cartographer, nautical expert, maker of instruments, globes, and atlases at Louvain, later Duisburg.

¹⁴) A specimen of it is to be found in the National Maritime Museum, Greenwich.

¹⁵) The English translation of all the Latin legends on the chart is to be found in *The Hydrographic Review*, Vol. IX-2 (1932).

achieved it by mathematical means has to be rejected; probably it has to be assumed that he did it by a graphical method, *viz.* by transferring the points of intersection of loxodromes and meridians, constructed on the globe, to the system of meridians in the chart ¹⁶).

It is to be noted incidentally that although this chart was destined for use at sea, in spite of its useful indications it was not yet very suitable for this purpose. Indeed, the seaman has greater need of detailed charts of limited regions than of a chart comprising all the oceans of the world. Much time had to elapse and a good deal of opposition had to be overcome before the seaman realized the importance of the system on which the chart is based and used it regularly at sea. Claes Heyndericks Gietermaker (1621-1669), writer of the widely used Dutch textbook *'t Vergulden licht der zeevaart* (The Golden Light of Navigation) of 1660, put his finger on the sore spot by ascribing the seaman's failure to promptly adopt improved methods to "inveterate habit, and it is said there is nothing more powerful than habit". In the eighteenth century the octant, which yet made for much greater accuracy in the measurement of the altitude of celestial bodies, met with no better reception. Seamen are conservative, or at any rate they tend to cling to the old familiar ways and methods.

The important problem of determining the distances of the parallels from the equator, as measured in the Mercator chart, was transferred to the mathematical sphere by two English nautical experts of great fame, *viz.* Thomas Hariot (1560-1621; the name is sometimes spelled Harriot) and Edward Wright (1558-1615). In the legend from the Mercator chart quoted above, Mercator states that he had increased the degrees of latitude in the same proportion in which the parallels in his chart have been lengthened, *i.e.* in proportion to the secant of their latitude. These words form the basis of the calculations undertaken by Hariot and Wright.

Of the former a manuscript has been preserved which bears the title: *The Doctrine of Nautical Triangles Compendious* ¹⁷). It dates from before 1596, in all probability from 1594. The bulk of this work is formed by a *Canon nauticus*, being a "table of meridional parts", from minute to minute, calculated by constant addition of the secants from one minute to the next. Hariot had based these calculations on the table of Clavius (1537-1612) ¹⁸), dating from 1586, which contained the values of the sine, the tangent, and the secant for every minute and to seven places of decimals, as we should now say. Hariot's work was original, for he constructed his table before Wright, but no writings of Hariot have been published, nor was the manuscript in question printed. Thus the credit of having been the first to construct such a table is indeed due to

¹⁶) This is the conclusion reached by D. Gernez in a paper entitled "*Quel procédé Mercator employa pour tracer le canevas de sa carte de 1569 à l'usage des marins?*" published in the Communications of the Académie de Marine de Belgique, Tome I, 1936-37. In this paper the writer reviews the numerous hypotheses about this matter and gives his own opinion.

¹⁷) The manuscript was described by Prof. Eva G. R. Taylor in The Journal of the Institute of Navigation, Vol. VI, 1953, p. 131. For Hariot, reference may also be made to her book *The Mathematical Practitioners*, Cambridge 1954.

¹⁸) Clavius, whose real name was Christoph Schlüssel, a Jesuit and a teacher of mathematics in Rome as well as Vatican astronomer, was one of the most diligent commentators on Sacrobosco.

Hariot, but since his *Canon* was not published, neither science nor navigation benefited by his work.

It is quite a different matter with Wright. He, too, calculated a table, which is known from the textbook by Thomas Blundevile (1560-1602), entitled: *His Exercises, containing sixe treatises . . . to be read and learned of all young gentlemen* (London 1594). This book was used and esteemed for fifty years, by the educated reader rather than by the simple seaman learning his trade. The treatise devoted to navigation comprises a chapter on the sea-chart, in which Cortes and Coignet are quoted and the loxodrome is illustrated and described. At the end there is a reference to Mercator and his worldmap with its increasing distances between the parallels. How this increase was brought about, "by what rule I knowe not", says Blundevile, "unlesse it be by such a Table as my friende M. Wright of Caius Colledge in Cambridge at my request sent me (I thanke him) not long since for that purpose, which Table, with his consent, I have here plainlie set downe together with the use thereof as followeth". It gives the meridional parts from latitude 1° to 80° in sixtieths of a degree of the equator, at intervals of 1° . For 1° , 30° , 52° , and 80° it gives respectively 60, 1883, 3668, and 8399, as against 60, 1888, 3665, and 8375 in the present-day nautical tables. The difference increases gradually with the latitude. Blundevile explains how the network of the chart is to be made. The parts of the equator are measured along the meridian, after which by means of the table the parallels are drawn at the correct distances from the equator. It is true that Blundevile owes this table to Wright, but it differs from the one which Wright himself published in 1599. This table, in its original form, has also been in the hands of others. It has been used indiscreetly, but it is beyond the scope of this introduction to go into this.

The appearance of the previously mentioned textbook, *Certaine Errors in Navigation . . . detected and corrected by Edw. Wright* (London 1599), forms an important landmark in the development of scientific navigation¹⁹⁾. The author, who had gained knowledge and experience at sea and who was an eminent mathematician, prepared a second and amended edition, which appeared in 1610. Many years after Wright's death, Joseph Moxon edited the third edition (1657), to which he added the translation of *The Haven-Finding Art*. The fact that the book continued to be sold and used until the end of the seventeenth century bears witness to the great influence of Wright's work on the deepening of nautical knowledge in the seventeenth century.

In his "Praeface to the Reader", Wright enumerates the many imperfections inherent in the art of navigation (which was already "some thousands of yeeres" old), in particular those relating to the chart. His treatise contains the means to improve the subjects in question. In the present context attention will be paid only to his tables of meridional parts and those of loxodromes. Of the latter he says with justifiable pride: "with help of which table, the Rumbes may in any chart, mappe or globe much more truely be described then by those maechanicall wayes long since published by Petrus Nonius, or latily practised by some globe-makers in England" (p. ggg 3).

¹⁹⁾ The title-page of the book, the "Praeface", and "A table for the true dividing of the meridians in the sea chart" are reproduced in *The Hydrographic Review*, May 1931, under the title of : Origin of Meridional Parts.

Wright gives a clear exposition of the construction of the network of the Mercator chart and concludes with the following words, which seem to echo those of Mercator: "the parts of the meridian at every poynt of latitude must needs increase with the same proportion wherewith the secantes or hypotenusae of the arke, intercepted between those pointes of latitude and the aequinoctiall do increase. Now then wee have an easie way layde open for the making of a table (by help of the Canon of Triangles) whereby the meridians of the Mariners Chart may most easily and truely be divided into parts, in due proportion from the aequinoctiall towards either pole". (p. D) The basis for the table had thus been described. He made it "by perpetuall addition of the secantes answerable to the latitude of each point or parallel unto the summe compounded of all the former secantes, beginning with the secans of the first parallels latitude and thereto adding the secans of the second parallels latitude" (p. D).

It was originally Wright's intention to publish the table of meridional parts with an interval of 1' of latitude, up to a maximum latitude of 89°59' and expressed in units of 0.0001 minute of the equator. But on second thoughts, for the sake of simplification, he chose an interval of 10' and for the unit he took 0.1 minute of the equator, which did not result in any "sensible error". In this form the "Table for the true dividing of the meridians in the sea chart" is to be found in his book. It covers a range from 0°10' to 89°50'.

He considered it necessary to drop three digits (p. D verso) "not onely for the easier, but also for the truer making of the table". "Truer", because he may have doubted the accuracy of the last decimal places. Wright was aware that the values in his table were a little too great, but he considered it sufficiently accurate. If anyone wished to attain greater accuracy and to work with a smaller interval, he might construct a table with the aid of the *Canon magnus triangulorum* of Rheticus (1514-1576)²⁰.

Next Wright describes (p. F verso) how this table has to be used for the construction of the "table of rumbs". In a few words it is said that this is done by "perpetuall addition", but no further explanation is given of why this is so. These words, however, have already been explained by the proof given above (p. 484), that the meridional parts of each of the points of intersection form a multiple of the meridional parts of the first point of intersection.

The table, which takes up twenty pages, now follows. It gives the points of intersection for seven loxodromes with meridians at a difference of longitude of 1°, and goes as far as latitude 89°59'. This great range shows that theory went further than practice. In fact, for practical purposes it was not necessary to carry the calculation thus far. Wright reckons the loxodromes from the equator, so that E by N is the first. The construction of the table must have taken a great deal of time. The number of points of intersection which had to be calculated was 3,600. The calculation of the table of the seventh loxodrome (N by E) alone — the longest of all — necessitated as many as 1,440 determinations of latitude. This loxodrome passes exactly six times round the earth before reaching latitude 89°59'.

It is this table which has been faithfully copied in its entirety by Stevin, with

²⁰ George Joachim Rheticus, whose real name was Joachim von Lauchen, born at Feldkirch (Vorarlberg, Austria), in the former province of Rhätikon and called Rheticus on this account. Mathematician and astronomer: 1537 Professor at Wittenberg; cooperated with Copernicus; 1542 Professor at Leipzig.

the exception of the order in which the loxodromes were taken.

In § 1 it has already been shown that Apian must also be reckoned among Stevin's predecessors in this field.

§ 3

CONCLUSION

It has already been stated that Stevin, when copying Wright's tables, wrote that he had found "some imperfection" in them, words which cannot but be regarded as a kind of accusation. Recurring to his objections in the Appendix, Stevin says that he has made random tests with the fourth loxodrome — not with the others — and that he has found differences. They were not great and not of a nature to prevent him assuming that Wright's tables were reasonably accurate. He therefore drew attention to differences which in his opinion were to be found in them, but he did not go to the bottom of the matter. He was moderate in his criticism and took over the tables, in spite of his objections to them.

It is only natural that Wright, who had seen the *Wisconstighe Gedachtenissen*, did not leave the matter there, although he esteemed Stevin's work. In the second edition of *Certaine Errors* he included a detailed reply, under the title: "Simon Stevin his errors, in blaming me of error in my tables of Rumbes" ²¹), from which it is quite evident how strongly he resented Stevin's criticism. He expresses this in his words: "but we shall (I make no doubt) find a greater fault in his fault-finding. Though I never durst presume to imagine I could set forth anything without some fault. It sufficeth me if the fault be so little that it cannot sensibly be discerned, which I hope I shall hereafter shew." These words at the same time characterize the object his argument was intended to achieve.

Wright evidently considers it exaggerated to criticize a difference in the latitude of no more than 12' and writes: "surely therefore Master Stevin might have eased me of some trouble and have saved himself some labour and reputation both, if he had spared his pains in seeking to find fault with me for so small a matter, whereby navigators could incur neither dammage nor danger, although mine error were so much as he supposeth. For in the practice of navigation what inconvenience can arise to a seaman by such an error as shall cause him to goe wide of his course no more then 12 minutes, in sailing so far that he must alter his longitude 78 degrees and his latitude 61 degrees and more?" On the enormous distance this difference was very small indeed, from the practical point of view.

Wright went more thoroughly into the matter and recalculated his table of meridional parts twice, aided by two calculators, this time from minute to minute. He found no differences that might result in any "sensible error". He also recalculated the table of the eighth loxodrome, which Stevin had taken from Apian, this time with an interval of 10' in the latitude. He checked the values for the fourth loxodrome, found by Stevin — who had worked with the table of Apian — and he applied

²¹) In the third edition this reply to Stevin is again included, in the same words. The only difference is that the tables have now been printed in a more compact form.

the newly calculated table, the result being that the difference of 12' was reduced to a very small one. "The truth is . . . thinking it would be sufficient if my Table of Rumbs erred not any where so much as a minute in the latitude of any rumb, which by this triall already made, I assure my self they do not." Wright is of the opinion that Stevin's error is in "his own grosse manner of triall, much more then in my table". If he had worked with smaller intervals, he would have found smaller deviations. And thus the duel was ended. Wright openly spoke his mind. He showed the lack of justification of Stevin's criticism and called his checking method too gross. But that Stevin's "reputation" was injured is a statement which is not in keeping with the general tone of his reply. It was just a little too sharp, and certainly not in accordance with his esteem of Stevin when he translated *The Haven-Finding Art* and wrote an introduction to it.

Paragraphs 1 and 2 have clearly confirmed that the substance of *The Sailings* forms no original work, as we already stated at the outset. Stevin, wishing to become informed about a subject which had never before been discussed in Holland and which was of great import to navigation, sought this information in the works of the few foreign scholars who had already studied the matter. He went back to the sources, as scientists are accustomed to do, and he properly mentioned his sources. He thoroughly studied the writings cited above and took over the main points. He acted in the same way with regard to the system of drawing the loxodromes on the globe by means of models — a system which had already become known among the globemakers of his day —, the old reduction table of Apian, and the two new tables of Wright. He incorporated these matters in his treatise and in his own words elaborated the whole into a logical and conveniently arranged discourse. The form into which he cast the treatise commands our admiration. The clarity of his words proves how familiar he had made himself with the subject. The criticism — directed in particular against Nunes and Wright — which Stevin considered himself justified in making, was included in the Appendix. Wright reprimanded him by showing that the argument was unsound and unfounded. He might also have pointed out the error in thought made by Stevin, which the latter might have avoided by following the example. But this was left by Wright to posterity.

Stevin's predecessors had developed the theory in their writings in the form of more or less elaborate expositions, incorporating the definitions in the text, so that they were virtually hidden in it and the reader had to discover them for himself. Moreover, the reader was faced with difficulties because the authors, though they did give indications as to the way in which some of the calculations had to be performed, omitted to give an explanation of the reason why. Stevin, on the other hand, renders his reader the great service of putting definitions foremost, formulating them clearly, and giving them in large print in his book. This made it much easier for the reader to understand the gist of the matter. The systematic composition of the treatise was of great value to students and must have aroused their gratitude to the author. Stevin's work was exceptional, since long after his day the textbooks on navigation were still deficient in point of proper arrangement of the subject-matter, systematic composition, rigour, and briefness of the argument. We refer, for instance, to the two popular textbooks of Gietermaker and Klaas de Vries, which were used in Holland almost to the exclusion of all other books, the former from 1660 to about 1800 and the latter from 1702 to about 1820. It is not

surprising that from such books the navigator was unable to learn spherical trigonometry, for instance.

Stevin's book of 1605 was of quite a different nature. The reader who was acquainted with the language of mathematics and the derivation of formulas was able to learn a good deal from it, thanks to Stevin's search in foreign sources, his able and careful elaboration of the subject-matter, and his lucid presentation of it. Here again — as with *The Haven-Finding Art* — it may be said that his work was of great use to his contemporaries. There is more: the practice of Dutch shipping, too, owes him a debt of gratitude. This will become evident from the following pages, in which we shall briefly describe how his work was continued in the Netherlands.

To Willebrord Snellius (1591-1626)²²⁾, who translated the *Wisconstighe Gbedachtenissen* into Latin²³⁾, and who was therefore intimately acquainted with their contents, the credit is due of having widened the knowledge of the loxodrome by his book entitled *Tiphys Batavus*^{24,25)}. It contained a long introduction, a treatise on the loxodrome, another devoted to the determination of the speed and the dead reckoning, and finally two tables. The first of these is entitled *Tabulae canonicae parallelorum*. It is a table of meridional parts from 0° to 70°, to four places of decimals, with an interval of 1', made by adding up the secants from minute to minute. Verification has shown it to be very accurate. It is not identical with that of Wright/Stevin. The second is a table of loxodromes, entitled *Canones loxodromici*. It has quite a different character from that of Stevin's Table of Loxodromes. The object of Stevin's table was primarily to become acquainted with the true shapes of the loxodromes on the earth's surface. It is true that Stevin had described how sailing problems could be solved by means of his table. However, not only was this method inconvenient, but it could not be followed because, but for an occasional exception, Stevin had not included the calculation of the distance in his table. It was thus incomplete. The table of Snellius was destined for practical use at sea. It is divided into quarter points and with the difference in latitude from minute to minute as argument gives the distance and the departure, the latter two magnitudes in miles of 15 to the degree. In this book Snellius showed how sailing problems are solved by means of the table. His system undoubtedly marked an advance on that of Stevin. Nevertheless this again was not altogether suitable for practice as yet, because the search in the column of the difference in latitude was not satisfactory. This book moreover, being of a scientific character and being written in Latin, was above the head of the seaman.

We shall pass by the achievements in this field of Ezechiel de Decker (ca 1595-1667), surveyor and mathematician²⁶⁾, and of Adriaan Metius (1571-1635),

²²⁾ Willebrord Snellius, mathematician and physicist, Professor at Leiden, famed for his method of measuring a degree of latitude and for his law of refraction.

²³⁾ *Works* XIb.

²⁴⁾ Willebrord Snellius van Royen, *Tiphys Batavus, sive histiodromice, de navium cursibus et re navali* (Leiden 1624).

²⁵⁾ Tiphys was the steersman of the ship "Argo", known from Greek mythology through the voyage of the Argonauts. Tiphys Batavus presumably means: the Dutch Tiphys, or the Dutch steersman.

²⁶⁾ Ez. de Decker lived at Gouda, later at Rotterdam. He admired Stevin and is known to have published the first Dutch table of logarithms: *Nieuwe Telkonst, inhoudende de logarithmi* (Gouda 1626).

Professor of mathematics and astronomy at Franeker, but direct our attention to Cornelis Jansz. Lastman (died before 1653). The latter had been born in Vlieland, had followed the sea, and around the middle of the seventeenth century at Amsterdam, in the Haarlemmerstraat, ran a nautical college, which was called *In de vergulde Graed-Boogh* (In the Golden Cross-staff) and which was continued after his death by his son Simon.

Lastman compiled a table of meridional parts as well as *Tafelen der Compas-streecken* (Traverse Table), which are to be found in the textbooks of navigation published by him²⁷). As with Stevin, this table applies to seven loxodromes, while there is also a table for the eighth loxodrome, which gives the reduction of departure into difference in longitude. It covers a range up to latitude 80° and the interval is 1'.

Whilst Stevin had furnished the latitude of the points of intersection with the meridians for the seven loxodromes and his table was intended to advance science, Lastman's table was so arranged that it was destined and suitable for practical application. Its character, therefore, is different. For the seven loxodromes, commencing at the equator and extending as far as latitude 75°, it gives the longitude and the latitude of the points through which the ship passes as she sails upon the loxodrome, at intervals of one geographic mile (1 geographic mile = 4 nautical miles). The argument is the distance run. The table takes up 50 pages, each of which contains 325 calculated places. The seventh loxodrome alone takes up 18 pages and necessitated the calculation of 5,760 places. The figure-work must have been enormous indeed.

Although an interval of one point in the course was large and consequently inconvenient — because a true course obtained by reduction of the course steered seldom falls on a full point — still it could be used in practical navigation at sea in order to determine the position when the course and the distance run were known, or *vice versa* — though this was slightly more difficult — to determine the course and the distance between two known places. Lastman's table was taken over by others. It is found again in many books, and was used for a long time, even to the early part of the nineteenth century. It was ousted by the traverse table which Cornelis Douwes (1712-1773)²⁸) included in his *Zeemans-Tafelen*²⁹). On the

²⁷) Cornelis Jansz. Lastman, *Lastmans beschrijvinge van de Kunst der Stuerlieden* (Amsterdam). Many editions are known: 1642, 1648, 1653, 1657, 1661, 1675, 1714. In addition: Vlissingen 1659.

²⁸) Corn. Douwes was mathematician to the Admiralty, to the East India Company, and to the city of Amsterdam, examiner of naval officers and navigators, teacher at the "Zeemans-Collegie", on Oude Zijds Achterburgwal, Amsterdam.

Douwes made the seaman independent of the determination of latitude which was based on the observation of the sun at the moment of its meridian passage. He discovered a simple scheme of calculation, which could be applied by the common seaman, with the aid of which the latitude could be figured out from two altitudes of the sun outside the meridian, the time interval between the observations being known. His method was applied from 1750 to about 1850 by all seafaring nations of the world, in the Netherlands to the end of the nineteenth century. The possibility created by Douwes greatly promoted the safety at sea. From the English, Douwes received a remuneration for his discovery (cf. my book *Cornelis Douwes, 1712—1773, zijn leven en zijn werk* (Haarlem 1941).

²⁹) *De noodige en bij ondervinding beproefde nieuw uitgevondene Zeemans-Tafelen en voorbeelden tot het vinden der breedte buiten den middag, door Cornelis Douwes* (Amsterdam 1761). Up to 1858 this table passed through 16 editions in the Netherlands. It is met with in numerous tables, e.g. in English and American tables.

English model he constructed a table in which, the course and the distance run being known, the difference in latitude and the departure were found. The interval in the course was small. A point was divided into eight parts, thus: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, a division which did not hold its ground and which was not used in England.

We will conclude with two opinions about Lastman's table, pronounced by colleagues of his.

Pieter Rembrantsz van Nierop (died 1708) prepared an amended edition of the textbook written by his uncle, the shoemaker-astronomer Dirk Rembrantsz van Nierop³⁰), author of a large number of books on navigation and astronomy. In the preface he says: the art of navigation, as described formerly by the Portuguese, the Spaniards, Medina, Coignet, Zamorano, and William Bourne, "was no more than an *A-B* in comparison with that of the present day, for when our C. J. Lastman in 1621³¹) first brought forward his Tables of Sines, Tangents, and Secants as well as his Table of Meridional Parts and eight Tables of Loxodromes, for use in navigation on a spherical earth, people made fun of him, saying: What does the man mean by these things? And now see how he has augmented and amended it up to 1640, and indeed, how he is now being followed by other writers, so that it is evident from this how various errors in navigation can be corrected more and more, some of them by the correct application of the art and others by diligent observation."

The second opinion was given by Simon Pietersz (born in 1601), a nautical teacher at Medemblik. In the textbook written by him³²) he concludes each discussion of a given subject with an interrogation of the pupil. After the discussion of the calculation of the position by dead reckoning we read (p. 111): "Question: What do you make use of when you want to indicate the position in the chart?

Answer : Now one thing, now another. But in general I have recourse to the miles and degrees which I find in the 8 *Compassstreken van Lastman*³³).

Question : Why, are they so excellent?

Answer : They are so wonderful, so commendable and infallible that they surpass all other means and their use."

In the paragraphs 2 and 3 we have shown that slightly more than one hundred years elapsed between the moment at which the scholar Nunes began to study the loxodrome and that at which the Dutch seaman could avail himself of the knowledge that had been gathered about it and in day-to-day practice at sea was able to perform calculations concerning the distance run or the course to be followed. The way was long. Stevin was one of those who helped to pave it.

This statement applies to the Netherlands. In England this part of the art of navigation developed along different lines. Richard Norwood (1590-1675), who had followed the sea and later became a nautical teacher in London, famous for his method of measuring a degree of latitude (1635), calculated a table of courses

³⁰) Pieter Rembrantsz van Nierop, *Verbeterde en vermeerderde Nieroper Schat-Kamer* (Amsterdam 1697).

³¹) Probably this has to be 1631.

³²) Simon Pietersz. *Stuermans Schoole, in welke de navigatie ofte Konst der Stuerluyden seer ordentelick en bequamelijck voorgestelt en geleert wert. Oock heel gediensstigh om in de schoolen der navigatie ghebruyckt te worden* (Amsterdam 1659).

³³) Traverse Table of Lastman.

and distances, which, the interval in the course being 1° and the distances being expressed in nautical miles, gave the difference in latitude and the departure, the latter two magnitudes not in miles but in tenths of miles³⁴). In later editions they were given in miles as well as tenths of miles, so that the construction of this table was identical with the one that is now known and used at sea.

³⁴) This table is to be found in his textbook *The Sea-Mans Practice* (London 1637). The book, which was very popular and the 15th edition of which appeared in 1682, was reprinted and used up to the early eighteenth century (in 1716, for instance).

VIERDE
BOECK DES
EERTCLOOTSCHRIFTS,
VANDE
ZEYLSTREKEN.

Argumen-
tum.

C O R T B E G R Y P

DER ZEYLSTREKEN.



Hydrogra-
phie.
Mathemati-
carum ar-
tium.

Ant de menichvuldighe wyde zeylagen deser landen
versheyden souckers veroirsaeckt hebben, van wonden
streckende tot voordering der groote zeevaerden, die elck
verthoonde an sijn VORSTELICKE GHENADE als
Admiral, om daer me tot hun voordeel te gheraken: Soo is
de stof des * Zeeschrifts een der besonder oirsaken gheveest, die hem track
totte begheerte en oeffening der * VViskonsten: Sulcx dat hy deursien heeft
al het oirboirste en diepsinnichste dat van die stof mijns wetens gbehandelt
wort. Van t selve Zeeschrift nemen wy voor ons hier te beschrijven dit
vierde bouck vande Zeylstreken, waer achter noch volghen sal den handel
vande Harvenvinding, en oock van Ebbenvloet, overmits wy daer in
wat besonders hebben, dat in dese visconstige ghedachtnissen sijn plaets
vereychr. Angaende de rest des Zeeschrifts, daer toe verstrecken hem tot
ghedachtis versheyden boucken van die stof handelende, en door hem
over sien.

Definiciones. Dese beschrijving der Zeylstreken, sal na vier noodighe* bepalinghen
der eyghen voordien, begrypen 11 voorstellen, welcker twee eerste sijn
van rechte Zeylstreken, d'ander van crome. VV aer achter noch volgen
sal een Anhang der Cromstreken.

BEP A-

SUMMARY OF THE SAILINGS

Since the numerous voyages from these parts to remote countries have induced many investigators to find aids for ocean navigation, which each by himself, striving after his own profit, showed to His Princely Grace as Admiral, the subject of hydrography was one of the causes which urged him especially to study and practise mathematics, so that he investigated all the most useful and subtle things that have to my knowledge been said about that subject. Of this hydrography we here intend to describe this fourth book, of the Sailings, which will be followed by the treatise on the Haven-Finding Art, and also that of Ebb and Flow, since we have made some remarkable discoveries about this, which require their place in these mathematical memoirs. As regards the remainder of hydrography, various books dealing with that subject, which he has examined, serve to aid his memory.

This description of the Sailings is to contain, after four necessary definitions of the special terms, 11 propositions, the first two of which refer to great-circle tracks ¹⁾, the others to rhumb lines ²⁾. This is to be followed by an Appendix about loxodromes.

¹⁾ Orthodromes.

²⁾ Loxodromes. This is the term that will be used in the translation.

BEPALINGHEN.

1 BEPALING.

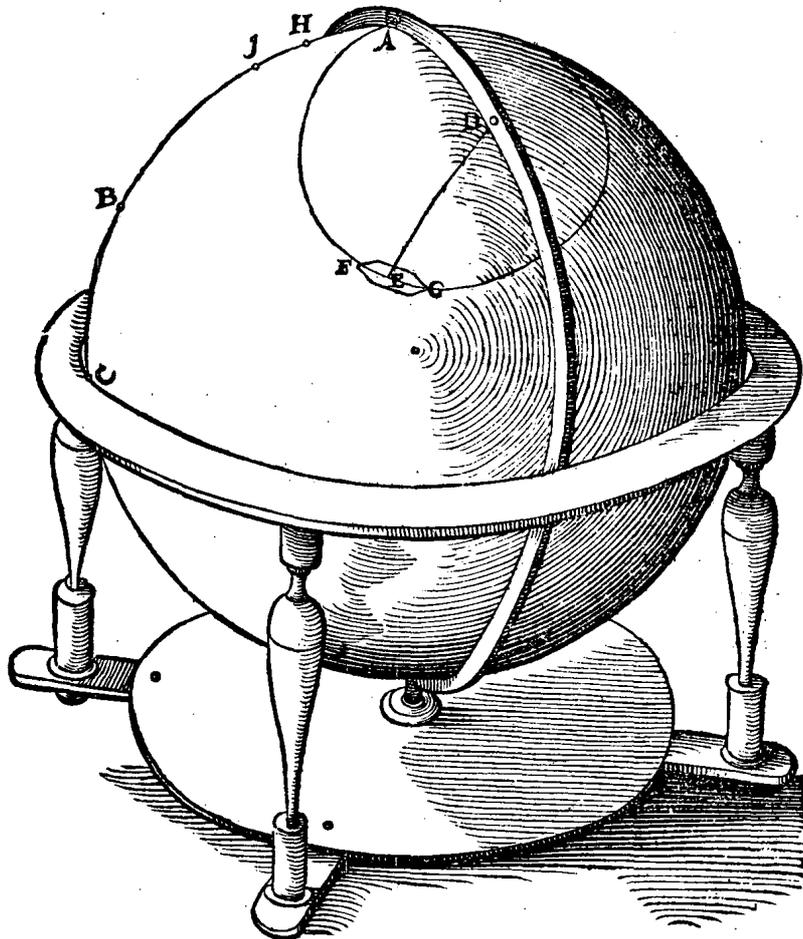
Zeylſtrekē ſijn de linien die ſeylende ſchepen beſchrijvē.

Als een ſchip ſeylende van ooft na weſt, de verdochte lini of ſtreck diet int varen beſchreven heeft, heet int ghemeen zeylſtreck: Int beſonder ooftenweſt ſtreck, en van ander winden of oirten crijchtſe ander namen.

2 BEPALING.

Rechte ſtreec noemen vvy des Eertcloots cortſte booch tuffchen tvvee punten.

Laet op den Eertcloot A B C, tuffchen de twee punten A en B, ghetrocken ſijn de booch A B, weſende de cortſte neem ick die daer tuffchen ghetrocken



H 2

can

DEFINITIONS

1st DEFINITION.

Sailing tracks are the lines which ships describe when they are sailing.

Thus, when a ship sails from east to west, the imagined line or track which it has described in sailing is in general called sailing track, in particular east and west track, and according to the other points of the compass or places it receives other names.

2nd DEFINITION.

Great-circle track we call the shortest arc on the earth between two points.

On the earth ABC let there be drawn, between the two points A and B , the arc AB , being the shortest, I assume, which can be drawn between them, which must be the arc of a great circle. Such directions are nowadays marked in mariner's

88 4 BOECK DES EERTCLOOTSCHRIFTS,

Efficiente. can worden, t welck sijn moet de booch eens grootste rondts: Soodanighe streken worden nu ter tijt beteyckent inde zeecompassen mette 32. linien daer in beschreven, welcke deur t ghedacht vanden * doender af langs t'vlack des eertcloots voortghetrocken tot inden sichteinder, of anders opde gheboorste eertclooten alsoo ghetyckent, bedien de 32 ghemeene streken, of winden.

Angaende ymant dencken mocht hoe dat cromme boghen eyghentlick ghenouch rechte streken ghenoecht worden, die sal weten dat dese rechttheyt geseyt wort int ansien datse noch ter rechter noch ter sliicker sijde en wijcken, gelijk wel doen de cromme streken, diens bepaling volghet.

3 BEPALING.

Een schip buyten t middelront en middachront so sey-
lende, dat de booch ghetrocken vande kiellini totten af-
punt, altijt op de kiellini een selven houck maectt: De lini
die t schip dan gheseylt heeft noemen vvy Cromstreec.

Laet inde form der 2 bepaling D des eertcloots afpunt sijn, E een schip, t welck gheseylt hebbe van A tot E, soo dat de booch E D, getrocken vande kiellini F G als van E totten afpunt D, altijt op de kiellini F G maecte een selven houck als F E D, t welck soo ghebeuren soude als t schip altijt op een selve streec seyde die t zeecompas anwijst, en dat de lini altijt recht noort wese. Dit soo sijnde, de lini of booch A F E die t schip gheseylt heeft heet cromstreec. Nu ghenomen dat den houck F E D recht sy, soo sal het schip altijt recht oost of recht west anghევaren hebben, en de booch A E sal deel eens kleenronts sijn: En dattet gheduerlick soo voortseyde, het soude weerom commen ter plaets van A daert begoft, volschrijvende het rondt. Hier uyt canmen verstaen dat de rechte oost streec A B, en de cromme A E, veel verschillen: Om van t welck breeder verclaring te doen, soo laet C beteyckenen het recht oostpunt, wesende de gemeene sine des sichteinders en middelronts, en t punt A sy onder het sop, en t punt B inde booch A C, en de booch A F E sy even met A B. Dit soo sijnde, ghenomen dat een schip seyde van A na C altijt op de booch A C, het sal int ansien des gheens die an A is, altijt recht oostwaert anseylen, maer niet int ansien des gheens die int schip is, welcke geduerlick grooter en grooter verschil sal vinden, ja ten eynde soo groot, dat genomen t punt A te liggen op de breedte van 50 tr. soo sal den seylder ontrent C commende, hem bevinden te seylen oock ontrent de 50 tr. van oosten na zuyden. Wederom, hoe wel B recht oogst light van A, nochtans een schip seyde van A af altijt recht oost an int ansien des seylders, en sal niet geraken tot B, maer verre van daer tot E, wel verstaende dat de booch A E even ghenomen wort met A B alsvooren.

Merckt noch dat hoe wel B recht oost light van A, nochtans so en ligt A niet recht west van B, t welck op groote boghen veel verschillen can. Laet by voorbeeld van een plaets diens breedte 45 tr. ghenomen worden een booch van 90 tr. recht oostwaert: De rechte streec van die oostersche plaets na d'ander, en sal niet sijn recht west, maer so veel noorderlicker als de breedte bedraecht, te weten 45 tr. dats recht noordwest. Sulcx datmen om vande westlicker plaets te seylen na de oostelicke, beginnen moet recht oost an, maer vande oostelicke na d'ander (om op een rechte streec te seylen, want op cromstreken heeft het weerkeeren altijt de naem des reghenoverwint vant wechvaren) noordwest an: En waerde
plact-

compasses by the 32 lines described therein, which, being produced mentally by the observer over the surface of the earth to the horizon, or otherwise drawn in this way on the model globes, designate the 32 common points of the compass or "winds" 1).

Since one might wonder how arcs can properly be called straight tracks, one is to know that this straightness means that they deviate neither to the right nor to the left, as do the curved tracks, the definition of which follows.

3rd DEFINITION.

When a ship sails outside the equator and the meridian in such a way that the arc drawn from the ship to the pole always makes the same angle with the keel-line 2), the line on which the ship has then sailed we call loxodrome.

In the figure of the 2nd definition let D be the pole of the earth, E a ship which shall have sailed from A to E , so that the arc ED , drawn from the keel FG , viz. from E to the pole D , shall always make the same angle with the keel FG , viz. FED , which would happen thus if the ship always sailed on the same point indicated by the mariner's compass and if the fleur-de-lys invariably pointed due north. When this is the case, the line or arc AFE on which the ship has sailed is called loxodrome. If it is now assumed that the angle FED is right, the ship will always have sailed due east or due west, and the arc AE will be part of a small circle. And if it sailed continually on in this way, it would come again to the place A where it began, completing the circle. From this it can be understood that the orthodromic course east AB and the loxodromic course east AE are widely different things. To give a fuller explanation of this, let C designate the true east, being the common intersection of the horizon and the equator, and let the point A be in the zenith and the point B in the arc AC , and let the arc AFE coincide with AB in A . This being so, if it is assumed that a ship sailed from A to C always along the arc AC , to one who is in A it will always appear to be sailing due east, but not so to one who is in the ship; he will find a constantly greater and greater difference, even finally so great that if the point A is assumed to lie in latitude 50° , the man in the ship, coming near C , will find he is sailing about 50° south from the east. Again, though B lies due east of A , nevertheless a ship appearing to the man in the ship always to be sailing from A due east will not arrive at B , but, far from there, at E , it being understood that the arc AE coincides with AB in A as above.

Note further that although B lies due east of A , nevertheless A does not lie due west of B , which may differ a good deal on large arcs. For example, from a place whose latitude is 45° let an arc of 90° due east be taken: the great-circle course from that more easterly place to the other will not be due west, but so much more to the north as the latitude amounts to, viz. 45° , that is due northwest, so that, in order to sail from the more westerly to the easterly place, one first has to sail due east, but from the more easterly to the other place (to sail on a great-circle course, for on loxodromes the home voyage is always called after the point of the compass opposite to that of the departure) due northwest. And

1) Winds = Spanish *vientos*, the word commonly used in sixteenth-century Spanish textbooks, meaning "directions".

2) Fore-and-aft line. See pag. 368.

plaetsens breedte van 57 tr. sulck verschil sou dan oock vast 57 tr. sijn, dats over de vijf ghemeeene streken.

En hoemen naerder den aspunt seylt, hoemen sulck verschil op groote seylagen merckelicker bevint, en dat om bekende redenen, die op een Eertcloot met haer behoorlike reetschappen openbaer sijn. Inder voughen dat Stierlynden die daer ontrent varen en landen soucken, noodich is van dese saeck goe kennis te hebben, want t'ghebeurt den onverdochten wel, datse haer schip op een ander plaets vindende dan hun gissing me brengt, sulcx ten eersten wijten onbemerkelicke afleydende stroomen, daer afnochtans d'oirsaek mach sijn het boveschreven niet gheslaghen te hebben na t'behooren.

Tot hier toe is gheseyt vande cromstreec recht oost en west angheseylt, die altijd een ront is, maer d'ander (uytghenomen inde * middachronden en int middelront) sijn altemael * slangtrecken, wiens form en ghedaente deur de volgende voorstellen openbaer sal worden.

*Meridiana
circulo et
aequatore.
Spirales.*

4 B E P A L I N G.

Eerste cromstreec noemtmen die in yder vierendeel des sichteinders naest het middachront is, d'ander volgende heet de tvveede, en soo oirdentlick voort totte achtste, die altijd een * ewevijdich ront is.

*Circulus pa-
rallelus.*

Als by voorbeelt int vierendeel des sichteinders van noort tot oost, de cromstreec naest het middachront, of anders geseyt naest het noorden, diemen oock heet noort ten oosten, wort d'eerste cromstreec ghenoeemt, noornoortooft de tweede, noortooft ten noorden de derde, en so voort met d'ander totte achtste, dat is d'oostcromstreec die altijd een ront is ewevijdich metter middelront: En foodanich is oock d'oiden in d'ander drie vierendeelen des sichteinders.

De reden waerom de cromstreken benevens de naem die sy hebben na de winden, noch geseyt worden eerste, tweede, derde, &c. is dusdanich: Anghesien vier * lijckstandige strekē, als by voorbeelt de streec van noort ten oosten, noort ten westen, zuyt ten oosten, en zuyt ten westen, in form malcander heel gelijk, en van grootheyt heel even sijn, sulcx dat deur de leering van een, de ghedaente over alle vier verstaen wort, soo vallet oirboir om oirdentlick van dese stof te handelen, datmen die als * afcomst een ghemeeene naem gheeft, te weten eerste, als haer * gheslacht, om niet elcke mael vier winden t'samen te moeten noemen, of maer een ghenoeemt sijnde, dat d'ander niet vergheten en schijnen.

Homologa.

*Species.
Gemma.*

N V D E
V O O R S T E L L E N.

Alsoo sijn VORSTELICKE GHENADE int lesen van *Cosmographia Petri Appiani & Gemma Frisij*, ghecommen was tot *Cap. 13 prima partis*: Daer na tot *7 Cap. in libello de locorum scribendorum ratione*, Alwaer stont de manier om deur ghetalen te vinden op wat streec d'een plaets van d'ander light, heeft het selve aldoen overgheslaghen, om twee redenen, d'ene dat den gront uyt welcke de wercking ghetrocken was daer niet by en stont, ten anderen dat

H 3 hy doen

where the latitude of the place is 57° , such difference would then also be 57° , *i.e.* more than five points.

And the nearer one gets to the pole, the more appreciable will this difference be found on long voyages, for familiar reasons, which are clear on a globe by means of its accessories. In such a way that it is necessary for navigators, sailing in those regions and trying to discover unknown lands, to be well acquainted with this matter, for it sometimes happens to those who are not prepared for it that, finding their ship to be in another place than according to their conjecture, they attribute this at once to imperceptible diverting currents, though the cause of it may be that they have not observed the above properly.

