THE PRINCIPAL WORKS

OF

SIMON STEVIN

EDITED BY

ERNST CRONE, E. J. DIJKSTERHUIS, R. J. FORBES,
M. G. J. MINNAERT, A. PANNEKOEK

AMSTERDAM
C.V. SWETS & ZEITLINGER


NATUS
BRUGIS
Anno
1548
DENATUS
HAGÆ CO-
MITIS
Anno
1620.
SIMON STEVIN,
MATHEMATICUS INSIGNIS
CONSILIAR. CEL SISS. MAURITII PRINCIPIS
ARAUSIO NENSIUM.

THE PRINCIPAL WORKS
OF
SIMON STEVIN

VOLUME I

GENERAL INTRODUCTION
MECHANICS

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E. J. DIJKSTERHUIS

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SIMON STEVIN

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ERNST CRONE,
Chairman of the Netherlands Maritime Museum, Amsterdam

E. J. DIJKSTERHUIS,
Professor of the History of Science at the Universities of Leiden and Utrecht

R. J. FORBES,
Professor of the History of Science at the Municipal University of Amsterdam

M. G. J. MINNAERT,
Professor of Astronomy at the University of Utrecht

A. PANNEKOEK,
Former Professor of Astronomy at the Municipal University of Amsterdam

The Dutch texts of STEVIN as well as the introductions and notes have been translated into English by Miss C. Dikshoorn.

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GENERAL INTRODUCTION

THE LIFE AND WORKS

OF

SIMON STEVIN

1. INTRODUCTION

Modern science was born in the period beginning with Copernicus' *De Revolutionibus Orbium Coelestium* (1543) and ending with Newton's *Philosophiae Naturalis Principia Mathematica* (1687).

For reasons we shall not enter into here medieval Scholasticism had not succeeded in finding an effective method for the investigation of natural phenomena. Nor had Humanism been able to find the new paths science had to follow, though it was indirectly instrumental in promoting natural science by fostering the study of Greek originals on mathematics, mechanics and astronomy. The conviction shared by both movements that science was something mankind had once possessed but lost since, led to the conviction that it had to be rediscovered in ancient books, and so turned men's eyes toward the past instead of to the future, where it was to be found.

The creation of modern science required a different mental attitude. Men had to realize that, if science were to grow, each generation had to make its own contribution and that all the wisdom of Antiquity was useful only as a starting point for new research.

In the development of this view the universities, which had always been the bulwarks of medieval science, could play none but a minor part. Naturally inclined to conservatism, they on the whole exerted a retarding influence. For the greater part the revival of learning was the work of individual scholars who, in full possession of traditional science, took the initiative of transcending its boundaries and venturing into unexplored realms of scientific thought.

During the sixteenth century these pioneers of modern science are to be found all over Europe. The Italians Tartaglia, Cardano, Benedetti took the lead in the domains of mathematics and mechanics. A new era in astronomy was opened by Copernicus in Prussia and by Tycho Brahe in Denmark. In France the mathematician Vieta prepared the way for the great progress in algebra that was to be accomplished in the seventeenth century.

The work of these prominent scholars was supplemented by the activity of numerous craftsmen who, urged on by economic necessity, tried to put science to practical use. Some of them were well-known artists (like Leonardo da Vinci and Albrecht Dürer), who at the same time worked as engineers planning or constructing canals, locks, dikes and fortifications. For the greater part, however, their names do not survive; they were the numerous makers of clockworks and nautical or astronomical instruments, the cartographers and, somewhat later, the grinders of lenses and the makers of telescopes and microscopes.

Simon Stevin acquired his honourable position in the history of civilization by working both in theoretical science and in engineering. This combination of faculties was prophetical: modern science truly required the cooperation of theory and practice. It could only come into being by theoretical speculation on data furnished by experience. Matter, being obstinate and unwilling to yield its secrets to pure reasoning, can only be forced to disclose its properties if submitted to experimental research. However, to perform experiments, technical skill in constructing and using instruments is wanted. On the other hand the accumulation of empirical data is not in itself sufficient. Only mathematical formulation of quantitative relations leads to theories, the consequences of which can be put to the test in newly devised experiments. Thus the evolution of science can only proceed by a constant interaction between theory and practice.

The role of the technician is by no means exhausted with his contributions to the experimental side of science. His help is needed again when the results achieved are to be applied for the benefit of humanity.

In later centuries the various departments of scientific work were as a rule separated, most scientists concentrating either on theoretical or on experimental research or on the application of science in technical inventions. In the age of pioneers, however, their concentration in one person was not yet uncommon. Stevin was an example of this, and he appears to have been fully aware of the significance the combination had for the growth of natural science.

We shall not endeavour to depict in this biography the intellectual atmosphere in which he accomplished his work as a scientist and an engineer; this will be done, as far as necessary, in the introductions to the works that will be published in this edition. A few words on the political background of his career in the Low Countries will be given in the description of his life, to which we will now pass on.

2. ORIGIN AND YOUTH

Stevin seldom published a work without mentioning on its title-page that he was a native of Bruges (Flanders) [1]. In doing so he provided us with one of the very few facts on his origin which are beyond all doubt. Another datum which is at least fairly certain was his birth in 1548 [2]. We derive it from the legend of a portrait in oils which is the property of the Library of Leyden University. It is confirmed (at least not contradicted) by one of four documents [3] which contain all the further information available on his birth and parentage. These are four deeds of the year 1577, in which his majority is declared and certain financial affairs are settled [4]. They reveal that he was the natural child of one Antheunis Stevin by Cathelyne van der Poort. It is rather perplexing that the only particulars which have become known about his parents refer to other natural children

[1]) As a rule he writes: Simon Stevin van Brugge; only in Work I: Brugghelinck. The mention of his origin is lacking in Works VII and X, in the latter of which the name of the author is not given at all.

[2]) In older biographies (Valerius Andreas 719; Sweertius 677; Foppens 1102) no year of birth is given. Voorduin 28, on the authority of the *Encyclopédie d'Yverdon*, has 1555.

[3]) Schouteet 140-142.

[4]) These deeds were discovered by Schouteet in the municipal archives of Bruges; they bear the date Oct. 30,1577 and their contents, which Schouteet gives in full, may be summarized as follows: Simon Stevin, natural son of Antheunis Stevin by Cathelyne van der Poort, is relieved from guardianship and declared independent, so that he can use and administer his own goods. He further promises to indemnify certain persons for giving bail on his behalf to Jan de Brune, a tax-official in the „Vrije van Brugge", now that he is going to occupy a position in his office. In one of the documents his age is given as twenty-eight or thereabouts. This may tally with the year of his birth as given above, though it leaves room for 1549 also. The „Vrije van Brugge" was a rural district surrounding the town; it was one of the four „members" of Flanders whose representatives, together with those of the nobility and the clergy, constituted the States.

of both the one and the other [5]). There are reasons to suppose that young Stevin was reared by his mother. All the anecdotes told by biographers on the traits he displayed as a child, and on the scholarly education he received are the fruit of pure fancy.

In 1577 we find him occupying a position in the financial administration in the "Vrije van Brugge" [6]). From a casual remark in one of his books we gather, further, that earlier he had worked as a bookkeeper and cashier in Antwerp [7]).

Most of Stevin's biographers tell us a good deal about the motives which are supposed to have prompted him to leave Bruges some time after 1571 and to set out on extensive travels in Poland, Prussia and Norway in the period between 1571 and 1581 [8]). However, it has proved impossible to check any of these statements. In particular there is no ground for the assertion that his departure had anything to do with the religious persecution becoming more intensive under the Duke of Alva [9]). Unless new documents be discovered, we shall have to put up with the deplorable lack of facts on the first three decades of his life.

This situation, though far from satisfactory, is somewhat ameliorated from the moment of his settlement in the Northern Netherlands. It is certain that he established himself at Leyden in the year 1581 [10]), and that he was matriculated at the University on February 16, 1583, under the name of Simon Stevinius Brugensis [11]).

3. IN THE NORTHERN NETHERLANDS

At that moment he had already written some of the long series of works he was to publish, some of which were to win him immortal fame. In 1582 the renowned printer and publisher Plantijn of Antwerp had published his *Tafelen van Interest (Tables of Interest)* (I), [12]), to be followed in the next year by a geometrical work, *Problemata Geometrica (Geometrical Problems)* (II), published by Joannes Bellerus, also of Antwerp. Henceforth all his works were to be published in the Northern Netherlands. In 1585 Plantijn, who in the meantime (1583) had transferred his business to Leyden, published his works *Dialektike ofte Bewysconst (Dialectics or the Art of Demonstration)* (III), *De Thiende (The Disme)* (IV) and *l'Arithmétique (Arithmetic)* (V). In the next year Plantijn's son-in-law Frans van Ravelingen continued the series with Stevin's most famous works: *De Beghin-*

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5) For further particulars cf. Dijksterhuis 3.

6) See Note 4 and a remark made by Stevin himself in Work XI (v; 2; dedication to M. de Bethune 6).

7) ibidem.

8) It is certain that he visited these countries; in his works he refers to experiences gained in Poland, Prussia and Norway, and the elaborate plans for the improvement of the harbour works and waterways and for the drainage system of such Prussian towns as Danzig, Braunsberg and Elbing testify to his having been there.

9) J. J. van Hercke; reprint 2-3.

10) The municipal registers of Leyden for the year 1581 contain the enrolment of Symon Stephani van Brueg as a student having taken residence at Nicolaas Stochius' at the Pieterskerkgracht. Stochius was the head-master of the Latin school.

11) In the registers of the University his name is found up to 1590 with the addition stud. art. apud Stochium. It may be mentioned that his name is given by Andreas Valerius 719 as Simon Stevinus, sive Stephanus.

12) The Roman numerals between parentheses refer to the list of Stevin's works given below.

selen der Weeghconst (The Principles of the Art of Weighing) (VI), *De Weeghdaet (The Practice of Weighing)* (VIa) and *De Beghinselen des Waterwichts (The Elements of Hydrostatics)* (VIb).

As apparent from this survey, the first years of Stevin's residence at Leyden must have been crowded with scientific work. This, however, did not prevent him from being active in technical science also. As early as 1584 we find him starting negotiations with the municipality of the town of Delft on the application of one of his inventions in the field of drainage. In the same year he is granted patents by the States General and the States of the province of Holland for various inventions, which are followed by several others during the ensuing years [13]. The majority of these patents refer to dredging and drainage; in particular he applies himself to improve the marshmill (i.e. a wind-driven scoop wheel used for pumping out water), a very important form of engine in a country the greater part of which had been literally wrested from the sea. There would be hardly any reason to mention, besides these highly practical inventions, his mechanically driven spit, which was no more than one of the countless mechanical toys the period revelled in, were it not that he marked this piece of work with the sign of the *clootcrans* (wreath of spheres), which was to become famous later on [14].

In 1588 in order to apply his hydraulic inventions in practice he entered into partnership with his friend Johan Cornets de Groot [15], the father of the boy who was to become the world-famous jurist Grotius. Together they built watermills in several places or improved existing ones by applying their new system.

It is characteristic of Stevin's wide range of interest that in this same year, in which all his energy seemed to be concentrated on technical problems, he published a book on quite a different subject: *Het Burgherlick Leven (Vita Politica) (The life of the burgher)* (VII).

Four years later, in 1594, the pamphlet *Appendice Algébraique* (VIII) proved that he still occupied himself with things mathematical. In the same year his *Sterctenbouwing (The building of fortresses)* (IX) saw the light, a work that ensured him a prominent position in the history of the art of fortification.

4. STEVIN'S IDEAS ON LANGUAGE

Though it is not uncommon to find mathematicians interested in linguistics, it is rather unusual to meet with one who as a scientist exerted a powerful influence on the language of his people and became no less famous in this respect than by his scientific achievements. Of this rare phenomenon Stevin is an example. Being strongly convinced that the Dutch language was singularly suitable for the rendering of scientific reasoning, and endowed with a peculiar gift of finding or coining words fitting this purpose, he became the founder of scientific and technical Dutch. Without being aware of the fact, everyone in the Netherlands

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[13]) The texts of these patents are given by Doorman (1) 82, 86-88, 274; (2) 17-19.

[14]) See below: *Art of Weighing* Prop. 19.

[15]) Johan Hugo Cornets de Groot (Janus Grotius; March 8, 1554-May 3, 1640), burgomaster of Delft (1591-1595), was a close friend of Stevin, who speaks of him with great admiration and gratitude (Work V; Dedication). De Groot wrote in Work V a Latin poem and in Work VI a Latin and a Greek one. They collaborated in an experiment on falling bodies (Work VIb 66).

daily uses terms and expressions which, if not introduced by Stevin, were at least brought into vogue by him.

It cannot be denied that in his digressions on the history and the qualities of the Dutch language he often exceeds the boundaries his intellectual soberness and scientific turn of mind should have imposed on him, and that his action for purity of language sometimes degenerates into fanatical purism. On the whole, however, his influence on the Dutch language must be considered beneficial.

His ideas on the superiority of Dutch as a scientific language are developed at length in the Memoir *Uytspraeck van de Weerdicheyt der Duytsche*[16] *Tael (Discourse on the Worth of the Dutch Language),* which serves as an introduction to the *Weeghconst* (VI). They will be summarized in the Introduction to this Memoir. It will then be seen that they are of biographical rather than scientific interest. One of them, however, deserves general attention. It is directed against the exclusive use of Latin for scientific purposes, entailing that those who in their youth lacked the opportunity of a scholarly education are for ever prevented from participating in scientific activity. In order to promote science, Stevin argues, all available forces should be released, and this is possible only if nobody is unnecessarily hampered by linguistic difficulties.

In using this argument Stevin associates himself with a number of similarly minded authors in various countries who during the sixteenth century were advocating the dethronement of Latin by the vernacular. So what at first view seems to be no more than a chauvinistic overestimation of Dutch, turns out to be a particular instance of a general plea for the good rights of the vernacular as the language of science, now applied to the special case of the Low Countries.

In accordance with his conviction of the superiority of his own language as a medium for scientific reasoning, Stevin after the publication of *l'Arithmétique* wrote all his works in Dutch. In doing so he deprived himself of the chance of being read in other countries. Indeed, his ideas and achievements were only made known there through Latin and French translations of some of his works [17]. The same circumstance has made it necessary to add an English translation to each of the works published in this edition.

Stevin's extensive digressions in the field of language can only be fully understood if considered in connection with his phantastical theory of the *Wijsentijt* (Age of the Sages). Indulging in the old dream of mankind that in a remote past all things, which we now only know in a deficient and incomplete state, were in perfect order, he wanders away into an elaborate discussion of the means by which this primordial Golden Age might be brought back again. One of these means consists in a systematic cultivation of natural science, which requires the ordered collaboration of all persons able to do scientific work, regardless of their previous training and social status. However, this will be possible only if all scientific ideas and reasonings are expressed in the vernacular. And this will meet with greater success according as the vernacular itself proves more fit for the purpose. Here the matter of the superiority of Dutch turns up again, culminating in the argument that the language of the Sages in the *Wijsentijt* can have been none other than Dutch.

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[16]) In Stevin's time this was the name of the language in use in the Low Countries. It has to be translated by Dutch, not by German.

[17]) Works XIa, XIb, XIII.

The whole theory forms a typical example of how the most rational and scientific of minds may at the same time foster the most irrational and phantastical ideas on topics lying outside the sphere of his specific competence.

5. THE POLITICAL BACKGROUND OF STEVIN'S LIFE [17a])

In the period dealt with above Stevin must have come into contact with Maurice, Count of Nassau, later Prince of Orange and Stadtholder of the United Provinces [18]). As his activity now shifts from that of a private scholar and engineer to that of a person of importance in the young Republic, we must interrupt his life-history for a moment to give a short survey of the political background.

In the sixteenth century the so-called Low Countries, out of which in the long run the present states of Belgium and the Netherlands were to emerge, consisted of seventeen provinces, owing allegiance to the descendant of the House of Habsburg-Burgundy, Philips II, who was also king of Spain. In the sixties of the sixteenth century a movement of opposition to the absolutist, centralizing and alien tendencies of Philip's government sprang up, to which the activities of the small groups of Calvinists, scattered over all the provinces, soon imparted a revolutionary character. In 1567 William, called the Silent, Count of Nassau and Prince of Orange, had to emigrate with many others in view of the arrival of the Duke of Alva. In 1572 the latter's repressive policy led to a second revolutionary attempt. At first only the provinces of Holland and Zealand managed to free themselves under the leadership of the determined Calvinist minority and of the Prince of Orange. In 1576 the other provinces joined in (Pacification of Ghent), but soon afterwards a war of reconquest was undertaken by the Spaniards and conducted by the Duke of Parma. The Walloon (French-speaking) provinces made their peace with the King in 1579. Parma's success was greatly favoured by dissension between the Catholics, to whom the aims of the revolution were primarily political, and the Calvinists, but the determining factor in the campaigns, which swayed forwards and backwards for a number of years, was the strategic barrier of the rivers traversing the Netherlands from East to West. In 1581, when Flanders and Brabant were already gravely threatened, but still represented on the States-General, the latter solemnly deposed Philip II. However, owing allegiance to the throne of Habsburg-Burgundy, they did not yet venture to take the sovereignty into their own hands. The feeling prevailed that they could not do without foreign help. An attempt to enlist the help of France by investing the Duke of Anjou, brother to the French King, with the sovereignty, led to failure. A movement set up in Holland to invest William the Silent with the sovereign power and the title of Count, was frustrated by the murder of the Prince on June 10, 1584.

[17a]. We are indebted for this paragraph to Prof. Dr P. Geyl, Professor of History at the University of Utrecht.

[18]) The Stadtholder, originally the Central Ruler's representative in a province, since the rebellion was appointed by the States of the province. Nevertheless, the tradition of a sovereign position still clung to the office, and the Stadtholder had a say in the appointment of town magistrates. Maurice was born in 1567 and died in 1625. He was Stadtholder from 1585 to the year of his death. There is no ground for the wide-spread belief that he was a pupil of Stevin at Leyden University. Indeed, there is no evidence that Stevin ever taught at this University.

At this moment the whole territory of the States consisted of no more than the Provinces of Holland, Zealand, Utrecht, parts of Guelders, Overijsel and Friesland and a few towns in Brabant and Flanders. In 1579 the famous Union of Utrecht had been concluded, and all the provinces still holding out had acceded to it. That in the end seven provinces only should remain, was decided by the fortunes of war. In 1585 Antwerp, the largest and wealthiest city of all the Low Countries, always a bulwark of the rebellion, had to surrender to Parma. Realizing the danger that the Low Countries might be entirely subdued by Spain, Queen Elizabeth of England now declared herself willing to send an auxiliary force under the command of the Earl of Leicester. However, Leicester did not succeed in improving the situation. When he resigned in 1587, even the provinces of Holland and Zealand, the real stronghold of the Netherlands, were in danger.

In these provinces, however, the spirit of resistance remained unbroken, and especially the great statesman Oldenbarneveldt, the advocate of the States of Holland and in that capacity their virtual leader [19]), did not waver one moment in his resolution to continue the struggle for independence. Full of confidence in the magic power the name of Orange had over the people, he made the States of Holland and Zealand invest William the Silent's son Maurice with the Stadtholderate of these provinces in 1585. In 1589 he persuaded the States of Utrecht, Guelders and Overijsel to follow this example and entrust the military power to the young prince.

The choice proved to be an excellent one. Together with his cousin, brother-in-law and intimate friend, William Louis of Nassau, who held the Stadtholderate of Friesland and Groningen, Maurice set about reorganizing the States Army and soon revealed himself as a military genius in using it as an instrument in the struggle for independence. Several successful sieges of towns occupied by the Spaniards, conducted as it were according to scientific methods unknown before that date, made his name as a commander famous all over Europe. He succeeded in liberating the whole territory of the remaining seven of the United Provinces. After ten years of hard fighting the "fence" of the Republic (to use a popular expression of the time) was closed and its domain was extended even beyond the boundaries. Thanks to his military achievements and the energetic politics pursued by Oldenbarneveldt the international position of the Dutch Republic underwent a radical change in the course of the same ten years. The States General, in 1585 still in quest of foreign help, in 1596 concluded an alliance with France and England, in which they were recognized on a footing of equality with these European powers.

6. STEVIN AND MAURICE

Since Stevin served as an engineer in the States Army and acted as a tutor to Prince Maurice, he doubtlessly played some part in this formidable change of things. Unfortunately we do not know exactly the nature, the extent and the

[19]) The Advocate of Holland, later called the Grand Pensionary, was an official appointed on an instruction by the States of the Province. He may be described as the leading minister of the province; he presided over the meeting of its States and was a permanent member of its delegation in the States-General. At the same time he acted in effect as the foreign secretary of the Union.

relative importance of the role he played, but we may surmise that it was by no means negligible. Outwardly, it is true, his position always remained rather modest. Up to 1603 his title was no other than that of "engineer". It was only then that, upon the recommendation of Maurice, he was appointed Quartermaster of the States Army with the special commission of planning the army camps. There is no evidence that he ever held the position of Quartermaster-General, assigned to him by his son Hendrick on the title-page of an edition of posthumous papers, the *Materiae Politicae* or *Burgherlicke Stoffen* (XIV). In historical works on the military operations of Maurice his name is not mentioned, and only a few documents give any particulars of his activity [20]).

Nevertheless he must doubtlessly have exercised a certain influence on the course of events in the United Provinces because of his intimate relation to the Prince as his tutor in mathematics and natural science and, later, as superintendent of his financial affairs. It was generally known that the Prince held him in great respect, and his reputation grew with Maurice's fame. He frequently sat on committees investigating matters of defence and navigation, and he was entrusted with the organization of a school of engineers to be incorporated into the University of Leyden [21]).

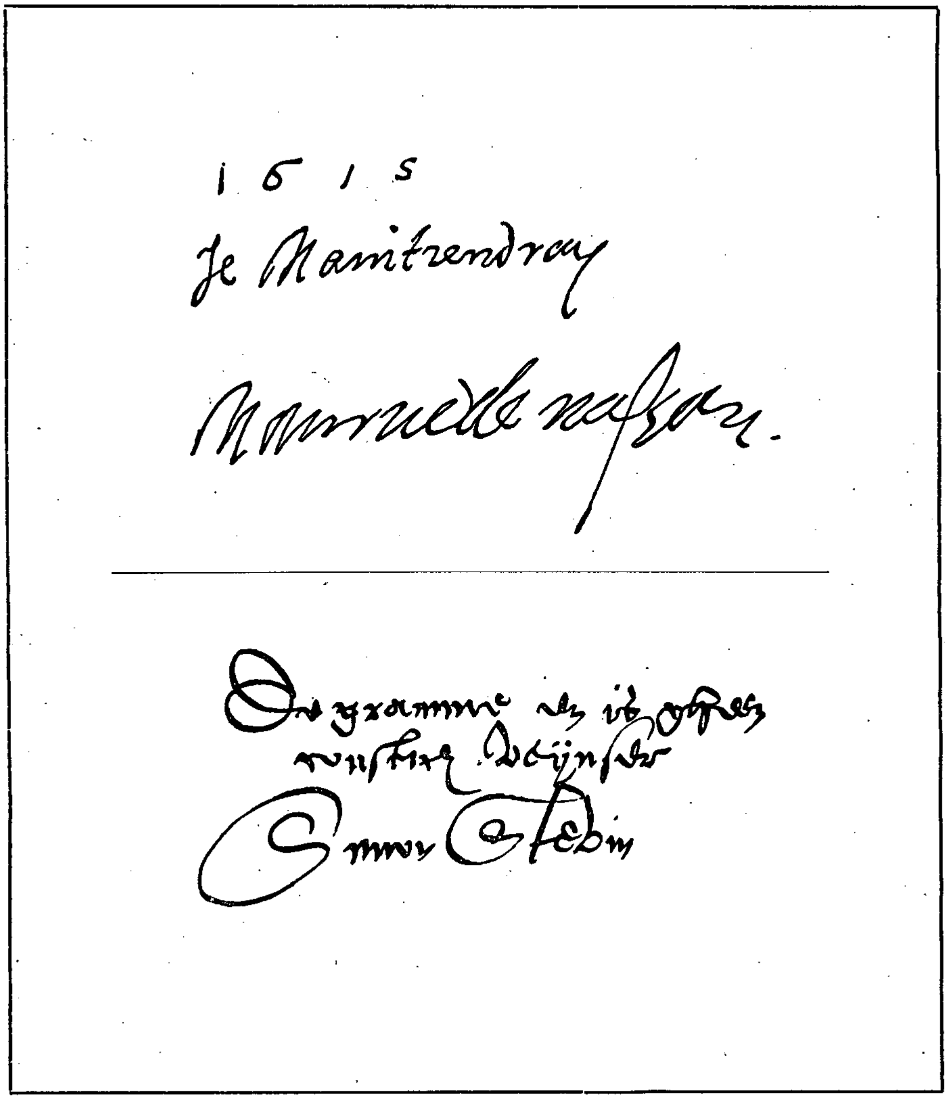
Whatever influence Stevin's cooperation with Maurice may have had on the latter's achievements, the effect of his activity as a tutor on his own scientific development is manifest enough. He had to compose textbooks on all the subjects the Prince wanted to study, and he was too original a thinker ever to confine himself to mere reproduction of what he found in existing works. He always managed to add some invention of his own or at least to improve the method of treatment.

After having compiled a considerable number of textbooks for the instruction of the Prince, Stevin took the initiative of publishing the whole corpus in a comprehensive edition. Thus between 1605 and 1608 the immense volume of his *Wisconstighe Ghedachtenissen (Mathematical Memoirs)* was formed (XI), to be followed by a partial French translation, the *Mémoires Mathématiques* (XIa), and a complete Latin one, the *Hypomnemata Mathematica* (XIb).

The version presented here of the origin of these magnificent editions shows a marked deviation from the current story [22]). According to this version the publication was entirely due to the initiative of Maurice, who, being accustomed to carry the manuscripts with him in his campaigns and afraid of losing them, decided to have them published. It is suggested that he paid for the publication, too. There is, however, no evidence to support the legend of this noble gesture. Stevin tells us, it is true, that he had sometimes seen the Prince anxious lest he should lose the manuscripts, but he leaves no doubt that he acted entirely on his own initiative when he undertook the publication [23]). And Snellius, who made the Latin translation, explicitly states that the idea of this translation occurred to him spontaneously and that the publishing firm had to meet all the expenses [24]).

It appears that Stevin's intimate relation to Maurice was not always regarded without some misgivings. To this Stevin himself alludes in a passage of the

[20]) The documentary evidence for the above: Dijksterhuis 10-16.
[21]) Dijksterhuis 14.
[22]) See e.g. Sarton 256. For further particulars: Dijksterhuis 330.
[23]) Work XI. Preface.
[24]) Work XIb. Dedication.



Autographs of Count Maurice and Stevin.
(From *Overzigt ener verzameling Alba Amicorum uit de XVIe en XVIIe eeuw* door Jhr F. A. Ridder van Rappard. Album B, p. 86).
The text of Stevin's autograph means: A man in anger is no clever dissembler.

Wisconstighe Ghedachtenissen [25]). Another hint is furnished by a letter written by the Dutch theologian Ubbo Emmius [26]) after the publication of this work. In the treatise on Astronomy, which forms part of it, Stevin frankly and whole-heartedly adopted the Copernican doctrine of the mobility of the earth, which in the opinion of many scholars of the time had a taint of heterodoxy. Emmius qualifies the astronomical theories held by Stevin as worse than absurd and preposterous, and seriously regrets that the name of the Prince should be stained by this "dirt".

After accomplishing the *Wisconstighe Ghedachtenissen,* Stevin published only two more works, which appeared in one volume in 1617: *Castrametatio (Marking out of army camps)* and *Nieuwe Maniere van Sterctebou door Spilsluysen (A new manner of fortification with the help of pivoted locks)* (XII). In the dedication to the first-mentioned work he styles himself a Castrametator (Measurer of Camps). This has given rise to the opinion that he here refers to a new post, instituted on his behalf. However, this opinion is unfounded. There is no evidence that his official position had undergone any change since his appointment as quartermaster in 1604; the term of Castrametator is no more than a personal way of describing the special duties he had to perform.

It appears [27]) that in the long run he felt dissatisfied by the lack of opportunity to show his capacities in a more important function than that of a Castrametator. His son Hendrick tells us that he petitioned the States General, advocating the institution of the office of Superintendent of the fortifications and recommending himself for this post. This petition, however, seems to have met with no more success than another, in which he asked for an increase of his salary as quartermaster; that, too, was refused by the States in 1620.

7. MARRIAGE, OFFSPRING AND DEATH

Returning now to Stevin's personal life, we have to relate some facts about his late marriage and his offspring. The data, however, are again disconcertingly scarce. It has been established [28]) that in the second decade of the seventeenth century he married a young woman from Leyden, called Catherine Cray (day and year of birth unknown), that she bore him four children, and that he bought a house at The Hague. The dates of these events, however, do not, as given, tally with one another. The house was bought on March 24, 1612 (it is 47, Raamstraat, The Hague, which in 1897 was adorned with a bust of Stevin, made after the Leyden portrait [29]); the eldest son, Frederick, was probably born in 1612, the second, Hendrick, probably in 1613, the eldest daughter, Susanna, on April 19, 1615 and the youngest, Levina, at some unknown date. Notification of the marriage, however, was not given until April 10, 1616. No date of the ceremony could be traced. If all these dates are correct (all of them have been derived from

[25]) Work XI; i, 21; p. 40. There are now many, he tells us, who cannot believe that the scientific occupations of the Prince are not detrimental to many affairs which might otherwise have been done more efficiently.

[26]) Ubbo Emmius (Dec. 5, 1547-Dec. 5, 1625) was afterwards the first Rector of Groningen University (founded in 1614).

[27]) Dijksterhuis 16.

[28]) Dijksterhuis 18-20.

[29]) Dijksterhuis 30.

authentic documents), we must conclude that at least three out of the four children were born out of wedlock.

Stevin's marriage was not to last long, as he died in 1620. Again we know no particulars; we can only prove [30] that he was still alive on February 20, and that his death occurred before April 18. No single further detail on the exact dates of the decease and the funeral is available. His widow remarried on March 14, 1621, her second husband being Maurice de Viry (or de Virieu), bailiff of Hazerswoude near Leyden. She died on January 5, 1672.

Out of the four children only the second son, Hendrick [31], is of interest to the reader of a biography of Stevin. He followed a career which outwardly resembled his father's, but he lacked the latter's genius. He studied mathematics at Leyden University. After having travelled through Europe, he became an engineer in the army and later held the post of quartermaster. A wound having obliged him to retire from military service, he married the widow of the lord of a manor at Alphen-on-Rhine (in Southern Holland) [32] in 1642, and obtained the manorial title for himself after the death of his wife. He died without issue in January 1670.

It is greatly to Hendrick's credit that he considered it a debt of honour to his father's memory to edit the latter's posthumous papers, which had been very carelessly dealt with by the widow. Thanks to his pious care we possess the volume of the *Materiae Politicae,* in which some thirty years after Stevin's death several of his unpublished treatises saw the light, whilst others were published some eighteen years later in Hendrick's own work, *Wisconstich Filosofisch Bedrijf (Mathematico-Philosophical Activity)* [33].

8. STEVIN'S LEGACY

Stevin had to wait long for recognition of his real value to the history of civilization. In the Netherlands he was remembered for a long time almost exclusively as the tutor of Maurice and as the builder of two sailing-chariots, with which the Prince would occasionally amuse himself and his guests. Works on the history of mathematics and natural science, it is true, mentioned his name, but did not do full justice to his achievements. When on the occasion of the tercentenary of the year of his birth a movement was started in his native city to erect a monument to his memory (the statue which now actually adorns the "Simon Stevin-plaats" at Bruges), he became the subject of a heated controversy [34], in which, through a curious twist of historic perspective, his loyalty to his native country was impugned (he was represented as a "Belgian", serving his country's enemy, Maurice). In answer to doubts which were expressed in the debate as to his merits as a scientist, one of his defenders collected all the testimonies on his achievements he had been able to lay hands upon in encyclopedias

[30]) Dijksterhuis 20.
[31]) Dijksterhuis 23.
[32]) Not Alphen near Breda, as Sarton (1) 246 supposes.
[33]) This work would deserve a closer study. As Van Zutphen points out, Hendrick had remarkable ideas on the drainage of the Zuider Zee.
[34]) Dijksterhuis 26-29.

and historical works [35]. This miscellaneous collection appears to have been impressive enough to contemporaries. Nevertheless it now strikes us as singularly inadequate.

It was only in the first decades of the twentieth century that the study of Stevin as a scientist was undertaken in a thorough and systematic way, the leader of this movement being the meritorious Belgian historian of mathematics, Father Henry Bosmans S.J.

We conclude this short introductory sketch of Stevin's life with a discussion of the correct pronunciation of his name: Stévin or Stevín. In Holland people are generally inclined to stress the second syllable, sometimes even to pronounce it as a French ending. Yet there is no doubt that the correct pronunciation is Stévin: the Flemish have always pronounced it so, and they still do so to this day.

[35]) van de Weyer (1).

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STEVIN'S ACHIEVEMENTS

The present edition does not attempt to embrace the totality of Stevin's writings. The bulk of the *Wisconstighe Ghedachtenissen* made it practically impossible to include this work as a whole. Moreover, several of his works are too much in the nature of a textbook to justify re-publication. Consequently the editors were obliged to make a selection and to publish only such works as contain mainly original contributions to science. This policy, however, entails that the image of Stevin evoked in the forthcoming volumes must needs be slightly distorted.

In order to remedy this unavoidable defect, we give as a prelude a concise survey of all Stevin's achievements, hoping thereby to convey a general impression of the wide range of his activities. We classify them under the following headings.

I. MATHEMATICS

A. *Arithmetic and Algebra.*

From the very beginning Stevin's desire is evidently to put science to practical use in daily life. Long before his time, tables of interest were used by bankers, but they were kept secret as tools of trade. In his *Tafelen van Interest* (I) Stevin now published a complete set of these tables and supplemented them with a textbook showing their application to problems of interest.

De Thiende (IV) contains a systematic treatise on decimal fractions and their application, with which this highly important improvement was formally introduced into arithmetic. Neither the fact that the idea of these fractions had been applied before him in goniometrical tables nor the circumstance that his so-called "thiendetalen" or "ghetalen van den tienden voortganck" (numbers of the tenth progress) were, properly speaking, no fractions at all, but integers introduced to avoid fractions, need prevent us from associating with the invention Stevin's name before all others. Undoubtedly the new technique was marred in the beginning by a cumbersome notation, which, however, was substantially improved in later works. In a series of examples the practical value of the "thiendetalen" for various categories of craftsmen is demonstrated. In an appendix Stevin advocates the introduction of the decimal principle in all human accounts and measurements, thereby anticipating the (partial) realization of this simple idea by two centuries.

The work *l'Arithmétique* (V) with its appendix, *La Pratique d'Arithmétique,* in which French translations of Works I and IV were incorporated, mainly deals with widely known subjects, leaving little room for originality. Stevin, however, succeeds in improving the symbolism in many respects and in contributing to the formulation of general rules for the solution of equations. The mainly practical character of the work does not prevent him from delving rather deeply into some highly theoretical topics and taking part in some fundamental controversies concerning the principles of arithmetic. Algebra, which is still considered as one of the numerous rules taught in arithmetic, is advanced by the above-mentioned rules on equations, by the solution of the problem how to find the greatest common divisor of two polynomials and, in the *Appendice Algebraique* (VIII), by a method for the approximation of a numerical root of an equation of any degree. Moreover, the application of the "rule of algebra" is illustrated in a translation (or rather paraphrase) of the first four books of Diophantus, the first to appear in any European vernacular.

B. *Geometry*

The work *Problemata Geometrica* (II) deals with problems in pure mathematics, such as division of a polygon, construction of regular and semi-regular polyhedra, construction of solids to satisfy certain conditions. The subject is resumed in a less strict form with many practical applications in *De Meetdaet* (XI; ii) (forming part of the *Wisconstighe Ghedachtenissen).* In this work various geometrical instruments are described in detail. The construction of an ellipse by lengthening the ordinates of a circle in the same proportion, which is taught here, seems to be Stevin's invention.

C. *Trigonometry*

The *Wisconstighe Ghedachtenissen* contain a very elaborate systematic treatise on plane and spherical trigonometry under the title *Van den Driehouckhandel* (XI; i, 1). It is shown that the trigonometrical formulae relative to the right-angled spherical triangle can be reduced to six, and how the various practical problems may be solved by means of them.

D. *Perspective*

The desire, expressed by Maurice, to become familiar with perspective drawing induced Stevin to compose a textbook on the subject (*Van de Verschaeuwing* XI; iii, 1), which is remarkable for the care with which all the terms to be used are defined, for the writer's inventiveness in coining Dutch scientific words, and for some personal contributions (construction of a perspective drawing on a picture plane not perpendicular to the ground plane, and solution of the so-called inverse problem of Perspective: given an object and a perspective drawing of it, to determine the place of the observer's eye).

II. MECHANICS

With one single exception all Stevin's contributions to this branch of science, which are to be found in the *Weeghconst* (VI) and the *Weeghdaet* (VIa), refer to statics. In the first book of the *Weeghconst* Stevin continues the work of Archimedes in the latter's *De Planorum Aequilibriis* by giving a mathematical demonstration of the condition of equilibrium of a horizontal lever. In the same book he proves in a most ingenious and interesting way the law of equilibrium on an inclined plane, basing himself on the conviction of the impossibility of perpetual motion. From this theorem the rule for composition and decomposition of a force acting on a point is deduced, by which the study of the equilibrium of a rigid body with one fixed point is made possible. It should be noted that Stevin for reasons of principle rejects the method of virtual displacements. The second book is devoted to the determination of centres of gravity. Here, too, Stevin applies the Archimedean method; he succeeds, however, in simplifying it in a way which is of some importance in the history of the Calculus. The *Weeghdaet* contains practical applications of the theorems of the *Weeghconst* in various instruments. In an appendix to the latter work we find Stevin's only contribution to dynamics: an experiment performed in collaboration with J. de Groot, in which falling bodies are proved to traverse the same distance in the same time regardless of their weight. A *Byvough der Weeghconst,* included in the *Wisconstighe Ghedachtenissen* (XI; iv, 7), brings new applications of theoretical statics in problems on cords and pulleys and in an investigation on the horsebit.

III. HYDROSTATICS

Here again Stevin acts as the immediate successor to Archimedes. He proves the latter's theorem on the force exerted by a fluid on a solid immersed in it in a more satisfactory way, and evaluates the forces which by its weight a liquid exercises on the bottom and the walls of the enclosing vessel. This leads him to the hydrostatic paradox, which is tested by experiment. The *Waterwicht* (VI b) with its appendix, the *Waterwichtdaet,* in which his hydrostatic theories are developed, must be considered as a valuable step towards the complete systematization of hydrostatics given by Pascal. The theory is applied in the problem of the diver and in a discussion on the stability of a floating vessel (*Van de vlietende Topswaerheyt;* XI; iv, 73).

IV. ASTRONOMY

In the section of the *Wisconstighe Ghedachtenissen* which bears the title *Van den Hemelloop* (XI; i, 3) Stevin first expounds the classical Ptolemaic theory of the structure of the universe and then shows how it can be transformed into the modern Copernican theory by a shift of the observer's standpoint. It should be borne in mind that his aim is no other than to bring home to his pupil as clearly as possible the two existing theories without pretending at all to enrich them with findings of his own. That he treats the Copernican system on a footing of equality with the Ptolemaic is in itself remarkable enough, when one considers that at the time the *Hemelloop* was composed this innovation in astronomy was still far from being generally accepted, that the authority of the greatest astronomer of the second half of the sixteenth century, Tycho Brahe, was against it, and that none of the leading scholars of the time had pronounced himself in favour of it. More remarkable still, Stevin not only explains the Copernican system, but also states his intimate conviction that this theory represents the real structure of the world, and endeavours to make this acceptable. In doing so, he shows his independence from the founder of the theory by rejecting the latter's hypothesis of a third motion of the earth besides the daily rotation about its axis and the annual revolution round the sun.

A second astronomical topic is dealt with in the *Eertclootschrift* under the title

Van de Spiegeling der Ebbenvloet (XI; i, 26). Here Stevin develops a theory of the tides, based on the assumption of an attraction exercised by the moon on the water, which for simplicity's sake is supposed to cover the whole of the earth's surface.

V. GEOGRAPHY

In the whole section *Eertclootschrift* of the *Wisconstighe Ghedachtenissen* there is only one subsection which can be brought under the heading of this paragraph, and this treatise, entitled *Vant stofroersel des Eertcloots* (XI; i, 22), deals neither with topography nor with cartography (as might have been expected), but with the gradual changes in the materials constituting the earth's surface. It discusses the accretion of land by sedimentation, the formation of dunes, the origin of mountains, and the gradual changes in the courses of sinuous rivers. It is supplemented by a number of hydrographical considerations in the memoir *Van de Waterschuyring* in Hendrick Stevin's *Wisconstich Filosofisch Bedryf* (XVIB 3).

VI. NAVIGATION

Stevin's nautical treatises refer to two different subjects. In the *Eertclootschrift* is found a treatise *Van de Zeylstreken* (XI; i, 24), in which the doctrine of courses and distances is taught. The only two methods of navigation susceptible of scientific treatment, viz. great circle sailing and loxodromic sailing, are explained both on mathematical lines and with the aid of a globe.

In *De Havenvinding* (X; abridged version XI; i, 25) Stevin discusses the possibility of making a landing on the basis of the known latitude and the magnetic declination of the harbour. The longitude can then be dispensed with.

VII. TECHNOLOGY

On Stevin's work as an engineer his own writings give none but the scantiest information, which is only partially supplemented by the study of various patents granted or applied for. The subjects of major interest are:

A. *Mills,* especially marshmills. He tried to make various improvements in the mechanism of these engines, and in his treatise *Van de Molens* (XV) corroborated them by a theoretical study based on his statical and hydrostatical theories. This seems to be the oldest scientific work on the subject, anticipating Smeaton's famous researches by some 150 years.

B. *Sluices and Locks.* This subject is treated in the first of the two chapters of the work *Nieuwe Maniere van Sterctebou, door Spilsluysen* (XIIB), which, as far as we know, constitutes the oldest extant printed treatise on sluices. The various types of sluices and possible improvements in their construction are discussed.

C. *Hydraulic Engineering.* In a posthumous paper on waterscouring, published by his son Hendrick (XVIB; 3), Stevin develops detailed plans for the improvement of the waterways of the town of Danzig and other towns in Prussia and the Netherlands.

D. *Sailing Chariots.* For years and years Stevin's fame has rested entirely on this invention, which plays, however, only a minor part among his technological inventions and is nowhere mentioned in his works. The reports of contemporaries are incomplete and bear obvious features of exaggeration and phantasy.

VIII. MILITARY SCIENCE

A. *Art of Fortification.* As in the history of mathematics, here again Stevin carries out the dual task of systematizing the existing knowledge and enriching it with personal contributions. Because of his work *Stercktenbouwing* (IX) his name ought to be associated before all others with the so-called old Dutch method of fortification, which is here explained systematically for the first time. The work *Nieuwe Maniere van Sterctebou, door Spilsluysen* (XIIB) deals with the use of sluices for defensive purposes and gives plans for the fortification of various towns in the Netherlands by these means.

B. *Castrametatio.* In the work of this title (XIIA) Stevin describes in detail the method of laying out camps and their internal organization, which was in use in the States Army. Several other writings on military science, destined for a *Crijchsconst,* were published in Work XIV.

XI. BOOK-KEEPING

Stevin's works on this subject (XI; v, 2; XIV B) can be brought under the two headings of mercantile and princely financial administration. In order to persuade Maurice to have his affairs as a prince and an army-commander organized by the Italian method of double-entry book-keeping, which had been in use in commerce for many years, he first explains this method in a textbook on commercial administration (XI; v, 21) and subsequently argues why it is desirable and in how far it is possible to apply it to the Prince's own affairs (XI; v, 22). He does this in the form of an extremely lively dialogue between Maurice and himself, which he reports to be a faithful record of a real conversation. Another system of administrating the princely domains is explained in the Work *Verrechting van Domeine mette Contrerolle* (XIVB).

X. ARCHITECTURE

The work *Materiae Politicae* (XIV) contains a number of posthumous papers on town planning and house building, and a treatise on the aesthetic aspects of architecture.

XI. MUSIC

In Stevin's time the traditional connection between music and arithmetic, a heritage of the medieval quadrivium, still survived. It was by no means necessary for a writer to be musical in the emotional sense of the word to feel interested in the theory of intervals. The important question of characterizing the various intervals by ratios in order to obtain a practicable temperament for keyed instruments is solved by Stevin in his *Spiegeling der Singconst* (XV). He boldly asserts that an octave consists of six equal intervals of a tone and twelve equal intervals of a semi-tone. In doing so, he anticipates the system of equal temperament, which was to be introduced about a century later.

XII. CIVIC MATTERS

The work *Het Burgherlick Leven* (VII) gives rules for the conduct of a citizen in cases where the divine and natural laws that determine his status are no longer mutually concordant or clash with his personal conviction. Moreover, it discusses the position of a prince (the highest citizen in the state) and sets the limits of his competence.

The work *Materiae Politicae* (XIV) contains an appendix to this treatise and develops plans for the organization of the various Councils which are to assist the prince of a great empire in his government.

This is but one of the numerous examples of Stevin's predilection for the organization of all kinds of military and civic matters.

XIII. LOGIC

This subject is dealt with in the booklet *Dialectike ofte Bewysconst* (III). Though it does not enrich this classical branch of science with any substantial innovation, it is remarkable for two reasons: it is one of the two oldest treatises on logic written in Dutch, and the method of exposition deviates greatly from that followed in the traditional textbooks. By both means Stevin hoped to make logic accessible to all readers, regardless of their previous training. The work is concluded by a *Tsamespraeck,* a dialogue, in which one of the interlocutors defends Stevin's views on the superiority of the Dutch language.

CONCLUSION

The description of Stevin's life and the survey of his achievements given above can still be supplemented with some general considerations on his personality. In the history of civilization Stevin figures as the prototype of the engineer, of the perfect technologist, who deals with practical problems in a scientific way. Being well acquainted with the work already done by others, he freely applies their results wherever possible, but in fields where nobody preceded him he seeks and finds his own paths. His scientific turn of mind is strong enough to make him pay full attention to purely theoretical problems without feeling unduly anxious about their useful effect in practical life; but he is too practically minded to let himself be entirely absorbed by theory. Thus he continually oscillates between what he calls *spiegeling* (speculation, i.e. theoretical investigation) and *daet* (practical activity).

This two-sidedness of his natural bent does not prevent him from appreciating one-sidedness in one special case; *daet* without *spiegeling* is, he thinks, absolutely worthless, but he is ready to accept *spiegeling* without *daet,* if only other men's activities are promoted by it, as was the case with the work of great mathematicians like Euclid, Archimedes and Apollonius.

The combination of theoretical interest and sense of the practical, though complete in itself, may exhibit itself in a single domain only and thus leave a man one-sided in another sense. This, however, is not the case with Stevin. It may be true that his preference and natural ability draw him particularly towards things mathematical, but in each case presenting itself he does everything that has to be done with the same rational deliberation. His is the precious gift of always considering the subject actually dealt with as the most important in the world. The foregoing survey will have demonstrated clearly to the reader how numerous these subjects were.

Stevin's versatility indeed is astonishing as long as one looks at the range of his achievements, not if one considers their nature and pays attention to the method, the style of his thinking. Here the mathematical character predominates. During long periods of his life he did not occupy himself with mathematical problems, but there was not a single moment at which he ceased thinking like a mathematician. To define carefully all terms to be used; to pay the utmost attention to the choice of words; to enounce exactly all assumptions to be accepted without demonstration; after having done so, to take for granted all that has been logically derived from these principles, and nothing else; this in a nutshell is the style of thought he never abandoned.

It is a method which in some fields is indispensable and in others highly useful, but like every method, limited in its applicability. On reading Stevin's works, we realize that he does not always abstain from transcending its natural boundaries; his inability to see in religion anything else but a slyly contrived means for making men behave decently (VII) may serve as one example; his lack of appreciation of all architectural beauty that does not consist in mathematical symmetry (XIVC, i) as an other. To him music appears to have been no more than that hidden problem in arithmetic Leibniz speaks of, in which the human soul—Heaven knows why — is said to take so intense a pleasure (XV). He determines the value of a language by statistical methods (VI). It would be interesting to know what his lost work on poetry (XIV D; ix) may have contained.

However this may be, the domain in which his mathematical way of tackling problems is correctly applied is wide enough for us not to over-emphasize the cases in which he failed to see its limitations. The more so, because it worked so admirably and always safeguarded him from its inherent natural dangers.

From the most serious of these dangers, dogmaticism, to which his younger contemporary Descartes fell a victim in such a notorious way, Stevin was safeguarded by the equilibrium he purposely maintained between theory and practice. If an assertion proves to be at variance with the facts, it has to be rejected as deliberately as when it is contrary to reason.

Just as he does not value practical ability without a theoretical foundation, he has no use for experience which does not stand the test of rational reflection. Having demonstrated in the *Toomprang* (XI; iv, 74) that, contrary to an opinion current among horsemen, the curves in the cheeks of a bit cannot have any influence on its action but that of increasing its weight, he rejects in advance the argument that all horsemen, grooms and bridlemakers hold the contrary on the ground of experience only.

As a natural consequence of this attitude he repeatedly opposes the appeal to authority which, after having been of the utmost importance in the Middle Ages, was still widely current in his age. This does not, however, prevent him from appealing to authority himself in cases in which he was specially interested. This happens e.g. in the *Huysbou* (XIVC, i), where a whole chapter on the building of houses in Antiquity is inserted solely for the reason that the authority of the classics helps him to combat those who opposed his exclusive appreciation of symmetry.

But here indeed one of his most deeply rooted convictions is at stake. Though he is generally willing to recognize the good right of an opinion different from his own, on this point he is apt to grow intolerant. There are a few more of these issues: one should not try to maintain that irrational numbers are in the least absurd or inexplicable; or that it is possible to build a good fortress not all the parts of which can be exposed to flanking fire; nor that scientific views could be expressed in any foreign language as clearly and concisely as in Dutch.

Anyone carefully restraining his personal opinions on these special points was likely to encounter a very reasonable, reliable and benevolent man, ready to give everybody credit for his own merits and striving after the advancement of the public welfare rather than personal honours and privileges. He repeatedly voices his opinion that science should be cultivated for the sake of the commonwealth only, and more particularly so with a view to the speedy restoration of the *Wijsentijt* era. Accordingly, when he feels obliged to point out mistakes in the works of others, he does so exclusively because of the retardation of the return of this Golden Age caused by any imperfection in human science. On the same grounds he urgently requests the reader in several passages of his works not to spare him his criticisms and to correct him wherever this is possible.

The advancement of learning for the sake of the commonwealth being his highest aim, nothing is more alien to his habits than letting another man's intellectual property pass for his own. This does not mean that his references are always as complete as we should like them to be. Here, however, it should be remembered, firstly, that the custom of the age did not impose on a writer the stringent obligation to mention all his sources, and, secondly, that this obligation does not even now apply to authors of textbooks; and, as we have seen, Stevin's works were textbooks to a high degree. However this may be, our curiosity as to his sources is by no means always satisfied.

The unselfishness with which Stevin devotes his scientific activity to the service of the community, and his efforts to release all available spiritual forces for the same purpose, regardless of social class, are signs of a strong social conscience. It is in accordance with all we know about him that his opinions in this field are of an intellectual rather than an emotional character. The same applies to various passages in which he advocates the interests of the poor (XIVA; i and iii); what at first sight appears to be a symptom of humanity not infrequently turns out to be a mere piece of rather commonplace utilitarianism.

As pointed out before, there is one element in the charm exercised upon the reader of Stevin's works which cannot possibly be brought home to anyone who has not mastered the Dutch language; it is the lucidity of his style and the peculiar flavour of the words of his own making in which he expresses his views. Like Galileo in Italy, he is one of the classical authors of Dutch national literature; as such he should be read in the schools, but this idea is still far from being realized in the Netherlands.

Here, however, we have to leave aside the national traits of his character and to consider him from an international point of view only. Undoubtedly he does not belong to the limited group of those great scientific geniuses who ring in a new era of human thought. But among the historical figures of the second rank his name as a mathematician and an engineer may be mentioned with honour, and his personality is always sure to arouse human interest in any reader making his acquaintance.

A BIBLIOGRAPHY OF STEVIN'S WORKS

The following bibliography contains the titles of all Stevin's works, with reprints and translations. For readability's sake no attempt at bibliographical correctness has been made. The reader interested in typographical details should consult the bibliography of Stevin's works in the *Bibliotheca Belgica* [1]), which also gives an account of pagination and foliation.

The Roman numerals preceding the titles are used throughout this edition in quotations of Stevin's works.

The numbers between [ ] refer to the libraries in the Netherlands and Belgium that possess the original works. Frequently occurring vignettes are indicated by capital letters. The meaning of these numbers and letters is explained in the footnote [2]).

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[1]) *Bibliotheca Belgica. Biographie générale des Pays-Bas par le bibliothécaire en chef et les conservateurs de la bibliothèque de l'université de Gand.*
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Libraries:

[2]) 1. Library of the Royal Academy of Sciences, Amsterdam.
2. Royal Library, The Hague.
3. Library of the Netherlands Society of Sciences, Haarlem.
4. Library of the University, Amsterdam.
5. Library of the University, Groningen.
6. Library of the University, Leyden.
7. Library of the University, Utrecht.
8. Library of the Technical University, Delft.
9. Library of the Province of Zealand, Middelburg.
10. Library of the Military Academy, Breda.
11. Library of the War Ministry, The Hague.
12. Athenaeum Library, Deventer.
13. Municipal Library, Rotterdam.
14. Municipal Library, Arnhem.
15. Historical Museum of Navigation, Amsterdam.
16. Royal Museum for the History of Natural Science, Leyden.
I. Royal Library of Belgium, Brussels.
II. Library of the University, Ghent.
III. Library of the University, Liège.
IV. Municipal Library, Antwerp.
V. Municipal Library, Bruges.
VI. Library of the Museum Plantin-Moretus, Antwerp.

Vignettes

A.	Hand with a pair of compasses.	Legend: Labore et Constantia:
B.	Idem.	Ribbon with Legend: Labore et Constatia.
C.	Idem.	The figure of a man with a spade on the left and that of a woman with a cross-staff on the right hold a a ribbon with legend: Labore et Constantia.
D.	Wreath of spheres.	Legend: Wonder en is gheen Wonder.
E.	A pair of compasses	The Patroness of the Netherlands in an enclosure with four arms. Legend: Labore et Constantia.
F.	Idem.	A laurel wreath. Legend: Labore et Constantia.

I. *Tafelen van Interest, midtsgaders de constructie derselver, ghecalculeert door SIMON STEVIN Brugghelinck.* — T'Antwerpen. By Christoffel Plantijn in den gulden Passer. 1582. 92 pp. Vignette A.
[13.I, II, VI]
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II. *Problematum geometricorum in gratiam D. Maximiliani, Domini Cruningen etc. editorum, Libri V. Auctore* SIMONE STEVINIO Brugense. — Antverpiae, Apud Ioannem Bellerum ad insigne Aquilae aureae. 118 pp. Vignette: Commerce in a vessel. Legend: In Dies Arte ac Fortuna.
[6, I, II, III, IV, V]

III. *Dialectike ofte Bewysconst. Leerende van allen saecken recht ende constelic oirdeelen; oock openende den wech tot de alderdiepste verborghentheden der Natueren. Beschreven int Neerduytsch door* SIMON STEVIN *van Brugghe.*
— Tot Leyden, By Christoffel Plantijn. 1585. 172 pp. Vignette C.
[3, 4, 5, 13, IV, VI]
Reprint: Rotterdam 1621. [2, 6, 13, I, II].

IV. *De Thiende leerende door onghehoorde lichticheyt allen rekeningen onder den menschen noodich vallende afveerdighen door heele ghetalen sonder ghebrokenen. Beschreven door* SIMON STEVIN *van Brugghe.* — Tot Leyden, By Christoffel Plantijn. 36 pp. Vignette C.
[13, IV, VI]
Reprints:
Gouda 1626, as an appendix to Ezechiel de Dekker, *Eerste Deel van de Nieuwe Telkonst.*
Gouda 1630, as an appendix to Ezechiel de Dekker, *Nieuwe Rabattafels.*
Anvers—La Haye 1924. Facsimile. With an introduction by H. J. Bosmans.
French translations:
La disme in V, XIII. Facsimile-reprint in G. Sarton, *The first explanation of decimal fractions and measures* (1585). *Together with a history of the decimal idea and a facsimile of Stevin's Disme.*—Isis 65. Vol. 23, 1 (1935) Nr. 65 153-244.
English translations:
Robert Norton, *Disme, the Art of Tenths, or Decimall Arithmetike. Invented by Simon Stevin.* — London 1608.
Henry Lyte, *The Art of tens, or decimall arithmeticke.* — London 1619.
Vera Sanford, *The Disme of Simon Stevin* — The Mathematics Teacher 14 (1921) 321—333.

V. *L'Arithmetique de* SIMON STEVIN *de Bruges: Contenant les computations des nombres arithmetiques ou vulgaires: Aussi l'Algebre, avec les equations de cinq quantitez. Ensemble les quatre premiers livres d'Algebre de* DIOPHANTE *d'Alexandrie, maintenant premierement traduicts en François.*

*Encore un livre particulier de la Pratique d'Arithmetique, Contenant entre autres, Les Tables d'Interest, La Disme; Et un traicté des Incommensurables grandeurs: Avec l'Explication du Dixiesme Livre d'*EUCLIDE.
— A Leyde, De l'Imprimerie de Christophle Plantin. 1585. 642 + 203 pp. Vignette B.
[3, 5, 6, I, III, V, VI]
Reprint, augmented and corrected by Albert Girard. Leiden 1625. [9, I, IV, VI]. This edition contains also Work VIII and a translation of the books V and VI of Diophantus by Albert Girard. Reprint of this edition in XIII.

VI. *De Beghinselen der Weeghconst beschreven duer* SIMON STEVIN *van Brugghe.* — Tot Leyden, In de Druckerye van Christoffel Plantijn. By Françoys van Raphelingen. 1586. 34 + 95 pp. Vignette D.
[1, 2, 3, 4, 6, 7, 9, 10, 15, I, III, IV, V, VI]

VIa. *De Weeghdaet beschreven duer* SIMON STEVIN *van Brugghe.* Title-page as in VI.

VIb. *De Beghinselen des Waterwichts beschreven duer* SIMON STEVIN *van Brugghe.* Title-page as in VI.
The works VI, VIa and VIb, which are always found bound together, are reprinted in XI. A Latin translation is contained in XIb, a French one in XIII.
Partial English translation of VIb by A. Barry in J. H. B. and A. G. H. Spiers, *The Physical Treatises of Pascal.* — New York 1937. 133—158.

VII. *Vita Politica, Het Burgherlick Leven, beschreven duer* SIMON STEVIN — Tot Leyden, By Franchoys van Ravelenghien. 1590. 56 pp. Vignette A.
[5, 6, I, II]
Reprints:
Delft 1611 [1, 2, 6, I, II, IV, VI]; Amsterdam 1646 [1, 4, I]; In XIV, with Appendix; Haarlem 1649 [1]; Middelburg 1658; Harlingen 1668 [V]; Amsterdam 1684 [2].
Amsterdam 1939 (with an introduction by A. Romein-Verschoor and G. S. Overdiep).

VIII. *Appendice Algebraique, de* SIMON STEVIN *de Bruges, contenant regle generale de toutes equations.* 1594.
(Leiden, Frans van Ravelingen). 6 pp.

The only extant copy of this booklet, which was the property of the Library of the University at Louvain, was lost when the library was burnt in 1914. The contents appear as a corollary to Prop. LXXVII of *L'Arithmétique* in the edition of 1625 and its reprint.

IX. *De Stercktenbouwing, beschreven door* SIMON STEVIN *van Brugge.* — Tot Leyden, By Françoys van Ravelenghien. 1594. 91 pp. Vignette C.
[2, 6, I, VI]
Reprint: Amsterdam 1624 [6, 10, I]
German translation:

Festung-Bawung. Das ist, kurtze und eygentliche Beschreibung, wie man Festungen bawen, und sich wider allen gewaltsamen Anlauff der Feinde zu Kriegszeiten auffhalten sichern und verwahren möge: Auff jetziger Zeit Zustand und Gelegenheit gerichtet, und auss Niderländischer Verzeichnusz SIMONIS STEVINI *Brugensis, Unserm geliebten Vatterland Teutscher Nation zu besondern Nutzen in hochteutscher Sprach beschrieben durch* Gothardum Arthus von Dantzig. — Getruckt zu Frankfort am Mayn, durch Wolffgang Richtern, In Verlegung Levini Hulsii Wittib. 1608.
8 + 132 pp. [2, II].
Reprint of this translation: Frankfort am Main 1623.
French translation in XIII.

X. *De Havenvinding.* — Tot Leyden, In de druckerye van Plantijn, By Christoffel van Ravelenghien, Gesworen drucker der Universiteyt tot Leyden. 1599. Vignette E.
[2, 4]
Reprint: a shortened version in XI; i, 25
Latin translations:
LIMENEVPETIKH, *sive, Portuum investigandorum ratio. Metaphraste* Hug. Grotio Batavo. — Ex Officina Plantiniana. Apud Christophorum Raphelengium, Academiae Lugduno-Batavae Typographum. 1599.
21 pp. Vignette F. [2, 6]
Limenheuretica; in XIb, translation of the version XI; i, 25 likewise by Grotius.
French translation:
Le Trouve-Port; in XIII.
English translation by Edward Wright: *The Haven-finding Art.* — London 1599. Inserted in the translator's work: *Errors in navigation detected.* — London 1657.
Partial reprint of X in *Rara Magnetica. Neudrucke von Schriften und Karten über Meteorologie und Erdmagnetismus.* No. 10. — Berlin 1898.

XI. *Wisconstighe Ghedachtenissen, inhoudende t'ghene daer hem in gheoeffent heeft den Doorluchtichsten Hoochgheboren Vorst ende Heere, Maurits, Prince van Oraengien, Grave van Nassau, Catzenellenbogen, Vianden, Moers &c. Marckgraef Van der Vere, ende Vlissinghen, &c. Heere der Stadt Grave ende S'landts van Cuyc, St. Vyt, Daesburgh &c. Gouverneur van Gelderlant, Hollant, Zeelant, Westvrieslant, Zutphen, Utrecht, Overyssel &c. Opperste Veltheer vande vereenichde Nederlanden, Admirael generael van der Zee &c. Beschreven duer* SIMON STEVIN *van Brugghe.*
Tot Leyden, In de Druckerye van Jan Bouwensz. Int Jaer 1608. Vignette D.
[1, 2, 3, 4, 6, 7, 8, 9, 11, 15, I, II, V, VI]
In folio with a very complicated division and pagination.
The principal division is into five parts:
i *Vant Weereltschrift*
ii *Van de Meetdaet*
iii *Van de Deursichtighe*
iv *Van de Weeghconst*
v *Van de Ghemengde Stoffen*

Part i. *Vant Weereltschrift*

i, 1 *Van den Driehouckhandel.*

i, 11 *Vant maecksel der tafels der Houckmaten.*
i, 12 *Van de platte driehoucken.*
i, 13 *Van de clootsche driehoucken.*
i, 14 *Van de hemelclootsche werckstucken duer rekeninghen der clootsche driehoucken ghewrocht.*

i, 2 *Vant Eertclootschrift.*

i, 21 *Van syn bepalinghen int ghemeen.*
i, 22 *Vant stofroersel des Eertcloots.*
i, 23 *Van de Eertclootsche Damphooghde.*
i, 24 *Van de Zeylstreken.*
i, 25 *Van de Havenvinding.*
i, 26 *Van de Spiegeling der Ebbenvloet.*
The treatises i, 24—26 constitute the *Zeeschrift.*

i, 3 *Van den Hemelloop.*

i, 31 *Van de vinding der Dwaelderloopen en der vaste sterren deur ervaringsdachtafels met stelling eens vasten Eertcloots.*
i, 32 *Van de Dwaelderloop deur wisconstighe wercking ghegront op de oneyghen stelling eens vasten Eertcloots.*
i, 33 *Van de vinding der Dwaelderloopen deur wisconstighe wercking ghegront op de wesentlicke stelling des roerenden Eertcloots.*

Part ii. *Van de Meetdaet*

ii, 1 *Van het teyckenen der grootheden.*
ii, 2 *Van het meten der grootheden.*
ii, 3 *Van de vier afcomsten, als vergaring, aftrecking, menichvuldiging en deeling der grootheden.*
ii, 4 *Van de everedenheytsreghel der grootheden.*
ii, 5 *Van de everedelicke snyding der grootheden.*
ii, 6 *Van 'tverkeeren der grootheden in ander formen.*

Part iii. *Van de Deursichtighe.*

iii, 1 *Van de Verschaeuwing.*
iii, 2 *Van de beghinselen der Spiegelschaeuwen.*
iii, 3 *Van de Wanschaeuwing.* (lacking)

Part iv. *Van de Weeghconst.*

iv, 1 *Van de beghinselen der Weeghconst.*
iv, 2 *Van de vinding der swaerheytsmiddelpunten.*
iv, 3 *Van de Weeghdaet.*

iv, 4 *Van de beginselen des Waterwichts.*
iv, 5 *Van den anvang der Waterwichtdaet.*
iv, 6 *Anhang der Weeghconst.*
iv, 7 *Byvough der Weeghconst.*

iv, 71 *Van het Tauwicht.*
iv, 72 *Vant Catrolwicht.*
iv, 73 *Van de Vlietende Topswaerheyt.*
iv, 74 *Van de Toomprang.*
iv, 75 *Van de Watertrecking.* (lacking)
iv, 76 *Vant Lochtwicht.* (lacking)

Part v. *Van de Ghemengde Stoffen.*

v, 1 *Van de Telconstighe Anteyckeningen.*
v, 2 *Van de Vorstelicke Bouckhouding in Domeine en Finance Extraordinaire.*

v, 21 *Coopmans Bouckhouding op de Italiaensche Wijse.*
v, 22 *Vorstelicke Bouckhouding op de Italiaensche Wijse.*
v, 221 *Bouckhouding in Domeine op de Italiaensche Wijse.*
v, 222 *Bouckhouding in Vorstelicke Dispense op de Italiaensche Wijse.*
v, 223 *Bouckhouding in Finance Extraordinaire op de Italiaensche Wijse.*

v, 3 *Van de Spiegheling der Singconst.* (lacking)
v, 4 *Van den Huysbou.* (lacking)
v, 5 *Van den Crijchshandel.* (lacking)
v, 6 *Van verscheyden Anteyckeningen.* (lacking)

XIa. *Memoires Mathematiques, contenant ce en quoy s'est exercé...... Maurice, Prince d'Orange...... descrit premierement en Bas Alleman par* SIMON STEVIN *de Bruges, translaté en François par* Jean Tuning, Licentié és Loix, & Secretaire de Monseigneur le Prince Henry, Comte de Nassau &c. — A Leyde, Chez Jan Paedts Iacobsz. Marchand Libraire, & Maistre Imprimeur de l'Université de la dite Ville. L'An 1608. Vignette: An angel with a book and a scythe.

[6, I]

French translation of XI, except the works i, 2; i, 3; ii, 5; ii, 6; iv.

XIb. *Hypomnemata Mathematica, hoc est eruditus ille pulvis, in quo se exercuit...... Mauritius Princeps Auraicus...... SIMONE STEVINO conscripta & è Belgico in Latinum à* Wil.Sn. *conversa.* — Lugduni Batavorum, Ex Officina Ioannis Patii, Academiae Typographi. Anno 1608. Vignette D.

[4, 5, 6, I, II, VI]

Complete Latin translation of XI by Willebrord Snellius.

XII. A. *Castrametatio, Dat is legermeting, Beschreven door* SIMON STEVIN *van Brugghe. Na d'oordening en 't ghebruyc van...... Maurits, Prince van Oraengien......* — Tot Rotterdam, By Jan van Waesberghe, in de Fame. Anno 1617. Legend: Literae immortalitatem pariunt. Vignette: Fame. 4 + 55 pp.

B. *Nieuwe Maniere van Sterctebou, door Spilsluysen. Beschreven door* SIMON STEVIN *van Brugghe.* 4 + 59 + 2 pp.
Title-page as above.
The two works are always found bound together.
[1, 2, 4, 6, 7, 9, 10, 11, 13, I, II, IV, V]
Reprint Leiden 1633. [4, 6, 9, V]
French translations:
La Castramétation. Nouvelle Maniere de Fortification par Escluses. Leiden 1618 [2, 4, 5, 6, 7, I, IV, V]
Idem. Rotterdam 1618. [2, 4, 10, 13, III]
Reprint of this translation in XIII.
German translations:
Castrametatio Auraico-Nassovica, das ist: Gründtlicher und auszführlicher Bericht, welcher Gestalt ein vollkommenes Feldtläger abzumessen...... seye: Erstlich in Niderländischer Sprach beschrieben durch Simonem Stevinum: *Anjetzo aber durch einen Liebhaber ins Hoch Teutsch übersetzt.* Franckfurt, Frid. Hulsii. 1631.
Wasser-Baw, das ist Eygentlicher und vollkommener Bericht von Befestiging der Stätte durch Spindel-Schleussen. — Frankfurt, in Verlegung Friderici Hulsii. Im Jahr 1631.
[2, I]

POSTHUMOUS EDITIONS

XIII. *Les Oeuvres Mathematiques de* SIMON STEVIN *de Bruges. Ou sont insérées les Memoires Mathematiques Esquelles s'est exercé le Tres-Haut & Tres-illustre Prince Maurice de Nassau, Prince d'Aurenge, Gouverneur des Provinces des Pais-bas unis, General par Mer & par Terre, &c. Le tout reveu, corrigé, & augmenté par* Albert Girard *Samielois, Mathematicien.*
— A Leyde Chez Bonaventure & Abraham Elzevier, Imprimeurs ordinaires de l'Université, Anno 1634. Vignette: Le solitare. Legend: Non Solus.
[2, 3, 4, 5, 6, 7, 8, 9, 11, I, II, III, V]
The work contains six parts.
i. *l'Arithmétique* (reprint of the edition of 1625).
ii. *Cosmographie.*
ii, 1 *Doctrine des Triangles* (translation of XI; i, 1 with corrections and additions by A. Girard; the tables have been omitted).
ii, 2 *Geographie* (translation of XI; i, 2).
ii, 3 *Astronomie* (translation of XI; i, 3).
iii. *La Practique de Geometrie* (translation of XI; ii).

iv. *L'Art Pondéraire ou La Statique* (translation of XI, iv).
v. *L'Optique* (translation of XI; iii).
vi, 1 *La Castramétation* (reprint of the translation of XIIA, 1618).
vi, 2 *La Fortification par Escluses* (idem of XIIB).
vi, 3 *La Fortification* (translation of IX by A. Girard).

XIV A. *Materiae Politicae. Burgherlicke Stoffen. Vervanghende Ghedachtenissen der Oeffeninghen des Doorluchtichsten Hoogstgheboren Heere Maurits by Gods Genade Prince van Oraengie &c. Ho:LO: Ghedachtenisse. Beschreven deur zal.* SIMON STEVIN *van Brugghe, desselfs Heeren Princen Superintendent van de Finance &c. En uyt sijn naghelaten Hantschriften bij een ghestelt deur Sijn Soon* HENDRICK STEVIN *Ambachtsheere van Alphen.*
Tot Leyden, Ter Druckerye van Iustus Livius, tegen over d'Academie. Vignette D.

B. *Verrechting van Domeine mette Contrerolle en ander behouften van dien. 't Welck is Verclaring van ghemeene Regel, waer deur verhoet worden alle abuysen mette swarichheden uytte selve spruytende, die men tot noch toe uyt geen Rekencamers van Domeine en Finance heeft connen weren. Wesende Oeffeningen &c.* as above. — Tot Leyden, Ter Druckerye van Iustus Livius, In 't tweede Iaer des Vredes. Vignette D.

[1, 2, 4, 6, 7, 8, I, II]

Reprint 1660, preceded by
Loochening van een Ewich Roersel, gesecht Perpetuum Mobile, by HENDRIK STEVIN.

A contains eight memoirs on administrative and military matters.
i *Van de oirdening der steden. Van de oirdening der deelen eens huys met 't gheene daer ancleeft.*
ii *Het Burgherlick Leven, vermeerdert met een Anhang van de Regiering des Vorsten, tegen Machiavel. Mitsgaders des Keysers Octaviaens gevoelen en ander getuygenissen angaende Phalaris.* (Augmented reprint of VII).
iii *Van der Raden oirden.*
iv *Van de amptlienkiesing en ghemeene anclevinghen der ampten.*
v *Ghemeene Regel op Gesantterie.*
vi *Van de Verdrucking.*
vii *Van de geduerige verlegghing des Crijchsvolcx.*
viii *Van de Crijchspiegeling.*

In B parts of XI; v have been reprinted.
Reprint of XIV: 's-Gravenhage 1686.

C. In some copies of XIV one finds a list of titles of treatises, which were destined for XI, but were not inserted in this work:
i *Van de Crijchconst.*

1 *Van de Crijch te Lande.*
2 *Van de Crijch te Water.*
Parts of this work were inserted in XIVA; iii, iv, viii and in XIIA.

ii *Van den Huysbau... waerby noch gevoucht is Weechdadelicken Handel van Cammen en Staven in Watermolens en Cleytrecking.*
Parts of this work were inserted in XIVA and XVIB.

iii *Spiegeling der Singconst. Byvough der Singconst.*
Published in XV.

iv *Van de tweede oneventheyt na myn gevoelen.*
Supplement to XI; i, 3.

v *Van de metael-prouf.*

vi *Van ettelicke wisconstige voorstellen en aenteyckeningen.*

vii *Nederduytsche Dialectica dats Bewysconst, anders geseyt Redenstryt.*
New Version of III.

viii *Nederduytsche Retorica dats Redenconst, anders geseyt Welsprekenheyt.*
1 *Van de eygenheyt des spraecx.*
2 *Van duysterheyt en claerheyt.*
3 *Van d'oirden des uytspraecx.*
4 *Vant cieraet.*
5 *Vant wesen.*

ix *Nederduytsche Dichtconst ghegront op de Françoysche Dichtconst, die daerom eerst beschreven wort. En hier is by gevoucht een verhael van Letterconstige geschillen.*
1 *Van de spelling.*
2 *Vant geslacht der namen.*
3 *Op seker* E, EN *en* DER.
4 *Van de buyging en vervouging.*

XV. *„Van de Spiegeling der Singconst" et „Van de Molens"*. Deux traités inédits. Réimpression par Dr. D. Bierens de Haan. L.L.D. — Amsterdam 1884.

FRAGMENTS IN WORKS OF OTHERS

XVI A. *Journal tenu par Isaac Beeckman de 1604 à 1634 publié avec une introduction et des notes par* C. de Waard. Tome II. — La Haye 1942.
Appendice I (394—438) contains fragments on the following subjects:
1 *Huysbou.*
2 *Spiegeling der Singconst.*
3 *Cammen ende Staven, Watermolens ende Cleytrecking.*
4 *Waterschueringh.*
5 *Van de Crijchconst.*

B. *Wisconstich Filosofisch Bedryf, van HENDRIC STEVIN, Heer van Alphen, van Schrevelsrecht, &c. Begrepen in veertien Boeken.*

— Tot Leyden, Gedruct by Philips de Cro-y, in 't Jaer 1667.
Plaetboec. Vervangende de figuren of formen gehorig tottet Wisconstich Filosofisch Bedryf van HENDRIC STEVIN, *Heer van Alphen, van Schrevelsrecht &c.*
Gedruct in 't Jaer 1668.

1	Boek VI, Prop. 2	*Van den handel der cammen en staven onses Vaders als bewegende oirsaec van dese.*
2	Boek X	*Van den handel der Watermolens onses Vaders* SIMON STEVIN.
3	Boek XI	*Van den handel der Waterschuyring onses Vaders SIMON STEVIN.*

DE BEGHINSELEN DER WEEGHCONST

THE ELEMENTS OF THE ART OF WEIGHING

INTRODUCTION

§ 1. HISTORICAL INTRODUCTION

Just as all other branches of mathematics and natural science, theoretical mechanics is rooted in Greek antiquity. Its roots are twofold, and of quite different origin. They are associated with the names of two great ancient thinkers, Aristotle and Archimedes.

In the former's Work, *Mechanica Problemata,* the statical problem of the equilibrium of a balance is dealt with from a dynamical point of view, a seemingly paradoxical idea, which was, however, to prove extremely fruitful. Archimedes on the other hand treated mechanics as a branch of mathematics; modelling himself on Euclid's foundation of geometry, he formulated a number of axioms on statics from which, with the aid of certain implicit suppositions concerning the theory of the centre of gravity, he logically derived the fundamental rule for the equilibrium of a lever. With him, statics came to be an autonomous science.

In the Middle Ages only the former of these two methods was applied. In the school which is named after Jordanus Nemorarius it developed from its original form, which may be characterized as a germ of the principle of virtual velocities, into that of virtual displacements. This was applied not only in the theory of the lever, but also for the derivation of the law of the inclined plane. In the 16th century this current was continued by the Italian scholars Tartaglia and Cardano [1]).

In the forties of that century the Archimedean approach became known through the publication of his works. The Italian mathematicians Commandino, Maurolyco, Guido Ubaldo del Monte, Benedetti, and Luca Valerio followed this method. However, they mainly made use of it for the determination of centres of gravity, thus enriching the science of Statics with new results, without finding new possibilities for its further development.

It is the abiding merit of Stevin that he did find these new possibilities. He not only studied and supplemented the work of Archimedes, but he also continued it.

In accepting the Archimedean method he radically rejects that of Aristotle and Jordanus. He thinks it perfectly absurd to derive a condition of equilibrium from the consideration of a situation which cannot possibly present itself as long as the state of equilibrium continues, viz. the simultaneous displacements of the balancing bodies. This severe and rather unjust criticism, however, does not prevent him from appealing sometimes to rules proper to the theory condemned by him.

In accordance with the prevalent custom of his day, Stevin as a rule quotes other authors only in order to dispute their opinions, and carefully conceals any sources from which he has drawn. This makes it extremely difficult to appreciate the degree of his originality. However, as long as no other works besides the Archimedean treatise *On the Equilibrium of Planes* have come to light in which his particular treatment of the subject is anticipated, we may credit him

[1]) For more detailed information on this period of the history of Mechanics the reader may consult: P. Duhem, *Les origines de la statique,* 2 vol., Paris, 1905-06.

with the merit of having been the first to take over and pass on the torch lit by the great Syracusan.

§ 2. SUMMARY OF THE WORK

We now proceed to give a summary of the contents of the *Art of Weighing* in present-day terminology.

In Part I of Book I the premisses of the *Art of Weighing* are set forth in 14 Definitions and 5 Postulates. In the same way as Archimedes had done, Stevin presupposes a theory of the centre of gravity without specifying its logical foundations and its results. His definition of centre of gravity (Def. 4) is substantially identical with that given by Pappus in his *Collectio Mathematica* [2]): a point such that if the solid is conceived to be suspended from it, the solid remains at rest in any position given to it. It is assumed that any body has such a point, and only one such point. In a Note to Postulate 5, Stevin is seen to be aware of the fact that this supposition is valid only if the verticals through the different points of the body are considered to be parallel.

In Definition II the fundamental concept of *evenstaltwichtigheid* is introduced. Bodies balancing one another at unequal arms of a lever are not really *evenwichtig* (of equal weight), but they only appear to be so. Stevin therefore carefully distinguishes between *evenwicht* or *evenwichtigheid* (equality of weight) and *evenstaltwichtigheid*. Since the first term has become current for denoting the second concept, whereas the term proposed by Stevin never penetrated into scientific terminology, it is rather difficult to translate the latter and its derivatives. Neither Snellius nor Girard succeeded in finding a Latin or French equivalent of one word; bodies called *evenstaltwichtig* by Stevin are designated by the former as *ex situ equilibria* [3]), by the latter as *équilibres selon leur disposition* [4]). Starting from Stevin's explanation that bodies balancing one another have a *ghelaet* (appearance) *van evenwichticheyt* (Def. II, Explanation), we translate *evenstaltwichtig* by "of equal apparent weight" and *evenstaltwichtigheid* by "apparent equality of weight" or "equality of apparent weight". It will be seen that Stevin not only speaks of bodies of equal apparent weight, but also attributes to a body a certain *staltwicht* depending on the circumstances (Prop. 19). This term, which is nowhere defined explicitly, was translated by Snellius by *sacoma* [5]) (from Greek Σήκωμα = weight) and by Girard by *puissance or pouvoir* [6]). In accordance with the above we render it by "apparent weight", though this is not a current term in modern physics. It would, however, be impossible to find a modern equivalent, since the concept which Stevin denoted by the word "staltwicht" had not yet taken a definite form in his mind. It may denote the moment of a force with regard to a point, but also the component of a force along a line, while in other cases the meaning is not quite clear. It should be remembered that in the 16th century the science of mechanics was only just coming into being,

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2) *Pappi Collectionis Mathematicae quae supersunt,* ed. Hultsch, 3 vols, Berlin, 1875-78. VIII 5; III 1030.

3) XIb. *De Staticis Elementis* 8.

4) XIII 435 b.

5) XIb. *De Staticis Elementis* 34.

6) XIII 448a. The current term in the Middle Ages is *gravitas secundum situm.* Tartaglia has *gravita secondo el luoco over sito. Quesiti et Inventioni diverse.* VIII Def. 13. Venetiis, 1546.

and that it was to take a couple of centuries before its fundamental conceptions could be fixed with a reasonable degree of exactitude.

Part II of Book I contains 27 propositions, which may be divided into two groups.

The first group (Prop. 1—18) refers to the theory of the lever and its applications, the second (Prop. 19—27) to the theorem of the inclined plane and its consequences.

Group I. The first of this group is to be found in Prop. 1, which contains a mathematical demonstration of the state of equilibrium of a straight lever. Since the method is closely akin to that of Archimedes *(Equilibrium of Planes,* Prop. 6), we first summarize the latter in a modern form. It is to be proved that, if two weights are suspended from a horizontal lever at distances from the fulcrum which are inversely proportional to the weights, the lever will be in equilibrium. Translating Archimedes' argument into modern symbols, we may say that he supposes the weights G_1 and G_2 to be suspended from a lever with fulcrum O at the points A_1 and A_2 respectively. Putting $OA_1 = l_1$, and $OA_2 = l_2$, he supposes

$$G_1 : G_2 = l_2 : l_1.$$

He now puts: $G_1 = n_1.G$

$G_2 = n_2.G$ (n_1 and n_2 are integers; G is a common measure of G_1 and G_2)

and accordingly

$$l_1 = n_2.\ l$$
$$l_2 = n_1.\ l.$$

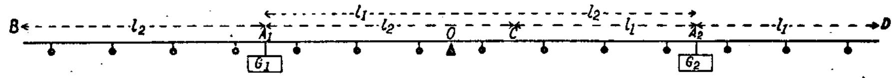
He now makes $A_1B = A_1C = l_2$

$A_2D = A_2C$,

which implies

$$A_2C = A_2D = l_1.$$

He now replaces the weight G_1 by $2n_1$ weights $G/2$ hanging in the middle points of the $2n_1$ segments l into which BC can be divided, and likewise G_2 by $2n_2$ weights $G/2$ in the middle points of the $2n_2$ parts of CD. Since $OB = OD$, the distribution of weights is now symmetrical with respect to O, and the lever is therefore in equilibrium (the validity of this inference has been granted by Postulate I).



The case that G_1 and G_2, and consequently l_1 and l_2 are incommensurable requires a separate demonstration, which is given by *reductio ad absurdum.*

It is to be noted that Archimedes also makes use of the converse theorem: if the lever is in equilibrium when the weights G_1 and G_2 are hanging from it at distances l_1 and l_2 from the fulcrum, then $G_1 : G_2 = l_2 : l_1$, but he does not prove this.

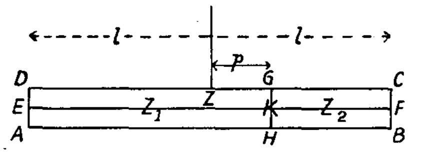
Stevin now modifies the Archimedean demonstration in two respects:

1) Starting with a symmetrical distribution of the weight along the lever, he proves the converse theorem.

2) By replacing the discrete magnitudes $G/2$ of Archimedes by continuous ones he eliminates the necessity of distinguishing between commensurable and incommensurable weights.

His demonstration may be translated into modern symbols as follows:

Let the rectangular parallelepiped $ABCD$ be suspended from a point in the vertical through its centre of gravity Z. $AB = 2\,l$. Cut the body by a plane GH at right angles to the axis EF and meeting this axis in K. $ZK = p$. The centres of gravity of the parts $AHGD$ and $BHGC$ are Z_1 and Z_2 respectively. Now suppose the weights G_1 and G_2 of these parts to be concentrated in their centres of gravity. Since



$$ZZ_1 = \frac{l - p}{2} \qquad ZZ_2 = \frac{l + p}{2}$$

and

$$G_1 : G_2 = (l + p) : (l - p)$$

it follows that

$$G_1 : G_2 = ZZ_2 : ZZ_1$$

so the weights are inversely proportional to the arms.

The converse theorem (which is the original one of Archimedes) is enunciated, but not proved.

Obviously Stevin's demonstration is open to the same objection as was raised by E. Mach [7]) against the proof given by Archimedes: the argument is based on the assumption that an existing state of equilibrium of the lever will not be disturbed, if a weight hanging at a given point is so distributed along the lever that the centre of gravity of this distribution remains in the original position or, conversely, if a body attached to a lever is replaced by its weight acting at its centre of gravity. This, however, is by no means evident. In the simplest case, in which a weight G at a distance l from the fulcrum is replaced by two weights $G/2$ at distances $l \pm a$, the assumption amounts to the functional equation.

$$f(G,l) = f(G/2, l - a) + f(G/2, l + a)$$

where $f(G,l)$ denotes the influence exerted on the lever by a weight G at the distance l from O, while moreover it is assumed that the influences of two separate weights can be combined by addition. Now it is clear that if the form of $f(G, l)$ were, for example, $G.l^2$ instead of $G.l$, the equality would not hold. The problem, however, consists in determining the form of the function $f(G, l)$, and the assumption is therefore unwarranted.

It has been urged [8]) against Mach's argument that at all events Archimedes did make this assumption explicitly in Postulate 6 of *Equilibrium of Planes,* which states that any body suspended from the lever may be replaced by any other having the same weight and hanging at the same place. It is then contended that this body may also consist of a number of bodies, the common centre of gravity of which is in the vertical through the original point of suspension. However, it is doubtful in the first place whether this is the real meaning of the text, and secondly whether it is permissible to make so far-reaching a statement in the form of a postulate. In any case this justification of Archimedes' procedure — if it be one — does not apply to that of Stevin, who has no such postulate.

After having satisfied himself as to the truth of the first proposition, Stevin

[7]) E. Mach, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt.* Leipzig, 1912, p. 14.

[8]) W. Stein, *Der Begriff des Schwerpunktes bei Archimedes.* Quellen und Studien zur Geschichte der Mathematik, der Astronomie und der Naturwissenschaften, Studien B I (1931) 221.

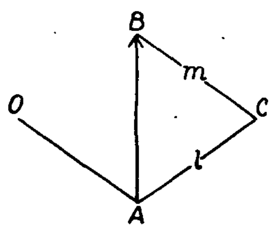
finds no difficulty in solving a number of simple statical problems: given two weights balancing one another at a lever, to determine the fulcrum (Prop. 2); given one of said weights and the fulcrum, to find the other (Prop. 3); given two weights and one of the arms, to find the other arm (Prop. 4); given a prism, to determine a weight in a given ratio to the weight of the prism by means of statics (Prop. 5).

The propositions 6—8 deal with the difference between stable, indifferent, and unstable equilibrium (without, however, making use of these terms). By way of introduction to the following problems, Prop. 9 states that if two weights are hanging in equilibrium from a lever at right angles to their lines of action, this lever may be replaced by another inclined to these lines, all the fulcrums remaining in the original vertical lines. After this it has become possible to solve various problems: to determine whether the equilibrium of a prism from which two weights are hanging so as to balance one another is stable, indifferent or unstable (Prop. 10); to determine the common centre of gravity of a prism and certain weights attached to it (Prop. 11); to find the weight which should be attached at a given point in a given prism loaded with known weights in order to keep the prism in a given position (Prop. 12).

Hitherto the prism has been subjected to forces directed vertically downwards only. Proposition 13 now introduces upward vertical forces, and shows how to replace them by equivalent downward forces. Proposition 14 shows that a body with one point fixed in the axis may be kept in equilibrium by an upward force acting at another given point on the axis. The magnitude of this force is determined (Prop. 15) and shown to be independent of the position of the body (Prop. 16). Proposition 17 shows how the weight of the prism is distributed between two points of support, both in the axis; the same problem is solved in Prop. 18 for the case that the two points are arbitrarily chosen.

Group II. Proposition 19 contains the famous demonstration of the so-called law of the inclined plane by means of the *"clootcrans"* (wreath of spheres). Since the argument is perfectly clear, it does not seem necessary to reproduce it here in a modern form; critical remarks about its validity and about the corollaries will be given in the notes.

It is remarkable that in the *Weeghconst* the inclined plane is not considered at all as a mechanical instrument; this will only be done in the *Weeghdaet.* In the *Weeghconst* it is used as a lemma for a theory of the equilibrium of a body with one fixed point; the transition is brought about by considering the point in which the body rests on the inclined plane as fixed, and omitting the plane. The main contents of the following propositions may be summarized as follows: given a rigid body, one point O of which has been fixed, the body is to be held in equilibrium by a force acting along a given line l in the vertical plane through both the fixed point and the centre of gravity of the body; to determine the magnitude of this force. Let the vertical force at A which keeps the body in equilibrium be represented by AB; this force has been determined in Prop. 14. If now the line m is drawn through B parallel to OA to meet l in C, then AC will represent the required force. The truth of this is evident to us, the forces AB and AC having equal statical moments about O.



This fundamental theorem is proved in Prop. 20 for upward, and in Prop. 21 for downward forces. In Prop. 23 it is shown that the value of *AC* is the same for the two positions of *AC* which make equal angles with *OA*. In Prop. 24 the minimum value of *AC* is found to be perpendicular to *OA*.

In the remaining propositions, no point of the body is supposed to be fixed; it is now suspended from two lines. It is proved that if these lines are non-parallel, they will meet in the vertical through the centre of gravity (Prop. 25); further that either both lines must be vertical or neither of them, and finally that in the latter case they incline one to the right and the other to the left of the vertical (Prop. 26). By considering as fixed either of the points of the body at which the lines are attached, it is proved that the fundamental proposition holds. Finally, in Prop. 28 the prisms hitherto considered are replaced by bodies of arbitrary form.

Book II deals with the determination of centres of gravity a) in plane figures (Prop. 1—13); b) in solids (Prop. 14—24). In an introductory note to the first group it is observed that such terms as weight, centre of gravity, etc. with reference to plane geometrical figures are to be understood metaphorically; a similar note to group b), referring to geometrical solids, is, however, lacking.

Prop. 1 shows, with reference to some examples, that if a plane figure has a geometrical centre, this point is at the same time its centre of gravity. It is then proved that the centre of gravity of a triangle is in a median (Prop. 2), from which follows its determination as the point of intersection of two medians (Prop. 3) and the ratio of the segments into which it divides a median (Prop. 4). Prop. 5 amounts to no more than a simple corollary to the preceding theorem. It is then shown how to determine the centre of gravity of a plane polygon (Prop. 6), of a trapezium (Prop. 7, 8), and of the remainder of a plane figure after removal of a given part (Prop. 9). The propositions 10—12 deal with the centre of gravity of a parabolic segment: this is proved to be in the diameter (Prop. 10), and to divide it in a ratio which is the same for any parabola (Prop. 11), this ratio (3 : 2) being determined in Prop. 12. Finally, in Prop. 13 the centre of gravity of a portion of a parabolic segment, cut off by a line parallel to the base, is determined.

In group b), Prop. 14 repeats Prop. 1 for solids. The centre of gravity of a prism is determined in Prop. 15. The propositions 16—18 (centre of gravity of a pyramid) correspond to Prop. 2—4 for the triangle, Prop. 19 to Prop. 9. In Prop. 20 the centre of gravity of a truncated pyramid is found, in Prop. 21 that of any solid. Prop. 22—23 deal with the centre of gravity of a segment of a paraboloid of revolution, Prop. 24 with that of a truncated segment.

Stevin's method of dealing with centres of gravity is substantially identical with that introduced by Archimedes in his work *On the Equilibrium of Planes* [9]) and subsequently used by Federigo Commandino in his *De centro gravitatis solidorum* [10]). It amounts to approximating a plane or solid figure, the centre of gravity of which has to be found, by means of a series of inscribed polygonal (or polyhedral) figures with known centres of gravity, and then determining the limiting position of the latter when the number of the sides (or faces respectively) of the inscribed figure is indefinitely increased. Whereas,

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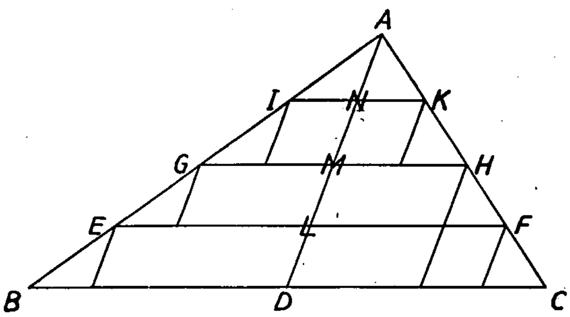
[9]) *Archimedes Opera Omnia,* ed. J. L. Heiberg. Vol. II, 124-213. Leipzig, 1913.
[10]) Federici Commandini *Liber de centro gravitatis solidorum.* Bononiae, 1555.

however, Archimedes and Commandino felt obliged to prove on each fresh occasion the correctness of the result obtained by the *reductio ad absurdum,* which is characteristic of the ancient method of treating infinite processes, Stevin makes use of the general consideration that two quantities, the difference between which can be proved to be less than any assigned magnitude, are equal to one another.

The working of the method may be illustrated by the following rendering of Prop. 2, in which it is proved that the centre of gravity of a triangle is in a median.



Let ABC be a triangle, D the middle point of BC. The median AD is divided into n equal segments (in the drawing $n = 4$). Through the points of division L, M, N are drawn lines parallel to BC, intersecting AB in E, G, I respectively and AC in F, H, K respectively. Through these points on AB and AC are drawn lines parallel to AD which, in a manner sufficiently clear from the drawing produce a figure Π_n consisting of $(n - 1)$ parallelograms. It is easily seen that the difference between the area of Π_n and that of the triangle ABC (Δ) is $\frac{\Delta}{n}$ and can therefore, through the choice of n, be made less than any assigned area.

By applying Prop. 1 it is seen that the centre of gravity of Π_n is in AD, so that, if Π_n be suspended from A, the line AD will be vertical, and the two parts into which the median AD divides Π_n will balance one another, or, in Stevin's terminology, will have the same "staltwicht" (apparent weight). It is now contended that the "staltwichten" of ADB and ADC differ less from one another than any given quantity, from which it is inferred that they are equal to one another. The final conclusion that the centre of gravity of Δ ABC is in AD is then easily reached.

The decisive point of the demonstration obviously consists in the contention relating to the "staltwichten" of the parts ADB and ADC; this is based on the preceding observation that the areas of Π_n and Δ ABC can be made to differ less than any given quantity. It is, however, to be doubted whether this is a sufficient reason. It is indeed obvious that the areas (and therefore the weights) of these two triangles can, through the choice of n, be made to differ less than any assigned quantity from the areas (and weights respectively) of the two parts into which Π_n is divided by AD, but the transition from this statement to that on "staltwichten" seems unwarranted, since it has not been shown anywhere how the "staltwicht" of a part of the figure relatively to AD is to be determined. At the back of Stevin's mind there seems to be a consideration, which may be expressed in modern terms by saying that the "staltwicht" of a part of the figure relatively to AD is a continuous function of the variables on which it depends (viz. area and distance of the centre of gravity from AD).

The method used by Stevin in Prop. 2 and repeated without any substantial modification in the propositions 10, 15, 16, 18, 22 forms an important contribution to the historical development of the treatment of infinite processes. As far as we know, he was the first to emancipate himself from the obligation to give each time again a proof by *reductio ad absurdum,* which the rigorism of the great Greek predecessors still imposed on mathematicians, and thus to pave the way for the newer and simpler methods which the Calculus was to provide. It is, however, characteristic of the powerful influence exerted by the Greek tradition even on those who strove to grow independent of it that Stevin cannot yet bring himself to formulate his innovation in the form of a general proposition to be applied in each individual case presenting itself, but repeats it *in extenso* each time again, just as Archimedes had done with the *reductio ad absurdum.* Just as in important passages of Book I, he clothes his reasoning in the classical form of a syllogism, the mood of which is *Baroko;* so, for example, in Prop. 2 of Book II which has been rendered above, he argues as follows:

A. Beside any different "staltwichten" there may be placed a gravity less than their difference.

O. Beside the present "staltwichten" *ADC* and *ADB* there cannot be placed any gravity less than their difference.

O. Therefore the present "staltwichten" *ADC* and *ADB* do not differ.

The fact that he uses this form of exposition is undoubtedly due to his desire to stress the importance of his innovation. He certainly had a right to do so: 16th century mathematics had profited immensely by the Greek source of knowledge, but it could not develop beyond the ancient boundaries unless it succeeded in emancipating itself from the burdensome Greek style of demonstration, even if this movement were to result — which indeed it did — in a temporary decline of mathematical rigour.

Stevin deserves to get credit for his clear insight into this necessity, and for the resolution with which he took the new road. It is yet another manifestation of his strong desire to make mathematics a practical tool for the investigation of nature, fit to be handled by all clear-minded people.

The immediate result was an enormous simplification of the treatment of centres of gravity. This becomes clear at once when his work is compared with that of Commandino, his only predecessor in this field [11]) besides Archimedes. The modern reader, impatient at Stevin's prolixity, need only compare his Prop. 23 on the centre of gravity of a segment of a paraboloid with the corresponding Prop. 29 in Commandino's *Liber de centro gravitatis solidorum* to see what remarkable progress Stevin had made.

§ 3. DISCOURSE ON THE WORTH OF THE DUTCH LANGUAGE [12])

In § 4 of the biographical introduction to this edition we mentioned Stevin's considerable influence on the development of the Dutch language. That which

[11]) Commandino expressly states in the preface to his work that he is the first to write on centres of gravity of solids. He cannot indeed believe that no one should have dealt with the subject before him, seeing that Archimedes in his work *On Floating Bodies* considers the position of the centre of gravity of a paraboloid a thing of common knowledge. He has not, however, succeeded in tracing any treatise about it.

[12]) We owe the following discussion of Stevin's philological ideas to Prof. Dr C. G. N. de Vooys, Former Professor of Dutch Language and Literature at the University of Utrecht.

now follows is the most important of the passages in which he expounds his linguistic ideas, the *Uytspraeck van de Weerdicheyt der Duytsche* [13]) *Tael* (Discourse on the Worth of the Dutch Language) [14]).

Sympathetic consideration and great esteem of the vernacular is an international feature of the 16th century. Italy took the lead, followed by France, which in turn stimulated the movement in the Netherlands and in Germany. In order to understand Stimon Stevin's curious linguistic views it is necessary to have regard to this historical background. On a first view it appears strange that the same Renaissance artists and humanists who were such fervent admirers of classical Latin should also have advocated the elaboration and the use of the vernacular. That this inconsistency is only apparent has been shown very clearly by F. Brunot in his *Histoire de la langue française* II, 1 : *L'émancipation du français.*. The reversion to Ciceronian purity and the close imitation of the classical style rendered Latin useless as a living language. Mediaeval Latin, with its greater simplicity of structure and its capacity of adapting itself to every requirement, could no longer find favour with the Renaissance scholars. The consequences of this were inevitable: "On cherchait l'élégance; on perdit la commodité". It began to be realized that the only suitable medium for the dissemination of knowledge and art in wide circles was the vernacular, which, however, had to be made as effective as possible for the purpose in view [15]). Attention was therefore paid to an adequate systematization of spelling, to syntax, to purification from foreign elements, and to extension of the vocabulary by new word formations.

The new appreciation of the vernacular sometimes resulted in overestimation. Thus the Antwerp scholar, Johannes Goropius Becanus, in his *Origines Antwerpianae* (1569) believed he could prove Dutch to be the oldest language of the world; nay, he even held that this language had been spoken by the inhabitants of Paradise and their immediate descendants. The name *Adam* was none other but the Dutch word *adem* (breath), for had not God breathed into his nostrils at the Creation? *Noach* (Noah) was the man who "acht op de noot" (minds the distress), *Babel* had a very apt meaning, for *babelen* "est tam confuse et inarticulate loqui, ut non intelligatur" [16]). The very name of *Duyts* furnishes evidence in support of the theory, for it means *Douts,* i.e. "the oldest". Philology

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[13]) As has been remarked in Note 16 to the biographical introduction, *Duytsch* has to be translated by Dutch, not by German. Sometimes Stevin distinguishes the language of the *Overlanders* (Germans) as *Hoogduytsch* from the Neerduytsch spoken by the *Neerlanders.* He considers the former to be a variety of the latter, which is spoken in its purest form in the province of North Holland (cf. p. 46 below).

[14]) He had dealt with this subject before in the *Dialectikelicke Tsamespraeck* at the end of the work *Dialectike* (III), and he returns to it once more in the introduction to the *Waterwicht* (cf. p. 385 below), the *Stercktenbouwing* (IX, p. 87), and the *Spiegeling der Singconst* (XV or XVIA, p. 56-57). A second, somewhat modified version of the *Uytspraeck* is given in the *Wisconstighe Ghedachtenissen* (XI; i, 21, Bepaling 6), where it is supplemented with an elaborate exposition of Stevin's theory of the *Wijsentijt* (Age of the Sages) (cf. General Introduction, § 4). This passage is to be inserted in our Volume III.

[15]) Brunot's argument has been given more fully by the writer in *De Nieuwe Taalgids* 1917. This article was reprinted in *Verzamelde Taalkundige Opstellen* I, p. 255. Cf. also the introduction to K.W. de Groot's article on *Het purisme van Simon Stevin* (Simon Stevin's Purism) in *De Nieuwe Taalgids* 1919.

[16]) Vide Dr. K. Kooiman: *Twespraack van de Nederduitsche Letterkunst* (1913) pp. 77 et seq.

of such kind seems rather naïve to us, moderns, but it is remarkable that among the writer's contemporaries men such as Coornhert and Spieghel took it seriously, and even had implicit confidence in the "irrefutable arguments".

Simon Stevin did not go as far as that: he does not go back to the language of Paradise, nor does he look for arguments in the Old Testament, but in his own way he assumes a certain evolution of linguistic history in order to account for the great antiquity of Dutch. Language as an invention of the human mind — which is to Stevin a miracle — presupposes an advanced stage of civilization. Consequently, in remote times, which he calls the "Wijsentijt" (Age of the Sages), the "Duytsen", i.e. the Germanic tribes in general and our ancestors, the Dutch, in particular, must have been a very powerful race with a highly developed culture. Through all sorts of causes this race must have fallen into a condition of barbarism, which lasted to the days of Julius Caesar. After that, another age of progress dawned, and the Germans grew more and more to be the masters of Europe. Stevin finds evidence for this in the fact that the Gauls — i.e. the French —, who conquered Southern Europe, must originally have spoken Dutch or at least held this language in great esteem. The Spaniards, too, have either been Dutch or have modelled their language on that of the Dutch.

Besides its venerable antiquity, Stevin also points out the inherent excellency of Dutch. His principal aim is to show that Dutch is more suitable for scientific purposes than any other language. This claim is based on four arguments:

1) Since the end of language consists in expressing thoughts by words, the more it is capable of denoting single things by monosyllables, the greater will its value be. By means of a statistical investigation Stevin proves Dutch to be superior in this respect to both Latin and Greek.

2) A second criterion for judging the merits of a language is the facility with which compound words are formed. In this respect, too, Dutch is found to excel.

3) The *Art of Weighing* contains examples of concise formulation of mechanical theorems not equalled in any other language.

4) The Dutch language possesses in a superlative degree the faculty which Stevin calls *beweeghlicheyt,* i.e. the power to move, the emotional appeal of which it is capable. In a most remarkable passage (cf. p. 87 below) this quality is illustrated by the strong influence exercised by religious orators on the people in the Low Countries.

A curious consequence of Stevin's views is that on account of its preference for monosyllables he considers the language of North Holland to be of older origin than his own native Western Flemish, whereas in reality it is the longer nouns ending in -e which are the earlier forms.

From a statement in some editions of *Materiae Politicae* (XIV) [17]) it appears that Stevin had also intended to write some treatises on rhetoric and poetry, in which he was to have laid down his views about moot points of grammar, in particular spelling. Since these treatises, if written at all, at any rate have not come down to us, we cannot make any statement about his views on these subjects. We only know of his aspirations after purification of the language and the creation of a practical, well-considered terminology, so that the vernacular might in due time be fully appreciated for its services to the nation and to science.

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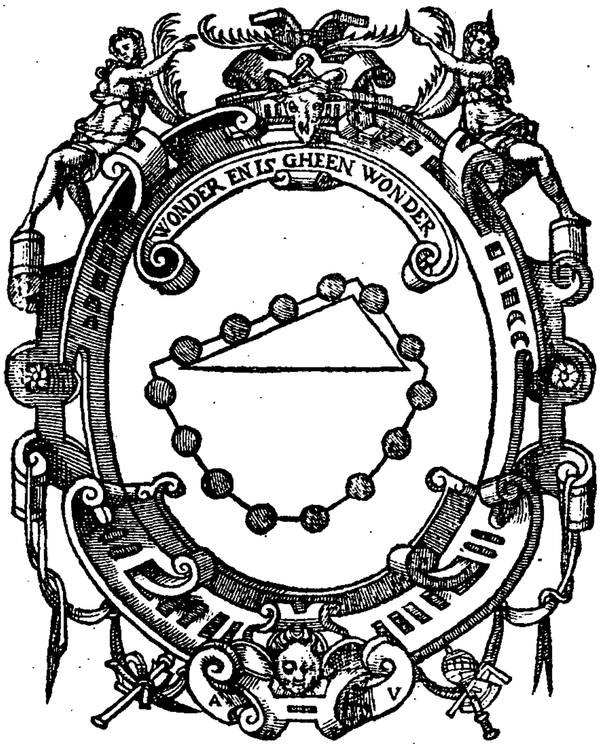
[17]) Dijksterhuis p. 60.

DE BEGHINSELEN DER WEEGHCONST

BESCHREVEN DVER

SIMON STEVIN

van Brugghe.



TOT LEYDEN,

Inde Druckerye van Chriſtoffel Plantijn,

By Françoys van Raphelinghen.

CIƆ. IƆ. LXXXVI.

OM DE WEEGHCONST,

WATERWICHT, ENDE

*LOCHTWIGHT.

Liber hic hodieq. lucifuga, paucorumq. adhuc testium.

CLARA Rhodos Clario, clarum celebrare Colossum
Desine, miraclo tanto nil tantula moles
Dignum habet, annorum labor ut fuerit duodenûm.
* Quintuplum tantum haud steterat, cùm viscera terræ
Spiritus intus agens vastos disuerberat artus.
Corruit illa, iacet ignobile littore corpus,
Pésque, capútque, aliæque incerto nomine partes.
Stulte quid inspectas, admiratusque iacentem
Quisnam erexerit, & quanam ratione modóque
Quæris? at heus! quàm mirandùm, multóque tremendum
Hoc magis est, totam leuis ut difflauerit aura
Molem? adeon' totam leuis ut difflauerit aura?
Sed neque mirandum fuit hoc, stultéve tremendum.
Si potis es rerum penetrantes discere caussas,
Et maiora videbis, & hæc mirabere nullus.
„ Pondera ponderibus nitantur maxima paruis,
„ Nutibus ut minimis firmissima quæque trahantur.
Hoc Natura parens, Naturáque anterior Mens
Omnibus in rebus statuit, seruátque statutum.
Ecce onus hoc Matris diuûm, & quicquid parit illa,
Cæruleus * Pater innixum sibi baiulat: illum

LVI, & quod excurrit, annos steterat, cùm terræmotu concidit.