Up to this point, mention has been made of the loxodrome on which the ship sails due east and west, which is always a circle, but the others (except in the meridians and in the equator) are all spirals, the appearance and character of which will become clear from the following propositions.

4th DEFINITION.

First loxodrome we call the one which in each quarter of the horizon is nearest to the meridian, the next is called the second, and so on in due order up to the eighth, which is always a parallel.

Thus, for instance, in the quarter of the horizon from north to east the loxodrome nearest to the meridian, or in other words nearest to the north, which is also called north by east, is called the first loxodrome, north northeast the second, northeast by north the third, and so on with the others up to the eighth, *i.e.* the east loxodrome, which is always a circle parallel to the equator. And such is also the order in the other three quarters of the horizon.

The reason why, besides the name they have in accordance with the points of the compass, the loxodromes are also called first, second, third, etc., is as follows. Since four homologous tracks, such as *e.g.* the tracks of north by east, north by west, south by east, and south by west, are quite similar in form and equal in size, so that when one has learned one of them, one understands the character of all four, it is expedient, in order to deal with this subject in the proper way, to give them a common name indicating their genus, *viz.* first loxodrome, so that we need not each time mention four points of the compass together or, if only one is mentioned, it may not appear that the others have been forgotten.

NOW THE PROPOSITIONS.

When His Princely Grace, in reading *Cosmographia Petri Appiani*¹⁾ & *Gemmae Frisij*, had come to *Cap. 13 primae partis*, and afterwards to *7. Cap. in libello de locorum scribendorum ratione*, which described the manner in which to find by numbers in what direction one place lies in respect to the other, he skipped this part, for two reasons, the one because the ground on which the operations were based was not mentioned, secondly because at that time he was not yet skilled in

¹⁾ Cf. *Introduction*, p. 482.

hy doen noch niet ervaren en was inden handel der platte en clootsche drie-
houcken: Maer hem daer na inde selve gheoeffent hebbende, en ghedachtich
sijnde t'gene inde boveschreven hoofsticken overgheslaghen was, heeft in die
plaets ten selven eynde ander manier van wercking ghedaen deur kennis der
oorsaken, en dat niet alleen op de vinding der streck van d'een plaets tot d'an-
der, maer oock op al d'onbekende palen dieder vallen in sulck voorstel, t'welck
hier vervought is als volght.

1 VOORSTEL.

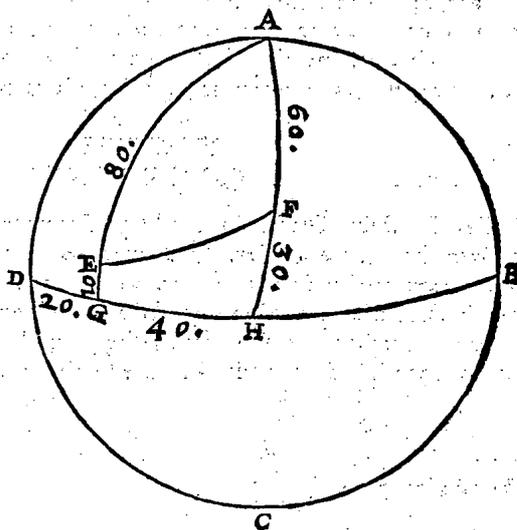
Wesende van tvvee plaetsen ghegheven drie palen de-
ses: Rechte streck van d'eerste plaets totte tvveede:
Rechte streck vande tvveede plaets tot d'eerste: langde-
schil: Schilbooch der breedte van d'eerste plaets: Schil-
booch der breedte vande tvveede plaets: En verheyte der
plaetsen: Te vinden d'ander drie onbekende palen.

T G E G H E V E N. Laet A B C D den eertcloodt sijn diens middelront B D, des
selfden begin D, aspunt A, eerste plaets t'punt E, tweede plaets t'punt F, tusschen
welcke getrockē is een grootste ronts booch E F, als verheyte: Deur de selve twee
plaetsen E, F, sijn getrockē de twee vierdeelenronts A E G, A F H, en des punts E
breede E G sy van 10 tr. en sal sijn schilbooch A E doen 80 tr. Voort des punts F
breede sy F H van 30 tr. en sal sijn schilbooch A F doen 60 tr. De langde van E sy
D G 20 tr. en de langde van F sy D H 60 tr. sulcx dattet verschil der langde vande
twee plaetsen E F is G H doende 40 tr. Inder voughen dat hier vande ses palen,
ghegheven of bekend sijn de drie, te weten A E, en A F, schilbogen der breedte,
en G H langdeschil. T B E G H E E R D E. Wy moeten de drie onbekende palen
vinden, te weten de rechte streck E F, dat is op wat rechte streck datmen van
E na F moet seylen, of anders de grootheyte des houcx A E F: Ten anderen de
streck F E, dat is de
grootheyte des houcx
A F E: Ten derden de
verheyde, te weten de
langde des boochs E F.

Merckt noch tot bree-
der verclaring der saeck
dat de ses palen int voor-
stel verhaelt, sijn de ses
ghemeene palen eens
cloodtschen driehoucx, te
weten drie houcken en
drie sijden, welcke in
dese stof sulcke namen
hebben.

T W E R C K.

Anghesien de booch
G H 40 tr. is, voor de



groot-

plane and spherical trigonometry. But having afterwards trained himself therein and remembering what he had skipped in the above-mentioned chapters, he proposed instead, for the same purpose, operations of another kind by knowledge of the causes, not only about the finding of the direction of one place in respect to the other, but also about all the unknown elements occurring in this proposition, which is here given as follows.

1st PROPOSITION.

If of two places three out of the following six elements are given: course of the great-circle track from the first place to the second; course of the great-circle track from the second place to the first; difference of longitude; complement of latitude of the first place; complement of latitude of the second place; and distance ¹⁾ between the places: to find the other three unknown elements.

SUPPOSITION. Let $ABCD$ be the earth, whose equator is BD , its beginning D , the pole A , the first place the point E , the second place the point F , between which has been drawn an arc of a great circle EF , for their distance. Through the said two places E and F have been drawn the two quarter circles AEG , AFH , and let the latitude EG of the point E be 10° , then its complement will be 80° . Further let the latitude of the point F be $FH = 30^\circ$, then its complement AF will be 60° . Let the longitude of E be $DG = 20^\circ$, and let the longitude of F be $DH = 60^\circ$, so that the difference of longitude between the two places E and F is GH , making 40° . In such a way that here out of the six elements three are given or known, *viz.* AE and AF , the complements of latitude, and GH , the difference of longitude. REQUIRED. We have to find the three unknown elements, *viz.* the course of the great-circle track EF , *i.e.* what course one has to sail from E to F , or otherwise the magnitude of the angle AEF . Second, the course FE , *i.e.* the magnitude of the angle AFE . Third, the distance, *viz.* the length of the arc EF .

Note also, as a fuller explanation of the matter, that the six elements mentioned in the proposition are the six common elements of a spherical triangle, *viz.* three angles and three sides, which in this subject are so called.

¹⁾ Stevin invariably uses the word "distance", no matter whether it is taken along the great circle or the loxodrome.

VANDE ZEYLSTREKEN.

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grootheyt des houck E A F, soo heeft de driehouck A E F drie bekende palen, te weten den selven houck E A F 40 tr. Voort de sijde A E 80 tr. en A F 60 tr. deur t'ghegheven: Hier me ghesocht d'ander drie onbekende palen, worden bevonden deur het 40 voorstel der clootsche driehoucken voor t'begheerde, te weten den houck A E F voor de streeck E F 55 tr. 51 ①, wijckende so verre vant noorden na t'oosten: Ende den houck A F E voor de streeck F E 109 tr. 44 ①, wijckende soo verre vant noorden over t'westen na het zuyden: Of anders gheseyt van westen na zuyden 19 tr. 44 ①. Ende de verheyt E F 42 tr. 15 ①.

VERVOLGH.

Tis openbaer hoemen deur elcke drie ghegheven bekende palen, d'ander drie onbekende vinden sal, sulcx dattet niet noodich en is besonder voorbeelden te beschrijven van die verscheydenheden in menichte seer veel vallende, te weten ses op elcke begheerde pael der ses palen. **T B E S L V Y T.** Wefende dan van twee plaetsen ghegheven drie palen deser ses: Rechte streeck van d'eerste plaets totte tweede: Rechte streeck vande tweede plaets tot d'eerste: langdeschl: Schilbooch der breede van d'eerste plaets: Schilbooch der breede vande tweede plaets: verheyt der plaetsen: Wy hebben ghevonden d'ander drie onbekende palen, na den eysch.

2 VOORSTEL.

Op rechte streken te seylen.

Nadien sijn **VORSTELICKE GHENADE** grondelick verstaen hadde den handel der seyling op cromstreken die hier na beschreven sal worden, en daer by verlijckende de rechte streken datse de cortste wech gheven, soo heeft hem behoerlick ghedocht, en d'oirden te vereyffchen, reghelen beschreven te worden hoemen die cortste streken soomen wilde seylen soude: Twelck oirsaeck was dat wy daer op leeten, en t'gene ons van dies ontmoete by ghedachtenis stelden, daer af beschrijvende twee voorbeelden, t'eerste * tuychwerckelick, *Mechanick*, t'ander wiskonlich.

1 Voorbeelt tuychwerckelick.

T G H E G H E V E N. Laet inde form der 1 bepaling A en B twee plaetsen op den certcloot beteyckenen, A daer t'schip af vaert, B daert sijn moet.

T B E G H E E R D E. Men wil een rechte streeck seylen vande plaets beteyckent met A, totte plaets beteyckent met B.

T W E R C K.

Men sal van A tot B trecken een verborgen oft uytvaghelicke grootsteronts booch, beteyckenende de rechte streeck die t'schip seylen moet, daer na t'punt A ghestelt sijnde onder het * toppunt, en dan de sopbooch gheleyt over B, sy wijst *Punsto verticuli* inden sichteinder, neem ick, dat B recht west van A light. Dit soo sijnde, men sal van A na B seylen recht westwaert an, by giffing drie of vier * trappen verre, *Grados*, die commen, neem ick, van A tot H: Alwaer t'punt H gheteyckent sijnde, men salt brenghen int middachront onder het toppunt, den aspunt soo veel verlegghende als de saeck vereyft, daer na de sopbooch andermael geleyt over t'punt

H 4

B sy

PROCEDURE.

Since the arc GH is 40° , for the magnitude of the angle EAF , the triangle AEF has three known elements, *viz.* the said angle $EAF = 40^\circ$; further the side $AE = 80^\circ$, and $AF = 60^\circ$, by the supposition. If by means of these we seek the other three unknown elements, the required values are found by the 40th proposition of spherical trigonometry ¹⁾, *viz.* the angle AEF for the course $EF = 55^\circ 51'$, deviating thus far from the north to the east. And the angle AFE for the course $FE = 109^\circ 44'$, deviating thus far from the north *via* the west to the south. Or in other words: from the west to the south, $19^\circ 44'$. And the distance $EF = 42^\circ 15'$.

SEQUEL.

It is evident how from any three given known elements we must find the other three unknown elements, so that it is not necessary to describe special examples of those various cases, of which there are a great many, *viz.* six for each required element of the six.

CONCLUSION. If of two places three out of the following six elements are given: course of the great-circle track from the first place to the second; course of the great-circle track from the second place to the first; difference of longitude; complement of latitude of the first place; complement of latitude of the second place; distance between the places; we have found the other three unknown elements; as required.

2nd PROPOSITION.

To sail on great-circle tracks.

After His Princely Grace had thoroughly understood the method of sailing on loxodromes to be described hereafter and, having compared therewith the great-circle tracks, had found that they give the shortest route, he considered it expedient and required by the order of things that rules should be described of how those shortest tracks would have to be sailed, if this were desired. Which induced us to attend to this matter and to make a note of what we had found about it, describing two examples of it, the first mechanical and the second mathematical.

1st Example, Mechanical.

SUPPOSITION. In the figure of the 1st definition let A and B denote two places on the earth, A the place from which the ship sails, B the place for which it is bound.

REQUIRED. It is desired to sail on a great-circle track from the place denoted by A to the place denoted by B .

PROCEDURE.

Draw from A to B an erasable arc of a great circle, denoting the great-circle track to be sailed by the ship; if then the point A is placed under the zenith and the vertical circle is made to pass through B , it indicates in the horizon, I assume, that B lies due west of A . This being so, one has to sail from A to B due west, by conjecture three or four degrees further, which I assume to be from A to H . And when the point H has been marked there, one has to bring it into the meridian under the zenith, lowering the pole as much as is required. If then

¹⁾ Stevin's *Trigonometry* (Work XI; i, 13), p. 295. See Vol. II B, p. 755.

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sy wijft inden fichteinder dat B van H light neem ick 3 tr. van weften nae zuyden, en daerom falmen op sulcken ftreec van H na B feylen weerom by giffing eenighe 4 of 5 tr. verre, t welck sy neem ick tot I, alwaer t ſchip ghecommen sijnde, men fal daer weerom doen sulcx alsmen an H dede, alwaermen oock bevinden fal datmen dan noch zuydelicker an nae B fal moeten feylen danmen van H dede: En ſghelijcx doende ſoo dickwils tot datmen ter plaets B comt, men fal de begheerde rechte ftreec A B ghefeylt hebben: Mits welverftaende dat de ſtucken als A H, H I, en diergelijcke cleen genouch genomen ſijn, Want hoe wel in plaets van A H wefende een grootſte ronts ſtuck, ghefeylt wiert een comme ftreec wefende cleenronts deel wat noordelicker uytcommende, voort dat in plaets van d'ander ſtucken des boochs A B, ghefeylt wierden ander crome ſtreken wefende ſlangtreckdeelen, doch met ſulcke ſtucken cleen genouch te nemen, canmen maken dat ſoodanich verſchil van gheender acht en is.

Merckt noch datter vant werck duſdanighe proef can genomen worden: Het ſchip ghecommen wefende tot neem ick H, en datmen dan deur dadelicke ervaring mette Son of ſterren d'eertcloots breede bevint t'overcommen mette breede die H op den gheboften eertcloon anwijft, dat geeft met reden vermoeden het ſchip de rechte ftreec wel ghefeylt te hebben, t welck volghen moet alsmen wel ghegift heeft.

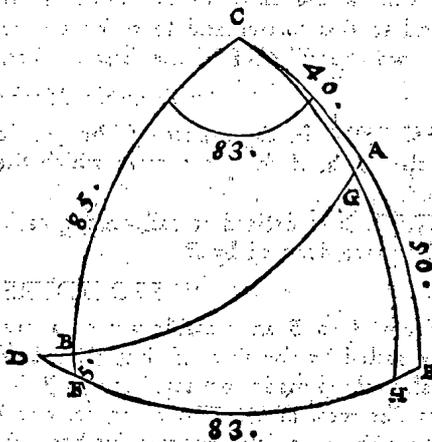
VERVOLGH.

Soo de vaert moet ghedaen ſijn opt middelront, t'is kennelick datmen altijd ſoude moeten varen recht ooft of weſt; Maer moerende op een middachront gheſchien, datmen dan altijd recht zuyt of recht noort ſoude moeten varen.

2 Voorbeeds wiſconſtich.

T O H E G H E Y E N. Laet A en B twee plaetſen op den eertcloon beteyckenen, A daer t ſchip af vaert, B daer ſijn moet, C den apunt, D E het middelront, de breede van A is E A 50 tr. en van B is F B 5 tr. en t verſchil haerder langden is F E 83 tr.

T B E G H E E R D E. Men wil een rechte ftreec feylen vande plaets beteyckent met A, totte plaets beteyckent met B, en dat wiſconſtelick vinden, te weten deur rekening der clootſche driehoucken.



Bereyſel van t eerſte deel des overcx.

Ick treck van A tot B een grootſte ronts booch beteycknende de rechte ftreec die t ſchip feylen moet: Daer nae de ſelve A B voorwaert tot datſe het middelront onmoet, t welck ſy in D: Daer nae den booch E A voorwaert tot

the vertical circle is again made to pass through the point B , it indicates in the horizon that B lies from H , I assume, 3 degrees from west to south, and accordingly one has to sail this course from H to B again by conjecture some 4 or 5 degrees further, I assume as far as I . And when the ship has arrived there, one has to do there again as at H , where it will be found that one will have to sail to B even more to the south than one did at H . And if the same is done until one arrives at the place B , one has sailed on the required great-circle track AB , provided the distances AH and HI and the like have been taken small enough. For even if instead of on AH , which is a part of a great circle, one sailed on a loxodrome, which is a part of a small circle, arriving slightly more to the north, and if further, instead of on the other parts of the arc AB , one sailed on other loxodromes, which are parts of spirals, yet by taking these parts small enough one can cause this difference to be of no account.

Note that the procedure may be checked as follows: When the ship has arrived, I assume, at H , and when it is then found by observation of the sun or the stars that the latitude on the earth corresponds to the latitude of H on the globe, this gives one reason to assume that the ship has rightly sailed on the great-circle track, which has to follow if the conjecture has been right.

SEQUEL.

If the sailing had to take place on the equator, it is obvious that one ought always to sail due east or west. But if it had to take place on a meridian, it is obvious that one ought always to sail due south or due north.

2nd Example, Mathematical.

SUPPOSITION. Let A and B denote two places on the earth, A the place from which the ship sails, B the place for which it is bound, C the pole, DE the equator; the latitude of A is $EA = 50^\circ$, and that of B is $FB = 5^\circ$, and their difference of longitude is $FE = 83^\circ$.

REQUIRED. It is desired to sail on a great-circle track from the place denoted by A to the place denoted by B , and to find this mathematically, *viz.* by means of a calculation of spherical trigonometry.

Preliminary of the First Part of the Procedure.

I draw from A to B an arc of a great circle, denoting the great-circle course on which the ship has to sail. After this I produce the said AB until it meets the equator; let this be in D . After this I produce the arc EA until it meets the pole C ; then AC will be 40° , for when we subtract $EA = 50^\circ$ from $EC = 90^\circ$,

tot datse den aspunt C ontmoet, en sal A C doen 40 tr. want van E C 90 tr. ghetrocken E A 50 tr. blijft voor A C 40 tr. S'ghelijcx treck ick F B voorwaert tot datse den aspunt C ontmoet, en sal B C doen 85 tr. want van F C 90 tr. ghetrocken F B 5 tr. blijft voor B C 85 tr. en den houck B C A, diens grootheyt megebracht wort van F E 83 tr. doet als de selve oock 83 tr.

1 *Deel des wercx.*

Om eerst te vinden op wat streck men sal beginnen te seylen van A na B, so moet ick weten de grootheyt des houcx C A B, want soo veel salmen moeten seylen van noorden na westen. Om daer toe te commen, soo heeft den selven driehouck C A B drie bekende palen, deur t'bereytsel, te weten den houck B C A 83 tr. de sijde A C 40 tr. en B C 85 tr. Hier me ghesocht de drie onbekende palen, worden bevonden deur het 40 voorstel der clootsche driehoucken, te weten den houck C A B 92 tr. 8 ①, de langde A B van d'een plaets tot d'ander 81 tr. 41 ①, en den houck C B A 39 tr. 45 ①. Nu soo veel als doet den voorschreven houck C A B, te weten 92 tr. 8 ①, soo veel salmen van A af moeten beginnen te seylen van noorden over westen na zuyden, dat is 87 tr. 52 ① van zuyden na westen, welcke seyling gheduert neem ick 4 tr. verre tot G toe, sulcx dat A G doet de selve 4 tr.

2 *Bereytsel dienende tottet 2 deel des wercx.*

Angheven dat de vinding der onbekende palen eens driehoucx sonder gheheven rechthouck als de voorgaende, moeyelicker valt dan met een ghegeven rechthouck, so sullen wy een bereytsel stellen om int volgende te wercken deur driehoucken met een ghegeven rechthouck, aldus: De driehouck B F D heeft drie bekende palen, te weten den houck B F D recht, mette sijde F B 5 tr. deur t'ghegeven, en den houck D B F even sijnde metten houck C B A, doet deur t'eerste deel des wercx 39 tr. 45 ①: Hier me ghesocht den houck D, en de sijde D B, worden bevonden deur het 34 voorstel der clootsche driehoucken, te weten den houck D 50 tr. 26 ①, en de sijde B D 6 tr. 27 ①, die vergaert tot A B 81 tr. 41 ①, comt voor A D 88 tr. 8 ①.

2 *Deel des wercx.*

Om te vinden op wat streck men sal beginnen te seylen van G na B, ick treck van C deur G tot int middelront E F de booch C G H als middachront, waert me G H D een rechthouckich driehouck is, hebbende drie bekende palen, te weten den houck G H D recht, den houck D 50 tr. 26 ① deur het 2 bereytsel, en de sijde G D 84 tr. 8 ①, want A D doet 88 tr. 8 ① deur het 2 bereytsel, daer af ghetrocken A G doende 4 tr. deur t'eerste deel des wercx, blijft alsboven voor G D 84 tr. 4 ①: Hier me ghesocht den houck H G D, wort bevonden deur het 34 voorstel der clootsche driehoucken van 87 tr. 45 ①: En op sulcken streck van zuyden na westen moetmen van G seylen na B, t'wele 7 ① zuydelicker is danmen van A tot G seyde, want ghetrocken 87 tr. 45 ①, van 87 tr. 52 ①, blijft de selve 7 ①. Nu dan van G aldus gheseylt hebbende soo verre men oirboir verstaet, men sal om voorder te seylen daer weerom doen als an G gedaen wiert, en dergelijcke tot ander plaetsen soo lang datmen tot B comt.

MERCKT ten 1 dat hoewel de boveschreven 7 ① zuydelicker te seylen so weynich is, dattet met een seylende schip niet gageflagen en can worden, doch

ver-

the remainder is 40° for AC . In the same way I produce FB until it meets the pole C ; then BC will be 85° , for when we subtract $FB = 5^\circ$ from $FC = 90^\circ$, the remainder is 85° for BC ; and the angle BCA , whose magnitude follows from $FE = 83^\circ$, like the latter is also 83° .

1st Part of the Procedure.

In order to find first what course one has to start sailing from A to B , I have to know the magnitude of the angle CAB , for thus much one will have to sail from north to west. To find this, the said triangle CAB has three known elements, by the preliminary, *viz.* the angle $BCA = 83^\circ$, the side $AC = 40^\circ$, and $BC = 85^\circ$. When herewith the three unknown elements are sought, they are found by the 40th proposition of spherical trigonometry¹⁾, *viz.* the angle $CAB = 92^\circ 8'$, the distance AB from one place to the other $81^\circ 41'$, and the angle $CBA = 39^\circ 45'$. Now as much as the aforesaid angle CAB amounts to, *viz.* $92^\circ 8'$, so much one will have to start sailing from A , from the north *via* the west to the south, *i.e.* $87^\circ 52'$ from south to west, which sailing continues, I assume, 4° further to G , so that AG is the said 4° .

2nd Preliminary, Serving for the 2nd Part of the Procedure.

Since the finding of the unknown elements of a triangle without a given right angle, like the foregoing, is more difficult than with a given right angle, we shall give a preliminary to operate in the following by means of triangles with a given right angle, as follows. The triangle BFD has three known elements, *viz.* the angle $BFD = 90^\circ$, with the side $FB = 5^\circ$, by the supposition, and the angle DBF , being equal to the angle CBA , by the first part of the procedure is $39^\circ 45'$. When herewith the angle D and the side DB are sought, they are found by the 34th proposition of spherical trigonometry²⁾, *viz.* the angle $D = 50^\circ 26'$ and the side $BD = 6^\circ 27'$; when we add the latter to $AB = 81^\circ 41'$, we get $88^\circ 8'$ for AD .

2nd Part of the Procedure.

In order to find what course one should start sailing from G to B , I draw from C through G to the equator EF the arc CGH as meridian, in consequence of which GHD is a right-angled triangle having three known elements, *viz.* the angle $GHD = 90^\circ$, the angle $D = 50^\circ 26'$, by the 2nd preliminary, and the side $GD = 84^\circ 8'$, for AD is $88^\circ 8'$ by the 2nd preliminary, and when from this we subtract AG , being 4° by the first part of the procedure, the remainder is, as above, $84^\circ 4'$ for GD . When herewith the angle HGD is sought, it is found by the 34th proposition of spherical trigonometry³⁾ to be $87^\circ 45'$. And that course from south to west one has to sail from G to B , which is $7'$ more to the south than the sailing from A to G , for when we subtract $87^\circ 45'$ from $87^\circ 52'$, this $7'$ is left. Now therefore, after having sailed thus from G as far as is considered expedient, in order to continue one has to do again as one did at G , and similarly at other places until one arrives at B .

NOTE in the first place that although the sailing of the above-mentioned $7'$ more to the south is so short a distance that it cannot be observed in a sailing

¹⁾ Stevin's *Trigonometry* (Work XI; i, 13), p. 295.

²⁾ *ibid.*, p. 245.

³⁾ *ibid.*, p. 245.

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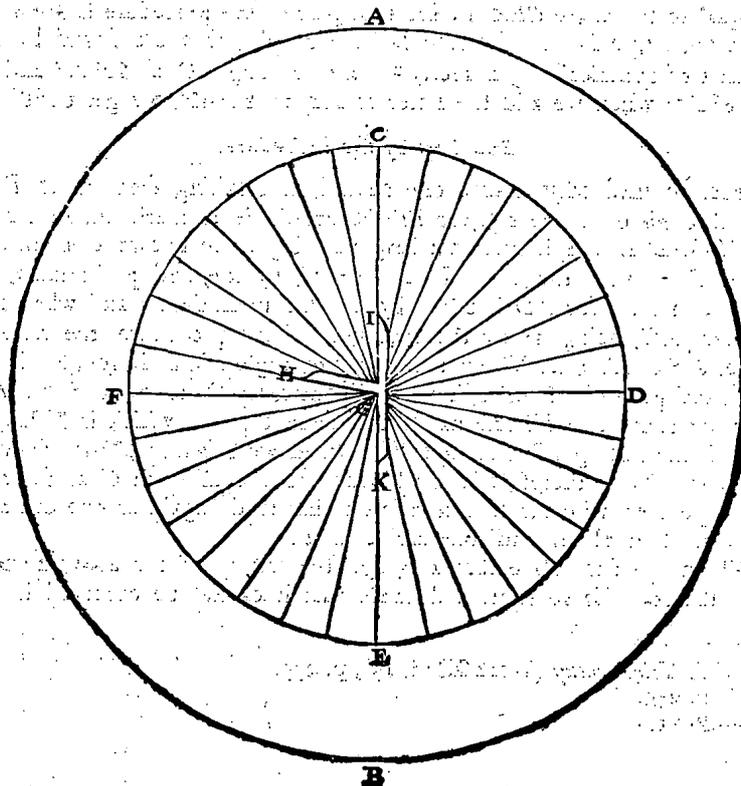
verfaetmen daer deur datmen de volghende booch van G voorwaert groeter mach nemen dan 4 tr. Maer soo d'eerste booch te groot had genomen geweest, sulcx datmen op een minder met sekerheyt seylen can, tis kennelic datmen dan de rekening op een minder booch behoort te maken. Noch staet te gedencken datter in dit voorbeeld op even boghen meerder verandering valt by t'punt B, dan verder daer af, want by B commende, men sal moeten van zuyden na westen seylen alleenlick 39 tr. 45 ^①, (deur dien den houck D B F soo groot is) t'welck 48 tr. 7 ^② zuydelicker is dan doenmen an A begoft, alwaer den houck B A E bevonden wiert van 87 tr. 52 ^③.

MERCK T ten 2 dat soomen begeerde te weten de breedte van t'punt G, om t'onderfoucken offe deur dadelicke ervaring soo bevonden wort, als van dergelike int i tuychwerckelick voorbeeld gheseyt is, men soude hier boven benevens den houck D G H des driehoucx D G H, noch vinden de sijde GH, wantse de begheerde breedte anwijst. T B E S L V Y T. Wy hebben dan op rechte streken gheseylt na den eyfch.

3 VOORSTEL.

Cromstreken tuychverckelick te teyckenen.

De dadelicke Eertcloodmakers ghebruycken verscheyden middelen en reetschappen totte teyckening der cromstreken elck dat hem best bevalt: Een van



dicn

ship, yet it is understood thereby that the next arc from G on may be taken larger than 4° . But if the first arc had been taken too large, so that a smaller arc appears necessary in order to sail with accuracy, it is obvious that one must then make the calculation on a smaller arc. It should also be borne in mind that in this example with equal arcs the change is greater at the point B than further away, for when one gets to B , one will have to sail from south to west only $39^\circ 45'$ (because the angle DBF has this magnitude), which is $48^\circ 7'$ more to the south than when one started at A , where the angle BAE was found to be $87^\circ 52'$.

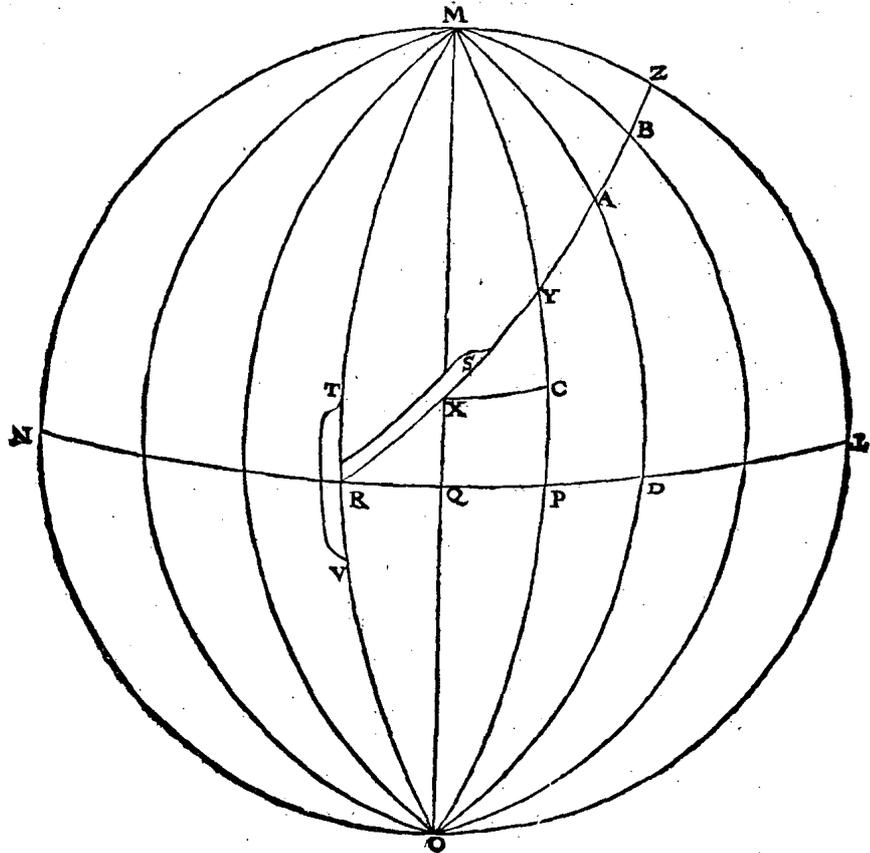
NOTE in the second place that if it were required to know the latitude of the point G , so as to ascertain whether it is found the same by observation as it has been said in the 1st mechanical example, in the above one would have to find, in addition to the angle DGH of the triangle DGH , the side GH , because it designates the required latitude. CONCLUSION. We have thus sailed on great-circle tracks; as required.

3rd PROPOSITION.

To draw loxodromes mechanically.

Practical globe-makers use various means and tools for drawing loxodromes, each taking that which suits him best. We shall here explain one of them, not in

dien sullen wy hier verclaren, niet om inde daer naghevolght te worden, maer om dattet wel uytdruckt den gront van t'ghene begheert is, en daer na beter gedaen moet sijn. Laet A B een cloot sijn, hier op beschrijf ick eenich cleender rondt C D E F, diens middelpunt G, welck rondt ick deel in 32 even deelen, treckende van daer tottet middelpunt G 32 boghen, die my de 32 ghemeene streken beteycken. Hierin an sien ick C G E voor de booch van noort na zuyt, en F G D daer op rechthouckich voor de booch van west na oost. Dit soo sijnde ick maeck een coper clootsche scheefhouck G H I K op den cloot passende, en hebbende de scheefheyte van een streck, want soo veel doet den houck F G H. Nu ghelijck hier ghemaect is de clootsche scheefhouck van een streck, alsoo sal mender meer maken tot seven toe, te weten voor elcke streck een die tuschen F C commen. Dese seven clootsche scheef houcken bereyt sijnde, men sal nemen een ander cloot L M N O vande selve grootheyte als A B, alwaer LN het middelront beteyckent, M den noortschen aspunt, O den zuytschen, tuschen dese twee aspunten sijn middachronden ghetrocken als M P O, M Q O,



M R O, snyende het middelront van trap tot trap inde punten P, Q, R. Hier op teycken ick de cromstreecken als volght: Ghenomen dat ick eerst wil hebben de cromme noortooststreck, soo neem ick uyt de boveschreven seven coperen cloot.

order that it may be imitated in practice, but because it shows very well the foundation of what is required, and because hereafter it can be done better. Let AB be a globe; on this I describe a small circle $CDEF$, the centre of which is G , which circle I divide into 32 equal parts, drawing from there to the centre G 32 arcs, which denote the 32 common points of the compass. Herein I look upon CGE as the arc from north to south, and FGD , at right angles thereto, as the arc from west to east. This being so, I make an oblique spherical angle of copper $GHIK$, fitting on the globe and having the obliquity of one point, for that is the magnitude of the angle FGH . Now just as here the oblique spherical angle of one point has been made, in the same way others have to be made, up to seven — *viz.* one for each point — which come between F and C . When these seven oblique spherical angles have been made, another globe $LMNO$ of the same size as AB must be taken, where LN denotes the equator, M the north pole, O the south pole; between these two poles have been drawn meridians, *viz.* MPO , MQO , MRO , intersecting the equator from degree to degree in the points P , Q , R . Upon this I draw the loxodromes as follows: Assuming that I first want to have the loxodrome of northeast, I take out of the above-mentioned seven oblique

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clootsche scheefhoucken dien welke de noortoofstreeck beteyckent, de selve sy R S T V, diens sijde T V vervought is op een der middachronden, als op M R O, soo dat den houck des coperen cloothoucx past op R ghemeene sine des middachronts en middelronts L N, en treck van R langs R S een liniken tottet naeste middachront als tot X: vervough daer na den coperen scheefhouck opt middachront M Q O, en also dattet houckpunt R dan comme an X: Treck daer na van X langs de voorschreven scheefhoucx sijde het liniken X Y. En alsoo voortgaende na Z tot datmen den aspunt na ghenouch is, of gheraect; men sal de cromme noortoofstreeck op den Eertcloot gheteyckent hebben: Wy segghen hier boven tot datmen den aspunt na ghenouch is, of gheraect, doch wisconstelick ghesproken en can niet gherocht worden, want de slangtreck soude oneyndelick daer rontom loopen en alijt naerderen sonder gheraken; Maer

Mechanic.

* tuychwerckelick can een sichtbaer aspunt gherocht worden.

Deur t'ghene wy tot hier toe gheseyt hebben vande teyckening der noortoofstreeck, is openbaer de teyckening van al d'ander cromstreken, en kennelick hoemen tot alle plaetsen eens Eertcloots de cromme seylstreken teyckenen sal na sijn wille. T B E S L V Y T. Wy hebben dan cromstreken tuychwerckelick gheteyckent, na den eysch,

Vande onsekerheyt inde voorgaende wyjse van teyckening.

Want deur de gheduerighe en menichvuldighe versetting van desen coperen scheefhouck R S T V onsekerheyt int werck can volghen, alsoo om dergelijcke redenen oock can in meer ander tuych tot sulcken eynde ghemaect, of alwaerder sekerheyt in dat sulcx onbewesen blijft: Soo ist te weten dat wy die manier alleenlick hier ghestelt hebben, eensdeels op datmen daer deur verstaet de onsekerheyt dieder is inde cromstreken alsoo op Eertclooten gheteyckent. Ten anderen om dattet wel verclaert de gront en eyghenschappen der cromstreken, daermen op bouwen mach wisconstighe wercking, deur welke de tuychwerckelicke meerder sekerheyt can krijghen, als blijcken sal, eerst beschreven sijnde de tafels als volght.

4 VOORSTEL.

Tafel der cromstreken te maken.

De somme deses voorstels is, dat wy moeten vinden in ghetalen, hoe lanck dat sijn de boghen als inde form des 3 voorstels Q X, P Y, en dergelijcke, want die ghetalen bekennt wesen, en na den eysch van dien punten gheteyckent als X, Y, a, b, Z, en van t'een punt tottet ander linikens ghetrocken, men krijcht de begheerde cromstreeck. Het vinden deser bogen soude mcugen aldus toegaen:

EERSTE MAECKSEL VANDE
TAFELS DER CROMSTREKEN.

Laet R Z noch eens de vierde cromstreeck beteyckenen, daer af wy vinden willen de boghen Q X, P Y: Tot desen eynde segh ick dat de driehouck X Q R drie bekende palen heeft, te weten den houck X R Q 45 tr. den houck X Q R recht, en de sijde R Q 1 tr. Hier me ghefocht de sijde Q X, wort bevonden deur het 36 voorstel der clootsche driehoucken van 59 ① 59 ②. Om nu te vinden de lini

spherical angles of copper the one which denotes the loxodrome of northeast; let this be $RSTV$, whose side TV has been placed on one of the meridians, *viz.* on MRO , so that the vertex of the oblique spherical angle of copper comes in R , the point of intersection of the meridian and the equator LN , and I draw from R along RS a short line to the next meridian, *viz.* X . Then place the oblique angle of copper on the meridian MQO , in such a way that the vertex R then comes in X , thereafter draw from X along the aforesaid side of the oblique angle the short line XY . And proceeding in this way to Z until one is near enough to the pole or reaches it, one has drawn the loxodrome of northeast on the globe. We said above: until one is near enough to the pole or reaches it, but mathematically speaking it is impossible to reach it, for the spiral must pass around it infinitely and always approach it without reaching it. But mechanically speaking it is possible to reach a visible pole.

From all that we have hitherto said about the drawing of the loxodrome of northeast it is evident how all the other loxodromes have to be drawn and it can be known how in any place of a globe we shall draw the loxodromes we desire. **CONCLUSION.** We have thus drawn loxodromes mechanically; as required.

Of the Uncertainty in the Foregoing Method of Drawing.

Because the constant and frequent displacement of this copper oblique angle $RSTV$ may result in uncertainty in the procedure, as may also happen for the same reasons with other tools made for this purpose — or even if the procedure were certain, this remains unproved — it is to be noted that we have only given this method here, on the one hand in order that the uncertainty in the loxodromes thus drawn on globes may be understood thereby; on the other hand because it explains very well the foundation and the properties of loxodromes on which mathematical operations can be based, owing to which the mechanical method may become more certain, as will appear when first the tables have been described, as follows.

4th PROPOSITION.

To make a table of the loxodromes.

The gist of this proposition is that we have to find the numerical values of the lengths of the arcs, *viz.* in the figure of the 3rd proposition QX , PY , and the like, for when these values are known and points have been marked accordingly, *viz.* X , Y , A , B , Z , and short lines have been drawn from one point to the other, the required loxodrome is obtained. The finding of those arcs might take place as follows.

FIRST METHOD OF MAKING THE TABLES OF THE LOXODROMES.

Let RZ once again denote the fourth loxodrome, from which we want to find the arcs QX and PY . To this end I say that the triangle XQR has three known elements, *viz.* the angle $XRQ = 45^\circ$, the angle $XQR = 90^\circ$, and the side $RQ = 1^\circ$. When herewith the side QX is sought, it is found by the 36th proposition of spherical trigonometry ¹⁾ to be $59'59''$. Now in order to find the line PY , I draw the arc XC parallel to QP ; then PC will also be $59'59''$, like QX ,

¹⁾ Stevin's *Trigonometry* (Work XI; i, 13), p. 255.

de lini PY, ick treck de booch X c ewijdich met QP, en sal P c dan oock doen 59 ① 59 ②, gelijk Q X: Sulcx datter vanden driehouck Y c X, gevonden moet worden de sijde c Y, om die te vergaren tot P c, en dan te hebben de booch P Y: Hier toe heeft den selven driehouck Y c X drie bekende palen, te weten den houck Y X c 45 tr. den houck Y c X recht, en de sijde X c 59 ① 58 ②, want so veel doet dien langdetrap buyten t' middelront deur de ghemeene tafel diemen daer af maeckt, en hier na oock volghen sal; daer me ghesocht de sijde c Y, wort bevonden deur het 36 voorstel der clootsche driehoucken van 59 ① 57 ②, die vergaert tot P c 59 ① 59 ②, comt voor P Y 1 tr. 59 ① 56 ②, en soo voort met dan d'ander.

TWEDE MAECKSEL VANDE
TAFELS DER CROMSTREKEN.

Anghesien het maken van volcommen tafels na de voorgaende eerste wijze, langher soude vallen dan my den tijt toelaet, soo sullen wy een ander stellen, beschreven en onlanx uytghegheven deur *Edward Wright*, want hoewelse eenige onvolcommenheit hebben daer wy inden Anhang der cromstreken breeder af segghen sullen, nochtans connense tot verclaring des voornemens dienen.

Tottet makèn vande volgende tafels der cromstreken wort eerst beschreven als bereytsel een tafel der versaemde snylijnen van 10 ① tot 10 ② aldus:

De snylijn van 10 ① doet.	10000042.
Daer toe de snylijn van 20 ① doende 10000168 comt	20000210.
Daer toe de snylijn van 30 ① doende 10000381 comt	30000591.

En so voort; maer eyntlick salmen overal de vijf laetste letters affynen, en sal een tafel sijn als volght:

I TAFEL

so that of the triangle YCX the side CY has to be found, in order to add it to PC and then have the arc PY . For this, the said triangle YCX has three known elements, *viz.* the angle $YXC = 45^\circ$, the angle $YCX = 90^\circ$, and the side $XC = 59'58''$, for that is the value of this degree of longitude outside the equator by the common table which is made thereof and which is to follow hereafter. When herewith the side CY is sought, it is found by the 36th proposition of spherical trigonometry ¹⁾ to be $59'57''$. When we add this to $PC = 59'59''$, we get $PY = 1^\circ 59'56''$, and so on with the others.

SECOND METHOD OF MAKING THE TABLES OF THE LOXODROMES.

Since the making of complete tables by the foregoing first method would take longer than time permits me, we shall give another method, described and recently published by *Edward Wright* ²⁾, for although they have certain imperfections, which we shall discuss more fully in the Appendix of Loxodromes, yet they may serve to explain our intention.

With a view to the making of the following tables of loxodromes, by way of preliminary a table is first described of the assembled secants, increasing by $10'$, as follows:

The secant of $10'$ is	10,000,042
If to this we add the secant of $20'$, being 10,000,168, we get	20,000,210
If to this we add the secant of $30'$, being 10,000,381, we get	30,000,591

And so on; but finally the five last digits must be discarded, and then the table will be as follows:

TABLE OF ASSEMBLED SECANTS

degrees minutes . secants

¹⁾ Stevin's *Trigonometry* (Work XI; i, 13), p. 255.

²⁾ Cf. *Introduction*, p. 482-483.

tr.	①	snjlinen.	tr.	①	snjlinen.	tr.	①	snjlinen.
0	10	100	5	10	31	10	10	6132
0	20	200	5	20	3205	10	20	6234
0	30	300	5	30	3305	10	30	6335
0	40	400	5	40	3405	10	40	6437
0	50	500	5	50	3506	10	50	6539
1	0	600	6	0	3606	11	0	6641
1	10	700	6	10	3707	11	10	6743
1	20	800	6	20	3808	11	20	6845
1	30	900	6	30	3908	11	30	6947
1	40	1000	6	40	4009	11	40	7049
1	50	1100	6	50	4110	11	50	7151
2	0	1200	7	0	4210	12	0	7253
2	10	1300	7	10	4311	12	10	7355
2	20	1400	7	20	4412	12	20	7458
2	30	1500	7	30	4513	12	30	7560
2	40	1601	7	40	4614	12	40	7662
2	50	1701	7	50	4715	12	50	7765
3	0	1801	8	0	4815	13	0	7868
3	10	1901	8	10	4916	13	10	7970
3	20	2001	8	20	5018	13	20	8073
3	30	2101	8	30	5119	13	30	8176
3	40	2201	8	40	5220	13	40	8279
3	50	2302	8	50	5321	13	50	8382
4	0	2402	9	0	5422	14	0	8485
4	10	2502	9	10	5523	14	10	8588
4	20	2602	9	20	5625	14	20	8691
4	30	2703	9	30	5726	14	30	8794
4	40	2803	9	40	5827	14	40	8897
4	50	2903	9	50	5929	14	50	9001
5	0	3004	10	0	6030	15	0	9104

VERSAEMDE SNYLINEN.

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nr.	①	snylinen.	nr.	①	snylinen.	nr.	①	snylinen.
15	10	9208	20	10	12358	25	10	15610
15	20	9312	20	20	12464	25	20	15721
15	30	9415	20	30	12571	25	30	15832
15	40	9519	20	40	12678	25	40	15942
15	50	9623	20	50	12785	25	50	16053
16	0	9727	21	0	12892	26	0	16165
16	10	9831	21	10	12999	26	10	16276
16	20	9935	21	20	13106	26	20	16388
16	30	10039	21	30	13213	26	30	16499
16	40	10144	21	40	13321	26	40	16611
16	50	10248	21	50	13429	26	50	16723
17	0	10353	22	0	13537	27	0	16835
17	10	10457	22	10	13645	27	10	16947
17	20	10562	22	20	13753	27	20	17060
17	30	10667	22	30	13861	27	30	17173
17	40	10772	22	40	13969	27	40	17285
17	50	10877	22	50	14078	27	50	17398
18	0	10982	23	0	14186	28	0	17512
18	10	11087	23	10	14295	28	10	17625
18	20	11192	23	20	14404	28	20	17738
18	30	11298	23	30	14513	28	30	17852
18	40	11403	23	40	14622	28	40	17966
18	50	11509	23	50	14731	28	50	18080
19	0	11615	24	0	14840	29	0	18194
19	10	11720	24	10	14950	29	10	18309
19	20	11826	24	20	15060	29	20	18423
19	30	11932	24	30	15170	29	30	18538
19	40	12038	24	40	15280	29	40	18653
19	50	12145	24	50	15390	29	50	18768
20	0	12251	25	0	15500	30	0	18884

nr.	①	snjlinen.	nr.	①	snjlinen.	nr.	①	snjlinen.
30	10	18999	35	10	22565	40	10	26358
30	20	19115	35	20	22688	40	20	26489
30	30	19231	35	30	22811	40	30	26621
30	40	19347	35	40	22934	40	40	26752
30	50	19464	35	50	23057	40	50	26884
31	0	19580	36	0	23180	41	0	27017
31	10	19697	36	10	23304	41	10	27149
31	20	19814	36	20	23428	41	20	27282
31	30	19931	36	30	23552	41	30	27416
31	40	20048	36	40	23677	41	40	27549
31	50	20166	36	50	23802	41	50	27683
32	0	20284	37	0	23927	42	0	27818
32	10	20402	37	10	24052	42	10	27953
32	20	20520	37	20	24178	42	20	28088
32	30	20639	37	30	24304	42	30	28223
32	40	20757	37	40	24430	42	40	28359
32	50	20876	37	50	24556	42	50	28495
33	0	20995	38	0	24683	43	0	28632
33	10	21115	38	10	24810	43	10	28769
33	20	21234	38	20	24938	43	20	28906
33	30	21354	38	30	25065	43	30	29044
33	40	21474	38	40	25193	43	40	29182
33	50	21594	38	50	25321	43	50	29320
34	0	21715	39	0	25450	44	0	29459
34	10	21836	39	10	25579	44	10	29598
34	20	21957	39	20	25708	44	20	29738
34	30	22078	39	30	25837	44	30	29878
34	40	22199	39	40	25967	44	40	30018
34	50	22321	39	50	26097	44	50	30159
35	0	22443	40	0	26228	45	0	30300

VERSAEMDE SNYLINEN. 101

nr.	①.	snylinen.	nr.	①.	snylinen.	nr.	①.	snylinen.
45	10	30442	50	10	34902	55	10	39857
45	20	30584	50	20	35058	55	20	40032
45	30	30726	50	30	35215	55	30	40208
45	40	30869	50	40	35373	55	40	40385
45	50	31013	50	50	35531	55	50	40563
46	0	31156	51	0	35690	56	0	40741
46	10	31301	51	10	35849	56	10	40921
46	20	31445	51	20	36009	56	20	41101
46	30	31590	51	30	36169	56	30	41282
46	40	31736	51	40	36330	56	40	41463
46	50	31882	51	50	36491	56	50	41646
47	0	32028	52	0	36654	57	0	41829
47	10	32175	52	10	36816	57	10	42013
47	20	32322	52	20	36980	57	20	42198
47	30	32470	52	30	37144	57	30	42384
47	40	32618	52	40	37308	57	40	42570
47	50	32767	52	50	37473	57	50	42758
48	0	32916	53	0	37639	58	0	42946
48	10	33066	53	10	37806	58	10	43135
48	20	33216	53	20	37973	58	20	43325
48	30	33367	53	30	38141	58	30	43516
48	40	33518	53	40	38309	58	40	43708
48	50	33670	53	50	38478	58	50	43901
49	0	33822	54	0	38648	59	0	44095
49	10	33975	54	10	38819	59	10	44289
49	20	34128	54	20	38990	59	20	44485
49	30	34282	54	30	39162	59	30	44681
49	40	34436	54	40	39334	59	40	44879
49	50	34591	54	50	39508	59	50	45078
50	0	34746	55	0	39682	60	0	45277

nr.	①.	snjlinen.	nr.	①.	snjlinen.	nr.	①.	snjlinen.
60	10	45478	65	10	52030	70	10	59960
60	20	45679	65	20	52269	70	20	60257
60	30	45882	65	30	52510	70	30	60555
60	40	46085	65	40	52752	70	40	60856
60	50	46290	65	50	52995	70	50	61159
61	0	46496	66	0	53241	71	0	61465
61	10	46703	66	10	53487	71	10	61774
61	20	46911	66	20	53736	71	20	62085
61	30	47120	66	30	53986	71	30	62399
61	40	47330	66	40	54237	71	40	62716
61	50	47541	66	50	54491	71	50	63035
62	0	47754	67	0	54746	72	0	63357
62	10	47967	67	10	55003	72	10	63682
62	20	48182	67	20	55262	72	20	64011
62	30	48398	67	30	55522	72	30	64342
62	40	48616	67	40	55784	72	40	64676
62	50	48834	67	50	56049	72	50	65014
63	0	49054	68	0	56315	73	0	65354
63	10	49275	68	10	56583	73	10	65698
63	20	49497	68	20	56853	73	20	66045
63	30	49720	68	30	57124	73	30	66396
63	40	49945	68	40	57398	73	40	66750
63	50	50171	68	50	57674	73	50	67107
64	0	50399	69	0	57953	74	0	67468
64	10	50628	69	10	58233	74	10	67833
64	20	50858	69	20	58515	74	20	68202
64	30	51090	69	30	58800	74	30	68574
64	40	51323	69	40	59086	74	40	68950
64	50	51557	69	50	59375	74	50	69331
65	0	51793	70	0	59667	75	0	69715

VERSAEMDE SNYLINEN.

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tr.	①.	snylinen.	tr.	①.	snylinen.	tr.	①.	snylinen.
75	10	70104	80	10	84354	85	10	108865
75	20	70497	80	20	84945	85	20	110075
75	30	70894	80	30	85546	85	30	111328
75	40	71296	80	40	86158	85	40	112630
75	50	71703	80	50	86781	85	50	113982
76	0	72114	81	0	87415	86	0	115389
76	10	72530	81	10	88061	86	10	116856
76	20	72951	81	20	88719	86	20	118389
76	30	73377	81	30	89389	86	30	119993
76	40	73808	81	40	90073	86	40	121675
76	50	74245	81	50	90771	86	50	123444
77	0	74687	82	0	91483	87	0	125209
77	10	75134	82	10	92210	87	10	127180
77	20	75588	82	20	92952	87	20	129272
77	30	76047	82	30	93711	87	30	131498
77	40	76512	82	40	94486	87	40	133879
77	50	76984	82	50	95280	87	50	136437
78	0	77462	83	0	96091	88	0	139200
78	10	77947	83	10	96923	88	10	142205
78	20	78438	83	20	97775	88	20	145497
78	30	78937	83	30	98648	88	30	149139
78	40	79442	83	40	99544	88	40	153213
78	50	79955	83	50	100464	88	50	157834
79	0	80476	84	0	101409	89	0	163176
79	10	81004	84	10	102380	89	10	169501
79	20	81541	84	20	103380	89	20	177259
79	30	82083	84	30	104409	89	30	187284
79	40	82639	84	40	105471	89	40	201513
79	50	83201	84	50	106565	89	50	226223
80	0	83773	85	0	107696	90	0	000000

104 4 BOVCK DES EERTCLOOTSCHRIFTS,

Dit bereytsel vande tafel der versaemde snijlijnen aldus ghedaen sijnde, en om nu tottet maken vande tafels der cromstreken te commen, soo laet inde voorgaende form RZ beteyckenen d'eerste cromstreeck, sulcx dat den houck X R Q des driehoucx X R Q nu doe 78 tr. 45 ①. Om te vinden de booch Q X, ick aensie de driehouck X R Q voor plat, om de cleenheyt der sijden, en segh datse drie bekende palen heeft, te weten den houck X Q R recht, X R Q 78 tr. 45 ①, de sijde Q R 1 tr. Hier me ghesocht de sijde Q X wort bevonden deur het 4. voorstel der platte driehoucken van 5 tr. 1 ①, die ick inde volghende tafel van d'eerste Cromstreeck stel byde breeden nevens 1 tr. der langde. Om nu al de volghende breeden deses tafels met cortheyt te vinden, ick sie inde voorgaende tafel der versaemde snijlijnen wat ghetal datter overcomt mette boveschreven 5 tr. 1 ①, en bevinde 3014, want de 5 tr. hebben 3004, en noch 10 sijn het everedelic deel voor de 1 ①. Dit ghetal van 3014 dient my int gemeen tottet vinden der ghetalen van P Y, 44, en al d'ander diergheelicke, t'welck aldus toegaet: Totte 3014, vergaert ander 3014, comt 6028, daer op vinde ick t'overcommen inde voorgaende tafel der versaemde snijlijnen 10 tr: De selve stel ick inde volghende tafel van d'eerste cromstreeck byde breeden nevens den 2 tr. der langde, als voor P Y. Daer na vergaert ick totte 6028, andermael 3014, comt 9042, daer op vinde ick t'overcommen inde voorgaende tafel 14 tr. 54 ①, de selve stel ick inde volghende tafel nevens den 3 tr. der langde, als voor d 4, En so voort mette rest der seven Cromstreken.

MERCKT dat ick totte voorgaende langden en breeden der cromstreken, noch vervoughe haer verheden, dat sijn de langden der bogen R X, R Y, R 4, en diergheelijcke om deur t'behulp der selve sonder eertclood of platte caert, maer alleenlick deur ghetalen, te beantwoorden de voorstellen die vanden handel der cromstreken omgaen, en int volghende beschreven sullen sijn. Dese verheden worden aldus bekend. Om ten eersten te vinden de verheyt R X, ick segh den driehouck X Q R te hebben drie bekende palen, te weten den houck X Q R recht, den houck X R Q 78 tr. 45 ①, en de sijde Q X van 5 tr. 1 ①. Hier me ghesocht de sijde R X, wort bevonden deur het 4. voorstel der platte driehoucken van 5 tr. 6 ① 54 ②, die ick stel in d'eerste tafel by de verheden nevens 1 tr. der langde. Ten anderen om te vinden de verheyt R Y, ick segh den driehouck Y ɛ X te hebben drie bekende palen, te weten den houck Y ɛ X recht, Y X ɛ 78 tr. 45 ①, en de sijde ɛ Y 4 tr. 59 ①, als blijkt deur de tafel, want treckende P ɛ 5 tr. 1 ①, als even sijnde met Q X, van P Y 10 tr. blijft voor ɛ Y alfooren 4 tr. 59 ①: Met dese drie bekende palen dan, ghesocht de sijde X Y, wort bevonden deur het 4. voorstel der platte driehoucken van 5 tr. 12 ① 54 ②, die vergaert tot R X 5 tr. 6 ① 54 ②, comt voor R Y 10 tr. 19 ① 48 ②, die ick stel in d'eerste cromstreeck byde verheden nevens 2 tr. der langde. En alsoo sal ghevonden worden de verheyt van R 4, met al d'ander.

Merckt dat wy dese verheden niet overal berekent noch ghestelt en hebben, maer alleenlick soo veel als tot ons volghende voorbeelden noodich sijn, eenfdeels dat de tafelen selgheen ghenouchsaem volcommenheyt en schijnen te hebben, ghelijck inden Anhang breeder gheseyt sal worden, als oock dat belet van ander saken ons t'selve niet toe en laet: Sulcx dat hier alleenlick de wijse ghetoot is, en open plaets ghelaten om die te meughen volmaeckct worden, by de ghene dieder lust en ghelegentheyt toe mochten hebben.

TAFEL

This preliminary of the table of the assembled secants thus having been made and in order to come now to the making of the tables of loxodromes, in the foregoing figure let RZ denote the first loxodrome, so that the angle XRQ of the triangle XRQ is now $78^{\circ}45'$. In order to find the arc QX , I look upon the triangle XRQ as a plane triangle, on account of the smallness of the sides, and say that it has three known elements, *viz.* the angle $XQR = 90^{\circ}$, $XRQ = 78^{\circ}45'$, the side $QR = 1^{\circ}$. When herewith the side QX is sought, it is found by the 4th proposition of plane trigonometry ¹⁾ to be $5^{\circ}1'$, which I put in the following table of the first loxodrome in the column of the latitudes, against 1° of longitude. In order to find all the subsequent latitudes of this table in a brief way, I look up in the foregoing table of assembled secants what value corresponds to the above-mentioned $5^{\circ}1'$ and find this to be 3,014, for the 5° makes 3,004, and 10 more is the proportional part for the $1'$. This value of 3,014 serves me in general to find the values of PY , DA , and all other similar elements, which takes place as follows. When to the 3,014 we add another 3,014, we get 6,028. Corresponding to this I find in the foregoing table of assembled secants 10° . I put this in the following table of the first loxodrome in the column of the latitudes, against 2° of longitude, *viz.* for PY . Then to 6,028 I add once more 3,014, which gives 9,042. Corresponding to this I find in the foregoing table $14^{\circ}54'$. I put this in the following table, against 3° of longitude, *viz.* for DA , and so on with the rest of the seven loxodromes.

NOTE that I further place behind the foregoing longitudes and latitudes of the loxodromes their distances, *i.e.* the lengths of the arcs RX , RY , RA , and the like, in order to solve by these means, without a globe or a plane chart, but only by numbers, the propositions which concern the subject of loxodromes and which are to be described in the sequel. These distances become known in the following way. To find first the distance RX , I say that the triangle XQR has three known elements, *viz.* the angle $XQR = 90^{\circ}$, the angle $XRQ = 78^{\circ}45'$, and the side $QX = 5^{\circ}1'$. When herewith the side RX is sought, it is found by the 4th proposition of plane trigonometry ¹⁾ to be $5^{\circ}6'54''$, which I put in the first table in the column of the distances, against 1° of longitude. Second, to find the distance RY , I say that the triangle YCX has three known elements, *viz.* the angle $YCX = 90^{\circ}$, $YXC = 78^{\circ}45'$, and the side $CY = 4^{\circ}59'$, as appears from the table; for when we subtract $PC = 5^{\circ}1'$, as being equal to QX , from $PY = 10^{\circ}$, the remainder is, as before, $4^{\circ}59'$ for CY . When with these three known elements the side XY is then sought, this is found by the 4th proposition of plane trigonometry ¹⁾ to be $5^{\circ}12'54''$. When we add this to $RX = 5^{\circ}6'54''$, we get for RY $10^{\circ}19'48''$, which I put in the table of the first loxodrome in the column of the distances, against 2° of longitude. And in the same way the distance RA and all the others have to be found.

Note that we have not calculated or noted these distances everywhere, but only as far as they are necessary for our subsequent examples, on the one hand because the tables themselves do not seem to be sufficiently accurate, as will be discussed more fully in the Appendix, but also because we are hindered by other matters from doing so, so that here the method only has been shown, and a blank has been left, to be completed by those who have a mind and an opportunity to do so.

¹⁾ Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

T A F E L S

DER

C R O M S T R E K E N .

TABLES OF LOXODROMES ¹⁾.

First Loxodrome

Second Loxodrome

Longitude (degr.)	Latitude (degr., min.)	Distance (degr., min.)
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¹⁾ We reproduce the tables for the first and for the second loxodrome, and (on p. 539) the end of the table for the seventh loxodrome.

Eerste Cromstreeck.

Tweede Cromstreeck.

lang. Breede verhedē														
tr.	tr. ①	tr. ①												
1	5	1	5	7.	31	82	27		61	89	27	1	2	24
2	10	0	10	20.	32	83	5		62	89	29	2	4	49
3	14	54			33	83	39		63	89	32	3	7	13
4	19	42			34	84	11		64	89	34	4	9	36
5	24	22			35	84	40		65	89	36	5	11	58
6	28	51			36	85	7		66	89	38	6	14	20
7	33	10			37	85	32		67	89	40	7	16	39
8	37	16			38	85	54		68	89	41	8	18	57
9	41	9			39	86	15		69	89	43	9	21	13
10	44	50			40	86	33		70	89	44	10	23	27
11	48	17			41	86	51		71	89	46	11	25	39
12	51	31			42	87	7		72	89	47	12	27	48
13	54	32			43	87	21		73	89	48	13	29	55
14	57	21			44	87	35		74	89	49	14	31	59
15	59	58			45	87	47		75	89	50	15	34	1
16	62	23			46	87	58		77	89	51	16	35	59
17	64	38			47	88	8		79	89	52	17	37	55
18	66	42			48	88	17		81	89	53	18	39	48
19	68	36			49	88	26		83	89	54	19	41	37
20	70	22			50	88	34		85	89	55	20	43	24
21	71	59			51	88	41		87	89	56	21	45	8
22	73	29			52	88	47		90	89	56	22	46	49
23	74	51			53	88	53		93	89	57	23	48	26
24	76	6			54	88	59		96	89	58	24	50	1
25	77	16			55	89	4		99	89	58	25	51	32
26	78	19			56	89	9		102	89	58	26	53	1
27	79	18			57	89	13		105	89	58	27	54	27
28	80	11			58	89	17		108	89	59	28	55	49
29	81	0			59	89	20		111	89	59	29	57	9
30	81	46			60	89	24		114	89	59	30	58	26

DER CROMSTREKEN.

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lang. Breedte verbede											
tr.	tr. ①	tr. ①									
215	89	37	6	89	46	190	89	52	195	89	57
220	89	37	12	89	46	200	89	52	210	89	57
225	89	38	18	89	47	210	89	53	225	89	57
230	89	38	24	89	47	220	89	53	240	89	57
235	89	39	30	89	47	230	89	53	255	89	57
240	89	39	36	89	47	240	89	53	270	89	58
245	89	39	42	89	48	250	89	53	285	89	58
250	89	40	48	89	48	260	89	54	300	89	58
255	89	40	54	89	48	270	89	54	315	89	58
260	89	40	60	89	48	280	89	54	330	89	58
265	89	41	66	89	48	290	89	54	345	89	58
270	89	41	72	89	49	300	89	54	360	89	58
275	89	41	78	89	49	310	89	54	20	89	58
280	89	42	84	89	49	320	89	55	40	89	58
285	89	42	90	89	49	330	89	55	60	89	58
290	89	42	96	89	50	340	89	55	80	89	58
295	89	42	102	89	50	350	89	55	100	89	58
300	89	43	108	89	50	360	89	55	120	89	58
305	89	43	114	89	50	15	89	55	140	89	58
310	89	43	120	89	50	30	89	56	160	89	58
315	89	44	126	89	50	45	89	56	180	89	58
320	89	44	132	89	51	60	89	56	200	89	59
325	89	44	138	89	51	75	89	56	220	89	59
330	89	44	144	89	51	90	89	56	224	89	59
335	89	45	150	89	51	105	89	56	260	89	59
340	89	45	156	89	51	120	89	57	280	89	59
345	89	45	162	89	51	135	89	57	300	89	59
350	89	45	168	89	52	150	89	57	320	89	59
355	89	46	174	89	52	165	89	57	340	89	59
360	89	46	180	89	52	180	89	57	360	89	59

M 2

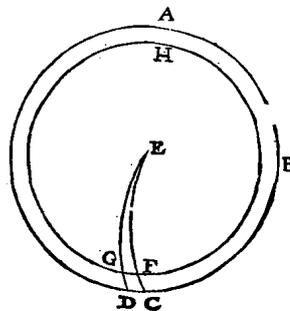
136 4 BOVCK DES EERTCLOOTSCHRIFTS,

Tot hier toe sijn beschreven de seven tafels der seven cromstreken. Angaende de achtste die is alijt een ront ewewijdich vant middelront, sulcx dattet in breede gheen verandering crijghende, soo en valter van sijn breedeschil niet te segghen, maer alleenlick van sijn verheyden, tot welcx eynde de volghende tafel dient.

VANT MAECKSEL DES

tafels der achtste cromstreeck.

Om eerst deur een form te verclaren den sin des volghenden tafels, soo laet ABCD een cloot sijn diens middelront ABCD, en aspunt E, waer op beschreven is een cleender rondt FGH, voort sy de booch CD van 1 tr. als langdeschil tusschen C en D, en ghetrocken de twee vierendeelen ronts ED, EC, snijende het cleender rondt in F en G, soo doet GD neem ick 10 tr. als breedeschil tusschen D en G. Dit soo sijnde de booch FG doet 1 tr. der langde, en dat 10 tr. verre buyten t'middelront, welke booch openbaerlick cleender moet sijn dan DC 1 tr. des middelronts. Nu is t'voornemen hier te vinden hoe veel 1 tr. FG langdeschil buyten t'middelront (te weten op 10 tr. der breede DG) doet in ① en ② des middelronts: Dat is, de langde FG vervought sijnde opt middelront, van hoe veel ① en ② die daer bevonden sal worden. Dit verstaen wesende wy sullen tottet voorbeelt commen.



TGHEGHEVEN. Het sy 1 tr. langdeschil buyten t'middelront op 10 tr. der breede. TBEGHEERDE. Wy moeten sijn grootheyt vinden in ① en ② des middelronts.

TWERCK.

Rechthoucx houckmaet.	10000000.
Gheeft schilhoucx houckmaet vande ghegheven 10 tr. doende	9848078.
Wat 1 tr. des middelronts doende.	60 ①?
Comt voor t'begheerde.	59 ① 5 ②.

Ghelijck inde volghende tafel staet, en alsoo met allen anderen.

TBEWYS.

Ghelijck de halfmiddellijn van een cloots grootste ront, als t'middelront;
Totte halfmiddellijn van haer cleender rondt, als het ewewijdich rondt op de 10 tr. der breede;

Alsoo de booch eens traps des grootste of middelronts;
Totte booch eens traps des cleender ronts op de 10 tr. der breede.

Maer de halfmiddellijn eens middachronts, is even ande halfmiddellijn des middelronts, die ghenomen wort op 10000000, als rechthouckmaet: En de halfmiddellijn des rondts beschreven op de breede van 10 tr. is int middachront houck-

Up to this point the seven tables of the seven loxodromes have been described. As to the eighth, this is always a circle parallel to the equator, so that, since its latitude does not undergo any change, nothing can be said about its difference of latitude, but only about its distances, for which purpose the following table serves.

OF THE MAKING OF THE TABLE OF THE EIGHTH LOXODROME.

In order first to explain by means of a figure the sense of the following table, let $ABCD$ be a globe, whose equator is $ABCD$ and the pole E , on which a small circle FGH has been described. Further let the arc CD be 1° , *viz.* the difference of longitude between C and D , and when the two quarter circles ED and EC intersecting the small circle in F and G have been drawn, I assume that GD is 10° , *viz.* the difference of latitude between D and G . This being so, the arc FG is 1° of longitude, 10° outside the equator, which arc must obviously be smaller than $DC = 1^\circ$ of the equator. Now the object is to find how much 1° (FG) of difference of longitude outside the equator (*viz.* at 10° of latitude, DG) makes in minutes and seconds of arc of the equator, *i.e.* when the longitude FG has been placed on the equator, to how many minutes and seconds of arc this will be found to amount there. This having been understood, we shall pass on to the example.

SUPPOSITION. Let there be 1° of difference of longitude outside the equator in latitude 10° . **REQUIRED.** We have to find its magnitude in minutes and seconds of arc of the equator.

PROCEDURE.

Sine of a right angle	10,000,000
Gives sine of complement of the given 10° , being	9,848,078
What does 1° of the equator give, being	60' ?
The required value is	59'5"

As is to be found in the following table; and the same applies to all the others.

PROOF.

As the semi-diameter of a great circle on a sphere, *viz.* the equator,
 To the semi-diameter of its smaller circle on said sphere, *viz.* the parallel in latitude 10° ,
 So the arc of one degree of the great circle or equator
 To the arc of one degree of the small circle in latitude 10° .

Now the semi-diameter of a meridian is equal to the semi-diameter of the equator, which is taken to be 10,000,000, *viz.* the sine of a right angle. And the semi-diameter of the circle described in latitude 10° is in [the plane of] the meridian the sine of the complement of the arc of 10° , *i.e.* the sine of 80° , being 9,848,078. Therefore, as 10,000,000
 to 9,848,078,

so the arc of one degree of the great circle or equator
 to the arc of one degree of the small circle in latitude 10° .

houckmaet des schilboochs van 10 tr. dats houckmaet van 80 tr. doende

9848078: Daerom

Ghelijck 1000000.

Tot 9848078:

Alsoo de booch eens traps des grootste of middelronts,

Tottē booch eens traps des cleender ronts op de 10 tr. der breedte.

Maer 10000000 ghevende 9848078, soo sal 60 ① gheven 59 ② 5 ③ deut t'werck; daerom 59 ① 5 ② is t'begheerde, t'welck wy bewijfen moesten.

M E R C K T.

Alsoo sijn VORSTELICKE GHENADE t'voorgaende besluyt ghesien hadde en ghesien volcommen te wesen, seyde daer op hem ghedachtich te sijn dat hy voormael derghelijcke grootheyte eens boochs als van F tot G, ghesocht hadde deur rekening der clootsche driehoucken, om datse als van clootsche stof volkommender is dan rekening ghelijck de voorgaende: Maer overdenckende de reden van dit verschil, seyde met onderscheyt d'een wijze te dienen tot besluyt op een vraegh van cromstreken, d'ander van rechtstreken. Als by voorbeeld begeert sijnde hoe lanck de wech valien sal om te varen van F tot G op een cromstreeck; dats hier de booch eens cleender ronts, de volcommen wercking sal ghedaen worden alfooren: Maer sulcken langde begheert wese van F tot G op een rechtstreeck, dats de booch eens grootste ronts, de volcommen wercking bestaet dan in rekening der clootsche driehoucken. T'welck ick hier heb willen anteyckenen, eensdeels tot ghedachtich: Ten anderen op dat de ghene dien sulcx duyfterder ontmoeten mocht, hier me verlicht sy.

T'maeckfel des tafels dan aldus verclaert sijnde, wy sullen de tafel beschrijven, ghetrocken uyt *Cosmographia Petri Apiani parte prima cap. 13.* als volgt.

M 3 ACHT-

But since 10,000,000 gives 9,848,078, by the procedure 60' will give 59'5"; therefore 59'5" is the required value; which we had to prove.

NOTE.

When His Princely Grace had read the foregoing conclusion and had seen it to be correct, he said he remembered having formerly sought such a magnitude of an arc as that from *F* to *G*, by a calculation of spherical trigonometry, because, the matter being concerned with spheres, this is more correct than a calculation like the foregoing. But reflecting on the reason of this difference, he said that a distinction had to be made here, *viz.* that the one method served as a solution of a question concerned with loxodromes, the other of one concerned with great-circle tracks. Thus, for instance, when it is required to know the length of the route when sailing from *F* to *G* on a loxodrome, *i.e.* in this case the arc of a small circle, the correct procedure has to be as before. But when this length is required from *F* to *G* on a great-circle track, *i.e.* the arc of a great circle, the correct procedure then consists in a calculation of spherical trigonometry. I wanted to note this here, on the one hand to show that it has been thought of, on the other hand in order that those to whom this should appear obscure might be enlightened by it.

The making of the table thus having been explained, we shall describe the table, taken from *Cosmographia Petri Apiani parte prima cap. 13*, as follows:

EIGHTH LOXODROME, which is a table of the magnitude of 1° of difference of longitude outside the equator in minutes and seconds of arc of the equator.

In this latitude	1° of differ. of longitude gives the following distance
(degr., min.)	(min., sec.)

ACHTSTE CROMSTREECK, TWELCK
 is een tafel van t'ghene 1 tr. langdeschil buyten t'mid-
 delront, doet in ① en ② des middelronts.

Op dese breedten.		Doet 1 tr. langdeschil dese werbe.		Op dese breedten.		Doet 1 tr. langdeschil dese werbe.		Op dese breedten.		Doet 1 tr. langdeschil dese werbe.		
tr.	①	①	②	tr.	①	①	②	tr.	①	①	②	
0	30	59	59	0	16	0	57	9	31	0	51	16
1	0	59	59	1	16	30	57	9	31	30	51	17
1	30	59	58	1	17	0	57	9	32	0	50	17
2	0	59	57	1	17	30	57	10	32	30	50	17
2	30	59	56	1	18	0	57	10	33	0	50	17
3	0	59	55	1	18	30	56	10	33	30	50	18
3	30	59	53	2	19	0	56	10	34	0	49	18
4	0	59	51	2	19	30	56	11	34	30	49	18
4	30	59	48	2	20	0	56	11	35	0	49	18
5	0	59	46	2	20	30	56	11	35	30	48	18
5	30	59	43	3	21	0	56	11	36	0	48	18
6	0	59	40	3	21	30	55	12	36	30	48	19
6	30	59	36	3	22	0	55	12	37	0	47	19
7	0	59	33	3	22	30	55	12	37	30	47	19
7	30	59	29	4	23	0	55	12	38	0	47	19
8	0	59	24	4	23	30	55	13	38	30	46	20
8	30	59	20	4	24	0	54	13	39	0	46	20
9	0	59	15	5	24	30	54	13	39	30	46	20
9	30	59	10	5	25	0	54	13	40	0	45	20
10	0	59	5	5	25	30	54	14	40	30	45	20
10	30	58	59	6	26	0	53	14	41	0	45	21
11	0	58	53	6	26	30	53	14	41	30	44	21
11	30	58	47	6	27	0	53	14	42	0	44	21
12	0	58	41	6	27	30	53	14	42	30	44	21
12	30	58	34	7	28	0	52	15	43	0	43	22
13	0	58	27	7	28	30	52	15	43	30	43	22
13	30	58	20	7	29	0	52	15	44	0	43	22
14	0	58	13	7	29	30	52	16	44	30	42	22
14	30	58	5	8	30	0	51	16	45	0	42	22
15	0	57	57	8	30	30	51	16	45	30	42	22
15	30	57	49	8				16				23

ACHTSTE CROMSTREECK, TWELCK
 is een tafel van t'ghene 1 tr. langdeschil buyten t'mid-
 delront, doet in ① en ② des middelronts.

Op dese breedem.		Doet 1 tr. langdeschil dese verbe.		Op dese breedem.		Doet 1 tr. langdeschil dese verbe.		Op dese breedem.		Doet 1 tr. langdeschil dese verbe.		
tr.	①	①	②	tr.	①	①	②	tr.	①	①	②	
46	0	41	40	61	0	29	5	76	0	14	30	30
46	30	41	18	61	30	28	37	76	30	14	0	31
47	0	40	55	62	0	28	10	77	0	13	29	31
47	30	40	32	62	30	27	42	77	30	12	59	31
48	0	40	8	63	0	27	14	78	0	12	28	31
48	30	39	43	63	30	26	46	78	30	11	57	31
49	0	39	21	64	0	26	18	79	0	11	26	31
49	30	38	58	64	30	25	49	79	30	10	56	31
50	0	38	34	65	0	25	21	80	0	10	25	31
50	30	38	9	65	30	24	52	80	30	9	54	31
51	0	37	45	66	0	24	24	81	0	9	23	31
51	30	37	21	66	30	23	55	81	30	8	52	31
52	0	36	56	67	0	23	26	82	0	8	21	31
52	30	36	31	67	30	22	57	82	30	7	49	31
53	0	36	6	68	0	22	28	83	0	7	18	31
53	30	35	41	68	30	21	59	83	30	6	47	31
54	0	35	16	69	0	21	30	84	0	6	16	31
54	30	34	50	69	30	21	30	84	30	5	45	31
55	0	34	24	70	0	20	31	85	0	5	13	31
55	30	34	59	70	30	20	1	85	30	4	42	31
56	0	33	33	71	0	19	32	86	0	4	11	31
56	30	33	6	71	30	19	2	86	30	3	39	31
57	0	32	40	72	0	18	32	87	0	3	8	31
57	30	32	14	72	30	18	2	87	30	2	37	31
58	0	31	47	73	0	17	32	88	0	2	5	31
58	30	31	21	73	30	17	2	88	30	1	34	31
59	0	30	54	74	0	16	32	89	0	1	2	31
59	30	30	27	74	30	16	2	89	30	0	31	31
60	0	30	0	75	0	15	31	90	0	0	0	31
60	30	29	32	75	30	15	1					31

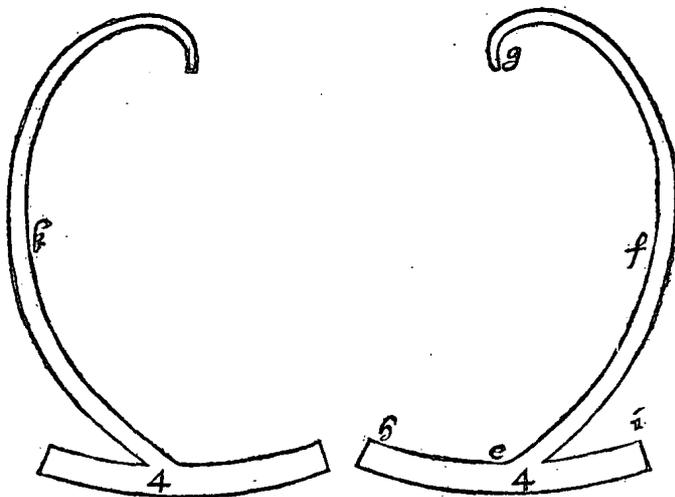
140 4 BOVCK DES EERTCLOOTSCHRIFTS,

Aldus dan inde voorgaende tafels beschreven sijnde der seven cromstreken breedten in yder middachbooch die van trap tot trap getrocken sijn, so ist openbaer hoemen daer me met groote sekerheyt op een Eertclood de cromstreken sal meughen teyckenen, want de punten der breedte ghestelt na t'behooren, en van d'een tot d'ander linikens ghetrocken, men comt tottet begheerde. Oock dienen de selve tafelen om te sien of cromstreken op Eertclooten of platte cacten wel ghereyckent sijn.

VANT MAECKSEL DER COPER CROMSTREKEN.