Non improbanda Thaletis opinio, à qua faciũt & sacræ litteræ, ipsáque adeò minimè fallax experientia.

aA 2 Pondere

Pondere iam tanto grauidum in se sustinet aër.
Spiritus hic lenísque leuísque his corporibus par,
Spirituíque leui faciunt hæc corpora. Tantum
Hic Natura potest. Naturæ Ars æmula, tantum.

Het Almachtich multo maiorum viriũ quàm Archimedis Trispastus, multo & vtibile magis.

*Fare age * Pantocrator cæco sit carcere clausum,*
Incipiantque foris ædes ac templa moueri,
Incipiant siluæ, montes migrare, videnti
Quæ tibi mens animi, quæ sit constantia? rectè
Cum Natura agitur, tecum & Natura agit, hæc si
Credideris licitis Naturæ legibus ipse.
Legibus hæc quòd eant licitis, etiam hoc cape. Tellus
Tellurem hanc præter sit, si licet, altera: firmo

* qui tamen immobilis. † Inferos, scilicet centrũ quod etiam immobile. * Ita diuinus Plato, in cõtrarium iuit Aristoteles.

Hîc mihi me sistas talo, ne vixero, si non
*Hunc ego cum Tellure * Polum, atque † Acheronta mouebo.*
Hoc Trutinaria nos docet. hæc eadem monet omnia
Vndique ponderibus consistere, pondere cassum
*Esse * nihil, non hanc nostram, non ætheream auram.*
Vt maris, & terræ, numeríque potentis arenæ,
Sint, fuerint aliàs, mensores; non tamen auræ
Dimensi spatium, aut vim ponderis appenderunt.
Te certè, Alhazene, loci quem nubila tranent;

Aliis arcus pluuius reipsa coloratur, Aristoteli in speciem. quod & magis est.

*Qua regione color * appareat Iridis; vtrum*
Hanc supra Notus, & Zephyrus, Boreásve, vel Eurus;
Sítne Cometarum certus locus; vtque sit, eius
Mensorem video, veneror. potiora monentem
Credo secuturus facili ratione fuisses.
Non bene permensum spatium tibi. iustius illud

In de Locht-wicht.

*Iusta pensauit * trutina Steuinius, ille*
Ipse tui studiose senex studiosus amator.

L. M. huic gloriolæ cess-

*Ergo per hæc * princeps graditur loca nullius ante*

Trita

Trita solo, venit, ecce, videt, penitósque penetrat
Naturæ anfractus, vt quæ miranda videntur
Illorum trepida soluamur relligione.
„ Nil admirari res maxima, Lector, at illa
„ Hæc erat, auctrices miri cognoscere caussas.

ferit auctor, si quis dicatur hanc viā munisse, strauisse, cogitasse prior. Ick spreeck vāde Lochtwicht.

OM DE SELFDE.

ΑΡΧΟΜΕΝΟΙΣ ἔργοις τηλαυγέος χρεία προσώπου.
Τοῦτο μελίφθογγος μοῦσα τὺ μοῦσα διδαύη.
Ἔστ' ἄρα, εἴ ποτ' ἔην μεγάλου τ' ἀνδρὸς μέγα τ' ἔργον
Μεῖζον ἂν ἢ δῖον Στεύινιος ἔργον ἔθη;
Καὶ γὰρ χαλδαῖοι σοφίαν λάχον, ἢ δ' ἄρ ἑβραῖοι,
Ἤ ποτὲ ῥωμαῖοι. Μοῦσα τὺ δ' ἕλλας ἔφυς,
Πυθαγόρα ἄντε, πλάτωνα τ', ἀριστοτέλην τέ περ οἶδας
Ἄστρων ἀρίζηλον τῆς φυσικῆς σοφίας.
Τοὶ δὲ λέγουσι τί γαῖα, τί δ' οὐρανὸς εὐρὺς ὕπερθεν,
Πόντος τ' ἥλιος τ', ἠερόεντε χάος.
Οἶδας δ' εὐκλείδην τε Μαθηματικῆς φίλον ἦτορ,
Ἄστρων ἀρίζηλον μουνοσόφης σοφίας,
Ὅστις πάντα μέτρησε. καὶ εὗρε δὲ ψάμμος ἀριθμὸν
Ἄλλοθεν, ἀλλὰ μάτην*, τ' οὔνομ' ἔπεσσιν ἐρῶ.

Archimedem dico.

Ειπ' ἄρα, εἴ ποτ' ἔην μεγάλυ τ' ἀνδρὸς μέγα τ' ἔργον
Μεῖζον ἂν ἢ οἷον Στεύνιος ἔργον ἔθη;

„Εἴπερ ἀριστεύει μὲν ὕδωρ, κτεάνων δ' ὅγε χρύσος
„ Λαμπρότατός τε, κρατεῖ θ' ἅλιος ὠρανόθι,
Ευκλείδης γε μέγας, μηδεὶς δὲ τὺ μεῖζον' ἐπαίνοις,
Ἦρα γ' ἂν Ευκλείδυ Στεύνιος εἶχε νόον;

IAN DE GROOT.

Simon Steuin wenſcht

RVDOLF DEN II[en]

ROOMSCH KEYSER

VEEL GHELVCX.

DAT Ghetal Grootheyt ende Ghewicht, in yder weſentlicke ſaeck * onſcheydelicke ancleuinghen ſijn, vol diepe ende nutte eyghenſchappen, en betuyghen niet alleen verſcheyden gheleerden, maer tis duer d'eruaring in allen an elcken bekent. Tis oock openbaer, dat d'eerſte twee tot grooten voordeele der menſchen, ter form van beſchreuen conſten gherocht ſijn, namelick * Telconſt ende Meetconſt, maer niet alſo t'Ghewicht, om dat ſijn oirſproncklicke eyghenſchappen den voorighen verborghen bleuen. Wel is waer dat inde Rechtwichten duer eruaring bemerckt is, twee eueſtaltwichtighe met haer ermen * eueredнich te weſen. Doch ſy hebben ghemeent * ſoodanighe eueredenheyt te ſchuylen onder de ronden beſchreuen op t'vaſtpunt duer d'uyterſten der ermen; Uyt het welck, na den ghemeenen aert der dwaling, gheen kennis der oirſaken en volghde. Wat de Scheefwichten belangt, daer en is niet met allen af ghewеten. Inder voughen dat deſe * ſtof gheen form van Conſt als d'ander ghecrighen en conde. Maer doen t'gheual anders luċte, ende dat ſulck langverborghen hem duer ſijn uyterſte beghinſelen openbaerde, ſy is eintlick daer toe ghecomen, in ſulcker ghedaente als die uwe Keyſerlicke M. hier toegheeyghent wort. Maer angheſien byde voordachtighe niet ſonder oirſaeck ende beſtaende reden anghe-

Inſeparabilia accidentia.

Arithmetica & Geometria.

Proportionales.

Als Ariſtoteles in Mechanicis met ſijn navolghers.

Materia.

SIMON STEVIN WISHES MUCH HAPPINESS TO RUDOLPH II, HOLY ROMAN EMPEROR

That number, magnitude, and weight are in all essential things inseparable attributes, full of profound and useful properties, is attested not only by several scholars, but it is also known to all by experience in all things. It is also known that the first two, to the great profit of man, have reached the status of recorded arts, viz. arithmetic and geometry; but not so weight, because its fundamental properties have remained hidden from our predecessors. It is true that with regard to vertical weights it has been observed by experience that two gravities of equal apparent weight are proportional to their arms [1]). But they thought that this proportionality was due to the circles described about the fixed point by the extremities of the arms [2]). From which, as is usual with errors, there followed no knowledge of the causes. As to the oblique weights, these were not known at all, with the result that this subject matter could not be shaped into an art like the others. But when the situation changed, and this long-hidden matter was revealed through its fundamental elements, it at last reached this status, in the form in which it is here being dedicated to Your Imperial Majesty. But since, by the thoughtful, nothing is started without any cause and reason, the question

1) Read: inversely proportional.
2) Cf. the Introduction to the *Art of Weighing;* p. 37.

angheuanghen en wort, ſoo mocht hier de vraegh van t'einde mijns doens ſijn, te weten of ick na de ghebruyck van velen, uwe K. M. tot beſchermer mijns wercx verſouck? Verre van daer, ſo doch de beſcherming ende regiering des Rijcx, niet alleen tot ſulcx, maer tottet uytleſen der voorredens an haer eyghentlick gheſchreuen, ſelden eenighe tijdt toelaet: Te meer dat ick van meyning was (wie can ſijn vermoeden weerſtaen?) ſoo wel Form als Stof gheen verdediging te behouuen. Ten is oock niet om met een groote Conſt, in een grooter ſpraeck eerſt uytghegaen, den grootſten van Europa te vereeren, hoe lijckformich ſulcx nochtans de reden ſoude mueghen weſen.

Effecta. Waerom dan? Op dat de Weeghconſtens *daden ſtreckende tot merckelicke verbetering der Ghemeenſaeck, int werck ghebrocht worden, van ſulcx als daer ick duer beſonder brieuen van v Octroy af verſouck. Waerom ſal yemant mueghen ſegghen, dit niet beſtelt duer leegher (naer de ghebruyck) totten Hoochſten vrie toeganck hebbende? Yghelick, om dattet t'onghehoort is, ſoude vreeſen

Propoſitione. niet alleen met een lacherlick *voorſtel te verſchijnen, maer ſelfs oock belacht te worden: Nu op dat der ſpotters ſchamp tot ghetuych haerder onwetenheyt ſtrecke, wy hebbent duer t'ghene willen verſoucken, dat voor den verſtandighen alſulcx ende meerder veruaet. daeraſmen wyder en breeder ſoude connen ſegghen; Maer want ons einde tot Saken ſtreckt, niet tot Woorden, ſullen deſe verlatende en die verwachtende, uwe K. M. in alle ootmoedighe eerbieding veel ghelucx wenſchen. Uyt Leyden in Oogſtmaent des 1586e Iaers.

might here arise what my object in this is, to wit whether, after the custom of many, I request Your Imperial Majesty to be the protector of my work? Far be it from me, seeing that the protection and government of the Empire seldom leaves any time, not only for this, but also for reading through the prefaces written to Your Imperial Majesty. The more so because I was of opinion (who can resist his own supposition?) that neither the form nor the matter needs any defence. Nor is it my object to honour the greatest man of Europe with a great art, first published in a greater language, however much in accordance with reason this might be. Why then? In order that the effects of the Art of Weighing tending to a considerable improvement of the commonwealth may be realized, to wit those for which I request a patent from you in special letters. Why, someone might say, do not you (according to the custom) set about this by means of inferiors, who have free access to the Greatest? Because it is unheard-of, everyone would be afraid not only to be coming with a ridiculous proposition, but also to be scoffed at. Now in order that the taunts of the scoffers may bear witness to their ignorance, we have attempted to do it by means of that which for the intelligent comprises all this and more. About which we might speak more amply and fully, but because our end is things, not words, we will, leaving the latter and waiting for the former, wish Your Imperial Majesty in all humility and respect much happiness. From Leyden, in Harvest Month of the year 1586.

SIMON STEVINS

VYTSPRAECK

VANDE WEERDICHEYT

DER DVYTSCHE TAEL.

Elegium.

Tis wel waer datter inde Natuer niet wonderlick en is, nochtan tot onderscheyt der dinghen die wy duer de oirsaken verstaen, vande ghene welcker redenen ons onbekent sijn, soo gheuen wy dese met recht de naem van wonder, niet dat sijt eyghentlick sijn, maer om dattet hem voor ons alsoo ghelaet. T'welck soo wesende, wy sullen ons in desen ansien billichlick mueghen verwonderen, duer wat middel de Natuer mocht wercken, doen sy ons vooroûders sich haer spraeck dede maken; ouermidts ons van soo constighen werck, der oirsaken ghenouchsaem wetenschap ghebreeckt. Maer want een beclaghelicke verblintheyt, als duer * TSCHICSEL veroirdent, t'verstant van velen alsoo verduystert ofte betoouert heeft, dat sy t'licht vande Sonne bouen dat der Sterren, ick meen de weerdicheyt deses Taels bouen al d'ander, niet en connen bemercken, tot groot achterdeel des Duytschen gheslachts; Ghemerckt daerbeneuen, dat wy voorghenomen hebben inde selue te beschrijuen de WEEGHCONST, wiens diepsinnighe *ghedaenten duer slechter spraken ten eersten niet wel bedietlick en sijn, soo sullen wy naer ons vermueghen daer wat af segghen, versouckende of t'uyterste des SCHICSELS bestemden tijts noch niet en naeckt. ende eerst van haer oudtheyt als volght:

Fatum.

Qualitates.

Tis te weten dat de Duytschen in die seer oude tijden vande welcke ter weerelt gheen opentlicke schriften ghebleuen en sijn, gheweest hebben een treffelick seermachtich Gheslacht. t'welck duer sulcke verderuende oirsaken als meer ander machtighe volcken weeruaren sijn, als oirloghe en dierghelijcke, voorderende uytroeying der wetten, breking van goe oirdens, verwoesting der steden, &c. tot manier van wiltheyt gherocht is, doch niet soo volcommen, of den ouden aert der grootmoedicheyt, rechtueerdicheyt, ende ghetrauheyt, daer Tacitus oock af betuycht, en bleef altijt in hemlien ghewortelt. Dese haer woestheyt heeft gheduert tot ontrent de tijden van Iulius Cæsar, welcke daer naer tot beteren staet begon te keeren, soo dat sy eintlick weder ghecommen sijn ter regiering ouer t'eertrijcxdeel Europa, als kenlick is. Maer want yemandt van haer voornomde eerste macht twyffelen mocht, ouermits wy

Li. de Morib. Germanor.

bB daer

SIMON STEVIN'S
DISCOURSE
ON THE WORTH OF THE DUTCH LANGUAGE.

It is true indeed that in Nature there is nothing mysterious; nevertheless, in order to differentiate between the things we understand through their causes and those whose causes are unknown to us, we rightly call the latter miraculous, not because they are really so, but because they appear so to us. This being so, we may justly wonder in this respect by what means Nature may have operated when she caused our ancestors to frame their language, seeing that we lack sufficient knowledge of the causes of this ingenious creation. But since a deplorable blindness, as if ordained by Fate, has so clouded and bewitched the minds of many people that they cannot perceive the superiority of the light of the sun to that of the stars — I mean the superiority of this language to all others — to the great detriment of the Dutch race; considering further that we intend to describe in this language the Art of Weighing, the profound nature of which cannot well be expounded at once in inferior languages, we will say something about this to the best of our ability, studying to discover whether the time ordained by Fate is not yet drawing to an end, and first we will discuss its antiquity, as follows.

The reader should know that in those very ancient times, of which no public records have been preserved in the world, the Dutch were an excellent, very powerful race, which fell into barbarism through such corruptive causes as other powerful nations have also experienced, such as wars and the like, increasing abolition of laws, disturbance of order, destruction of the cities, etc.; however, not so completely but the ancient character of magnanimity, justice, and loyalty, to which Tacitus also bears witness, always remained rooted in them. This their barbaric state lasted approximately until the days of Julius Caesar, after which their condition began to improve again, so that at last they have regained dominion over the part of the world called Europe, as is apparent. But because someone might doubt their former power, since we have no public records about it, daer af, soo weynich als van veel ander volcken diens tijts, gheen openlicke schriften en hebben, soo sullen wy die aldus beuestighen.

Tis openbaer dat de Gallen die by ons Walen ghenoemt worden, ende int ghemeen nu Françoysen heeten, ouer oude tijden een machtich volck gheweest sijn, welcke Griecken, Spaeigne, Italie, strenghelick becrijghden ende ouerwonnen: vande welcke noch Gallogræcia ofte Galatia in Griecken, ende Celtiberia in Spaeignie, de naem behouden hebben: In Italie stichten sy Milaen, Coma, Brescia, Verona, Bergamo, Trente, Vicentia. De selue Françoysen hebben ofte voormael Duytsch ghesproken, ofte het Duytsch in grooter eere ghehadt, ende voor hun wit ghehouden ghelijck sy nu t'Latijn doen. T'eerste wort aldus bethoont: Tis yder spraeck ghemeen datse inden eenen oirt des landts wat anders uytghesproken wort als op den anderen. By voorbeelt, daer de Neerlanders segghen *Dat, Wat, Vat*, d'Ouerlanders segghen *Das, Was, Vas*: Voor der Parisienen *Chanter, Charbon, Chaleur*, de Picarden ghebruycken *Canter, Carbon, Caleur*; Daer de Castilianen segghen *Hazer, Hierro, Harina*, in Portugael seytmen *Fazer, Ferro, Farina*, &c. Nu alsulck verschil van het Duytsch dat de Françoysen voormael sprakẽ, tottet Duytsch van dese landen, was, dat sy voor ons *W* int ghemeene ghebruyčten *Gu*. Als daer wy segghen *Ick winde*, sy seyden ende segghen noch *Ie Guinde*. Ende voor ons *Windas* (t'welck een ghecoppelt woort is van Windt en as, als oftmen wilde segghen een as die windt) sy ghebruycken *Guindas*, sommighe *Guindal*, vande welcke oock commen hun *Guindement, Guindant, Guindeur*, &c. Wederom voor *Wincket, Wimpel, Want, Wesp, Weet, Wincken, Melcwey, Wildemalue*, sy segghen *Guichet, Guimple, Guant, Guespe, Guedde, Guigner, Megue, Guimalue*. Voor *Waren*, ofte *Bewaren*; *Guarder*, daer af commen la *Guarde, Guardeur, Guardebras, Guardemenger, Guarderobbe, &c.* oock *Guarir, Guarison*, dat me bewaren ende bewaring beteeckent, want de * Ghenesers achten dat sy duer drancken, cruyden, saluen &c. alleenlick t'ghebreck bewaren voor ongheual, ende dat sy gheensins en heelen, maer dat de Natuer altijt haer seluen gheneest: Van t'voornomde commen oock *Guarnir, Guarnison, Guarniture*, &c. welcke oock Bewaren ende bewaring bedien: Sghelijcx *Warande*, daer sy *Guarenne* voor nemen. Wederom *Gue* (achterlatende *ch*, die sy soo als wy niet noemen en connen) dat is by ons *Wech*, te weten den wech daer t'water van een riuier ouer loopt, waer af hun *Gueer* ende meer ander commen. Wederom *Guerdonner*, als oft sy wilden segghen *Weerdonner* dat is *Weergheuen* oft verghelden, daer af gheseyt wort *Guerdon, Guerdonnement, Guerdonneur, &c.* Voorts *Mot de Guet*, dat is *Woort vande Wet*, ouermits sulck woort inde steden vande wet comt, ende duer haer ghegheuen wort: Oft andersins mach *Guet* van *Wacht* commen, achterlatende *ch* die sy so niet en ghebruycken als voor ghesevt is, ende *e* voor *a* ghenomen, t'welck by ons

* *Medici.*

no more than about many other nations of that time, we will prove it as follows.

It is generally known that the Gauls, who are named Walloons among us and are now usually called French, in ancient times were a powerful nation, which waged fierce wars against Greece, Spain, and Italy, and conquered them; from which Gallograecia or Galatia in Greece and Celtiberia in Spain still have their names. In Italy they founded Milan, Como, Brescia, Verona, Bergamo, Trento, Vicenza. These same Frenchmen formerly either spoke Dutch or greatly esteemed the Dutch language, and considered it their example, as they now do with Latin. The first is proved as follows: It is a common feature of all languages that they are pronounced somewhat differently in different parts of the country. For example, where the Dutch say *Dat, Wat, Vat,* the Germans say *Das, Was, Vas;* for the Parisians' *Chanter, Charbon, Chaleur,* the Picards use *Canter, Carbon, Caleur;* where the Castilians say *Hazer, Hierro, Harina,* in Portugal they say *Fazer, Ferro, Farina,* etc. Now a similar difference between the Dutch language formerly spoken by the French and the Dutch of these regions was that they generally used *Gu* for our *W*. Thus, where we say *Ick winde,* they said and still say *Ie Guinde.* And for our *Windas* (which is a compound of *Windt* and *as,* as who should say an *as* that *windt*) they use *Guindas,* some of them *Guindal,* from which are also derived their *Guindement, Guindant, Guindeur,* etc. Again, for *Wincket, Wimpel, Want, Wesp, Weet, Wincken, Melcwey, Wildemalve* they say *Guichet, Guimple, Guant, Guespe, Guedde, Guigner, Megue, Guimalve.* For *Waren,* or *Bewaren,* they use *Guarder,* from which are derived *Guarde, Guardeur, Guardebras, Guardemenger, Guarderobbe,* etc., and also *Guarir, Guarison,* which also means to preserve and preservation, for the physicians deem that by means of potions, herbs, ointments, etc. they merely "preserve" the disease from becoming fatal, and that they do not cure men at all, but that Nature always cures itself. From the aforesaid are also derived *Guarnir, Guarnison, Guarniture,* etc., which also mean preserve and preservation; similarly *Warande,* for which they take *Guarenne.* Again *Gue* (omitting *ch,* which they cannot pronounce as we do), that is with us *Wech,* to wit the course taken by the water of a river, from which their *Gueer* and others are derived. Again there is *Guerdonner,* as if they would say *Weerdonner,* that is *Weergheven* or requite, from which are derived *Guerdon, Guerdonnement, Guerdonneur,* etc. Further *Mot de Guet,* that is *Woort vande Wet,* because this word takes the place of the law and is given by it. Or otherwise *Guet* may be derived from *Wacht,* omitting *ch,* which they do not use in this way, as has been said before, and taking *e* for *a,* which is common with us, for ons ghemeen is, want men seght so veel Bert, Swert, als Bart, Swart: Tis ooc kennelick dat sommighe *Wecht* voor *Wacht* ghebruycken: Van *Guet* commen *Guetter*, *Guette*, *Guetteur*, *&c.* Wederom voor *Ter Weere*; *A la Guerre*, daer af ghesëyt wort *Guerroyer*, *Guerroyeur*, *&c.* Voor *Op de Wyse*; *A la Guise*, daer af ghemaeckt wordt *Guisarme*, *Deguiser*, *Deguisement*, ende soo met meer anderen die wy om cortheyt achterlaten. Vyt dese ghemeene reghel dan van *W* tot *Gu* (bouen de groote menichte van d'ander ghemeene woorden die sy ghewislick uyt het Duytsch hebben, welcke wy om cortheyt verswyghen, te meer dattet bouesc hreuen an t'voornemen voldoet) schijnt ghenouch te mueghen besloten worden, de Françoysen voormael Duytsch ghesproken te hebben, dat is Duytschen gheweest te sijne, ende veruolghens dat de Duytschen eertijts een bekent machtich volck waren.

Doch so v dat niet en gheuiel, maer dat sy die woorden voormael uyt het Duytsch vergaert hebben, ghelijck sy nu sulcx uyttet Latijn doen (want een van tween is nootsaeclick) t'selue valter uyt te besluyten. Want dat soo ghenomen, tis ghe schiet naer hemlien verwoestheyt, daer in, ofte daer vooren: Niet daer naer, want de Duytsche tael by haer sedert in gheen acht gheweest en heeft, maer de Latijnsche, daer sy de hunne naer verandert hebben: Oock niet daer in, want dat een machtich volck t'welck Spaeigne, Griecken, Italie, conde beuechten ende verwinnen, haer spraeck souden gheformt hebben naer de wildens tael, ten sluyt niet; Dat sulcken Gheslacht vande wilden soude leeren an t'windaes een naem gheuen, tis te belachelick, soo sy doch seluer eer dan wilden, windassen ghebruyckten. Tis dan nootsaeclick dat sy dese woorden na der Duytschen ghemaeckt hebben voor haer verwoesting, te weten doen sy groot en machtich waren, ende dat yder Gheslacht d'ooghe op hun had.

Hier toe helpt noch dit, dat hemlieder tael wyder streckte als ander, t'welck sy in haer woestheyt daer toe niet ghebrocht en hebben, want dat wilden die niet en handelen, noch verre en reysen, haer tael wyder souden doen verbreyden als ander machtighe volcken, die groote landen en Koninckrijcken onder haer ghebiedt hadden, het strijt teghen den ghemeenen loop des weerelts, t'waer ongheschickt sulcx toe te laten. Dese wyde verbreyding dan der tael is gheschiet voor de verwoesting: Waer uyt oock te bemercken is in wat macht sy doen moesten wesen, anghesien t'verstroeyde eentalich ouerblijfsel na soo grooten menichte van iaren, hem wyder streckte dan die teghenwoordelick in groote macht waren: T'sijn voorwaer oirsaken die metgaders d'ander redenen, ons dwinghen te ghelooueń, dat de Spaengnaerden voormael oock, of Duytschen gheweest sijn, of dat sy hun tael naer het Duytsch gherecht hebben. want sy, ghelijck de Françoysen, oock segghen *Guante*, *Guardar*, waer af commen we say both *Bert, Swert* and *Bart, Swart.* It is also known that some people use *Wecht* instead of *Wacht.* From *Guet* are derived *Guetter, Guette, Guetteur,* etc. Again, for *Ter Weere* they use *A la Guerre,* from which are derived *Guerroyer, Guerroyeur,* etc. For *Op de Wyse,* they use *A la Guise,* from which are derived *Guisarme, Deguiser, Deguisement,* and in the same way with several others, which we omit for brevity's sake. From this general rule therefore of *W* into *Gu* (in addition to the great number of other common words which they have undoubtedly borrowed from Dutch and which we omit for brevity's sake, the more so because the above is sufficient for this purpose) it seems we may safely conclude that the French formerly spoke Dutch,that is that they were Dutch, and consequently that in former days the Dutch were a well-known and powerful nation.

But if this should not satisfy you, and you should think that they formerly borrowed these words from Dutch as they now do from Latin (for one of the two is necessary), the same conclusion can be drawn from it. For on this assumption it must have happened after, during or before their period of barbarism. Not after this period, for the Dutch language has not been held in esteem by them since that time, but rather the Latin tongue, on which they have modified their own. Not during this period either, for it is not plausible that a powerful nation, which could fight Spain, Greece, and Italy, and conquer them, should have modelled their speech on the language of the barbarians; it is all too ridiculous to assume that such a race should have learned from the barbarians to give a name to the windlass, since they themselves surely used windlasses before the barbarians did. Therefore it cannot be but that they have modelled these words on those of the Dutch before their period of barbarism, that is when they were great and powerful, and the eyes of all nations were upon them.

This is supported by the fact that their language was more widely distributed than any other, which they have not achieved in their period of barbarism, for it is contrary to the common course of the world that barbarians, who do not trade or make long voyages, should succeed in propagating their language more widely than other powerful nations, which ruled over large countries and kingdoms; it would be absurd to admit such a thing. Therefore this wide propagation of the language took place before the period of barbarism; from which it may also be gathered how powerful they must have been at that time, since the dispersed monolingual remnant is scattered further afield after such a long time than the nations which are powerful in our days. These are truly causes which, combined with the other reasons, compel us to believe that formerly even the Spaniards either were Dutch or modelled their language on Dutch, for — just like the men *Guarda, Guardador, Guardoſo,* oock *Guarida,* ende *Guarnecer, Guarnicion, &c.* Wederom *Guerra,* daer ſy af ſegghen *Guerrear, Guerreador, Guerrero, &c.* Wederom *Guinar.* Voort *Guiſa,* daer *Guiſar* af comt, &c, Ia dat etlicke Indianen welcke men ſegt veel duytſche woorden te ghebruycken; oock verſcheyden ander contreyen in Aſie wiens ſpraeck Hieronymus betuycht tſijnder tijt bycans de ſelue gheweeſt te ſijne met die van Trier; ende meer ander volcken wiens talen met Duytſch ghemengt ſijn, ſulcx uyt het Duytſch hebben, al van die oude tijden af, dat ſy in haer eerſte groote macht waren.

Effectum. T'voornomde wort noch ſtercker, opentlicker, ende nootſakelicker beueſticht, duer haer talens conſtich maecſel, voorwaer gheen werck van ſlechte wilden, maer te verwonderen hoe gheleerde tammen, ſulcx hebben connen ter * daet brenghen. daer af wy breeder ſpreken, ende haer weerdicheyt bouen al d'ander, met merckelicke redenen beueſtighen moeten, aldus: T'einde der ſpraken is, onder anderen, te verclaren t'inhoudt des ghedachts, ende ghelijck dat cort is, alſo begheert die verclaring oock cortheyt, de ſelue can bequamelicxt gheſchien, duer ynckel ſaken met ynckel gheluyden te beteeckenen; Oock ſoodanighe, datſe oueral de T'ſaemvoughing bequamelick lijden; Datſe de Conſten grontlick leeren; Ende den Hoorders heftelick beweghen tot des ſprekers voornemen. Nu of deſe alle vier, byden Duytſchen beter ghetroffen ſijn dan by eenighe ander, dat ſullen wy oirdentlick verclaren, eerſt bethoonende, ende dat metter daet, op datment ghelooue, der Duytſchen 742 eenſilbighe woorden inden eerſten perſoon; daerder de Latinen alleenlick 5 hebben; De Griecken gheen eyghentlicke, maer langhe vercort tot 45. Daernaer ſullen wy ſegghen vande namen, bynamen, &c. welcke wy alle metter haeſt vergaert hebben yder uyt ſijn Woortbouck als volght.

DVYTSCHE

French — they also say *Guante, Guardar,* from which are derived *Guarda, Guardador, Guardoso,* also *Guarida* and *Guarnecer, Guarnicion,* etc. Again *Guerra,* from which they derive *Guerrear, Guerreador, Guerrero,* etc. Again *Guinar.* Further *Guisa,* from which is derived *Guisar,* etc. Nay, we are even bound to believe that several tribes in India, which are said to use many Dutch words, also many other regions in Asia, whose speech is stated by St. Jerome [1]) to have been almost identical with that of Treves in his day, and other nations whose languages are interspersed with Dutch, have borrowed this from the Dutch, already from those ancient times when they were at the height of their power.

The above is proved even more strongly, clearly, and inevitably by the ingenious structure of their language, which is certainly not the work of simple barbarians; it is even to be wondered at how learned civilized beings have succeeded in effecting this, a subject which we must discuss more fully, while confirming with clear arguments its superiority to all the other languages, as follows. The object of language is, among other things, to expound the tenor of our thought, and just as the latter is short, the exposition also calls for shortness; this can best be achieved by denoting single things by single sounds [2]); also in such a way that in every respect they properly admit of composition; that they thoroughly teach the arts; and that they violently move the hearers to act after the speaker's intention. Now we will set forth systematically that all these four points have been hit off better by the Dutch than by any other people, first showing — such with facts, so that the reader may believe it — the 742 Dutch monosyllabic words in the first person, where the Latins have only 5 and the Greeks have no monosyllables proper, but only 45 long words that have been contracted. After that we will deal with the nouns, adjectives, etc., all of which we have hurriedly collected from the respective dictionaries, as follows:

[1]) As is well known, St. Jerome (347-c.420) passed a part of his youth at Treves. The passage from St. Jerome that Stevin has in mind seems to be: Hieronymus, *Comment. in epist. ad Galatas lib. II*; prol. cap. 3 (Migne, Patrologia Latina Vol. XXVI. col. 382 C): Unum est quod inferimus .... Galatas excepto sermone Graeco, quo omnis Oriens loquitur, propriam linguam eandem pene habere quam Treviros, nec referre, si aliqua exinde corruperint, cum et Afri Phoenicum linguam nonnullam ex parte mutaverint, et ipsa Latinitas et regionibus quotidie mutetur et tempore (communicated by Dr J. W. Ph. Borleffs, The Hague).

[2]) The term „single sound" is naturally used here in a different sense from that of modern phonetics. It here means: what is pronounced in a single effort of speech.

TABLE OF CONTENTS
OF THE FOLLOWING LISTS OF WORDS

DVYTSCHE EENSILBIGHE VVOORDEN.

Ick

Acht. I'estime. Existimo.
Aes. I'apaste. Inesco.

Back. Ie cuis. Pinso.
Baeck. Pono pharum.
Baen. Ie prepare le chemin. Præparo viam.
Ban. Ie banne. Proscribo.
Baer. I'enfante. Pario.
Bas. I'abbaie. Latro.
Baet. Ie prouffite. Commodus sum.
Bel. Ie tire la clochette. Tintinno.
Ben. Ie suis. Sum.
Berst. Ie creue. Crepo.
Bey. I'attens. Expecto.
Bid. Ie prie. Precor.
Biecht. Ie confesse. Confiteor.
Bied. I'offre. Præsento.
Bies. Ie beze. Mugio.
Bijt. Ie mors. Mordeo.
Bind. Ie lie. Ligo.
Blaeck. Ie flamboye. Flammo.
Blaes. Ie souffle. Flo.
Blaeu. Ie coleure de bleu. Colore cæruleo pingo.
Bleck. Ie couvre de lames. Bracteo
Bleeck. Ie pasli. Palleo.
Bleet. Ie beelle. Balo.
Bleyck. Ie blanchi du linge. Candefacio.
Blijf. Ie demeure. Maneo.
Blinck. Ie reluys. Resplendeo.
Block. Ie labeure assiduellement. Assiduè laboro.
Bloe. Ie saigne. Sanguino.
Bloey. Ie fleuronne. Floreo.
Bloot. Ie mects nud. Nudo.
Bloos. Ie vermeillonne. Rubeo.
Blusch. I'estain. Extinguo.
Bluts. Ie froisse. Collido.
Boerd. Ie bourde. Nugor.
Boet. Ie remedie. Medeor.
Boey. Ie mects des pieges aux pieds Compedio.
Boock. Ie bats. Cudo.
Bol. Ie boule. Voluo globum.
Boord. Ie borde. Fimbrio.
Boor. Ie fore. Perforo.
Borg. Ie pleige. Fide iubeo.
Bot. Ie rebouche. Hebeto.
Bot. Ie boutonne. Gemmo.
Bou. I'edifie. Ædifico.
Bra. Ie rosti. Asso.
Braeck. Ie vome. Euomo.
Brand. Ie brusle. Ardeo.
Bras. Ie bauffre. Epulor.
Bree. Ie fay large. Dilato.
Breeck. Ie romps. Rumpo.
Brey. Ie entrelache. Reticulo.
Briesch. Ie rugis. Rugio.
Bring. Ie apporte. Adporto.
Brock. Ie coupe des morceaux de pain. In frusta frango.
Brod. Ie radoube. Resarcio.
Broe. Ie couue. Incubo ouis.
Broey. Ie fourboulle. Subferuefacio
Brou. Ie brasse. Coquo cereuisiam
Brul. Ie murle. Mugio.
Bruyck. I'vse. Fruor
Buck. Ie plie le dos. Curuo
Buet. Ie troque. Commuto
Buyg. Ie plie. Flecto
Buysch. Ie frappe. Pulso

Caerd. Ie carde. Carmino.
Caert. Ie ioue aux cartes. Ludo chartis.
Caets. Ie ioue à la paume. Ludo pila
Cap. Ie hache. Concîdo
Cier. I'orne. Orno
Claer. Ie fai clair. Clarifico
Colf. Ie croche. Ludo claua
Cop. Ie scarifie. Scarifico
Cost. Ie couste. Consto
Coot. Ie ioue aux os. Talis ludo.
Coock. Ie cuisine. Coquo
Crab. Ie racle. Rado
Craey. Ie crie. Cornicor
Craeck. Ie croque. Crepito
Croon. Ie courronne. Corono.
Cruys. ie crucifie. Crucifigo
Cuyp. Ie fai cuues. Vieo dolia.

Dab. Ie patrouille. Palpo
Daeg. I'aiourne. Cito
Dael. Ie descens. Descendo.
Danck. Ie remercie. Habeo gratiam
Dans. Ie danse. Tripudio
Dau. Ie fai rosee. Roro
Deck. Ie couure. Tego
Delf. I'enfoui. Fodio
Denck. Ie pense. Cogito
Derf. I'oste. Audeo
Dersch. Ie bats en grange. Trituro
Derf. I'ay besoing. Egeo
Deys. Ie recule. Recedo
Dicht. Ie compose en rime. Cõpono
Dick. I'espessis. Denso
Dien. Ie sers. Seruio
Diep. Ie fay profond. Profundum facio.
Dijck. Ie fay vne dique. Iacio aggerẽ
Ding. Ie barguine. Liceor
Ding. Ie plaide. Litigo
Doem. Ie damne. Damno
Doe. Ie fai. Facio
Dool. I'erre. Erro
Doo. Ie tue. Occido.
Doog. Ie vaux. Valeo
Doop. Ie baptize. Baptizo.
Dor. Ie deuiens aride. Aresco
Dorst. I'ay soif. Sitio
Dou. ie presse. Premo
Draey. Ie tourne. Torno
Draeg. Ie porte. Porto
Drael. Ie tarde. Tardo.
Draef. Ie trote. Succusso.
Dreich. Ie menace. Minor
Drijf. Ie chasse. Agito.
Drinck. Ie boy. Bibo
Dring. Ie poulse. Penetro turbam
Drooch. Ie seiche. Sicco
Droom. Ie songe. Somnio
Droop. Ie arrouse quelque chose de gresse. Conspergo pinguedine.
Druck. I'imprime. Imprimo
Dub. Ie doubte. Dubito
Ducht. I'ai doubte. Vereor
Duer. Ie dure. Duro
Duld. Ie souffre. Patior
Dun. Ie tenue. Extenuo
Dvvael. I'erre. Erro
Dvving. Ie contrains. Cogo

Eer. Ie honnore. Honoro
Eg. I'herse. Occo
Eind. Ie fine. Finio
Eet. Ie menge. Edo
Ets. Ie mors en cuiure de l'eau forte Inedo cuprum aqua forti.
Eysch. Ie demande. Peto

Fael. Ie faille. Fallor
Fluyt. Ie ioue à la flute. Cano fistula.
Fryt. Ie fricasse. Frigo
Frons. Ie fronse. Rugo

Gaen. Ie voy. Eo
Gaep. Ie baye. Oscito
Geck. Ie mocque. Lasciuio
Gheef. Ie donne. Do
Gheel. Ie fay iaune. Rufo
Gheeu. Ie baaille. Oscito
Gheld. Ie vaux. Valeo
Ghiet. Ie fonds. Fundo
Ghisp. Ie fouette. Flagello
Ghis. Ie souspeçonne. Suspicor
Glat. Ie polie. Polio.
Glie. Ie glisse. Labor.
Gloey. ie deuien rouge. Candesco.
Gom. Ie goumme. Linio gummi.
Gord. Ie ceinds. Cingo.
Graef. I'engraue. Sculpo.
Greys. Ie refrongne. Obduco frontẽ
Grijp. Ie gripe. Capio.
Grim. Ie rugi. Rugio.
Groey. Ie verdoye. Verno.
Grou. I'ai en horreur. Abominor
Groen. Ie paind verd. Virido.
Gruet. Ie salue. Saluto.
Gun. Ie fauorise. Faueo.

Haeck. Ie hache. Concîdo.
Haest. I'haste. Festino.
Haeck. I'accroche. Iunco.
Hael. Ie porte. Adfero.
Hang. Ie pends. Pendo.
Harp. I'harpe. Lyram pulso.
Haet. Ie hay. Odio habeo.
Heb. I'ai. Habeo.
Hecht. Ie attache. Figo.
Heel. Ie guarri. Sano.
Heet. Ie cauffe. Calefacio.
Heet. Ie nomme. Nomino.
Heet. Ie commande. Iubeo.
Hef. Ie leue. Leuo.
Hel. Ie panche. Acclino.
Heel. Ie cele. Celo.
Help. I'aide. Iuuo.
Herd. Ie durcis. Duro.
Hey. Ie hie. Fistuco.
Hygh. I'ahaine. Anhelo.
Hinck. Ie cloche. Claudico.
His. I'incite. Instigo.
Hoed. Ie garde. Custodio.
Hoest. Ie tousse. Tussio.
Hol. Ie creuse. Cauo. (Cumulo.
Hoop. Ie comble en monceaux.
Hoor. I'oy. Audio.
Hoop. I'espere. Spero.
Houd. Ie tiens. Teneo.
Hou. Ie coupe. Seco.
Hou. Ie marie. Nubo.
Hoy. Ie fene du foin. Sicco fœnũ sole
Huc. Ie croupe. Sido.
Huer. Ie loue. Conduco.
Hul. Ie coeffe. Orno caput.
Huts. I'hoche. Quatio.
Huyl. I'hurle. Vlulo.

Iaech. Ie chasse. Venor.
Ianck. Ie glappe. Gannio.
Iock. Ie mocque. Iocor.

Kaeck. Ie cabasse. Suppilo.
Kan. Ie sçay. Scio.

Ick

Kau. Ie masche. Mando.
Keer. Ie tourne. Verto.
Keer. Ie ballie. Scopo.
Kem. Ie peigne. Pecto.
Ken. Ie cognoy. Cognosco.
Kerm. Ie lamente. Lamentor.
Kern. ie bats le beure. Butyrũ pulso
Keur. Ie visite. Censeo.
Kick. Ie gronde. Mussito.
Kies. I'elis. Eligo.
Kijck. Ie voy. Intueor.
Kijf. ie tense. Litigo.
Kip. I'esclos poussins. Pullulo.
Klack. Ie creuasse auec vn son esclatant. Cum fragore rimas ago.
Klad. Ie crotte. Penicillo vestem à luto detergo.
Klaeg. Ie plains. Queror.
Klap. Ie babille. Fabulor.
Klee. Ie vests. Vestio.
Klem. Ie pince. Premo.
Klets. Ie frappe du fouet le faisant sonner. Scutica ferio.
Kleef. I'attache. Visco.
Klein. I'appetisse. Minuo.
Klief. Ie fends. Findo
Klim. Ie monte. Scando.
Klinck. Ie sonne. Sono.
Kluen. Ie frappe. Pulso.
Kloot. Ie boule. Voluo globum.
Kloop. Ie hurte. Pulso.
Klos. Ie ioue à la boule par trauers d'vn anneau. Ludo globo per annulum.
Klucht. Ie plaisante. Facetias narro
Knaeg. Ie ronge. Rodo.
Knau. Ie masche. Mando.
Kne. Ie pestris. Depso.
Kners. Ie grince les dens. Dentibus strideo.
Knich. I'hoche la teste. Nuo.
Kniel. I'agenouille. Geniculor.
Knip. Ie chiquenaude. Talitra infligo.
Knoop. Ie noue. Necto.
Knor. Ie gronde. Grunnio.
Koel. Ie fai tied. Tepido.
Kom. Ie viens. Venio.
Koop. I'achapte. Emo.
Kout. Ie deuise. Fabulor.
Kraets. Ie gratte. Scabo.
Krau. Ie gratte. Scabo.
Kranck. Ie debile. Debilito.
Kriel. Ie remue comme entre les fourmis. Mobilito per turbas.
Krijg. I'acquiers. Acquiro.
Krijt. Ie pleure. Eiulo.
Krimp. Ie retrecis. Arcto.
Kroch. Ie geins. Queror.
Kroock. Ie fronce. Rugo
Krol. Ie crespille. Crispo.
Krom. Ie courbe. Curuo.
Krop. I'emple le gouion. Ingluuiem farcio auium.
Kruy. Ie poulse. Vi pello.
Kruy. I'espice. Aromatibus condio
Kruyp. Ie rampe. Repo.
Kuch. Ie tousse. Tussito.
Kus. Ie baise. Osculor.
Kuysch. Ie nettoye. Nitido.
Lach. Ie ris. Rideo.
La. Ie charge. Onero.
Laeck. Ie diminue. Diminuo.
Laeck. Ie desprise. Contemno.
Lang. I'alonge. Prolongo.
Lang. I'aueins. Porrigo.
Lap. Ie rapiece. Interpolo.
Laet. Ie laisse Linquo.
Laef. Ie rafreischis. Foueo.
Laeu. Ie fai tiede. Tepido.
Leck. Ie liche. Lambo.
Leeg. I'abbaisse. Humilio.
Leem. I'enduis d'argille. Luto.
Leen. Ie preste. Mutuo.
Leer. I'enseigne. Doceo.
Leg. Ie mects. Pono.
Leeck. Ie coule. Stillo.
Leen. I'appuis. Cubito.
Lees. Ie lis. Lego.
Lesch. I'esteins. Extinguo.
Let. I'empesche Impedio.
Let. Ie considere. Considero.
Leef. Ie vis. Viuo.
Ley. Ie meine. Duco.
Licht. I'esclaircis. Luceo.
Licht. Ie leue. Leuo.
Lieg. Ie ments. Mentior.
Lig. Ie couche. Iaceo.
Lijd. I'endure. Fero.
Lisp. Ie begaye. Balbutio.
Lock. I'alliche. Allicco.
Loer. Ie lorne. Obseruo.
Lol. Ie grongnonne Eiulo ad instar felis.
Lol. Ie me chauffe comme les vielles qui vsent d'vn pot plein de feu le mettant soubs elles.
Lonck. I'oeillade. Oculo.
Loon. Ie baille de loyer. Remunero
Loop. Ie cours. Curro
Los. Ie delasche. Laxo.
Loot. Ie iette le sort. Sortior.
Loof. Ie loue. Laudo.
Lub. Ie chastre. Castro.
Lul. Ie finge le son. Sonum imitor.
Luym. Ie lorne. Insidiantibus oculis intueor.
Luys. I'espluche des pouils. Pediculos lego.
Lie. Ie passe. Transeo.
Mach. Ie puis. Possum.
Maey. Ie fauche du foin. De. fseco prata.
Maeck. Ie fay. Facio.
Mael. Ie peins. Pingo.
Mael. Ie moule. Molo.
Maen. Ie stipule. Stipulor.
Maen. Ie coniure. Adiuro.
Melck. Ie traicts le laict. Mulgeo.
Meng. Ie mesle. Misceo.
Men als peerden oft vvaghen. Ie meyne. Duco.
Merck. Ie marque. Noto.
Mest. I'engraisse. Sagino.
Meet. Ie mesure. Metior.
Mets. Ie massonne. Extruo muros.
Mein. Ie cuide. Opinor.
Mick. I'ay l'oeil à quelque chose. Collimo.
Mye. Ie contregarde. Caueo.
Min. I'ayme. Amo.
Moet. Il me faut. Debeo.
Moey. Ie moleste. Molesto.
Moord. Ie meurtris. Trucido.
Mor. Ie murmure. Murmuro.
Muf. Ie sens le relant. Sitũ redoleo
Munt. Ie monnoye. Cudo nummos.
Muyl. Ie rechigne. Contraho vultũ
Muyt. Ie mutine. Seditionem facio
Naey. Ie couse. Suo.
Naeck. I'approche. Propinquo
Nau. Ie fay estroict. Angusto.
Neem. Ie preus. Accipio.
Nest. Ie niche. Nidifico.
Nyg. I'encline. Inclino.
Nies. I'esternue. Sternuo.
Nieu. Ie fay nouueau. Nouo.
Nijp. Ie pince. Vellico.
Noem. Ie nomme. Nomino.
Noo. I'inuite. Inuito.
Noop. I'aguilonne. Stimulo.
Oogst. Ie moissonne. Messem facio.
Oos. Ie vuide l'eau. Exhaurio.
Pael. Ie borne. Termino.
Paer. Ie mects pair à pair. Binos pono.
Paert. Ie partis. Partior.
Paey. I'appayse. Paco.
Pand. Ie mects en gaige. Pignero.
Pap. Ie colle. Glutino.
Pas. Ie le fais accorder. Apto.
Peck. Ie poisse. Pico.
Peel. Ie pele. Decortico.
Pers. Ie persse. Premo.
Pick. Ie beque. Rostro.
Pijp. Ie pipe. Pipo.
Pis. Ie pisse. Meio.
Plaeg. Ie vexe. Vexo.
Plasch. Ie pateline en l'eau. In aqua palpo.
Pleck. Ie macule. Maculo.
Pleeg. Ie soulois. Soleo.
Pleyt. Ie plaide. Litigo.
Ploeg. I'are. Aro.
Plomp. Ie rebouche. Hebeto.
Plomp. Ie plonge en l'eau. Mergo
Ploy. Ie plie. Plico.
Pluck. Ie cueille. Carpo.
Pluys. I'espluche. Polio.
Poch. Ie vante. Iacto.
Pomp. Ie vuide l'ossee. Sentinam expurgo.
Poog. Ie tasche. Nitor.
Por. I'incite. Incito.
Pot. I'amasse en pot. In ollulis coaceruo.
Poy. Ie boy. Indulgeo potationibus.
Praem. I'oppresse. Opprimo.
Prang. Ie oppresse. Opprimo.
Praet. Ie babille. Fabulor.
Prick. I'esguillonne. Stimulo.
Preeu. Ie derobbe finemẽt Surripio.
Pris. Ie prise. Laudo.
Print. I'imprime. Imprimo.
Proef. I'esprouue. Probo.
Proef. Goutte. Gusto.
Pronck. Ie tiens grauité comme vne espousee.
Put. Ie puise. Haurio aquam.
Quel. Ie fasche. Molesto.
Quets. Ie blesse. Lædo.
Quist. Ie degaste. Dissipo.
Quijl. Ie baue. Stillo pituitã ex ore
Ra. I'aduinne. Diuino.
Raeck. Ie touche. Tango.
Raep. I'amasse. Colligo.
Raes. Ie rage. Lasciuio.
Reck. Ie tens. Tendo.
Ren. Ie cours. Cursito.
Reul. Ie troque. Commuto.
Rey. Ie danse. Duco choreas.
Reyck. Ie tends. Porrigo.
Reys. Ie chemine. Proficiscor.
Richt. I'erige Erigo.
Rieck. Ie sens. Sentio.
Riem. Ie rame. Remigo.
Rie. Ie cheuauche. Equito.
Rijm. Ie rhime. Versifico.

Rijp

Ick

Ryp. Ie meuris. Præcoque.
Rys. Ie me leue. Subleuo.
Rips. Ie route. Ructo.
Reck l'eſtrique la quenouille. Penſum ſtruo.
Roem. Ie me vante. Iacto.
Roep. Ie crie. Clamo.
Roer. Ie remue. Moueo.
Roeſt. Ie enrouille. Rubiginē traho
Roey. Ie rame. Remigo.
Rol. Ie roulle. Voluo.
Ronck. Ie ronfle. Sterto.
Rond. I'arondis. Rotundo.
Roof. Ie rauis. Spolio.
Rot. Ie pourris. Putreo.
Ruck. I'arrache. Auello.
Ruet. I'engraiſſe. Sebo.
Run. Ie coagule. Coagulo.
Run. Ie flotte. Fluo.
Ruſt. Ie repoſe. Quieſco.
Ruym. I'amplifie. Amplifico.
Ruyſch. Ie bruys. Strepo.
Saey. Ie ſeme. Semino.
Saeg. Ie ſie. Serro.
Saelf. I'oings. Vngo.
Scha. I'endomage. Noceo.
Schaf. Ie traicte. Tracto.
Schaeck. Ie prens vne fille par force. Rapio virginem.
Schaem. I'hontis. Erubeſco.
Schamp. Ie gliſſe. Labor.
Schants. Ie fortifie rempars. Munio vallis.
Schat. I'eſtime. Aeſtimo.
Schaef. Ie rabotte. Dolo.
Scheer. Ie tonds. Tondeo.
Scheld. Ie tenſe. Obiurgo.
Schel. I'eſcorche. Decortico.
Schenck. Ie verſe. Infundo.
Schend. Ie gaſte. Corrumpo.
Schep. Ie cree. Creo.
Scherm. I'eſcrime. Digladior.
Scherp. I'aguiſe. Acuo.
Scherf. Ie hace. Concido.
Schuer. Ie deſchire. Lacero.
Scheyd. Ie ſepare. Separo.
Schick. I'ordonne. Ordino.
Schiet. Ie tire. Sagitto.
Schijn. Ie luis. Luceo.
Schil. Ie differe. Differo.
Schimp. Ie brocarde. Iocor.
Schock. Ie ſecoue. Succuſſo.
Schors. Ie trouſſe. Succingo.
Schau. Ie contemple. Contemplor
Schrab. I'eſgrattigne. Vnguibus ſcabo.
Screeu. Ie crie eſclatant. Exclamo.
Screy. Ie pleure. Lachrymo.
Schrick. Ie ſaillis. Diſſilio.
Scroef. Ie vire. Torqueo cochleam
Scroem. I'effraye. Horreo.
Schrie. I'aiambe. Facio gradum.
Schrijf I'eſcripts. Scribo.
Schud. Ie ſecoue. Quaſſo.
Schup. I'houe. Palea leuo.
Schur. I'eſcure. Tergo. (occludo
Schut. Ie contregarde. Aſſamentis
Schu. I'euite. Euito.
Schuil. Ie m'embuſche. Lateo.
Schuyf. Ie coule. Trudo.
Seep. Ie ſauonne. Sapone linio.
Seg ie di. Dico.
Send. I'enuoie. Mitto.
Seng. Ie bruſle quelque peu. Suburo
Set. Ie mets. Pono.
Stel. Ie mets. Pono.