Maer wantet tot verclaring der nabeschreven voorstellen bequamer soude vallen, en oock tottet ghebruyck in veel voorbeelden mijns bedunckens niet ongerievich, datmen de seven cromstreken van koper maecte, om die te leggen op yder punt des cloots daermen wil, en alsoo ten eersten t'begheerde te sien, sonder noodich te wesen de cromstreken op een Eertclood te teyckenen: Of ghereyckent sijnde sonder te moeten doen de moeylicker wercking daer uyt volghende, soo sullen wy daer af wat breeder uytlegging doen als volghet:

Op een Eertclood ghereyckent sijnde een cromstreeck na de wijze als boven, ick neem de voorschreven noortooststreeck R X Y Z, wesende een vierde, men sal maken een dergelijcke van koper, welke anghewesen wort mette volgende form *efg*, welverstaende dat de cant daer *efg* an comt, de eyghen lini als R X Y Z bediet, waer an noch vervought is het stick *hi*, op wiens uysterste cant *hi*, de lini *efg* sulcken houck maect, als R X Y Z opt middelront L N. Dese koper cromstreeck wort oock verstaen te hebben een clootsche hollicheyt, sulcx datse op den cloot gheleyt die overal gheraect.



Voort ghelijck hier gemaect is dese koper noortooststreeck, oock dienende voor zuytweststreeck, alsoo salmen maken d'ander ses, t'samen seven: En noch sulcke seven ander, van verkeerde ghestalt der voorgaende, als by voorbeeld de form

After the description in the foregoing tables of the latitudes of the seven loxodromes in each of the meridians drawn from degree to degree, it is clear how with the aid of these we shall be able to draw the loxodromes with great certainty on a globe, for when the points of latitude have been properly marked and small lines have been drawn from one to the other, the required loxodrome is obtained. The said tables also serve to ascertain whether loxodromes have been drawn correctly on globes or plane charts.

OF THE MAKING OF THE COPPER LOXODROMES.

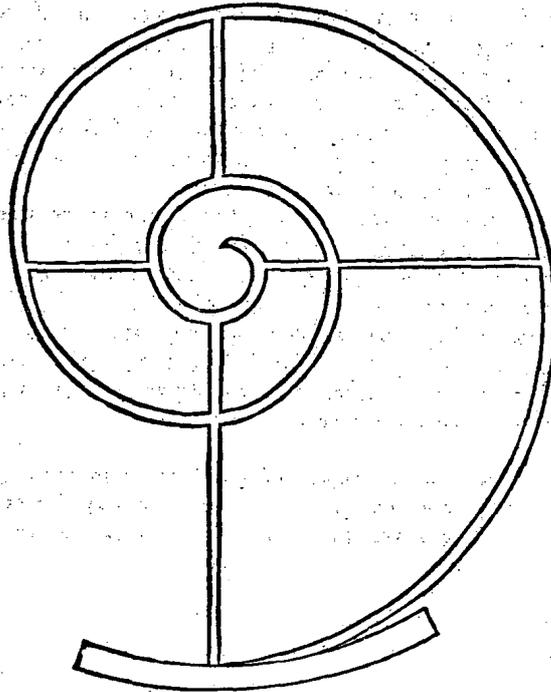
But since it would be more suitable for the explanation of the succeeding propositions — and further in my opinion not inconvenient for use in many examples — to make the seven loxodromes of copper, so that we may put them on any desired point of the globe and thus see at once what was required, without the necessity of drawing the loxodromes on a globe, or, if they have been drawn, without having to perform the more difficult operations following therefrom, we shall explain this somewhat more fully, as follows:

After a loxodrome has been drawn on a globe in the above way — I assume the aforesaid loxodrome of northeast $RXYZ$, which is a fourth loxodrome — we should make one like it of copper, which is denoted by the figure efg below, it being understood that the side on which efg comes stands for the line $RXYZ$ itself, while the piece hi has been attached thereto, with whose outer edge hi the line efg forms the same angle as $RXYZ$ with the equator LN . This copper loxodrome is also understood to be spherically concave, so that, when it is placed on the globe, it touches the latter everywhere.

Further, just as this copper loxodrome of northeast, also serving for the loxodrome of southwest, has here been made, in the same way the other six should be made, seven in all. And seven more of them, having the opposite shape to

form *k*, wefende van verkeerde gefalt der noortoofstreeck *efg*, dienende totte noortweftstreeck, en oock totte zuytoofstreeck. Noch machmen dese koper cromstrecken teyckenen met ghetalen, d'eerste met 1, de tweede met 2, en soo voort, om daer deur met een op sien te weten de hoemenichste elck is.

Angaende de 7 cromstreeck veel keeren soude hebben, en daerom flap sijn, men soude se (met oock ander streken diet noodich hadden) meughen verslijven



ghelijck dese form anwijst, of anders mochtmen tot een cromstreeck verscheyden stucken maken. **T B E S L Y T.** Wy hebben dan cromstrecken gheteyckent, na den eysch.

5 V O O R S T E L.

Wefende ghegeven langdeschilen breedte van twee plaetsen evener breedte: Te vinden haer cromstreecken verhey.

M E R C K T.

Wy sullen in elck der volghende voorstellen drie werckinghen beschrijven, d'eerste mette koper cromstreeck, de tweede mette gecromstreeckten Eertcloot, die beyde * tuchwerckelick sijn, de derde * wisconstich deur getalen, uytgenomen in dit en t'eerstvolghende voorstel, alwaer geen werck mette koper cromstreeck en valt, om datter gheen achtste koper cromstreeck als onnoodich sijnde ghemaect en wiert. **T G H E G H E V E N.** Laet het langdeschil sijn van 30 tr. en haet

*Mechanic.
Mathematic.*

the foregoing ones, as *e.g.* the figure *k*, having the opposite shape to the loxodrome *efg* of northeast, serving for the loxodrome of northwest, and also for the loxodrome of southeast. Further these copper loxodromes may be marked with the number of each, the first with 1, the second with 2; and so on, so that one may know at a glance which of them is meant.

Since the 7th loxodrome would have many windings and accordingly would be limp, we might stiffen it (as well as other loxodromes which might need it) as shown in the figure below, or otherwise we might combine several pieces to form one loxodrome. CONCLUSION. We have thus designed loxodromes; as required.

5th PROPOSITION.

Given the difference of longitude and the latitude of two places of equal latitude: to find their loxodrome and their distance.

NOTE.

In each of the following propositions we shall describe three operations, the first with the copper loxodrome, the second by means of the globe with the loxodromes drawn thereon, both of which are mechanical operations, the third mathematical, by means of numbers, except in the present and the next proposition, where no operation with the copper loxodrome takes place, because no eighth copper loxodrome has been made, this being unnecessary.

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en haer breedten 24 tr. TBEGHEERDE Wy moeten haer cromstreeck en verheynt vinden.

1 *Verck metten ghecromstreeckten Eertclood.*

Anghefien de twee ghegheven breedten even sijn, soo moet deur ghemeene reghel de begheerde cromstreeck de achtste wesen, dat is de oost en weststreeck, sulcx dat daer toe niet anders te doen en is.

Angaende de verheynt ick segh aldus, soo de twee ghegheven plaetsen op den Eertclood gheteyckent stonden, en vallende op een ewewijdich rondt metten middelront; men sal den passer sonau openen, dat de rechte lini tusschen de twee voeten verdocht, gheen hinderlick vershil en hebbe vande cromme des ronts diemen meten moet, als opening van 1 tr. meer of min, na dat de cromheyt des voorgheffelden ronts vereyscht, want meerder cromte hebben de bogen byden afpunt dan by t' middelront: T welck soo sijnde, men sal sien hoe dickwils die openheyt des passers comt inde booch tusschen de twee plaetsen, daer nae de trappen t' samen vergaert, men sal in dit voorbeeld welghenouch gewrocht sijnde, vinden voor begheerde verheynt 27 tr. 24 ①.

Maer soo de twee ghegheven plaetsen op den Eertclood niet en stonden, men sal twee uytvaghelicke punten stellen opt ghegheven langdeschil en breedten, die ghebruyckende voor ghegheven.

Maer soose niet recht en vallen op een gheteyckent ewewijdich ront, men sal int meten des boochs de passer altijt doen gaen ewewijdich van het naefte ewewijdich ront, t' welck soot u byder ooghe niet nauwe genouch ghedaen en wierde, deur t' behulp van een ander passer gheschicn can,

M E R C K T.

De vraech mocht nu sijn hoe veel mijlen de boveschreven 27 tr. 24 ① maken, maer insiende de verscheydenheyt der mijlen in verscheyden landen, so en canmen daer af int ghemeen niet sekens segghen. Sulcx dat wy hier en oock int volghende, de verheynt alleenlick deur tr. en ① beschrijven, die elck verkeeren mach in sulcke mijlen alst hem belieft. Den trap wort van velen gheacht lanck te wesen ontrent 18 uyren gaens eens ghemeenen ganck: Een dier uyren wort ghenomen op 8000 stappen, oock op 1500 Rijnlantische roeden, dat comt den stap op 2½ Rijnlantische voeten. Doch t' waer te wenschen dat Stierlien int gemeen trappen en ① ghebruyckten, om malcander int ghemeen te verstaen.

2 *Verck door ghetalen.*

Anghefien de twee ghegheven breedten even sijn, soo moet deur ghemeene regel, als oock int 1 werck gheseyt is, de begheerde cromstreeck de achtste wesen.

Om nu de verheynt te vinden, ick souc inde tafel des 8 cromstreeck int 4 voorstel de ghegheven breedten van 24 tr. en daer nevens wat verheynt tot die plaets overcomt op 1 tr. langdeschil, bevinde 54 ① 48 ②: Hier me segh ick, 1 tr. langdeschil, doet tot dese plaets 54 ① 48 ② verheynt, wat de ghegheven 30 tr. langdeschil? Comt voor begheerde verheynt 27 tr. 24 ①: waer af t' bewijs deur t' werck openbaer is. TBESLVYT. Wesende dan ghegheven langdeschil en breedten van twee plaetsen evenbreede, wy hebben ghevonden haer cromstreeck en verheynt, na den cyfch.

M E R C K T.

SUPPOSITION. Let the difference of longitude between the places be 30° and their latitude 24° . REQUIRED. We have to find their loxodrome and their distance.

1st Procedure, by means of the Globe with the Loxodromes Drawn thereon.

Since the two given latitudes are equal, by the general rule the required loxodrome must be the eighth, *i.e.* the east and west loxodrome, so that nothing else has to be done about this.

As to the distance, I say as follows: if the two given places are marked on the globe and fall on a circle parallel to the equator, one must open the compasses so little that the straight line imagined between the two points does not differ appreciably from the arc of the circle that has to be measured, *viz.* an opening of about one degree, as the curvature of the circle in question requires, for arcs near the pole have a greater curvature than arcs near the equator. This being so, one must ascertain how often this opening of the compasses is included in the arc between the two places; the degrees afterwards being added up, in the present example — if the procedure has been exact enough — one will find $27^\circ 24'$ for the required distance.

But if the two given places are not marked on the globe, one must mark two erasable points having the difference of longitude and the latitude given, using them as if they were given.

But if they do not fall exactly on a drawn parallel, in measuring the arc one must always keep the compasses parallel to the nearest parallel, which, if it should not be done accurately enough at sight, can be effected with the aid of another pair of compasses.

NOTE.

The question might now be asked to how many miles the above-mentioned $27^\circ 24'$ corresponds, but considering the inequality between miles in different countries, in general nothing can be said with certainty about this. So that here as well as in the sequel we shall only describe the distance in degrees and minutes, which everyone may reduce to such miles as he pleases. A degree is considered by many people to be equivalent to about 18 hours' walk at an ordinary pace. One such hour is taken to be 8,000 paces, also 1,500 Rhineland roods, so that one pace is $2\frac{1}{4}$ Rhineland feet. But it were to be wished that navigators generally used degrees and minutes, so as to understand each other generally.

2nd Procedure, by Numbers.

Since the two given latitudes are equal, by the general rule, as has also been said in the 1st procedure, the required loxodrome must be the eighth.

Now in order to find the distance, I look up in the table of the 8th loxodrome in the 4th proposition the given latitude of 24° , and against it I see what distance corresponds in that place to 1° of difference of longitude, for which I find $54'48''$. I now say: 1° of difference of longitude gives in this place a distance of $54'48''$; what does the given difference of longitude of 30° give? We get $27^\circ 24'$ for the required distance; the proof of which is evident from the procedure. CONCLUSION. Given the difference of longitude and the latitude of two places of equal latitude, we have thus found their loxodrome and their distance; as required.

M E R C K T .

Soo de ghegeven ghetalen niet gantschelick en overquamen mette ghetalen inde tafel beschreven, men soude (soo hier als inde volghende voorstellen) om t'begheerde seer na te krijghen, vinden wat het voorgestelde ghetal eygentlick toecomt, ghelijck in ander tafels ghebruyckt wort, en soo daer voorbeeldt af is int 11 voorstel des honckmaetmaeckfels.

6 V O O R S T E L .

Wesende ghegeven verheyte en breede van twee plaetsen evener breede: Te vinden haer cromstreeck en langdeschil.

T G H E G H E V E N . Laet twee plaetsen verheyte sijn van 27 tr. 24 ①, en haer breedten 24 tr. T B E G H E E R D E . Wy moeten haer cromstreeck en langdeschil vinden.

1 *Verck metten ghecromstreekten Eertclood.*

Anghesien de twee ghegeven breedten even sijn, soo moet deur gemeene reghel als int 1 werck des 5 voorstels gheseyt is, de begheerde cromstreeck de achtste sijn.

Angaende het langdeschil, ick segh aldus: Soo de twee ghegeven plaetsen op den Eertclood gheteyckent stonden op een ewewijdich ront, men sal op den passer nemen een deel des middelronts als 1 tr. meer of min en meten daer me de ghegeven 27 tr. 24 ①, int voorschreven ront dat deur de twee plaetsen streck, brengende voorts onder het middachront t'eynde der selve, daer nae t'begin, en sal alsdan de booch des middelronts, begrepen tusschen foodanige twee middachronten, t'begheerde langdeschil sijn, en bevonden worden van 30 tr.

Maer soo de twee ghegeven plaetsen op den Eertclood niet en stonden, men sal daer me doen alsoo van derghelijcke gheseyt is int 5 voorstels eerste werck.

2 *Verck door ghetalen.*

Anghesien de twee ghegeven breedten even sijn, soo moet deur gemeene reghel als int 1 werck des 5 voorstels gheseyt is, de begheerde cromstreeck de achtste sijn.

Om nu het langdeschil te vinden, ick souck inde tafel des 8 cromstreeck int 4 voorstel de ghegeven breede van 24 tr. en daer nevens wat verheyte tot die plaets overcomt op 1 tr. langdeschil, bevinde 54 ① 48 ②: Hier me segh ick 54 ① 48 ②, gheven tot dese plaets 1 tr. langdeschil, wat de ghegeven verheyte 27 tr. 24 ①? Comt voor begeert langdeschil 30 tr. waer aft'bewijs deur t'werck openbaer is. T B E S L V Y T . Wesende dan ghegeven verheyte en breede van twee plaetsen evener breede, wy hebben ghevonden haer cromstreeck en langdeschil, na den eyfch.

7 V O O R -

NOTE.

If the given numerical values do not correspond altogether to the numerical values given in the table, one must (both here and in the subsequent propositions), in order to get the required value very accurately, find what the value in question properly corresponds to, as is commonly done in other tables and in the way of the example given thereof in the 11th proposition of the work on the making of tables of sines ¹⁾.

6th PROPOSITION.

Given the distance and the latitude of two places of equal latitude: to find their loxodrome and their difference of longitude.

SUPPOSITION. Let the distance between two places be $27^{\circ}24'$ and their latitude 24° . REQUIRED. We have to find their loxodrome and their difference of longitude.

1st Procedure, by means of the Globe with the Loxodromes Drawn thereon.

Since the two given latitudes are equal, by the general rule, as said in the 1st procedure of the 5th proposition, the required loxodrome must be the eighth.

As to the difference of longitude, I say as follows. If the two given places are marked on the globe on a parallel, one must take between the compasses a part of the equator, *viz.* about one degree, and measure therewith the given $27^{\circ}24'$ in the aforesaid circle passing through the two places, bringing first one end thereof and then the other under the meridian; then the arc of the equator contained between these two meridians will be the required difference of longitude and will be found to be 30° .

But if the two given places are not marked on the globe, one must proceed in the same way as has been said of a similar case in the first procedure of the 5th proposition.

2nd Procedure, by means of Numbers.

Since the two given latitudes are equal, by the general rule, as said in the 1st procedure of the 5th proposition, the required loxodrome must be the eighth.

Now in order to find the difference of longitude, I look up in the table of the 8th loxodrome in the 4th proposition the given latitude of 24° , and against it I see what distance corresponds in that place to 1° of difference of longitude, which I find to be $54'48''$. I now say: $54'48''$ gives in this place 1° of difference of longitude; what does the given distance of $27^{\circ}24'$ give? We get 30° for the required difference of longitude, the proof of which is evident from the procedure. CONCLUSION. Given the distance and the latitude of two places of equal latitude, we have thus found their loxodrome and their difference of longitude; as required.

¹⁾ Stevin's *Trigonometry* (Work XI; i, 11), p. 58, where he explains how one is to find the sine of a given angle by means of the tables.

7 VOORSTEL.

Wesende ghegheven tyveer plaetsen cromstreeck en breedten: Te vinden (midts dat de ghegheven breedten niet even en sijn) haer langdeschil en verheynt.

Tis kennelick soo de twee ghegheven breedten even waren, dat de cromstreeck soude de 8 vallen, of waer de ghegheven cromstreeck de achtste, dat de breedten souden moeten even sijn: Maer want langdeschil en verheynt gevonden worden deur breeddeschil, en dat hier gheen en is, so en canmen deur sulck ghegheven het langdeschil en de verheynt niet vinden, en daerom ist dat int voorstel dese uytneeming staet, namelijk, *midts dat de ghegheven breedten niet even en sijn*, waer uyt kennelick is, dat wanneer in voorbeelden deses voorstels de ghegheven cromstreeck seer nae de achtste staet, datmen inde daer t'besluit niet seer seker en heeft.

M E R C K T.

T'ghebeurt wel dat een Sierman seyende van d'een plaets tot d'ander op een selve streeck, deur dadelicke ervaring ghenomen heeft die twee plaetsen breedte, en dat hy hier uyt wil vinden de verheynt, om te sien hoe sijn giffing die hy int seylen daer op ghenomen mach hebben, overcomt met dese reghelen: Voort hoe daer me overcomt het langdeschil dat op Eertclooten en in tafels daer af ghestelt mach sijn: Oft anders soo de voornoemde giffing en langdeschil onbekent waren, hoemen die bekent sal maken, en tot sulcken eynde dient dit voorstel. **T'GHEGHEVEN.** Laet der twee plaetsen cromstreeck sijn de 4, en de westelicker breedte 5 tr. 59 ①, d'ander breedte 28 tr. 42 ①.

T'BEGHEERDE. Wy moeten haer langdeschil en verheynt vinden.

1 *Verck mette copen cromstreeck.*

Ick neem een Eertclood welcke A B C D sy, diens aspunt A, en middelront D B, teycken daer op een verborghen of uytvagelick punt E, sulcx dat sijn breedte F E doe 5 tr. 59 ① der ghegheven westelicker plaets; neem voort na t'inhout van t'ghegheven de vierde copen cromstreeck, welcke sy G H I K, vervough haer gront G H opt middelront D B, die daer langs henen schuyvende tot dat de wijslijn I K comt op E; keer daer na den Eertclood westwaert (om dat d'ander plaets oostlicker is) tot dat de cromstreeck het middachront deursnijt in d'ander ghegheven breedte van 28 tr. 42 ①, t'welck valt neem ick an L, en de gemeeone sine des middachronts en middelronts sy alsdan M. Dit soo wesende F M is der twee plaetsen langdeschil, t'welck op den Eertclood in dit voorbeelt bevonden moet worden voor t'begheerde van 24 tr.

Voort is des cromstreecx lini E L de verheynt, welcke gemeten als int 5 voorstels eerste werck, moet in dit voorbeelt bevonden worden voor t'begheerde van 32 tr. 8 ①.

2 *Verck metten ghecromstreecken Eertclood.*

Ick verkies op den Eertclood eenighe cromstreeck als de ghegheven, dats de 4, en breng die onder het middachront, snyende t'felve inden 5 tr. 59 ① der ghegheven westelicker breedte; ick keer daer na den Eertclood westwaert (om dat d'ander

7th PROPOSITION.

Given the loxodrome and the latitudes of two places: to find (provided the given latitudes are not equal) their difference of longitude and their distance.

It is obvious that if the two latitudes were equal, the loxodrome would be the eighth, or if the given loxodrome were the eighth, the latitudes would have to be equal. Now because the difference of longitude and the distance are found by means of the difference of latitude and there is none in this case, it is not possible to find the difference of longitude and the distance by means of this datum, and that is why the proposition contains this condition, *viz. provided the given latitudes are not equal*, from which it is obvious that if in examples of this proposition the given loxodrome is very near to the eighth, in practice the solution is not very accurate.

NOTE.

It sometimes happens that a navigator, sailing from one place to another on the same course, has found the latitudes of those two places by practical observation and that from these he wants to find the distance, in order to see how far his conjecture, which he may have made during the voyage, agrees with these rules. Further, how far the difference of longitude that may be given thereof on globes and in tables agrees with it. Or else, if the aforesaid conjecture and difference of longitude were unknown, how they have to become known; and it is for this purpose that the present proposition serves.

SUPPOSITION. Let the loxodrome of the two places be the fourth, and the latitude of the more westerly place $5^{\circ}59'$, the latitude of the other $28^{\circ}42'$.

REQUIRED. We have to find their difference of longitude and their distance.

1st Procedure, with the Copper Loxodrome.

I take a globe, which shall be $ABCD$, whose pole is A and the equator DB , and mark on it an erasable point E so that its latitude FE is the $5^{\circ}59'$ of the more westerly of the given places. Further, in accordance with the supposition, I take the fourth copper loxodrome, which shall be $GHIK$, place its base GH on the equator DB , moving it along the latter until the index line IK falls through E , then turn the globe to the west (because the other place is further to the east) until the loxodrome intersects the meridian in the other given latitude of $28^{\circ}42'$, which is, I assume, at L ; and let the point of intersection of the meridian and the equator then be M . This being so, FM is the difference of longitude between the two places, which on the globe in this example, to satisfy the requirement, must be found to be 24° .

Further the part EL of the loxodrome is the distance, which, when measured as in the first procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be $32^{\circ}8'$.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

I choose on the globe a loxodrome for the one given, which is the 4th, and bring it under the meridian, so that it intersects the latter in the $5^{\circ}59'$ of the

given more westerly latitude. I then turn the globe to the west (because the other place is further to the east) until the loxodrome intersects the meridian in the $28^{\circ}42'$ of the other given latitude. This being so, the part of the equator between the first and the second position is the difference of longitude, which in this example must be found to be 24° .

Further the part of the loxodrome between those two positions is the distance, which, when measured as in the 1st procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be $32^{\circ}8'$.

3rd Procedure, by means of Numbers.

I seek in the table of the 4th proposition the given 4th loxodrome and look up what longitudes and distances correspond to the two given latitudes. I find that the values corresponding to the smaller latitude $5^{\circ}59'$ are the longitude 6° and the distance $8^{\circ}29'$, and those corresponding to the greater latitude $28^{\circ}42'$ I find to be the longitude 30° and the distance $40^{\circ}37'$. When from this we subtract the longitude and distance first mentioned, we get for the required difference of longitude 24° , for the distance $32^{\circ}8'$; the proof of which is evident from the procedure. CONCLUSION. Given the

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ghegheven twee plaetsen cromstreeck en breedten : Wy hebben ghevonden (midts dat de ghegheven breedten niet even en waren) haer langdeschil en verhey, na den eyfch.

8 VOORSTEL.

Wesende ghegheven tvveer plaetsen breedten en langdeschil: Te vinden haer cromstreeck en verhey.

TGHEGHEVEN. Laet de breedte der westlicker plaets sijn van 5 tr. 59 ①, dander plaetsens breedte 28 tr. 42 ①, en haer langdeschil 24 tr.

TBEGHEERDE. Wy moeten de cromstreeck en verhey vinden.

1 Werck mette coper cromstreeck.

Soo de ghegheven plaetsen op den Eertclood niet gheteyckent en waren, men salder uyt vaghelicke punten setten, volghende t' ghegheven, die ick inde form des 7 voorstels neem te wesen de twee punten E en L : Daer na salmen uyt de seven coperen cromstreken een verkiesen die ons uyster ooghdunckt t' begheerde ten naesten te commen, legghende haer gront G H mette wijslijn opt middelront B D, die daer langs henen schuyvende tot dat des cromstreecx wijslijn I K, comme op d'een der twee ghegheven plaetsen, ick neem op E: Soo d'ander plaets L aldan comt te gheraken de selve wijslijn I K, so is die coper streeck de begheerde; maer sulcx niet ghebeurende, men sal een ander nemen daer op passende, of die nemen welcke ten naesten comt, de selve moet in dit voorbeelt bevonden worden voor t' begheerde de vierde.

Voort is des cromstreecx lini E L de verhey, welcke ghemeten als int 5 voorstels eerste werck, moet in dit voorbeelt bevonden worden voor t' begheerde van 32 tr. 8 ①.

2 VVerck metten ghecromstreeckten Eertclood.

Soo de twee plaetsen op den Eertclood gheteyckent stonden, en datse by ghevalle op een selve gheteyckende cromstreeck lagen, tis kennelick dat die cromstreeck de begheerde soude sijn: Maer want dat selden ghebeurt, so sulden wy de saeck hier by voorbeelt nemen datse beyde in gheen selve gheteyckende cromstreeck en vallen, waer toe den voortganck dusdanich is: Keert den clood tot dat eenige cromstreeck het middachront deursnijt op de breedte van d'een plaets, ick neem de westlicker: Daer nae anghesien d'ander plaets oostelicker is, soo keert den clood westlicker, tot datter een booch des middelronts verlopen is van 24 tr. te weten het ghegheven langdeschil: Siet dan of de ghenomen cromstreeck het middachront deursnijt, op de ghegheven grootste breedte der 28 tr. 42 ① van d'ander plaets, want dat soo ghebeurende, die cromstreeck is de begheerde: Dies niet, neemt een ander cromstreeck, en doet daer me alfooren en derghelijcke soo dickwils tot dat ghy de begheerde cromstreeck crijcht, of de begheerde ten naesten, welcke in dit voorbeelt bevonden sal worden de vierde.

Om daer na de verhey te hebben, men sal tusschen de twee boveschreven plaetsen inde vierde cromstreeck ghevonden, meten de langde des selven cromstreecx met een passer, als int 5 voorstels 1 werck, en moet in dit voorbeelt bevonden worden voor t' begheerde van 32 tr. 8 ①.

3 Werck

loxodrome and the latitudes of two places, we have thus found (provided the given latitudes are not equal) their difference of longitude and their distance; as required.

8th PROPOSITION.

Given the latitudes and the difference of longitude of two places: to find their loxodrome and their distance.

SUPPOSITION. Let the latitude of the more westerly place be $5^{\circ}59'$, the latitude of the other place $28^{\circ}42'$, and their difference of longitude 24° .

REQUIRED. We have to find the loxodrome and the distance.

1st Procedure, with the Copper Loxodrome.

If the given places are not marked on the globe, one must mark thereon erasable points, in accordance with the supposition, which in the figure of the 7th proposition I assume to be the two points *E* and *L*. Thereafter one must choose from the seven copper loxodromes one which at sight appears to approximate the required loxodrome most, placing its base *GH* with the index line on the equator *BD*, moving it along the latter until the index line *IK* of the loxodrome falls through one of the two given places, I assume through *E*. If the other place *L* then falls on the said index line *IK*, this copper loxodrome is the one required; but if this is not the case, one must take another, which does fit on it; or take the one that approximates it most; in this example, to satisfy the requirement, this must be found to be the fourth.

Further the part *EL* of the loxodrome is the distance, which, when measured as in the first procedure of the 5th proposition, in this example, to satisfy the requirement, must be found to be $32^{\circ}8'$.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

If the two places were marked on the globe and happened to lie on the same drawn loxodrome, it is obvious that this loxodrome would be the one required. But because this rarely happens, we shall here give an example where the two do not fall on the same drawn loxodrome, for which the procedure is as follows. Turn the globe until a certain loxodrome intersects the meridian in the latitude of one of the places, I assume the more westerly one. Then, since the other place lies further to the east, turn the globe further to the west until an arc of the equator of 24° has been traversed, *viz.* the given difference of longitude. Then see whether the chosen loxodrome intersects the meridian, in the given greater latitude of the $28^{\circ}42'$ of the other place, for if this is the case, that loxodrome is the one required. If not, take another loxodrome and proceed therewith as before, and similarly until you get the required loxodrome, or approximately the one required, which in this example will be found to be the fourth.

In order to find next the distance, one must measure, between the two above-mentioned places found in the fourth loxodrome, the length of this loxodrome by means of a pair of compasses, as in the 1st procedure of the 5th proposition; in this example, to satisfy the requirement, this must be found to be $32^{\circ}8'$.

3 *Verck door ghetalen.*

Ick fouck inde tafel des 4 voorstels de cleenste gegeven breedte van 5 tr. 59 ①, in eenighe der 7 cromstreken, ick neem inde vierde, en sie wat langde daer me overcomt, bevinde 6 tr.

Daer na fouck ick inde selve vierde cromstreeck wat langde datter overcomt op d'ander ghegheven breedte 28 tr. 42 ①, bevinde 30 tr.

Daer af ghetrocken 6 tr. eerste in d'oirden, blijft langdeschil. 24 tr.

Byaldien nu t'selve langdeschil niet even en waer, oft immers niet na ghenouch even metter ghegheven langdeschil, so en soude die ghenomen tafel der 4 cromstreeck de begheerde niet sijn, en daerom soudemen dierghelijcke moeten versoucken op een ander cromstreeck, en dat soo dickwils tot datmen het derde des oirdens even vonde, of ten naesten even metter ghegheven langdeschil: Maer t'valt indit voorbeelt even, daerom de begheerde cromstreeck is de vierde.

Om nu de verheyte te vinden, ick fouc inde boveschreven vierde cromstreeck d'een ghegheven breedte 5 tr. 59 ①, vinde daer op te overcommen verheyte van 8 tr. 29 ①,

En inde selve vierde cromstreeck fouck ick d'ander ghegheven breedte 28 tr. 42 ①, vinde daer op te overcommen verheyte van 40 tr. 37 ①.

Diens verschil vande verheyte 8 tr. 29 ① vijfde in d'oirden, is voor de begheerde verheyte. 32 tr. 8 ①.

Waer af t'bewijs deurt'werck openbaer is. T B E S L V Y T. Wefende dan ghegheven tweer plaetsen breedten en langdeschil, wy hebben ghevonden haer cromstreeck en verheyte, na den eyfch.

9 V O O R S T E L.

Wefende ghegeven tvveer plaetsen breedte en verheyte: Te vinden de cromstreeck en langdeschil.

M E R C K T.

T'gnebeurt dat ymant bekent sijn tweer plaetsen breedten, en verheyte van d'een tot d'ander, deur gissing dieint seylen daer op mach hebben ghenomen gheweest, en dat hy begheert te weten de cromstreeck en langdeschil, om sijn toecommende seyling daer na te rechten, of om sulcke twee plaetsen op den Eertcloot recht te teyckenen, of daer op gheteyckent sijnde, om te sien hoe sijn rekening daer me overcomt, en tot sulcken eynde can dit voorstel dienen.

T G H E G H E V E N. Laet de westlicker plaetsens breedte sijn van 5 tr. 59 ①, van d'ander 28 tr. 42 ①, en haer verheyte 32 tr. 8 ①. T B E G H E E R D E. Wy moeten haer cromstreeck en langdeschil vinden.

1 *Verck mette copen cromstreeck.*

Om eerst de cromstreeck te vinden, ick neem een Eertcloot welcke beteyckent sy mette form des 7 voorstels, set daer op d'een ghegheven plaets, later sijn de westlicker, diens breedte 5 tr. 59 ① als t'punt E: ick neem daer na een der seven copen cromstreken die my uytter ooghe dunckt t'begheerde ten naesten te commen, legghende haer gronts wijfslijn opt middelront B D, die daer langs henen schuyvende tot dat des cromstreeckx wijfslijn I K komme opt punt E: ick meet

N 2

3rd Procedure, by means of Numbers.

I look up in the table of the 4th proposition the smaller of the given latitudes $5^{\circ}59'$, under one of the 7 loxodromes, I assume under the fourth, and see what longitude corresponds thereto, which I find to be 6°

Thereafter I look up in the said table of the fourth loxodrome what longitude corresponds to the other given latitude $28^{\circ}42'$, which I find to be 30°

When from this we subtract 6° , the first in the present list, the difference of longitude is 24°

Now if the said difference of longitude were not equal, or at least almost equal, to the given difference of longitude, the chosen table of the 4th loxodrome would not be the one required, and consequently we should have to try similarly with another loxodrome, until the third figure in the present list were found to be equal, or approximately equal, to the given difference of longitude. But in this example it is equal, consequently the required loxodrome is the fourth.

Now in order to find the distance, I look up in the above-mentioned table of the fourth loxodrome the one of the given latitudes $5^{\circ}59'$, and find as the distance corresponding thereto $8^{\circ}29'$

And in the said table of the fourth loxodrome I look up the other given latitude $28^{\circ}42'$, and find as the distance corresponding thereto $40^{\circ}37'$

The difference of the latter from the distance $8^{\circ}29'$, the fifth in the present list, is, for the required distance, $32^{\circ} 8'$

The proof of which is evident from the procedure. **CONCLUSION.** Given the latitudes and the difference of longitude of two placés, we have thus found their loxodrome and their distance; as required.

9th PROPOSITION.

Given the latitudes of and the distance between two places: to find the loxodrome and the difference of longitude.

NOTE.

It sometimes happens that a man knows the latitudes of two places and the distance from one to the other, by a conjecture which he may have made in practice, and that he wants to know the loxodrome and the difference of longitude, in order to direct his future sailing thereby, or in order to mark these two places correctly on the globe, or, if they are marked thereon, to see how his reckoning agrees therewith, and for this purpose the present proposition may serve.

SUPPOSITION. Let the latitude of the more westerly of the places be $5^{\circ}59'$, that of the other $28^{\circ}42'$, and their distance $32^{\circ}8'$. **REQUIRED.** We have to find their loxodrome and their difference of longitude.

1st Procedure, with the Copper Loxodrome.

In order to find first the loxodrome, I take a globe, on which the figure of the 7th proposition shall have been drawn. I mark thereon the one given place, let it be the more westerly one, whose latitude is $5^{\circ}59'$, viz. the point *E*. I then take one of the seven copper loxodromes which at sight appears to me to approximate the required loxodrome most, putting its base on the equator *BD*, moving it along the latter until the index line *IK* of the loxodrome falls through the point *E*.

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meet voort met een passier na de wijze als int 5 voorstel van E langs de wijslijn der cromstreeck, tot dat ick heb de ghegeven verhey 32 tr. 8 ①, die comt neem ick an L, welck punt soo vallende, dat sijn breede de ghegeven is van 28 tr. 42 ①, die ghenomen coper cromstreeck is de begheerde: Maer sulcx niet ghebeurende, men sal een ander nemen, en daer me dergelijcke doen, en dat so dickwils tot datmen een crijcht waer in sulcke breede alsoo even valt, of ten naesten comt, t'welck in dit voorbeelt moet sijn voor t'begeerde de vierde cromstreeck.

Om daer na het langdeschil te hebben, men salt soucken vande twee punten E, L, t'welck sy F M, en moet in dit voorbeelt bevonden worden van 24 tr.

2 *Werk metten ghecromstreekten Eertclood.*

Om eerst de cromstreeck te vinden, ick verkies op een ghecromstreekten Eertclood eenighe cromstreeck die my uyer oogh dunckt t'begheerde ten naesten te commen: Later inde form des 7 voorstels sijn de cromstreeck I K, teycken daer op d'een ghegeven plaets, later sijn de westelicker als t'punt E, diens breede F E 5 tr. 59 ①: Ick meet voort met een passier na de wijze als int 5 voorstel, van E langs de cromstreeck tot dat ick heb de ghegeven verhey van 32 tr. 8 ①, die comt neem ick an t'punt L, welck punt soo vallende, dat sijn breede L M de ghegeven is van 28 tr. 42 ①, de ghenomen cromstreeck is de begeerde. Maer sulcx niet ghebeurende, men sal een ander nemen, en daer me dergelijcke doen, en dat so dickwils tot datmen een crijcht waer in sulcke breede alsoo even valt, of ten naesten comt, t'welck in dit voorbeelt moet sijn voor t'begheerde de vierde cromstreeck.

Om daer na het langdeschil te hebben, men salt soucken vande twee punten E, L, t'welck sy F M, en moet in dit voorbeelt bevonden worden voor t'begeerde van 24 tr.

3 *Werk door ghetaalen.*

Om eerst de cromstreeck te vinden, ick souck inde tafels des 4 voorstels d'een ghegeven breede in een der cromstrecken, later sijn de cleenste van 5 tr. 59 ①, voor t'eerste inde 4 cromstreeck, en sie wat verhey daer me overcomt, bevinde

8 tr. 29 ①.

Daer toe ghedaen de ghegeven verhey (ick segh daer toe ghedaen om dat d'eerste breede de cleenste was, want waerse de grootste gheweest men soude moeten afrecken) doende

32 tr. 8 ①.

Comt verhey

40 tr. 37 ①.

Daer na sien ick inde selve vierde cromstreeck, wat breede datter overcomt op de verhey van 40 tr. 37 ① derde in d'oirden, bevinde

28 tr. 42 ①.

Dese breede even sijnde mette ghegeven breede der tweede plaets, soo is dese ghenomen vierde cromstreeck de begheerde: Waerse daer me oneven gheweest, men soude dergelijcke moeten met een ander versoucken, tot datmen sulcx bevonde te overcomen, of ten naesten.

Om nu het langdeschil te vinden, ick souck eerst de cromstreeck alfooren, en sie wat langde datter overcomt op d'een ghegeven breede van 28 tr. 42 ①, bevinde

30 tr.
Sghe.

I next measure by means of a pair of compasses, by the method described in the 5th proposition, from *E* along the index line of the loxodrome until I have got the given distance $32^{\circ}8'$, which I assume to be in *L*; and if this point falls in such a way that its latitude is the given latitude $28^{\circ}42'$, the chosen copper loxodrome is the one required. But if this is not the case, one must take another and proceed similarly therewith, until one gets a loxodrome in which this latitude is equal or nearly so, which, in this example, to satisfy the requirement, must be the fourth loxodrome.

In order to get next the difference of longitude, one must seek it for the two points *E* and *L*, which shall be *FM*, and this in the present example must be found to be 24° .

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

In order to find first the loxodrome, I choose on a globe with the loxodromes drawn thereon a loxodrome which at sight appears to me to approximate the required one most. Let it be, in the figure of the 7th proposition, the loxodrome *IK*. I mark thereon the one given place, let it be the more westerly one, *viz.* the point *E*, whose latitude *FE* is $5^{\circ}59'$. I next measure by means of a pair of compasses, by the method described in the 5th proposition, from *E* along the loxodrome until I have got the given distance of $32^{\circ}8'$, which I assume to be in the point *L*; and if this point falls in such a way that its latitude *LM* is the given latitude $28^{\circ}42'$, the chosen loxodrome is the one required. But if this is not the case, one must take another and proceed similarly therewith, until one gets a loxodrome in which this latitude is equal or nearly so, which, in this example, to satisfy the requirement, must be the fourth loxodrome.

In order to get next the difference of longitude, one must seek it for the two points *E* and *L*, which shall be *FM*, and this in the present example, to satisfy the requirement, must be 24° .

3rd Procedure, by means of Numbers.