Suer. Ie trompe. Impono.
Seyl. Ie vogue. Velifico.
Sie. Ie boullis. Ferueo.
Sie. Ie voy. Video.
Sift. Ie crible. Cribro.
Sijp. Ie degoute. Mano.
Sinck. I'enfonſe. Mergo.
Sing. Ie chante. Canto.
Sit. I'aſſis. Sedeo.
Slab. Ie baue. Squaleo.
Sla. Ie frappe. Verbero.
Slacht. Ie reſemble de condition. Aſſimilor.
Slaep. Ie dors. Dormio.
Slap. Ie laſche. Laxo.
Slaef. Ie trauaile comme eſclaue. Laboro.
Slem. Ie fai bonne chere. Comeſſor.
Sleur. Ie traine. Traho.
Sleyp. Ie traine. Traho.
Slicht. Ie fais vni. Plano.
Slijp. I'aguiſe. Acuo.
Slijt. I'vſe. Tero.
Slis. I'appaiſe. Paco.
Slick. I'aualle. Gurgito.
Slof. Ie trouſſe. Replico.
Slorp. I'hume. Sorbeo longo tractu.
Sluym. Ie ſommeile. Dormito.
Sluyp. Ie voy en cachette. Clanculum ingredior.
Sluyt. Ie ferme. Claudo.
Smacht. Ie ſuffoque. Suffoco.
Smack. Ie rue de roideur. Proijcio
Smaeck. Ie gouſte. Guſto.
Smal. I'attenue. Extenuo.
Sme. Ie forge. Cudo.
Smeeck. Ie amadoue. Adulor.
Smeer. I'oins. Linio.
Smelt. Ie fonds. Fundo.
Smet. Ie macule. Maculo.
Smets. Ie bauſtre. Comeſſor.
Smoeck. Ie fume. Fumo.
Smoor. I'eſtouffe. Suffocor.
Smijt. Ie bats. Verbero.
Snap. Ie babille. Garrio.
Snau. Ie parle rudement. Duriter loquor.
Snie. Ie coupe. Scindo.
Snoep. Ie friande en cachette. Clam cupedias edo.
Snoer. I'enfile. Filo traiicio.
Snoey. I'esbranche. Frondo.
Snorck. Ie ſanglotte. Sterto.
Snuyt. Ie mouche. Mungo.
Soeck. Ie cherche. Quæro.
Soen. Ie reconcilie. Reconcilio.
Soog. I'alecte. Lacto.
Sorg. I'ay ſoing. Curo.
Sout. Ie ſale. Salio.
Spa. I'houe. Fodio.
Span. Ie tends. Tendo.
Spaer. I'eſpargne. Parco.
Speck. Ie larde. Lardo.
Speel. Ie ioue. Ludo.
Spen. Ie ſeure. Ablacto.
Speer. Ie tends. Tendo.
Speet. I'embroche. Figo veru.
Speur. Ie trache. Indago.
Spie. Ie cheuille. Clauis ligneis figo
Spie. I'eſpie. Inſidior.
Spin. Ie file. Neo.
Spits. Ie fai pointu. Acuo.
Spit. Ie foue. Fodio.
Splijt. Ie fends. Findo.
Spoey. Ie me haſte. Accelero.
Spoel. Ie deuuide du fil pour tiſtre. Glomero. (concito.
Spoor. I'eſperonne. Calcaribus
Spot. Ie mocque. Derideo.
Spou. Ie craiche. Spuo.
Spreeck. Ie parle. Loquor.
Sprey. I'eſtens. Tendo.
Spring. Ie ſaute. Salto.
Spruyt. Ie germe. Germino.
Spuel. Ie reinſe. Eluo.
Spuyt. Ie iecte l'eau par vn eſcliſſoire. Eijcio aquam ſyringe.
Staeck. Ie paiſſele. Palo.
Sta. Ie me tiens ſur les pieds. Sto
Staeck. Ie ceſſe. Deſiſto.
Stamp. Ie pile. Tundo.
Stap. I'aiambe. Pleno gradu incedo.
Steeck. Ie pique. Pungo.
Steel. Ie derobbe. Furor.
Stek. Ie poſe. Pono.
Stelp. I'eſtanche le ſang. Siſto ſanguinem.
Stem. Ie baille ma voix. Suffragium fero.
Steen. Ie ahenne. Gemo.
Sterf. Ie meurs. Morior.
Steun. I'appuye. Fulcio.
Steyl. Ie dreſſe contremont. Erigo.
Sticht. Ie fonds. Fundo.
Stijg. Ie monte. Scando.
Stijf. Ie roidis. Rigeo.
Stil. I'appaiſe. Paco.
Stinck. Ie ſens mal. Oboleo.
Stoor. Ie trouble. Turbo.
Stoot. Ie hurte. Trudo.
Stop. I'eſtoupe. Obthuro.
Stort. I'eſpans. Effundo
Stoof. I'eſtuue. Foueo.
Stou. I'incite de faire ou d'aller. Propello.
Strael. Ie rayonne. Radio.
Straf. Ie punis. Punio.
Streck. Ie tens. Tendo.
Streel. Ie peine. Pecto.
Strick. Ie noue. Nodo.
Stroop. I'eſcorche. Deglubo.
Stroey. I'eſpars. Spargo.
Strijck. Ie frotte. Linio.
Strie. Ie combats. Pugno.
Stier. Ie conduicts. Duco.
Stuyp. I'encline. Inclino.
Stuyt. Ie vante. Iacto.
Stuyt. Ie bonds. Reſulto.
Stuyf. Ie pouldroye. Puluero.
Sucht. Ie ſouſpire. Suſpiro.
Suf. Ie radotte. Deliro.
Suyg. Ie tette. Sugo.
Suym. Ie chome. Moror.
Suyp. I'hume. Sorbeo.
Svvack. I'affoiblis. Infirmo.
Svvaer. I'appeſantis. Grauo.
Svvalp. Ie flote. Affluo.
Svvert. ie noircis. Nigro.
Svveet. Ie ſue. Sudo.
Svvelg. I'aualle. Glutio.
Svveel. I'enfle. Tumeo.
Svvem. Ie nage. Nato.
Svveer. Ie iure. Iuro.
Svverm. I'eſcheme. Examino.
Svvicht. Ie ceſſe. Ceſſo.
Svvig. Ie tais. Taceo.
Tael. I'enquiers. Inquiro.
Tap. Ie tire. Promo.
Tas. I'entaſſe. Acerbo.
Taſt. Ie taſte. Palpo.
Tees. I'eſpluche. Carpo.
Tel. Ie nombre. Numero.

Tem

Tem. Ie dompte. Domo.
Tems. Ie tamise. Cribro.
Teer. Ie digere. Coquo cibum.
Tier. Ie tempeste. Tumultuo.
Toef. I'attens. Expecto.
Tol. Ie donne gabelle. Tributũ do.
Ton. I'entonne. Infundo in dolia
Toog. Ie monstre. Ostendo.
Toom. Ie bride. Freno.
Toon. Ie sonne. Tono.
Top. Ie ioue de la toupie. Trocho ludo.
Tau. Ie tanne. Coria perficio.
Tracht. Ie delibere. Delibero
Traen. Ie larmoye. Lachrymo.
Treck. Ie tire. Traho.
Terd. Ie marche. Gradior.
Tref. Ie touche. Tango.
Troost. Ie console. Consolor.
Trau. I'espouse. Duco vxorem.
Ick Treur. Ie contriste. Mœreo.
Tuyg. Ie tesmoigne. Testor.
Tuyn. I'enuironne de hayes. Sepio
Tuysch. Ie ioue à dets. Alea ludo.
Tvvist. I'estriue. Alterco.
Tvvyn. Ie tors du fil. Torqueo filũ
Val. Ie tombe. Cado.
Valsch. Ie faulce. Falso.
Vang. Ie prens. Capio.
Vaer. Ie voy par chariot ou par nauire. Meo.
Vast. Ie ieune. Ieiuno.
Vaet. i'entonne. Infundo modios
Vaet. i'empoigne. Comprehendo
Vecht. ie combats. Pugno.
Vel. I'abbats. Prosterno.
Verg. Ie mects au deuant. Propono
Vest. Ie confirme. Confirmo.
Veyl. Ie mects en vente. Venale propono.
Veins. Ie dissimule. Dissimulo.
Vier. ie fai feu de ioye. Celebro Vulcania.

Vyl. ie lime. Limo.
Vys. ie vire. Verto cochleam.
Vil. i'escorche la peau. Deglubo.
Vind. Ie trouue. Inuenio.
Visch. Ie pesche. Piscor.
Vlack. ie planis. Planum facio.
Vlam. ie flamboye. Flammo.
Vlecht. i'entrelasse. Vieo.
Vleck. Ie macule. Maculo.
Vley. Ie flatte. Adulor.
Vlie. ie fui. aufugio.
Vlieg. ie vole. Volo.
Vliem. ie lance. Scalpello lancino
Vliet. ie flotte. Fluo.
Vloeck. ie maudis. Execror.
Vloey. ie flotte. Fluo.
Vlooy. i'espluche. Capto pulices
Vlucht. ie prens la fuite. Agito fugam.
Vocht. ie ramoitis. Humecto.
Voed. ie nourris. Nutrio
Voel. ie sens. Sentio.
Voer. Ie meine. Veho.
Volg. ie sui. Sequor.
Vau. ie plie. Plico.
Vracht. Ie charge voicture. Onero vectura.
Vraeg. ie demande. Interrogo.
Vrees. ie crain. Timeo.
Vries. i'engele. Gelo.
Vroom. Ie corrobore. Corroboro
Vrye. Ie fay l'amour. Procor
Vul. i'emplis. Impleo.
Wacht. ie garde. Custodio.
VVaeg. ie hasarde. Aleam omnem iacio.
VVaeck. ie fai le guet. Vigilo.
Vvaey. ie vente. Spiro.
Vvalg. i'ay appetit de vomir. Nauseo.
Vval. ie fourboulis. Subferueo.
Vvaen. ie presume. Præsumo.

Vvan. ie vanne. Vanno.
Vvasch. ie laue. Lauo.
Vvas. ie crois. Cresco.
Vvas. ie couure de cire. Cero.
Vved. ie gage. Certo.
Vveyck. i'amollis. Mollio.
Vveen. ie pleure. Ploro.
Vveer. ie defends. Defendo.
Vvend. ie tourne. Verto.
Vven. i'accoustume. Assuefacio.
Vverck. ie besongne. Operor.
Vverm. ie chauffe. Calefacio.
Vverp. ie iette. Iacio.
Vver. i'empestre. implico.
Vveet. ie scay. Scio.
Vvet. i'aguise. Acuo.
Vveef. ie tis. Texo.
Vveyd. ie pasture. Pasco.
Vvie. ie sarcle. Exstirpo herbas.
Vvieg. ie berche. Ventilo.
Vvil. ie veux. Volo.
Vvinck. ie cile. Nuo.
Vvind. i'enuelope. inuoluo.
Vvin. ie gaigne. Lucror.
Vvip. ie branfle haut & bas. Sursum & deorsum mobilito.
Vvisch. ie troche. Tergo.
Vvit. ie blanchis. Albo.
Vvoel. ie fais tumulte. Tumultuo.
Vvoest. ie desolé. Desolo.
Vvond. ie naure. Vulnero.
Vvoon. ie demeure. Moror.
Vvreck. ie venge. Vindico.
Vvring. ie tords. Torqueo.
Vvroet. ie fouille. Volutor.
Vvrijf. ie frotte. Frico.
Vvurg. i'estrangle. Strangulo.
Vvyck. ie fui place. Cedo.
Vvyd. i'eslargis. Amplifico.
Vvye. ie dedie. Dedico.
Vvys. ie monstre. Monstro.

LATYNSCHE EENSYLLABIGHE VVOORDEN, *die in het Duytsch oock al een silbich siin.*

Do. ie donne. ick gheef.
Flo. ie souffle. ick blaes.
No. ie naige. ick svvem.
Sto. ie suis debout. ick stae.
Sum. ie suis. ick ben.

GRIECSCHE EENSILBIGHE VVOORDEN *die uyt langhe vercort siin.*

Βδῶ	Βδέω of Βδῆμι	Κρῶ	Κρέω ende Κέρω	Σχῶ	Σχέω
Βλῶ	Βλῆμι	Κτῶ	Κτάω ende Κτέω	Σῶ	Σάω ende Σέω
Βῶ	Βόω	Κῶ	Κάω ende Κόω	Τλῶ	Τλάω
Γνῶ	Γνόω	Λῶ	Λάω	Τμῶ	Τέμνω
Γρῶ	Γρόω	Μνῶ	Μνάω of Μνέω	Τρῶ	Τρέω
Γῶ	Γάω ende Γέω	Νῶ	Νάω of Νέω	Τῶ	Τάω
Δμῶ	Δμάω of Δμέω	Ξῶ	Voor Ξέω	Φθῶ	Φθάω Φθέω Φθόω
Δρῶ	Voor Δράω	Πλῶ	Πλέω of Πλεόω	Φλῶ	Voor Φλάω ende Φλέω
Δῶ	Δέω ende Δίδωμι	Πνῶ	Πνέω of Πνεύω	Φρῶ	Φρῆμι
Ζῶ	Ζάω ende ζέω	Πτῶ	Πτάω of Πτῆμι	Φῶ	Φάω Φῆμι
Θλῶ	Θλάω	Πῶ	Πάω Πῆμι	Χρῶ	Χράω χρέω χρόω
Θνῶ	Θνάω of Θνήσκω	Ρῶ	Ρέω Ρόω eñ Ερέω	Χῶ	Χέω χέυω {χρείω
Θῶ	Θέω	Σκλῶ	Κλέω	Ψῶ	Ψάω ende Ψέω
Κλῶ	Κλάω, κλέω, κλείω	Σμῶ	Σμέω of Σμόω	Ω῀	Ἔω
Κνῶ	Κνάω of κνῆμι	Σπῶ	Σπάω	Ὧ	Ἕω

Dander

D'ANDER DVYTSCHE YNCKEL

GHELVYDEN, ALS DER NAMEN, BYNAMEN, VOORSETtinghen, *&c. ſiin in ghetale tot* 1418 *de Latijnſche (tot de tſaemvouging onbequaem) alleenlick* 158 *de Grieſche* 220 *Als volght.*

DVYTSCHE EENSILBIGHE NAMEN, BYNAMEN, &c.

Acht. Huict. Octo.
Ael. Anguille. Anguilla.
Aem. Caque. Cadus.
An. Aupres. Apud.
Aep. Sing. Simia.
Aer. Eſpie. Spica.
Aert. Complexion Complexio.
Aes. Apaſt. Eſca.
Aex. Hache. Aſcia.
Af. Ius. De.
Al. Tout. Totus.
Alf. Fee. Fatifer.
Als. Quand. Cum.
Am. Nourrice. Nutrix.
Ampt. Office. Officium.
Angſt. Anxieté. Anxietas.
Arm. Bras. Brachium.
As. Eſſieu. Axis.
Back. Auge. Linther.
Badt. Bain. Balneum.
Baeck. Machoire. Maxilla.
Baeck. Pharus.
Bael. Bale. Sarcina.
Baen. Parterre. Sphæriſterium.
Baer. Biere. Feretrum.
Baer. Onde. Vnda.
Baert. Barbe. Barba.
Baerſch. Perche. Perca.
Baes. Hoſte. Herus.
Baet Gaing. Commodum.
Baeg. Bague. Monile.
Bal. Eſteuf. Pila.
Balch. Panche. Beſtiarum venter.
Balck. Poutre. Trabs.
Bald. Incontinent. Breui.
Ban. Excommunication. Excommunicatio.
Banck. Banc. Scamnum.
Bandt. Lien. Vinculum.
Bang. Angoiſſeux. Anxius.
Bar. Preſent. Præſens.
Barg. Porceau chaſtré. Maialis.
Bas. Abbay. Latratus.
Baſt. Canepin. Scheda.
Baſt. Har. Laqueus.
Bat. Mieux. Melior.
Bay. Bayette. Badius color.
Beck. Bec. Roſtrum.
Bed. Lict. Lectus.
Be. Petition. Petitio.
Beelt. Image. Imago.
Beemt. Prai. Pratum.
Been. Os. Os.
Been. Iambe. Crus.
Beer. Verrat. Verres.
Beer. Ours. Vrſus.
Beeſt. Beſte Beſtia.
Bel. Clochette. Tintinnabulum.
Ben. Banne. Sporta.
Berdt. Ais. Aſſer.
Berg. Mont. Mons.
Beſt. Mieux. Melior.
Bey. Tous deux. Ambo.
Biecht. Confeſſion. Confeſſio.
Bie. Mouche à miel. Apes.
Bier. Biere. Cercuiſia.
Bies. Ionc. Iuncus.
Bieſt. Caille. Coloſtra.
Bladt. Foeuille. Folium.
Blas. Soufflement. Flatus.
Blaes. Veſſie. Veſica.
Blaeu. Bleu. Cærulus.
Bleck. Foeuille ou lame de quelque metal. Lamina.
Bleeck. Palle. Pallidus.
Blein. Empoulle. Puſtula.
Blie. Ioyeux. Hilaris.
Blindt. Aueugle. Cæcus.
Block. Tronc. Truncus.
Bloet. Sang. Sanguis.
Bloem. Fleur. Flos.
Blont. Blont. Flauus.
Bloo. Timide. Timidus.
Bloot. Nud. Nudus.
Bock. Bouc. Hircus.
Bo. Meſſagier. Nuncius.
Boeck. Liure. Liber.
Boef. Ribaud. Nebulo.
Boel. Amoureuſe. Amica.
Boer. Villageois. Ruſticus.
Boerd. Bourde. Nugæ.
Boet. Penitence. Pœnitentia.
Boey. Piege. Pedica.
Boog. Arc. Arcus.
Bolck. Molue. Molua.
Bol. Boule. Globus.
Bom. Bedon. Tympanum.
Bont. Fourrure. Pelles.
Boom. Arbre. Arbor.
Boon. Febue. Faba.
Boord. Bord. Margo.
Boos. Mauuais. Malus.
Boot. Bateau. Scapha.
Borg. Bourg. Caſtrum.
Boor. Vibrequin. Terebrum.
Bors. Bourſe. Burſa.
Borſt. Poictrine. Pectus.
Boſch. Bois. Sylua.
Bot. Petoncle. Paſſer.
Bot. Stolide. Stolidus.
Bot. Bouton de fleur. Gemma.
Bou. Hardi. Audax.
Bout. Bougeon. Sagitta capitata.
Brack. Sentant la marine. Marinum
Brack. Bracque. Canis ſagax.
Braey. Le gras de la iambe. Sura.
Braem. Ronce. Rubus.
Brand. Vn grand feu brulant vne maiſon où ſemblable. Incendium.
Brau. Sourcil. Supercilium.
Breet. Large. Latus.
Breuck. Amande. Mulcta pecuniaria
Brief. Lettre. Literæ.
Brijn. Saumure. Muria.
Bril. Lunette. Specillum.
Brim. Geneſt. Geniſta.
Brock. Vn petit morceau du pain taillé ou rompu. Fruſtum.
Broeck. Marez. Palus.
Broeck. Brayette. Subligaculum.
Broer. Frere. Frater.
Broot. Pain. Panis.
Brooſch. Fragile. Fragilis.
Brug. Pont. Pons.
Bruyck Vſage. Vſus
Bruydt. Eſpouſe. Sponſa.
Bruyn. Brun. Beticus color.
Bry. Boullie de farine de panis. Puls
Buel. Bourreau. Carnifex.
Buer. Voiſin. Vicinus.
Buyt. Butin. Præda.
Bult. Boſſe. Gibbus.
Burn. Fontaine. Fons.
Bus. Canon. Tormentum.
Bus. Boite. Pyxis.
Buyck. Ventre. Venter.
Buil. Gibeciere. Marſupium.
Buil. Boſſe. Tuber.
Buis. Canal. Canalis.
By. Pres. Propè.
Cael. Chauue. Caluus.
Caen. Caniſſure. Canus.
Caerd. Chardon. Virga Paſtoris.
Caets. Chaſſe. Meta.
Caf. Paille. Acus.
Calck. Chaux. Calx.
Cant. Bord. Extremitas.
Cap. Cappe. Cuculla.
Car. Chariot. Carrus.
Caes. Fourmage. Caſeus.
Cas. Caſſe. Capſa.
Cat. Chat. Felis.
Cau. Chucas. Monedula.
Cijs. Cens. Cenſus.
Cier. Chere. Vultus lætus.
Claer. Clair. Clarus.
Clerck. Clerc. Clericus.
Cloof. Fente. Fiſſura.
Cluit. Farce. Facetiæ.
Cock. Cuiſinier. Coquus.
Colf. Maſſue. Claua.
Com. Eſcueille. Scutella.
Comſt. Venue. Aduentus.
Cond. Notoire. Notus.
Coord. Corde. Chorda.
Cop. Chef. Caput.
Cop. Coupe. Calix.
Corf. Corbeille, Corbis.
Corſt. Crouſte. Cruſta.
Cort. Court. Curtus.
Coſt. Couſt. Sumptus.
Cot. Taniere. Cauus.
Coot. Oſſelet. Talus.
Cous. Chauſſe. Caliga.
Coy. Eſtable à brebis. Ouile.
Crab. Eſcreuiſſe. Cammarus.
Craem. Boutique. Officina.
Craen. Robinet. Epiſtomium.
Craen. Grue. Grus.
Craey. Corneille. Cornix.
Crap. Garance. Erythrodanum.
Croon. Couronne. Corona.

Cruyn. Sommet. de la teste. Vertex capitis.
Crays. Croix. Crux.
Cuyp. Cuue. Cupa.
Cuyt. Oeufs d'vn poisson. Oua pisciū
Dach. Iour. Dies.
Dack. Toict. Tectum.
Daet. Effect. Effectus.
Daer. La. Ibi.
Dal Val. Vallis.
Dam. Terrain. Agellus.
Damp. vapeur. vapor.
Dan. Donc. Tunc.
Danck. Gre. Gratia.
Dans. Danse. Saltatio.
Darm. Boiau. Intestinum.
Das. Daim. Dama.
Dat. Ce. Hoc.
Dau. Rosee. Ros.
De. Le. Illa.
Deech. Paste. Massa.
Deel. Part. Pars.
Deern. Seruante. Ancilla.
Den. Le.
Des. Du.
Dees. Cestui. Hic.
Deucht. vertu. virtus.
Dicht. Solide. Solidus.
Dicht. Rim. Rithmus.
Dick. Espes. Densus.
Die. Cuisse. Femur.
Die. Le. Ille.
Dief. Larron. Latro.
Diep. Profond. Profundus.
Dier. Animal.
Dier. Cher. Carus.
Dies. A telle condition. Sub conditione.
Dijc. Dique. Agger.
Dinc. Chose. Res.
Disch. Table. Mensa.
Dit. Ceci. Hoc.
Doch. Aumoins. Saltem.
Doe. Alors. Tunc.
Doec. Toile. Tela.
Doel. But. Scopus.
Door. Par. Per.
Door. Huis. Fores.
Donst. Duuet. Plumulæ molliores.
Doot. Mort. Mors.
Doof. Sourd. Surdus.
Doop. Baptesme. Baptismus.
Door. Fol. Stultus.
Doos. Boitte. Capsa.
Dop. Escaille entier d'vn œuf quand le dedens est osté. Ouum exinanitū.
Dorp. village. Pagus.
Dorst. Soif. Sitis.
Doy. Degel. Regelatio.
Dra. Incontinent. Statim.
Draec. Dragon. Draco.
Draet. Fil. Filum.
Draf. Bran. Furfur.
Draf. Trot. Succussatio equi
Dranc. Beuvrage. Potus.
Drec. Boue. Lutum.
Dreef. Vne longue rangee d'arbres plantees. Series arborum.
Droef. Triste. Tristis.
Droes. vne enflure venant à la gorge derriere les aureilles ou es eines. Panus.
Drom. Le fil de la treme du tisserant. Licium.
Dronc. Yvre. Ebrius.
Drooch. Sec. Siccus.
Droom. Songe. Somnium.
Druc. Tristesse. Tristitia.
Drop. Goute. Gutta.
Druys. Grappe. Vua.
Drie. Trois. Tres.
Dul. Enragé. Furiosus.
Dun. Tenue. Tenuis.
Duyst. Mille. Mille.
Duym. Poulce. Pollex.
Duyn. Dune. Agger arenosus.
Duys. Colomb. Columba.
Dvvael. Touaille. Mantile.
Dvvaes. Sot. Stultus.
Dvvanc. Contraincte. Vis.
Dvvee. Mol. Mollis.
Dvveers. De trauers. Extransuerso
Dvverch. Nain. Nanus.
Dy. Toi. Tibi.
Dijn. Tien. Tuus.
Echt. Mariage. Matrimonium.
Eec. Vinaigre. Acetum.
Eedt. Serment. Iuramentum.
Een. Vn. Vnus.
Eer. Auant. Prius.
Eer. Honneur. Honor.
Eerd. Terre. Terra.
Eerst. Premier. Primus.
Eg. Herse. Occa.
Elf. Onze. Vndecim.
Elst. Alose. Alosa.
El. Aulne. Vlna.
Eind. Fin. Finis.
Eng. Estroict. Angustus.
Erch. Maling. Malignus.
Erm. Pauure. Pauper.
Ernst. A bon escient. Seriò.
Erf. Heritage. Hæredium.
Esch. Fresne. Fraxinus.
Ey. Oeuf. Ouum.
Eyc. Chesne. Quercus.
Eysch. Petition. Petitio.
Fael. Faulte. Error.
Faem. Renom. Fama.
Feest. Feste. Festum.
Fel. Felon. Crudelis.
Feil. Faute. Error.
Fielt. Gueu. Mendicus.
Fier. Fier. Ferus.
Fijn. Fin. Exilis.
Flesch. Flacon. Lagena.
Fluex. Subit. Subito.
Fluym. Fleume. Phlegma.
Fluyt. Fluite. Fistula.
Foc. Trinquet. Artemon.
Foey. Phi.
Form. Forme. Forma.
Fraey. Ioli. Lepidus.
Fret. Foret. viuerra.
Frisch. vigoreux. viuidus.
Fruit. Fruict. Fructus.
Gaef. Pur. Purus.
Gaer. Totalement. Omnino.
Gay. Gai. Psittacus.
Gal. Fiel. Fel.
Galgh. Gibbet. Patibulum.
Galm. Retentissemēt de la voix. Echo
Ganc. Allure. Incessus.
Ganc. Allee. Ambulacrum.
Gans. Oie. Anser.
Gants. Entier. Integer.
Garst. Ranci. Rancidus.
Gast. Hoste. Hospes.
Gat. Trou. Foramen.
Gaef. Don. Donum.
Gau. Prompt. Promptus.
Gec. Badin. Sannio.
Geel. Iaune. Rufus.
Gheen. Nul. Nullus
Gheest. Esprit. Spiritus.
Ghelt. Argent. Pecunia.
Ghelt. Porceau chaitrè. Maialis.
Ghelt. Lot. Quatuor heminæ.
Ghent. Iar. Anser mas.
Geut. Fonte. Fusura.
Gheit. Cheure. Capra.
Ghi. vous. Tu.
Ghier. vautour. vultur.
Ghift. Don. Donum.
Ghild. Homme liberal. Prodigus.
Ghins. vers la. Illuc.
Ghist. Lie. Fæx.
Glants. Resplendeur. Splendor.
Glas. verre. vitrum.
Glat. Poli. Politus.
Godt. Dieu. Deus.
Goedt. Bon. Bonus.
Gom. Gomme. Gummi.
Gort. Orge seiche. Hordeum aridum.
Goot. Ruisseau. Aquæductus.
Goudt. Or. Aurum.
Gracht. Fosse. Fossa.
Graef. Comte. Comes.
Graen. Grain. Granum.
Graet. Areste. Arista.
Graf. Sepulchre. Sepulchrum.
Gram. Courroucé. Iratus.
Gras. Herbe. Gramen.
Graeu. Gris. Glaucus.
Grents. Frontiere. Ora.
Greep. Poignee. Manipulus.
Griel. Gripaille.
Grijs. Gris. Canus.
Groef. Fosse. Fouea.
Grof. Gros. Grossus.
Grond. Fond. Fundum.
Groot. Grand. Grandis.
Grou. Horreur. Horror.
Gruen. verd. viridis.
Gruet. Salutation. Salutatio.
Gruis. Moilon. Rudus.
Gunst. Faueur. Fauor.
Guit. Gueu. Mendicus.
Haec. Croc. Hama.
Haeg. Haie. Seps.
Haen. Coc. Gallus.
Haer. Poil. Pilus.
Haest. Haste. Properatio.
Haet. Haine. Odium.
Half. Demi. Dimidius.
Hals. Col. Collum.
Ham. Iambon. Perna.
Handt. Main. Manus.
Harp. Harpe. Lyra.
Hars. Resine. Resina.
Haes. Lieure. Lepus.
Hecht. Manche. Capulus.
Hec. Claie. vacerra.
Heel. Tout. Totus.
Heerdt. Fouier. Focus.
Heer. Seigneur. Herus.
Heesch. Enroué. Raucus.
Heet. Chaud. Calidus.
Hel. Clair. Clarus.
Held. Homme Noble. Herus.
Hel. Enfer. Infernus.
Helm. Heaume. Galea.
Hem. Lui. Illi.
Hend. Chemise. Indusium.
Hengst. Estallon. Caballus.
Hert. Dur. Durus.
Herst. Autumne. Autumnus.
Herst. Eschinee. Spina porci.

Here

Hert. Cœur. Cor.
Hesp. Iambon. Perna.
Het. Ce. Id.
Heur. Sienne. Sua.
Heusch. Courtois. Ciuilis.
Heus. Anse. Ansa.
Hex. Sorciere. venefica.
Hey. Lande. Campus sterilis.
Hey. Hie. Fistuca.
Heyl. Salut. Salus.
Heir. Armée. Exercitus.
Hic. Hoquet. Singultus.
Hiel. Talon. Talus.
Hier. Ici. Hic.
Hind. Biche. Cerua.
Hin. Poulle. Gallina.
Hirs. Mil. Milium.
Hoe. Comment. Quomodo.
Hoeck. Coing. Angulus.
Hoen. Poulle. Gallina.
Hoer. Paillarde. Meretrix.
Hoest. Toux. Tussis.
Hoet. Chapeau. Pileus.
Hoef. Metaire. Villa.
Hof. Iardin. Hortus.
Hoir. Heritier. Heres.
Hol. Caue. Cauus.
Hondt. Chien. Canis.
Hooch. Hault. Altus.
Hooft. Teste. Caput.
Hoop. Monceau. Aceruus.
Hoop. Espoir. Spes.
Hop. Houbelon. Lupulus.
Hop. Hupe. Vpupa.
Hord. Caye. Crates.
Hoos. Chausse. Caliga.
Hout. Bois. Lignum.
Hou. Coup de taille. Ictus.
Hoy. Foin. Fœnum.
Hulp. Aide. Auxilium.
Hulst. Boux. Aquifolia.
Hupsch. Elegant. Elegans.
Hut. Loge. Mapale.
Huych. Luette en la gorge. Angina.
Huyck. Hucque. Cucullus.
Huyl. Cheueche. Vlula.
Huys. Maison. Domus.
Hy. Il. Ille.
Ia. Oui. Ita.
Iacht. Chasse. Venatus.
Iaer. An. Annus.
Ick. Ie. Ego.
Iet. Quelque chose. Aliquid.
Iuecht. Ieunesse. Iuuentus.
Ys. Glace. Glacies.
In. En. In.
Inct. Encre. Atramentum
Iock. Ioug. Iugum.
Iock. Raillerie. Iocus.
Ionck. Ieune. Iuuenis.
Is. Est. Est.
Kaeck. Machoire. Maxilla.
Kaey. Cai. Acta.
Kaeck. Pilori. Numellæ versatiles.
Kalf. Veau. Vitulus.
Kam. Peigne. Pecten.
Kan. Pot. Amphora.
Kans. Chansse. Casus aleæ.
Kant, als brood. Chanteau. Frustum
Keel. Garderobbe. Supparum.
Keer. Tour. Circuitus.
Keers. Chandelle. Candela.
Kelck. Calice. Calix.
Keel. Gorge. Guttur.
Kees. Fourmage. Caseus.

Kemp. Chanure. Cannabis.
Kerck. Eglise. Templum.
Kerf. Crenne. Crena.
Kern. Pepin. Semen.
Kers. Guine. Cerasa.
Kers. Cresson. Narstutium.
Kert. Cren. Crena.
Keur. Chois. Optio.
Keurs. Corset. Cyclas.
Key. Caillou. Cautes.
Kiel. Carine. Carina.
Kim. Cipeau d'vn tonneau. Ora vasis.
Kindt. Enfant. Puer.
Kin. Menton. Mentum.
Kist. Coffre. Cista.
Kit. Boisson. Brochus.
Klacht. Querelle. Querela.
Klack. Creuasse. Crepitatio.
Klad. Crote. Macula luti.
Klamp. Membrure d'vn huis. Membrum asseris.
Klanck. Tintement. Tinnimentum.
Klap. Babil. Loquacitas.
Klau. Patte. Vnguis.
Kleet. Vestement. Vestis.
Klef. Attachant comme glu. Tenax.
Kley. Argille. Argilla.
Klein. Petit. Paruus.
Klier. Aposteume. Tonsilla.
Klinck. Clichet. Pessulus.
Klis. Grateron. Aparine.
Klock. Cloche. Campana.
Kloeck. Hardi. Audax.
Kloet. Rable. Rutabulum.
Klomp. Billot. Massa.
Kloot. Boule. Sphæra.
Klop. Coup. Ictus.
Kloof. Creuasse. Rima.
Klucht. Farce. Facetiæ.
Kluys. Heremitage. Sacellum.
Knaep. Seruiteur. Seruus.
Knecht. Garson. Seruus.
Knick. Hocement de la teste. Nutus
Knie. Genouil. Genu.
Knip. Chiquenaude. Talitrum.
Knol. Naueau. Napus.
Knoop. Neud. Nodus.
Knop. Bouton. Bulla.
Koe. Vache. Vacca.
Koeck. Gateau. Libum.
Koel. Tiede. Tepidus.
Koen. Hardi. Audax.
Koets. Couche. Cubile.
Kool. Charbon. Carbo.
Kool. Chou. Brassica.
Koop. Achapt. Emptio.
Korck. Liege. Suber.
Koudt. Froid. Frigus.
Kout. Deuis. Fabula.
Kracht. Force. Virtus.
Krack. Son esclatant. Crepitus.
Kraegh. Gouion. Iugulus.
Kramp. Crampe. Spasmus.
Kram. Crampon. Fibula.
Kranck. Debile. Debilis.
Krans. Chapeau de fleurs. Sertum.
Kreeft. Escreuisse. Cancer.
Krib. Creche. Præsepium.
Krieck. Cerise. Cerasus.
Krijch. Guerre. Bellum.
Krijt. Croie. Creta.
Krijt. Braiement. Eiulatio.
Kroes. Goblet. Scyphus.
Krom. Tortu. Tortus.
Krop. Cropion. Ingulum.
Kruyck. Cruche. Vrna.

Kruym. Mie. Mica.
Kruydt. Herbe. Herba.
Rud. Troupeau de bestes. Grex.
Kund. Notoire. Notus.
Kunst. Art. Ars.
Kus. vn baiser. Basium.
Kuyl. Spelonque. Spelunca.
Kuysch. Chaste. Castus.
Kuyt. Le mol derriere la iambe. Sura.
Kuyt. Petite biere.
Lach. Ris. Risus.
Lacy. Layette. Capsa.
Laet. Tard. Sero.
Laf. Fade. Flaccidus.
Laegh. Rang. Series.
Lam. Affoible. Paralyticus.
Lam. Agneau. Agnus.
Lamp. Lampe. Lampas.
Lanc. Flanc. Femen.
Lanc. Long. Longus.
Landt. Terre. Terra.
Lap. Piece de drap. Segmentum.
Last. Charge. Moles.
Lat. Late. Assula.
Laeu. Tiede. Tepidus.
Lec als Lec schip.
Leer. Cuir. Corium.
Leech. Oisif. Otiosus.
Ledt. Membre. Membrum.
Leech. Bas. Humilis.
Leec. Lay. Laicus.
Leer. Eschelle. Scala.
Leedt. Desplaisir. Luctus.
Leem. Argille. Argilla.
Leen. Fief. Prædium beneficiarium.
Leep. Chassieux. Lippus.
Leep. Cauteleux. Astutus.
Leer. Doctrine. Doctrina.
Leest. Forme de corduanier. Forma.
Leeu. Lion. Leo.
Lil. Le mollet du bout de l'aureille. Cartilago.
Leen. Appuy. Podium.
Lest. Dernier. vltimus.
Lets. Laisse. Lorum.
Leur. Rauauderie. Res nullius valoris
Ley. Ardoise. Ardosia.
Licht. Lumiere. Lux.
Licht. Legier. Leuis.
Licht. Poulmon. Pulmo.
Liet. Chanson. Cantio.
Lief. Cher. Charus.
Lief. Ami. Amicus.
Lier. Lire. Lyra.
Lijc. Funerailles. Exequiæ.
Lijf. Corps. Corpus.
Lijm. Colle. Colla.
Lijn. Lin. Linum.
Lijst. Bordure. Limbus.
Lind. Tillet. Tilia.
Lint. Ruben. vitta.
Lip. Leure. Labium.
Lis. Ranse. Carex.
List. Finesse. Astutia.
Loen. Lourdaut. Idiota.
Lof. Los. Laus.
Lont. Meiche.
Looc. Des aulx. Allium.
Loof. Foeuille. Frons
Loogh. Lexiue. Lixiuium.
Loon. Salaire. Salarium.
Loop. Cours. Cursus.
Loos. Subtil. Subtilis.
Loos. Poulmon. Pulmo.
Loot. Plomb. Plumbum.

Loof. Armier d'vne maiſon. Vmbraculum.
Loos. Mot de guet. Teſſera.
Los. Deſlie. Laxus.
Loſch. Louſche. Strabo.
Lot. Sort. Sors.
Lucht. Air. Aer.
Lul. Reſonance d'vne chanſon.
Luſt. Volupté. voluptas.
Luy. Pareſſeux. Ignauus.
Luys. Pouil. Pediculus.
Luyt. Luc. Teſtudo.
Macht. Puiſſance. Poteſtas.
Maech. Affin. Affinis.
Maecht. vierge. virgo.
Mael. Malle. Mantica.
Mael. Fois.
Maen. Lune. Luna.
Maent. Mois. Menſis.
Maer. Mais. Sed.
Maer. Bruict. Rumor.
Maet. Meſure. Menſura.
Maet. Compaignon. Socius.
Maegh. Eſtomach. Stomachus.
Mal. Fol. Stultus.
Mals. Tendre. Tener.
Mam. Mammelle. Mamma.
Man. Homme. vir.
Manck. Boiſteux. Claudus.
Maſt. Matz. Malus.
Mat. Las. Defeſſus.
Me. Auec. Cum.
Mee. Garance. Rubra.
Meel. Farine. Farina.
Meeps. Fragile. Fragilis.
Meer. Mer. Mare.
Meers. Hune. Carcheſium.
Meer. Plus. Plus.
Meerſch. Marez. Palus.
Mees. Mauſangle. Parix.
Meeſt. Tout le plus. Plurimus.
Meeu. Oiſeau marin. Aquila marina.
Melck. Laict. Lac.
Mem. Nourrice. Nutrix.
Men. On.
Menſch. Homme. Homo.
Merch. Moelle. Medulla.
Merck. Marque. Signum.
Merc. Marché. Forum.
Mes. Couſteau. Culrer.
Met. Auec. Cum.
Mey. May. Frons feſta.
Mier. Fourmi. Formica.
Mijl. Lieue. Milliare.
Mijn. Mon. Meus.
Mijne. Mine. Fodina.
Mijt. Mite. Mita.
Milt. Liberal. Liberalis.
Min. Moins. Minus.
Mis. Faute. Defectus.
Meſt. Fiens. Fimus.
Miſt. Bruyne. Bruma.
Moe. Las. Laſſus.
Moer. Mere. Mater.
Moes. Porée. Holus.
Moet. Courage. Animus.
Moey. Tante. Matertera.
Mol. Taulpe. Talpa.
Mondt. Bouche. Os.
Moort. Meurtre. Internecio.
Mos. Mouſſe. Muſcus.
Moſt. Mouſt. Muſtum.
Mot. Teigne. Tinea.
Mout. Grain appareillé pour braſſer de la biere. Polenta.
Mau. Manche. Manica.
Moy. Orné. Ornatus.
Muer. Mur. Murus.
Muf. Relant. Situs.
Mug. Moucheron. Culex.
Munt. Monnoie. Moneta.
Muts. Bonet. Pileus.
Muil. Mulet. Mulus.
Muyl. Muſeau. Roſtrum.
Muys. Souris. Sorex.
Muyt. Mue. Cauea.

Na. Apres. Poſt.
Nacht. Nuict. Nox.
Naeckt. Nud. Nudus.
Naem. Nom. Nomen.
Naen. Nain. Nanus.
Naeſt. Tout le plus prochain. Proximus.
Naet. Couſture. Sutura.
Nap. Plat creux. Catinus.
Nues. Nez. Naſus.
Nat. Mouillé. Madidus.
Nau. Eſtroict. Strictus.
Neck. Chainon. Ceruix.
Neer. Bas. Inferus.
Neef. Nepveu. Nepos.
Neen. Non. Non.
Neep. Pinſure. Compreſſio.
Neſt. Nid. Nidus.
Net. Net. Nitidus.
Net. Retz. Rete.
Neet. Lende. Lens.
Nicht. Niepce. Neptis.
Nier. Rein. Ren.
Niet. Rien. Nihil.
Nieu. Nouueau. Nouus.
Nijdt. Enuie. Inuidia.
Noch. Encore. Adhuc.
Noen. Midi. Meridies.
Noo. A regret. Inuitus.
Noort. North. Septentrion.
Noot. Neceſſité. Neceſſitas.
Nop. Floc. Floccus.
Nuet. Noix. Nux.
Noit. Iamais. Nunquam.
Nu. Maintenaut. Nunc.
Nut. Vtile. Vtilis.
Och. Ah. Hei.
Oft. Ou. Vel.
Olm. Orme. Vlmus.
Om. Pour. Ob.
Ons. Noſtre. Noſter.
Oock. Auſſi. Etiam.
Oog. Oeil. Oculus.
Oogſt. Moiſſon. Meſſis.
Oom. Oncle. Patruus.
Oor. Aureille. Auris.
Oort. Lieu. Locus.
Ooſt. Orient. Oriens.
Op. Deſſus. Super.
Os. Beuf. Bos.
Oudt. Viel. Vetus.
Oyt. Onques. Vnquam.

Pacht. Ferme. vectigal.
Pack. Fardeau. Sarcina.
Pael. Pau. Palus.
Paer. Paire. Par.
Paert. Part. Pars.
Palm. Paulme. Palma.
Pand. Hypoteque. Pignus.
Pandt. Pand. Lacinia.
Pan. Paelle. Sartago.
Pap. Papin. Pappa.
Pas. En poinct. Commodum.
Pat. Sentier. Semita.
Paeu. Paon. Pauo.
Paeis. Paix. Pax.
Peck. Poix. Pix.
Peert. Cheual. Equus.
Peerſch. Pers. Cœruleus.
Pels. Peau. Pellis.
Pen. Plume. Calamus.
Pens. Trippe. Inteſtina.
Perck. Parc. Septum.
Peer. Poire. Pirum.
Pers. Preſſe. Torcular.
Peez. Corde d'arc. Chorda arcus.
Pijp. Tuyau. Tubus.
Pier. Vers de terre. Lumbricus.
Pijck. Pique. Haſta.
Pie. Manteau à marinier, Nautica penula.
Pijl. Fleſche. Sagitta.
Pijn. Doleur. Dolor.
Pin. Baſton poinctu. Veruculum.
Pips. Pepie. Pituita.
Plack. Ferule. Ferula.
Plaet. Planche. Syrtes.
Plaets. Place. Locus.
Plaegh. Vexation. Vexatio.
Planck. Planche. Planca.
Plas. Marée. Lacuna.
Plat. Plat. Planus.
Pleck. Taſche. Macula.
Pleit. Bateau large & plat. Stlata.
Plicht. Office. Officium.
Ploech. Charrue. Aratrum.
Plomp. Rebouché. Hebes.
Ploy. Plis. Plica.
Pluym. Plume. Pluma.
Pock. Verolle. Lues venerea.
Poer. Pouldre. Puluis.
Poel. Lac. Lacuna.
Pol. Concubinaire. Concubinus.
Pols. Pouls. Pulſus.
Pomp. Oſſée. Sentina.
Pondt. Liure. Pondo.
Poort. Porte. Porta.
Poos. Petit eſpace de temps. Momentum.
Poot. Patte. Palma pedis.
Pop. Pouppée. Pupa.
Poſt. Poſteau. Poſtis.
Poſt. La poſte. Curſor.
Poot. Sion. Talea.
Pot. Pot. Olla.
Pracht. Magnificence. Magnificentia.
Prat. Fier. Arrogans.
Prick. Lampreie. Muſtula.
Priem. Poinſon. Pugiunculus.
Prijs. Pris. Laus.
Proef. Preuue. Proba.
Pruym. Prume. Prunum.
Pruts. Superbe. Superbus.
Prye. Charogne. Cadauer.
Punt. Poinct. Punctum.
Put. Puis. Puteus.
Puyſt. Empoule. Puſtula.
Quaet. Mauuais. Malus.
Quael. Langueur. Languor.
Quant. Gallant. Scitus homo.
Quijt. Quicte.
Radt. Roue. Rota.
Raedt. Conſeil. Conſilium.
Raem. Chaſſis. Fulcrum feneſtræ quadratum.
Raep. Naueau. Rapum.
Ram. Belier. Aries.
Ramp. Malheur. Infelicitas.
Ranck. Branche. Ramus.
Ranck. Fineſſe. Aſtutia.
Ranck. Greſle. Gracilis.

Randt

Randt. Bord. Ora.
Raſch. Soudain. Cito.
Raſp. Rape. Scalprum.
Rat. Rat. Glis.
Raef. Corbeau. Coruus.
Raeu. Cru. Crudus.
Recht. Droict. Rectus.
Ree. Biche. Cerua.
Reep. Cercle. Circulus.
Reſt. Reſte. Reſiduum.
Rueck. Odeur. Odor.
Rue. Chien maſle. Canis mas.
Rues. Geant. Gigas.
Rey. Danſe. Chorea.
Reyn. Pur. Purus.
Reys. Fois.
Reys. voiage. Profectio.
Reb. Coſte. Coſta.
Riet. Canna. Arundo.
Riem. Ceinture. Cingulum.
Riem. Rame. Remus.
Rijck. Riche. Diues.
Rijm. Gelée. Pruina.
Rijm. Rhitme. Rhitmus.
Rijp. Meur. Maturus.
Rijs. Riz. Oriza.
Rijs. Branche. Ramus.
Rinck. Anneau. Annulus.
Rindt. Beuf. Bos.
Ring. viſte. velox.
Rin. Cage. Cauea.
Rinſch. Aucunement ſur. Subacidus
Roch. Raye. Raia.
Rock. Saie. Toga.
Rock. Quenouille. Colus.
Roe. verge. virga.
Roedt. Suie de cheminée. Fuligo.
Roef. Poupe. Puppis.
Roem. vanterie. Iactantia.
Roep. Cri. Clamor.
Roer. Gouuernal. Gubernaculum.
Roeſt. Enrouillure. Rubigo.
Roogh. Oeuf de poiſſon. Oua piſcium
Rog. Seigle. Siligo.
Rol. Roulle. Phalanga.
Rondt. Rond. Rotundus.
Roock. Fumée. Fumus.
Root. Rouge. Ruber.
Roof. Butin. Præda.
Room. Creime. Cremor.
Roos. Roſe. Roſa.
Ros. Roux. Rufus.
Ros. Ceual. Equus.
Rot. Pourri. Putris.
Rot. Bende de gens. Claſſis.
Rou. Rude. Rudis.
Ruet. Süif. Seuum.
Rug. Dos. Dorſum.
Rups. Chenille. Bruchus.
Ruſt. Repos. Quies.
Ruim. Ample. Amplus.
Ruyn. Hongre. Cantherius.
Ruyt. Rue. Ruta.
Ruyt. Lozenge. Teſſera.
Rie. Rang. Series.
Sacht. Mol. Mollis.
Sac. Sac. Saccus.
Sael. Selle. Ephippium.
Sael. Salle. Atrium.
Saen. Toſt. Statim.
Saen. Cremme. Cremor.
Saet. Semence. Semen.
Saey. Saiette.
Saeg. Sie. Serra.
Saec. Cauſe. Cauſa.
Salf. Onguent. vnguentum.
Salm. Saulmon. Salmo.
Sandt. Arene. Arena.
Sap. Suc. Succus.
Sarck. Tombe. Cippus.
Sadt. Saul. Satür.
Saus. Saulſe. Condimentum.
Saut. Sel. Sal.
Schacht. Fleſche. Sagitta.
Scha. Dommage. Damnum.
Schaeu. Ombre. Vmbra.
Schaeck. Eſchetz. Alueus.
Schael. Taſſe. Patera.
Schaep. Brebis. Ouis.
Schaerd. Teſt. Ruma.
Schaers. A peine. Vix.
Schaets. Eſchaſſe. Gralla.
Schalck. Caut. Cautus.
Schamp. Brocard. Scomma.
Schand. Deſhonneur. Ignominia.
Schants. Rempart. Vallum.
Schaer. Grande multitude de gens. Caterua.
Schat. Threſor. Theſaurus
Schaef. Rabot. Dolabra.
Schee. Gaine. Vagina.
Scheef. Bihay. Obliquus.
Scheel. Louche. Luſcus.
Scheel. Greue de la teſte. Separatio comæ.
Scheel. Couuercle. Operculum.
Scheer. Force. Forfex.
Schel. Sonnette. Tintinnabulum.
Schel. Eſcorce. Cortex.
Schelm. Meſchant. Nequam.
Schelp. Coquille. Calix.
Schenck. Don. Donum.
Scheen. Creue de la iambe. Tibia.
Scherf. Teſt. Teſta.
Scherp. Agu. Acutus.
Scheur. Fente. Fiſſura.
Scheut. Scion. Surculus.
Schicht. Dard. Iaculum.
Schier. Tantoſt. Mox.
Schijn. Lueur. Splendor.
Schijf. Tableau. Menſa.
Schil. Difference. Differentia.
Schilt. Eſcu. Scutum.
Schimp. Brocard. Iocus.
Schip. Nauire. Nauis.
Schoe. Soulier. Calceus.
School. Eſcole. Schola.
Schol. Sole. Solea.
Schoof. Gerbe. Faſcis ſpicarum.
Schoon. Beau. Pulcher.
Schoot. Giron. Gremium.
Schors. Eſcorce. Cortex.
Schout. Preteur. Prætor.
Schrab. Eſgrattigneure. Laceratio vnguium.
Schraegh. Treſteau. Fulcrũ menſariũ.
Schram. Berlaſſe. Vibex.
Schre. Adiambée. Paſſus.
Schreeu. Cri. Clamor.
Schreef. Traict. Tractus lineæ.
Schrift. Eſcripture. Scriptura.
Scroef. Eſcrouue. Cochlea.
Schub. Eſcaille de poiſſon. Squamma
Schud. Vautneant. Scurra.
Schult. Debte. Debitum.
Schup. Pelle. Pala.
Schuer. Grange. Granarium.
Schurft. Rongneux. Scabioſus.
Schu. Saunage. Agreſtis.
Schuym. Eſcume. Spuma.
Schuyt. Naſſelle. Nauicula.
Se. Couſtume. Mos.
See. Mer. Mare.
Seel. Groſſe corde. Funis.
Seem. Sameau. Corium hædinum.
Seep. Sauon. Sapo.
Seer. Vlcere. Vlcus.
Seer. Fort. Valdè.
Self. Meſme. Ipſum.
Ses. Six. Sex.
Seyl. Voile. Velum.
Seys. Faux. Falx.
Sicn. Soy. Se.
Sieck. Malade. AEgrotus.
Siel. Ame. Anima.
Sift. Crible. Cribrum.
Sijn. Son. Suus.
Sim. Singe. Simia.
Sin. Sens. Senſus.
Sint. Depuis. Poſt illa.
Slab. Bauette. Faſcia pituitaria.
Slach. Coup. Ictus.
Slaep. Tempes de la teſte. Tempora.
Slaep. Somne. Somnus.
Slang. Couleuure. Coluber.
Slap. Laſche. Laxus.
Slaef. Eſclaue. Seruus emptitius.
Slecht. Simple. Simplex.
Sleck. Limaçon. Limax.
Sle. Traineau. Traha.
Slee. Prune. Acatium.
Slet. Torchon. Peniculamentum.
Sleyp. Longue queue de veſtement.
Slijck. Boue. Lutum. (Syrma.
Slijm. Limon. Limus.
Slim. Abihay. Obliquus.
Slincx. Gauche. Siniſter.
Slip. Pand. Peniculamentum.
Slot. Serrure. Sera.
Sluys. Eſcluſe. Cataracta.
Smaet. Calumnie. Calumnia.
Smaeck. Gouſt. Guſtus.
Smal. Eſtroict. Arctus.
Smeer. Graiſſe. Abdomen.
Smert. Doleur. Dolor.
Smet. Macule. Macula.
Smis. Forge. Fabrica ferraria.
Smit. Mareſchal. Faber ferrarius.
Smout. Greſſe. Pinguedo.
Snap. Babil. Garrulitas.
Snaer. Corde de luc. Fides
Snau. Mot dit auec deſpit. Iracunda locutio.
Sne. Coupure. Sciſſura.
Snee. Neige. Nix.
Snel. Viſte. Celer.
Snip. Beccaſſe. Gallinago.
Snick. Souſpir. Suſpirium.
Snoeck. Brochet. Lupus.
Snoer. Cordon. Chorda.
Snuf. Rume. Rheuma.
Snoo. Meſchant. Vilis.
Suot. Morue. Pituita.
Soo. Ainſi. Sic.
Soch. Laict. Succus.
Sock. Chauſſon. Soccus.
Soch. Truye. Porca.
Soet. Doulx. Dulcis.
Sool. Semelle. Solea.
Soon. Fils. Filius.
Son. Soleil. Sol.
Soom. Bord. Limbus.
Sop. Feſte. Faſtigium.
Sop. Ius. Ius.
Sorg. Soing. Cura.
Sot. Fol. Stultus.
Spa. Houe. Ligo.
Spaey. Tard. Tardus.

Spaen. Esclat. Affula.
Span. Extenfion de la paulme. Spithama.
Specht. Piemar. Picus.
Speck. Lard. Lardum.
Spel. Ieeu. Lufus.
Speer. Lance. Lancea.
Speur. Trace du pas qui demeure apres auoir marché. Veftigium.
Spie. Cheuille. Impages.
Spie. Efpieur. Infidiator.
Spier. La chair blanche qui eft à la poictrine d'vn oifeau. Pulpa.
Spies. Pique. Hafta.
Spil. Fufeau. Fufus.
Spind. Huche. Penarium.
Spin. Araigne. Aranea.
Spint. Picotin. Corbula.
Spit. Broche. Veru.
Spits. Haultain.
Spoet. Hafte. Properatio.
Spoel. Nauette. Glomus textorius.
Spond. Chaflit. Sponda.
Spoor. Efperon. Calcar.
Sport. Efchellon. Climacter.
Spot. Mocquerie. Irrifio.
Spraeck. Langaige. Lingua.
Spreuck. Diction. Sententia.
Spreeu. Eftourneau. Sturnus.
Spriet. Iauelot. Venabulum.
Sproet. Lentille. Lentigo.
Spronck. Sault. Saltus.
Sprot. Harangade. Membras.
Sprau. Pepie des poules. Pituita in gallinis.
Spruyt. Ietton d'arbre. Germen.
Speu. Excluse. Cataracta.
Speut. Efcliffoire. Syrinx.
Spijs. Viande. Cibus.
Spijt. Defpit. Contumelia.
Stadt. Ville. Vrbs.
Staeck. Pali. Palus.
Stael. Acier. Chalybs.
Staet. Eftat. Status.
Stacy. Loifir. Otium.
Staf. Bafton. Baculus.
Stal. Eftable. Stabulum.
Stam. Lignage. Genus.
Stanck. Puanteur. Fœtor.
Standt. Cuue. Cupa.
Stang. Perche. Pertica.
Stand. Eftat. Status.
Stap. Pas. Paffus.
Steck. Bafton. Baculus.
Steeds. Affiduel. Affiduè.
Steech. Obftiné. Obftinatus.
Steeck. Coup. Ictus.
Steel. Tige. Caulis.
Steen. Pierre. Lapis.
Steert. Queue. Cauda.
Stel. (als ftel bier) Eftale. Vetus.
Stelt. Efchaffe. Gralla.
Stem. Voix. Vox.
Sterck. Fort. Fortis.
Steur. Eftourgeon. Turfio.
Steyl. Contremont. Surfum.
Stier. Taureau. Taurus.
Stijf. Roide. Rigidus.
Stijl. Pofteau. Poftis.
Stil. Quoy. Quietus.
Stock. Bafton. Baculus.
Stoel. Selle. Sedes.
Stof. Poudre. Puluis.
Stom. Muet. Mutus.
Stompt. Rebouché. Obtufus.
Stoop. Lot. Gelta.
Stoot. Poulfement. Concuffus.
Storm. Tempefte. Tempeftas.
Stoof. Eftuue. Hypocauftum.
Stout. Hardi. Audax.
Stracx. Incontinent. Quamprimum
Strael. Ray. Radius.
Straet. Rue. Platea.
Straf. Rigoreux. Durus.
Strang. Riuage de la mer. Littus.
Streeck. Traict. Tractus.
Streng. Afpre. Seuerus.
Streep. Traict. Stria.
Strick. Lacs. Laqueus.
String. Ridelle. Reftis.
Stronck. Tronchet. Truncus.
Stroo. Eftrain. Stramen.
Stroom. Cours de l'eau. Fluxus aquæ
Strop. Har. Vinculum.
Struyck. Plançon. Frutex.
Struys. Auftruche. Strutiocamelus
Struyf. Crefpes. Laganum.
Strijt. Bataille. Prœlium.
Stuck. Piece. Fruftum.
Stuer. Seuere. Seuerus.
Suer. Aigre. Acer.
Sulck. Tel. Talis.
Sus. Ainfi. Sic.
Sus. Tout quoy. Silentium.
Suydt. Midi. Meridies.
Suyl. Pilier. Columna.
Svvack. Debile. Debilis.
Svvaer. Pefant. Grauis.
Svvart. Noir. Niger.
Svveem. Becaffon. Rufticula minor
Svveep. Fouet. Flagrum.
Svveert. Efpee. Enfis.
Svveer. Vlcere. Vlcus.
Svveet. Sueur. Sudor.
Svverm. Iection de mouches. Examen apum.
Svvyn. Porceau. Porcus.
Sy. Elle. Illa.
Taeck. Rameau. Ramus.
Taeck. Certain oeuure par iour. Penfum.
Tael. Langue. Lingua.
Taert. Tarte. Scriblita.
Taey. Coriace. Lentus.
Tal. Nombre. Numerus.
Tam. Dompte. Manfuetus.
Tang. Tenaille. Forceps.
Tant. Dent. Dens.
Tap. Broche d'vn tonneau. Embolum vafis.
Tas (als hoytas) Fenil. Fenile.
Teen. Ofier. vimen.
Teen. Orteil. Digitus pedis.
Teer. Tendre. Tener.
Tel. Haquenée. Gradarius equus
Temft. Tamis. Cribrum.
Teuch. Traict. Hauftus.
Teyl. vn plat creux de terre. Gabata figlina.
Thien. Dix. Decem.
Tob. Cuue. Cupa.
Toch. Certes. verè.
Tocht (als tocht des heyrs) Le marcher de l'armée. Agmen.
Toe (als tot daer toe) A. Ad.
Tol. Gabelle. vectigal.
Tong. Langue. Lingua.
Ton. Tonneau. Dolium.
Toom. Reine d'vne bride. Habena.
Toon. Monftre. Demonftratio.
Toon. Son. Tonus.
Top. Toupie. Trochus.
Torn. Ire. Ira.
Torfch. Grappe. Racemus.
Torts. Torche. Fax.
Tot. Iufques. Vfque.
Tau. Corde. Funis.
Traech. Lente. Lentus.
Traen. Larme. Lachryma.
Trap. Degré. Gradus.
Treck. Traict. Tractus.
Treeft. Trefpie. Tripes.
Troch. Auge. Linter.
Tromp. Trompe.
Tronck. Tronck. Truncus.
Trooft. Solas. Solatium.
Tros. Bagage qu'on porte à la guerre. Impedimenta exercitus.
Trau. Fidele. Fidelis.
Trijp. Tripe.
Tucht. Modefteté. Modeftia.
Turf. Tourbe. Cefpes.
Tuych. Hardes. Arma.
Tuygh. Tefmoing. Teftis.
Tuyn. Iardin. Hortus.
Tvvaelf. Douze. Duodecim.
Tvvee. Deux. Duo.
Tvvift. Difcord. Difcordia.
Tvvyn. Filtors. Filum retortum.
Tijck. Coutil. Culcitra.
Tijt. Temps. Tempus.
Vaem. Toife. Hexappus.
Vaer. Pere. Pater.
Vaeck. Sommeil. Sopor.
Vael. Baillet. Heluus.
Vaen. Baniere. Vexillum.
Vaer. Peril. Periculum.
Vaert. Foffe nauigable. Foffa.
Vaert. Allure. Profectio.
Valck. Faucon. Falco.
Val. Cheute. Cafus.
Val. Trebuchet. Decipulum.
Valfch. Faulx. Falfus.
Van. De. A.
Vaft. Ferme. Firmus.
Vat. Vaiffeau. Vas.
Vee. Beftial. Pecus.
Veel. Beaucoup. Multus.
Veer. Paffage. Traictus.
Veers. Vers. Verfus.
Veldt. Champ. Campus.
Vel. Peau. Pellis.
Veint. Garfon. Infans.
Verfch. Frez. Recens.
Veft. Muraille d'vne ville. Mœnia.
Vet. Graiffe. Pinguedo.
Veul. Poulain. Pullus equinus.
Veych. Qui eft prochain de fa mort.
Veyl. Expoíé en vente. Venalis.
Vier. Quatre. Quatuor.
Vier. Feu. Ignis.
Vies. Facheux. Morofus.
Vijf. Cinc. Quinque.
Vijg. Figue. Ficus.
Vijl. Lime. Lima.
Vijs. Vis. Cochlea.
Vilt. Feultre. Cento.
Vinck. Becfigue. Frigilla.
Vin. Lopin de chair. Offa.
Vin. Aifle de poiffon. Pinna.
Vifch. Poiffon. Pifcis.
Vlack. Plain. Planus.
Vlaey. Flan. Scriblita.
Vlaegh. Ondée de pluye. Nimbus.
Vlam. Flambe. Flamma.
Vlas. Lin. Linum.
Vleck. Village. Pagus.
Vleefch. Chair. Caro.

Vlieg

Vlieg. Mouche. Muſca.
Vlien. Lancette à chirurgien. Scalprũ
Vlier. Sureau. Sambucus.
Vlies. Toiſon. Vellus.
Vliet. Riue. Ripa.
Vlijt. Diligence. Diligentia.
Vlock. Floc Floccus.
Vloeck. Maudiſſon. Imprecatio.
Vloet. Flot. Fluctus.
Vloer. Aire. Area.
Vloi. Pulce. Pulex.
Vlucht. Fuite. Fuga.
Vocht. Humide. Humidus.
Voet. Pied. Pes.
Voor. Deuant. Ante.
Volck. Peuple. Populus.
Vol. Plain. Plenus.
Vonck. Etincelle. Scintilla.
Vondt. Inuention. Inuentio.
Voocht. Tuteur. Tutor.
Voort. Auant. Vltra.
Voos. Corrompu. Inſipidus.
Vorck. Fourche. Furca.
Vorſch. Grenouille. Rana.
Vorſt. Gellée. Gelu.
Vort. Pourri. Putridus
Vos. Regnard. Vulpes.
Vau. Pli. Plicatura.
Vracht. Voicture. Vectura.
Vraegh. Demande. Interrogatio.
Vranck. Franck. Liber.
Vreck. Chiche. Parcus.
Vre. Paix. Pax.
Vrees. Craincte. Timor.
Vreemt. Eſtrange. Extraneus.
Vreucht. Ioye. Gaudium.
vriendt. Ami. Amicus.
vroech. Tempre. Mane.
vroet. Sage. Prudens.
vroet. Eſcars. Parcus.
vro. Dehait. Hilaris.
vroom. Preux. Probus.
vrau. Femme. Femina.
vrucht. Fruict. Fructus.
vrij. Libre. Liber.
vyr. Heure. Hora.
vyt. Hors. Ex.
vyl. Chatuan. Bubo.
v. vous. Tibi.
vurſt. Prince. Princeps.
vuyl. Ord. Sordidus.
vuyſt. Poing. Pugnus.
Wacht. Garde. Cuſtodia.
vvaeck. veille. vigilia.
vvaen. Preſomption. Præſumptio.
vvaer. Ou. vbi.
vvaer. Marchandiſe. Merx.
vvaer. vray. verus.
vvaegh. Balance. Libra.
vval. Rempars. vallum.
vvandt. Paroy. Paries.
vvang. Ioue. Mala.
vvan. van. vannus.
vvant. Car. Nam.
vvant. Gand. Manica.
vvas. Cire. Cera.
vvat. Quoi. Quid.
vveb. Fil pour tiſtre. Textura.
vvech. Chemin. Iter.
vveer. Belier. Aries.
vveer. Temps. Tempus.
vveer. Derechef. Iterum.
vvee. Malheur. væ.
vveech. Paroy. Paries.
vveyck. Mol. Mollis.
vveet. Guedde. Glaſtum.
vveeld. Delice. Delitiæ.
vveer. Toutes armes de defence. Arma.
vveret. Hoſte. Hoſpes.
vvees. Orphelin. Pupillus.
vveeck. Sepmaine. Septimana.
vvelck. Quel. Quis.
vvel. Bien. Bene.
vvenſch. Souhait. Optio.
vverck. Eſtoupe. Stupa.
vverck. Oeuure. Opus.
vverf. Cay. Acta.
vverm. Chaud. Calidus.
vverp. Iect. Iactus.
vveſp. Gueſp. Veſpa.
vveſt. Occident. Occidens.
vvet. Loy. Lex.
vvey. Megue. Serum.
vvicht. Enfant. Puer.
vvicht. Pois. Pondus.
vvie. Qui. Quis.
vvieck. Tente. Pannus.
vviegh. Berceau. Cunæ.
vviel. voile de Nonnain. velum.
vviel. Roue. Rota.
vvilt. Sauuage. Silueſter.
vvil. volonté. voluntas.
vvinck. Cil d'œil. Nictus oculi.
vvint. vent. ventus.
vvinſt. Gain. Quæſtus.
vvip. Baſcule. Tollenon.
vviſch. Trochon. Penicillus.
vvis. viorne. D'oiſier. vimen.
vvis. Certain. Certe.
vvit. Blanc. Albus.
vvoeſt. Deſert. Deſertus.
vvolck. Nuée. Nubes.
vvolf. Loup. Lupus.
vvol. Laine. Lana.
vvond. Plaie. Plaga.
vvoort. Mot. verbum.
vvorſt. Sauciſſe. Botulus inteſtinarius
vvoudt. Foreſt. Silua.
vvraeck. vengeance. vindicta.
vvrat. verrue. verruca.
vvreet. Cruel. Crudelis.
vvronck. Torſement. Torſio.
vvelp. Ieuſne Chien. Catulus.
vvulps. Folaſtre. Laſciuus.
vvurm. ver. vermis.
vvy. Nous. Nos.
vvydt. Large. Amplus
vvyf. Femme. Mulier.
vvyl. Temps vacant. Spatium.
vvyn. vin. vinum.
vvys. Sage. Sapiens.
Zier. Ciron. Chiron.

LATYNSCHE EENSILBIGHE NAMEN, BYNAMEN, &c.

Ab Abs. De. van.
Ac. Et. Ende.
Ad. A. Tot.
AEs. Cuire. Coper.
Ah. Ach. Ach.
An. Aduerbium interrogantis.
Ars. Art. Conſt.
Arx. Chaſteau. Borch.
As. Liure. Pont.
Aſt. Mais. Maer.
At. Mais. Maer.
Au. Interiectio conſternati animi.
Aut. Ou. Oft.
Bis. Deux fois. Tvveemael.
Bos. Bœuf. Os.
Calx. Chaulx. Kalck.
Cis. Deça. Op dees ſijde.
Clam. En cachette. Heymelic.
Cor. Cœur. Hert.
Cos. Queue. vvetſteen.
Cras. Demain. Morghen.
Crus. Iambe. Been.
Crux. Croix. Cruys.
Cum. Auec. Met.
Cur. Porquoy. vaerom.
De. De. van.
Dos. Doſt. Houvvelicke ghift.
Dux. Duc. Leydtſman.
E. De. vyt.
En. voici. Siet hier.
Et. Et. Ende.
Ex. De. vyt.
Fœx. Lie. Ghiſt.
Falx. Faulx. Sickel.
Fas. Licite. Toeghelaten.
Fax. Fallot. Torts.
Fel. Fiel. Gal.
Flos. Fleur. Bloem.
Fons. Fonteine. Born.
Frons. Fueille. Blat.
Frons. Front. Stirn.
Fur. Larron. Dief.
Git. Genus ſeminis.
Glans. Gland. Eeckel.
Glos. Seur de mon mari. Mijns mans ſuſter.
Grex. Troupeau de beſtes. Kud.
Grus. Grue. Craen.
Ha. A. A.
Hac. Parci. Lancx hier.
Heu. Helas. Eylas.
Heus. He. Hau.
Hic Hæc Hoc Hunc Hanc Hi Hæ Hos Has His.
Hinc, D'ici. Hieraf.
Huc. Ici. Hervvaert.
Hiems. Yuer. vvinter.
Iam. Ia. Nu.
Id. Cela. Dat.
In. En. In.
Is. Ea. Id.
Ius. Ius. Sop.
Ius. Droict. Recht.
Lac. Laict. Melc.
Lanx. Baſſin. Schael.
Lar. Fouyr. Heerdt.
Laus. Los. Lof.
Lex. Loy. vvet.
Lis. Noiſe. Tvviſt.
Lux. Lumiere. Licht.
Me.
Mel. Miel. Honich.
Mens. Sens. Sin.
Merx. Marchandiſe. vvaer.
Mons. Montaigne. Berch.
Mors. Mort. Doot.
Mox. Tantoſt. Terſtont.
Mus. Souris. Muys.
Næ. Certainement. vvaerlic.
Nam. Car. vvant.
Ne. Non. Niet.

Nil. Rien. Niet.
Nix. Neige. Sneeu.
Non. Non. Neen.
Nos. Nous. vvy.
Nox. Nuict. Nacht.
Num. Aduerb.
Nunc. Maintenant. Nu.
Nux. Noix. Nuet.
O. O. O.
Ob. Pour. Om.
Oh. Interiect.
Os. Bouche. Mondt.
Os. Os. Been.
Par. Paire. Paer.
Pax. Pais. Paeys.
Per. Par. Door.
Pes. Pied. voet.
Phy. Interiect.
Pix. Poix. Pec.
Plebs. Peuple. Ghemeinte.
Plus. Plus. Meer.
Pons. Pons. Brug.
Post. Depuis. Nae.
Præ. Deuant. Voor.
Pro. pour. voor.
Proh. Interiect.
Puls. Papin. Pap.
Pus. Boue. Etter.
Quis Qui Quæ Quod Quid. Quin. Que ne. Dat niet.
Quot. Combien. Hoeveel.
Quum. Quand. Als.
Res. Chose. Dinc
Ros. Rosée. Dau.
Rus. Les champs. Velt.
Sal. Sel. Saut.
Sat. Asses. Ghenouch.
Scobs. Sciure. Saegmeel.
Scrobs. Fosse. Gracht.
Se. Accus.
Sed. Mais. Maer.
Seps. Haye. Tuyn.
Seps. Serpens.
Seu. Ou. Oft.
Sex. Six. Ses.
Si. Si. Ist dat.
Sic. Ainsi. Soo.
Sin. Mais si. Maer ist dat.
Sol. Soleil. Son.
Sons. Coupable. Misdadich.
Sors. Fortune. Fortune.
Spes. Esperance. Hoop.
Splen. Rate. Milt.
Stips. Denier. Pennine.
Stirps. Racine. Struyc.
Sub. Soubs. Onder.
Sus. Porc. Soch.
Tam. Tant. Soo seer.
Tax. Son de fouet. Clets.
Ter. Troisfois. Driemael.
Thus. Encens. Vvierooc.
Tot. Autant. Soo veel.
Trabs. Poultre. Balc.
Tres. Trois. Drie.
Trux. Cruel. vvreet.
Tu. Toy. Ghy.
Tunc. Adonc. Dan.
Væ. Vvee.
Vas. vaisseau. Vat.
ve. Ou. Oft.
vel. ou. Oft.
ver. Printemps. Lenten.
vir. Homme. Man.
vis. Force. Stercte.
vix. A grand paine. Naulicx.
vos. vous. Ghylien.
vox. voix. Stem.
vrbs. ville. Stadt.
vt. Afin. Op dat.

GRIECSCHE EENSILBIGHE NAMEN BYNAMEN, &c.

Α℃, mensis September.
Αἴξ. Capra.
Ἅλς. Sal, mare.
Ἄλξ. Potentia.
Ἄν. Si.
Ἄς. ἕως. Vsquequó.
Αὖ. Autem.
Αἶψ. Aduerbium & coniunct.
Βεκ. Panis.
Βὴξ. Tussis.
Βλὰξ. Mollis.
Βλὴρ. Musca.
Βλὶξ. Assiduè.
Βοῦς. Bos.
Βρὶξ. Lactuca.
Βὺξ. Profunditas.
Βὼξ. Genus piscis.
Βῶς. Tergus bubulum.
Γὰρ. Nam.
Γῆ. Terra.
Γλὰξ. Herbæ genus.
Γλαὺς. Noctua.
Γνὺξ. Genu.
Γοῦν siue γᾶν pro γ' οὖν. Igitur.
Γραῦς Anus.
Γρὺξ. Sordes vnguium.
Γύψ. Vultur.
Δαὶ, vel Δὲ. Autem.
Δαὶς. Conuiuium.
Δὰς. Fax.
Δεῖ. Oportet.
Δὶν. Corpus.
Δὴ. Sanè.
Δὴν. Diu.
Δὴξ. Vermis lignũ corrodens.
Δὶς. Bis.
Δμὼς. Seruus.
Δόρξ. Caprea.
Δράξ. Manipulus.
Δρὶς. Virtus.
Δρῦς. Quercus.
Δῶ pro δῶμα. Domus.
Δὼς. Dos.
Εἴ siue ἤν. Si.
Εἶρ. Procella.
Εἷς. Vnus.
Ἐκ. siue Ἐξ. Ex.
Ἐν siue Ἐνὶ, Ἐἰς Ἐς. In.
Ἕξ. Sex.
Εὖ. Bene.
Ζὰψ. Mare.
Ζείρ. Genus vestis.
Ζεὺς siue Δεὺς. Ζὰν. Ζὴν. Iupiter
Ζῶς. Viuus.
Ζῶν. Animal.
Θὴν. Cumulus.
Θὴς. Debitor.
Θὴρ. Fera.
Θὴς. Mercenarius.
Θὶν. Littus.
Θὶς. Nomen piscis.
Θρὰξ. Nomen auis.
Θρὶς. Capillus.
Θρὶψ. Vermiculus.
Θρὶψ. Parcus.
Θῶθ. September apud Ægyptios.
Θὼς. Genus lupi.
Θὼψ. Adulator.
Ἶν siue ἶς. Genus mensuræ.
Ἴξ. Vermis.
Ἶς. Neruus.
Ἴψ. Vermis.
Καὶ. Nam.
Κἂν quamuis pro Καὶ ἐὰν
Κὰρ. Caput.
Κήξ. Genus auis.
Κῆρ. Cor.
Κὶς. Vermis.
Κλᾶξ. Clauis.
Κλὼν. Ramus.
Κλὼψ. Fur.
Κνὶψ. Culex.
Κνὼψ. Cæcus.
Κὶξ. Plantæ genus.
Κοῦ. Vbi.
Κρέας. Caro.
Κρᾶς. caput.
Κρῖ pro Κριθὴ. Hordeum.
Κτεὶς. Pecten.
Κτὶρ. Possessio.
Κτὶς. Viuerra.
Λὰξ. Aduerbium cum calcibus

Λᾶς

Λᾶς. Lapis.
Λὶς. Leo.
Λῖς. Pannus lineus.
Λύγξ. Inanis singultus.
Λύγξ. Feræ genus.
Λὺξ. Lux.
Λῶψ. Chlamys.
Μὰ. Aduerbium iurandi.
Μὰν. Quidem.
Μὰψ. Frustrà.
Μεὶς pro Μὴν mensis.
Μὶν. Tamen, quidem.
Μὴ. aduerb. Ne.
Μὴν. Mensis & aduerbium tamen.
Μνᾶ. Mina.
Μνοῦς. Lana tenerrima cum qua nascuntur agni.
Μῦς. Mus.
Μῶν. An.
Μὼψ. Cui hebes acies oculorū
Ναὶ. Certè.
Ναῦς. Nauis.
Νοῦς. Mens.
Νῦν. Nunc.
Νὺξ. Nox.
Νῶψ. Lusciosus.
Ξὺν pro σὺν Cum.
Ὁ. Hic.
Οἶ pro ὅπου Vbi.
Ὅς. Qui.
Οὐκ. Non.
Οὖν. Ergo.
Οὖς. Auris.
Ὤψ. Vox.
Πᾶ. Qua, Quò, Vbi.
Παῖ. Idem.
Παῖς. Puer.
Πᾶς. Omne.
Πάξ. Genus calceamenti.
Πὰρ pro Παρὰ. A, Ab, Ex.
Παῦ pro Παῦσαι. Pausa.
Πῇ. Qui.
Πλὰξ. Tabula.
Πλὴν. Præter.
Πλῆξ. Stimuli genus.
Πλὶξ. Gressus.
Πλοῦς. Nauigatio.
Πνὶξ. Suffocatio.
Ποῖ. Quodammodo.
Ποῦ, Vbi, Partim, Alibi.
Ποὺς. Pes.
Πρὰν pro πρώην Dudum.
Πρὶν. Prius.
Πρὸ. Antè.
Πρὸξ. Animal seruo simile.
Πρὸς. Per, Ad.
Πρὼν. Eminentia montis.
Πρὼξ. Ros.
Πτύγξ. Genus auis.
Πτὺξ. Plicatura.
Πτὼξ. Timidus.
Πὺξ. Aduerb. pugnis.
Πῦρ. Ignis.
Πῶς. Quò.
Πω vbi pro πόθεν.
Πῶς. Quomodo.
Ῥὰ. Genus radicis.
Ῥὰξ. Acinus vuæ.
Ῥὴν. Ouis.
Ῥὶν. Naris.
Ῥὶς. Nasus.
Ῥὶψ. Vimen flexile.
Ῥοῦς. Fluxus.
Ῥὼξ. Rupes.
Ῥὼψ. Virgultum.
Σᾶ. Incolumia.
Σὰρξ. Caro.
Σεὶρ. Sol.
Σκὺς. Vermis.
Σὴς. Tinea.
Σὴψ. Serpens.
Σκῶρ. Merda.
Σκὼψ. Auis loquax.
Σὸς. Tuus.
Σπλὴν. Splen.
Σταὶς. Farina.
Στὶξ. Turma continens viros trecentos.
Στρὶγξ. Auis.
Στροῦς, id est, Στρουθὸς Passer auis.
Σὺ. Tu.
Σῦς. Sus.
Σφὴν. Cuneus quo ligna scinduntur.
Σφὴξ. Vespa.
Σφὶγξ. Sphinx animal.
Σῶς. Saluus.
Τὰρ. Autem.
Τὶς. Quis, Aliquis.
Τρεῖς. Tres.
Τρὶς. Ter.
Τρὺξ. Fæx.
Τρὼξ. Gurgulio.
Τὺ. Tu.
Τὼς. Sic.
Ὗς. Sus, & piscis nomen.
Φὰψ. Auis genus.
Φεῦ. Heu.
Φὴρ pro Θὴρ Fera.
Φθεὶρ. Pedunculus, etiam medium claui.
Φθοῖς. Genus placentæ.
Φλὸξ. Flamma.
Φρὴν. Præcordia.
Φρὶξ. Maris vel fluctuum fremitus.
Φὺξ. Aduerbialiter cum fuga.
Φὼρ. Fur.
Φὼρ. Genus apium.
Φὼς. Inustio ab igne facta in cruribus.
Φὼς. Vir.
Φῶς. Lux.
Χεὶρ. Manus.
Χρὴ. Oportet.
Χλοῦς. Herba.
Χνοῦς. Lanugo.
Χοῦς. Agger.
Χοῦς. Congius.
Χρῶς, & χρὼς. Corpus.
Ψὶξ. Mica.
Ὦλξ. Sulcus.
Ὥς. Vt.
Ὢψ. Facies.

Dd ANGAENDE

ANGAENDE ymandt totte voorschreuen Latijnsche ende Griecsche ynckel gheluyden, die metter haest vergaert sijn, noch eenighe derghelijcke mocht vinden, aldaer niet beschreuen, sulcx soudemen int Duytsch oock connen doen, ende onghelijck in al veel meerder menichte, want wy de schandelicke om noemen, ende ander die ons buyten *Den schat der Duytscher talen*(welcke t'Woortbouck was daerwyse uyt vergaerden) ons wel inden sin quamen, moetwillens uytghelaten hebben, ons daer in vernoughende, dat duer de voorgaende opentlick blijct, d'oude Duytschen met voorset, d'uyterste volmaectheyt in desen, meer dan eenighe ander, naghetracht, ende ghetroffen te hebben.

Grammatica elementis. Merckt noch dat sy t'selue oock ghedaen hebben in des * Letterconsts beghinselen, dat is inde boucstaffen ofte letteren, die sy al met eensilbighe gheluyden noemen, t'welck voorwaer d'uyterste volcommenheyt naerder is, dan de contrari; want ghelijct inde * Meetconst ongheschict *Geometria.* waer, t'punt, beghin der grootheyt, meerder te stellen dan grootheyt, alsoo ist oock inde Letterconst ombetaemlick, t'beghin van meer gheluyden te sijn, dan t'ghene van verscheyden beghinselen ghemaect wort. Als by voorbeelt int spellen van *Dal*, datmen opt Griecsche seght Delta, Alpha, Lambda, Dal; ofte opt Hebreusch Daleth, Aleph, Lamed, Dal; alwaer yder beghin ongheschictelick van meer gheluyden is, dan t'ghene vande drie beghinselen ghemaect wort. Daerom segghen wy veel natuerlicker ende aerdigher, De, A, El, Dal: want t'ghene inde Consten beghin is, moet daerin het alder eenvoudichste sijn, t'welck hier, soot de Duytschen ghetroffen hebben, ynckel gheluyt is. Daerom deden de Latinen wel, doe sy leerden lesen en schrijuen, dat sy in desen d'ander lieten varen, eñ de Duytschen volghden. Wat de onghegronde meyning vam hemlien belangt, die souden duruen segghen de Duytschen sulcx eer vande Latinen te hebbẽ, die en spreken niet duer beweeghnis der reden, maer ghedreuen van eensinnighe moetwillicheyt, soo doch de Latinen na sulcke cortheyt niet ghetracht en hebben, maer ter contrari, t'gheen by ons cort en goet was, dat hebben sy naer huerlieder ghebruyck gheern verlangt: als Angst, Caes, Beest, Put, Muer, Recht, Cael, Graen, Heer, &c. daer sy voor segghen *Anxietas, Caseus, Bestia, Puteus, Murus, Rectus, Caluus, Granum, Herus.* Tis dan vande Duytschen dat de letteren de volmaecste namen hebben.

Wat de Fransche, Italiaensche, Spaensche, ende meer talens eensilbighe gheluyden belangt, welcke hier yemandt begheeren mocht, wy en hebben die niet ghestelt, om dattet Griecx ende Latijn in volcomenheyt d'ander te bouen gaende, tottet voornemen voldoen; want als wy bewesen hebben, het Duytsch volmaecter dan dese twee te sijne, soo volght uyt noch stercker reden, dattet veel volmaecter is dan eenighe van dien. Wel is waer dat de Fransche eensibilghe gheluyden, de Latijnsche

In case the reader should find, in addition to the above-mentioned, hurriedly collected Latin and Greek single sounds, some more of this kind which are not included in the list, this might also be done in Dutch, and even in much greater number, for we have intentionally omitted those that were shameful to mention and others that occurred to us outside *Den schat der Duytscher talen* [1]) (which was the dictionary from which we collected them), being satisfied that it is evident from the preceding lists that the ancient Dutch, more than any others, purposely strove after and achieved the highest perfection in this matter.

It is further to be noted that they have also done this in the elements of grammar, i.e. in the letters, all of which they denote by monosyllables, which is certainly nearer to the highest perfection than the contrary; for just as in geometry it would be absurd to consider the point, the element of magnitude, greater than magnitude itself, in the same way it is also improper in grammar for the element to consist of more syllables than that which is made of several elements. Thus for example in spelling the word *Dal,* which is said in Greek: Delta, Alpha, Lambda, *Dal;* or in Hebrew Daleth, Aleph, Lamed, *Dal;* in which each element improperly consists of more syllables than that which is made up of the three elements. Therefore *we* say, much more naturally and peculiarly: De, A, El, *Dal,* for the elements in the arts should be simplest of all, which in this case, as the Dutch have achieved it, are single sounds. Therefore the Latins did well, when they learned to read and write, to abandon the other method in this and imitate the Dutch. As to the unfounded opinion of those who should dare to say that the Dutch have rather borrowed this from the Latins, such people are not moved to say so by reason, but by obstinate wilfulness, for the Latins did not aim at such brevity: on the contrary, they liked to lengthen in accordance with their custom that which was short and good with us: for example *Angst, Caes, Beest, Put, Muer, Recht, Cael, Graen, Heer,* etc., for which they say *Anxietas, Caseus, Bestia, Puteus, Murus, Rectus, Calvus, Granum, Herus.* It is therefore from the Dutch that the letters have received their most perfect names.

As regards the monosyllables in the French, Italian, Spanish, and other languages which the reader might desire to be mentioned here, we have not mentioned them because Greek and Latin, being superior to the others, suffice for the purpose; for if we have proved Dutch to be more perfect than these two, it follows *a fortiori* that it is much more perfect than any of the former. It is true indeed that the French monosyllables are greater in number than the Latin ones, since the

[1]) This is a work by Ian van den Werve, a lawyer at Antwerp, who advocated purification of the Dutch language.

tijnsche in menichte te bouen gaen, ouermidts de Françoysen dickmael snoeyen ende vercorten, t'ghene sy vande Latijnen ontleenen, als voor *Facio, Seruio, Venio, Rideo, Sentio,* &c. te segghen *Ie Fay, Sers, Vien, Ri, Sens*; welcken aert der vercorting sy noch schijnen behouden te hebben van weghen dat sy, als vooren ghesevt is, eens Duytsch spraken; maer wat isser af? sy en lijden gheen binding, sy sijn ter Tsaemvoughing onbequaem, ende veruolghens van cleinder weerde.

TEN tweeden soo volghter van der woorden voornomde T'saemvoughingh ghesevt te worden, welcke niet t'onrecht voor een der voornaemste ende nutste eyghenschappen die in talen begheert worden, gheacht is; wiens voordering ende nootlicheyt den ghenen die hun inde Consten oefnen, niet onbekent en is, ouermidts der dinghen namen daer duer oock haer corte * bepalinghen sijn. Hier in wort byden gheleerden het Griecx gheluckigher gheacht als d'ander, dat is, als de ghene die by haer verleken wierden, onder welcke het Duytsch gheen plaets en had; anders ten waer gheen oirdeel van gheleerden, maer van verkeerden gheweest, want ghelijck gheen menschen die wel by haer sinnen sijn drie grooter ghetal en achten dan Duyst, maer veel cleender; alsoo oock de Griecsche Tsaemvoughing niet bouen de Duytsche, maer verre daer onder, want in die sijn hier en daer sommighe woorden diese lijden, maer in dese oueral, ende dat met een ander besonder cortheyt, ghe schictheyt, ende eyghentlicker beteeckening haers grondts, welcke nootsakelick volghen uyt de voorgaende groote menichte der ynckel gheluyden, daerenbouen ter bequame T'saemvoeghing wonderlick ghetroffen. Ymant mocht nu van desen eenighe voorbeelden begheeren; maer wanttet licht ende te slicht waer, uyt de oneindelicke een groote menichte te vergaren (als inde T'saemspraeck der * Bewysconst beghonnen is) soo gheuen wy hem seluer eenighe voor te stellen die hem ter coppeling onbequaemst duncken. Ick neem dat hy daertoe verkiest (om haer aldermerckelicste verscheydenheyt, ende gheduerighen strijt) Water en Vier: voorwaer soot den noot erghens voorderde dese te vergaren, als by ghelijcknis, ymandt willende segghen, *Tot d'incomst des Kueninex waren vieren ghemaect die van selfs int water ontstaken,* hy sal die noemen (ghelijck wy anders segghen Turfvieren, Eyckevieren) *Watervieren:* ende daer toe en behouft hy gheen gheleerde te sijne, noch hem lang te bedencken, maer de leecken worden, duer de wonderlicke eyghenschap des taels, van selfs daertoe ghedronghen. Ten is den hoorenden oock gheen nieu noch vreemt woort, hoewel hy dat van te vooren noyt ghehoort en had, reden dat sulcke niet alleen duer de ghewoonte verstaen en worden, maer uyt den ghemeenen aert der gheluyden, welcke d'oude Duytschen soo constelick

Definitiones.

Dialectica.

dD 2 daertoe

French often curtail and shorten that which they borrow from the Latins, saying for example, for *Facio, Servio, Venio, Rideo, Sentio,* etc.: *Ie Fay, Sers, Vien, Ri, Sens;* which shortening tendency they seem to have retained because, as has been said above, they once spoke Dutch; but what of that? They do not admit of combination, they are unfit for composition, and consequently have less value.

The second point to be discussed is the aforesaid composition, which is not unjustly deemed one of the principal and most useful properties required in languages; the advantage and need of which is not unknown to those who exercise themselves in the arts, since the names of things are thus also short definitions thereof. In this respect Greek is considered by the scholars to be more felicitous than the other languages, that is the languages compared with it, among which Dutch did not figure; otherwise it would not have been a judgment of scholars, but of fools, for just as no man in his senses will deem three to be a greater number than one thousand, but much less, thus Greek composition is not superior to Dutch, but far inferior, for in the former there are occasionally a few words admitting of it, but in the latter it is always possible, and such with a special brevity, suitability, and proper denotation of their fundamental meaning which are the necessary consequences of the above-mentioned multitude of single sounds, which are also wonderfully suited for composition. The reader might now require some examples of this, but because it would be easy and too simple to collect a great many from the infinite multitude (as we started to do in the dialogue in the Dialektike [1]), we suggest that he himself should propose some, which seem to him least suited for composition. I assume that he chooses for this (because of their highly obvious difference and continuous conflict): *Water* and *Vier* [2]). Indeed, if circumstances should require these to be combined, for example if anyone should wish to say: *Tot d'incomst des Kuenincx waren vieren ghemaect die van selfs int water ontstaken* [3]), he would call them *Watervieren* [4]) (just as we also say *Turfvieren* [5]), *Eyckevieren* [6])). And for this he need not be a scholar, nor need he take thought about it long, but the unlearned are automatically induced to do this owing to the wonderful character of the language. Nor is it a new or strange word for the hearer, even though he had never heard it before, because such words are understood not only through usage, but owing to the common character of the sounds, which the ancient Dutch have found so ingeniously for the purpose that I, and all those who know no more of the origin,

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[1]) Work III, *Dialectikelicke Tsamespraeck.* This „dialectical dialogue" in part already develops the ideas of the *Uytspraeck.*

[2]) i.e. water and fire.

[3]) For the King's entry fires had been arranged, which kindled out of themselves in the water.

[4]) water fires.

[5]) peat fires.

[6]) literally: oak fires.

daertoe gheuonden hebben, dat ick, met al de ghene die van d'oirſaeck niet meer en weten, ons alſvooren noch met recht mueghen verwonderen duer wat middelen dat mach gheſchiet ſijn. Merckt bouen al dit noch een beſonder, ende weerdighe eyghenſchap, by hemlien conſtelick inde T'ſaemvoughing veroirdent, ia ſulcke, dat gheen Griecx, noch Latijns * Letteraer, ſoodanighe uyt die talen perſſen en ſal, al wrong hy tot ſweetens toe: Te weten dattet laetſte der ghecoppelde altijdt * Grondt is, ende t'voorgaende * Ancleuing; Als wanneermen ſeght, *Putwater*, ſo is water grondt, ende put ancleuing, want onſen ſin dan voornamelick tot water ſtreƈt, om t'welck t'onderſcheyden van ſtroomwater, reghewater, &c. men vougter Put voor: Maer als wy dit verkeeren, ſegghende *Waterput*, dan is den ſin (hoewel het de ſelue woorden ſijn) al een ander, want Put is dan grondt, ende Water ancleuing, ouermidts de voornamelicke meyning dan is van een Put, om welcke t'onderſcheyden van een Meſput, Calckput, &c. men ſtelter water voor. Alſoo oock is *Glasveinſter*, een veinſter van glas, maer *Veinſterglas*, is glas niet daermen uyt drinckt, maer plat daermen veinſters af maeƈt. Wederom *Olinuet*, is een nuet des gheſlachts daermen olie uyt perſt, maer *Nuetolie*, is olie van nueten. Sghelijcx *Iachthondt*, is een hondt daermen mede iaecht, maer *Hondiacht*, een iacht niet met voghelen, dan met honden, &c. Wat den ghenen belangt die noch van meyning mueghen ſijn, het Griecx in deſen gheualle voor het Duytſch te gaen, wy achten dat ſulcx gheſchiet duer dat hem het Duytſch onbekent is, of datter verſtandt der oirdeling ghebreeƈt, of dat hy hartneckich ſy, of eenich der ghelijcke beletſel heb, niet weerdich een woort daer af meer te roeren.

Grammaticus. *Subieƈtum.* *Adiunƈtum.*

TEN derden moeten wy ſegghen vande bequaemheyt deſes taels tot de leering der Conſten, waer af wy (bouen dien ſulcx nootſaeckelick volght uyt het voorgaende toeghelaten) de volghende Weeghconſt, ſulck ſy is, tot voorbeelt ſtellen; welcke ghy, ghemerƈt de groote rijcheyt onſes taels, uyt welcke alles veel beter behoort ghedaen te ſijne, daertoe miſſchien niet weerdich en ſult achten, te meer datter duyſenden by ons ſijn, dieſe veel beter, ende met beuallicker woorden beſchrijuen ſouden: Maer niet teghenſtaende al dit, ſo iſt doch ſoo ghedaen, dat gheen van al d'ander Gheſlachten der volcken wie hy ſy, t'ſelue, ſoo veel des ſpraecx grondelicke beteeckening, ende uytbeelding der Saeck belangt (ick en wil niet ſegghen, ſoude connen verbeteren, daer gheen vreeſe voor te hebben en is, want hemlien ghebreeƈt Stof * welcke gheweert ſoo wort oock gheweert de daet) ſoude connen ſoo nauolghen. want waer wildy ſpraken halen daermen duer ſegghen ſal, Eueſtaltwichtich, Rechthefwicht, Scheefdaellini, en dierghelijcke daer de Weeghconſt vol af is? ſy en ſijnder niet, de Natuer heeft daer toe aldereyghentlicxt het Duytſch veroirdent.

Sublata materia tollitur effeƈtus, *4. v. 2. ſtrijtreden der Bewyſconſt.*

TEN

may justly wonder, as said before, by what means this may have happened. In addition to all this, a special and valuable property should also be noted, which was ingeniously disposed by them in composition, a property such that no Greek or Latin grammarian can squeeze it from those languages, though he should wring them until he sweat: to wit, that the last member of the compound is always the head word and the preceding one the attribute. For example, when we say *Putwater* [1]), *water* is the head word and *put* [2]) the attribute, for then we chiefly mean *water,* to distinguish which from *stroomwater* [3]), *reghewater* [4]), etc. we prefix *put* to it. But if we invert the order, saying *Waterput* [5]), the meaning is quite different (though the words are the same), for then *put* is the head word and *water* the attribute, since the principal meaning of it is then that of a *put*, to distinguish which from a *Mesput* [6]), *Calckput* [7]), etc. we prefix *water* to it. In the same way, *Glasveinster* [8]) is a *veinster* of *glas*, but *Veinsterglas* [9]) is *glas* not such as we drink from, but plates from which *veinsters* are made. Again, *Olinuet* [10]) is a *nuet*, of the genus from which *olie* is pressed, but *Nuetolie* [11]) is *olie* from *nueten.* Similarly, a *Iachthondt* [12]) is a *hondt* with which one *iaecht,* but *Hondiacht* [13]) is a *iacht* not with birds, but with *honden,* etc. As for him who should still be of opinion that Greek is superior to Dutch in this respect, we consider that this is because he ignores Dutch or because he is not competent to judge or is obstinate or has some similar defect not worth wasting any more words about.

Thirdly we have to discuss the suitability of this language for the teaching of the arts, of which (apart from the fact that it follows of necessity from the above assumptions) we are taking the following Art of Weighing, such as it stands, as an example; which, considering the great wealth of our language, in view of which everything should have been done much better, you may not deem worthy, the more so as there are thousands among us who would describe it much better and in more pleasing words. But in spite of all this, it is a fact that none of the other nations, whichever it be, could imitate it as far as the fundamental meaning of the language and the representation of the thing are concerned (I would not say: could improve upon it, which need not be feared, for they lack the material, in the absence of which the effect is also absent); for where would you find any languages in which one can say *Evestaltwichtich, Rechthefwicht, Scheefdaellini* and the like, in which the Art of Weighing abounds? They do not exist, Nature has specially designed Dutch for it.

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1) well-water.
2) well.
3) river water.
4) rain water.
5) literally: water well.
6) dung pit.
7) lime-pit.
8) glass window.
9) window glass.
10) literally: oil-nut.
11) nut-oil.
12) literally: hunting-hound.
13) literally: hound-hunting.

TEN laetsten moeten wy, na t'voornemen, deses taels beweeghlicheyt bethoonen, waer toe ons onder anderen tot voorbeelt dienen can, Hendrick Glareaen, in sijn Latijnsche * uytspraeck te Friburg op Suetonius ghedaen, alwaer hy heftelick ontsteken op der Keysers boosheden, ende gheen Latijnsche noch Griecsche woorden (hoewel hy in die spraken seer eruaren was) bequaem ghenouch vindende, om den hoorders haer afgrijslicheden tot een walghe te maken, heeft dat ondertusschen door duytsche bestelt, als daer hy segt: *Quid enim de Tiberio dicam? vlceroso in omnem inuidiam animo, quo nihil vmquam fucatius toto terrarum orbe, nihil nocentius, nihil turpius vixit. De eo sanè, quod vix Latinè dixeris, nostra lingua ornatißimè dici poterit*: Ein abgfeimpter, eerloser, znichtigher boesswicht. *Si licet Graca immiscere Latinis, sape etiam apud non intelligentes Graca; cur non liceat inserere Celtica ac Germanica non minus vetusta lingua verba, apud intelligenteis? Sed pudet plura de eo Diuo: dixissem libentius*, von dem leidighen Tüfel. *Producatur Caligula Imperator, merdosus ille pusio*, Das schantlich physickguckly, *pudenda Germanici Casaris progenies, &c.* Ende corts daer na sprekende van Nero, Galba, Otto, Vitellius: *Cui enim monstro potius comparabuntur belluones illi, bibones, comedones, lurcones, abdomines, ventres*, brasser, schlemmer, pfuser, schlucker?

* *Oratione.*

Neemt noch merckelicker voorbeelt, ande prekinghen ofte verscheyden leeringhen der gheloouen, die inde Duytsche landen gheschien: waer vindtmen ander contreyen daer de ghemeenten alsoo ghetrocken worden, den eenen tot dit, den anderen tot dat, ende elck tot t'ghene hy hoort? wat is d'oirsaec? de beweeghlicheyt der Duytsche woorden, al veel heftelicker des menschen sin ende ghemoet tot des Redenaers voornemen dringhende, als eenighe ander; want soo hy de tong wel t'sijnen beuele heeft, ende dat hem maer int hooft quaem een bessem de bruyt te sijne, hy sal de ghemeente beweghen ter bruyloft te commen; Ia noch al slimmer dinghen doen bestaen, streckende niet alleen tot ellende van wyf en kinderen, tot verlies van lijf en goedt, maer oock tot ghemeene verderfnis des landts, als metter daet, dat beclaghelick is, te veel blijct; Ende dit al door die heftighe beweeghlicheyt deses taels: Daerom waert wel te wenschen, dat gheen ander begaefde der Duytsche tong, sulck ampt ten deele en viele, dan diens einde tot de ghemeene welvaert strect; want soodanigher menschen Duytsche woorden, vaten inde hoorders herten als clissen an wolle, sy sijn als den breydel des peerts, als t'roer eens schips, duer t'welck de ghemeente gheuoert wort daert den stierman belieft. Angaende yemandt sulcx der Duytschen lichtveerdicheyt soude willen toeschrijuen, seker t'waer teghen d'oude oirconden van Cesar, Tacitus, ende veel ander des huydighen daechs, welcke, rekenende int ghemeene Gheslacht teghen Gheslacht, hun voor t'stantvastichste ende ghestadichste achten: Daerom soo wy ghesegt hebben, Lastly we have to prove, as we intended, the emotional appeal [1]) of this language, as an example of which may serve, among other things, the case of Hendrick Glareaen [2]), in his Latin oration made at Friburg on Suetonius, where, being greatly incensed about the vices of the emperors, and not finding any Latin or Greek words (though he was greatly versed in those languages) suitable enough to inspire his hearers with horror of their hideous deeds, he uses German words for it now and then, as where he says: *Quid enim de Tiberio dicam? ulceroso in omnem invidiam animo, quo nihil umquam fucatius toto terrarum orbe, nihil nocentius, nihil turpius vixit. De eo sanè quod vix Latinè dixeris, nostra lingua ornatissime dici poterit:* Ein abgfeimpter, eerloser, znichtigher boesswicht. *Si licet Graeca immiscere Latinis, saepe etiam apud non intelligentes Graeca, cur non liceat inserere Celtica ac Germanicae non minus vetustae linguae verba, apud intelligenteis? Sed pudet plura de eo Divo: dixissem libentius,* von dem leidighen Tüfel. *Producatur Caligula Imperator, merdosus ille pusio.* Das schantlich physickguckly, *pudenda Germanici Caesaris progenies,* etc. And a little further on, speaking of Nero, Galba, Otto, Vitellius: *Cui enim monstro potius comparabuntur helluones illi, bibones, comedones, lurcones, abdomines, ventres,* brasser, schlemmer, pfuser, schlucker?

Take an even more obvious example, viz. the preachings or different teachings of creeds which take place in the Dutch countries. Where do we find any other regions where the congregations are so much fascinated, one by this, the other by that, and each by that which he hears? What is the cause? The emotional appeal of the Dutch words, which cause men's minds and hearts to be persuaded by the orator's intentions much more vehemently than any other, for if he has his tongue well in his command and should get it into his head that a broom was to be the bride, he will induce the congregation to come to the wedding. Nay, he will provoke even worse things, tending to cause the misery of wife and children, the loss of life and property, but also the general ruin of the country, as is only too evident, a thing to be deplored. And all this is due to the vehement emotional appeal of this language. Therefore it were to be wished that such a function were to fall to no persons expert in the use of the Dutch language but those who have the welfare of all in view; for the Dutch words of such men cling to the hearts of the hearers like burs to wool, they are like a horse's bridle, like a ship's rudder, by means of which the congregation is led as it pleases the steersman. If the reader should be inclined to attribute this to the frivolity of the Dutch, this would be contrary to the ancient records of Caesar, Tacitus, and many others of the present day, who, comparing in general one race with another, consider the Dutch to be the most steadfast and constant. Therefore, as we have

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[1]) Stevin's term is „beweeghlicheyt" (mobility), but this means the power to move.

[2]) Henricus Glareanus, a Swiss scholar (1488-1563), author of the well-known work *Dodekachordon* (1547), was professor at Freiburg im Breisgau from 1529 to 1560.

tis duer de heftighe beweeghlicheyt der Duysche woorden, nootsakelick volghende uyt haren voorschreuen constighen grondt.

MAER wat hebben doch d'uytheemsche verachters der Duytsche spraek; die schampweerdighe schampers, die oirdeelders als blinden vande verwe, voor * strijtredens by te brenghen? Ia, segghen sy, als wy al veel iaren die tael gheleert hebben, soo spreken wy noch soo erbarmelick, dat de Duytschen lachen moeten wanneer syt hooren, maer d'onse hebben sy terstont gheleert, hoe can d'hare dan goedt sijn? O aerme ongheuallighe ghedierten! O iammerlicke wysheyt! Om dat een witte muer beter om schilderen is dan Paris oirdeel, is sy daerom oock constigher? Om dat den volmaeckten omtreck eens naeckten menschen lichaems, onder de formen de aldermoeylicste is die den schilder ontmoet, is sy daerom de verachtste? Om dat een * Singconstich stick met vier of vyf stemmen, vol schoonder * vluchten, bequamer * vallen, lieflicker * teghepunten, den leerenden luytslaghers moeylicker valt, als danskens, ende ghemeene straetlijkens, ist daerom oock het verworpenste? Iaet voor verworpen plompaerts, die haer grofheyt bedecken souden costen sy swyghen: Alsoo oock, om dat de Duytsche spraeck, welcke de diepe verborghentheden der natuer grondelick uytbeelden can, lastigher om leeren is als d'ander diese verswyghen moeten, is sy daerom de slichtste? Ia sy voor slichter dan slichte slichthoofden, die niet en weten waerin goetheyt of soetheyt van talen gheleghen is. Voorwaer souden woorden an woorden hanghen om eenwoordighe redenen te maecken, ghelijck letteren an letteren woorden baren, sy moeten als de letteren constighe gheluyden hebben, niet naer t'gheual van hier en daer t'samen gheschrapt, als de hare, maer sulcke als ons voorouders ghetroffen hebben, ende dat duer middelen, daer alle verstanden (soomen uyt het fijne van eens anders oirdeelen mocht) voor rusten moeten; Reden is dese, dat de spraken niet duer eenen, maer duer velen van verscheyden gheuoelen ghemaeckt worden, d'een sus, d'ander soo, dese beter, die ergher willende; maer de voorighe Duytschen hebben ghedaen, als of sy altemael de saken eruaren daer de talen toe dienen, met een selfde gheneghentheyt aldus eendrachtelick ghedocht hadden: *Anghesien wy duer t'behulp van tong, lippen, tanden, verhemelt, keel, bycans oneindelicke verscheyden eensilbighe gheluyden connen uyten, soo ist billich dat wy yder ynckel saeck een eensilbich gheluyt toeyghenen (want min is onmueghelick, meer is onnut) ende van sulcker aert, dat sy de Tsaemvoughing bequamelick lijden, op dat wy daer duer niet alleen de ghemeene dinghen, maer oock de wonderlicke die de Natuer daghelicx baert, beuallick ende verstaenlick uytbeelden mueghen.* Wat der woorden langhe silben belangt, welcke int Griecx, Latijn, ende meer ander talen, sonder grondt ghenomen

Margin notes: *Argumenta.* (strijtredens) — *Musicalis.* (Singconstich) — *Fuga.* (vluchten) — *Cadentia.* (vallen) — *Cõtrapuncta* (teghepunten) said, the cause is the vehement emotional appeal of the Dutch words, which is an inevitable consequence of their above-mentioned ingenious character.