In order to find first the loxodrome, I look up in the table of the 4th proposition the one given latitude — let it be the smaller one of $5^{\circ}59'$ — under one of the loxodromes, to begin with under the 4th loxodrome, and I see what distance corresponds thereto, which I find to be $8^{\circ}29'$

When we add thereto the given distance (I say: add, because the first latitude was the smaller one, for if it had been the greater one, we should have to subtract), being

$32^{\circ} 8'$

The distance becomes

$40^{\circ}37'$

Thereafter I see in the said table of the fourth loxodrome what latitude corresponds to the distance of $40^{\circ}37'$, the third in the present list, which I find to be

$28^{\circ}42'$

Since this latitude is equal to the given latitude of the second place, this chosen fourth loxodrome is the one required. If it had not been equal to it, we should have to try similarly with another, until we found it to correspond, or nearly so.

In order to find now the difference of longitude, I first seek the loxodrome, as before, and see what longitude corresponds to the one given latitude $28^{\circ}42'$, which I find to be

30°

S'ghelijcx wat langde datter overcomt op d'ander ghegeven breedte
de van 5 tr. 59 ①, bevinde 6 tr. 6 tr.
Die ghetrocken vande 30 tr. vijfde in d'oirden, blijft voor begheert
langdeschil 24 tr.
Waer af t'bewijs deur t'werck openbaer is. T B E S L V Y T. Wefende dan
ghegeven tweer plaetsen breedte en verhey, wy hebben ghevonden de crom-
streeck en langdeschil, naden eyfch.

10 V O O R S T E L.

Wefende ghegeven tvveer plaetsen cromstreeck, lang-
deschil, en d'een plaetsens breedte: Te vinden d'ander plaet-
sens breedte en verhey.

T G H E G H E V E N. Laet de cromstreeck sijn de vierde, langdeschil 24 tr. en
d'een plaetsens breedte wefende de westlickste en cleenste 5 tr. 59 ①.

T B E G H E E R D E. Wy moeten vinden d'ander plaetsens breedte en verhey.

1 *Uerck mette copen cromstreeck.*

Om ten eersten te vinden d'ander plaetsens breedte, ick teycken op eenighen
Eertcloot als die des 7 voorstels een punt ghelijck E; op de ghegeven breedte van
5 tr. 59 ①: Daer na, want de ghegeven cromstreeck de vierde is, so vervough
ick de vierde copen cromstreeck alsoo, dat haer gronts wijflijn G H passe opt
middelront D B, en dat de wijflijn I K gerake t'punt E: Tselve gebrocht on-
der het middachront, ick teycken des middachronts sine F int middelront
D B, keer daer na den Eertcloot westwaert tot datter int middelront van F
af deurloopen sijn 24 tr. des ghegeven langdeschils, t'welck valt neem ick
van F tot M, en teycken aldan des middelronts sine inde cromstreeck, als ter
plaets van L: Dit soo sijnde, L M is d'ander plaetsens begheerde breedte, die
in dit voorbeelt bevonden moet worden van 28 tr. 42 ①. En E L is de be-
gheerde verhey, welke ghemeten meteen passer na de wijze des 5 voorstels,
moet in dit voorbeelt bevonden worden van 32 tr. 8 ①.

2 *Uerck metten ghecromstreeckten Eertcloot.*

Om eerst te vinden d'ander plaetsens breedte, anghesien de vierde crom-
streeck de ghegeven is, ick teycken op den ghecromstreeckten Eertcloot in
eenighe 4 cromstreeck welke I K sy, een punt als E, op de ghegeven breedte
van 5 tr. 59 ①: Tselve punt E ghebrocht onder het middachront, ick tey-
cken des middachronts sine F int middelront D B: keer daer na den Eertcloot
westwaert, tot datter int middelront van F af deurloopen sijn 24 tr. des ghe-
geven langdeschils, t'welck valt neem ick van F tot M, en teycken aldan des
middelronts sine inde cromstreeck, als ter plaets van L. Dit soo sijnde L M is
d'ander plaetsens begheerde breedte, die in dit voorbeelt bevonden moet wor-
den van 28 tr. 42 ①: En E L de begheerde verhey, welke ghemeten na de
wijfe des 5 voorstels moet in dit voorbeelt bevonden worden van 32 tr. 8 ①.

Likewise what longitude corresponds to the other given latitude $5^{\circ}59'$, which I find to be 6°

When we subtract the latter from the 30° , the fifth in the present list, the remainder is, for the required difference of longitude, 24°
 The proof of which is evident from the procedure. CONCLUSION. Given the latitudes of and the distance between two places, we have thus found the loxodrome and the difference of longitude; as required.

10th PROPOSITION.

Given the loxodrome and the difference of longitude of two places, and the latitude of one place: to find the latitude of the other place and the distance.

SUPPOSITION. Let the loxodrome be the fourth, the difference of longitude 24° , and the latitude of one place, being the more westerly and smaller one, $5^{\circ}59'$.

REQUIRED. We have to find the latitude of the other place and the distance.

1st Procedure, with the Copper Loxodrome.

In order to find first the latitude of the other place, I mark on a globe, *viz.* the one of the 7th proposition, a point, *viz.* *E*, in the given latitude of $5^{\circ}59'$. Thereafter, because the given loxodrome is the fourth, I place the fourth copper loxodrome in such a way that the base *GH* fits on the equator *DB* and the index line *IK* passes through the point *E*. The latter having been brought under the meridian, I mark the point of intersection *F* of the meridian with the equator *DB*, then turn the globe to the west until the 24° of the given difference of longitude have been traversed in the equator from *F*, which I assume to be from *F* to *M*, and then I mark the point of intersection of the equator with the loxodrome, *viz.* *L*. This being so, *LM* is the required latitude of the other place, which in this example must be found to be $28^{\circ}42'$. And *EL* is the required distance, which, when measured with a pair of compasses by the method described in the 5th proposition, in this example must be found to be $32^{\circ}8'$.

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

In order to find first the latitude of the other place, since the fourth loxodrome is the given loxodrome, I mark on the globe with the loxodromes drawn thereon, on a 4th loxodrome, which shall be *IK*, a point, *viz.* *E*, in the given latitude of $5^{\circ}59'$. The said point *E* having been brought under the meridian, I mark the point of intersection *F* of the meridian with the equator *DB*, then turn the globe to the west until the 24° of the given difference of longitude have been traversed in the equator from *F*, which I assume to be from *F* to *M*, and then mark the point of intersection of the equator with the loxodrome, *viz.* *L*. This being so, *LM* is the required latitude of the other place, which in this example must be found to be $28^{\circ}42'$. And *EL* must be the required distance, which, when measured by the method described in the 5th proposition, in this example must be found to be $32^{\circ}8'$.

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3 *Verck met gbetalen.*

Ick fouck de ghegheven breede 5 tr. 59 ① inde tafel des 4. voorstels inde ghegheven vierde cromstreeck, en sie wat verheyte en langde daer op overcomt, bevindende verheyte

	8 tr. 29.
En langde	6 tr.
Daer toe vergaert het ghegheven langdeschil (ick segh vergaert om dat de ghegheven breede de cleenste is, waerse de grootste men sou moeten afstrecken) doende	24 tr.
Comt langde	30 tr.
Daer op vinde ick te overcommen inde bovenschreven 4 streeck, de breede der tweede plaets voor t'begheerde	28 tr. 42 ①.
En vinde oock daer nevens de verheyte van	40 tr. 37.
Diens verschil vande verheyte 8 tr. 29 ①, eerste in d'oiden, doet voor begheerde verheyte	32 tr. 8.

Waer af t'bewijs deur t'werck openbaer is. **T B E S L V Y T.** Wefende dan gegeven twee plaetsen cromstreeck, langdeschil, en d'een plaetsens breede, Wy hebben ghevonden d'ander plaetsens breede en verheyte, na den eysch.

11 VOORSTEL.

Wefende ghegheven twee plaetsen cromstreeck, d'een plaetsens breede en verheyte: Te vinden d'ander plaetsens breede en het langdeschil.

M E R C K T.

Genomen an ymant kennelick te sijn op wat cromstreeck dat men van d'een plaets tot d'ander seylt, voort d'een plaetsens breede en de verheyte, die deur giffing int seylen mach ghevonden sijn, waer me hy begheert te vinden d'ander plaetsens breede, en het langdeschil, om die op een Eertclood te teyckenen, of daer op wefende te meughen sien hoe sijn rekening daer me overcomt, en tot sulcken eynde can dit voorstel dienen. **T G H E G H E V E N.** Laet de cromstreeck sijn de 4, de breede der westelicker plaets de cleenste wefende van 5 tr. 59 ①, en de verheyte 32 tr. 8 ①. **T B E G H E E R D E.** Wy moeten vinden d'ander plaetsens breede, en het langdeschil.

1 *Verck mette copen cromstreeck.*

Ick teycken op den Eertclood des 7 voorstels eenich punt als E, op de ghegheven breede van 5 tr. 59 ①, vervough daer op de vierde copen cromstreeck, te weten alsoo dat den gront G H passe opt middelront D B, en de wijflijn I K op E: Ick meet voorts met een passer langs de selve wijflijn I K, van E oostwaert (om dat E deur t'gegeven westelicker is) na K, de ghegheven verheyte van 32 tr. 8 ①, na de wijfse des 5 voorstels, welcke comt neem ick tot L. Dit soo sijnde, ick fouck de breede van L, welke is L M, die in dit voorbeeld bevonden moet worden voor t'begheerde van 28 tr. 42 ①.

Daer na fouck ick der twee plaetsen langdeschil F M, t'welck men in dit voorbeeld bevinden moet voor t'begheerde van 24 tr.

2 *Werck*

3rd Procedure, by means of Numbers.

I look up the given latitude of $5^{\circ}59'$ in the table of the 4th proposition under the given fourth loxodrome, and see what distance and longitude correspond thereto, and I find the distance to be $8^{\circ}29'$

And the longitude 6°

When we add thereto the given difference of longitude (I say: add, because the given latitude is the smaller one; if it were the greater, we should have to subtract), being 24°

The longitude becomes 30°

Corresponding thereto in the table of the above-mentioned 4th loxodrome I find the required latitude of the second place $28^{\circ}42'$

And I also find against it the distance $40^{\circ}37'$

The difference between the latter and the distance $8^{\circ}29'$, the first in the present list, is the required distance $32^{\circ} 8'$

The proof of which is evident from the procedure. **CONCLUSION.** Given the loxodrome and the difference of longitude of two places and the latitude of one place, we have thus found the latitude of the other place and the distance; as required.

11th PROPOSITION.

Given the loxodrome of two places, the latitude of one place, and the distance: to find the latitude of the other place and the difference of longitude.

NOTE.

Assuming that a man knows on what loxodrome one sails from one place to the other, further the latitude of one place and the distance, which may have been found by conjecture in practice, from which he wants to find the latitude of the other place and the difference of longitude, in order to mark them on a globe or, if they are marked thereon, to see how his reckoning agrees therewith, for this purpose the present proposition may serve. **SUPPOSITION.** Let the loxodrome be the 4th, the latitude of the more westerly place, being the smaller one, $5^{\circ}59'$, and the distance $32^{\circ}8'$. **REQUIRED.** We have to find the latitude of the other place and the difference of longitude.

1st Procedure, with the Copper Loxodrome.

I mark on the globe of the 7th proposition a point, *viz.* E , in the given latitude of $5^{\circ}59'$, place thereon the fourth copper loxodrome, *viz.* in such a way that the base GH fits on the equator DB and the index line IK passes through E . I next measure with a pair of compasses along the said index line IK from E eastward (because E by the supposition is more westerly) to K the given distance of $32^{\circ}8'$, by the method of the 5th proposition, which I assume to come as far as L . This being so, I seek the latitude of L , which is LM , which in this example, to satisfy the requirement, must be found to be $28^{\circ}42'$.

I next seek the difference of longitude FM of the two places, which in this example, to satisfy the requirement, must be found to be 24° .

2 *VVerckmetten ghecromstreecken Eertclood.*

Ick verkies eenighe gheteyckende vierde cromstreeck daer ick de ghegheven breedte in teyckenen can: Latet inden Eertclood des 7 voorstels sijn de cromstreeck I K, waer in gheteyckent is t'punt E, soo dat sijn breedte FE doe de ghegheven 5 tr. 59 ①: Ick meet voort met een passier langs de wijslijn IK van E oostwaert (om dat E deur t'ghegheven de westlicker plaets is) na K, de ghegheven verheyte van 32 tr. 8 ①, na de wijze des 5 voorstels, welke comt neem ick tot L. Dit soo sijnde ick souck de breedte van L, welck is L M, die in dit voorbeelt bevonden moet worden voor t'begheerde van 28 tr. 42 ①.

Daer na souck ick der twee plaetsen langdeschil F M, t'welckmen in dit voorbeelt bevinden moet voor t'begheerde van 24 tr.

3 *VVerck door ghetalen.*

Ick souck de ghegheven breedte 5 tr. 59 ① inde ghegeven vierde cromstreeck der tafels des 4 voorstels, en sie wat langde en verheyte daer op overcomt, bevinde langde

6 tr.

En verheyte

8 tr. 29 ①.

Daer toe vergaert de ghegheven verheyte (ick segh vergaert om dat de ghegeven breedte de cleenste is, waer se de grootste men soude moeten afrekenen) doende

32 tr. 8 ①.

Comt verheyte

40 tr. 37. ①.

Die ghesocht inde boveschreven vierde cromstreeck, ick vinde daer nevens te overcommen voor begheerde breedte der tweede plaets

28 tr. 42 ①.

En vinde oock daer nevens de langde van

30 tr.

Diens verschil vande langde 6 tr. eerste in d'oirden, doet voort begheert langdeschil.

24 tr.

Waer aff'bewijsdeur t'werck openbaer is. T B E S L V Y T. Wesende dangegeven twee plaetsen cromstreeck, d'een plaetsens breedte en verheyte: Wy hebben ghevonden d'ander plaetsens breedte, en het langdeschil, na den eyfch.

M E R C K T.

Wy hebben hier vooren gheseyt vant seylen op rechte en cromme seyfstreken, maer wantter in groote zeevaerden, noch ghebruyckt wort daert de gheleghentheyte toelaet een wijze van seyling ghemengt van beyden, te weten een achtste streeck met een middachront, soo sullen wy daer aff' hier wat vermaen doen. Laet inde volghende form A B den Eertclood beteyckenen, diens middelront A B, en noortschen aspunt C, voort sijn D, E, twee plaetsen van verscheyden langden en breedden, deur welcke ghetrocken sijn de twee ewewijdige ronden D F en E G. Om nu van E tot D te seylen, niet op een rechte noch cromme streeck alfooren, maer op een middachront met een achtste streeck: Men seylt eerst van E af recht noortwaert an, opt middachront E F, tot datmen sich vint op de breedte diemen weet D te hebben, t'welck sy neem ick tot F: daer na keertmen westwaert, blijvende gheduerlick op de selve breedte, dat is geduerlick varende op de achtste cromstreeck tot datmen ter begheerde plaets D comt.

Merckt noch datmen soude meughen eerst beginnen te seylen van E recht

N 4

west-

2nd Procedure, by means of the Globe with the Loxodromes Drawn thereon.

I choose a certain drawn fourth loxodrome on which I can mark the given latitude. Let it be, on the globe of the 7th proposition, the loxodrome *IK*, on which the point *E* has been marked, so that its latitude *FE* is the given $5^{\circ}59'$. I next measure with a pair of compasses along the index line *IK* from *E* eastward (because *E* by the supposition is the more westerly place) to *K* the given distance of $32^{\circ}8'$, by the method of the 5th proposition, which I assume to come as far as *L*. This being so, I seek the latitude of *L*, which is *LM*, which in this example, to satisfy the requirement, must be found to be $28^{\circ}42'$.

I next seek the difference of longitude *FM* of the two places, which in this example, to satisfy the requirement, must be found to be 24° .

3rd Procedure, by means of Numbers.

I look up the given latitude of $5^{\circ}59'$ under the given fourth loxodrome of the table of the 4th proposition, and see what longitude and distance correspond thereto, and I find the longitude to be

6°

And the distance

$8^{\circ}29'$

When we add thereto the given distance (I say: add, because the given latitude is the smaller one; if it were the greater, we should have to subtract), being

$32^{\circ} 8'$

The distance becomes

$40^{\circ}37'$

This being looked up under the above-mentioned fourth loxodrome, I find against it, as the value corresponding thereto, the required latitude of the second place

$28^{\circ}42'$

And I also find against it the longitude

30°

The difference between the latter and the longitude 6° , the first in the present list, is the required difference of longitude

24°

The proof of which is evident from the procedure. **CONCLUSION.** Given the loxodrome of two places, the latitude of one place, and the distance, we have thus found the latitude of the other place and the difference of longitude; as required.

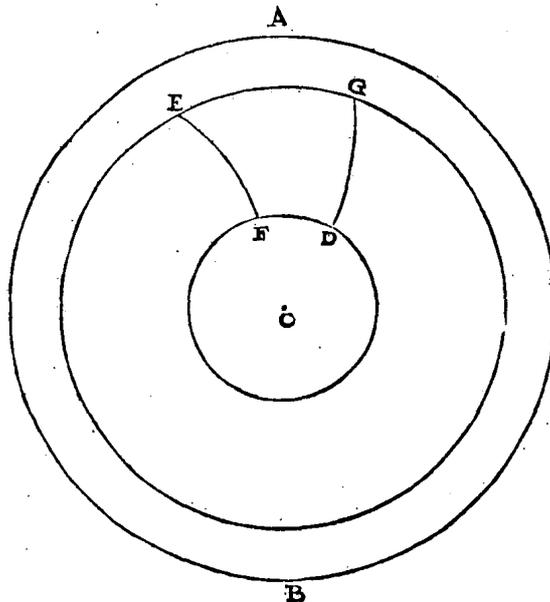
NOTE.

We have spoken above of sailing on great-circle tracks and loxodromes, but because on long voyages use is also made, as occasion permits, of a method of sailing that is a combination of the two, *viz.* an eighth loxodrome and a meridian, we shall say something about this here. In the figure overleaf let *AB* designate the earth, whose equator is *AB* and the north pole *C*; further *D* and *E* are two places of different longitudes and latitudes, through which have been drawn the two parallels *DF* and *EG*. Now in order to sail from *E* to *D*, neither on a great-circle track nor on a loxodrome, as above, but on a meridian and an eighth loxodrome, one first sails from *E* straight to the north, on the meridian *EF*, until one is in the latitude which one knows *D* to have; let this be, I assume, at *F*. Thereafter one turns to the west, remaining constantly in the same latitude, *i.e.* constantly sailing on the eighth loxodrome until one arrives at the desired place *D*.

Note further that one might first begin to sail from *E* straight to the west,

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westwaert an, dats gheduerlick op de achtste cromstreeck, tot datmen hadde de langde van D, als tot G, en daer na van G recht noortwaert an, tot datmen op D comt: Doch alst niet en waert uyt oirfaeck van winden of stroomen, diemen van te vooren wist dat hinderlick souden sijn, so waert beter eerst te seylen van



E op F, en van F op D, dan eerst van E op G, en van G op D, uyt oirfaeck datmen met meerder sekerheyte van F tot D op een selve breede can blijven, dan van G tot D op een selve langde, inder voughen dat soo veel men deur afleydende stroomen of qua rekening, in langde misgiste, soo verre souden men te oostlick of westlick van D commen: la sulcx, datmen somwijlen als D een cleen Eylant waer, niet weten en soude (ghelijckt metter daet dickwils ghebeurt (of men daer afoost of west waer, niet teghenstaende men de breede versekert hadde: Maer van E eerst noortwaert an varende, t' sy datmen wat oostlicker of westlicker dan op F ter begheerde breede comt, ten doet daer sulcken hinder niet, want blijvende int seylen nae D altijt op de behoerlicke breede, men moet D ontmoeten.

Tot dese wijze van seyling op een achtste cromstreeck met een middachront, en behouft men gheen rekeninghen als de voorgaende van rechte en cromme seyfstreken, maer ten is soo corten wech niet, gelijk men inde form openbaerlick siet.

ANHANG

i.e. constantly on the eighth loxodrome, until one is in the longitude of *D*, *viz.* at *G*, and then from *G* straight to the north, until one arrives at *D*. But if it were not because of winds or currents which one knew beforehand would be troublesome, it would be better first to sail from *E* to *F* and then from *F* to *D*, rather than first from *E* to *G* and then from *G* to *D*, because one can remain with greater certainty in the same latitude from *F* to *D* than in the same longitude from *G* to *D*, so that as much as the conjecture concerning the longitude was wrong owing to diverting currents or miscalculation, so much one would arrive too far to the east or west of *D*, even to the extent that sometimes, if *D* were a small island, one would not know (as often happens in actual fact) whether one was to the east or west of it, in spite of the fact that one had made sure of the latitude. But if one first sails from *E* to the north, it does not matter so much if one arrives in the required latitude somewhat further east or west than *F*, for if in sailing to *D* one always remains in the proper latitude, one is bound to come to *D*.

For this method of sailing on an eighth loxodrome and a meridian one requires no calculations like the foregoing, of great-circle tracks and loxodromes, but it is not so short a route, as is plainly seen in the figure.

A N H A N G

DER CROMSTREKEN.

1 HOOFSTICK.

Verhael op de oirden der cromstreken.

Sommige als *Robert Flues*, nemen de cromstreken te beginnen vant middachront af na t'middelront toe, noemende die van noort ten oosten d'eerste, noort noorrooft de tweede, en soo voort. Ander als *Edward Wright*, tellen van t'middelront af na t'middachront, noemende de cromstreeck van oost ten noorden d'eerste, oost noordoost de tweede, en soo voort. Maer wanttet om twijffeling te weeren, goet waer dat alle die van dese stof handelen, int noemen der cromstreken een selve oirden volghden, soo heeft my oirboir ghedocht mijn ghevoelen te verclaren, waerom ick meyn datter meerder reden is te beginnen van het middachront, gheleijck wy hier vooren oock ghedaen hebben, dan van t'middelront, als volght:

Voor al wanneermen seght d'eerste cromstreeck die te wesen, welcke eerst na t'middelront volght, soo en is dat niet eyghentlick gesproken den * Doender buyten t'middelront sijnde, dadelick of deur t'ghestelde, wantmen in sulck *Efficiens.* ansien dan niet en soude moeten segghen d'eerste cromstreeck vant middelront af, dan d'eerste cromstreeck van een ewewijdich rondt mettet middelront: Maer dat soo ghenomen, t'selve ewewijdich ront en sal in d'oirden der cromstreken niet vallen, t'welck nochtans gheen rechte streeck sijnde een cromstreeck is, volghende haer bepaling. Dit anghemerckt, de natuerlicke oirden schijnt te vereyffchen datmen t'middachront neme voor begin, want van daer af tellende, soo valt de oosten weststreeck me onder de cromstreken, wefende al tijt de achtste na t'behooren, en daerom hebben wy hier vooren die wijse ghevolght.

2 HOOFSTICK.

Van *Petrus Nonius* feyl, angaende de ghetalen der cromstreken.

Nadien de Portuguijfen en Spaengnaerden eerst ernstelick de groote zeevaerden anghევანghen hadden, soo viel by hun anmercking op de gedaente en eyghenscapen der cromstreken: Waer af den vermaerden *Wisconstnaer Petrus Nonius* handelende, heeft gheschreven vande ghetalen dienende tottet formen der selve, maer sy en wierden by hem niet recht ghenouch getroffen, t'welck ick niet en segh tot sijn verachting, want den gront daer hy op boude, hadde uytterlick soo vasten ansien, dat d'alder ervarenste voor t'eerste lichtelick souden gheimeent hebben de saeck soo te wesen, en by aldien hem sulcke oirsaeck van proef ontmoet hadde, als anderen na hem wel bejeghende, hy soude soo wel als anderen t'ghebreck bemerckt hebben.

Tis dan te weten dat hy int 2; Hoofsttick sijns 2 boucx *de Reg. & instr.* besluyt de houckmaten der boghen vanden aspunt totte cromstreeck in *ghe-

APPENDIX OF THE LOXODROMES.

1st CHAPTER.

Account about the order of the loxodromes.

Some people, like *Robert Hues* ¹⁾, regard the loxodromes as starting from the meridian and proceeding to the equator, calling that of north by east the first, north northeast the second, and so on. Others, like *Edward Wright* ²⁾, count from the equator to the meridian, calling the loxodrome of east by north the first, east northeast the second, and so on. But since, to avoid doubt, it would be advisable for all those dealing with this subject to follow the same order in naming the loxodromes, it seemed expedient to me to set forth my view why I think there is more reason to start from the meridian, as we have done in the foregoing, than from the equator, as follows:

Especially if we say that the first loxodrome is the first, reckoning from the equator, this is not properly expressed, when the observer is outside the equator, in actual fact or by the supposition, because in this respect we ought not to say the first loxodrome from the equator, but the first loxodrome from a parallel to the equator. But even if this is done, this parallel will not fall under the order of the loxodromes, and yet, not being a great-circle track, it is a loxodrome according to the definition. Considering this, the natural order of things seems to demand that the meridian be taken as the starting-point, for, reckoning thence, the east and west track also falls under the loxodromes, being always the eighth, as it should be, and that is why we have followed this method in the foregoing.

2nd CHAPTER.

Of the error of *Petrus Nonius* ³⁾, concerning the numerical values of the loxodromes.

When the Portuguese and the Spaniards had seriously started on long sea-voyages, their attention was caught by the appearance and properties of the loxodromes. The famous mathematician *Petrus Nonius*, dealing with this, wrote about the numerical values serving to form them, but he did not fix these values accurately enough, which I do not say to disparage him, for the foundation on which he built seemed outwardly so secure that even the most experienced people would at first easily have thought that the matter was like this. And if he had had the same opportunity of testing it as others after him found, he would have perceived the error just as well as others.

Thus it is to be noted that in the 23rd Chapter of his 2nd book *de Reg. & instr.* ⁴⁾ he concludes that the sines of the arcs from the pole to the loxodrome

¹⁾ Cf. *Introduction*, p. 488.

²⁾ Cf. *Introduction*, p. 482, 483, 494.

³⁾ Cf. *Introduction*, p. 491-492.

⁴⁾ Cf. *Introduction*, p. 491.

*Continua
proportione.*

* gheduerighe everedenheyt te wesen; Als inde voorgaende form des 3 voorstels, (alwaer wy nemen R Z de vierde te sijn) dat ghelijck houckmaet van M R, tot houckmaet van M X, also houckmaet van M X, tot houckmaet van M Y, en van M Y tot die van M a, en van M a tot die van M b; en van M b, tot die van M Z, en daerom oock ghelijck houckmaet van M R, tot houckmaet van M X, also houckmaet van M b, tot houckmaet van M Z: Doch dat gemist te wesen blijkt aldus: De driehouck M R X heeft drie bekende palen, te weten de sijde M R 90 tr. den houck M R X 45 tr. en den houck R M X 1 tr. Hier me ghefocht de sijde M X, wort bevonden van 89 tr. diens houckmaet doet 9998: Sulcx dat de houckmaet van M R, in foodanighen reden is totte houckmaet van M X, als 10000 tot 9998. Laet nu de cromstreeck van R tot b, soo na den * aspunt ghecommen wesen, dat M b doe 10 tr. diens houckmaet is 1736. Dit soo ghenomen, de houckmaet van M Z soude moeten doen 1736, want seggende houckmaet van M R 10000, geeft houckmaet M X 9998, wat houckmaet van M b 1736? Comt houckmaet die sijn soude voor M Z alsovooren 1736: Maer de selve soo niet te wesen wort aldus bethoont: De driehouck M b Z heeft drie bekende palen, te weten de sijde M b 10 tr. den houck M b Z 45 tr. en den houck b M Z 1 tr. Hier me ghefocht de sijde M Z, wort bevonden deur het 42 voorstel der clootsche driehoucken van 8 tr. 33 ①, diens houckmaet 1487, groot verschil heeft vande voorschreven 1736, t'welck sy soude moeten doen om met d'ander everedenich te sijn. Sulcx dat de booch M Z, die volghende t'ghestelde maer en doet 8 tr. 33 ①, wort na d'ander reghel bevonden van 10 tr. t'welck 1 tr. 27 ① te veel is. Maer sulcken feyl vallende op alleenelick een everedenheytswercking, soo machmen dencken wat grooter feyl datter op M Z boven dat noch moet commen, deur al de voorgaende versaeinde feylen der werckingen commen totten boveschreven 10 tr. vanden aspunt gheraect.

Polum.

3 HOOFSTICK.

Vant feyl inde tafels der cromstreken deur *Edwart Wright*.

Na de Portuiguysen en Spaengnaerden sijn in groote zeevaerden de Enghelschen ghevolght, welcke op dese ghedaente der cromstreken oock acht nemende, hebben t'feyl van *Nonius* bemerckt, en tot verbetering van dien soo sijnder onlanx uytghegheven tafelen der cromstreken deur *Edwart Wright*, als die des 4 voorstels van desen, welcke de saeck naerder commen: De proef daer ick sulcke meerder naerheyt deur vermoede, was dat ick na d'eerste wijze des maecksels der tafels vande cromstreken int boveschreven 4 voorstel, socht de breeden des vierden cromstreecx, (in welcke t'werck licht valt, deur gheduerighe vergaring sonder menichvuldig of deeling, om dat raecklijn en rechthouckmaet van 45 tr. daer even vallen) en dat tot op den 78 tr. der langde, waer op ick bevant t'overcommen 61 tr. 26 ①: Maer *Wrights* tafels hebben tot sulcken plaets 61 tr. 14 ①, die op soo grooten langde maer 12 ① en schillen: Boven dien was my noch bekend datter ware ghetal, volghende sulck ghestelde, minder moest sijn dan die 61 tr. 26 ①: Sulcx dat my dit, als gheseyt is, vermoen gaf van die tafels de fake na te commen: (Maer hoe de ghetalen der tafels van ander cromstreken met sulcke rekeningen overcommen, en heb ick deur ander belet niet versocht) Doch en isser de rechte gront noch niet volcommelick ghetrossen. Om hier af verclaring te doen, soo schrijf ick eerst t'navolghende

VER.

form a continued proportion. Thus, in the foregoing figure of the 3rd proposition (where we assume RZ to be the fourth loxodrome), as the sine of MR is to the sine of MX , so is the sine of MX to the sine of MY , and that of MY to that of MA , and that of MA to that of MB , and that of MB to that of MZ ; and consequently also: as the sine of MR is to the sine of MX , so is the sine of MB to the sine of MZ . Now that this is wrong, appears as follows: The triangle MRX has three known elements, *viz.* the side $MR = 90^\circ$, the angle $MRX = 45^\circ$, and the angle $RMX = 1^\circ$. When herewith the side MX is sought, it is found to be 89° , the sine of which is 9,998, so that the sine of MR is to the sine of MX in the ratio of 10,000 to 9,998. Now let the loxodrome from R to B have come so near to the pole that MB is 10° , the sine of which is 1,736. Assuming this, the sine of MZ would have to be 1,736, for when we say: sine of $MR = 10,000$ gives sine of $MX = 9,998$, what does the sine of $MB = 1,736$ give? The sine, which would be that of MZ — as before — is 1,736. Now that this is not true, is proved as follows. The triangle MBZ has three known elements, *viz.* the side $MB = 10^\circ$, the angle $MBZ = 45^\circ$, and the angle $BMZ = 1^\circ$. When herewith the side MZ is sought, it is found by the 42nd proposition of spherical trigonometry ¹⁾ to be $8^\circ 33'$, the sine of which = 1,487 differs a good deal from the aforesaid 1,736, which it would have to be in order to be proportional to the other. So that the arc MZ , which according to the supposition is only $8^\circ 33'$, according to the other rule is found to be 10° , which is $1^\circ 27'$ too much. Now if such an error occurs in only one operation of proportion, one may consider how much greater will be the error that must occur in MZ in addition, from all the previous errors accumulated of the operations carried out before one reaches the above 10° from the pole.

3rd CHAPTER.

Of the error in the tables of the loxodromes by *Edward Wright*.

The Portuguese and the Spaniards were succeeded in long sea-voyages by the English, who, also paying attention to this appearance of the loxodromes, noticed the error of *Nonius*; and to correct this error, tables of the loxodromes were recently published by *Edward Wright*, *viz.* those of the 4th proposition of the present work, which are more accurate. The test on the ground of which I expected this greater accuracy was that I sought the latitudes of the fourth loxodrome according to the first method of making the tables of the loxodromes in the above-mentioned 4th proposition (in which case the operations are easy, owing to constant addition, without multiplication or division, because the tangent of an angle of 45° is equal to the sine of a right angle ²⁾), up to 78° of longitude, with which I found $61^\circ 26'$ to correspond. But *Wright's* tables have for this place $61^\circ 14'$, which differs only $12'$ on so great a longitude. Moreover it was known to me that the correct value, according to this supposition, must be less than this $61^\circ 26'$, so that — as I have said — this made me expect that those tables were rather accurate (But since I was prevented by other business, I have not examined how the values of the tables of other loxodromes agree with these

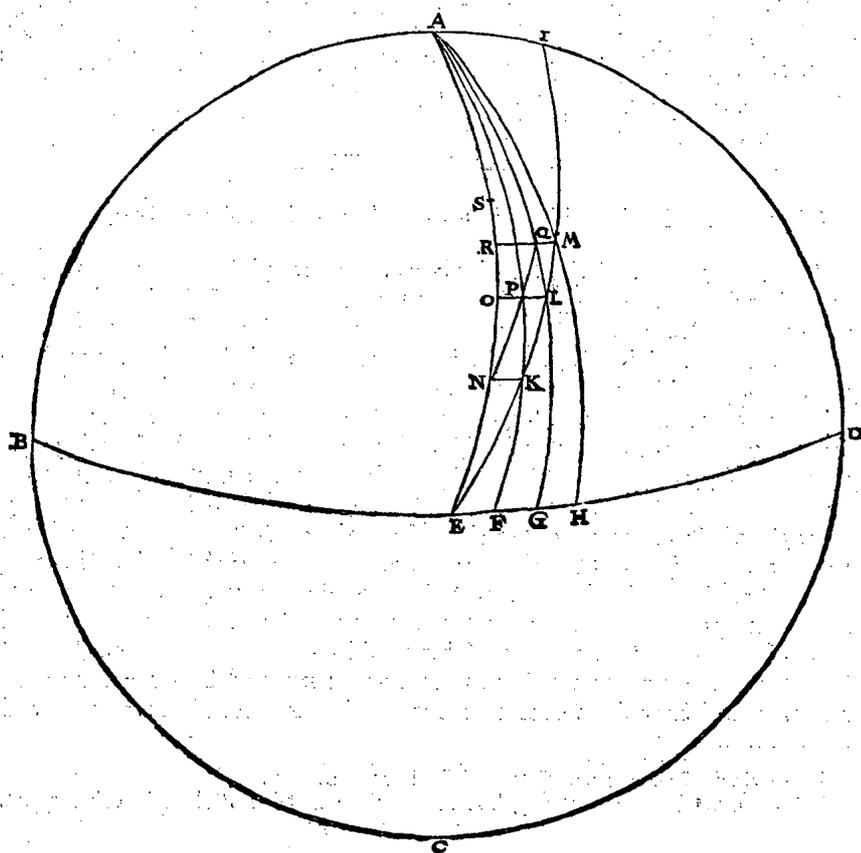
¹⁾ Stevins's *Trigonometry* (Work XI; i, 13), p. 302.

²⁾ This must be the meaning, but the original text is not clear in this respect.

VERTOCH.

Ghelijck des cromstreecx afvijcking vant middelront op haer voortganck eens langdetraps, tot haer afvijcking op haer voortganck van noch een langdetrap daer an volghende: Alsoo seer na snylijn door t'begin van die laetste voortganck, tot snylijn door t'begin deseerften voortganckx.

TGHEGHEVEN. Laet ABCD een Eertcloodt sijn diens aspunt A, middelront B ED, waer in gheteyckent sijn de vier punten E, F, G, H, sulcx dat E F, F G, G H, elck doen 1 tr. Op dese vier punten E, F, G, H, sijn ghetrocken de vier middachboghden A E, A F, A G, A H; voort sy E I een cromstreeck, ick neem d'eerste, snyende de vier middachboghden in K, L, M, I; Daer na sy ghetrocken de booch K N ewewijdich met F E, en L O met G E, snyende A F in P, sghelijcx sy ghetrocken de booch M Q R ewewijdige met H E en snyende A G in Q: Daer



na sy

calculations). But the correct basis has not yet been perfectly laid. To explain this, I first give the following

THEOREM.

As the deviation of the loxodrome from the equator during its advance through one degree of longitude is to its deviation during its advance through the next degree of longitude, so very nearly is the secant through the beginning of the latter advance to the secant through the beginning of the former advance.

SUPPOSITION. Let $ABCD$ be a globe, whose pole is A , the equator BED , in which are marked the four points $E, F, G,$ and H , so that $EF, FG,$ and GH are each 1° . On these four points $E, F, G,$ and H are erected the four meridians $AE, AF, AG,$ and AH . Further let EI be a loxodrome, I assume the first, intersecting the four meridians in $K, L, M,$ and I . Thereafter let the arc KN be drawn parallel to FE , and LO parallel to GE , intersecting AF in P ; similarly let the arc MQR be drawn parallel to HE and intersecting AG in Q . Thereafter let the first loxodrome be drawn from N via P to Q , which must be equal and similar to the part KLM .

This being so, FK is the deviation of the loxodrome from the equator FED during its advance through one degree of longitude; *i.e.* the loxodrome having advanced from E to K , where its change of longitude was one degree, EF , during this advance through one degree of longitude its deviation has become FK , or EN , as being equal thereto, because KN is parallel to FE by the supposition. And in the same way it is to be understood that NO is the deviation of the loxodrome during its advance through another degree of longitude, coming from K to L , where it has received as much deviation as from N to O , because LO is parallel to GE by the supposition. And for the same reason OR is the deviation of the loxodrome during its advance through yet another degree of longitude,

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na sy ghetrocken d'eerste cromstreeck van N deur P tot Q, die even en gelijk moet sijn metter deel K L M.

Dit soo wesende FK is des cromstreecx afwijking vant middelront FED op haer voortganck eens langdetraps: Dat is, de cromstreeck voortghegaen hebbende van E tot K, daerse in langde verandering ghecreghen heeft een trap EF; soo is op die voortganck eens langdetraps, haer afwijking gheworden FK, of EN als daer me even sijnde, om dat KN ewewijdich is met FE deur t'gegeven: En alsoo salmen verstaen dat NO is des cromstreecx afwijking op haer voortganck van een langdetrap meer, met te commen van K tot L, waer op sy afwijking ghecreghen heeft soo veel als van N tot O, om dat LO ewewijdich is met GE deur t'ghegheven: En om dergelijcke redenen is OR des cromstreecx afwijking op haer voortganck van noch een langdetrap meer, met te commen van L tot M. Sulcx dat NO is des cromstreecx afwijking op haer voortganck eens langdetraps, en OR afwijking op haer voortganck van noch een trap daer an volghende. **T B E G H E E R D E.** Wy moeten bewijfen dat ghelijck NO tot OR, alsoo seer na de snylijn van des cloots middelpunt deur L begin des laetsten voortganck LM, tot snylijn deur K begin des eersten voortganck KL: Maer de snylijn deur N, is even ande snylijn deur K, om dat KN ewewijdighe is met FE: En sghelijcx is de snylijn deur O, even ande snylijn deur L, om dat LO ewewijdighe is met GE, en daerom moeten wy bewijfen dat ghelijck NO tot OR, alsoo snylijn deur O, tot snylijn deur N.

Maer om hier te segghen tot wat eynde dit streckt, en de somme des voornehmens int cort te verclaren, soo is te weten dat wy bethoonen sullen, de ghealen inde boveschreven tafel ghevonden, niet te overcommen met dese everedenheyt, en daerom niet heel recht te wesen. Voort hoemen deur sulcken gront ghewisse tafels can maken, hoe wel deur een moeylicker wercking.

T B E W Y S.

Laet ons nemen de form in d'eerste bepaling des houckmaetmaeckfels, waer in teansen valt dat den driehouck A B I, ghelijck is metten driehouck A F C, diens *lijckstandighe sijden everedenich sijn, te weten

Homologa.