But what arguments have the foreign despisers of the Dutch language to adduce, those scorners deserving scorn, who judge as blind men judge of colours? Why, they say, when we have studied this language for many years, we still speak so lamentably that the Dutch cannot help laughing when they hear it, but they very quickly learn our language; so how can theirs be a good one? Oh, thou miserable, despicable vermin! Oh, lamentable wisdom! Because a white wall is easier to paint than the judgment of Paris, is it also more artistic for that? Because the perfect contour of a naked human body is the most difficult of all forms encountered by the painter, is it the most despicable for that? Because a piece of music in four or five parts, full of beautiful fugues [1]), apt cadences, pleasant counterpoints is more difficult for people learning to play the lute than dances and common street songs, is it the most abject for that? Yes, so it is for abject churls, who would conceal their coarseness if they could be silent. In the same way also, because the Dutch language, which is capable of thoroughly interpreting the profound secrets of Nature, is more difficult to learn than the other languages, which have to keep silent about them, is it the simplest because of that? Yes, so it is for simpler than simple simpletons, who do not know in what consists the excellence or sweetness of languages. In truth, if words are to be combined with other words into compounds forming one word, just as letters combined with letters produce words, like the letters they should consist of artful sounds, not scraped together at random, as in the languages of other nations, but such as our ancestors have created them, and this by means which pass all understanding (if one may judge of others by one's own). The reason is that languages are not made by one man, but by many with different ideas, one like this, another like that, one having better intentions, the other worse; but the ancient Dutch did as if, having all together learned for what purpose languages are meant, they had thought of one accord, and one and the same mind: *Since by means of the tongue, lips, teeth, palate, and throat we can utter an almost infinite variety of monosyllabic sounds, it is fit that we should assign to every single thing a monosyllabic sound (for less is impossible, and more is useless), and of such a nature that they are fit for composition, so that we may pleasingly and intelligibly represent by them not only ordinary things, but also the strange things which Nature creates daily.* As to the long syllables of the words which, having been groundlessly accepted in Greek and Latin, and other languages besides, have reduced Latin to

[1]) In the sixteenth century a „fugue" was what we now describe as a canon.

men ſijnde, t'Latijn daertoe ghebrocht hebben, dattet in twyffel is of de gheſproken woorden der ouden nu ter deghe ſoude connen verſtaen worden, daer ſchijnen ſy aldus gheſeyt te hebben: *Nadien de Natuer als duer ghemeen inſturting allen menſchen inghebeelt heeft, dat de gheſproken woorden een manier van gheſanck eyſſchen als * hoochbyclanck * leeghbyclanc en dierghelijcke onder welcke des woorts langhe ſilb van meeſten anſien is de reden wil dat wy des ſpraecx ſoo beſonder * ancleuing niet na t'gheual, maer na yet behoirlicx ende ſekers veroirdenen: wat ſal dat weſen? dit, datſe voor ghemeen reghel altijt comme op des * doende woorts ende dieder uyt ſpruyten, voornaemſte ſilb, als in Hōoren, Verhōorende, Ghehōort, Behōorende, Hōorende, dat de langhe ſilb. altijt op Hōor valle, die aldaer de weerdichſte is, wantmen inden eerſten perſoon ſeght ick Hōor, d'ander ſilben als en, ver, ende, ghe, be, en ſiin maer by gheſette, daermen alle woorden me verandert. Maer inde ghecoppelde, daer ſalſe altijt op d'eerſte vallen, als Sāutvat, Hāumes, V̄ytgaen, Ī̄nſien, en dierghelijcke.* Soo hebben ſijt afgheclaert, ende dit noch als ſommighe meenen, in haer wiltheyt, waeruytmen aldus ſtrijden mocht: *Hebben ſy ſulcx ghedaen in haer wiltheyt, wat connen ſy in haer temheyt!* Voorwaer wy ſouden deſe eere wel draghen, maer ghemerct de Natuer niet teghen Natuer en doet, de reden wil dat wy ons met een minder vernoughen, te weten, dattet voormael een ſeer wys, gheleert, ende ouertreflick Gheſlacht is gheweeſt, als vooren bethoont is, daertoe ghecommen ſijnde met langher tijdt, duer veel eruaringhen. Ende ſoo wy van deſe voorghanghers weerdighe navolghers willen gheacht ſijn, en ſullen niet duer een beeſtelick ghetuych van ondancbaerheyt, ſo groote gauen ons naghelaten, duer onwetenheyt verſmaen, noch, den laſteraers dieſe niet en kennen, ſottelick gheloouen, noch verlatende den ſpieghel der talen, ons dickmael behaghen in haer leelick ſchrapſel van ſchuym der vuyllicheyt; maer ſullen ter contrari die clouclick beſchermen, niet met ydel woorden als d'hare ſijn, noch na t'onuerſtandt van hemlien die de goetheyt der Saken in haer talen beſchreuen, onbeſcheydelick de goetheyt der talen meenen te weſen; maer ghelijck t'gout duer t'vier beproeft wort, alſoo ſalmen haer weerdicheyt duer de * daet bethoonen: Welcke ſal die ſijn? deſe, neemt voor * grondt t'ghene in al d'ander ſpraken tot noch toe der Naturen diepe verholentheden ſijn, welcke ſy niet ter deghe bedien en connen, als dat ſy v (onder duyſentich anderen daer het Duytſch vol af is) dit na ſegghen: *Ghelijck rechtheſlini tot ſcheefheſlini, alſoo rechthefwicht tot ſcheefhefwicht*; ende diet ſoo doen connen, belooftſe vrielick een koeck; Ia dat ſy t'auent op ſullen blijuen (voor kinderen dienen doch kinder prijſen) ende ick verſeker v dat ghyder ſonder ſchade ſult afcommen, want het is blijckelijck ghenouch wat ſy hier in vermueghen, te weten voor deſe woorden langhe redenen te ſtellen, die t'onderſcheyt der * palen, ende de form der * eueredenheyt oueral ſeer

Accentus acutus. *Accentus grauis.* *Adiunctum.* *Verbi actiui.*

Strijtreden van het onghelooflicker tot het ghelooflicker door de 13e ſtrijtredē des 4e voorſtels der Bewyſcōſt.

Effectum. *Subiecto.*

20, v. 1. B. der Begh. vande Weeghconſt.

Kinderen in kennis der weerdicheyt vande Duytſche tael.

Terminorum Proportionis.

dD 4 verduysuch a state that it is doubtful whether the speech of the Ancients would now be completely understood, the Dutch seem to have said as follows: *Since by universal inspiration Nature has impressed on all people the idea that the spoken words require a kind of intonation, such as strong and weak accent and the like, under the influence of which the long syllable of a word has the greatest importance, reason demands that we design this peculiar affix of language not by chance, but properly and methodically. What shall this be? That as a general rule the accent shall always fall on the chief syllable of the active verb and its derivatives, for example that in Hōoren, Verhōorende, Ghehōort, Behōorende, Hōorende the long syllable shall always be Hōor, which is the most important in these words, for we say in the first person Ick Hōor* [1]; *the other syllables, such as en, ver, ende, ghe, be are only affixes, with which all the wordt are altered. But in compounds the accent shall always fall on the first syllable, for example Sāutvat, Hāumes, Ūytgaen, Īnsien* [2], *and the like.* Thus they achieved it, and such, as some suppose, in their barbarian condition, from which it might be argued: *If they have done this in their barbarian condition, of what are they capable in their civilized state!* In truth, we should *the like.* Thus they achieved it, and such, as some suppose, in their barbarian condition, from which it might be argued: *If they have done this in their barbarian condition, of what are they capable in their civilized state!* In truth, we should accept this compliment, but seeing that Nature does not act contrary to Nature, reason demands that we shall be content with a lesser one, viz. that formerly there was a very wise, learned, and excellent race, as has been proved before, which reached this condition after a long time and through much experience. And if we wish to be considered worthy followers of these predecessors, we must not disdain with beastly ingratitude such great gifts bequeathed to us, through ignorance, nor foolishly believe the slanderers, who do not know them, nor, abandoning the mirror of languages, frequently revel in their ugly dregs; but on the contrary should valiantly protect it, not with idle words as are those of the others, nor according to the unwisdom of those who have so little humility as to deem the excellence of things described in their language to be the excellence of the language. But just as gold is tested by fire, so shall its superiority be proved by practice. What shall this be? Take for subject those things which have hitherto been Nature's profound secrets in all the other languages, which they cannot completely denote; for example let them imitate this (among thousands of other things in which Dutch abounds): *Ghelijck rechthefllini tot scheefheflini, alsoo rechthefwicht tot scheefhefwicht* [3]. You may safely promise a cake to those who succeed in this; nay, that they may stay up late (children should after all be given children's rewards), and I assure you that you will get off cheaply, for it is sufficiently evident of what they are capable in this respect, viz. that they can only replace these words by long circumlocutions, which greatly obscure the distinction between the terms and the form of the proportion throughout. But if

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[1]) I hear, of which the other words are derivatives.
[2]) salt-tub, chopping knife, to go out, to see.
[3]) Prop. XX of Book I of the *Art of Weighing*.

verduysteren. Maer soo sy vraghen wat sulcke woorden te bedien hebben, men mach antwoorden, dattet de opening is van t'ghene tot nochtoe den voorighen sterflicken seerbegheerde verborghentheden gheweest sijn, streckende tot groot voordeel van t'menschelick gheslacht, want hoewel yder lichaem in sijn eyghen plaets licht noch swaer en is, nochtans t'ghewicht des lochts is duer sulcx nu volcomelick ghelijck van ander stoffen, metgaders ettelicke sijn noytbekende * ancleuinghen, openbaer gheworden, soo de * daet van dies ende meer anderen cortelick betuyghen sal. Laetter maer cloeclick anuallen, want hebbender Reuchlinus, Valla, Erasmus, Barbarus, Picus, Politianus, &c. me duer gherocht, die maer Latijn en beschermden, Sghelijcx de Françoysen, wiens * strijtredenen ende talens * stof ons kennelick ghenouch sijn, wat sullen wy die het (O weerdighen * grondt!) DVYTSCH voorstaen? Seker niet alleen de spraeck ophelpen, noch ons seluen voorderen, maer oock ander volcken, welcke alsdan niet alleen huer wooninghen ende lichamen met der Duytschen constighe wercken vercieren sullen, maer oock haren gheest met wetenschap, want de Consten welcke ander met haer eyghen woorden niet uyten en connen, die sal den ghemeenen man alhier duer de beghinselen grondelick mueghen verstaen, ende door sijn ingheboren gheneghentheyt tot de selue, die tot yder volcx baet al anderssins connen voorderen dant den anderen mueghelick is.

Adiunctis. *Effectus.* *Argumenta.* *Materia.* *Subiectum.*

Dit is t'ghene wy vande weerdicheyt der Duytsche tael voorghenomen hadden te verclaren; Inde selue sullen wy de WEEGHCONST, die de wonderlicste der vrie is, eerst tot Constens form laten commen, als spraeck die der Natueren eyghenschappen grondelicxt beteeckenen can, ende als bequaemste wit, daer al d'ander die willen, tot yder ghemeentens grootste nut, na micken, ende haer * bepalinghen, daer inde Consten veel an gheleghen is, na rechten mueghen. Oock by aldien der Duytschen vliet daerin alsoo vermeerderde, ghelijck de reden wel eyscht, t'selue soude ons voornemen versterken om met ander angheuanghen voort te varen: Doch soo de contrarie ghesch iede, ick can my vernoughen in een eerlick voornemen mijn goede wille te verclaren, welcke in haer beroup tot yders dienst gheeyghent is.

Definitiones.

CORT-they ask what these words signify, it may be replied that it is the revelation of those things which had hitherto been secrets greatly coveted by earlier mortals, things which greatly benefit mankind, for though a body is neither light nor heavy in its own place, through this it has now become manifest that the air has weight just as well as other substances, while also several of its unknown attributes have thus become known, as the practical discussion of this and other matters will shortly prove. Let them attack valiantly, for what have Reuchlinus, Valla, Erasmus, Barbarus, Picus, Politianus, etc. [1]) achieved, who merely protected Latin, and likewise the French, whose arguments and linguistic material we know well enough? What then shall *we* achieve, who propagate *Dutch* (O worthy subject!)? Certainly not only bring the language on a higher level or advance ourselves, but also other nations, which will then adorn not only their houses and bodies with the artistic products of the Dutch, but also their minds with knowledge, for the arts which other nations cannot express in their own words, will here be thoroughly understood from the elements by the common man, and through his inborn disposition thereto he will be able to advance it, to the profit of all nations, in quite a different way from what is possible to the others.

This is what we had proposed to declare with regard to the worth of the Dutch language. In this language we will first cause the Art of Weighing, which is the most miraculous of the free arts, to attain to the status of an art, being a language which is capable of describing Nature's properties most thoroughly and the most suitable object at which all the others who may wish to are aiming, to the great profit of every community, and on which they may model their definitions, which are of great importance in the arts. Also, if the zeal of the Dutch in this art should increase, as reason demands, this would strengthen our resolve to continue with other planned studies. But if this does not happen, I can be content with an honest resolve to declare my good will, which is at everyone's service, if an appeal is made to it.

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[1]) Stevin here enumerates some famous humanists: Johann Reuchlin (1455-1522), Lorenzo Valla (c. 1406-1457), Desiderius Erasmus (1469?-1536), Ermolaus Barbarus (1454-1493), Giovanni Pico della Mirandola (1463-1494), Angelo Poliziano (1454-1494).

CORTBEGRYP.

Argumentū.

DE Beghinſelen der Weeghconſt, welcke van ſwaerheyt ſijn duer t'ghedacht van natuerlicke ſtof ghewеert, ſullen in twee boucken begrepen worden. Des eerſten boucx eerſte deel ſal van 14 * bepalinghen weſen: T'ander van 28 * voorſtellen, vande ghedaenten der ghewichten, die tweederhande ſijn, als Rechtwichten, ende Scheefwichten. Der Rechtwichten ſijn twee * afcomſten, te weten Rechtdaelwichten, ende Rechthefwichten, beſchreuen inde achtien eerſte voorſtellen. Der Scheefwichten ſijn oock twee afcomſten, als Scheefdaelwichten, ende Scheefhefwichten, verclaert inde reſt der voorſtellen.

Definitionib. *Propoſitionibus.* *Species.*

HET tweede bouck der Beghinſelen ſal vande vinding der * ſwaerheyts middelpunten ſijn, wiens eerſte deel vande * Platten is; T'ander vande lichamen. Twelck wy tot meerder claerheyt int corte ende tafelwys aldus vervaten:

Centrorum grauitatum. *Planorum.*

Der Beghinſelen vande Weeghconſt ſijn twee boucken, het
- ***1e Vande ghedaenten der ghewichten, wiens***
  - ***1e Deel van 14 Bepalinghen.***
  - ***2e Deel van 28 voorſtellen der***
    - ***Rechtwichten.***
      - ***Rechtdaelwichten.***
      - ***Rechthefwichten.***
      - *(both:)* ***Beſchreuen inde 18 eerſte voorſtellen.***
    - ***Scheefwichten.***
      - ***Scheefhefwichten.***
      - ***Scheefdaelwichten.***
      - *(both:)* ***Verclaert inde reſt der voorſtellen.***
- ***2e Vande vinding der ſwaerheyts middelpunten der***
  - ***Platten.***
  - ***Lichamen.***

THE ARGUMENT

The Elements of the *Art of Weighing* [1]), which deal with gravity, dissociated in thought from physical matter, are to be contained in two books. The first part of the first book is to consists of 14 definitions, the second part of 28 propositions about the properties of weights, which are of two kinds, viz. vertical weights and oblique weights [2]). There are two kinds of vertical weights, viz. vertical lowering weights and vertical lifting weights; these are described in the first eighteen propositions. There are also two kinds of oblique weights, viz. oblique lowering weights and oblique lifting weights; these are explained in the remaining propositions.

The second book of the Elements is to deal with the finding of the centres of gravity, to wit: in the first part those of plane figures, and in the second, those of solids. For the sake of greater clearness we summarize this shortly in a scheme as follows:

The Elements of the Art of Weighing comprise two books, the
- 1st of the properties of weights, of which
  - the 1st part contains 14 definitions
  - the 2nd part contains 28 propositions concerning
    - vertical weights
      - vertical lowering weights
      - vertical lifting weights

      described in the first 18 propositions.
    - oblique weights
      - oblique lifting weights
      - oblique lowering weights,

      explained in the remaining propositions,
- 2nd of the finding of the centres of gravity of
  - plane figures
  - solids

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1) *Weeghconst* is Stevin's translation of the Latin term *Ars ponderaria.*

2) i.e. weights by means of which vertical and oblique forces respectively are exerted.

HET EERSTE BOVCK VANDE BEGHINSELEN DER WEEGCONST,

Beſchreuen door Simon Steuin.

TEERSTE DEEL vande Bepalinghen.

I. BEPALING.

Definitio.

WEEGCONST is die, welcke leert de Redenen, Eueredenheden, ende ghedaenten vande ghewichten ofte ſwaerheden der lichamen.

VERCLARING.

GHELIICK de * Meetconſt anſiet der formen grootheden niet hare ſwaerheden, houdende die alleenelick voor euen ofte oneuen, diens grootheden euen ofte oneuen ſijn; Alſoo anſiet ter contrarie de Weegconſt haer ſwaerheden, niet haer grootheden, houdende die voor euen ende oneuen, diens ghewichten euen ofte oneuen ſijn: Ende ghelijck diens voornamelicke wercking beſtaet int onderſoucken der * Redenen, Eueredenheden, ende Ghedaenten haerder grootheden, Alſo deſens int onderſoucken der Redenen Eueredenheden, ende ghedaenten haerder ſwaerheden ofte ghewichten, welcker beſcriuing t'voornemen is deſes handels.

* Geometria

* Rationum, Proportionū & qualitatum.

II. BEPALING.

Swaerheydt eens lichaems, is de macht ſijnder daling in gheſtelde plaets.

VERCLARING.

DE ſwaerheydt ofte lichticheydt die wy ghemeenelick ſegghen een lichaem te hebben, en is niet ſijn eyghen weſendicke ghedaente, maer veroirſaeſt uyt ſijn ghemeenſchap met een ander (wiens breeder verclaring wy elders gheſchict hebben) want veel Stoffen die ſwaer ſijn inde locht, worden licht beuonden int water, ende de lichte inde locht, ſijn elders

Materia.

THE FIRST BOOK
OF THE ELEMENTS OF THE ART OF WEIGHING,
Described by Simon Stevin

THE FIRST PART OF THE DEFINITIONS

DEFINITION I.

The art of weighing is the art which teaches the ratios, proportions, and properties of the weights or gravities of solids.

EXPLANATION.

Whereas geometry relates to the magnitudes of figures, not their gravities, holding only those to be equal or unequal whose magnitudes are equal or unequal, the art of weighing on the contrary relates to their weights, not their magnitudes, holding those to be equal or unequal whose weights are equal or unequal. And just as the chief task of the former consists in examing the ratios, proportions and properties of their magnitudes, so the task of the latter consists in examining the ratios, proportions, and properties of their gravities or weights, the description of which is the object of this treatise.

DEFINITION II.

The gravity of a solid is the power of its descent in a given place.

EXPLANATION.

The gravity or levity which we commonly state a solid to have is not its own essential property, but is caused by reference to something else (the more detailed explanation of which is given elsewhere), for many substances which are heavy in the air are found to be light in water, and those which are light in the air are ders ſwaer; daerom als wy ſegghen een haudt te weghen hondert pondt, wy verſtaen daer by de macht ſijnder daling in gheſtelde plaets, dat is in dien* Grondt daert in gheweghen was.

* *Subiecto.*

DOOR tverkerde deſer bepaling is te verſtaen, dat lichticheyt eens lichaems de macht is ſijnder rijſing, maer in gheſtelde plaets, want eyghentlick is alle lichaem ſwaer.

III. BEPALING.

BEKENDE ſwaerheyt is diemen door bekent ghewicht uytet.

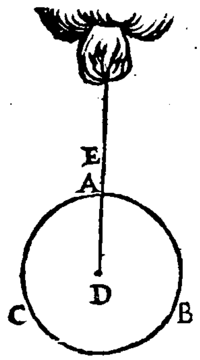
VERCLARING.

ALS wanneermen ſeght een lichaem ofte ſwaerheydt te weghen ſes pont, ofte acht marck, oft drie oncen, &c. Om datſe door ſulcke bekende ghewichten gheuytet wort, wy noemenſe Bekende ſwaerheydt.

IIII. BEPALING.

SWAERHEYDTS middelpunt is, an twelck het lichaem door ons ghedacht hanghende, alle gheſtalt houdt diemen hem gheeft.

VERCLARING.



LAET A B C een cloot ſijn, diens ſtof ouer al eueſwaer is, welcke wy met haer middelpunt D door ons ghedacht nemen te hanghen ande lini E D; Ende is kennelick dat dien cloot ghekeert wordende, ſal houden alle gheſtalt diemen haer gheeft, want ſoomen B keerde daer A is, B ſal daer blijuen, ende voort yder deel op ſijn plaets, want ſoo dat niet en gheſchiede, de ſtof ſoude an deen ſijde ſwaerder ſijn als an d'ander, twelck teghen tgheſtelde waer. D dan naer luyt deſer bepaling is Swaerheydts middelpunt des cloots A B C; Ende alſoo ſalmen verſtaen dat binnen alle lichamen ſoo wel ongeſchicter form ende van ſtof oneenuaerdigher ſwaerheydt als gheſchicter ende eenvaerdigher, is eenich ſulcken punt, waer an tlichaem alſo hanghende, alle gheſtalt houdt diemen hem gheeft, welck punt ghenoemt wort ſijn Swaerheydts middelpunt. Ende op dattet door eenighe ſijne eyghenſchappen kennelicker ſy, ſullender noch dit toe ſegghen: Het ſwaerheydts middelpunt der oirdentlicke lichamen als Pilaren, Clooten, *Lancworpighe Clooten, der Vijf gheſchicte lichamen, &c. ouer al euewichtigher

* *Sphæroidalium.*

heavy elsewhere. If we therefore state a piece of wood to weigh a hundred pounds, we understand by this the power of its descent in a given place, that is in the medium in which it has been weighed.

By the converse of this definition is to be understood that the levity of a solid is the power of its ascent, but in a given place, for properly speaking any solid is heavy.

DEFINITION III.

A known gravity is one expressed by a known weight.

EXPLANATION.

As when a solid or gravity is said to weigh six pounds, or eights marcks [1]), or three ounces, etc. Because this gravity is expressed by such known weights, we call it a known gravity.

DEFINITION IV.

The centre of gravity is the point such that if the solid is conceived to be suspended from it, it remains at rest in any position given to it.

EXPLANATION.

Let *ABC* be a sphere whose material is everywhere equally heavy, which is conceived to be suspended with its centre *D* from the line *ED*. It is evident that if this sphere is turned, it will remain at rest in any position given to it, for if *B* were turned to the position of *A*, still *B* would remain there, and every other part would also remain in its place, for if this did not happen, the material would be heavier on one side than on the other, which would be contrary to the supposition. According to this definition therefore *D* is the centre of gravity of the sphere *ABC*. And in the same way it shall be understood that within all solids, both those of an irregular form and made of a material of non-uniform weight and those of a regular form and made of a material of uniform weight, there is one and only one point such that the solid, if suspended from it, remains at rest in any position given to it, which point is called its centre of gravity. And in order that it may be better known through some of its properties, we shall add the following remarks. The centre of gravity of the ordinary solids such as prisms, spheres, spheroids, of the five regular solids, etc. — the material being everywhere equally

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[1]) The *marck* is a unit of weight of German origin. For particulars the reader is referred to K. M. C. Zevenboom and Dr D. A. Wittop Koning, *Nederlandse gewichten. Stelsels, ijkwezen, vormen, makers en merken.* Leiden, 1953, p. 15 et seq.

wichtigher Stof sijnde, is tselue der Form ofte grootheydt, datmen anders Meetconstich middelpunt noemt. Maer die niet ouer al euewichtigher Stof en sijn, en hebben dese twee punten niet nootsaeckelick tot een selfde plaets. Wat de *naelden, ende onghe schicte lichamen belangt, sy en hebben gheen formens ofte grootheydts middelpunt, maer alleen des swaerheydts. Het ghebuert oock in veel lichamen als Rynghen, Haecken, Beckens, ende dier ghelijcke, dat haer swaerheydts middelpunt niet en valt inde stof des lichaems, maer binnen tlichaem uyt de stof. *Pyramides.*

DAER wort inde bepaling ghe seydt *Duer ons ghedacht* reden datmen int bepalen moet nemen, tghene den aert van tbepaelde best verclaert, twelck Pappus (daer hy int 8^e^ bouck het swaerheydts middelpunt bepaelt) door tghedacht oock bequamelick ghedaen heeft. Men soudet oock mueghen aldus bepalen: *Swaerheydts middelpunt eens lichaems, is door twelck alle plat, tlichaem deelt in euestaltwichtighe deelen.* Wat Euestaltwichticheyt is sal door de 11^e^ Bepaling verclaert worden.

V. BEPALING.

SWAERHEYTS middellini eens lichaems, is de oneindelicke hanghende door sijn swaerheydts middelpunt.

VERCLARING.

ALS inde form der 4^e^ bepaling, de oneindelicke hanghende lini door tswaerheydts middelpunt D, daer an de swaerheyt door ons ghedacht hangt, ghelijck D E ouer beyden sijden oneindelick voortghetrocken, noemen wy de Swaerheydts middellini des lichaems A B C.

VI. BEPALING.

SWAERHEYTS middelplat eens lichaems, is alle plat hem deelende door sijn swaerheydts middelpunt.

VERCLARING.

ALS eenich plat snijende den Cloot der 4^e^ bepaling door sijn middelpunt D, wort des selfden Swaerheyts middelplat ghe seyt, ende alsoo met allen anderen. Sijn eyghenschap is tlichaem alsins te deelen in twee euestaltwichtighe stucken.

VII. BEPALING.

ALLE rechte lini begrepen tusschen twee swaerheyts middellinien, noemen wy dier swaerheden Balck.

A 2 VERCLAheavy — is identical with that of the figure or magnitude, which is otherwise called the geometrical centre. But those solids whose material is not everywhere equally heavy do not necessarily have these two points in the same place. As to the pyramids and irregular solids, they do not have a centre of figure or magnitude, but only a centre of gravity. It may also occur with many solids, such as rings, hooks, basins and the like, that their centre of gravity does not fall within the material of the solid, but inside the solid and outside the material.

The definition contains the word "conceived", because in formulating a definition we should make use of that which is best adapted to explain what is being defined, a method which has also been aptly applied by Pappus by the use of the term "conceived" (where in the 8th book [1]) he defines the centre of gravity). The definition might also be worded as follows [2]): *The centre of gravity of a solid is the point through which any plane divides the solid into parts of equal apparent weight.* The meaning of the expression "equal apparent weight" will be explained in Definition XI.

DEFINITION V.

Centre line of gravity of a solid is the infinite vertical through its centre of gravity [3]).

EXPLANATION.

Thus, in the figure of Definition IV, the infinite vertical through the centre of gravity *D*, from which the gravity is conceived to be suspended, as *DE*, produced indefinitely on either side, is called the centre line of gravity of the solid *ABC*.

DEFINITION VI.

Centre plane of gravity of a solid is any plane dividing it through its centre of gravity.

EXPLANATION.

Thus, any plane cutting the sphere of Definition IV through its centre *D* is called a centre plane of gravity of the said sphere, and the same applies to all the others. Its property is to divide the solid always into two parts of equal apparent weight.

DEFINITION VII.

Any straight line contained between two centre lines of gravity, we call the beam of these gravities.

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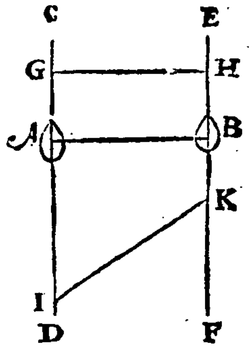
1) *Pappi Alexandrini Collectionis Mathematicae quae supersunt* ed. F. Hultsch. 3 vols, Berlin, 1875-78. VIII 5. III. 1030.

2) This definition is also given by Pappus l.c. On the meaning of the term „equal apparent weight", see p. 38.

3) This definition has been somewhat modified in XI; iv, 1 with a view to the applications to be made in the *Byvouch*. There any line through the centre of gravity is called centre line of gravity, whereas the vertical line through the centre of gravity is called vertical centre line of gravity. In the subsequent applications the term „centre line of gravity" is usually taken to mean the vertical through the point of suspension. See the note on Prop. 6.

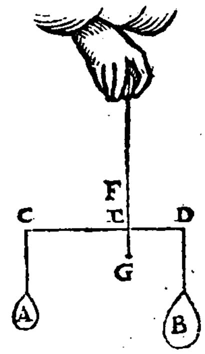


VERCLARING.

LAET A ende B twee lichamen wesen, ende haer swaerheydts middellinien C D ende E F, tusschen de welcke ghetrocken sijn, eenighe linien soot valt als G H, A B, I K, yder van dien, ende alle ander alsoo begrepen tusschen twee swaerheydts middellinien, noemen wy den Balck der swaerheden A B.

VIII. BEPALING.

WESENDE den Balck ghedeelt met de swaerheyts middellini daer de twee swaerheden euestaltwichtich an sijn, wy noemē de deelen Ermen.



VERCLARING.

LAET A B. twee lichamen wesen, diens balck sy C D, welcke ghedeelt is in E, met de swaerheydts middellini F G, daer de twee swaerheden euestaltwichtich an hanghen; de twee deelen des balcx als E C ende E D worden Ermen ghenoemt.

IX. BEPALING.

ENDE die swaerheydts middellini der twee swaerheden, heeten wy Handthaef.

VERCLARING.

ALS E F, der 8e bepaling wort Handthaef ghenoemt.

X. BEPALING.

ENDE des Handthaefs punt inden balck, Vastpunt.

VERCLARING.

ALS E, der 8e bepaling wort Vastpunt gheseyt.

XI. BEPALING.

ENDE die twee swaerheden noemen wy Eycstaltwichtighe.

VERCLA-

EXPLANATION.

Let *A* and *B* be two solids, and their centre lines of gravity *CD* and *EF*, between which there shall be drawn at random a number of lines, as *GH*, *AB*, *IK*; we call each of these, and any others contained in the same way between two centre lines of gravity, the beam of the gravities *A* and *B*.

DEFINITION VIII.

The beam being divided by the centre line of gravity at which the two gravities are of equal apparent weight, we call the parts arms [1]).

EXPLANATION.

Let *A* and *B* be two solids, and their beam *CD*, which is divided in *E* by the centre line of gravity *FG* at which the two gravities are of equal apparent weight. The two parts of the beam, as *EC* and *ED*, are called arms.

DEFINITION IX.

And the centre line of gravity of the two gravities is called the handle.

EXPLANATION.

Thus, *EF* of Definition VIII is called the handle.

DEFINITION X.

And the point of the handle on the beam is called the fixed point.

EXPLANATION.

Thus, *E* of Definition VIII is called the fixed point.

DEFINITION XI.

And the two gravities are said to be of equal apparent weight.

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[1]) Evidently this definition is based on the assumption that the centre of gravity of the system of the two bodies is somewhere between their centre lines of gravity, and on the application of the second definition of centre of gravity given in the Explanation of the 4th Definition.

VERCLARING.

ALS A ende B, inde form der 8ᵉ bepaling, tsy haer eyghenwichten euen ofte oneuen sijn, wy noemen die Eueſtaltwichtighe, ouermidts ſy naer de gheſtalt euewichtich ſijn, want A doet anden balck door tgheſtelde ſoo grooten ghewelt als B, ende B als A.

Deſe Eueſtaltwichticheydt dient nootſaeckelick verſtaen, ende onderſcheyden vande Eueneyghenwichticheydt, angheſien dit al wat anders is als dat, want om by voorbeelt daer af te ſpreken, tghewicht ande cortſte ſijde des onſels hanghende, is ſomtijts thienmael ſwaerder als tander, nochtan hebben ſy een ghelaet van euewichticheyt, maer ten is niet eyghen, dan alleenlick na de gheſtalt.

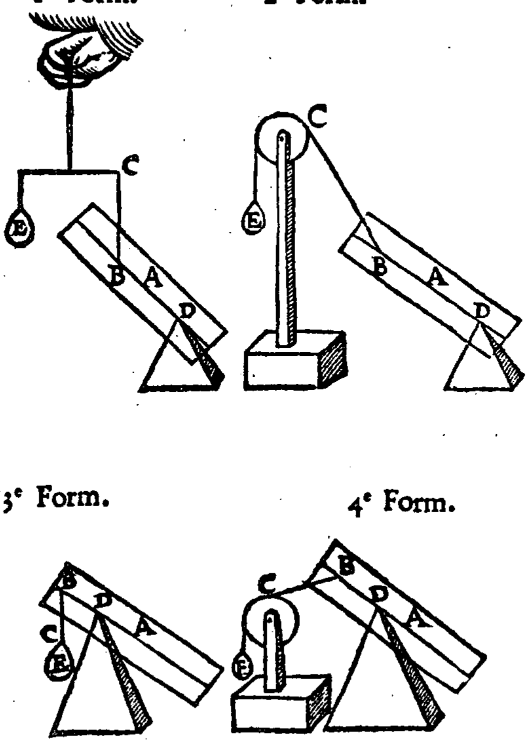
XII. BEPALING.

HEFWICHT is t'ghene oirſaeck is van eens ſwaerheydts verheffing, ende Daelwicht van eens ſwaerheydts daling.

VERCLARING.

LAET den pilaer A, een ſwaerheyt weſen, diens lini daer ſy alſoo by ghehouden wort ſy B C, eñ tpunt daer ſy op ruſt D, ende E, ſy t'ghewicht dat tlichaem A in die gheſtalt houdt. Wy noemen E der eerſte ende tweede Form Hefwicht, overmidts tſelve wicht, het lichaem A verheft, oft in die verheven gheſtalt houdt. Maer E der derde ende vierde Form, Daelwicht, om dattet het lichaem an ſijn ghehechte ſijde B doet dalen, ofte in die ghedaelde geſtalt houdt.

1ᵉ Form. 2ᵉ Form. 3ᵉ Form. 4ᵉ Form.



EXPLANATION.

Thus, *A* and *B* in the figure of Definition VIII, no matter whether their proper weights are equal or unequal, are said to be of equal apparent weight, since in appearance they have the same weight, for by the supposition *A* exerts the same force on the beam as *B*, and *B* as *A*.

It is essential that this term equality of apparent weight be understood, and be distinguished from equality of proper weights, since the latter is quite a different thing from the former. In fact, to give an example, the weight hanging at the shorter side of a steelyard is sometimes ten times heavier than the other weight, and yet they seem to have the same weight, but this is not actually so, but only in appearance [1]).

DEFINITION XII.

Lifting weight is that which causes the ascent of a gravity, and lowering weight is that which causes the descent of a gravity.

EXPLANATION.

Let the prism *A* be a gravity, and let the line by which it is thus held be *BC*, and the point on which it rests *D*, and let *E* be the weight keeping the solid *A* in that position. We call *E* in the first and the second figure lifting weight, since this weight lifts the solid *A* or keeps it in such a lifted position. But *E* in the third and the fourth figure we call lowering weight, because it lowers the solid on the side *B* where it is attached, or keeps it in such a lowered position.

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1) See the remarks in the *Introduction*, p. 38.

XIII. BEPALING.

ENDE de rechte lini vande verheven swaerheyt naer thefwicht, noemen wy Heflini, maer vande ghedaelde swaerheyt naer het daelwicht, Daellini, eñ alsulcke linien in t'gemeen, Trecklini.

VERCLARING.

ALS de rechte lini C B der 12^e bepaling noemen wy inde 1^e ende 2^e form Heflini, maer inde 3^e ende 4^e Daellini. Ende sulcke linien (die bouen de voorgaende ons oock euewidich vanden sichtender connen ontmoetem) in t'ghemeen Trecklini.

XIIII. BEPALING.

ENDE als de Heflini ofte Daellini rechthouckich is opden * Sichteinder, soo noemen wy die Rechtheflini, Rechtdaellini, ende hare ghewichten Rechthefwicht, Rechtdaelwicht: Maer opden Sichteinder scheefhouckich wesende, alsdan Scheefheflini, Scheefdaellini, ende hare ghewichten Scheefhefwicht, Scheefdaelwicht.

Horizon.

VERCLARING.

ALS de Heflini eñ Daellini C B der 1^e ende 3^e form vande 12^e bepaling, om dat sy door t'ghestelde rechthouckich sijn op den sichteinder, wy noemen die Rechtheflini, en dese Rechtdaellini, ende haer ghewichten E Rechthefwicht, Rechtdaelwicht: Maer wesende de Heflini ofte Daellini C B, scheefhouckich opden sichteinder, als inde 2^e ende 4^e form, dan heeten wy die Scheefheflini, ende dese Schefdaellini, ende haer ghewichten E Scheefhwicht, Scheefdaelwicht.

I^e MERCK.

WAER *Sichteinder by ons een woort soo ghemeen ende bekent als byden Griecken* Horizon, *t'welck de Latinen oock ghebruycken, ende daer vooren altemet* Finitor, *ofte* terminator visus, *wy en souden daer af hier niet segghen, ouermits sijn eyghen plaets inde * Sterconst is; Maer want den ongheualligen slaep des Spieghels der talen sulcx niet toeghelaten en heeft, oock dat dit woordt hier naer dickmael sal ghenoemt worden, sullen dat verclaren, doch niet als wesentlicke bepaling deses boucx, om de redenen als vooren, Aldus:* Sichteinder is des

Astrologia.

DEFINITION XIII.

And the straight line from the lifted gravity to the lifting weight we call lifting line, and the one from the lowered gravity to the lowering weight we call lowering line, and all such lines in general we call drawing lines [1]).

EXPLANATION.

Thus, the straight line *CB* of Definition XII is called lifting line in the 1st and the 2nd figure, and lowering line in the 3rd and the 4th figure. And such lines (which besides the above may also be parallel to the horizon) are called in general drawing lines.

DEFINITION XIV.

And when the lifting or the lowering line is at right angles to the horizon, we call it vertical lifting line and vertical lowering line, and the respective weights: vertical lifting weight and vertical lowering weight. But when it is oblique to the horizon, we call it oblique lifting line and oblique lowering line, and the respective weights oblique lifting weight and oblique lowering weight.

EXPLANATION.

As the lifting and the lowering line *CB* of the 1st and the 3rd figure of Definition XII; because by the supposition they are at right angles to the horizon, we call the former vertical lifting line and the latter vertical lowering line, and their weights E: vertical lifting weight and vertical lowering weight. But when the lifting line or lowering line *CB* is oblique to the horizon, as in the 2nd and the 4th figure, we call the former oblique lifting line and the latter oblique lowering line, and their weights *E* oblique lifting weight and oblique lowering weight.

NOTE I [2]).

If "sichteinder" were with us a word as common and familiar as "horizon" with the Greeks, which the Latins also use, and formerly sometimes "finitor" or "terminator visus", we should not mention it here, because its proper place is in astronomy. But because the regrettable sleep of the mirror of languages [3]) has not permitted this, and also because this word will frequently be used hereinafter, we shall explain it, but not as an essential definition of this book, for the reasons mentioned before, thus: Horizon is the world's greatest circle, which

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[1]) This definition has been somewhat changed in XI; iv, 1 in accordance with the change in Definition 5.

[2]) This Note has been omitted in XI; iv, 1

[3]) This expression can only be understood in connection with Stevin's theory about the superiority of the Dutch language, as developed in the *Uytspraeck van de Weerdicheyt der Duytse Tael.* Dutch is the mirror of languages, i.e. all languages have to take example by it; but unfortunately this mirror has long slept, i.e. it has been neglected.

des weerelts grootste rondt, dat haer sienlick deel scheydt van het onsienlick: *Dat is, onder veel ronden die inde Sterconst bepaelt worden, soo isser een het aldermerckelicste, scheydende ooghenschynlick den oppersten baluen weereltcloot vanden ondersten, ende in ons ansien den hemel met sijn omtreck naeckende, t'welck volcommentlicxt schijnt vande hoochste plaets eender contreyen, ofte op een water daer hem nerghens landt en vertoocht; Ende ouermits ons ghesicht langs der eerden ofte langs het water niet voorder strecken en can dan tot diens rondts voornoemden omtreck, ende daer in eindet, soo wort dat rondt ghenoemt den Sichteinder, dat is den Einder van t'ghesicht. Ende alle platten die op t'eertrick vanden Sichteinder * euewydich sijn (welcke by ons ghemeenelick ghesëyt worden op waterpas te ligghen) worden * lijckspreuckelick oock sichteinders ghenoemt. Ick seg lijckspreucklick want eyghentlick ofte * wisconstelick en isser gheen ander, dan dat door des weerelts middelpunt streckt.* *Parallela.* *Metaphorice.* *Mathematice.*

IIe MERCK.

DE *form vanden Weegconstighen* Pilaer, is de selue der *Meetconst, maer wy nemen hier sijn stof eenuaerdigher swaerheyt te wesen, ende sijn grondt ende decsel viercanten. Wat de ghemeene constwoorden belangt int Latijn aldus ghebruyckt.* *Columna.* *Geometria.*

Materia	Daer voor sullen wy soodanige Duytsche stellen	Stof
Forma		Form
Effectus		Daet
Subiectum		Grondt
Aiunctum		Ancleuing
Genus		Gheslacht
Species		Afcomst
Definitio		Bepaling
Propositio		Voorstel
Problema		Eysch
Theorema		Vertooch
Ratio		Reden
Proportio		Everedenheyt
Æquales		Even
Similes		Ghelijcke
Exemplum		Voorbeelt
Centrum grauitatis		Swaerheyts middelpunt
Axis		As
Diameter		Middellini
Circumferentia		Omtreck
Parallela		Euewydeghe
Homologa latera		Lijckstandighe sijden
Superficies		Vlack

A 4 Planum

separates its visible from its invisible part. That is, among many circles which are defined in astronomy there is one which is most notable, apparently separating the upper half of the celestial sphere from the lower half and touching in our view the heaven with its periphery, a fact which becomes most completely apparent from the highest place of a region or on an expanse of water where no land presents itself to our view. And since our sight cannot extend along the earth or the water beyond the aforesaid periphery of that circle, and ends therein, that circle is called the "sichteinder", that is the "einder van t' ghesicht" (terminator of sight). And all those planes which on the earth are parallel to the horizon (which with us are usually said to be level) are also called horizons metaphorically. I say metaphorically, for in the proper or mathematical sense there is no other horizon but that passing through the centre of the world.

NOTE II.

The form of the prism as considered in the art of weighing is the same as in geometry, but we here take its material to be everywhere equally heavy, and its base and top to be squares. As to the common technical words, used as follows in Latin:

materia		stof	— material
forma		form	— form, figure
effectus		daet	— practice
subiectum		grondt	— medium
adiunctum		ancleuing	— attribute
genus		gheslacht	— genus
species		afcomst	— species
definitio		bepaling	— definition
propositio		voorstel	— proposition
problema		eysch	— problem
theorema		vertooch	— theorem
ratio	we shall use the following Dutch words for them	reden	— ratio
proportio		everedenheyt	— proportion
aequales		even	— equal
similes		ghelijcke	— similar
exemplum		voorbeelt	— example
centrum gravitatis		swaerheyts middelpunt	— centre of gravity
axis		as	— axis
diameter		middellini	— diameter
circumferentia		omtreck	— circumference
parallelae		euewydeghe	— parallel lines
homologa latera		lijckstandighe sijden	— homologous sides
superficies		vlack	— surface

Planum	Plat
Columna	Pilaer
Arithmetica	Telconſt
Geometria	Meetconſt
Ars Mathematica	Wiſconſt
Mathematicus	Wiſconſtnaer
Mathematice	Wiſconſtlick.

WELCKE *Latijnſche met eenighe ander dieder by mueghen vallen Wy tot meerder claerheyt, ſomwylen inden cant ſullen ſcriven neven haer duytſche. Deſe drie letteren v. b. E. altemet inde cant gheſtelt beteeckenen om cortheydt, voorſtel, bouck, Euclides, als 2 v. 6. b. E. dat is te ſegghen het 2ᵉ voorſtel des 6ᵉⁿ boucx van Euclides.*

BEGHEERTEN.

ANGHESIEN ſommighe ſaken als beghinſelen door ghemeene wetenſchap bekendt ſijn, ende gheen bewijs en behouven; Ander bedectelicker den beriſpers tot ſtof ſouden dienen, om te ſtraffen t'ghene gheen ſtraf en verdient, Wy ſullen naer * Wiſconſtnaers ghebruyck, eer wy tot de voorſtellen commen, begheeren dat ons alſulcke toeghelaten worden.

** Mathematicorum more.*

I. BEGHEERTE.

WY begheeren datmen toelate euen ghewichten an euen ermen oock eueſtaltwichtich te ſijne.

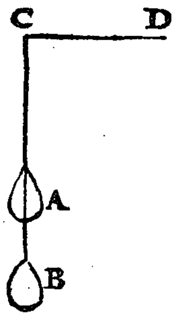
II. BEGHEERTE.

** Mathematica.*

ENDE ande* wiſconſtighe lini alle ghewicht te connen hanghen ofte daer op te connen ruſten, ſonder dat ſy breke ofte buyghe.

III. BEGHEERTE.

ENDE de ſwaerheydt hoogher ofte leegher hangende, altijt van een ſelfde gewicht te blijven.



VERCLARING.

ALS de ſwaerheydt A neerghetrocken ſijnde tot B, aldaer euen ſoo ſwaer te weſen, ofte ſulcken macht an C D te doen, als ſy ter plaets van A dede.

IIII. BE-

planum	plat	— plane (figure)
columna	pilaer	— prism
arithmetica	telconst	— arithmetic
geometria	meetconst	— geometry
ars mathematica	wisconst	— mathematics
mathematicus	wisconstnaer	— mathematician
mathematice	wisconstlick	— mathematically

We shall sometimes give these Latin words, with some more that may occur, in the margin by the side of the Dutch words, for the sake of greater clearness. The three letters v.b.E., sometimes found in the margin, signify for brevity's sake "voorstel" (proposition), "bouck" (book), "Euclides"; for example: 2v.6b.E. means the 2nd proposition of the 6th Book of Euclid.

POSTULATES.

Since some matters of an elementary nature are common knowledge, and need not be proved, while other matters of a more veiled character might give the critics cause to criticize that which does not deserve criticism, we shall, after the custom of mathematicians, before arriving at the propositions, postulate that the following things be granted.

POSTULATE I.

We postulate that it be granted that equal weights at equal arms are also of equal apparent weight.

POSTULATE II.

And that at the mathematical line any weight can hang or rest without its breaking or bending.

POSTULATE III.

And that the gravity always keeps the same weight, no matter whether it hangs higher or lower.

EXPLANATION.

And that the gravity *A*, being pulled down to *B*, has the same weight in that place or exerts on *CD* the same force as it did in the place *A* [1]).

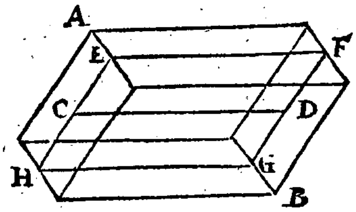
[1]) It is only in the Explanation that the meaning of the Postulate becomes manifest: it is not only the weight of the body which is unchanged when it is elevated or lowered, but also the influence it exerts on the lever.

IIII. BEGHEERTE.

ENDE datmen by des pilaers beſchreuen plat t'welck hem door de langde des as deelt, verſtaen ſal den voorgheſtelden pilaer.

VERCLARING.

ALS weſende A B een pilaer diens as C D, ende de ſelue doorſneen met eenich plat als E F G H, datmen door t'beſcreuen plat E F G H, al de reſt achterghelaten, verſtaen ſal den ghegheuen pilaer.

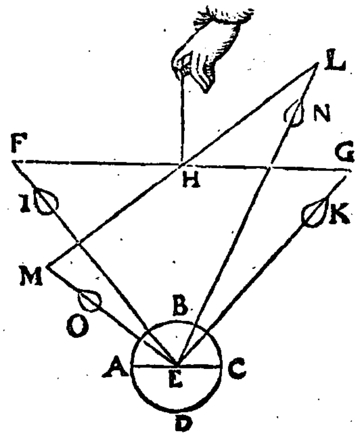


V. BEGHEERTE.

ENDE alle hanghende linien voor * evewydighe ghehouden te worden. *Parallelis.*

VERCLARING.

DE reden is deſe; Laet A B C D den eertſcloot ſijn, wiens middelpunt E, ende * ſichteinder A C, ende F G een balck, euewydich vanden ſichteinder A C, diens balcx even ermen H F, H G, ende euen ſwaerheden daer an I, K; alwaer het blyct, dat de hanghende linien F I, ende G K, gheen euewydighe en ſijn, maer onder naerder malcander dan bouen; Laet daer naer den balc F G ghekeert worden op t'vaſtpunt H, alſoo dat G comme daer nu L is, ende F daer M, ende K ſal commen daer nu N, ende I daer nu O is, ende den houck L M E is naerder den rechthouck dan M L E, waer duer O (als in het volghende 22e voorſtel blijcken ſal) naer de gheſtalt ſwaerder is dan N. Vyt deſen volght oock dat onder alle lichamelicke formen die inde natuer beſtaen, ſo en iſſer gheen ander, * wiſconſtelick ſprekende, dan den cloot, an wiens ſwaerheydts middelpunt het lichaem door ons ghedacht hanghende, alle gheſtalt houdt diemen hem gheeft; Ofte door t'welck alle plat, t'lichaem deelt in eve- *Horizon.* *Mathematice.*



B

POSTULATE IV.

And that a plane through the axis [1]) of the prism shall stand for the given prism.

EXPLANATION.

Let *AB* be a prism, its axis *CD*, and let the prism be cut by any plane, as *EFGH*. That the plane *EFGH*, all the rest omitted, shall stand for the given prism.

POSTULATE V.

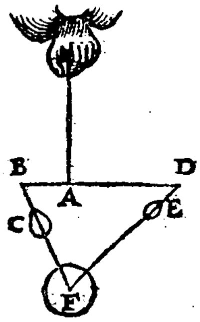
And that all verticals be held to be parallel lines.

EXPLANATION.

The reason is the following. Let *ABCD* be the terrestrial sphere, the centre of which is *E*, the horizon *AC*, and *FG* a beam parallel to the horizon *AC*, the equal arms of this beam *HF*, *HG*, and let there be hanging therefrom equal gravities *I*, *K*; from which it may be seen that the verticals *FI* and *GK* are not parallel lines, but are nearer to one another at the bottom than at the top. Let then the beam *FG* be turned about the fixed point *H* so that *G* comes where *L* is now, and *F* where *M*, and *K* will come where *N*, and *L* where *O* is now, and the angle *LME* is nearer to a right angle than *MLE*, owing to which (as will become apparent in the 22nd proposition hereinafter) *O* is heavier in appearance than *N*. From this it also follows that among all the corporeal forms existing in Nature there is, mathematically speaking, none but the sphere which remains at rest in any position given to it, when conceived to be suspended from its centre of gravity, or which is divided by any plane through the centre of gravity into parts

[1]) The axis of a prism or a cylinder is the straight line joining the centres of gravity of the parallel faces. The axis of a pyramid or a cone is the straight line joining the vertex with the centre of gravity of the base.

in eueſtaltwichtighe deelen, maer om de oneindelicke verſcheyden gheſtalten, ſullender oneindelicke verſcheyden ſwaerheyts middelpunten in ſijn. Oock en ſoude (teghen t'volgende 1ᵉ voorſtel) de ſwaerſte ſwaerheyt niet ſulcken reden hebben tot de lichtſte, als den langſten erm tot den cortſten, maer d'eene ſoude naer de gheſtalt ſwaerder ſijn, om dat haer houck plomper ende den rechthouck naerder is dan des anders houck. Maer om t'ſelue by voorbeelt te verclaren, laet A B den cortſten erm ſijn, diens ghewicht C, ende A D den langſten erm, diens ghewicht E in ſulcken reden ſy tot tghewicht C, als A B tot A D, ende F ſy t'ſweerelts middelpunt; Alwaer blijckt dat den houck F B A plomper ende den rechthouck naerder is, dan den houck A D F, waer uyt volght (door tvoornoemde 22ᵉ voorſtel) dat C naer de gheſtalt ſwaerder ſal ſijn dan E.



Alle deſe ongheuallen ſpruyten daer uyt, dat F E met G E in d'eerſte form, ofte B F met D F der tweede form, gheen evewydighe linien en ſijn: Maer ouermits dat verſchil in alle t'ghene de menſchen weghen, onbemerckelick is, want den balck ſoude al veel milen lanck moeten ſijn eer hem dat ſoude connen openbaren, ſoo begheeren wy datſe voor euewydighe ghehouden worden. Wel is waer dat wy die anſiende voor t'ghene ſy ſijn, volcommelick ſouden connen wercken na haerlieder ghedaente, maer want dat moeyelicker ſoude weſen, ende tot de ſaeck, dat is de WEEGDAET nochtans niet voorderlicker, ſo iſt beter ghelaten.

HET

of equal apparent weight, but because of the infinite variety of forms there will be an infinite number of different centres of gravity in them. Also (contrary to the 1st proposition hereinafter) the heavier gravity would not have to the lighter the same ratio as the longer arm to the shorter, but the one would be heavier in appearance than the other, because its angle is more obtuse and nearer to a right angle than the angle of the other. But in order to explain this by an example, let *AB* be the shorter arm, its weight *C*, and *AD* the longer arm, whose weight *E* shall have to the weight *C* the same ratio as *AB* to *AD*, and let *F* be the centre of the earth; then it appears that the angle *FBA* is more obtuse and nearer to a right angle than the angle *ADF*, from which it follows (by the aforesaid 22nd proposition) that *C* will be heavier in appearance than *E*.

All these difficulties result from the fact that *FE* and *GE* in the first figure, or *BF* and *DF* in the second figure, are not parallel lines. But since this difference is imperceptible in all things weighed by us — for the beam would have to be many miles long before it would become perceptible — we postulate that they be held to be parallel lines. It is true that, taking them for what they are, we should be able to operate exactly according to their nature, but because this would be more difficult, and yet would be of no advantage for the practice of weighing, it is better not to do this.

HET ANDER DEEL VANDE VOORSTELLEN.

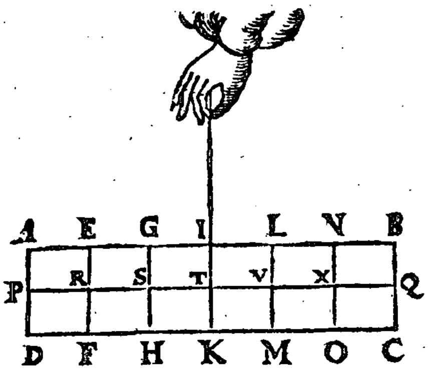
I. VERTOOCH. I. VOORSTEL. *Theorema. Propoſitio.*

WESENDE twee eueſtaltwichtighe ſwaerheden, de ſwaerſte heeft ſulcken reden tot de lichtſte, als den langſten erm tot den cortſten.

I^e^ VOORBEELT.

TGHEGHEVEN. Laet A B C D een pilaer ſijn weghende 6 ℔. welcke ghedeelt ſy in 6 euen deelen, door * platten euewydich van ſijn grondt A D, als E F, G H, I K, L M, N O, ſniende den as P Q in R, S, T, V, X: Laet ons nu nemen L M D A voor de ſwaerſte ſwaerheydt, wiens ſwaerheyts middelpunt is S, ende L M C B voor de lichtſte ſwaerheydt, wiens ſwaerheyts middelpunt is X, eñ SX is dier deelen balck door de 7^e^ bepaling, eñ T is t'ſwaerheyts middelpunt des heelen pilaers, ende T I d'hanthaef, waer an L M D A ende L M C B eveſtaltſwichtich hangen, ende T X is den langſten erm, ende T S den cortſten door de 8^e^ bepaling. TBEGHEERDE. wy moeten bewyſen dat ghelijck de ſwaerſte ſwaerheydt L M D A, tot de lichtſte L M C B, alſo den langſten erm T X, tot den cortſten T S. TBEWIIS. De ſwaerſte ſwaerheydt L M D A weeght 4 ℔, ende de lichtſte L M C B 2 ℔, ende den langſten erm T X heeft ſulcken reden tot de cortſte T S, ghelijck 2 tot 1 door t'ghegheven: Maer ghelijck 4 tot 2, alſoo 2 tot 1, ghelijck dan de ſwaerſte ſwaerheyt L M D A, tot de lichtſte L M C B, alſo den langſten erm T X, tot den cortſten T S.