Ghelijck A I tot A B, alsoo AC tot A F,

Maer A C is even met A B, en G C met A F, daerom

Ghelijck A I tot A B, alsoo A B tot G C.

Maer A I is snylijn, A B rechthouckmaet, en G C houckmaet van C E, te weten des schilboochs van B C, daerom segh ick by ghemeene reghel, dat

Ghelijck eens boochs snylijn tot rechthouckmaet, alsoo rechthouckmaet, tot schilboochs houckmaet.

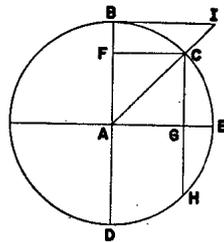
Dit soo sijnde, wy commen nu totte form deses vertoochs, waer me ick om dese bewesen everedenheyt aldus segh:

Ghelijck int vierendeelronts A E, snylijn deur N, dats snylijn des boochs N E, tot rechthouckmaet, also rechthouckmaet, tot schilboochs houckmaet van N E wesende houckmaet van A N: Dats andermael int cort gheseyt: Ghelijck snylijn deur N, tot rechthouckmaet, alsoo rechthouckmaet, tot houckmaet van A N. En weerom Ghelijck snylijn deur O, tot rechthouckmaet, alsoo rechthouckmaet, tot houckmaet van A O.

Maer

coming from L to M . So that NO is the deviation of the loxodrome during its advance through one degree of longitude, and OR the deviation during its advance through yet another subsequent degree. **REQUIRED.** We have to prove that as NO is to OR , so very nearly is the secant from the centre of the sphere through L , the beginning of the latter advance LM , to the secant through K , the beginning of the former advance KL . Now the secant through N is equal to the secant through K , because KN is parallel to FE . And similarly the secant through O is equal to the secant through L , because LO is parallel to GE , and consequently we have to prove that as NO is to OR , so is the secant through O to the secant through N .

Now in order to state here for what purpose this serves and to set forth briefly the sum of what we propose to say, it is to be noted that we are going to stress the fact that the numerical values found in the above-mentioned table do not agree with this proportion, and consequently are not quite exact. Furthermore, how it is possible to make accurate tables on this basis, though by more difficult operations.



PROOF.

Let us take the figure in the first definition of the work on the making of tables of sines¹⁾, where it can be seen that the triangle ABI is similar to the triangle AFC , whose homologous sides are proportional to each other, *viz.*:

As AI to AB , so AC to AF ;

Now AC is equal to AB , and GC to AF , therefore

As AI to AB , so AB to GC .

Now AI is the secant, AB the semi-diameter, and GC the sine of CE , *viz.* of the complement of BC ; I therefore say by the general rule that

As the secant of an arc is to the semi-diameter,

So is the semi-diameter to the sine of the complement.

This being so, we now come to the figure of the present theorem, about which on account of this proved proportion I say as follows:

As in the quarter circle AE the secant through N , *i.e.* the secant of the arc NE , is to the semi-diameter, so is the semi-diameter to the sine of the complement of NE , being the sine of AN . That is, stated briefly once again: As the secant through N is to the semi-diameter, so is the semi-diameter to the sine of AN . And again:

As the secant through O is to the semi-diameter, so is the semi-diameter to the sine of AO .

¹⁾ Stevin's *Trigonometry* (Work XI; i, 11), Fol. 1.

Maer rechthouckmaet aldus inde twee voorgaende everedenheden middel-everedenighe sijnde, soo volght daer uyt dat

Ghelijck snylijjn deur N, tot snylijjn deur O;

alsoo houckmaet van A O, tot houckmaet van A N:

Maer ghelijck houckmaet van A O, tot houckmaet van A N, alsoo het rondt diens halfmiddellijn de houckmaet is van A O, dats t'rondt daer L P deel af is, tottet rondt diens halfmiddellijn de houckmaet is van A N, dats t'rondt daer K N deel af is: Daerom

Ghelijck snylijjn deur N, tot snylijjn deur O, alsoo het rondt daer L P deel af is, tottet rondt daer K N deel af is.

Maer ghelijck het rondt daer L P deel af is, tottet rondt daer K N deel af is, alsoo het deel L P, tottet deel K N, want elck is i tr. sijns rondts deur t'ghegheven: Daerom

Ghelijck snylijjn deur N, tot snylijjn deur O, alsoo L P, tot K N.

Maer L P en K N sijn lijkstandighe sijden in twee driehoucken Q L P, P K N, die na ghenouch ghelijck sijn, (ick segh na ghenouch, uyt oirsaeck datse so na malcander staen, en so cleene sijden hebben, waer deur oock int vertooch gheseyt is *Alsoo seer na*) daerom sy sijn met d'ander lijkstandighe Q L, P K everedenich, dat is

Ghelijck L P tot K N, alsoo Q L tot P K: En vervolghens

Ghelijck snylijjn deur N, tot snylijjn deur O, alsoo Q L tot P K.

Maer R O is even met Q L, en O N met P K: Daerom

Ghelijck snylijjn deur N, tot snylijjn deur O, alsoo R O tot O N.

En deur verkeerde reden,

Ghelijck O N tot O R, alsoo snylijjn deur O, tot snylijjn deur N.

T B E S L V Y T. Ghelijck dan des cromstreecx afwijking: vant middelront, op haer voortganck eens langdetraps, tot haer afwijking op haer voortganck van noch een langdetrap daer na volghende: Alsoo seer na snylijjn door t'begin van die laetste voortganck, tot snylijjn door t'begin des eersten voortganck, t'welck wy bewijfen moesten.

Dese everedenheyt aldus vast en bekent sijnde, wy fullen tot ons voorghenomen verclaring van t'feyl der tafels commen.

Laet E I inde form deses vertoochs, beteyckenen d'eerste cromstreeck, daer in wy ons voorstellen te moeten vinden de twee breeden F K, G L. Om dan eerst te vinden F K, ick segh den driehouck K F E (die ick ten eersten voor plat ansie, om dat sulcke driehoucken inde tafels voor plat ghenomen wierden) te hebben drie bekende palen, te weten den houck K E F 78 tr. 45 ①, K F E recht, en F E i tr. Hier me ghesocht de sijde K F, en alles berekent met rechthouckmaet van 1000000, tot op ② 100, wort bevonden deur het 4 voorstel der platte driehoucken van 5 tr. i ① 38 ②. Maer E N is even an F K, daerom E N doet oock 5 tr. i ① 38 ②. Nu dan N E bekent sijnde, en om na de regel des voorgaenden vertoochs te vinden N O, ick seg aldus: Snylijjn deur N, dats van N E 5 tr. i ① 38 ② doende 10038616, gheeft snylijjn deur E doende 1000000, (die wy hier snylijjn heeten, hoe wel het eyghentlick gheen en is, maer om de naem na de reghel te ghebruycken) wat de booch N E 5 tr. i ① 38 ②? Comt voor de booch N O 5 tr. o ② 28 ②: Die vergaert tot N E 5 tr. i ① 38 ②, comt voor: E O, dats oock voor de begheerde G L 10 tr. 2 ① 6 ②: En soo veel wort oock volcommelick bevonden volghende d'eerste wijze des maeckfels vande tafels der cromstrecken int 4 voorstel, want N K als gront des rechthouckighen driehouck P K N, doet 59 ① 46 ②, waer me de reghel gevolght, deur rekening der platte driehoucken,

O

men

Now since thus the semi-diameter in the two foregoing proportions is the mean proportional, it follows from this that

As the secant through N is to the secant through O ,

So is the sine of AO to the sine of AN .

Now as the sine of AO is to the sine of AN , so is the circle whose semi-diameter is the sine of AO , *i.e.* the circle of which LP is a part, to the circle whose semi-diameter is the sine of AN , *i.e.* the circle of which KN is a part. Therefore:

As the secant through N is to the secant through O , so is the circle of which LP is a part to the circle of which KN is a part.

Now as the circle of which LP is a part is to the circle of which KN is a part, so is the part LP to the part KN , for each is 1° of its circle by the supposition. Therefore:

As the secant through N is to the secant through O , so is LP to KN .

Now LP and KN are homologous sides in two triangles QLP and PKN , which are nearly similar (I say: nearly, because they are so close to each other and have such small sides, on which account the theorem also spoke of *so very nearly*), therefore they are proportional to the other homologous sides QL and PK , *i.e.*:

As LP is to KN , so is QL to PK . And consequently:

As the secant through N is to the secant through O , so is QL to PK .

Now RO is equal to QL , and ON to PK . Therefore:

As the secant through N is to the secant through O , so is RO to ON .

And by the inverse proportion:

As ON is to OR , so is the secant through O to the secant through N .

CONCLUSION. As therefore the deviation of the loxodrome from the equator during its advance through one degree of longitude is to its deviation during its advance through the next degree of longitude, so very nearly is the secant through the beginning of the latter advance to the secant through the beginning of the former advance; as we had to prove.

This proportion thus being established and known, we shall come to our proposed exposition of the error of the tables ¹⁾.

Let EI in the figure of the present theorem designate the first loxodrome, on which we propose we have to find the two latitudes FK and GL . In order to find first FK , I say that the triangle KFE (which to begin with I regard as plane, because such triangles were taken as plane in the tables) has three known elements, *viz.* the angle $KEF = 78^\circ 45'$, $KFE = 90^\circ$, and $FE = 1^\circ$. When herewith the side KF is sought and everything is calculated with a semi-diameter = 10,000,000, to the nearest second of arc, it is found by the 4th proposition of plane trigonometry ²⁾ to be $5^\circ 1' 38''$. But EN is equal to FK , therefore EN is also $5^\circ 1' 38''$. Now NE thus being known, in order to find NO according to the rule of the foregoing theorem, I say as follows: Secant through N , *i.e.* $NE = 5^\circ 1' 38''$, being 10,038,616, gives secant through E , being 10,000,000 (which we here call secant, although properly speaking it is no secant, but we do so in order to use the name that accords with the rule); what does the arc $NE = 5^\circ 1' 38''$ give? The arc NO becomes $5^\circ 0' 28''$. When we add this to $NE = 5^\circ 1' 38''$, EO , *i.e.* also the required GL , becomes $10^\circ 2' 6''$. And exactly the same value is also found according to the first method of the making of tables of the loxodromes in

¹⁾ Cf. *Introduction*, p. 488-490.

²⁾ Stevin's *Trigonometry* (Work XI; i, 12), p. 147.

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men bevint K P, dats oock voor N O van 5 tr. 0 ① 28 ②, en vervolghens E O inde boveschreven volcommenheyt. Maer niet alsoo deur de tweede wijze, want alles met ② ghewrocht, daer comt voor G L 10 tr. 1 ①. Maer om de selve tweede wercking hier breeder te verclaren ick segh aldus: Ghesocht inde tafel der versaeemde snylijnen watter overcomt op 5 tr. 1 ① 38 ② van E N, wort bevonden 3020, daer toe vergaert noch eens 3020 comt 6040, waer op inde selve tafel overcommen voor G L of E O 10 tr. 1 ①, die 1 ① 6 ② verschillen van d'ander 10 tr. 2 ① 6 ②, en daerom en is dit niet heel volcommen, want hier me voortgaende men gheraect allenx tot noch wat meerder verschil.

Merckt noch wijder de reden te vereytschen, datmen int soucken van K F des driehoucx K F E, den selven driehouck niet en behoort te nemen voor plat, gemerckt de twee sijden daermen rekening me maeckt als E F, F K boghen sijn, sulcx dat F K gesocht deur rekening der clootsche driehoucken, wort bevonden van 5 tr. 0 ① 51 ②, die van d'ander 5 tr. 1 ① 38 ② verschillen 47 ②: Want hoe wel die in haer selven voor t'eerste cleen sijn, int vervolgh wordet seyl grooter.

4 H O O F T S T I C K.

Hoe t'maeckfel van gheviffe tafels der cromstreken soude meughen gheschien na t'ghevoelen des Schrijvers.

Ghelijck ghevonden is N O, deur een wercking getrocken uyt het vertoock int 3 Hooftstick deses Anhangs, alsoo sal ghevonden worden O R; segghende snylijn van C E, gheeft snylijn van N E, wat de booch N O? t'gene daer uyt comt is voor de booch O R, welke vergaert tot O E, men crijcht E R, dats oock H M breedte des 3 tr. der langde. En om daer nae te vinden de breedte van noch een langdetrap voorder, die ick neem te wesen R S, ick segh: Snylijn van R E, gheeft snylijn van O E, wat de booch O R? t'ghene daer uyt comt is voor de booch R S, en soo voort met d'ander.

M E R C K T nu noch dat deur t'nemen van cleender boghen der langde, sekerder werck valt dan deur grooter, om bekende oirsaken: Maer om versekert te sijn datmen de boghen cleen ghenouch neemt, dat sal gheschien deur een dobbel wercking, aldus toegaende: Benevens t'vinden der breedten deur t'nemen der langdeboghen van trap tot trap als hier vooren, soo salmen noch doen een ander wercking van halve trap tot halve trap, soo langhe alse gheen hinderlick verschil en hebben: Maer beginnende hinderlick verschil te crijghen, men sal d'eerste wercking van trap tot trap verlaten, blijvende by die van halve trap tot halve trap, en daer benevens noch vougende een wercking van vierendeel traps tot vierendeel traps, dats van 15 ① tot 15 ①, welke voorder commende, soose oock hinderlick verschil creghen, men sal die van halve trap tot halve trap verlaten, en voughen by d'ander een wercking van noch cleender boghen als van 10 ① tot 10 ①. En daer me canmen, hoe wel de wercking moeylick valt, tot sekerheyt commen, want gheen hinderlick verschil vallende tusschen twee werckingen, d'een deur t'nemen van heele langdetrappen, d'ander van halve, ten is niet noodich soo verre met noch cleender boghen te wercken dan met halve, want t'ghene men daer deur naerder vonde, soude door meerder moeyte gheschien sonder tot voordeel te strecken.

Merckt noch dat soomen dese wercking wilde doen na d'eerste wijze des maeckfels vande tafels der cromstreken int 4 voorstel, t'soude oirboir sijn eerst te maken een tafel van t'ghene 1 tr. langdeschil buyten t'middelront, doet
in

the 4th proposition, for NK , being the base of the right-angled triangle PKN , is $59'46''$, and when the rule is followed, by a calculation of plane trigonometry, KP , *i.e.* also NO , is found to be $5^{\circ}0'28''$, and consequently EO is found as exactly as above. But it is not like this by the second method, for when everything is calculated to the nearest second, GL becomes $10^{\circ}1'$. Now in order to set forth this second operation more fully, I say as follows: When one looks up in the table of the assembled secants what value corresponds to the $5^{\circ}1'38''$ of EN , this is found to be 3,020. When one adds thereto another 3,020, this becomes 6,040, to which there corresponds in the said table for GL or EO $10^{\circ}1'$, which differs $1'6''$ from the other value of $10^{\circ}2'6''$, and consequently this is not quite exact, for if one continues in this way, the difference gradually increases.

Note further that reason demands that when seeking KF of the triangle KFE , one ought not to regard this triangle as plane, since the two sides by means of which the calculation is made, *viz.* EF and FK , are arcs, so that FK , when sought by a calculation of spherical trigonometry, is found to be $5^{\circ}0'51''$, which differs $47''$ from the other, $5^{\circ}1'38''$. For although at first it is small in itself, the error will become greater in the sequel.

4th CHAPTER.

How the making of accurate tables of the loxodromes might be effected in the opinion of the author.

As NO has been found, by an operation derived from the theorem in the 3rd Chapter of this Appendix, so OR must be found, saying: secant of CE gives secant of NE , what does the arc NO give? The result of this is the value of the arc OR , and when we add this to OE , we get ER , *i.e.* also HM , the latitude at 3° of longitude. And in order to find then the latitude at yet one degree of longitude further, which I assume to be RS , I say: Secant of RE gives secant of OE , what does the arc OR give? The result of this is the value of the arc RS , and so on with the others.

NOTE further that when taking smaller arcs of longitude, the operations are more accurate than when taking larger ones, for known reasons. Now to ensure that the arcs are taken small enough, a double operation has to be performed, as follows. Besides the finding of the latitudes by taking the arcs of longitude from degree to degree, as hereinbefore, another operation must be performed from one half degree to the next half degree, as long as they have no appreciable difference. Now when an appreciable difference begins to appear, one must stop the first operation from degree to degree, keeping to that from one half degree to the next, and adding thereto an operation from one fourth degree to the next one fourth degree, *i.e.* from $15'$ to $15'$, and when this has gone on for some time, if again an appreciable difference arises, one must stop the operation from one half degree to the next, and add to the other an operation with even smaller arcs, *viz.* from $10'$ to $10'$. And thus, though the operation is difficult, one can attain to accuracy, for if there is no appreciable difference between two operations, the one by taking whole degrees of longitude and the other by taking half degrees, it is not necessary to operate further with arcs even smaller than half arcs, for what one might thus gain in greater exactness would be obtained by greater trouble without yielding any advantage.

Note further that if one wanted to perform this operation according to the

in ①, ② en ③ des middelronts, als de tafel des achtsten cromstreecx, en dat niet van 30 tot 30 ① der breedte, noch berekent tot op ② als die, maer van ① tot ③ der breedte, en berekent tot op ①.

De moeyte des maeckfels van sulcken tafel ghedaen wesende, tis kennelick dat de voorder wercking dan gheschien soude alleen deur menichvuldig sonder deeling, daermen na de voorgaende ander wijze met snylijnen, benevens de menichvuldig een deeling moet doen, doch en isser dan sulcken tafel niet te maken.

Dit is de sekerste wech die my nu te vooren comt, en hoe wel de wercking moeylick soude vallen, doch eens wel ghedaen wesende, men soude hem daer op meughen betrouwen. Maer sooder middeler tijt deur ymant ander lichter wercking met ghenouchsaem bewesen sekerheyt ghevonden wierde, t'waer billich die te aenvaerden.

5 HOOFSTICK.

Hoemen scherper opt Zeecompass soude connen seylen dan na t'ghemeen ghebruyck.

Het zeecompass placht eertijts ghedeelt te sijn in 8 streken, t'welck daer maer doen de wijder zeevaerden nauwer toeficht vereyschten, ghecommen is tot 32. En hoewel eenige die halven, voorder commende tot 64 streken, doch houden ander dese laetste deeling voor onnoodich, achtende smenschen gesicht in een varende schip op soo cleene ghedeelten des ronts gheen seker oordeel te connen hebben. Maer sijn VORSTELICKE GHENADE desen handel der scherpsyling grondelick overdenckende, heeft daer af middel voorgewent, om smenschen oordeel seker te connen sijn niet alleen op 64 streken, maer op streken van trap tot trap der 360 daermen de ronden in deelt, ja tot op ghedeelten eens traps, sulcx dat verscheyden menschen op een varende schip t'samen in het zeecompass siende, sullen al ghelijckelick uyt eenen mont noemen een selve trap, ja helft of vierendeel van dien, na dat den tuych groot en forchvuldelick mocht ghemaect sijn. Ende want dit stof is mette voorgaende ghemeenschap hebbende, soo vervough ick t'selve in desen Anhang, daer af een corte verclaring doende met twee voorbeelden,

1 Voorbeelt deur een zeecompass met te lely op stijf papier.

Laet A B C D E F G een zeecompass beduyen, waer in een rondt stijf papier sy, niet met 32 streken na de ghemeene wijze, maer met 360 tr. te weten elck vierendeelronts in 90 tr. gheteykent opt uysterste des cants van t'selve papier, en beginnende de telling in yder vierendeel vant middachrondt af: En daer onder noch een ander rondt stijf papier, waer in het yser gestreken met zeylsteen vast light, om alsoo na t'ghemeen ghebruyck de afwijcking des yfers vande lely soo groot te stellen, als de seylnaeldens afwijcking vant noorden op de tegenwoordighe plaets vereyscht.

Opden buytecant der casse van sulcken compass, sy ghetrocken een lini recht over eynde als C E, daer na het liniken C H, van C na t'middelpunt des compass streckende, en noch een ander van H recht neerwaert ewewijdich commende met C E: Voort sy het punt A soo, dat de verdochte lini van C tot A, strecke

first method for the making of the tables of the loxodromes in the 4th proposition, it would be expedient first to make a table of what 1° of difference of longitude outside the equator is equivalent to in minutes, seconds, and sixtieths of seconds of the equator, like the table of the eighth loxodrome, not from $30'$ to $30'$ of latitude, nor calculated to the nearest second, like the latter, but from minute to minute of latitude and calculated to the nearest sixtieth of a second.

The trouble having been taken to make this table, it is evident that the further operations would then have to be performed only by multiplication, without division, whereas according to the foregoing other method by means of secants it is necessary besides the multiplication to perform a division, but in that case no such table has to be made.

This is the most accurate method that I can think of now, and even though the operations should be difficult, once they had been performed satisfactorily, one might rely thereon. But if at any time an easier operation of sufficiently tested accuracy were found by someone else, it would be reasonable to adopt it.

5th CHAPTER.

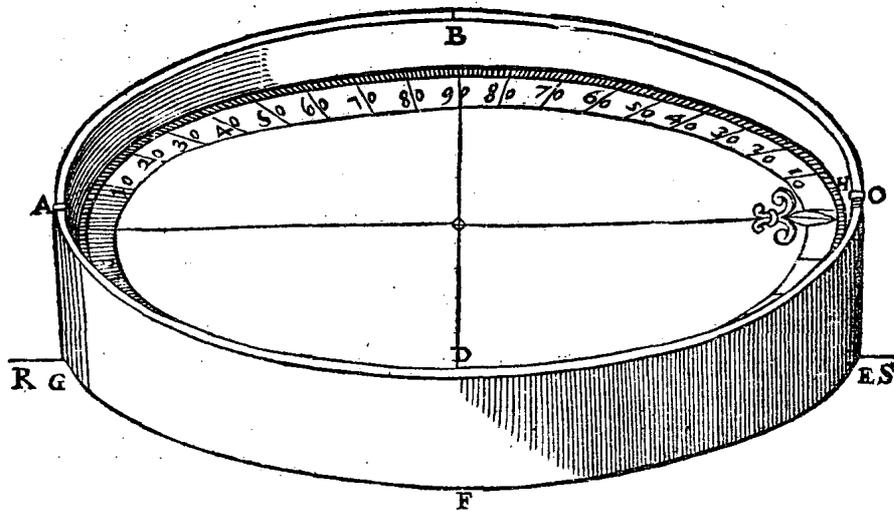
How one might steer more exactly by the mariner's compass than is usually done.

The mariner's compass was formerly divided into 8 points, which number was afterwards, when the longer voyages called for greater accuracy, increased to 32. And although some people halve these, thus arriving at 64 points, others consider this latter division unnecessary, being of the opinion that man's eyesight cannot judge with accuracy in a moving ship about such small parts of a circle. But His Princely Grace, thoroughly reflecting on this subject of accurate steering, put forward a means to ensure the accuracy of a man's judgment not only with 64 points, but with points from degree to degree of the 360 into which a circle is divided, and even to parts of a degree, so that several people in a moving ship, when looking together at the compass, will all with one accord and unanimously name the same degree, or even one half or one fourth thereof, according as the instrument has been constructed large and accurate. And because this is a subject that is connected with the foregoing, I include it in this Appendix, giving a short explanation thereof with two examples.

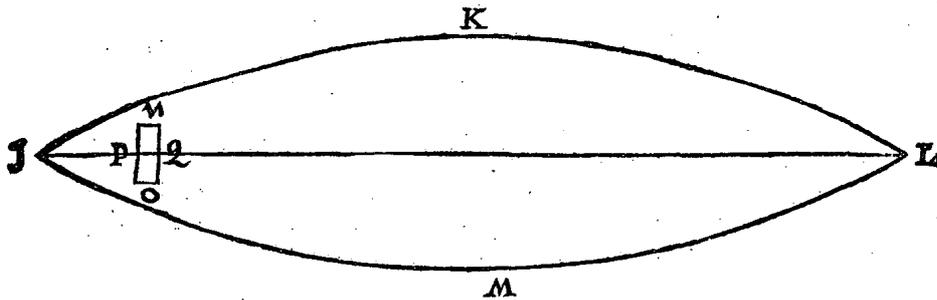
1st Example, by means of a Mariner's Compass with the Fleur-de-lys on Cardboard.

Let ABCDEFG denote a mariner's compass, containing a circular piece of cardboard, not with 32 points, as usual, but with 360 degrees, *viz.* each quarter of the circle marked in 90 degrees on the circumference of the said piece of cardboard, the counting commencing in each quarter from the meridian. And underneath it there is yet another circular piece of cardboard, on which the needle touched with loadstone is fixed, in order to adjust by the general custom the deviation of the needle from the fleur-de-lys to the value required by the deviation of the magnetic needle from the north at the place in question.

On the outside of the box of this compass let there be drawn a vertical line, *viz.* CE, then the small line CH extending from C to the centre of the compass, and yet another from H vertically downward, coming parallel to CE. Further let



recht over t'middelpunt vant compas: Daer na sy van A noch een lini recht neertwaert ghetrocken opden buytecant tot G, als A G, sulcx dat de verdochte lini van E tot G, strecken moet recht onder t'middelpunt vant compas. Dit soo sijnde, Laet nu I K L M de grontteyckening van een schip bedien, waer in N O is de plaats daermen al seyrende t'compas op stelt: Op t'plat van t'selve sal ghe-



trocken worden een lini P Q, ewewijdich streckende mettet kiel des schips, (die ick daerom kiellini noem) te weten in een lini of gespannen draet als I L, comende uyt het middel vant achterste des schips, totter middelste vant voorste. Maer want P Q in dese teyckening seer cort valt, soo sal ick om int volghende bequamelicker verclaring te meughen doen, andermael trecken een langher lini R S, die ick nu houde als voor kiellini in een schip ghetrocken te sijn, na de meyning alfooren. Op dese kiellini R S salmen het compas stellen, soo dat de twee punten E en G effen commen daer op te passen.

Om nu hier deur met sekerheyt te seylen tot op trappen of cleender gedeelte, als by voorbeeld te moeten gheseylt worden opden 17 tr. van westen na zuyden, men sal t'schip soo stieren, dat den selven 17 tr. altijd passe op de lini die geseyt is te strecken van H neerwaert ewewijdeghe met C E, want soo lang die overcommen, machmen seggen t'voorghenomen oordeel van nauwer sekerheyt gevonden te wesen, overmits dat so haest dien selven 17 tr. een trap of een halve wijck vande voorschreven lini onder H; al de ghene diet sien, sullen, al of sy uyt eenen mont spraken, terstont t'samen connen seggen hoe veel het buyten den wech is.

a Voor-

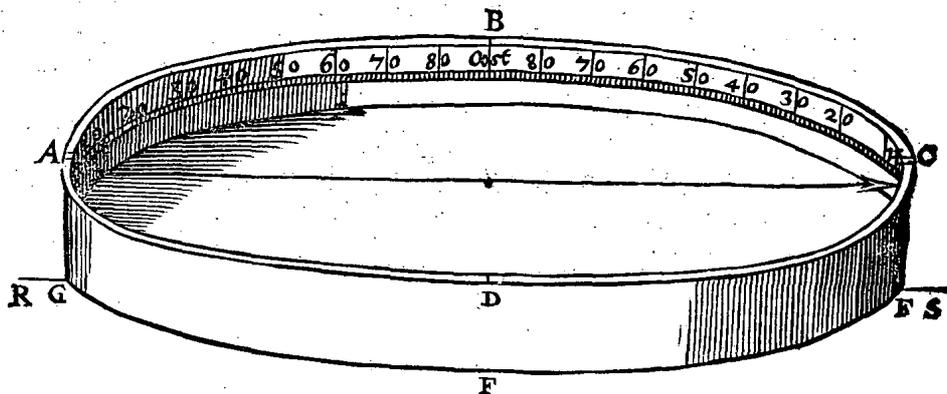
the point *A* be such that the imaginary line from *C* to *A* extends straight through the centre of the compass. Thereafter let there be drawn from *A* a line vertically downward on the outside to *G*, viz. *AG*, so that the imaginary line from *E* to *G* must pass straight below the centre of the compass. This being so, let *IKLM* denote the plan of a ship, in which *NO* is the place where during the voyage the compass is mounted. In the plane of this let a line *PQ* be drawn parallel to the keel of the ship (which I therefore call "keel-line"¹), viz. in a line or stretched thread, viz. *IL*, extending from the middle of the rear part of the ship to the middle of the front part. But because *PG* in this drawing is very short, in order to make possible a more convenient explanation in the following, I will draw once more a longer line *RS*, which I now consider as having been drawn for the keel-line in a ship, as meant before. One must place the compass in this keel-line *RS*, so that the two points *E* and *G* coincide exactly therewith.

Now in order to steer an exact course by this means to the nearest degree or smaller divisions, e.g. if one has to sail on the 17th degree from west to south, one must steer the ship in such a way that the said 17th degree always coincides with the line which has been said to extend from *H* downward parallel to *CE*, for as long as these coincide, we can say that the intended more accurate judgment has been found, since, as soon as the said 17th degree deviates one degree or one half degree from the aforesaid line through *H*, all those who see it will be able to say with one accord, as if they were speaking with one mouth, how far it is out of the course.

¹) Fore-and-aft line.

2 Voorbeeld deur een zeecompas met een feylnaelde.

Maer want de punt van een feylnaelde alleen draeyende sonder stijf papier, scherper en fekerder verthoont de naeldens natuerlicke wijfing, dan twee crome bestreken yferkens diemen ghemeenelick ruytwijs teghen t'papier plackt, soo can sulcx deur foodanighen bloote feylnaelde niet noch meerder fekerheyt gedaen worden, aldus: Dese A B C D E F G H R S van beteyckening sijnde als int eerste voorbeeld, men sal inden binnecant der casse daer de lini van H neerwaert inkomt, teyckenen de 360 tr. te weten viermael 90 tr. beginnende t'een vierendeel van H na B, t'ander van H na D, s'gelijcx van A na B, en van A na D.



Voort salmen an t'punt H schrijven noort, an A zuyt: Maer an B, t'welck na de ghemeene wijze west soude sijn, salmen oost stellen, en an D west. Daer na soo veel de feylnaeldens afwijcking ter teghenwoordighe plaats is, op sulcken verheyt salmen t'punt E vande kiellini R S stellen. Als by voorbeeld wesende de afwijcking 10 tr. van noort na oost, men sal den 10 trap van C na B doen overcommen op de kiellini R S, want alsdan moet t'punt E oock 10 tr. vande kiellini verschillen. Dit soo sijnde, en datmen by voorbeeld gheseyt wilde varen 17 tr. van west na zuyt, men sal t'schip soo stieren, dat de punt vande feylnaelde passe opden 17 trap van westen af na zuyt dat inde casse staet, en is openbaer datmen dan t'begheerde sal doen, met meerder fekerheyt dan na d'eerste wijze.

Maer soomen vreesde dat dese verkeerde stelling van oost en west, om d'onghewoonte wille dwaling mocht veroirsaken, voor de bootsghesellen die te roer staen: De Stierman soude in plaats vande namen der vier winden, meugen ghebruycken de vier letters A, B, C, D, en in plaats van hun te bevelen datse feylen soudén 17 tr. van west na zuyt, mach bevelen te feylen 17 tr. van D na A.

Voort om met meerder bequaemheyt het noort der casse of t'punt E, so verre vande kiellini te stellen, als de naeldens afwijcking vereyscht, men soude van de kiellini af over beyde sijden 40 of 50 tr. meughen teyckenen, om de cassens punt E te stellen op sulcken trap als de afwijcking mebrengt, makende int middelpunt van sulcken ront een pinneken, om de casse met haer bodems middelpunt (een putken daer in gheboort sijnde) op te draeyen.

DER SEYLSSTREKEN

EY N D E.

O 3

*2nd Example, by means of a Mariner's Compass with one
Magnetic Needle.*

Now because the point of a magnetic needle turning by itself, without a piece of cardboard, shows the natural pointing of the needle more sharply and accurately than two bent magnetized irons such as are usually stuck in the form of a rhomb against the paper, this can be done with even greater accuracy by means of a simple magnetic needle, as follows: The figure *ABCDEFGHRS* having the same letters as in the first example, in the inside of the box, where the line descends from *H* downward, the 360° must be marked, *viz.* four times 90° , starting one quarter from *H* to *B*, the other from *H* to *D*, similarly from *A* to *B* and from *A* to *D*. Further one must write north at the point *H*, south at *A*. But at *B*, which ordinarily would be west, one must place east, and at *D* west. Thereafter, as much as is the deviation of the magnetic needle in the place where the ship is at the moment, at such a distance ¹⁾ from the keel-line *RS* one must place the point *E*. Thus, for instance, if the deviation is 10° from north to east, one must cause the 10th degree from *C* to *B* to coincide with the keel-line *RS*, for then the point *E* must also deviate 10° from the keel-line. This being so, and if one wanted, for instance, to sail 17° from west to south, one must steer the ship in such a way that the point of the magnetic needle coincides with the 17th degree from west to south marked in the box; it is obvious that one will then perform the required operation with greater accuracy than by the first method.

But if one should be afraid that this reversed placing of east and west might cause errors, on account of its unusual nature, for the sailors who are at the helm, the navigator might use instead of the names of the four points of the compass the four letters *A*, *B*, *C*, and *D*, and instead of ordering them to steer 17° from west to south may order them to steer 17° from *D* to *A*.

Furthermore, in order that one may place the north of the box or the point *E* more conveniently as far from the keel-line as the deviation of the needle requires, one might mark 40° or 50° from the keel-line to both sides, in order to place the point *E* of the box on such a degree as the deviation calls for, fitting in the centre of the circle a small pin for the box to pivot on at the centre of its bottom (in which a small pit has been drilled).

END OF THE SAILINGS.

¹⁾ Stevin means a rotation through so many degrees.

DE WYSENTYT

THE AGE OF THE SAGES

From the *Wisconstighe Ghedachtenissen* (Work XI, i, 21)

This chapter forms the introduction to Stevin's Physical Geography. He gives an exhaustive exposition of his views about the "Age of the Sages", which was already discussed by us in the introduction to Vol. I (p. 44; cf. also the *Discourse*, pp. 59 ff.). In connection with the promise there made we now reproduce two fragments of this chapter, while a summary is given of the rest. This treatise gives a curious insight into Stevin's personality and into his opinions about language, the Renaissance, and the study of science generally.

WYSENTYT noemen vvy die, vvaer in by de menschen een seltsaem vvetenschap ghevveest heeft, t'vvelck vvy deur seker teyckens ghevviselick mercken, doch sonder te vveten by vvic, vvaer, of vvanneer.

Anghesien het int volghende dickwils te pas sal commen, t'onghewoonlick woort *Wysentyt* te noemen, en dat mijn voornemen is t'sijnder plaats te weten inde navolgende vernieuwing des *Wysentyt*, verclaring te doen, hoemen mijns bedunckens de saeck an soude meughen legghen, om weerom te gheraken tot alfulcken groote wetenschappen soo in Hemelloop als ander stoffen, ghelijck wy mercken by de menschen eertijts gheweest te hebben, so heeft my noodich ghedocht den selven tijt te bepalen alsvooren.

Maer om hier af breeder verclaring te doen ick segh aldus: Tis int ghebnyck damen den tijt van over ontrent neghen of thien hondert jaren, tot over ontrent 150 jaren, noemt *Barbarum saeculum*, soo veel te segghen als Leecketijt, om dat de menschen seven of acht hondert jaren lanck waren als leecke, sonder oeffening der letters of vrye consten: T'welck sijn oirspronck nam doen de Christenen d'overhant cregghen boven de Heydenen: Van welcke sy te vooren veel gheleden hebbende, en daer benevens de Heydensche Religie seer hatende, verbranden en vernielden niet alleenlick alle boucken der Religie, mette ghene daer eenich vermaen van hare Goden in stont, maer oock der vrye consten d'een metten anderen, waer syse crijghen conden. Ten laetsten heeft dit een eynde ghenomen, sulcx daimen heel verkeert de verborghen overbleven Heydensche boucken, weerom in allen houcken ghesocht heeft, int licht ghebrocht, en met groote neersicheyt en cost doen drucken, niet alleen van vrye consten, maer oock hun Goden aengaende, sulcx datter nu yder Christen vry staet die in sijn ghedichten te aenroepen, la ghedichten der Christelicke Religie te vermenghen met veersen vande rammeling der Heydensche Goden, en die daer in seer ervaren sijn, worden daerom oock seer gheleert ghenoomt. Nu alsoo benevens eenighe ydelheden, oock ernstighe dinghen voort quamen, en dat de letters en vrye consten weerom op de beenen gherochten, men heeft die voorschreven tijt van seven oft acht hondert jaren, tot onderscheyt des tijts diet nu is, en ontrent duyfent jaer daer te vooren was, ghenoomt alsgheseyt is Leecketijt, by welke d'ander verleken, *Wysentyt* soude meughen heeten: Doch tot sulcke *Wysentyt* en streckt ons meyning in de boveschreven bepaling niet, want die met d'ander seven of acht hondert jaren, al t'samen niet dan leecke tijt en sijn, verleken by den onbekenden tijt die wy deur teyckens sekerlick mercken gheweest te hebben. Maer om vande selve teyckens breeder verclaring te doen ick segh:

TEN EERSTEN datter byde menschen een groote ervaring en kennis des Hemelloops gheweest heeft, welke ten tijde van *Hypparchus* en *Ptolemus* bycans te niet en vergaen was, sulcx dat al t'ghene sy daer af beschreven hebben, maer voor overblijffels te houden en sijn van t'ghene datter gheweest hadde, want den eersten grondt waer deur het inhoudt dier overblijffels ghevonden wiert, te weten de ervarings dachtafels, sijn verloren, en men heeft sedert gheen ander ghemaect.

Angaende het ongheregelt roerfel der Dwaelers dat *Ptolemus* de * tweede

*Secundam in
equalitatem.*

oneven-

6th DEFINITION.

AGE OF THE SAGES we call that time in which exceptional learning was to be found among men, a fact which we perceive with certainty from certain signs, but without knowing among whom, where or when.

Since in the sequel the unusual term *Age of the Sages* will often be mentioned, while it is my intention to set forth in its place, to wit in the subsequent treatise on the restoration of the *Age of the Sages*, how in my view we might set out to attain once more to such great learning, both in astronomy and in other subjects, as we perceive was formerly found among men, it appeared necessary to me to define this time as above.

Now to set this forth more fully, I say as follows. It is customary to call the time from about nine hundred or one thousand years ago to about 150 years ago *Barbarum saeculum*, that is to say *Age of the Ignorant*, because for seven or eight hundred years men were ignorant, not practising letters or the liberal arts; which [condition] originated in the days when the Christians prevailed over the pagans. Having previously suffered a great deal from the latter and moreover deeply hating the pagan religion, they burned and destroyed not only all religious books, along with those which contained any information whatever about their gods, but also those on the liberal arts, one along with the other, wherever they could lay hands on them. Finally this came to an end, so that on the contrary the hidden pagan books that were left were again sought for in every corner, brought to light, and printed with great zeal and at great expense, not only those on the liberal arts, but also those concerning their gods, so that now every Christian is free to invoke them in his poems, nay, to introduce in Christian religious poetry verses concerned with the whole lot of the pagan gods, and those who are greatly proficient in it are called very learned on this account. Now since, besides some trivialities, also serious things emerged, and letters and the liberal arts were thus restored again, the aforesaid period of seven or eight hundred years was named — to distinguish it from the present time and that of about one thousand years ago — as has been said: *Age of the Ignorant*, in comparison with which the other period might be called *Age of the Sages*. But it is not this *Age of the Sages* which we mean in the above definition, for this, along with the other seven or eight hundred years, is nothing but an age of the ignorant in comparison with the unknown period which, judging from certain signs, we perceive with certainty to have existed. Now to set forth these signs in greater detail, I say as follows:

Firstly, that there existed among men great experience and knowledge of astronomy, which at the time of *Hipparchus* and *Ptolemy* had almost disappeared, so that all they have written about it is only to be regarded as remnants of what had existed, for the first foundation on which the content of those remnants was based, to wit the empirical ephemerides, have been lost, and no other such tables have since been made.