Datum. *Plana parallela.* *Quæſitum.* *Demonſtratio.*



MAER op datmen niet en dencke dit daer alſo by gheualle ghe- ſciedt te ſijne, wy ſullender * Wiſconſtich bewys af doen aldus: *Mathematicam demonſtrationem.*

II^e^ VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom een pilaer ſijn, ghedeelt

B 2 met een

THE SECOND PART
OF THE PROPOSITIONS

THEOREM I. PROPOSITION I.

Given two gravities of equal apparent weight, the heavier has to the lighter the same ratio as the longer arm to the shorter.

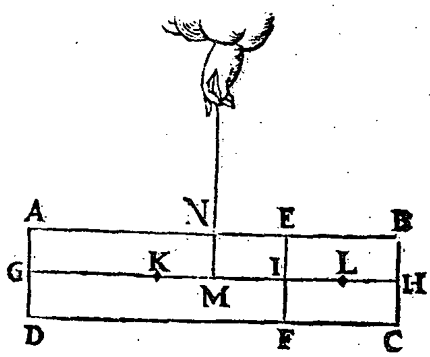
EXAMPLE I.

SUPPOSITION. Let *ABCD* be a prism weighing 6 lbs, which shall be divided into 6 equal parts by planes parallel to its base *AD*, as *EF*, *GH*, *IK*, *LM*, *NO*, meeting the axis *PQ* in *R*, *S*, *T*, *V*, *X*. Let us now take *LMDA* for the heavier gravity, whose centre of gravity is *S*, and *LMCB* for the lighter gravity, whose centre of gravity is *X*; then *SX* is the beam of these parts by the 7th definition, and *T* is the centre of gravity of the whole prism, and *TI* the handle at which *LMDA* and *LMCB* are hanging in equality of apparent weight, and *TX* is the longer arm and *TS* the shorter arm by the 8th definition. WHAT IS REQUIRED TO PROVE. We have to prove that as the heavier gravity *LMDA* is to the lighter *LMCB*, so is the longer arm *TX* to the shorter *TS*. PROOF. The heavier gravity *LMDA* weighs 4 lbs and the lighter *LMCB* 2 lbs, and the longer arm *TX* by the supposition has to the shorter *TS* the ratio of 2 to 1. But as 4 is to 2, so is 2 to 1; therefore, as the heavier gravity *LMDA* is to the lighter *LMCB*, so is the longer arm *TX* to the shorter *TS*.

But in order that it may not be thought that this happened only accidentally, we shall give a mathematical proof of it, as follows.

EXAMPLE II.

SUPPOSITION. Let *ABCD* again be a prism, divided by a plane parallel to met een plat euewydich van A D, als E F, fniende den as G H, waert fy in I, ende het fwaerheyts middelpunt van het deel E F D A fy K, int middel van G I, ende van het deel E F C B, fy L int middel van I H, eñ des heels A B C D fy M int middel van G H, ende M N fal der deelen E F D A ende E F C B handthaef fijn, daer an fy eueftaltwichtich hanghen.



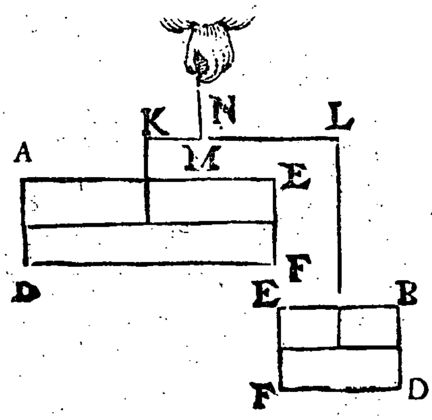
1. Gheftalt. TBEGHEERDE. Wy moeten bewyfen dat ghelijck het lichaem ofte de fwaerheydt (twelck hier een felfde is om *Proportioně.* * haer eueredenheydt, want ghelijck tlichaem E F D A, tot tlichaem E F C B, alfoo diens fwaerheyt tot defens, ouermits den pilaer door tgheftelde oueral eenuaerdigher fwaerheyt is) van E F D A, tot E F C B, alfo den langften erm M L, tot den cortften M K. TBEWYS, 1e LIDT. M H is euen an M G door tghegheuen, laet tot elck doen K M, foo fal dan K H euen fijn an M G met K M; daer naer van d'eene ghetrocken G K, ende van d'ander K I (welcke G K ende K I euen fijn door tghegheuen) foo fal K M met K M euen blijuen an I H; Ende haer helften als K M ende I L fullen oock euen fijn. II e LIDT. Laet tot elck (te weten K M ende I L) doen M I, Ende M L fal euen fijn an I K. III e LIDT. Ghelijck G I tot haer *Alternam proportionem* helft K I, alfo I H tot haer helft I L, ende door * oueranderde eueredenheyt ghelijck G I tot I H, alfo K I tot I L, maer K I is euen an M L door het 2e lidt, ende I L an M K door het 1e lidt, daerom ghelijck G I tot I H, alfoo M L tot M K; Maer ghelijck G I tot I H, alfo het lichaem ofte de fwaerheyt E F D A, tot E F C B. Ghelijck dan de fwaerfte fwaerheyt E F D A, tot de lichtfte E F C B, alfo den langften erm M L, tot den cortften M K.

NV mocht yemant fegghen, ghy hebt dat voorftel wel bewefen in deelen die tfamen een heel pilaer maken eenvaerdigher fwaerheyt, maer wie weet of dat alfo plaets fal houden in allen anderen verfcheyden deelen van onghefchicter form, ende oneuefwaerder ftof, daerom fullen wy de ghemeenheydt des voorftels aldus bethoonen: Laet ons achten dat den balck K L der 1e gheftalt hier bouen, in haer plaets bliue, ende dat het ftick E F D A neerghetrocken wordt, ende dat het blyue hanghende met een lini uyt fijn fwaerheydts middelpunt an tpunt K, ende dat infghelijcx oock neerghetrocken fy het ander ftick E F C B, ende dat het blijue hanghende by fijn fwaerheydts middelpunt an tpunt L, *AD*, as *EF*, meeting the axis *GH* in *I*, and let the centre of gravity of the part *EFDA* be *K* in the middle of *GI*, and that of the part *EFCB*, *L* in the middle of *IH*, and that of the whole *ABCD*, *M* in the middle of *GH*, and *MN* shall be the handle of the parts *EFDA* and *EFCB*, at which they are hanging in equality of apparent weight. WHAT IS REQUIRED TO PROVE. We have to prove that as the solid or the gravity (which is the same thing in this case on account of their proportionality, for as the solid *EFDA* is to the solid *EFCB*, so is the gravity of the former to that of the latter, since by the supposition the prism is everywhere equally heavy) *EFDA* to *EFCB*, so is the longer arm *ML* to the shorter *MK*. PROOF, FIRST SECTION. *MH* is equal to *MG* by the supposition; add to each *KM*, then *KH* will be equal to *MG* plus *KM*. If then *GK* is substracted from the one and *KI* from the other (which *GK* and *KI* are equal by the supposition), *KM* plus *KM* will remain equal to *IH*, and their halves, as *KM* and *IL*, will also be equal. SECOND SECTION. Add to each (viz. *KM* and *IL*) *MI*; then *ML* will be equal to *IK*. THIRD SECTION. As *GI* is to its half *KI*, so is *IH* to its half *IL*, and by taking the terms alternately: as *GI* is to *IH*, so is *KI* to *IL*. But *KI* is equal to *ML* by the second section, and *IL* to *MK* by the first section, therefore, as *GI* is to *IH*, so is *ML* to *MK*; but as *GI* is to *IH*, so is the solid or gravity *EFDA* to *EFCB*. Consequently, as the heavier gravity *EFDA* is to the lighter *EFCB*, so is the longer arm *ML* to the shorter *MK* [1]).

Now someone might say: You have proved this proposition indeed of parts which together constitute a complete prism made of material which is everywhere equally heavy, but who knows whether the proposition will also hold with regard to all other different parts of irregular form and made of material which is not everywhere equally heavy. Therefore we shall prove the general validity of the proposition as follows. Let us assume that the beam *KL* of the 1st figure above remain in its place, and that the part *EFDA* be pulled down, and remain hanging at the point *K* by a line from its centre of gravity, and that likewise the other part *EFCB* be also pulled down, and remain hanging at the point *L* in its centre

[1]) For a criticism of this argument, see the *Introduction*, p. 40.

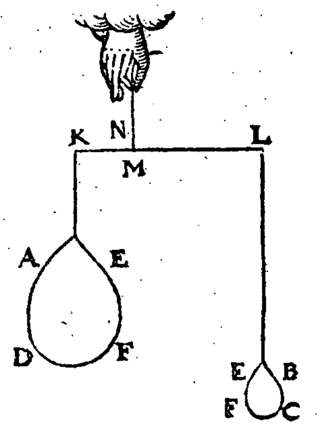
tpunt L, ende dat E F C B niet en ghenake an E F D A, ende haer gheſtalt ſy dan ſoo dees form uytwyſt. Nu doen het lichaem in d'eerſte gheſtalt hinck ande hanthaef M N, alſdoen was E F D A eueſtaltwichtich met E F C B; Maer tghewicht E F D A in dees tweede gheſtalt neerghetrocken ſynde, en brengt an K L gheen meerder noch minder ſwaerheyt dan in d'eerſte gheſtalt door de 3^e^ begheerte. Sghelijcx en brengt tghewicht



2. Gheſtalt.

E F C B der tweede gheſtalt, an L K gheen meerder ſwaerheydt dan in d'eerſte gheſtalt, daerom de ghewichten der tweede gheſtalt ſijn an K L de ſelfde die ſy in d'eerſte waren, daerom oock de balck K L blijft noch inde ſelue eerſte gheſtalt, waer duer E F D A noch eueſtaltwichtich blijft met E F C B. De ſticken dan des pilaers blijuen ſoo wel eueſtaltwichtich verſcheyden, als doen ſy an malcanderen waren, ende de ermen oock inde ſelue reden.

Dit ſo ſynde, laet ons de lichamen E F D A ende E F C B der tweede gheſtalt ander formen gheuen, die alſoo duwende (neemt dat de ſtof ſy van was, cleye, ofte yet ſoodanich t'welck ſulcx lijde) dat E F D A der tweede gheſtalt, ſy E F D A deſer derde gheſtalt, ende dat E F C B der tweede gheſtalt, ſy E F C B deſer derde gheſtalt; Ende is openbaer dat K L noch in haer ſelue gheſtalt ſal blyuen, ende de ermen M L, M K, inde ſelue reden, ende veruolgens E F D A noch eueſtaltwichtich met E F C B, want dees verandering der form (al de ſtof bliuende) en veroirſaeckt gheen verandering des ghewichts.



3. Gheſtalt.

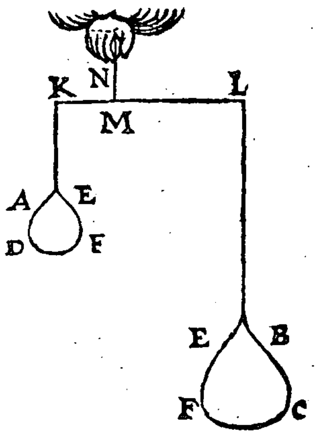
Laet ons ten laetſten weeren E F D A der derde gheſtalt ende hanghen in diens plaets een lichaem van loot des ſelfden ghewichts, ende inde plaets van E F C B een hauten lichaem des ſeluen ghewichts,

B 3

of gravity, and that *EFCB* do not touch *EFDA*; then their position will be such as this figure shows. Now when the solid in the first figure hung at the handle *MN*, *EFDA* was of equal apparent weight to *EFCB*. But when the weight *EFDA* is pulled down in this second figure, it does not cause any more or less gravity to hang at *KL* than in the first figure by the 3rd postulate. Likewise the weight *EFCB* of the second figure does not cause any more gravity to hang at *LK* than in the first figure. Therefore the weights of the second figure at *KL* are the same as in the first figure, and therefore also the beam *KL* remains in the position of the first figure, owing to which *EFDA* remains of equal apparent weight to *EFCB*. Thus the parts of the prism, when separated, remain of equal apparent weight just as when they were joined, and the arm also have the same ratio.

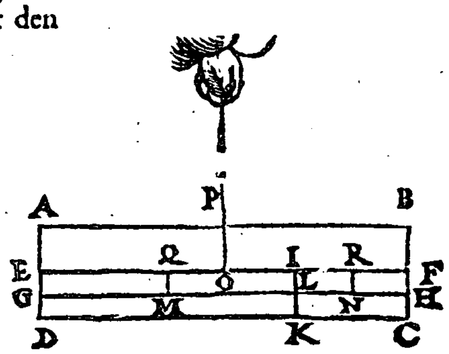
This being so, let us give other forms to the solids *EFDA* and *EFCB* of the second figure, moulding them in such a way (assuming the material to be wax, clay or something of the kind, which shall admit of it) that *EFDA* of the second figure shall become *EFDA* of this third figure, and that *EFCB* of the second figure shall become *EFCB* of this third figure. Then it is manifest that *KL* will remain in the same position, and the arms *ML*, *MK* will have the same ratio, and consequently *EFDA* will still be of equal apparent weight to *EFCB*, for this change of the form (all the material remaining) does not cause any change in the weight.

Let us finally take away *EFDA* of the third figure, and hang in its place a solid of lead of the same weight, and in the place of *EFCB* a wooden solid of the wichts, wiens vierde gheſtalt alſdan ſy als hier neuens. Ende is kennelick dat K L noch inde ſelue gheſtalt ſal blyuen, ende veruolghens E F D A noch 4. Gheſtalt. eueſtaltwichtich met E F C B, ende de ermen noch inde ſelue reden.

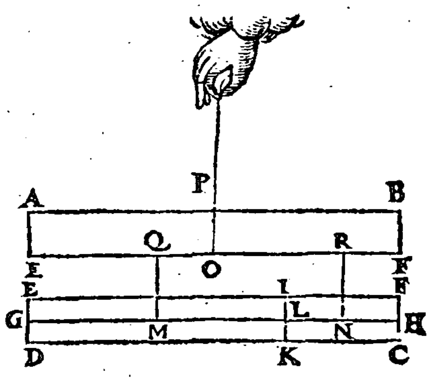


III. VOORBEELT.

MEN can tvoorgaende oock bethoonen, blyuende de twee ſwaerheden hanghende an eenen lichamelicken balck, in deſer voughen: Laet den pilaer A B C D gheſneen ſijn in twee deelen, met een plat door den as E F, ende den as des onderſten deels E C ſy G H, ende E C ſy doorſneen met een plat I K euewydich vanden grondt E D, ſniende den as G H in L, ende het ſwaerheydts middelpunt van het deel I K D E ſy M int middel van G L, ende van het deel I K C F ſy N int middel van L H, ende des heels A B C D ſy O in middel van E F, ende O P ſy ſwaerheydts middellini des heels A B C D, ende M Q van I K D E, ende N R van I K C F. Dit ſoo ſijnde tis kennelick dat des heels pilaers rechter ſijde, euewichtich is teghen haer ſlincker.



LAET ons nu het onderſte deel E F C D neertrecken, alſo dat het blyue hanghende ande linien M L ende N R, als hier neuens. Ende is openbaer dat den lichamelickē balc A B F E noch in haer eerſte gheſtalt ſal blyuen. Laet ons nu achten dat het deel I K D E, gheſneen ſy van I K C F, ende dat elck deel vallē mach daert wil, maer ſy hanghen an haer ſwaerheyts



middel-same weight, the situation then being as shown in the annexed fourth figure. It is then evident that *KL* will again remain in the same situation, and consequently *EFDA* will still be of equal apparent weight to *EFCB*, and the arms will still have the same ratio.

EXAMPLE III.

The above may also be shown when the two gravities remain hanging from a physical beam, in the following way: Let the prism *ABCD* be cut into two parts by a plane through the axis *EF*, and let the axis of the lower part *EC* be *GH*, and let *EC* be cut by a plane *IK* parallel to the base *ED*, meeting the axis *GH* in *L*, and let the centre of gravity of the part *IKDE* be *M* in the middle of *GL*, and that of the part *IKCF*, *N* in the middle of *LH*, and that of the whole *ABCD*, *O* in the middle of *EF*, and let *OP* be the centre line of gravity of the whole *ABCD*, and *MQ* of *IKDE*, and *NR* of *IKCF*. This being so, it is evident that the right side and the left side of the whole prism are of equal weight [1]).

Now let us pull down the lower part *EFCD*, in such a way that it shall remain hanging from the lines *ML* and *NR*, as shown in the annexed figure. Then it is manifest that the physical beam *ABFE* will still remain in the situation of the first figure. Now let us suppose that the part *IKDE* be cut from *IKCF*, and that either part is free to fall at will; but they are hanging at their centres of gravity

1) Read: of equal apparent weight.

middelpunten M,N,ſy houden dan haer eerſte ghegheuen gheſtalt door de 4e bepaling, daerom A B F E blijft oock noch in ſijn eerſte ghedaente. Maer I K D E, ſulcken reden te hebben tot I K C F, als den erm O R, tot den erm O Q, is vooren beprouft; Inder voughen dat tghene eerſt betoocht was anden weegconſtighen balck (dat is een lini) ſulcx hebben wy hier verclaert an een lichamelicken. TBESLVYT. Weſende dan twee eueſtaltwichtighe ſwaerheden, de ſwaerſte heeft ſulcken reden tot de lichtſte (van wat ſtof ofte form oock de lichamen ſijn) als den langſten erm tot den cortſten, twelck wy bewyſen moeſten. *Conclusio.*

VERVOLG.

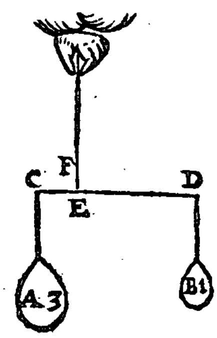
VYT het verkeerde des voorgaenden voorſtels volcht, dat hebbende de ſwaerſte ſwaerheydt ſulcken reden tot de lichtſte, als den langſten erm tot den cortſten, dat die twee ſwaerheden eueſtaltwichtich ſijn.

I. EYSCH. II. VOORSTEL. *Problema.*

WESENDE ghegheuen bekende ſwaerheden, haer handthaef te vinden.

1e VOORBEELT.

TGHEGHEVEN. Laet d'een ſwaerheyt A ſijn weghende 3 ℔, hanghende an C, d'ander B van 1 ℔ hanghende an D; ende C D ſi balck.



TBEGHEERDE Wy moeten haer hanthaef vinden. TWERCK. Men ſal C D alſo deelen, dat haer meeſte ſtick naeſt de ſwaerheydts middellini van de minſte ſwaerheydt, ſulcken reden hebbe tot het minſte ſtick, ghelijck de meeſte ſwaerheyt tot de minſte, twelck ſy in E, te weten dat E D ſulcken reden hebbe tot E C, als 3 ℔ van A, tot 1 ℔ van B. Ick ſeg dat de hanghende door E, als E F, d'hanthaef is.

II VOORBEELT.

TGHEGHEVEN. Laet d'een ſwaerheydt ſijn den pilaer A B C D weghende 6 ℔, ghedeelt als den pilaer int beghin des eerſten voorſtels; Ende an Q hanghe een ghewicht Y van 12 ℔. TBEGHEERDE. Wy moeten d'handthaef vinden. TWERCK. De ſwaerheydts middellini des pilaers is I T, eñ van tghewicht Y is B Q, ende T Q is balck, de ſelue

M, *N*, and they therefore remain in their first given situation by the 4th definition. Hence *ABFE* also still remains in its first situation. But it has been proved before that *IKDE* has to *IKCF* the same ratio as the arm *OR* to the arm *OQ*. Therefore, what was first proved with regard to a beam as considered in the art of weighing (that is a line), we have here explained with regard to a physical beam. CONCLUSION. Given therefore two gravities of equal apparent weight, the heavier one has to the lighter (no matter of what material the solids consist and what form they have) the same ratio as the longer arm to the shorter, which we had to prove.

COROLLARY.

From the converse of the preceding proposition it follows that if the heavier gravity has to the lighter the same ratio as the longer arm to the shorter, the two gravities are of equal apparent weight [1]).

PROBLEM [2]) I. PROPOSITION II.

Given two known gravities, to find their handle.

EXAMPLE I.

SUPPOSITION. Let the one gravity be *A*, weighing 3 lbs, hanging at *C*, the other *B* of 1 lb, hanging at *D*, and let *CD* be the beam. WHAT IS REQUIRED TO FIND. We have to find their handle. CONSTRUCTION. *CD* shall be divided in such a way that the longer segment of it, adjacent to the centre line of gravity of the lighter gravity, shall have to the shorter segment the same ratio as the heavier gravity to the lighter, and let the point of division be at *E*, to wit so that *ED* shall have to *EC* the same ratio as *A* (3 lbs) to *B* (1 lb). I say that the vertical through *E*, as *EF*, is the handle.

EXAMPLE II.

SUPPOSITION. Let the one gravity be the prism *ABCD*, weighing 6 lbs, divided like the prism at the beginning of the first proposition; and let there be hanging at *Q* a weight *Y* of 12 lbs. WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of the prism is *IT*, and that of the weight *Y* is *BQ*, and *TQ* is the beam. The latter shall

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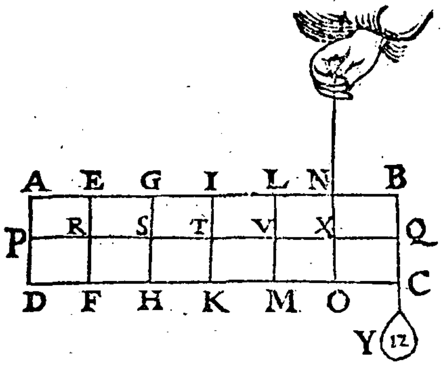
[1]) As has been remarked in the *Introduction*, p. 39-40, this converse proposition ought to have been proved as well.

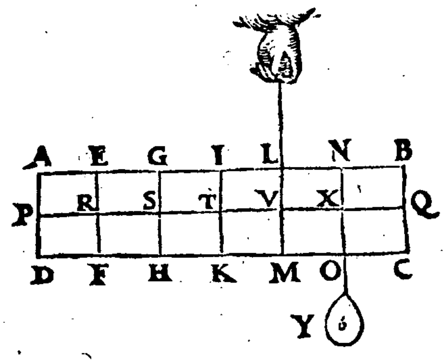
[2]) The Dutch term *Eysch* has been changed into *Werckstick* in XI; iv, i and may accordingly be translated by *Problem*.

de ſelue ſalmen in tween deelen, alſoo dat de ſticken de reden hebben als 12 ℔ van Y, tot 6 ℔ vanden pilaer, weluerſtaende tcortſte ſtick naer de ſwaerheydts middellini vande ſwaerſte ſwaerheydt Y, twelck vallen ſal in X, inder voughen dat N X de begheerde hanthaef is.



III. VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom den pilaer ſijn, ghedeelt als vooren, hanghende nu Y 6 ℔ an X. TBEGHEERDE. Wy moeten d'handthaef vinden. TWERCK. De ſwaerheydts middellini des pilaers is I T, ende van Y is N X, ende T X is balck: de ſelue ſalmen in tween deelen, alſoo dat de ſticken de reden hebben als 6 ℔ van Y, tot 6 ℔ des pilaers, twelck vallen ſal in V, indervoughen dat V L de begheerde handthaef ſijn ſal.



TVOORNOEMDE WERCK
OP EEN ANDER MANIER.

DE ſwaerheydts middellini van M L B C Y, is N X, ende van M L A D is S G, ende S X is balck, de ſelue ſalmen in tween deelen, alſo dat de ſtucken de reden hebben als 8 ℔ van M L B C Y, tot 4 ℔ van M L A D: welverſtaende tcortſte ſtick naer de ſwaerheyts middellini van tſwaerſte deel, twelck vallen ſal in V, inder voughen dat V L wederom de begheerde handhaef ſijn ſal als vooren.

IIII. VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom den pilaer ſijn, ghedeelt als vooren, hanghende Y 6 ℔ an X, ende Z 24 ℔ an R. TBEGHEERDE. Wy moeten d'hanthaef vinden. TWERCK. De ſwaerheyts be divided in two in such a way that the segments have the ratio of 12 lbs (*Y*) to 6 lbs (the prism), to wit: the shorter segment adjacent to the centre line of gravity of the heavier gravity *Y*; let the point of division be at *X*, so that *NX* is the required handle.

EXAMPLE III.

SUPPOSITION. Let *ABCD* again be the prism, divided as above, *Y* (6 lbs) now hanging at X. WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of the prism is *IT*, and that of *Y* is *NX*, and *TX* is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 6 lbs (*Y*) to 6 lbs (the prism); let the point of division be at *V*, so that *VL* will be the required handle.

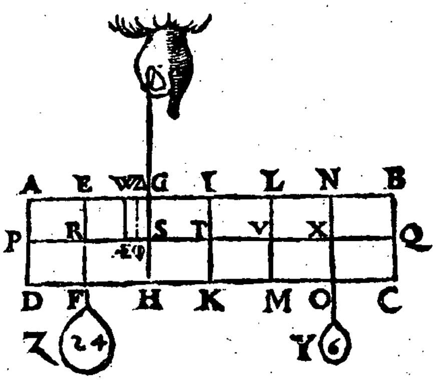
THE ABOVE CONSTRUCTION

in a different manner.

The centre line of gravity of *MLBCY* is *NX*, and that of *MLAD* is *SG*, and *SX* is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 8 lbs (*MLBCY*) to 4 lbs (*MLAD*), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at *V*, so that *VL* will again be the required handle, as before.

EXAMPLE IV.

SUPPOSITION. Let *ABCD* again be the prism, divided as before, *Y* (6 lbs) hanging at X, and *Z* (24 lbs) at *R*. WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of *ABCDY*



ſwaerheydts middellini van A B C D Y, is L V door het 3ᵉ voorbeelt, ende van Z is R E, daerom is R V balck: de ſelue ſalmen in tween deelen, alſoo dat de ſtucken de reden hebben als 12 ℔ van A B C D Y, tot 24 ℔ van Z: wel verſtaende tcortſte ſtic naer de ſwaerheyts middellini van tſwaerſte deel, twelck vallen ſal in S, inder voughen dat S G de begheerde handthaef ſijn ſal.

TVOORNOEMDE WERCK OP EEN ANDER MANIER.

DE ſwaerheydts middellini van A B C D Z is Æ W door het 3ᵉ voorbeelt, alſoo dat S Æ doet $\frac{2}{5}$ van S R, ende de ſwaerheydts middellini van Y is X N, ende Æ X is balck, de ſelue ſalmen in tween deelen, alſoo dat de ſticken de reden hebben als 30 ℔ van A B C D Z, tot 6 ℔ van Y: welverſtaende tcortſte ſtick naer de ſwaerheydts middellini van tſwaerſte deel, twelck vallen ſal in S, inder voughen dat S G wederom de begheerde handthaef is als vooren.

TVOORNOEMDE WERCK OP EEN ANDER MANIER.

DE ſwaerheydts middellini van Y Z, is (door het eerſte voorbeelt) Φ Δ, alſoo dat S Φ doet $\frac{1}{3}$ van S R, ende de ſwaerheydts middellini vande pilaer is T I, ende T Φ is balck: de ſelue ſalmen in tween deelen, alſo dat de ſticken de reden hebben als 30 ℔ van Y met Z, tot 6 ℔ vande pilaer, te weten tcortſte ſtick naer de ſwaerheydts middellini van tſwaerſte deel. twelck vallen ſal in S, inder voughen dat S G wederom de begheerde handthaef is als vooren.

Vᵉ VOORBEELT.

TGHEGHEVEN. Laet A B C D wederom den pilaer ſijn ghedeelt als vooren, hanghende Y 6 ℔ an X, ende Z 24 ℔ an R, eñ Æ 12 ℔ an Q. TBEGHEERDE. Wy moeten d'handthaef vinden. TWERCK. De ſwaerheydts middellini van A B C D Y Z is S G door het 4ᵉ voorbeelt, ende van Æ is Q B, ende S Q is balck: de ſelue ſalmen in tween deelen, alſo dat de ſticken de reden hebben als 36 ℔ vanden pilaer met Y

C ende

is LV, by the 3rd example, and that of Z is RE; therefore RV is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 12 lbs ($ABCDY$) to 24 lbs (Z), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S, so that SG will be the required handle.

THE ABOVE CONSTRUCTION
in a different manner.

The centre line of gravity of $ABCDZ$ is AEW, by the 3rd example, in such a way that SAE makes $^2/_5$ [1]) of SR, and the centre line of gravity of Y is XN, and AEX is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 30 lbs ($ABCDZ$) to 6 lbs (Y), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S, so that SG will again be the required handle, as above.

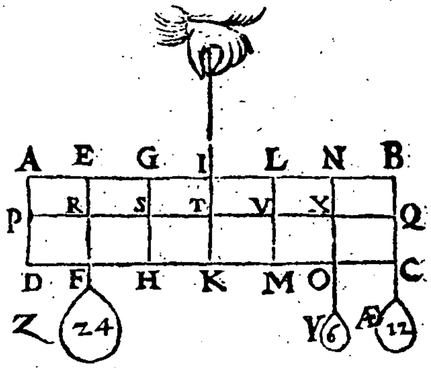
THE ABOVE CONSTRUCTION
in a different manner.

The centre line of gravity of YZ is (by the first example) $\Phi\Delta$, in such a way that $S\Phi$ makes $^1/_5$ of SR, and the centre line of gravity of the prism is TI, and $T\Phi$ is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 30 lbs (Y with Z) to 6 lbs (the prism), to wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at S, so that SG will again be the required handle, as above.

EXAMPLE V.

SUPPOSITION. Let $ABCD$ again be the prism, divided as above, Y (6 lbs) hanging at X, and Z (24 lbs) at R, and AE (12 lbs) at Q. WHAT IS REQUIRED TO FIND. We have to find the handle. CONSTRUCTION. The centre line of gravity of $ABCDYZ$ is SG, by the 4th example, and that of AE is QB, and SQ is the beam. The latter shall be divided in two in such a way that the segments have the ratio of 36 lbs (the prism with Y and Z) to 12 lbs (AE), to

[1]) Read 3/5.



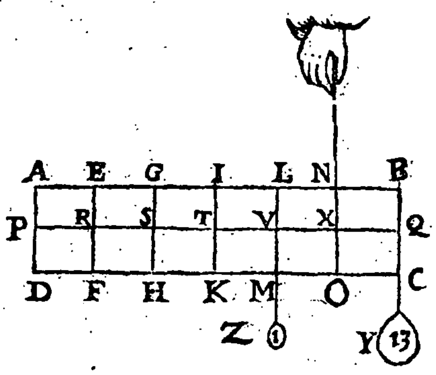
ende Z, tot 12 ℔ van Æ, te weten tcortste stick naer de swaerheydts middellini van tswaerste deel, twelck vallen sal in T, inder voughen dat T I de begheerde handthaef sal sijn.

Ende soomen noch hinghe an P 24 ℔, d'handthaef soude S G sijn, ende so voorts met allen anderen swaerheden diemen anden pilaer soude mueghen hanghen. TBEWYS. De swaerste swaerheydt A int eerste voorbeelt, heeft sulcken reden tot de lichtste B, als den langsten erm E D, tot den cortsten E C, daerom E F door de 9e bepaling is d'hanthaef. Sghelijcx sal oock tbewijs sijn van al dander voorbeelden, twelck wy om de cortheydt achterlaten.

TBESLVYT. Wesende dan ghegheuen bekende swaerheden, wy hebben haer handthaef gheuonden naer den eysch.

MERCKT.

SOOMEN tghewicht Y des 2en voorbeelts verswaerde van 1 ℔, ende datmen an V hinghe 1 ℔, inder voughen dat haer ghestalt dan waer als hier onder;



*Tis kennelick uyt het voorgaende dat X N noch handthaef blijft, ende alles an haer eueſtaltwichtich hangt. Tselue sal X N oock blijuen, soomen Z 1 ℔ hangt an T, ende dat Y doe 14 ℔, ofte Z 1 ℔ an S, ende dat Y doe 15 ℔, ofte Z 1 ℔ an R, ende dat Y doe 16 ℔, ofte Z 1 ℔ an P, ende dat Y doe 17 ℔, ende soo oirdentlick voort by aldien den pilaer langher waer; te weten, verswarende Y altijt van 1 ℔, voor elcke langde als X V, daermen Z voorder an verschuyft. Waer uyt de * Ghedaenten des Onsils bekent sijn, als inde Weegdaet breeder daer af sal ghehandelt worden.*

* Qualitates.

II. EYSCH. III. VOORSTEL.

WESENDE ghegheuen twee eueſtaltwichtighe swaerheden, d'een bekent dander onbekent, ende d'hanthaef: Die onbekende bekent te maken.

1. VOOR-

wit: the shorter segment adjacent to the centre line of gravity of the heavier part; let the point of division be at *T*, so that *TI* will again be the required handle.

And if in addition 24 lbs were hung at *P*, the handle would be *SG*, and so on with any other gravities that might be hung from the prism. PROOF. The heavier gravity *A* in the first example has to the lighter one *B* the same ratio as the longer arm *ED* to the shorter *EC*; therefore *EF* is the handle by the 9th definition. A similar proof can also be given of all the other examples, which we omit for brevity's sake. CONCLUSION. Given therefore two known gravities, we have found their handle as required.

NOTE.

If the weight Y of the 2nd example were made heavier by 1 lb, and 1 lb were hung at V, in such a way that the situation would be as below, it is evident from what precedes that XN still remains the handle, and that the whole hangs from it in apparent equality of weight. The handle will also remain XN, if Z (1 lb) is hung at T and Y is made 14 lbs, or Z (1 lb) is hung at S and Y is made 15 lbs, or Z (1 lb) is hung at R and Y is made 16 lbs, or Z (1 lb) is hung at P and Y is made 17 lbs, and so regularly on if the prism were longer, viz. always making Y heavier by 1 lb for every segment of the beam equal to XV along which Z is displaced; by which the properties of the steelyard are known, which will be dealt with more in detail in the Practice of Weighing [1].

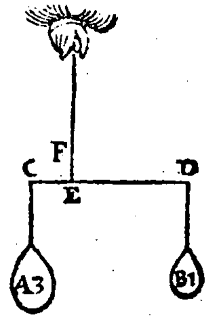
PROBLEM II. PROPOSITION III.

Given two gravities of equal apparent weight, one of them known and the other unknown, and the handle: to make known the unknown.

[1]) See *The Practice of Weighing*, Prop. 5

I. VOORBEELT.

TGHEGHEVEN. Laet A ende B twee eueſtaltwichtighe ſwaerheden ſijn, welcker A hanghende an C weeght 3 ℔, maer B hanghende an D is onbekent, ende E F ſy d'handthaef. TBEGHEERDE. Wy moeten tghewicht van B bekent maken. TWERCK. Men ſal onderſoecken wat reden den erm E D heeft, tot den erm E C, wort beuonden, neem ick, als van 3 tot 1, daerom ſeg ick, E D 3, gheeft E C 1, wat A 3 ℔? comt voor B 1 ℔.



II. VOORBEELT.

TGHEGHEVEN. Laet inde form des 2en voorbeelts van het 2e voorſtel den pilaer A B C D voor d'een ſwaerheyt weghen 6 ℔, ende dander onbekende ſwaerheyt ſy tghewicht daer an hanghende Y, ende d'hanthaef ſy X N. TBEGHEERDE. Wy moeten tghewicht van Y bekent maken. TWERCK. Anghesien T I ſwaerheydts middellini is des pilaers, ende Q B van Y, ſo ſal T Q balck ſijn, diens cortſten erm X Q, ende langſten X T; Daerom ſalmen onderſoucken wat reden den erm X Q, heeft tot X T, wort beuonden neem ick, als van 1 tot 2. Ich ſeg dan, X Q 1, gheeft X T 2, wat den pilaer 6 ℔? comt voor Y 12 ℔. Der ghelijcke voorbeelden mochten wy hier ſtellen op dander formen der voorbeelden des 2en voorſtels, ten waer die door de voorgaende kennelick ghenouch ſijn. TBEWYS. Laet B int eerſte voorbeelt, ſoot mueghelick waer, ſwaerder ſijn dan 1 ℔, de ſwaerſte ſwaerheydt dan en ſal niet ſulcken reden hebben tot de lichtſte, als den langſten erm tot den cortſten; twelck teghen het 1e voorſtel is; B dan en is niet ſwaerder dan 1 ℔. Sghelijcx ſalmen oock bethoonen dat ſy niet lichter en is, ſy weeght dan effen 1 ℔, twelck wy bewyſen moeſten. TBESLVYT. Weſende dan ghegheuen twee eueſtaltwichtighe ſwaerheden, d'een bekent dander onbekent, ende d'hanthaef: Wy hebben die onbekende bekent ghemaeckt, naer den eyſch.

III. EYSCH. IIII. VOORSTEL.

WESENDE ghegeuen twee bekende eueſtaltwichtighe ſwaerheden met de langde van d'eenen erm: de langde des anderen erms te vinden.

TGHEGHEVEN. Laet A ende B twee eueſtaltwichtighe ſwaerheden ſijn, welcker A hanghende an C weeght 3 ℔, ende B hanghende an D 1 ℔, ende de langde des erms D E ſy 6 voeten. TBEGHEERDE.

EXAMPLE I.

SUPPOSITION. Let *A* and *B* be two gravities of equal apparent weight, of which *A*, hanging at *C*, weighs 3 lbs, but *B*, hanging at *D*, is unknown, and let *EF* be the handle. WHAT IS REQUIRED TO MAKE KNOWN. We have to make known the weight of *B*. CONSTRUCTION. It shall be ascertained what ratio the arm *ED* has to the arm *EC*. I assume this is found to be 3 to 1. Therefore I say: *ED* 3 gives *EC* 1, what *A* 3 lbs? *B* becomes 1 lb [1]).

EXAMPLE II.

SUPPOSITION. In the figure of the 2nd example of the 2nd proposition let the prism *ABCD* be the one gravity, weighing 6 lbs, and let the other — unknown — gravity be the weight *Y* hanging therefrom, and let the handle be *XN*. WHAT IS REQUIRED TO MAKE KNOWN. We have to make known the weight of *Y*. CONSTRUCTION. Since *TI* is the centre line of gravity of the prism, and *QB* that of *Y*, *TQ* will be the beam, the shorter arm of which will be *XQ* and the longer *XT*. It shall therefore be ascertained what ratio the arm *XQ* has to *XT*. I assume this is found to be 1 to 2. I therefore say: *XQ* 1 gives *XT* 2, what the prism 6 lbs? *Y* becomes 12 lbs. We might give similar examples with regard to the other figures of the examples of the 2nd proposition, if these were not sufficiently evident from what precedes.

PROOF. Let *B* in the first example, if this were possible, be heavier than 1 lb; the heavier gravity will not then have to the lighter the same ratio as the longer arm to the shorter, which is contrary to the first proposition. *B* therefore is not heavier than 1 lb. In the same way it can also be shown that it is not lighter. It therefore weighs precisely 1 lb, which we had to prove. CONCLUSION. Given therefore two gravities of equal apparent weight, one of them known and the other unknown, and the handle, we have made known the unknown, as required.

PROBLEM III. PROPOSITION IV.

Given two known gravities of equal apparent weight, and the length of one arm: to find the length of the other arm.

SUPPOSITION. Let *A* and *B* be two gravities of equal apparent weight, of which *A*, hanging at *C*, weighs 3 lbs, and *B*, hanging at *D*, 1 lb, and let the length of the arm *DE* be 6 feet. WHAT IS REQUIRED TO FIND. We have to

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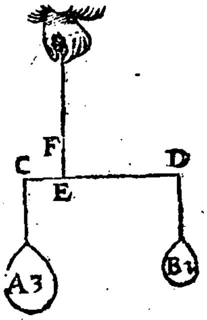
[1]) The meaning of this elliptical way of formulating the rule of three will be clear: If $ED = 3$, $EC = 1$. Therefore, if $A = 3$ lbs, $B = 1$ lb.

Wy moeten de langde des anderen erms vinden. TWERCK. Men sal segghen A 3 ℔, gheeft B 1 ℔, wat D E 6 voeten? comt voor E C 2 voeten. Ende der ghelijcke voorbeelden mochten wy stellen op de formen der voorbeelden des 2ᵉ voorstels, ten waer die duer tvoorgaende kennelick ghenouch sijn.



TBEWYS. Laet E C, soot mueghelick waer, langher sijn dan 2 voeten; den langsten erm sal dan minder reden hebben tot den corsten, dan de swaerste swaerheyt tot de lichtste, twelck teghen het eerste voorstel is, E C dan en is niet langher dan 2 voeten; Sghelijcx salmense oock bewysen niet corter te sijn, sy is dan effen van twee voeten, twelck wy bewysen moesten.

TBESLVYT. Wesende dan ghegheuen twee euestaltwichtighe swaerheden met de langde van d'eenen erm, wy hebben de langde des anderen erms gheuonden, naer den eysch.

IIII. EYSCH. V. VOORSTEL.

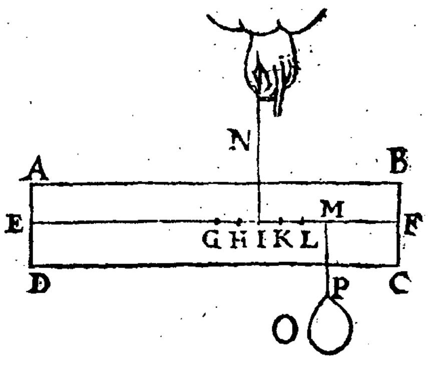
WESENDE ghegheuen een pilaer: te vinden een ghewicht in ghestelde reden tot des pilaers ghewicht.

TGHEGHEVEN. Laet ABCD een pilaer wesen, diens as E F, ende haer * middelpunt G, ende de ghestelde reden sy van 2 tot 3.

Centrum.

TBEGHEERDE. Wy moeten een ghewicht vinden in sulcken reden tot den pilaer, als van 2 tot 3, dat is euen an sijn $\frac{2}{3}$. MERCKT. Ghelijck de *Meetconstighe ende Telconstighe voorstellen verscheyden werckinghen hebben, alsoo oock de Weegconst, want men soude vanden pilaer een stuck connen snien in sulcken reden tot den heelen pilaer, als van 2 tot 3, Oft andersins om den pilaer heel te laten, men mocht hem teghen ander stof weghen, daer af nemende de $\frac{2}{3}$, maer wy willent Weegconstlicker doen in deser voughen. TWERCK. Men sal van tmiddelpunt G af, naer F, teeckenen eenighe vijf punten (te weten 5 voor de somme der

Geometricæ & Arithmeticæ propositiones.



ghegheuen

find the length of the other arm. CONSTRUCTION. It can be said: if *A* 3 lbs gives *B* 1 lb, what *DE* 6 feet? *EC* becomes 2 feet. We might give similar examples with regard to the figures of the examples of the 2nd proposition, if these were not sufficiently evident from what precedes.
PROOF. Let *EC*, if this were possible, be longer than 2 feet; the longer arm will then have to the shorter a ratio less than the heavier gravity to the lighter, which is contrary to the first proposition. Therefore *EC* is not longer than 2 feet. In the same way it can also be proved not to be shorter. It is therefore precisely 2 feet long, which we had to prove. CONCLUSION. Given therefore two gravities of equal apparent weight, and the length of one arm, we have found the length of the other arm, as required.

PROBLEM IV. PROPOSITION V.

Given a prism: to find a weight which shall have to the weight of the prism a given ratio.

SUPPOSITION. Let *ABCD* be a prism, its axis *EF* and its centre *G*, and let the given ratio be that of 2 to 3. WHAT IS REQUIRED TO FIND. We have to find a weight having to the prism the ratio of 2 to 3, i.e. being equal to $^2/_3$ of the latter.

NOTE.

Just as geometrical and arithmetical propositions have different operations, so also the Art of Weighing, for one might cut from the prism a piece having to the whole prism the ratio of 2 to 3. Or in another way, to keep the prism intact, it might be balanced against some other material, after which $^2/_3$ of the latter would be taken; but we will do it more in accordance with the Art of Weighing, as follows. CONSTRUCTION. There shall be marked from the centre *G*, towards *F*, five points (to wit 5, for the sum of the given terms 2 and 3), as *H*, *I*, *K*, *L*, ghegheuen palen 2. 3) als H, I, K, L, M, van malcanderen euewyt; Ende van het tweede punt I (van het tweede om dat 2 het ander der ghegeuen ghetalen is) salmen den pilaer ophangen byde swaerheyts middellini I N; Daer naer salmen an tvijfde punt M een ghewicht hanghen als O, euen so swaer dat alles in eueſtaltwichticheyt sy, twelck soo wesende, ick seg dat tghewicht van O, in sulcken reden is tot tghewicht des pilaers, als 2 tot 3, ofte dat O euen is ande $\frac{2}{3}$ des pilaers.

TBEWYS. G is * swaerheydts middelpunt des pilaers A B C D, ende M P swaerheyts middellini van O, daerom ghelijck den erm I G tot den erm I M, alsoo O tot den pilaer door het 1e voorstel, maer I G heeft sulcken reden tot I M, als 2 tot 3, daerom O heeft sulcken reden tot den pilaer, als 2 tot 3, twelck wy bewysen moesten. TBESLVYT. Wesende dan ghegheuen een pilaer, wy hebben gheuonden een ghewicht in ghestelde reden tot des pilaers ghewicht, naer den eysch. MERCKT. Wy souden oock mueghen voorbeelden stellen met Redenen van * onmetelicke palen, maer sulcx is openbaer ghenouch door tvoorgaende, metgaders tghene wy vande onmetelicke grootheden elders ghescreuen hebben.

Centrum grauitatis.

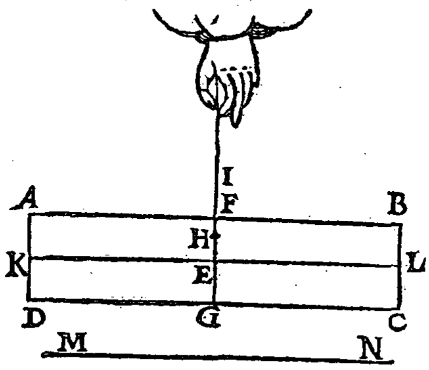
Incommensurabilium terminorum.

II VERTOOCH. VI VOORSTEL.

WESENDE een hanghende pilaer ghesneen door sijn swaerheydts middelpunt, met een plat euewydich vanden gront, eñ wesende tvastpunt in dat plat bouē het swaerheyts middelpunt: Den as des pilaers blijft euewydich vāden * sichteinder.

Horizon.

TGHEGHEVEN. Laet A B C D een pilaer sijn, ghesneen door sijn swaerheydts middelpunt met een plat F G, euewydich vanden grondt A D, ende laet H vastpunt inde swaerheydts middellini I G wesen, bouen het swaerheyts middelpunt E, ende K L sy as, ende M N sichteinder.



TBEGHEERDE. Wy moeten bewysen dat den as K L euewydich blijft vanden sichteinder. M N.

TBEWYS. Laet K L soot mueghelijck waer, oneuewydich sijn vanden sichteinder M N, als in

C 3 dees

M, equidistant from one another; and from the second point *I* (from the second because 2 is the second of the given numbers) the prism shall be hung by the centre line of gravity *IN*. After this, at the fifth point *M* a weight *O* shall be hung, just heavy enough for the whole to be of equal apparent weight. This being so, I say that the weight of *O* has to the weight of the prism the ratio of 2 to 3, or that *O* is equal to $^2/_3$ of the prism. PROOF. *G* is the centre of gravity of the prism *ABCD*, and *MP* is the centre line of gravity of *O*; therefore, as the arm *IG* is to the arm *IM*, so is *O* to the prism, by the 1st proposition. But *IG* has to *IM* the ratio of 2 to 3; therefore *O* has to the prism the ratio of 2 to 3, which we had to prove.
CONCLUSION. Given therefore a prism, we have found a weight in a given ratio to the weight of the prism, as required.

NOTE.

We might also give examples with ratios of incommensurable terms, but this is sufficiently manifest from what precedes and from what we have said elsewhere about incommensurable magnitudes [1]).

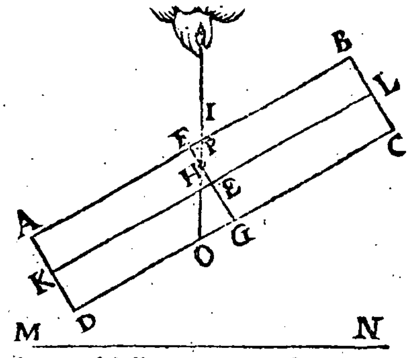
THEOREM II. PROPOSITION VI.

Given a hanging prism, cut through its centre of gravity by a plane parallel to the base, and the fixed point being in that plane above the centre of gravity, the axis of the prism remains parallel to the horizon.

SUPPOSITION. Let *ABCD* be a prism, cut through its centre of gravity by a plane *FG*, parallel to the base *AD*, and let *H* be the fixed point in the centre line of gravity *IG*, above the centre of gravity *E*; and let *KL* be the axis, and *MN* the horizon. WHAT IS REQUIRED TO PROVE. We have to prove that the axis *KL* remains parallel to the horizon *MN*.
PROOF. Let *KL*, if this were possible, be non-parallel to the horizon *MN*, as in

[1]) This remark refers to Stevin's ideas on irrational numbers, which are discussed in V *(Thèses mathématiques)*. In his opinion there is no reason to call certain numbers absurd or irrational; irrational ratios should be called numbers on a par with rational ratios.

dees tweede form, ende laet IH voortghetrocken worden tot in O, sniende AB in P, ende laet het stuck des pilaers POCB alsoo euewichtich blijuen hanghen teghen PODA, maer dat is grooter ende swaerder dan dit (want FGDA, is euen an FGCB, ende minder is den driehouck FHI ghesneen van FGCB, dan de driehouck OHG ghesneen van FGDA, daerom, &c.) het swaerder dan sal euewichtich sijn an een lichter twelck onghescȟict is, KL dan blijft euewydich vanden sichteinder MN, als in d'eerste form.



Tis oock te anmercken als voor ghemeenen Weegconstighen Reghel, dat

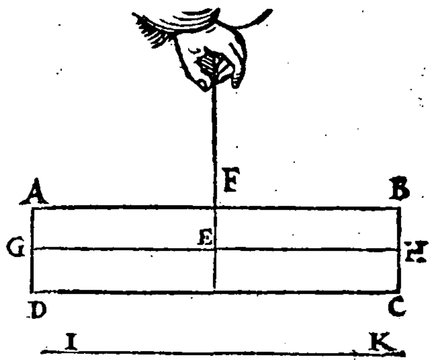
Alle swaerheyts middelpunt eens hanghenden lichaems is in sijn swaerheydts middellini.

Maer tswaerheydts middelpunt hier bouen E en is inde tweede form niet in sijn swaerheydts middellini IO, tis dan een onmueghelicke ghestalt. TBESLVYT. Wesende dan een pilaer ghesneen, &c.

III VERTOOCH. VII VOORSTEL.

WESENDE tvastpunt het swaerheydts middelpunt des hanghenden pilaers, hy houdt alle ghestalt diemen hem gheeft.

TGHEGHEVEN. Laet ABCD een pilaer wesen, diens swaerheydts middelpunt E vast sy, daer by hanghende ande lini EF, ende den as GH sy euewydich vanden sichteinder IK.



TBEGHEERDE. Wy moeten bewysen dat den pilaer ABCD alle ghestalt houdt diemen hem gheeft.

TBEWYS. Laet ons den ghegheuen pilaer (tpunt E vast blijuende) een ander ghestalt gheuen dan d'eerste, als in dees tweede form, ende this second figure, and let *IH* be produced to *O*, meeting *AB* in *P*, and let the part *POCB* of the prism remain hanging in equilibrium [1]) against *PODA*; now the former is greater and heavier than the latter (for *FGDA* is equal to *FGCB*, and the triangle *FHI* cut from *FGCB* is less than the triangle *OHG* cut from *FGDA*; therefore, etc.). The heavier part will therefore be of equal weight [2]) with the lighter part, which is absurd [3]). *KL* therefore remains parallel to the horizon *MN*, as in the first figure.

It is also to be considered a general rule in the Art of Weighing that: *The centre of gravity of a hanging solid is always in its centre line of gravity* [4]). But the centre of gravity *E* above is not in its centre line of gravity *IO* in the second figure; this is therefore an impossible situation. CONCLUSION. Given therefore a prism, cut etc.

THEOREM III. PROPOSITION VII.

The fixed point being the centre of gravity of the hanging prism, the latter remains at rest in any position given to it.

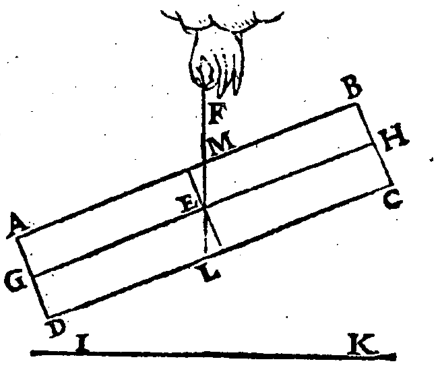
SUPPOSITION. Let *ABCD* be a prism, the centre of gravity *E* of which shall be the fixed point by which the prism is hanging from the line *EF*, and let the axis *GH* be parallel to the horizon *IK*. WHAT IS REQUIRED TO PROVE. We have to prove that the prism *ABCD* remains at rest in any position given to it. PROOF. Let us give the given prism (the point *E* remaining fixed) a different position from the first, as in this second figure, and let *FE* be produced to *L*,

[1]) Read: in apparent equality of weight.

[2]) Read: equal apparent weight.

[3]) As Girard (XIII 441a) rightly remarks, this conclusion is not justified. It does not matter at all that *OCBP* and *DOPA* are not of equal weight; it has to be proved that they are not of equal apparent weight.

[4]) According to Definition 5, centre line of gravity is the vertical through the centre of gravity. Evidently the word is taken here in the sense of vertical through the point of suspension. Here, as elsewhere, Stevin seems to make use of a certain theory of gravity, which, however, is explained nowhere.