As to the irregular motion of the planets, which *Ptolemy* calls the second inequality, about which he says in the 4th chapter of his 5th book, and even more clearly in the 2nd chapter of his 9th book, that he thinks he was the first to have observed it and that it had not been noted by his predecessors, frequently accusing them of carelessness in observing the positions and motions of the heavenly bodies — to this it is replied that his predecessors must have seen the said second

10 I BOVCK DES EERTCLOOTSCHRIFTS,

onevenheyt noemt, t'welck hy int 4 Hoofstlick sijns 5 boucx, en noch meer int 2 Hoofstlick sijns 9 boucx, meent selfs eerst gagheslaghen te hebben, en by sijn voorganghers niet bemerckt gheweest te sijn, beschuldighende dickmael hun onnachtsaemheyt int gaslaen vande plaetsen en loopen der Hemelsche lichten: Daer wort op gheantwoort, dat de voorgangers de selve tweede onevenheden ghesien hebben, eer hun meughelick was d'eerste onevenheyt, of Dwaelders middelloop soo aerdich te beschrijven ghelijcke *Ptolemens* daer nae gecreghen heeft, en datse om tot soo grooten vondt te gheraken niet onachtsaem en hadden gheweest, maer neerstigher danmen van *Hypparchus* tijt af, en voor hem hoe langhe en weer ick niet, tot nu toe gheweest heeft, of meughelick is gheweest te sijne, om datmen na den Wijsentijt, de saeck op sulcken voet niet aengherast en heeft ghelijckmen doen dede, soo wy terstont t'sijnder plaets daer af breeder verclaring sullen doen.

*Diversa
gentes.*

Het voorgaende wort bevesticht deur dienmē nae *Ptolemens* tijt in ettelicke Arabische schriften vernomen heeft, dat voor hem by verscheyden * gheslachten van volck int ghebruyck is gheweest, de vaste sterren op de Hemelclooten in ander formen te teyckenen dan de ghemeene Egypische, van welke *Ptolemens* noch *Hypparchus* nergkens eenich ghewach en doen: Soodanighe heeft my gheroont den Edelen Hoochghelcerden Heer *Iosephus Scaliger*, in boucken met verclaring der teyckens ghedaen in Arabische spracck, en dat niet op een wijze, maer wel tot drie clooten toe, elck verscheyden vanden anderen. De formen van een dier clooten wierden gheseyt Hemelteyckens der Indianen, van welke namen, doch sonder schilderie, my oock ghedenckt ghesien te hebben in een Latijns bouck van seer ouden druck, maer des schrijvers naem is my vergeten, ick en weet oock niet waer t'bouck bleven is. Een deelander teyckens heb ick geschildert gesien tegen de mueren van een camer op t'Coninx hof in Polen tot Craco, wefende monsters, diens leden gemengt waren uyt verscheyden afcomsten van ghedierten, en stont daer by geschreven **SIGNA HERMETIS**, dats teyckens van Hermes. Niet een van al de boveschreven: Hemelteyckens en vinde ick als gheseyt is by *Ptolemens* vermaent te sijn: Waer uyt schijnt te meughen besloten worden, die t'sijnder handt niet ghecommen te wesen, veel min de Hemelloopsche leeringhen die elck * gheslacht na sijn wijze daer op ghegront hadde. Voort hebben de Hemelmeters eertijts wel gheweten dat den Eertclood om de Son draeyde, sonder dat de eyghentlicke voorstellen van dien *Ptolemens*, soot schijnt, ter hant ghecommen sijn, want had hysse ghesien, tis daer voor te houden dat hy (soomen oirdeelen mach uyt sijn verstandt en redelickheyt inde rest) soude toeghelaten hebben het natuerlick roersel byde ervaren Hemelmeters beschreven, en dat der eerste leerlinghen verlaten, of alleenlick leerings halven by * stelling gebruyckt. Van dit roersel des Eertcloots hebben vernomen *Philolaus Pythagoricus* als *Plutarchus* seght, voort *Aristarchus Samius* so *Archimedes* int bouck des * santtals betuycht; Maer dat sulcx inden wijsentijt niet en ghebeurde, schijnt ghenouch uyt *Archimedes* woorden te meughen besloten worden, inhoudende dat *Aristarchus* schreef teghen de Hemelmeters die de weert seyden een cloot te sijn, wiens middelpunt des Eertcloots middelpunt is, en halfmiddellijn even ande lini tusschen t'middelpunt der Son, en t'middelpunt des Eertcloots, welke lini (benevens ander slechticheden) d'een tijt langher wefende als d'ander, soo en schijnter niet veel bescheys in.

Proporioni.

Voort soomen insiet de form der * everedenheydt by *Aristarchus* aldus gheselt:

Ghe-

inequalities before they were able to describe the first inequality, or the planets' mean motion, as nicely as *Ptolemy* afterwards succeeded in doing, and that, to make such a great discovery, they cannot have been careless, but more diligent than people were from the time of *Hipparchus* and I do not know how long before him up to the present, or may have been, because after the Age of the Sages the matter was not tackled on the same scale as it had been, as we shall presently set forth more fully in its place.

The foregoing is confirmed by the fact that after the time of *Ptolemy* it became known from some Arabic writings that before him it was customary among various nations to mark the fixed stars on celestial globes in figures different from the common Egyptian ones, of which neither *Ptolemy* nor *Hipparchus* makes mention anywhere. Such figures were shown to me by the Noble Very Learned Mr. *Josephus Scaliger*¹⁾, in books with an explanation of the figures given in the Arabic language, not in one way, but on as many as three globes, each different from the others. The figures on one of these globes were said to be constellations of the inhabitants of India, about which names, but without a representation, I also remember having read in a Latin book of a very old edition, but I have forgotten the author's name, nor do I know what has become of the book. Other signs I have seen painted on the walls of a room in the royal palace at Cracow in Poland²⁾; these were monsters whose members were a combination of those of different species of animals, with an inscription: *Signa Hermetis, i.e.* signs of Hermes. As has been said, I do not find any of the above-mentioned signs mentioned in *Ptolemy*, from which it may apparently be concluded that they had not been transmitted to him, and even less so the astronomical doctrines which every nation in its own way had based thereon. Further, astronomers formerly knew quite well that the earth revolved round the sun, but the actual propositions concerning this apparently had not come down to *Ptolemy*, for if he had seen them, it is to be assumed that (if we may judge by his intelligence and reasonability in the rest) he would have admitted the natural motion described by experienced astronomers and would have abandoned that according to the earlier doctrines³⁾ or used it merely for instructive purposes, by way of hypothesis. The following authors learned of this motion of the earth: *Philolaus Pythagoricus*, as *Plutarch* says⁴⁾, further *Aristarchus Samius*, as *Archimedes* states in the book of the number of the grains of sand⁵⁾. Now that this did not happen in the Age of the Sages apparently can be concluded sufficiently from *Archimedes'* words, which are to the effect that *Aristarchus* wrote against the astronomers who said that the world is a sphere whose centre is the earth's centre, while its radius is equal to the line joining the centre of the sun and the centre of the earth, this line (among other imperfections) being longer at one time than at another, and so it does

¹⁾ Joseph Julius Scaliger (Agen 1540–Leiden 1609), Italian humanist, philologist and chronologist, professor at Ghent and at Leiden. He published excellent editions of Latin authors and wrote the monumental works *De emendatione temporum* (1583), *Thesaurus Temporum* (1606). Cf. note 4 on p. 325.

²⁾ This interesting passage shows that Stevin visited Poland. Cf. Biographical introduction, Vol. I, p. 5. Such monsters belong to the world of Accadian and Sumerian mythology and astronomy.

³⁾ The word *leerlinghen* in the Dutch text is evidently a printer's error for *leeringhen*.

⁴⁾ *De placitis philosophorum* III, cap. 13.

⁵⁾ Commonly called: *the Sand Reckoner* (= *Arenarius*). *Opera*, ed. Heiberg (Leipzig 1913) II, 218.

VAN SYN BEPALINGHEN INT GHEMEEN. II

*Ghelyck cloos middelpunt,
 Tot cloorvlack,
 Alsoo Eertclootwech,
 Tot verheyte der vaste sterren.*

Alwaer tegen Wilconftenaers reghel verlijcking van * verſcheenſlachtighe ^{Heteroge-} ſijnde, te weten punt met vlack, ſoo ſchijnet ſelve als vooren gheſeyt is daer uyt ^{nich} te moghen beſloten worden. Angaende *Archimedes* de boveſchreven even-

denheyt uytleght, en ſeght aldus behooren verſtaen te worden:

*Ghelyck Eertcloot,
 Totzet ghene men de weerelt noemi,
 Alsoo Eertclootwechs cloot,
 Tot vaste sterrens cloot.*

Tmach daer me ſijn hoet wil, dan wilconſtighe redenen vereyſſchen gewiſſe woorden: In ſomme daer en is gheen teycken van wijsentijt.

Ander getuychnis van een groote oeffening dieder voor *Ptolemens* tijt inden Hemelloop geweest heeft, hebben wy door de verſcheyden manieren van leeringen der clootſche driehoucken op verſcheyden gronden geſicht, naderhant te voorchijne ghecommen in Arabiſche ſpraeck, en daer uyt int Latijn gherocht. Want nadien de menſchen eertijts ſaghen hoe noodich goede manier van rekening der clootſche driehoucken was, om na volcommen kennis des hemelloops te trachten, ſoo heeft hun verſtant in die ſtof wonderlick gearbeyt. De manier an *Ptolemens* ter hant ghecommen, en door hem beſchreven, is cort en aerdich, beſtaende in vergaring en afrecking van redens der linien, die verdocht worden in ſeker plat dat men * clootſne noemt, maer t'gebruyck is moeyelic, want den Doender ant werck commende, en vint in dien beſchreven drie-^{Sefioſpha-} houckhandel tot alle ontmoetende voorbeelden gheen geformde * werckſtuc-^{Problemata-} ken, om die met lichticheyt te volgen, maer moet geduerlick becommert ſijn; met t'overdencken wat manier van vergaring of afrecking der redens hy tot ſijn voorbeeld uyt de clootſne verkiefen ſal. Een ander manier iſt diemen oock ghebruyck met bedencking der gemeene ſneen van platten der ronden op den cloot gheteyckent. En noch een ander diemen by *Regiomontanus* bevint.

Angaende ſommighe achten de vonden des Hemelloops niet ſoo ſeer oudt te weſen, maer dat de meeſte beſonderheden van dien deur *Hypparchus* tot ſmenſchen kennis ſouden ghecommen ſijn. En dat *Timochares* gheleest heb- bende 30 jaren na den Grooten *Alexander*, onder de ſterflicke den eerſten was, die t'vinden en opſchrijven der plaerſen vande vaste ſterren beneerſtichde, mijn gevoelen is daer af anders: Wel wil ick toeſtae, dat ſo *Hypparchus* uyt de boucken van ſijn voorgangers niet beſchreven en hadde het bouck daer na *Ptolemens* ter handt ghecommen, en van hem tot ons gherocht, dat wy nu vanden loop der Dwaelers weynich kennis ſouden hebben: Maer dat hy van die ſelſame voorſtellen een vinder ſoude gheweest ſijn, t'en ſchijnt niet om onder anderen deſe reden: *Ptolemens* ſegt int 11 Hoofſtick ſijns 4 boucx, dat *Hypparchus* een ſwaricheyt ontmoete, om dat hy deur * ſtelling des Maenloops in een * inrondt, tot ander beſluit quam dan deur ſtelling in een * uytmiddelpuntichron: Bewijſt ^{Positionem, Epicyclo, Eccentrico circulo.} voort dat ſulck verſchil niet en quam deur de verſcheydenheyt die *Hypparchus* inde twee ſtellinghen vermoet, dan deur ſijn miſrekening, of ander ongheval: Dit ſoo ſijnde, het ſchijnt te meughen beſloten worden, dat *Hypparchus* onder ander vonden diemen hem toeſchrijft, vooral gheen vinder en was des * ver-^{Theorema-} toochs, deur t'welck bewezen wort d'een en d'ander ſtelling een ſelve beſluit te gheven, maer veel eer dat hy ſulck vertooch (t'welck wy niet en ſegghen tot ſijn

ver-

not seem to make us much wiser. Further, if we understand the form of the proportion, given as follows in *Aristarchus*:

As the centre of the sphere is to the surface of the sphere,
so is the earth's orbit to the distance of the fixed stars,

where, contrary to the rule of mathematicians, heterogeneous things are compared, to wit a point and a surface, it seems that the aforesaid statement may be inferred therefrom. As to the fact that *Archimedes* explains the above-mentioned proportion and says it ought to be understood as follows:

As the earth is to that which is called the world,
so is the sphere of the earth's orbit to the sphere of the fixed stars,

however this may be, mathematical arguments call for exact words. To sum up, there is no sign of the Age of the Sages there.

Other evidence of the fact that people were greatly versed in astronomy before *Ptolemy's* time we have through the various doctrines about spherical triangles, established on different foundations, which afterwards appeared in Arabic and thus got into Latin. For since people formerly saw the necessity of a good method for calculations on spherical triangles in striving to attain perfect knowledge of the heavenly motions, their minds worked marvellously at this subject. The method which came into *Ptolemy's* hands and was described by him ¹⁾ is short and elegant, consisting in addition and subtraction ²⁾ of the ratios of the lines which are imagined in a given plane, which is called "section of the sphere", but the practical application of it is difficult, for when the practician sets to work, in this treatise on trigonometry he does not find for all the examples encountered by him worked-out problems, which can easily be followed, but he has to be continually concerned to think which manner of addition or subtraction of the ratios he has to choose for his example from the section of the sphere. Another method is that also used in studying the common intersections of the planes of the circles drawn on the sphere. And yet another method is that found in *Regiomontanus*.

As to the fact that some people think the astronomical discoveries are not so very ancient, but that most of the particulars about the subject have come to men's knowledge through *Hipparchus*, and that *Timocharis* ³⁾, who lived 30 years after *Alexander* the Great, was the first among mortals to have diligently practised the finding and writing down of the positions of the fixed stars, I am of a different opinion about this. I am prepared to admit indeed that, if *Hipparchus* had not written — on the basis of his predecessors' books — the book which afterwards came into *Ptolemy's* hands, and through him has reached us, we should now have little knowledge of the motion of the planets. But that he was one of the discoverers of these extraordinary propositions does not seem to be true, among other things for the following reason. *Ptolemy* says in the 11th chapter of his 4th book that *Hipparchus* encountered a difficulty because by assuming the moon to move on an epicycle he reached another result than by assuming it to move on an eccentric circle. He further proves that this difference was not due to the difference between the two hypotheses as suspected by *Hipparchus*, but

¹⁾ *Syntaxis* I, cap. 13.

²⁾ By "addition and subtraction of ratios" Stevin means what we should call „multiplication and division of ratios“.

³⁾ In the Dutch original the orthography *Timocharis* is used. This Greek astronomer lived at Alexandria around 230 B.C.

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*Mathemati-
can demon-
strationem.*

*Solfutio
effi vale.*

Algebram.

Arithmetici.

Unitatem.

vermindering maer om t'voornemen te bewijfen) niet begrepen en heeft, wan t
andere het waer so veel al oft ymant wel verstaende * t'wifconftich bewijs vant
laetste voorftel des eerften boucx van *Euclides*, nochtans daer na totte dadelicke
ervaring commende, twifelde of de twee viercanten der twee cortfte fijden
eens rechthouckigen driehoucx, even sijn ant viercant der langfte fide, om dat
hyfe dadelick bevint te verſchillen, ſonder te weten dat ſulcx niet en comt deur
ghebreck des voorftels, maer deur feyling der handen, ooghen, deur miſreke-
ning, of ander ongeval. Angaende dat *Ptolemeus* int 2 Hoofſtick sijns 3 boucx,
de manier van *Hipparchus* prijft voor die der ouden, nopende het vinden der
Sonnens plaets tuſſchen de vaſte ſterren, om daer uyt de lanckheyte des jaers te
berekenen, welcke plaets ſy meynen de ouden gheſocht te hebben deur de Son-
nens evenaer breede doenſe was in haer * Noortſtant, welke onderſoucking
Hipparchus niet meerder ſekerheyte dede doenſe inde lentſne was. Hier op ſeg-
men, dat ſy die door ſoodanige langduerighe ſelſaem oeffeninghen en ſteeroo-
ginghen, gherocht waren ter kennis van der Dwaelders middelloopen, niet en
fouden bemerckt hebben ſoodanighe ervaringhen gantsch onbequamelick in
Sonſtant ghedaen te worden, ick en ſiender gheen teycken af: Maer by aldienſe
haer ervaringhen in Sonſtant ghedaen hebben, ſoo *Hipparchus* ſeght, nis te ver-
moeden dat ſulcx niet en ghebeurde deur onderſoucking vande Sonnens eve-
naerbrede, maer veel eer met meerder ſekerheyte deur t'nemen der ſchijnbaer
duyſteraerlangde tuſſchen haer en eenige der vaſte ſterren. Wanſe beneven de
Son, alſmen vlietelick ghenouch ſteerooght, gheſien connen worden: T welck
Ptolemeus onbekent weſende, ſo en ſchijnt ſijn verheffing van *Hipparchus* vont,
boven die van ſijn oude voorganghers in gheen ghenouchſaem reden gegront,
ſoo veel *Timochares* angaet, dat hy als gheſeyt is onder de ſterflicke den eerſten
foude gheweest hebben, die t vinden en opſchrijven der plaetſen vande vaſte
ſterren beneerſchide: Angheſien hemden die dat ſegghen, en *Timochares* me,
onbekent ſchijnen gheweest de beſchrijvinghen der Hemelclooten, op welke
haer oude voorganghers van verſcheyden gheſlachten, de ſterren in ander for-
men vervinghen als vooren gheſeyt is, ſo en ſchijnt *Timochares* ſulcken eerſten
gaſlaghet der vaſte ſterren niet gheweest te hebben. Te meer dat der Dwael-
ders middelloopen *Hipparchus* ter hant ghecommen, ghevonden ſchijnen deur
ſekerder beſchrijving der vaſte ſterren dan van *Timochares*, die 10 \odot voor cleen-
ſte maet ghebruyckte, ſoo in *Ptolemeus* tafelen blijktt.

HET TWEEDE TEYCKEN is de wonderlicke ervarentheyte die wy ſien
eertijts by de menſchen inde Telconſt geweeſt te hebben, waer afmen een van
de vreemde ſelſaemheden houden mach de * Sielreghel, die over weynich ja-
ren deur Arabiſche boucken weer te voorſchijne gecommē is, daer afmen deur
naghelaten ſchriften niet en merckt gheweten te hebben Caldeen, Hebreen,
Griecken (want *Diophantus* is jonck) of Romeynen, al welke gheen * Telders
diemen deur weerdicheyte Telders noemt, gheweest en sijn. Oock inſiende dat
hemden daer toe noodighe reetſchap ghebrack, namelick talleters, met ſooda-
nighe thiende voortganck als de uytgheſproken getalen hebben, ſo waſt hem-
den onmeughelick: Maer mochten ſulcke telders sijn, als die nu met pennin-
ghen legghen, of met erijſchreefkens rekenen, of die ghelijcke. Dete taltey-
kens van thiende voortganck sijn inde Arabiſche ſpraeck weerom voortghe-
commen, in ſulcker voughien dat men daer me anvanghende, mach leeren wat
t'begin of punt des ghetals is, t'welck de leeckenrijt (ick meen vanden anvang
der vermaerde Griecken tot nu toe) qualick verſtaende, gheſeyt heeft de * een-
heyte te weſen. Want den Edelen Hoochgeleerden Heer *Josephus Scaliger*, heeft
myge-

to his faulty calculation or some other accident. This being so, it may apparently be concluded that *Hipparchus*, though many other discoveries were attributed to him, was not the discoverer in particular of the theorem by means of which it is proved that the two hypotheses give the same result, but much rather that he did not understand this theorem (which we do not say to belittle him, but to prove our intention), for otherwise it would be as if a man, though properly understanding the mathematical proof of the last proposition ¹⁾ of the first book of *Euclid*, yet, when afterwards it comes to practical work, should doubt whether the two squares on the two shortest sides of a right-angled triangle are [together] equal to the square on the longest side, because he finds them to differ in actual practice, without knowing that this is not due to a defect in the proposition, but to errors of the hands or eyes or to faulty calculation or some other accident. As to the fact that *Ptolemy* in the 2nd chapter of his 3rd book praises the method of *Hipparchus* above that of the ancients, concerning the finding of the sun's position among the fixed stars, in order to calculate therefrom the length of the year, which position they thought the ancients had sought by means of the sun's equatorial latitude when it was at its summer solstice, which research *Hipparchus* carried out with greater accuracy when it was at the vernal equinox, — to this it is said: I see no evidence that those who through such prolonged and exceptional practice and watching had arrived at knowledge of the planets' mean motions should not have noticed that such observations are made quite improperly at the solstice. Now if they made their observations at the solstice, as *Hipparchus* says, it may be suspected that this was not done by seeking the sun's equatorial latitude, but rather with greater accuracy by taking the apparent ecliptic longitude between the sun and some of the fixed stars. For if we watch diligently enough, they can be seen by the side of the sun; and since this was not known to *Ptolemy*, his exaltation of *Hipparchus'* discovery above those of his ancient predecessors seems to be founded on insufficient grounds. As regards *Timocharis*, viz. that — as has been said — he should have been the first among mortals to have diligently practised the finding and writing down of the positions of the fixed stars: since those who say this, and *Timocharis* as well, seem to have been unacquainted with the descriptions of the celestial globes on which their ancient predecessors of different nations marked the stars in other figures, as has been said above, *Timocharis* does not seem to have been this first observer of the fixed stars. The more so because the planets' mean motions which were handed down to *Hipparchus* seem to have been found by a more accurate description of the fixed stars than that of *Timocharis*, who used 10' for the smallest measure, as appears from *Ptolemy's* tables.

The second sign is the wonderful experience in arithmetic which it has been found man formerly possessed, one of the curious peculiarities of which may be considered to be *Algebra*, which came to light again a few years ago from Arabic books, which subject, from the writings they left, is seen not to have been known to the Chaldeans, the Hebrews, the Greeks (for *Diophantus* is no ancient writer ²⁾) or the Romans, all of whom were no arithmeticians worth the name. Considering also that they lacked the necessary instruments, viz. numerals, with the same tenth progression as the numbers have when pronounced, it was impossible for

¹⁾ Actually this is the penultimate proposition of the first book.

²⁾ *Diophantus* lived at Alexandria about A.D. 250. Stevin seems to have a mistaken view of the epoch when this mathematician flourished. His *Arithmetica* is actually a textbook of algebra.

my getoont, dat de Arabiers daer voor teyckendē een punt, aldus . r'elve oock punt noemende, en wierden die punten onder de talletters ghebruyckt in plaets daer wy o stellen, overcommende metter ghene wy over eenighe laren in onse Fransche Arithmetique onder de 2 bepaling daer afseyden. D'oirfaeck waerom in plaets van dat punt, byde Europeanen nu een o gestelt wort, acht ick dese, dat wy ghetwoon sijn punten te ghebruycken int eynde en onderscheyding der geschreven redens, welcke punten oock dickwils achter ghetalen volghen, maer soomen daer punten stelde, sy soudē twijffeling maken oft een punt waer des ghetals, dat onbehoirlick vermeerderende, oft een punt als onderscheyt des redens, en om soodanighe twijffeling te voorkomen, heefstmen het punt verandert, en daer voor een o gheschreven. Nu dan o inden Wyfentijt punt gheheten hebbende, wy sullent om die te volghen nu voorraen oock dien naem gheven, en dat tot onderscheyt vant meerconstich punt, Talpunt noemen, verlatende de eerste naem *Begin*, die wy daer toe dus langhe gebruyckt hebben. Angaende dat sommighe niet ghenouch deur nauwerlicke reden connen oordeelen, hun ghedraghen totte * loofweerdicheyt der ghene die daer af ghehandelt hebben, ick acht die op een goeden wech te wesen, midts datse de loofweerdichste loofweerdicheyt volghen, dat sijn de ervaren Telders des Wyfentijts, en verlaten de Griecken die gheen telders en waren, noch volcommelick sijn en condē deur ghebrek van rechte talteyckens als vooren gheseyt is: want hoe wel *Euclides* schoone Telconstighe * vertooghen beschrijft, die uyt den Wyfentijt t'sijnder hant getocht waren, daer en sijn * werckstucken noch * Teldaet by, welcke als gheseyt is onlanx te voorschijn commen sijn in Arabische spraek. *Sulex* dat *Euclides* vertoogt, en ghetuyghenis gheven des Wyfentijts die te voren geweest hadde, en doen niet en was. De reden waerom wy hier so ernstelick van dit punt seggen, is dat bepaling der eenheyt voor punt des getals, onder anderen getuychnis geeft des Wyfentijts diet was doenment punt teyckende en daer voor hielt: En oock des Leeckenijts diet sedert geweest heeft, so onvolkommen * Telders makende, als bepaling des deels der grootheyt voor punt der grootheyt, onvolkommen Meters soude mebrenghe. Noch canmen merken, dat inden selven Wyfentijt veel Telconstige werkingen met besonder lichticheyt afgeveerdicht wierden deur rekening op thiende voortganck ghegront. Om van t'welck breeder reden te verclaren, het is te weten dat alsoo ick over eenige laren de thiende beschreef, en my inbeelde met groote lichticheyt te meugen gebruyct worden in deyling der houckmaten en bogen met thiende voortganck; En alsoo ick daer na de selve manier eyghentlick beschreef, in sulcker vougen als van dies inden volgenden Hemelloop t'sijnder plaets een hoofstuck ghemaect sal worden met sulcke cortheyt als blijcken sal: So heb ick daer na bemerct dat sgelijcx voor my al gedaen had geweest, oft immers gedaē schein geweest te sijne in ouden tijt, die ick meyne dat den Wyfentijt was, om dese redenē: De tafel der houckmaten van *Regiomontanus*, diens halfmiddellijn gedeelt is in 1000000, begreep in haer volcommelick den thienden voortganck die ick socht: Want nemende de halfmiddellijn gedeelt te sijn in 100 even deelen, in plaets van 60 der Egyptenaers, En daer nae my selven voorstellende de deeling te wesen van thiende voortganck, in plaets der t'festichde vande Egyptenaers, ick bevant met die tafel alghedaen werck, soo veel de houckmaten angaet: En meende doen *Regiomontanus* daer af een eerste vinder geweest te sijne, te meer dat hy int begin van sijn houckmaetmaecksel, seght dat die voor hem geweest sijn, de middellijn in weynich stucken deelden, als *Prolemeus* in 120, *Arzabel* in 300, deelende elck van dien in 60 ①, en elcke 1 ① in 60 ②. Maer naderthant is my yet te vooren ghecommen, waer uyt ick nu anders vermoede, t'welck aldus toeginck: Alsoo ick

B eens

them; but they may have been arithmeticians of the kind who now count by laying out pennies or reckon with chalk marks, or the like. These numerals of the tenth progression have come to light again from Arabic books, so that, beginning with this, we can learn what is the origin or point of number, which the Age of the Ignorant (I mean from the beginning of the famous Greeks up to the present), misunderstanding it, stated to be unity. For the Noble and Very Learned Mr. *Josephus Scaliger* has shown me that the Arabs drew a point for it, as follows: . , also calling it point, and these points were marked underneath the numerals, where we put 0, which corresponds to what we said about it some years ago in our French *Arithmétique*, in the 2nd definition ¹⁾. I consider that the cause why instead of this point a 0 is now used by Europeans is that we are accustomed to use points at the end and the demarcation of written sentences, which points also often succeed numbers; if there points were used, they would raise doubt as to whether it was a point belonging to the number, increasing the latter unduly, or a point destined to demarcate the sentence, and in order to prevent such doubt, the point was changed and replaced by a 0. Now then, since in the Age of the Sages 0 was called point, in order to imitate them we shall henceforth also give it this name and call it Numerical Point, so as to distinguish it from the geometrical point, abandoning the previous name of *Commencement*, which we hitherto used for it. As to the fact that some people, not being able to judge sufficiently by means of natural reason, act on the authority of those who have dealt with the subject, I consider that they are on the right road, provided they follow the most authoritative authority, *i.e.* the skilled arithmeticians of the Age of the Sages, and do not follow the Greeks, who were no arithmeticians, nor could perfectly be so, through lack of the proper numerals, as has been said above. For though *Euclid* writes elegant arithmetical theorems, which had come down to him from the Age of the Sages, they include neither problems nor practice of arithmetic, which, as has been said, have recently come to light from Arabic books, so that *Euclid's* theorems bear witness to the Age of the Sages which had previously existed and did not then exist. The reason why we here discuss this point so seriously is that the definition of unity as the point of number is evidence, among other things, of the Age of the Sages which existed when the point was drawn and looked upon as such; and also of the Age of the Ignorant which has existed since then, producing as imperfect arithmeticians as the definition of a part of a magnitude as a point of the magnitude would testify to imperfect geometers. It may also be noted that in this Age of the Sages many arithmetical operations were performed with extraordinary ease by means of computation based on the tenth progression. In order to explain this more fully, it is to be noted that while some years ago I described the Tenth and imagined that it might be used with great ease in the division of sines and arcs according to the tenth progression; and while thereafter I described this method properly, in such a way as in its place a chapter is to be made about it in the subsequent book on astronomy, with the succinctness that will appear there, I perceived afterwards that this had already been done before me, or at least seemed to have been done in ancient times, which I think was the Age of the Sages, for the following reasons. The table of sines of *Regiomontanus*, whose radius is divided into 10,000,000, completely comprised the tenth progression which I sought. For assuming the radius to be divided into 100 equal parts instead

¹⁾ Vol. II B, pp. 495 *et seq.*

of the 60 of the Egyptians, and subsequently imagining the division to be in tenths instead of in sixtieths of the Egyptians, I found that in this table the work had already been accomplished as far as the sines are concerned. And I then thought that *Regiomontanus* had been the first discoverer of it, the more so as at the beginning of his trigonometry he says that those who preceded him divided the diameter into a small number of parts, e.g. *Ptolemy* into 120, *Arzabel* ¹⁾ into 300, subdividing each of them into 60' and each minute into 60". But later on I hit upon something on account of which I now have different surmises; this happened as follows. When one day I wanted to investigate whether by means of the table of sines of *Regiomontanus* the ratio between the diameter of a circle and its circumference would be found so closely that the limits remained within the limits of *Archimedes'* ratio or not, I looked up to this end how much was the sine of 1' and found it to be 2,909, in the same parts as the radius has 10,000,000. But this sine is nearly equal to its arc, because of the smallness of the arc; consequently the 5,400' constituting a quarter circle are nearly equal to 5,400 times 2,909, i.e. equal to 15,708,600, so that the quarter circle is to the radius nearly in the ratio of 15,708,600 to 10,000,000. And consequently the whole circle, which is four times as great, i.e. 62,834,400, is to the whole diameter nearly in the ratio of 62,834,400 to 20,000,000. I found this ratio to fall within the aforesaid limits of the ratio of *Archimedes*, to wit less than 22 to 7 and greater than 223 to 71 ²⁾. Now the same idea which struck me, viz. that of wishing to seek the ratio of the diameter to the circumference by means of the tables of sines, also seems to have occurred formerly to the ancients, for the following reason. *Georgius Peurbachius* ³⁾ having laid hand on certain writings (as he says in his treatise on trigonometry) containing the opinions of different nations, such as inhabitants of India, Egyptians, and Arabs, about the ratio of the circle's diameter to its circumference, there were also some who put it at 20,000 to 62,832, almost the same figures as above, but a little closer still to the true ratio. Now the table of sines used by those who did so had the diameter 2 with some numerical points, whose number was not only seven, as in *Regiomontanus'* table, but apparently must have been one point more, because this number of 62,832 is accurate to five figures, whereas ours is accurate only to four. This becomes evident when the limits of the ratio are taken much narrower, as was done by the famous arithmetician Mr. *Ludolf van Ceulen* ⁴⁾, to wit that when the diameter is taken

	200,000,000,000,000,000,000
the circumference is shorter than	628,318,530,717,958,647,694
but longer than	628,318,530,717,958,647,690,

so that it may apparently be concluded that before the time of *Regiomontanus* ⁵⁾ the radius in the table of sines was divided into 10,000,000 or into a number with one point more, which time may reasonably be supposed to have been the unknown Age of the Sages, since there is no evidence that this happened in known times.

¹⁾ Abû Ishâq Ibrâhîm Ibn al-Zarqâla, usually called Abraham Arzachel, was a Jewish astronomer, who lived at Toledo in the second half of the eleventh century (c. 1029 to c. 1087).

²⁾ The figure 227 instead of 223 in the Dutch text appears to be a printer's error.

³⁾ See note 2 on p. 315.

⁴⁾ On this Dutch mathematician, who lived towards the end of the sixteenth century, see the notes in Vol. II, particularly on p. 3 and p. 767.

⁵⁾ See note 1 on page 319.

VAN SYN BEPALINGHEN INT GHEMEEN. 15

HET DERDE TEYCKEN is de Meetconst, want hoe wel de Griecten daer in seer ervaren sijn geweest, doch wort by velen bevesticht dat syse van ander gecregen hebben. Voorwaer een seer wonderbaerlicke const, vast getuychnis gevende van een seltfame wetenschap der gene, wiese oock meughen gheweest sijn, diese tot sulcken grootheyt gebrocht hebben. Hier af is onsr' meeste bescheyt ghebleven inde beginfelen van *Euclides*, waer in benevens de stof der Meetconst, noch wat seer besonders, seltfaems en nuttelicx te sien en leeren is, namelick des Wijfentijts oirden in beschrijving der Wisconsten, daer afick inde volghende vernieuwing des Wijfentijts breeder mijn gevoelen sal seggen.

VOOR VIERDE TEYCKEN schijnt datmen soude meugen houdē den handel der Damphooghde, onlanx inde Arabische spraeck weerom te voorschijn commen, en hier na verclaert int Eertclootschrifts derde bouck benuyghende datter eertijts by de * Wisconstenaers een seltfaem onderfoucking gheweest is vande weerelts ghefalt en natuerens verborghen eyghenschappen. *Mathematicos.*

HET VYFDE TEYCKEN is den wonderbaerlicken seer seltfamen handel der * Stoffscheyding, by de Griekē onbekent, die onlanx begoft heeft haer weerom te vertoonen, deur welcke de menschen het wesen der stoffen tot haren grooten voordeele, anders onderfoucken en kennen, dan hemlien sonder die groote const meughelick was te begrijpen. Hier in achtmen *Hermes Trifmegistus* den ervarensten gheweest te sijn daer schriftelick bescheyt afbleven is, doch onbekent wie hy was, uyt wat lant, of tot wat tijt hy leefde, hoewelmen hem voor seer oudt acht. *Alchimie.*

Angaende dat de Griecten met hun navolgers die *Philosophi* genoemt worden, handelen vande natuer, seggende alle stof te bestaen in vier beginfelen, als eerde, water, locht, en vyer, mette vervolgen diefer uyt trekken: Seker hun neersticheyt is lovelick geweest, als gedaen hebbende watse connen, maer wacharmen t' was al van hooren seggen, met weynich bescheyt, met veel dwalinghen, en sonder kennis der oirsaken, wantse veel ghaelt wort uyt de dadelicke Stoffscheyding daer sy niet af en wisten.

Angaende dat dese weerdighe Const, dese onuytputtelicke brun der wijfheyt, by velen in verachting gheroot is, deur dien ettelicke hun daer in oeffnende bedrieghers of missers sijn, brenghende ander lieden tot schade, hun belovende gout te maken van stof gheen gout wesende: Daer wort op gheseyt, sulck misbruyck te meughen strecken tot verachting der misbruyckers, maer niet der looflicke Const.

HET SESTE TEYCKEN is de * Gheesthandel, waer in men seght dat *Magia* over seer langhe tijt eenighe volcken met kennis der oirsaken hun vlietelick gheoeffent hebben, want hoewel sulcx schrickelick is, soo mercktmē nochtans wat groote wetenschappen datter uyt de * Wisconsten volghden, diemen van foodanighe grondelicke kennis voor oirfaeck houdt, en wat een seltfaem wijfheyt datter voormael byde menschen gheweest heeft, welcken tijt wy den Wijfentijt noemen. *Mathematicos.*

Maer hoemen de saeck na ons meyning soude meugen anleggen om weerom daer an te gheraken, dat sullen wy inde volghende VERCLARING intgemeen beschrijven: En hoemen weerom totte voorgeweten kennis der Dwaelderloopen soude meughen commen, dat sal inde beschrijving der Dwaelderloopen int besonder noch eyghentlicker gheseyt worden.

B 2 MERCKT.

Up to this point we have spoken of the Ancients' division of the sines according to the tenth progression, but that they may moreover also have divided the arc of a quarter circle in this way, in order thus to get the ease to be set forth in the Appendix to the Heavenly Motions, may be assumed from the division of the circle into 1,600, into which mathematical instruments were formerly divided, as stated by *Ptolemy* in the 2nd chapter of his 3rd book, from which it followed that, as a quarter circle is divided according to the Egyptian manner into 90 degrees, and in mathematical instruments each degree is often subdivided into four parts, in spite of the fact that in computations the sixtieth progression was followed, here each quarter circle was thus divided into 100 degrees, and in mathematical instruments each degree was often divided into four parts, in spite of the fact that in computations the tenth progression was followed. For it does not seem likely that they, having devised the numerals according to the tenth progression and the rules for computations which are performed with regard to the tenth progression, as we shall describe more precisely in the book on miscellaneous subjects, should not have perceived the great advantage of the tenth progression in the division of a circle so frequently occurring in calculations relating to the heavenly motions.

The third sign is geometry, for though the Greeks were greatly versed in this, it is yet confirmed by many that they got it from others. It is indeed a very wonderful science, giving sure evidence of exceptional learning in those who brought it to such a high level, whoever they may have been. About this the most complete information has been preserved in the elements of *Euclid*, in which besides the subject of geometry something very peculiar, extraordinary, and useful is also to be noted and learned, *viz.* the systematic order observed by the Age of the Sages in the description of mathematics, about which I will give my views more fully in the subsequent treatise on the restoration of the Age of the Sages.

The fourth sign may apparently be considered to consist in the discussion on the height of the atmosphere, recently brought to light again by a treatise in Arabic and hereafter set forth in the third book of Geography, which states that in former times among mathematicians an exceptional inquiry into the form of the world and of nature's hidden properties took place.

The fifth sign is the marvellous and very unusual subject of alchemy, unknown among the Greeks, which recently began to make its appearance again, by means of which people are able to inquire into the nature of substances, to their great profit, in another way than was possible for them to understand without this great science. In this, *Hermes Trismegistus* ¹⁾ is considered to have been the most skilled of those of whom written information has come down to us, but it is unknown who he was, from what country, or in what time he lived, though he is thought to be very ancient.

As to the fact that the Greeks, with their imitators, who are called *Philosophi*, treat of nature, saying that every substance consists of four elements, *viz.* earth, water, air, and fire, with the conclusions they draw therefrom, their diligence was indeed praiseworthy, for they did what they could, but alas, they had it all from hearsay, with little real knowledge, with many errors, and without

¹⁾ *Hermes Trismegistos* is the mythical author of several treatises on alchemy, magic, and philosophy. He has many traits in common with the ancient Egyptian god *Thoth*, the god of the craftsmen, scribe of the gods, and inventor of writing.

VANDE VERNIEUWING DES

*VVysentijts, t'welck is verclaring hoer schijnt dat men de
saeck mocht anlegghen, om allencx weerom te ghe-
raken an sulcke groote wetenschappen alffer
inden VVysentijt geveest sijn.*

ANghe sien het seker is de mensch tot eenighe tijt soo seltsame groote wetenschap ghehad te hebben als inde 6 bepaling verclaert is, en dat mijns wetens nergens an en blijkt sijn verstant een haer vermindert te wesen, so geuet met reden vermoen, meughelick te sijn dat hy daer an weerom soude connen gheraken, by aldien hyder sulck middel als voormael toe gebruyckte. T'selve heeft my veroirsaeckt te schrijven mijn ghevoelen, van t'gene de mensch nu ghebreeckt dat hy doen hadde: Alles vervatende in vier leden, waer af de somme, die ick daer nae van elcke breeder verclaren sal, dusdanich is.

B 3 I L I D T.

knowledge of the causes, because these are frequently deduced from practical alchemy, with which they were not acquainted.

As to the fact that this worthy science, this inexhaustible source of wisdom, has fallen into disgrace among many people because some of those who study it are cheats or evildoers, causing damage to others by promising them to make gold out of matter which is no gold, to this it is said that such abuse tends to disgrace the men committing it, but not the praiseworthy science.

The sixth sign is magic, which is said to have been diligently practised a very long time ago by certain nations with knowledge of the causes, for though this is frightening, it is yet perceived what great learning has resulted from mathematics, which is regarded as the cause of such thorough knowledge, and what extraordinary wisdom existed among men in former ages, which time we call the Age of the Sages.

Now how in our view we can set out to attain thereto again, will be described generally by us in the subsequent *Exposition*. And how one might attain to the former knowledge of the planets' motions again, will be said even more properly in particular in the description of the planets' motions ¹).

[In connection with a conversation he had about the Age of the Sages with the jurist and humanist Hugo Grotius, Stevin received from the latter a list of references to classical authors who asserted that a very long time ago mankind possessed marvellous scientific knowledge. He includes this list in his text.]

Of the restoration of the Age of the Sages, being an exposition of how it seems one might set out to attain gradually to such great learning again as existed in the Age of the Sages.

Since it is certain that at one time mankind possessed the exceptional, great learning that has been set forth in the 6th definition, while as far as I know there is no evidence at all that human intelligence has the least bit deteriorated, there is reason to assume that it is possible for man to attain thereto again if he uses the same means to this end as formerly. This induced me to write down my view as to what man now lacks which he then had, comprising it all in four sections, the sum of which — to be set forth in greater detail hereinafter for each of them — is as follows.

¹) See the note on p. 608.

18 I BOVCK DES EERTCLOOTSCHRIFTS,

1 L I D T.

Ten eersten ghebreken ons seer veel dadelicke ervaringen daermen de con-
sten een vasten gront opgheeft. Om tot sulcke ervaringhen te gheraken, soo
souden hun seer veel menschen t' samen daer toe moeten begheven.

2 L I D T.

Genem. Om te gheraken tot soogrootten menichte van menschen als hier toe noo-
dich sijn, soo souden de voorschreven ervaringhen en oeffeningen der con-
stent ghehandelt moeten worden by een * gheslacht in sijn eyghen angeboren tael,
welcke om wat besonders daer in uyt te rechten, besonderlick goet soude moe-
ten wesen, t'welck ick sedert den Wyfentijt niet en merck gheschiet te sijn, uyt-
ghenomen byde Griecken, maer dat alleenlick int stick der Meetconst, want de
rest en tref niet.

3 L I D T.

Na dien goede talen noodich sijn, men soude om na goede talen te connen
trachten, voor al moeten weten waer in talens goetheyt bestaet, want die rechte
kennis nu by soo weynich menschen is, datse met d'ander wetenschappen des
Wyfentijts verloren schijnt.

4 L I D T.

Anghesien goede oirden in beschrijving en leering der consten, tot haer be-
voordering seer behulpich is, t'waer oirboir daer op vlietich te letten, en met goe-
de oirdeel van dies t'beste te verkiefen. Tot welck eynde ick in stofder wisconsten
gheen beter en merck, dan d'oirden des Wijfentijts.

VERCLARING DES I LIDTS.

Maer om van elck deser vier leden breeder reden te geven: En ten eersten dat
ons veel dadelicke ervaringhen gebreken, daermen de consten een vasten gront
op geeft, ick sal tot verclaring van dien beginnen met voorbeeld des Hemelloops,
tot kennis van welcke die des Wyfentijts openbaerlick gherocht sijn deur een
grootte menichte van ervaringhen, als breeder daer af inde 6 bepaling gheseyt is,
en noch breeder gheseyt sal worden int eerste voorstel des volghenden boucx
vande *Sonloop*, voort inde boucken der Dwaelderloopen, daer wy berekende
dachtafels voorbeeldsche wijze ghebruycken sullen al offe deur ervaringhen be-
commen waren, en bethoonen hoe t'vinden der Dwaelderloopen daer deur al
een ander lichte anvancken voortganck crijcht, danmen sedert den Wyfentijt
ghebruyckt heeft.

*Terrain-
cognita Au-
stralis.* Al dit overleyt wefende, ick acht openbaerghenouch te sijn, ghebreck van
overvloet der ervaringen, oirsaeck te wesen dat de menschen met grootte moey-
te en hoofbreking, hun tijt overbrenghen met te soucken Hemelloopsche ge-
daenten die alsoo niet vindelick en sijn. Tgaet hier me, op dat ickt deur voor-
beeldt van Eertclootsche stof noch opentlicker verclare, als met eenen varende
langde cant vant * onbekende Zuytlandt, en siende de mont van een groote
rivier daer uyt aldus beslote: Langs groote rivieren sijn vruchtbaer landen: In
vruchtbaer landen langs groote rivieren verkiefen veel menschen haer woo-
ning: Daer veel menschen woonen graken goede Steden: Daerom an die rivier
legghen

1st SECTION.

In the first place, what we lack is a large body of data obtained by practical experience, on which the sciences can be firmly founded. In order to arrive at such a body of data, a great many people would have to apply themselves jointly to this task.

2nd SECTION.

To arrive at so great a number of men as is needed for this, the aforesaid experiences and pursuit of the sciences would have to be practised by a nation in its own native language, which, if it is to accomplish something exceptional therein, would have to be exceptionally good, which I am not aware has been the case since the Age of the Sages, except among the Greeks, but this in the field of geometry alone, for to the rest it does not apply.

3rd SECTION.

Since good languages are needed, one would, to be able to strive after good languages, have to know before all what the excellence of a language consists in, for this proper knowledge is now possessed by so few men that it seems to have been lost along with the other sciences of the Age of the Sages.

4th SECTION.

Since good order in the description and teaching of the sciences is very conducive to their advance, it would be expedient to attend diligently thereto and judiciously to choose the best, for which I am not aware of any better order in the matter of mathematics than that of the Age of the Sages.

EXPOSITION OF THE 1st SECTION.

Now to speak in greater detail about each of these four sections, and in the first place that what we lack is a large body of data obtained by practical experience, on which the sciences can be firmly founded, by way of illustration I will begin with the example of astronomy, the knowledge of which was evidently attained by the people of the Age of the Sages by means of a large body of empirical data, as has been stated more fully in the 6th definition and will be stated even more fully in the first proposition of the subsequent book of the sun's motion ¹⁾, further in the books on the planets' motions, where we shall use calculated ephemerides, for instance, as if they had been obtained by experience and show how the finding of the planets' motions thus starts and proceeds more easily than has been customary since the Age of the Sages.

All this being considered, I think it is evident enough that the lack of plenty of empirical data is the cause that people spend their time with great trouble and pondering in seeking for astronomical schemes which cannot thus be found. It is with this — if I may explain it even more clearly by a geographical example — as with a man sailing by the shores of unknown Australia, who, seeing the mouth of a large river, should draw from this the following conclusion: Along large rivers lie fertile lands. In fertile lands along large rivers big numbers of people

¹⁾ From this sentence it appears that originally the treatise on the Age of the Sages was intended to precede *The Heavenly Motions*, whereas it is now the introduction to the *Physical Geography*.

VAN SYN BEPALINGHEN INT GHEMEEN. 19

legghen grootewelvarende Steden. En of hy voort op sulck ghestelde (deur een ghesien deel vant heel bestuytende) sulcke Landen en Steden in caerte teyckende, denckt eens wat sekerheyt of ghelijckheyt die mette Landen soude hebben, en hoe sulcke caerten en schriften soudent overcommen mettet ghene men daer na dadelick sage, want daer deur machmen met een verstaen, wat sekerheyt datter can wesen in besluyt van eens Dwaelders heelen loop, uyt een ghesien deel ghetrocken, en hoe dat sulcke reghelen en schriften soudent connen overcommen mettet ghene wy daer na dadelick sien.

Nu sulcx als hier gheseyt is vande noodighe ervaringhen in stof des Hemelloops, dergelijcke is oock te verstaen van ander, als Ebbenvloet, tot wiens grondelicke kennis ons louter ervaringhen ghebreken, daer int volghende 6 bouck afgheseyt sal worden: S'gelijcx vant Eertcloots Stofroersel int volgende 2 bouck beschreven: Voort Ervaringhen der * Steroidceelen, of voorfegginghen deur Sterren: Oock der * Stoffcheyding: En Gheneising, waer inmen (om niet te bewijfen met, *Hippocrates* segt) al een ander dadelicke oefening soude moeten wesen soo in * opsnijding, als onderfoucking vande ghedaenten der cruyden en * Gheneestoffen, die veel door stoffcheyding ghevonden worden. Maer want van elck van dese int besonder te schrijven, meer tijts soude behouven dan my te pas comt, en dan noodich schijnt daer an te besteden, so sal ick cortheys halven overflaen.

*Indicarie
Astrologie.
Alchimie.*

*Anatomia.
Medicamentorum.*

Ick houde dan voor openbaer, dat ghelijck int eerste lidt gheseyt is, ons seer veel dadelicke ervaringhen ghebreken, daermen de consten een vaste grondt op gheeft. Maer om nu te verclaren dat (ghelijck daer voorder staet) om tot sulcke ervaringhen te gheraken, seer veel menschen t'samen hun daer toe soudent moeten begheven, ick sal weerom met voorbeeld des Hemelloops beginnen. Ten eersten, een mensch en can niet gheduerlick by dage by nachte, laer uyt laer in, galsaen de Dwaelders plaetsen en alles datter noodich is: Maer een seer groote menichte sulcx doende, t'ghene by d'een ghebreeckt, wort by d'ander bevonden. Ten tweeden, de ervaringhen van eenen, al waren se in haer selven gewis, soo en verstrecken se anderen nochtans tot gheen secker gront, om int veroidenen der * spiegelinghen daer op te werck te gaen, deur diender gheen proef af en is: Maer seer veel verscheyden menschen ervaringhen, die daer nae teghen malcander overleyt sijnde, bevonden worden so na t'overcommen als de saeck vereyscht, daer machmen op steunen. Tot voorbeeld van desen connen verstrecken de ervaringhen onlanxghedaen tusschen den Doorluchtighen Vorst *Willem* Landtgraef van Hessen, en den vermaerden * Gaslagher *Tuychobrade* in druck uytgaende: Ervaringhen voorwaer diens ghelijcke ick federt den Wyfentijt niet en merck ghebeurt te wesen. En sulcke soudemen dan met groote menichte vinden. Ten derden, soo isser dickwils tot sommighe plaetsen oertogen locht, inder voughen datmender in ettelicke weken geen Hemelsche lichten en siet: In sulcken ghevalle canmen dan hebben d'ervaringhen van anderen ghedaen inde Landen daert clare locht was. Ten vierden soo foudertusschen de Gaslaghers een cergiericheyt en twift te verwachten staen, willende elck het sijne voor best bewijfen, waer me de consten (hoe wel de menschen daerentussche int stuck der seden hun dickwils misgaen) gemeenlick gheen cleene voortganck en krijghen: Daer anders den handel by weynich menschen bestaende, elck sijn vonden bewaert en verberghet.

Theoriarum.

Observationum.

Angaende sommighe achten de stof te weerdich te sijn om vande gemeente ghehandelt te worden, en alleen den Vorsten toestaet: Mijn ghevoelen is daer anders, want de Vorsten des Eerrijcx sijn weynich, en onder die weynighe isser

B 4 luttel

choose to live. Where there live many people, good cities arise. Therefore on this river there are large and prosperous cities. And if further on this hypothesis (drawing a conclusion about the whole from the part he had seen) he were to draw a map of such lands and cities, just think what accuracy or similarity to the lands this would have and how such maps and descriptions would agree with what would be actually seen afterwards, for from this it may at the same time be understood what degree of certainty there may be in a conclusion about the whole motion of a planet drawn from the sight of a part, and how such rules and descriptions might agree with what we actually see afterwards.

The same statement that has here been made about the necessary empirical data in astronomical matters also applies to other subjects, such as ebb and flow, for the thorough knowledge of which we merely lack empirical data, about which something will be said in the subsequent 6th book. Likewise about the motion of the earth's matter, described in the subsequent 2nd book. Further empirical data concerning astrological judgments or predictions by means of the stars. Also empirical data of alchemy, and medical science, in which (to avoid proving things by means of the statement: *Hippocrates* says) there would have to be much more practical work, both in anatomy and in the examination of the properties of herbs and medicines, which are often found by means of alchemy. But because it would take more time to write about each of these subjects in particular than suits me and seems necessary to spend on it, I will, for the sake of brevity, omit this.

I therefore regard it as evident that, as has been said in the first section, what we lack is a large body of empirical data, on which the sciences can be firmly founded. Now to explain that (as is further said there), in order to arrive at such a body of data, a great many people would have to apply themselves jointly to this task, I will again begin with the example of astronomy. In the first place, one man cannot continually, by night and by day, year in and year out, observe the positions of the planets and all that is necessary; but when a large number of people do this, what is lacking in the observations of one man will be found in those of another. Secondly, the data obtained by one man, even if they were exact in themselves, yet do not serve others as a sure basis on which to proceed in framing the theories, because they have not been checked. But when the data obtained by a great many different people, having been compared with each other, are found to agree as closely as the matter requires, one can rely thereon. As an example of this may be taken the data recently obtained on the one hand by the illustrious Prince William, Landgrave of Hesse¹⁾, and on the other hand by the famous observer Tycho Brahe, whose work was then being published: data indeed which I am not aware have been equalled since the Age of the Sages. And such data would then be found very plentifully. Thirdly, the sky will often be overcast in some places, so that for some weeks no heavenly bodies are seen; in such a case one may then rely on the data obtained by others in the countries where the sky was clear. Fourthly, ambition and rivalry would be apt to arise among observers, each wishing to prove his own work best, owing to which the sciences (although men meanwhile often misbehave in moral respects) usually make no inconsiderable progress, whilst on the other hand, if the branch of science is

¹⁾ Wilhelm IV von Hessen (Kassel 1532 - *ibid.* 1592) was a diligent observer up to 1567, when he succeeded his father; later he was able to attract excellent collaborators to Kassel. He gave special care to the determination of time.

luttel dieder een natuerlicke gheneghentheyte toe hebben. Siet eens tot voorbeeld van desen t'gene den Coninck *Alfonfus* weervoer, die tottet maken der tafels op sijn naem uytgaende, over de vier hondert duyfent ducaten onkosten dede: Seker sijn yver tot sulcken const was lovelick, maer wat isser eyntlick uytgherecht? niet besonders, want sijn Wisconftnaers hebben sonder nieuwe ervaringhen, te werck ghegaen op *Ptolemus* bloote stelling, waer uyt met die groote schat niet dan wat onsekers ghemaect en conde worden. Maer dese const byde ghemeente ghehandelt wefende, men can alles met meerder sekerheyte en kennisder oirsaken voor niet krijghen.

Nu sulcke voorbeeld alsick hier tot verclaring des voornemens ghetrocken heb uyt Hemelloopsche stof, derghelijcke soudemen oock meughen bybrengen uyt ander consten: Maer achtende hier me ghenouch te blijcken dat om in elck veel ervaringhen te krijghen, noodich is dat seer veel menschen t'samen hun daer toe souden moeten begheven, ick sal die om cortheydts wille ooverslaen, en voortvaren.

VERCLARING DES 2 LIDTS.

Angaende het tweede lid, te weten dat om te gheraken tot soo grooten menichte van menschen als hier toe noodich sijn, dese oeffening der consten soude moeten ghehandelt worden by een gheslacht in sijn eyghen angeboren tael, welcke om wat besonderlicx daer in uyt te rechten, besonderlick goet soude moeten wesen, hier op segh ick aldus: Soude een ghemeente haer in een const oeffenen, sy soude moeten de tael verstaen daerse in gheschreven is, t'welck haer eyghen tael moet wesen, want hoe wel ettelicke ouders hun kinders t'Latijn doen leeren, waer in men de vrye consten nu meest handelt, de selve sijn weynich int anfsien vande gemeente. Ten anderen leertmen de Ionckheyt t'Latijn, om hun eyntlick te begheven totte Rechten, Godheyt, of ghenefing: Isser onder de sulcke ymant die hem daer na gantschelic totte Wisconften schickt, dat ghebeurt seer selden, en ghemeenelic teghen hun ouders danck, ghelijck onder anderen den vermaerden Gaslager *Tuychbrabe* hem schrijft ghebeurt te sijn: Daer isser onder die oock een grooten deel, welcke hun tijt voort deurbrengghen in oeffening der Latijnsche spraek, leerende veersen der * Dichters van buyten, om op alle dingen die in gemeene t'saemspraek voorvallen, een Latijns veers te connen vervoughen: Soucken voort bloemkens van woorden en spreucken om in haer brieven en schriften te pas te brengghen: En hoe wel sulcke opghetoyde stijl an sommighe mis haecht, soo isser nochtans veel ander dieder hun niet me verfaen en connen. In somme men vintter onder soodanige weynich die hun volcommelic totte wisconften begheven, en daerom ist noodich ghelijck wy gheseyt hebben dese stof ghehandelt te worden inde ghemeentens aengheboeren tael. Doch moets daer benevens noch goet sijn, connende alles uytbeelden dat totte saeck noodich is. Maer want dit stuck der talen een vande voornaemste punten is, die my doen wan hopen van weerom tot een wysetijt te meughen gheraken, om dat als vooren gheseyt is, metten onderganck van veel wenschappen des selfden wyfentijts, oock t' ondergegaen schijnt s' menschen kennis of oirdel van goetheyt der talen, en datter swaricheyt sal hebben hun dat te doen verstaen, soo moet ick na mijn vermeughen daer af breeder mijn ghevoelen verclaren. Tis openbaer dat de Francoysen (op dat wy mer voorbeeld der Franfche tael beginnen) de vrye consten in haer spraek meer beschrijven als ander

Polierum.

practised by few people, each of them will keep his findings to himself and conceal them.

Concerning the fact that some people consider the matter to be too sublime to be studied by ordinary people and to be fit only for princes, I hold a different view, for the princes of the earth are few, and among those few there are few who have a natural inclination for it. Just see, for instance, how King Alphonsus fared, who spent more than four hundred thousand ducats on the compilation of the tables published under his name ¹⁾. His zeal with respect to this science was indeed laudable, but what has been ultimately achieved? Nothing much, for his mathematicians proceeded without new-found empirical data on *Ptolemy's* mere hypothesis, from which with that large sum only something uncertain could be accomplished. Now when this science is studied by ordinary people, everything can be obtained for nothing with greater certainty and knowledge of the causes.

Now examples similar to the one I have here taken from astronomy, to explain my intention, might also be adduced from other sciences. But since I consider it is now evident enough that to obtain a large number of empirical data in each of them it is necessary that a great many people should apply themselves jointly to it, I will omit them, for brevity's sake, and continue.

EXPOSITION OF THE 2nd SECTION.

As to the second section, to wit that to arrive at so great a number of men as is needed for this, the aforesaid pursuit of the sciences would have to be practised by a nation in its own native language, which, if it is to accomplish something exceptional therein, would have to be exceptionally good, about this I say as follows: if ordinary people were to study a science, they would have to understand the language in which it is written, which should be their own language, for though some parents have their children taught Latin, in which the liberal arts are now usually treated, these are few in comparison with the people at large. On the other hand young people are taught Latin in order that they may ultimately study law, divinity or medicine. It rarely happens that among these there is anyone who thereafter devotes himself wholly to mathematics, and then it is against his parents' wishes, as *e.g.* the famous observer Tycho Brahe reports it happened to him. There are also among them a great many who further spend their time in practising the Latin language, learning verses of the poets by heart in order that they may apply a Latin verse to anything occurring in an ordinary conversation. They further collect flowers of speech and aphorisms in order to quote them in their letters and writings. And although such an ornate style displeases some people, there are nevertheless many others who cannot get enough of it. Briefly, there are found few among them who devote themselves wholly to mathematics and for this reason it is necessary, as we have said, that this subject should be discussed in the native language of the community. But in addition this language also has to be good and to be able to render everything that is required for the matter. Now because this question of the language is one of the chief points which make me despair we shall ever reach an Age of the Sages again, because — as has been said before — along with the decline of many sciences of the said Age of the Sages men's knowledge or appreciation of the excellence of a language also seems to have declined, and it will be difficult to

¹⁾ See note 2 on p. 315.

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der volckē, t'welck wel oirfaeck is datter hemlien in haer gemeente meer menschen daer in oeffenē, dant soudē by aldien ser niet af en handeldē, maer want se daer in seer veel Griekische en Arabische constwoordē stellē, de groote voortganck met kennis der oirfaekē, om weerom tot een Wijsentijt te geraken, en can daer uyt niet volgen: Want als mē de constwoorden niet grondelick en verstaet, als by voorbeelt inde Fransche tael te seggē van *Prostapheresē*, *Parallaxe*, *Nadir*, *Almicanarat*, en veel dergelijcke, haer beteyckening geduerlick ronthouden valt de gemeente lastich, de oeffening moeylick, verdrietich, en de faeck vā slappe voortganck, sy en connē niet verbeteren gebreckige constwoorden die dickwils beter bepaling vereytschen, noch de dwalingen mercken dieder uyt volgē, dan verblijden hun te connen spreken woordē die ander haer lantslieden niet en verstaen, en datmen hun met verwondering voor Hoochgeleerden hout.

make them understand this, I have to set forth my view of this more fully to the best of my ability. It is known that the French (to start with the example of the French language) describe the liberal arts more frequently in their own language than do other nations, which is probably the cause that more of the community study them than would be the case if they did not discuss them, but great progress use in their studies very many Greek and Arabic technical terms, the great progress involving knowledge of the causes, necessary to reach an Age of the Sages again, cannot result therefrom. For when the technical terms are not thoroughly understood, as when, for instance, in the French language the words Prostaphérèse, Parallaxe, Nadir, Almucantar and the like are used, it is difficult for ordinary people to remember their meaning constantly, the study of the sciences is difficult and annoying, and the progress of the matter is poor; they cannot correct defective technical terms, which often call for a better definition, nor note the errors resulting therefrom, but they are glad that they can use words which others of their countrymen do not understand and that they are looked upon with admiration as very learned men.

[In order to show how the use of good technical terms affects the pursuit of science, Stevin points to the word "proportion", which failed to be understood by students of musical theory, thus leading to confused and unpractical systems of tuning. If, however, the Dutch word "evenredigheid" ("equi-ratio") had been used, equal temperament would automatically have been adopted.

The admixture of foreign words with a language, such as it takes place in French, is an impoverishment rather than an enrichment. Contrary to the general view therefore Stevin asserts that the French language is neither rich nor suitable for science. There are indeed charming French poems, which Stevin himself has read with great pleasure. But this merely proves that France counts many great poets; the language itself is extremely defective, because it contains so large a number of foreign words which ordinary people do not understand: — The same applies to Italian and Spanish, though to a less extent.

In order to assess the value of a language, Stevin wants to ascertain to what extent it is able to serve as a vehicle for science, in particular for mathematics. French, Italian, Spanish or Latin cannot express the mathematical sciences unless by using Greek words. Greek on the contrary is a very suitable language, because compound words are formed very easily in that language. Even better is Dutch, which is able to forge even shorter and plainer compounds of this kind from mostly monosyllabic basic words.

To prove this, Stevin gives lists of more than two thousand monosyllabic Dutch words, whilst on the other hand he finds only small numbers of monosyllabic Latin or Greek words. After this he shows by means of four instances how the meaning of Dutch compounds is always unambiguous and immediately clear to anyone. He who gains an insight into the character of the Dutch language cannot but marvel at its excellence. In historical times there has been no nation clever enough to grasp the great importance of such a language structure and to produce such a language. This language accordingly might have arisen in the Age of the Sages. If the Dutch should properly appreciate again their language (which is spoken in its purest form in North Holland), they might help to bring about the speedy restoration of the Age of the Sages.

Those who do not know what the language of science is, think that the acquisition of learning makes our lives gloomy and distracts us from important practical matters. Thus there are many people who think that Prince Maurice by his devotion to the sciences is distracted from the business of government. But to this it has to be answered that the study of sciences in the systematic formulation and style of the Age of the Sages makes things easier and aids us in accomplishing many great things.

The modes of expression of scientific style are next discussed by Stevin in five chapters.

1. The style of Euclid, in which a rigorous distinction is made between proposition and problem. Each of them is subdivided into the supposition, the property to be proved, the construction, and the proof (*datum, quaesitum, constructio, demonstratio*). Such a subdivision presents great advantages. One can first fully study the theoretical proposition, which one has to understand thoroughly; afterwards one can then give one's undivided attention to the practical rules (example: with multiplication, division, extraction of roots). Ptolemy did not make use of this mathematical style, which undoubtedly dates back to the

Age of the Sages. Peurbach, Regiomontanus, and Copernicus on the other hand did make use of it, just like Stevin himself in his Mathematical Memoirs.

2. *The propositions should be preceded by definitions. Stevin relates how in certain circumstances he had to get to know more about earthwork; wattleing, carpentry, brick-laying, forging, etc. For this, the workers in these trades were his best teachers. He began by asking them for the meaning of the words he did not understand, wrote down the definitions of these words, and learned them by heart. It appeared that he could then talk quite easily with the workers and that he thus quickly improved his knowledge. In Stevin's „Arithmétique” all the definitions are given at the beginning; another time he would prefer to open each chapter with those definitions which pertain to that chapter.*
3. *Dichotomy.*
By this, Stevin means the classification of the subject under discussion into two (or more) possible cases; in the discussion of the spherical triangle, for instance, the following cases are to be distinguished:
the side AC is less than 90 degrees;
the side AC is equal to 90 degrees;
the side AC is more than 90 degrees.
4. *Anaphora: the same concepts should invariably be referred to by the same terms*¹⁾.
5. *The separate discussion of theory and practical application. In a final chapter Stevin shows that theory and practice are both necessary.]*

¹⁾ In rhetoric, this word designates a figure consisting in the repetition of a word or a phrase at the beginning of two or more successive sentences.

Theorie &
praxis.

VANT MENGHEN DER *SPIE- GHELING EN DAET.

Per Hypote-
sim.

Perpendicu-
laru.

Mathemati-
cas artes.

Mechani-
cam.

Vni-versitati-
bus.
Elementu.

WAntter int stic des oirdens noch een verschild valt vant menghen der spiegheling en daet, soo moet ick daer af mijn ghevoelen seggen, eerst haer beteyckening verclarende voor de ghene diet onbekent mocht sijn. Spiegheling is een verdochten handel sonder natuerlicke stof, ghelijck onder anderen sijn de Spieghelingen des Spieghelaers *Euclides*, handelende * deur stelling van grootheden en gheralen, maer elck ghescheyden van natuerlicke stof. Daet is een handel die wesentlick met natuerlicke stof gheschiet, als lant en wallen meten, de menichte der roen of voeten tellen dier in sijn, en dierghelijcke. T'besluyt vande voorstellen der Spiegheling is volcommen, maer der daet onvolcommen: Als by voorbeelt de Spiegheling vint en bewijst dat den helft des uytbrengs vande * hanghende en gront eens wisconstich driehouck, volcommelick gheeft herinhoudt des plats sonder eenich gebreck of overschot: Maer een wesentlick driehouck van lant of ander glatter stof dadelick ghemeten sijnde, t'besluyt is daer af onvolcommen, eensdeels om dat wy gheen langde soo nau meten en connen, dattet gheen duyfentste deel der dichte eens haers en schilt, of al waert by gevalle heel effen, tis onbewijfelic. Ten anderen om dat gheen natuerlicke linien soo heel recht, noch natuerlicke vlacken soo heel plat en sijn, als de wisconstighe bepalinghen vereytschen, of al waren sy soo heel recht en plat, ten is niet bewijfelic. De eyghenschap en t'eynde der Spiegeling is datse verstreckt tot seker gront vande manier der wercking inde daet, alwaermen deur nauwer en moeyelicker toeficht de volcommenheyt der Spiegheling so na mach commen, als de saccks einde tot Smenschen ghebruyck vereytscht.

Hier uyt is te verstaen, dat wanneer sommighe de * Wisconsten van onvolcommenheyt beschuldighen; deur dien veel dadelicke werckinghen niet heel effen uyt en commen, datter kennis gebreeckt des onderscheyts tusschen Spiegeling en Daet, tusschen Wisconstighen en tuychwerckelicken handel: Want de Daet of * tuychwerckelicken handel om de boveschreven redenen altijt onvolcommen moet wesen.

Deze twee deelen Spiegeling en daet sijn so verscheydé, dat menich mensch hem t'eenmael totter een begheeft, sonder van t'ander kennis te hebben, ghelijck menich leeraer met sijn toehoorders inde * ghemeen scholen ghebeurt, die hun gheduerlick in Spieghelingen oeffenen, als in *Euclides* * beginselen der Meetconst, sonder dadelick te meten landen, wallen, of vaten, of yet anders te doen daer de Daet in bestaet: En weerom verkeert so vintmen dadelicke Lantmeters, welcke alle reghels diese besighen gelooven, of toestaen waer te wesen, sonder inde Spiegeling t'ondersoucken de oirsaken en bewijs: Ia sommighen en weten niet datter sulcke oirsaken en bewijs af sijn.

Angaen-

ON THE COMBINATION OF THEORY AND PRACTICE.

Because, in the matter of orderly exposition, opinions differ with regard to the combination of theory and practice, I feel bound to give my view about this, first explaining their meaning for those who are not acquainted with it. Theory is a fictitious operation without natural material, such as *e.g.* the theories of the theoretician *Euclid*, which operate by the assumption of quantities and numbers, but each of them without connection with natural material. Practice is an operation which essentially takes place with natural material, such as the measurement of land and ramparts, counting the number of rods or feet contained therein, and the like. The conclusion of theoretical propositions is perfect, but that of practical propositions is imperfect. Thus, for instance, the theory finds and proves that half the product of the height and the base of a mathematical triangle perfectly gives the area of the plane surface, without any deficiency or surplus; but when a real triangle of land or smoother material is measured actually, the conclusion is imperfect, in the first place because we cannot measure any length so exactly that it does not differ by one thousandth part of a hair's breadth, or even if it is quite exact, it cannot be proved. In the second place because no natural lines are quite so straight nor natural plane surfaces quite so flat as the mathematical definitions require, or even if they are quite straight and flat, it cannot be proved. The property and the end of theory is that it furnishes a sure foundation for the method of practical operation, in which by closer and more painstaking care one may get as near to the perfection of the theory as the purpose of the matter requires for the benefit of man.

From this it is to be understood that if some people accuse the mathematical sciences of imperfection, because many practical operations do not produce results which are quite exact, they lack knowledge of the difference between theory and practice, between mathematical and mechanical operations, for practice or mechanical operation on the above account must always be imperfect.

These two sections, theory and practice, are so different that many people apply themselves altogether to the one, without being acquainted with the other, as is the case with many lecturers and their audience in the universities, where they constantly study theories, *e.g.* *Euclid's* elements of geometry, without actually measuring lands, ramparts or vessels or doing anything else in which practice consists. And, conversely, practical surveyors are to be found who take on trust all the rules they apply or regard them as true, without examining the causes and the proof in the theory; nay, some do not even know that such causes and proofs exist.

VAN SYN BEPALINGHEN INT GHEMEEN. 47

Angaende ettelicke segghen de Spiegheleling sonder Daet onnut te wesen, het schijnt datmen de saeck met beter onderscheyt soude meughen insien. Om hier af mijn ghevoelen te verclaren, ick seggh by voorbeelt aldus: Datmen t'werck eens aerbeyders die boomen int bosch afhout, soude voor onnut achten, om dat hyder self gheen huysen, schepen, molens, sluyfen, tonnen, kisten, beelden, en dierghelijcke me en maect, dat en waer openbaerlick niet wel gheseyt, want hoewel sulcke wetenschap in een mensch looffick is, nochtans ghemerckt hy met boomen af te houwen, an veel ander stof levert, om eick hem in sijn ambacht te oefnen, soo en is sijn aerbeyt niet te verachten: Maer dat hy die boomen afhieuwe om te laten verrotten, sonder nut daer af te verwachten, dat waer dwafelick ghedaen: En alsoo ist oock mette Spieghelelaers in vrye consten, sy connen den Doenders stof leveren en voorderlick sijn, sonder self Doenders te wesen: Als den Spieghelelaer *Euclides*, die wy niet en bevinden Doender gheweest te hebben, heeft nochtans voorstellen beschreven den dadelicken Boumeesters, Landmeters, en ander doenders seer voorderlick: Den Spieghelelaer *Ptolemus* en meer ander die gheen dadelicke Stierlien en waren, hebben nochtans reghelen beschreven den dadelicken Stierlien op groote Zeevaerden, en ander hun daer in oeffnende seer nut: la sulcx dat de dadelicke Stierlien self sich voor meesters achten, als sy verstaen de reghelen door sulcke Spieghelelaers beschreven, hoewelse nochtans gheen dadelicke Stierlien en waren. Daerom een Spieghelelaers Spieghelelinghen die ander Doenders te sta commen, en sijn niet onnut al en is hy self gheen Doender.

Tot hier toe deur ettelicke omstandighen verclaert hebbende wat Spiegeling en Daet beteyckent, soo is te weten dat d'oude Wisconsteners met oock ettelicke nieuwe, van yder dier twee deelen onvermengt int besonder handelen, welke oirden wy daert te pas comt oock volghen: Doch wantter by ettelicke een ander ghevoelen is, die Spiegheleling en daet met malcander menghen, om teen mette ander t'samen te leeren, so moet ick om breeder reden mijns doens te verclaren, daer af mijn ghevoelen segghen.

Voor al soo ist te weten dat der Menschen natuerlicke gheneghentheden angaende de Daet seer verscheyden sijn, waer toe noch helpen de oirsaken die desen anders ontmoeten en dringhen als dien: Den eenen heeft natuerlicke lust met eenighe dringhende oirsaken totte Stercktebou, en dinghen de crijch angaende: Den anderen tot Landmeten: De derde tot Wijnchroon: De vierde tot saken de groote Zeevaerden belanghende: De vijfde totten * Huys bou: De sefte totte Spiegheleling alleen: De sevende tot ettelicke van dese, of tot altermael, met noch oneyndelicke ander. Nu also een der voornaemste eynden vande beschrijving der vrye consten streckt, om daer deur te crijghen veel menschen, die ten ghemeenen oirboire met lichticheyt gheraken ter kennis van t'ghene daer sy hun toe begheven, soo willen wy eens overlegghen, of dat inde leering gheschien can door vermenging des daets mette Spiegheleling, tot welcken eynde ick aldus seggh: Alsmen onder spieghelighen voorstellen ettelicke des daets vermengt, en dat van een afcomst ghetrocken uyt de oneyndelicke menichte diemen daer af beschrijft, de selve dadelicke voorstellen en sullen misschien niet sijn van die afcomst daer den leerlinck na tracht: Als by voorbeelt, datmen inde Meetconst tusschen de spieghelighen voorstellen eenighe beschrijft des daets, waer in voorbeelden commen neem ick der meting deur het * schuyfcruyts van ongherakelike veynsters en pylaren van ghestichten; Maer t'can ghebeuren dat den leerlinck tot sulcke afcomst van Meetdaet gheen lust en sal hebben, denckende misschien dattet inde Daet lutel ghebruycx heeft, oock gheen ghenouchsaem seker-

Architectura-
tam.

Radium.

As regards the assertion of some people that theory without practice is useless, it would seem that this matter should be considered more critically. To set forth my view about this, I say, for instance, as follows: it would evidently not be right to regard the work of a labourer, who cuts down trees in the wood, as useless because he does not personally make houses, ships, mills, canal-locks, barrels, chests, sculptures, and the like therewith, for although such knowledge is praiseworthy in a man, yet considering that by cutting down trees he supplies many others with material with which each of them can pursue his trade, his work is not to be despised. But if he were to cut down those trees to let them rot, without expecting any benefit of them, that would be acting foolishly. And thus it is also with the theoreticians in the liberal arts: they are able to furnish the practicians with material and to be of use to them, without themselves being practicians. Thus the theoretician *Euclid*, whom we do not find to have been a practician, nevertheless described propositions which are of great use to practical architects, surveyors, and other practicians. The theoretician *Ptolemy* and several others, who were no practical navigators, nevertheless described rules which are of great use to practical navigators during long voyages and to others pursuing this art, even to the extent that practical navigators consider themselves masters when they are conversant with the rules described by such theoreticians, although nevertheless the latter were no practical navigators. Accordingly the theories of a theoretician which are of use to others who are practicians are not useless, even though he himself is no practician.

Having so far explained by means of some ample considerations what is the meaning of theory and practice, I would say that it is to be noted that the ancient mathematicians as well as some modern ones treat of each of these two sections in particular without combining them, an order which we also follow where this is proper. But because a different view is held by some people, who combine theory and practice, in order to teach the two things together, I have to give my views about this, so as to explain the reason of my conduct more fully.

Before all it is to be noted that people's natural inclinations towards practice vary widely, a fact which is also promoted by the causes, which occur to and impel some people in a different way from others. One man has a natural liking, with some causes impelling him thereto, for the art of fortification and things concerned with war, another for surveying, a third for displacing barrels of wine, a fourth for matters concerned with great voyages, a fifth for architecture, a sixth for theory alone, a seventh for some or all of them, with an infinite number of others as well. Now since one of the main ends of the description of the liberal arts is thus to get many people who, for the benefit of all, easily acquire the knowledge of the subject to which they apply themselves, we will consider for a moment whether in instruction this can take place by a combination of practice and theory, to which end I say as follows: If among theoretical propositions one included some practical ones, and that of a species chosen from the infinite number described, these practical propositions may not be of the species the pupil is aiming at; thus, for instance, if in geometry among the theoretical propositions one describes some practical ones, in which there occur instances — I assume — of measurements, by means of the cross-staff, of inaccessible windows and columns of buildings. But it may be that the pupil has no liking for this species of practical measurement, thinking perhaps that it is little used in practice nor produces

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Speciem. kerheyt, om op sulcke gevonden maten van pylaren in ander ghebou te werck te gaen, of ander inyallen die hem meughen te voor commen: Boven dien en sal hyder niet vinden *d'afcomft daer hy na tracht: Sulcx dat by aldien hy al wil verstaen watter int bouck is, sal moeten leeren dat hy niet en begheert te weten. Maer de Spiegheleling als een oirdentlick geketent werck alleen beschreven sijnde, en den leerlinck die verstaende, sy verstreckt hem tot ghemeene gront, om innerlick te begrijpen alsulcken deel der Daer als hy uyt verscheyden beschreven Daden int licht uytgaende, na sijn behaghen verkiesen sal.

Voor besluyt, ick heb mijn ghevoelen verclaert hoet schijnt daermen de saeck an mocht legghen, om metter tijt weerom te gheraken tot sulcke grootewetenschappen alser inden Wyfentijt gheweest sijn, ghelijck t'voornemen was. En met oplicht van sulcken gront, sal ick de volghende handelinghen beschrijven.

BERSTE BOVCK DES
EERTCLOOTSCHRIFTS
EYNDE.



sufficient certainty to proceed for another building on the basis of the measures of columns thus found, or for other reasons that may occur to him, while moreover he will not there find the species which he is aiming at, so that if he wants to understand all that the book contains, he will have to learn what he does not wish to know. But if the theory alone is described as a systematic chain of reasoning and if the pupil understands it, it will serve him as a general basis for grasping mentally any part of practice which he chooses at his own pleasure from different descriptions of practice that are published.

To conclude, I have set forth my view as to how it seems one might set out to attain gradually to such great learning again as existed in the Age of the Sages, as I intended to do. And with respect to this basis I will write the following treatises.

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