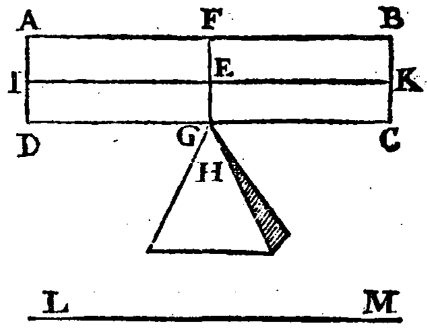


ende laet F E voortghetrocken worden tot in L, sniende A B in M, ende en laet den pilaer soot mueghelick waer niet in die ghestalt blijuen, dan het stick M L D A, ofte M L C B neervallen; Maer dees twee deelen sijn ghelijck euegroot, ende daerom oock eueswaer, het eene dan van euewichtighe sal swaerder sijn dan t'ander, twelc ongeschickt is: Den pilaer dan blijft in die ghestalt, eñ sgelijcx in allen anderen diemen hem soude mueghen gheuen. TBESLVYT. Wesende dan tvastpunt het swaerheydts middelpunt des pilaers, hy houdt alle ghestalt diemen hem gheeft, twelck wy bewysen moesten.

IIII VERTOOCH. VIII VOORSTEL.

WESENDE den pilaer ghesneen door sijn swaerheyts middelpunt, met een plat euewydich vanden gront, ende wesende tvastpunt in dat plat beneden het swaerheydts middelpunt: Den pilaer (natuerlick verstaen) keert om tot dat sijn swaerheydts middelpunt is in sijn swaerheydts middellini.

TGHEGHEVEN. Laet A B C D een pilaer wesen, ghesneen door sijn swaerheyts middelpunt E, met een *plat F G euewydich vanden grondt A D, ende laet G vastpunt sijn, beneden tswaerheydts middelpunt E, met welck punt G den pilaer ligt ofte rust op tpunt des pins H, ende I K sy as, euewydich vanden * sichteinder L M.



* Plano.

* Horizonte.

TBEGHEERDE. Wy moeten bewysen dat den pilaer omkeeren sal, tot dat sijn swaerheydts middelpunt is in sijn swaerheyts middellini.

maer

meeting *AB* in *M*. And let the prism, if this were possible, not remain in that position, but let the part *MLDA* or *MLCB* fall down. But these two parts are equal in size, and consequently also equally heavy; one of two parts of equal weight will therefore be heavier than the other, which is absurd [1]). The prism therefore remains in that position, and likewise in any other position that might be given to it. CONCLUSION. The fixed point therefore being the centre of gravity of the prism, the latter remains at rest in any position given to it, which we had to prove.

THEOREM IV. PROPOSITION VIII.

Given the prism, cut through its centre of gravity by a plane parallel to the base, and the fixed point being in that plane below the centre of gravity, the prism (physically speaking) turns upside down until its centre of gravity is in its centre line of gravity [2]).

SUPPOSITION. Let *ABCD* be a prism, cut through its centre of gravity *E* by a plane *FG* parallel to the base *AD*, and let *G* be the fixed point, below the centre of gravity *E*, with which point *G* the prism lies or rests on the point of the peg *H*, and let *IK* be the axis, parallel to the horizon *LM*. WHAT IS REQUIRED TO PROVE. We have to prove that the prism will turn upside down until its centre of gravity is in its centre line of gravity, such physically speaking, for, conceived

[1]) Of course this is not absurd at all: two bodies of equal weight need not be of equal apparent weight.
[2]) Here again, centre line of gravity is not taken in the sense of Definition 5; it again means vertical through the point of suspension.

Mathematice. maer dit natuerlick verstaen, want * Wisconstelick ghenomen soo can sy daer op rusten.

T B E W Y S.

A. Al dat ligt moet grondt hebben daert op rust,
E. Dees pilaer en heeft gheen grondt daer by op rust,
E. Dees pilaer dan en can soo niet ligghen.

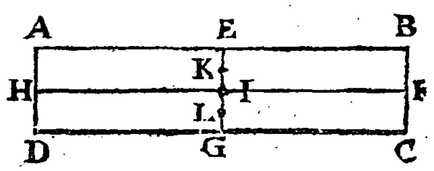
Syllogismi. DES * Bewysredens tweede voorstel is daer uyt openbaer, dat het punt gheen grootheyt en is, ende veruolghens gheen grondt: wel is waer dat wy dickmael nemen door tghestelde een lichaem alsoo te rusten, maer metter daet en connen wy dat niet te weeg brenghen. Inder voughen dat hoewel den as IK euewydich ghestelt is vanden sichteinder LM, soo sal nochtans den pilaer (tpunt G vast blyuende) omkeeren ouer die sijde daer hy eerst beghint. Maer dat hy so lang keeren sal tot dat sijn swaerheydts middelpunt inde swaerheydts middellini si, is door het 6ᵉ voorstel openbaer. TBESLVYT, Wesende dan den pilaer ghesneen, &c.

Iᵉ MERCK.

YEMANT *mocht hier noch de verclaring begheeren des verschils tusschen hanghen en ligghen, waer op d'antwoort is dat wy een lichaem voor hanghende houden, als sijn swaerheyts middelpunt is onder, ofs int tghenaecsel daert op rust; Maer tswaerheyts middelpunt daer bouen sijnde, alsdan houden wyt voor ligghen, staen, oft sitten; Ligghen, als de langste sijde des lichaems haer streckt langs den sichteinder: Staen, als sy daer op rechthoukich is; daerom ist oock dat wy den teerlinck (ouermits sijn sijden al euen lanck sijn) soo eyghentlick seggben te staen als te ligghen, ende te ligghen als te staen. Sitten is wat tusschen ligghen en staen.*

IIᵉ MERCK.

SOO *yemant thinhoudt der voorgaende drie voorstellen door eenighe eruaring wilde sien, hy mocht nemen een reghel van houdt ofte ander stof eenvaerdigher dicke ende swaerheyt, als ABCD, treckenende de punten E, F, G, H, inde middelen der linien AB, BC, CD, DA, treckende EG, ende HF, malcander sniende in I, maeckende daer naer een seer cleen gaetken an I, ende daer bouen een gaetken als K, ende onder I een gaetken als L. Ende stekende een naelde door tgatken K, die vrielick daerin drayen mach, d'eruaring sal bethoonen dat HF altijdt euewydich sal blijuen vanden sichteinder. Maer de naelde in I stekende, de reghel sal daerop alle ghestalt bouden diemen haer gheeft. Ende de naelde in L ghesteken, alles sal omkeeren ouer*



die

mathematically [1]), it can rest thereon.

PROOF.

A [2]). *Everything that lies must have a base on which it rests;*
E . *This prism has no base on which it rests;*
E . *Therefore this prism cannot lie in this way.*

The second proposition of the syllogism is apparent from the fact that a point is no magnitude, and consequently no base; it is true that by the supposition we often assume a body to rest in this way, but in actual fact we cannot bring it about. Therefore, though the axis *IK* be put parallel to the horizon *LM*, the prism (the point *G* remaining fixed) will nevertheless turn upside down on the side where it begins to turn. But it is manifest by the 6th proposition that it will turn until its centre of gravity is in the centre line of gravity. CONCLUSION. Given therefore the prism, cut, etc.

NOTE I.

If anyone should here desire the explanation of the difference between hanging and lying, the answer is that we hold a solid to be hanging when its centre of gravity is below or in the support on which it rests; but if the centre of gravity is above the latter, we hold the solid to be lying, standing or sitting: lying, if the longest side of the solid is parallel to the horizon; standing, if the said side is at right angles thereto. This is also the reason why we may just as well say of a cube (because all its sides are of equal length) that it stands as that it lies, and that it lies as that it stands. Sitting is something intermediate between lying and standing.

NOTE II.

If anyone should wish to see the contents of the preceding three propositions by some experience, he might take a ruler of wood or some other material which is everywhere equally thick and heavy, for example ABCD, marking the points E, F, G, H in the centres of the lines AB, BC, CD, DA, joining EG and HF, which meet in I, making thereafter a very small hole at I, and above it a hole K and below I a hole L. And if he puts through the hole K a needle which can freely pivot therein, experience will show that HF will always remain parallel to the horizon. But if he puts the needle in I, the ruler will remain at rest in any position given to it. And if the needle is put in L, the whole prism will turn

[1]) Stevin here introduces a distinction between a real or physical and a mathematical body. His proposition relates to the first one, which cannot in practice remain at rest in the position shown in the figure. The distinction is expressed by the terms ***natuerlick verstaen*** (physically speaking) and ***wisconstelick ghenomen*** (conceived mathematically). Evidently the consideration of physical possibility introduces quite a new element into the system, which disturbs its mathematical coherence.

[2]) Stevin here makes use of the symbolism of ancient formal logic, in which *A* denotes a universal affirmative proposition (All *X*'s are *Y*'s) and *E* a universal negative proposition (No *X*'s are *Y*'s). The mood of the syllogism is *Camestres*. Stevin makes use of the syllogistic formulation whenever he wants to stress the importance or the originality of his reasoning. Other examples are to be found in Prop. 24 of Book I and in Props 2, 10, 15, 16, 18, 22 of Book II of *The Art of Weighing*. There is not the slightest ground for Vailati's contention *(Il principio dei lavori virtuali da Aristotele a Erone d'Alessandria*, Atti R. Acc. d. Sc. di Torino 82 (1896-97), 949) that the syllogistic formulation was meant ironically.

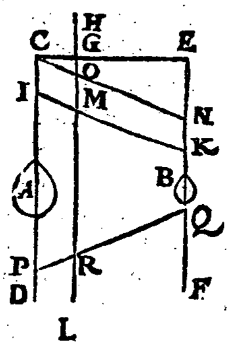
*die syde daert eerst beghint, tot dat I is in haer swaerheyts middellini, waer af d'oirsaeck inde voornoemde 6e, 7e, 8e, voorstellen * Wisconstlick blijckt.* *Mathematicè.*

V. VERTOOCH. IX. VOORSTEL.

D'HANTHAEF oneindelick voortghetrocken, deelt alle balcken tweer swaerheden in haer ermen.

TGHEGHEVEN. Laet A B twee swaerheden sijn ende haer middellinien C D, E F, ende haer balck C E, ende d'hanthaef G H, inder voughen dat C G is tot G E, als de swaerheydt B tot A, Laet I K noch een balck wesen, oneuewydich van C E, ende laet G H oneindelick voortghetrocken worden naer L, sniende den balck I K in M.

TBEGHEERDE. Wy moeten bewysen dat I M ende M K, oock de ermen sijn der swaerheden A B; dat is ghelijck B tot A, alsoo M I tot M K. TBEREYTSEL. Laet ghetroken worden C N, euewydich van I K, sniende H L in O. TBEWYS. Ghelijck C G tot G E, alsoo C O tot O N, Maer C O is euen an I M, ende O N an M K, daerom ghelijck C G tot G E, alsoo I M tot M K, maer ghelijck B tot A, alsoo C G tot G E, door tghegheuen, daerom ghelijck B tot A, alsoo M I tot M K, tselfde sal alsoo bewesen worden van allen balcken tusschen C D ende E F, als P Q, doorsneen in R, ende allen anderen diemen soude mueghen trecken. *2 v 6 B. E.* *34 v. 1 B. E.*



TBESLVYT. D'handthaef dan oneindelick voortghetrocken, deelt alle balcken tweer swaerheden in haer ermen, twelck wy bewysen moesten.

I. VERVOLGH.

HIER uyt blijct datmen om te vinden de swaerheydts middellini tweer swaerheden, niet nootsaeckelick en moet nemen een * euewydighe vanden * sichteinder, maer alsulcke alsmen wil, ende als best te pas comt. *Parallela Horizonti.*

II VERVOLGH.

ANGHESIEN alle swaerheydts middelpunt inde swaerheyts middellini is, soo volght dat alle rechte lini begrepen tusschen twee swaerheydts middelpunten, oock dier swaerheden balck is, ende het onderscheydt der ermen diens balcx, oock het swaerheydts middelpunt te wesen der twee swaerheden.

D 5 EYSCH.

upside down on the side where it begins to turn, until I is in its centre line of gravity, the cause of which appears mathematically from the aforesaid 6th, 7th, and 8th propositions.

THEOREM V. PROPOSITION IX.

The handle, produced indefinitely, divides any beam of two gravities into its arms.

SUPPOSITION. Let *A* and *B* be two gravities, and their centre lines of gravity *CD*, *EF*, and their beam *CE*, and the handle *GH*, so that *CG* is to *GE* as the gravity *B* is to the gravity *A*. Let *IK* be another beam, not parallel to *CE*, and let *GH* be produced indefinitely to *L*, meeting the beam *IK* in *M*. WHAT IS REQUIRED TO PROVE. We have to prove that *IM* and *MK* are also the arms of the gravities *A* and *B*; i.e. as *B* is to *A*, so is *MI* to *MK*. PRELIMINARY. Let *CN* be drawn parallel to *IK*, meeting *HL* in *O*. PROOF. As *CG* is to *GE*, so is *CO* to *ON*. But *CO* is equal to *IM*, and *ON* to *MK*, therefore as *CG* is to *GE*, so is *IM* to *MK*. But as *B* is to *A*, so is *CG* to *GE* by the supposition. Therefore, as *B* is to *A*, so is *MI* to *MK*. The same can also be proved of any beam between *CD* and *EF*, as *PQ*, cut in *R*, and any others that might be drawn. CONCLUSION. The handle therefore, produced indefinitely, divides any beam of two gravities into its arms, which we had to prove.

COROLLARY I.

From this it appears that in order to find the centre line of gravity of two gravities one need not take a line parallel to the horizon, but may take any line one likes and which suits best.

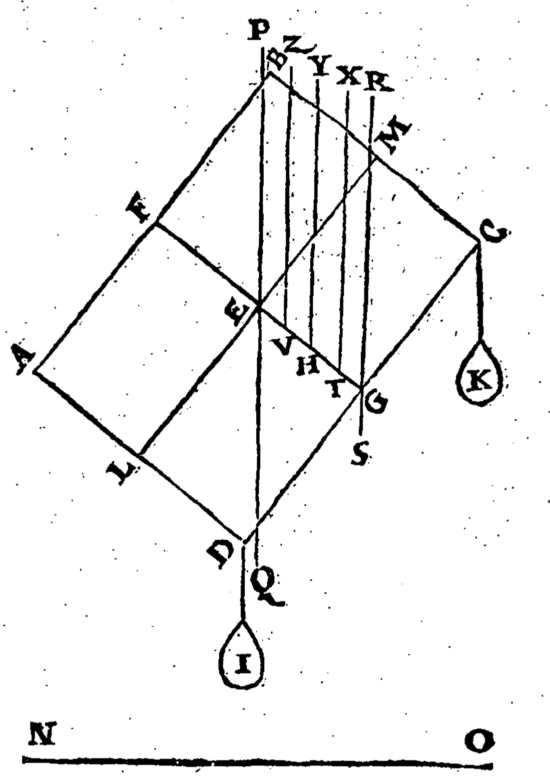
COROLLARY II.

Since the centre of gravity is always in the centre line of gravity, it follows that any straight line contained between two centres of gravity is also the beam of those gravities, and the dividing point of the arms of that beam is also the centre of gravity of the two gravities.

WESENDE ghegheuen een vaſtpunt des bekenden pilaers, ende bekende eueſtaltwichtighe ſwaerheden an hem hangende: Te vinden of den as euewydich ſal blijuen vanden * ſichteinder, oft alle gheſtalt houden diemen hem gheeft, ofte omkeeren tot dat ſijn ſwaerheydts middelpunt is in ſijn ſwaerheyts middellini.

Horizonte.

TGHEGHEVEN. Laet A B C D een pilaer ſijn weghende 4 ℔, ende gheſneen door ſijn ſwaerheydts middelpunt E, met een plat F G. euewydich vanden grondt A D, ende laet H vaſtpunt weſen beneden tmiddelpunt E int middel van E G; Ende anden pilaer twee ghewichten hanghen als I, K, elck weghende 4 ℔, welcker middellinien vaſtpunten ſijn D, C, ende laet L M den as, ende N O ſichteinder weſen.



TBEGHEERDE. Wy moeten vinden of den as L M euewydich ſal connen blijuen vanden ſichteinder N O; ofte alle geſtalt houden diemen haer gheeft; Ofte ommekeeré tot dat haer ſwaerheydts middelpunt E is inde ſwaerheyts middellini door H, welcke verſcheydenheden vallen connen naer de reden der ſwaerheyt des pilaers, tot de ghewichten dieder anhangen.

TWERCK. Men ſal trecken door E de ſwaerheyts middellini P O des pilaers, daer naer door G de-

PROBLEM V. PROPOSITION X.

Given a fixed point of the known prism, and known gravities of equal apparent weight hanging therefrom: to find out whether the axis will remain parallel to the horizon, or will remain at rest in any position given to it, or will turn upside down until its centre of gravity is in its centre line of gravity.

SUPPOSITION. Let *ABCD* be a prism weighing 4 lbs and cut through its centre of gravity *E* by a plane *FG* parallel to the base *AD*, and let *H* be the fixed point below the centre *E*, in the middle of *EG*. And let there hang from the prism two weights, as *I*, *K*, each weighing 4 lbs, whose centre lines are fixed points *D*, *C*, and let *LM* be the axis, and *NO* the horizon. WHAT IS REQUIRED TO FIND OUT. We have to find out whether it will be possible for the axis *LM* to remain parallel to the horizon *NO*, or whether it will remain at rest in any position given to it, or will turn upside down until its centre of gravity *E* is in the centre line of gravity through *H*, which different possibilities may occur according to the ratio of the gravity of the prism to the weights hanging therefrom. CONSTRUCTION. Through *E* there shall be drawn the centre line of gravity *PQ* of the prism, and

G de swaerheyts middellini R S der ghewichten I, K, ende E G sal balck sijn, daer naer salmen sien door het 2e voorstel waer tvastpunt der hanthaef valt: want commet onder H, soo keert L M tot sy euewydich blijft vanden sichteinder N O; Maer commet in H, sy houdt alle ghestalt die men haer gheeft; Commet bouen H, alles keert om. Maer den pilaer weeght 4 ℔, ende I, K, elck 4 ℔ tsamen 8 ℔ door tghegheuen, daerom ghedeelt E G in T, alsoo dat E T, sulcken reden heb tot T G, als 8 tot 4: Ick seg dat L M keeren sal (ouermits T onder H comt) tot sy euewydich is vanden sichteinder. Laet nu den pilaer weghen 4 ℔, ende I en K elck 2 ℔, tsamen 4 ℔, daerom ghedeelt E G in H (welcke H tmiddel van E G is door tghegheuen) alsoo dat E H sulcken reden heb tot H G, als 4 tot 4: ick seg dat L M (ouermits het in H viel) alle ghestalt sal houden diemen haer gheeft. Laet nu den pilaer weghen 4 ℔, ende I, K, elck 1 ℔, tsamen 2 ℔, daerom ghedeelt E G in V, alsodat E V sulcken reden hebbe tot V G, als 2 tot 4, Ick seg dat den pilaer met al de rest omkeeren sal (ouermitds V bouen H comt) tot dat H is in haer swaerheydts middellini. TBEWYS. Ten eersten I en K elck 4 ℔ weghende, dat dan L M keert tot sy euewydich is vanden sichteinder, blijct aldus: De hanghende door T ghelijck T X, is swaerheyts middellini des heels, daerom die latende, ende hanghende tgheheel ande * hanghende door H, als H Y (welcke H ons ghegheuen vastpunt is) so sal de sijde naer B C K, swaerder sijn dan naer A D I, daerom oock sal de sijde B C K neerdalen, tot dat H inde swaerheydts middellini is des heels, ende dan sal L M euewydich sijn vanden sichteinder N O.

Perpendicularem.

Ten tweeden I, K, elck 2 ℔ weghende, dat dan L M alle ghestalt houdt, wordt aldus bethoont: Laet ons achten dat I ende K opghe-schorst sijn, alsoo dat D tswaerheydts middelpunt sy van I, ende C van K, ende door de 3e begheerte sy en sullen 'anden pilaer gheen oirsaeck van verandering der swaerheydt wesen; Twelck soo sijnde, H is tswaerheyts middelpunt van soodanighen lichaem vergaert uyt den pilaer ende de twee ghewichten I K, ende door de 4 bepaling tsal daer op alle ghestalt houden diemen hem gheeft, tselfde sal also bewesen worden in alle ghestalten daermen L M in soude connen stellen.

Ten laetsten I, K, elck 1 ℔ weghende, dat dan alles omkeert, wort aldus bethoont: De hanghende door V ghelijck V Z, is swaerheyts middellini des heels, daerom die latende, ende hanghende tgheheel ande hanghende H Y door H ghegheuen vastpunt, so sal de sijde naer A D I, swaerder sijn dan naer B C K, daerom oock sal de sijde A D I neerdalen, tot dat H inde swaerheyts middellini is des heels, ende ofmen schoon L M (alles op tvastpunt H draeyende) euewydich stelde vanden sichteinder N O, sy en can so niet blyuen door het 8 voorstel, maer alles sal omkeeren, twelck wy bewysen moesten.

D 2 TBESLVYT

then through *G* the centre line of gravity *RS* of the weights *I*, *K*; *EG* will then be the beam. After this, it shall be ascertained by the 2nd proposition where the fixed point of the handle will fall. For if it comes below *H*, *LM* will turn until it remains parallel to the horizon *NO*. But if it comes at *H*, it will remain at rest in any position given to it. If it comes above *H*, the whole prism turns upside down. Now the prism weighs 4 lbs, and *I*, *K* each 4 lbs, together 8 lbs, by the supposition. Then, *EG* being divided in *T* so that *ET* has to *TG* the ratio of 8 to 4, I say that *LM* will turn (since *T* comes below *H*) until it is parallel to the horizon. Now let the prism weigh 4 lbs, and *I* and *K* each 2 lbs, together 4 lbs. Then, *EG* being divided in *H* (which *H* is the middle of *EG*, by the supposition) so that *EH* shall have to *HG* the ratio of 4 to 4, I say that *LM* (since it fell in *H*) will remain at rest in any position given to it. Now let the prism weigh 4 lbs, in *I*, *K* each 1 lb. together 2 lbs. Then, *EG* being divided in *V* so that *EV* shall have to *VG* the ratio of 2 to 4, I say that the prism with all the rest will turn upside down (since *V* comes above *H*) until *H* is in its centre line of gravity. PROOF. Firstly, the fact that if *I* and *K* each weigh 4 lbs, *LM* will turn until it is parallel to the horizon is proved as follows: The vertical through *T*, as *TX*, is the centre line of gravity of the whole; therefore, omitting this one and hanging the whole from the vertical through *H*, as *HY* (which *H* is the given fixed point), the part adjacent to *BCK* will be heavier than that adjacent to *ADI*. Therefore the part *BCK* will descend until *H* is in the centre line of gravity of the whole, and then *LM* will be parallel to the horizon *NO*.

Secondly, the fact that if *I*, *K* each weigh 2 lbs, *LM* will remain at rest in any position given to it is shown as follows: Let us suppose *I* and *K* to have been pulled up in such a way that *D* is the centre of gravity of *I*, and *C* of *K*; then, by the 3rd postulate, they will not be the cause of any change of the gravity at the prism. This being so, *H* is the centre of gravity of a solid made up of the prism and the two weights *I*, *K*, and by the 4th definition it will remain at rest in any position given to it; the same can likewise be proved of any position in which *LM* could be put.

Lastly, the fact that if *I*, *K* each weigh 1 lb, the whole turns upside down is shown as follows: the vertical through *V*, as *VZ*, is the centre line of gravity of the whole; therefore, omitting this one and hanging the whole from the vertical *HY* through *H*, the given fixed point, the side adjacent to *ADI* will be heavier than that adjacent to *BCK*, and therefore also the side *ADI* will descend until *H* is in the centre line of gravity of the whole. And though one should put *LM* (the whole turning about the fixed point *H*) parallel to the horizon *NO*, it cannot remain at rest in that position, by the 8th proposition, but the whole will turn upside down, which we had to prove.

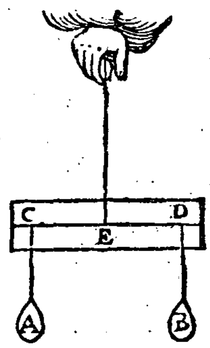
TBESLVYT. Wesende dan ghegheuen een vastpunt des bekenden pilaers, &c.

Uyt het voorgaende is ghenouch blijckelick den ghemeenen voortganck in allen anderen, als van pilaren welcker vastpunt is buyten de lini als F G, ende der ghewichten vastpunten op ander plaetsen dan D C; Maer ouermits wy hier voornamelick trachten de oirsaecken vande gedaenten des waegs grondelick te openbaren (daer af inde Weeghdaet breeder sal ghesëyt worden) so en gheuen wy van sulcke onghesc hicte ghestaltheden gheen besonder voorbeelden.

6. EYSCH. 11. VOORSTEL.

WESENDE ghegheuen een bekende pilaer, ende bekende swaerheden daer an hanghende: Te vinden het vastpunt daer op hy alle ghestalt houdt diemen hem gheeft.

I^e MERCK.



S o o *tweer euewychten als A, B, vastpunten C, D, waren in des pilaers as, euewyt van tmiddelpunt E, als in dees form, tis kennelick, door het tweede deel des bewys van het* 10^e *voorstel, dat E tbegheerde punt soude sijn, maer wy sullen tvoorbeelt van onghesc hicter ghestalt gheuen.*

II^e MERCK.

T i s *openbaer dat wesende de twee vastpunten der ghewichten als C D, ende tvastpunt des handthaefs als E, alle drie in een rechte lini als hier bouen, ende an C D euen ghewichten ghehanghen, soo groot ofte cleen alst valt: E sal altijt tvastpunt blijuen, daer sy alle ghestalt op houden diemen haer gheeft. Maer soo die drie punten als C E D in een rechte lini wesende C ende D niet euewyt en waren van E, ende datmen an haer ghewichten hinghe* * *euerednich met de ermen, dat E noch altijt tvastpunt sal blijuen daer sy alle ghestalt op houden diemen haer gheeft.*

* *Proportionales.*

TGHEGHEVEN. Laet A B C D een pilaer sijn, weghende 10 lb, diens swaerheydts middelpunt E, ende laet de ghewichten daer an hanghende wesen F 1 lb, diens vastpunt G, ende H 4 lb, wiens vastpunt I. TBEGHEERDE. Wy moeten het vastpunt vinden daerop sy alle ghestalt houden diemen haer gheeft. TWERCK. Men sal trecken

CONCLUSION. Given therefore a fixed point of the known prism, etc.

From the preceding the common procedure in all other cases is sufficiently clear, for example with prisms whose fixed points are outside the line *FG*, while the fixed points of the weights are in places other than *D*, *C*. But since we are here mainly attempting to disclose the causes of the properties of the balance in principle (of which we will speak more fully in the Practice of Weighing [1])), we do not give any specific examples of such irregular forms.

PROBLEM VI. PROPOSITION XI.

Given a known prism, and two known gravities hanging therefrom: to find the fixed point on which it remains at rest in any position given to it.

NOTE I.

If the fixed points C, D of two equal gravities A, B were in the axis of the prism, equidistant from the centre E, as in the annexed figure, it is evident from the second section of the proof of the 10th proposition that E would be the required point, but we will give the example with regard to an irregular form.

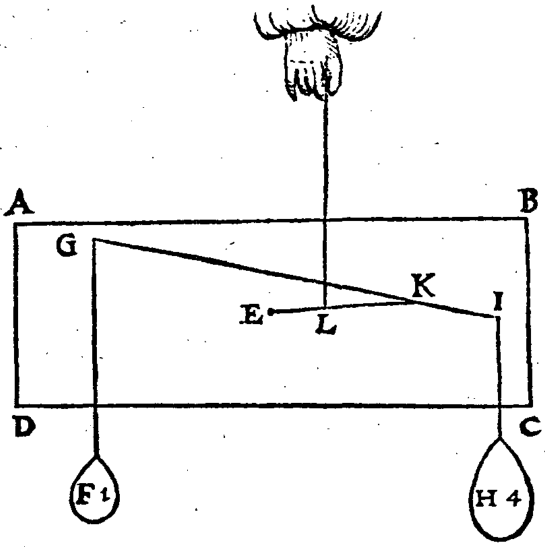
NOTE II.

It is manifest that if the two fixed points of the weights are C, D, and the fixed point of the handle is E, all three being on a straight line, as above, equal weights hanging at C, D, as great or small as the case may be, E will always remain the fixed point on which they remain at rest in any position given to them. But if, these three points, as C, E, D, being on a straight line, C and D were not equidistant from E, and weights were hung at them proportional to the arms, E will still remain the fixed point on which they remain at rest in any position given to them.

SUPPOSITION. Let *ABCD* be a prism weighing 10 lbs, its centre of gravity *E*; and let the weights hanging therefrom be *F* (1 lb), and its fixed point *G*, and *H* (4 lbs), and its fixed point *I*. WHAT IS REQUIRED TO FIND. We have to find the fixed point on which they remain at rest in any position given to them. CONSTRUCTION. There shall be drawn *GI*, the beam of the weights

[1]) See *The Practice of Weighing*, Prop. 2.

trecken G.I balck der gewichten F H, daer naer salmen vinden haer ermen door het 2e voorstel, dat is ghelijck F 1 ℔, tot H 4 ℔, also den erm K I, tot K G, daer naer salmen trecken E K balck des pilaers ter eender, ende der ghewichten F H ter ander sijden, de selue E K ghedeelt in L, also dat den erm E L sulcken reden hebbe tot L K, als



5 ℔ van F H, tot 10 ℔ des pilaers, L sal tbegheerde punt sijn op twelck sy alle ghestalt sullen houden diemen haer gheeft, waer af tbewys openbaer is door het 7e voorstel.

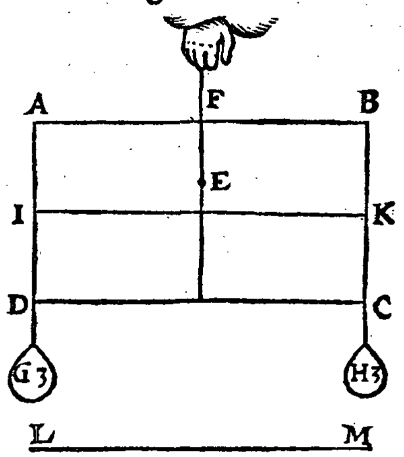
VII. EYSCH. XII. VOORSTEL.

WESENDE ghegheuen een bekende pilaer, met sijn vastpunt ende bekende ghewichten daer an hanghende die den as euewydich houden vanden * sichteinder: Te vinden een ghewicht hanghende ter begheerder plaets des pilaers, dat den as in ghegheuen ghestalt houde.

Horizonte.

I. VOORBEELT.

TGHEGHEVEN. Laet A B C D een pilaer sijn weghende 6 ℔, diens vastpunt E, ende handthaef E F, ende twee ghewichten G,H, elck 3 ℔ weghende, welcker vastpunten C, D; eñ I K, sy as, euewydich vanden sichteinder L M, ende D sy tpunt voor de begheer-



D 3 de plaets.

F, *H*, and then their arms shall be found by the 2nd proposition; i.e. as *F* (1 lb) is to *H* (4 lbs), so is the arm *KI* to the arm *KG*. After this, there shall be drawn *EK*, the beam of the prism on the one hand and of the weights *F*, *H* on the other hand. This beam *EK* being divided in *L* in such a way that the arm *EL* shall have to the arm *LK* the same ratio as *F*, *H* (5 lbs) to the prism (10 lbs), *L* will be the required point on which they will remain at rest in any position given to them. The proof of this will be manifest from the 7th proposition.

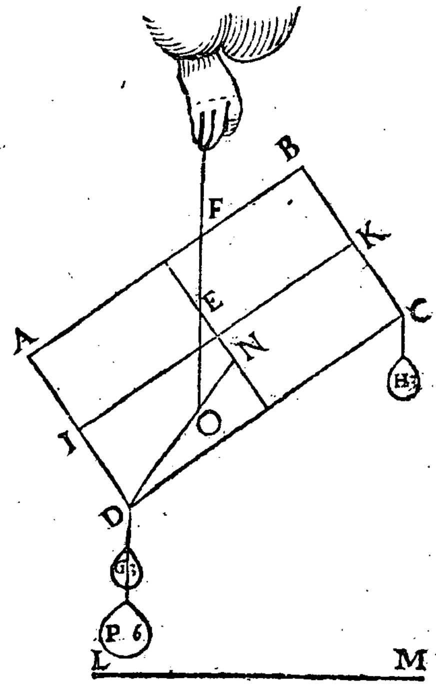
PROBLEM VII. PROPOSITION XII.

Given a known prism, with its fixed point, and known gravities hanging therefrom, which keep the axis parallel to the horizon: to find a weight hanging in a required place of the prism which shall keep the axis in the given position.

EXAMPLE I.

SUPPOSITION. Let *ABCD* be a prism weighing 6 lbs, its fixed point *E* and its handle *EF*, and let there be two weights *G*, *H*, each weighing 3 lbs, whose fixed points are *C*, *D*. And let *IK* be the axis, parallel to the horizon *LM*, and let *D* be the point indicating the required place. Then the axis *IK* (the whole turning de plaets. Daer naer wort den as I K (alles draeyende op E) verheuen als inde tweede form. TBEGHEERDE. Wy moeten een ghewicht an D vinden, dat den as I K in die ghestalt houde. TWERCK. Men sal vinden door het 11e voorstel, tvastpunt daer op den as alle ghestalt houde diemen haer gheeft twelck N sy: Daer naer salmen trecken D N, ende de * hanghende E O, sniende N D in O, daer naer salmen sien wat reden N O heeft tot O D, ick neme als van 1 tot 2, daerom hanghe ick an D een ghewicht P van 6 lb, te weten in sulcken reden tot den pilaer met de twee ghewichten G, I, al tsamen 12 lb, als van 1 tot 2; Ick seg P 6 lb, te wesen het begheerde ghewicht.

Perpendicularem.



TBEWYS. Tswaerste ghewicht 12 lb des erms O N, heeft sulcken reden tot het lichtste 6 lb des erms O D, ghelijck den langsten erm O D, tot den cortsten O N; Daerom hanghet al eueſtaltwichtich ande handthaef E F door het 1e voorstel. Ende veruolghens den as I K blijft in haer ghegheuen ghestalt.

11. VOORBEELT.

LAET ABCD een pilaer sijn weghende 6 lb, diens vastpunt E, ende handthaef E F, ende G een ghewicht van 2 lb, diens vastpunt H, ende I een ghewicht van 1 lb, diens vastpunt K, ende den as L M sy euewidich vanden * sichteinder N O, ende P sy een punt inden pilaer voor de begheerde plaets. Daer naer werdt den as L M (alles draeyende op E) verheuen, als inde tweede form.

Horizonte.

TBEGHERDE.

Wy moeten een ghewicht an P vinden, dat den as L M in die ghestalt houde.

TWERCK.

about *E*) is lifted, as in the second figure. WHAT IS REQUIRED TO FIND. We have to find a weight at *D* which shall keep the axis *IK* in that position. CONSTRUCTION. By the 11th proposition, there shall be found the fixed point on which the axis shall remain in any position given to it. Let this be *N*. Then there shall be drawn *DN* and the vertical *EO*, meeting *ND* in *O*. After this, it shall be ascertained what ratio *NO* has to *OD*. I take this to be that of 1 to 2. I therefore hang at *D* a weight *P* of 6 lbs, to wit in the ratio of 1 to 2 to the prism with the two weights *G*, *I*, being together 12 lbs. I say that *P*, weighing 6 lbs, is the required weight. PROOF. The heavier weight (12 lbs) of the arm *ON* has to the lighter weight (6 lbs) of the arm *OD* the same ratio as the longer arm *OD* to the shorter arm *ON*. Therefore the whole hangs, by the first proposition, from the handle *EF* in apparent equality of weight. And consequently the axis *IK* remains in its given position.

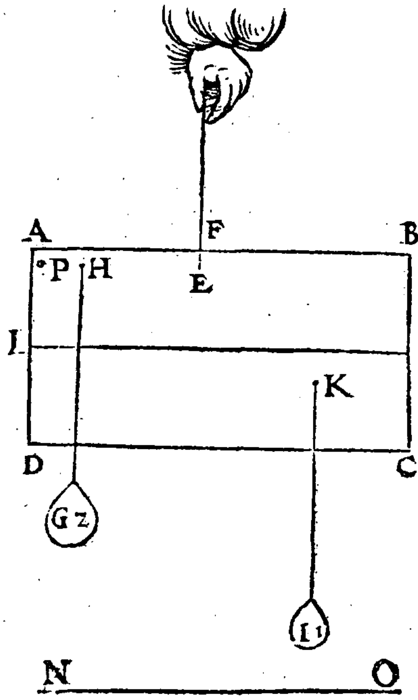
EXAMPLE II.

Let *ABCD* be a prism weighing 6 lbs, its fixed point *E* and its handle *EF*, and let *G* be a weight of 2 lbs and its fixed point *H*, and *I* a weight of 1 lb and its fixed point *K*. And let the axis *LM* be parallel to the horizon *NO*, and let *P* be a point in the prism for the required place. Then the axis *LM* (the whole turning

T WERCK.

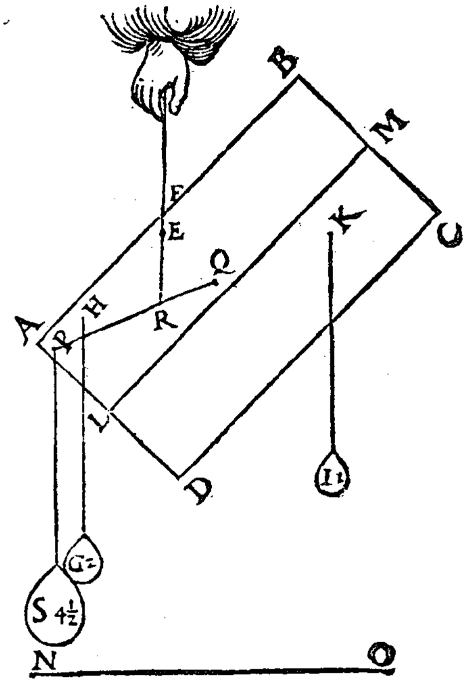
Men sal vinden door het 11e voorstel tvastpunt daerop tghegheuen alle ghestalt houdt diemen hem gheeft, twelck Q sy, daer naer salmen trecken P Q, ende de * hanghende E R, sniende P Q in R: siende daer naer wat reden R Q heeft tot R P, ick neem als van 1 tot 2, so hang ick an P een ghewicht S van 4 $\frac{1}{2}$ ℔, te weten in sulcken reden tot den pilaer met de twee ghewichten G, I, al tsamen 9 ℔, als van 1 tot 2; Ick seg S 4 $\frac{1}{2}$ ℔ te wesen het begheerde ghewicht.

Perpendicularem.



T BEWYS.

Tswaertste ghewicht 9 ℔ des erms R Q, heeft sulcken reden tot het lichtste ghewicht 4 $\frac{1}{2}$ ℔ des erms R P, ghelijck den langsten erm R P, tot den cortsten R Q, daerom hanghet al eueſtaltwichtich ande handthaef E F door het 1e voorstel, eñ veruolghens den as L M blijft in haer ghegheuen ghestalt, twelck wy bewysen moesten.



T BESLVYT.

Wesende dan ghegheuen een bekenden pilaer met sijn vastpunt,&c.

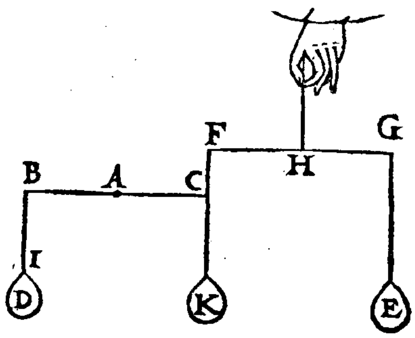
6 VER-

about *E*) is lifted, as in the second figure. WHAT IS REQUIRED TO FIND. We have to find a weight at *P* which shall keep the axis *LM* in that position. CONSTRUCTION. There shall be found, by the 11th proposition, the fixed point on which the given prism shall remain at rest in any position given to it. Let this point be *Q*. Then there shall be drawn *PQ* and the vertical *ER* meeting *PQ* in *R*. After this, it shall be ascertained what ratio *RQ* has to *RP*. I take this to be that of 1 to 2. I therefore hang at *P* a weight *S* of 4½ lbs, to wit in the ratio of 1 to 2 to the prism with the two weights *G*, *I*, weighing together 9 lbs. I say that *S*, weighing 4½ lbs, is the required weight. PROOF. The heavier weight (9 lbs) of the arm *RQ* has to the lighter weight (4½ lbs) of the arm *RP* the same ratio as the longer arm *RP* to the shorter arm *RQ*. Therefore the whole hangs, by the 1st proposition, from the handle *EF* in apparent equality of weight, and consequently the axis *LM* remains in its given position, which we had to prove. CONCLUSION. Given therefore a known prism, with its fixed point, etc.

EEN daelwicht ende een hefwicht an hem euen, doen met euen houcken an euen ermen euen ghewelden.

I^e. VOORBEELT met rechtwichten.

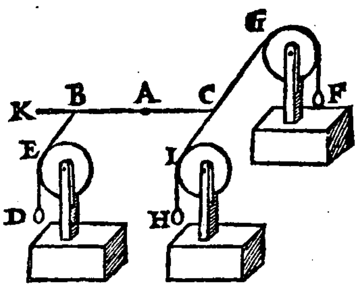
TGHEGHEVEN. Laet A des balcx BC vastpunt, ende AB met AC twee euen ermen sijn, ende an B hanghe het rechtreckwicht D, ende an C sy het rechthefwicht E, euewichtich an D, ende sijn balck sy FG, diens vastpunt H, ende euen ermen HF, HG, ende den houck ABI, sy euen anden houck ACF. TBEGHEERDE. Wy moeten bewysen dat het rechtdaelwicht D, ende trechthefwicht E, ande euen ermen AB, AC, euen ghewelden doen. TBEREYTSEL. Laet an C een ghewicht K hanghen, euen an D. TBEWYS. Laet ons weeren E, eñ is blijckelijck dat de macht van D is de ermen AB, AC, in die ghegheuen ghestalt te houden, want D is euen an K, ende AB an AC. Laet nu D weeren, ende E wederom anhanghen, ende de macht van E is oock de ermen AB, AC, in die ghegheuen ghestalt te houden, want K is euen an E, ende HF an HG, daerom E ende D doen an, euen ermen AB, AC, euen ghewelden.



II^e VOORBEELT met scheefwichten.

TGHEGHEVEN. Laet A des handthaefs vastpunt, ende AB met AC twee euen ermen sijn, ende an B hanghe tscheefdaelwicht D, diens scheefdaellini BE, eñde an C sy tscheefhefwicht F, euen an D, eñ sijn scheefheflini sy CG, ende den houck ABE, sy euen anden houck ACG.

TBEGHEERDE. Wy moeten bewysen dat het scheefdaelwicht D, ende tscheefhefwicht F, ande euen ermen AB, AC, euen ghewelden doen. TBEREYTSEL. Laet an C een scheefdaelwicht H



hanghen

THEOREM VI. PROPOSITION XIII.

A lowering weight and a lifting weight equal to it, acting at equal angles on equal arms, exert equal forces.

EXAMPLE I, with vertical weights.

Let *A* be the fixed point of the beam *BC*, and *AB* and *AC* two equal arms. And let there hang at *B* the vertical lowering weight *D* and let there be at *C* the vertical lifting weight *E* of equal weight to *D*; and let the beam of the latter be *FG*, its fixed point *H*, and the equal arms *HF*, *HG*, and let the angle *ABI* be equal to the angle *ACF*. WHAT IS REQUIRED TO PROVE. We have to prove that the vertical lowering weight *D* and the vertical lifting weight *E* exert equal forces on the equal arms *AB* and *AC*. PRELIMINARY. Let there hang at *C* a weight *K*, equal to *D*. PROOF. Let us take away *E*, then it is apparent that the power of *D* is to keep the arms *AB*, *AC* in that given position, for *D* is equal to *K*, and *AB* to *AC*. Now let us take away *D* and attach *E* again, then the power of *E* is also to keep the arms *AB*, *AC* in that given position, for *K* is equal to *D*, and *HF* to *HG*. Therefore *E* and *D* exert equal forces on equal arms *AB*, *AC*.

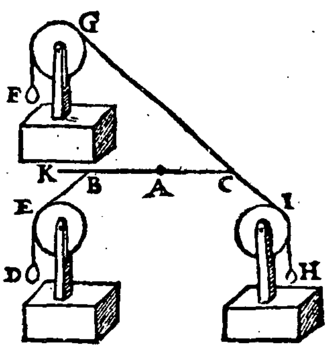
EXAMPLE II, with oblique weights.

SUPPOSITION. Let *A* be the fixed point of the handle, and *AB* and *AC* two equal arms. And let there hang at *B* the oblique lowering weight *D*, whose oblique lowering line shall be *BE*, and let there be at *C* the oblique lifting weight *F*, equal to *D*, whose oblique lifting line shall be *CG*, And let the angle *ABE* be equal to the angle *ACG*. WHAT IS REQUIRED TO PROVE. We have to prove that the oblique lowering weight *D* and the oblique lifting weight *F* exert equal forces on the equal arms *AB*, *AC*. PRELIMINARY. Let there hang at *C*

hanghen euen an D, diens scheefheflini C I,* euewydich sy van B E, *Parallela.*
ende C B sy wat voortghetrocken tot in K. TBEWYS. Laet ons weeren F, ende is kennelick dat de macht van D teghen H, is de ermen A B, A C, in die ghegheuen ghestalt te houden, want D is euen an H, ende den erm A B, an A C, ende den houck A C I, anden houck K B E. Laet nu D weeren, ende F wederom anhanghen, ende de macht van F is oock de ermen A B, A C, in die ghegheuen ghestalt te houden, ouermidts H euen is an F.

III VOORBEELT.

TGHEGHEVEN. Laet A des handthaefs vastpunt, eñ A B met A C twee euen ermen sijn, ende an B hanghe het scheefdaelwicht D, diens scheefdaellini B E, ende an C sy het scheefhefwicht F, euen an D, diens scheefheflini sy C G, ende den houck K C G, sy euen anden houck K B E. TBEGHEERDE. Wy moeten bewysen dat het scheefdaelwicht D, ende het scheefhefwicht F, ande euen ermen A B, A C, euen ghewelden doen. TBEREYTSEL. Laet an C een scheefdaelwicht H hanghen euen an D, diens scheefdaellini C I, also dat den houck A C I, euen sy anden houck A B E. TBEWYS. Laet ons weeren F, ende is kennelick dat de macht van D is de ermen A B, A C, in die ghegheuen ghestalt te houden, want D is euen an H, ende den erm A B an A C, ende den houck A C I, anden houck A B E. Laet nu D weeren, ende F wederom anhanghen, ende de macht van F is oock de ermen A B, A C, in die ghegheuen ghestalt te houden, ouermits H euen is an F.



TBESLVYT. Een daelwicht dan ende een hefwicht an hem euen, doen met euen houcken an euen ermen euen ghewelden, twelck wy bewysen moesten.

VIII. EYSCH. XIIII. VOORSTEL.

WESENDE ghegheuen een pilaer, ende twee punten inden as, t'een vast t'ander int langste deel verroerlick: Te vinden een rechthefwicht an tverroerlick, dat den pilaer in sijn ghegheuen standt houde.

TGHEGHEVEN. Laet A B C D een pilaer sijn, weghende 6 ℔, E ende an oblique lowering weight *H*, equal to *D*, whose oblique lifting line *CI* shall be parallel to *BE*, and let *CB* be somewhat produced to *K*. PROOF. Let us take away *F*; then it is evident that the power of *D* against *H* is to keep the arms *AB*, *AC* in that given position, for *D* is equal to *H*, and the arm *AB* to *AC*, and the angle *ACI* to the angle *KBE*. Let us now take away *D* and attach *F* again; then the power of *F* is also to keep the arms *AB*, *AC* in that given position, because *H* is equal to *F*.

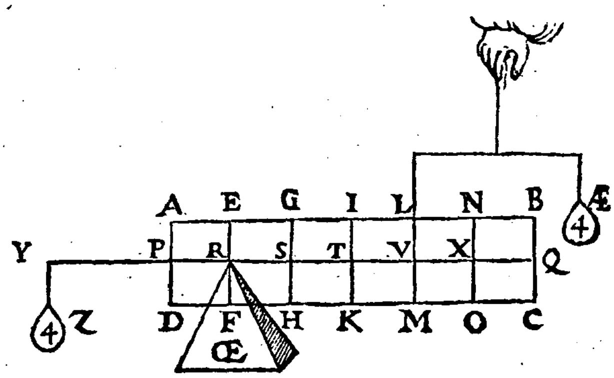
EXAMPLE III.

SUPPOSITION. Let *A* be the fixed point of the handle, and *AB* and *AC* two equal arms. And let there hang at *B* the oblique lowering weight *D*, whose oblique lowering line shall be *BE*, and let there be at *C* the oblique lifting weight *F*, equal to *D*, whose oblique lifting line shall be *CG*. And let the angle *KCG* be equal to the angle *KBE*. WHAT IS REQUIRED TO PROVE. We have to prove that the oblique lowering weight *D* and the oblique lifting weight *F* exert equal forces on the equal arms *AB*, *AC*. PRELIMINARY. Let there hang at *C* an oblique lowering weight *H*, equal to *D*, whose oblique lowering line shall be *CI*, in such a way that the angle *ACI* shall be equal to the angle *ABE*. PROOF Let us take away *F*; then it is evident that the power of *D* is to keep the arms *AB*, *AC* in that given position, for *D* is equal to *H*, and the arm *AB* to *AC*, and the angle *ACI* to the angle *ABE*. Let us now take away *D* and attach *F* again; then the power of *F* is also to keep the arms *AB*, *AC* in that given position, because *H* is equal to *F*. CONCLUSION. A lowering weight therefore and a lifting weight equal to it, acting at equal angles on equal arms, exert equal forces, which we had to prove.

PROBLEM VIII. PROPOSITION XIV.

Given a prism, and two points in the axis, one being fixed and the other in the longer part being movable: to find a vertical lifting weight at the movable point which shall keep the prism in its given position.

ende die ghedeelt als int beghin des 1e voorstels, ende vastpunt sy R, ende roerlick V, int langste deel des as R Q, want int cortste R P ist onmueghelick dat eenich rechthefwicht den as in haer ghegheuen stant houde. TBEGHEERDE. Wy moeten een rechthefwicht an V vinden, dat den pilaer in die standt houde. TWERCK. Men sal de lini Q R voorttrecken tot in Y, also dat R Y euen sy an R V: Daer naer salmen vinden tghewicht Z an Y, euestaltwichtich met dē pilaer, tselue (gedenckēde dat R vastpunt is) sal van 4 lb wesen door het 3e voorstel; Ick seg daerom dat het begheerde rechthefwicht twelck Æ sy, van 4 lb sal wesen.



TBEWYS. Ouermidts den erm R V des rechthefwichts Æ, euen is anden erm R Y des ghewichts Z, ende Æ euen an Z, soo is de ghewelt Æ euen an de ghewelt van Z door het 13e voorstel. Maer de ghewelt van Z is (Æ gheweert sijnde) den pilaer in die standt te houden, de ghewelt dan van Æ (Z gheweert sijnde) is oock den pilaer in die standt te houden, twelck wy bewysen mosten. TBESLVYT. Wesende dan ghegheuen een pilaer, ende twee punten inden as, t'een vast, t'ander int langste deel verroerlick: Wy hebben gheuonden een rechthefwicht an tverroerlick, dat den pilaer in sijn ghegheuen standt houdt naer den eysch.

MERCKT.

MEN *soude oock mueghen segghen metten cortsten V R 3, gheeft R T 2, wat den pilaer 6 lb? comt voor Æ 4 lb als vooren, waer af de reden int volghende 15 voorstel blijken sal.*

1e VERVOLGH.

ANGHESIEN den heelen pilaer door tghestelde 6 lb weeght, waer af Æ de 4 lb verheft, so volgt nootsaeckelick datter opt punt R, dat is op tlop des keghels OE, 2 lb rusten.

OPTE

SUPPOSITION. Let *ABCD* be a prism weighing 6 lbs, and let it be divided as at the beginning of the 1st proposition; and let the fixed point be *R* and the movable point *V* in the longer part of the axis *RQ*, for it is impossible for any vertical lifting weight in the shorter part *RP* to keep the axis in its given position. WHAT IS REQUIRED TO FIND. We have to find a vertical lifting weight at *V* which shall keep the prism in that position. CONSTRUCTION. The line *QR* shall be produced to *Y*, in such a way that *RY* shall be equal to *RV*. Then there shall be found the weight *Z* at *Y*, of equal apparent weight to the prism. Bearing in mind that *R* is the fixed point, this weight will be of 4 lbs, by the 3rd proposition. I therefore say that the required vertical lifting weight, which shall be *Æ*, will be of 4 lbs. PROOF. Since the arm *RV* of the vertical lifting weight *Æ* is equal to the arm *RY* of the weight *Z*, and *Æ* is equal to *Z*, the force of *Æ* is, by the 13th proposition, equal to the force of *Z*. But (*Æ* being taken away) the power of *Z* is to keep the prism in that position; therefore (*Z* being taken away), the power of *Æ* is also to keep the prism in that position, which we had to prove. CONCLUSION. Given therefore a prism, and two points in the axis, one being fixed and the other in the longer part being movable, we have found a vertical lifting weight at the movable point which shall keep the prism in its given position, as required.

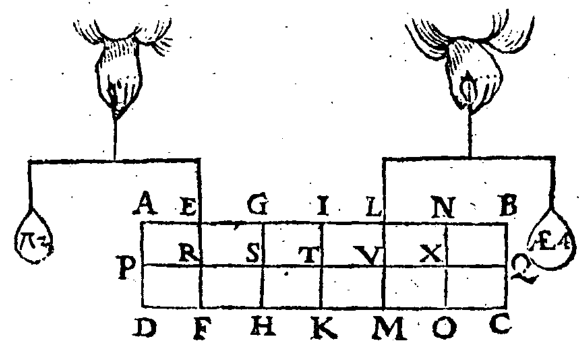
NOTE.

One might also say: with the shorter arm VR 3 gives RT 2, what the prism 6 lbs? Æ becomes 4 lbs, as above, the cause of which will become apparent in the 15th proposition hereinafter.

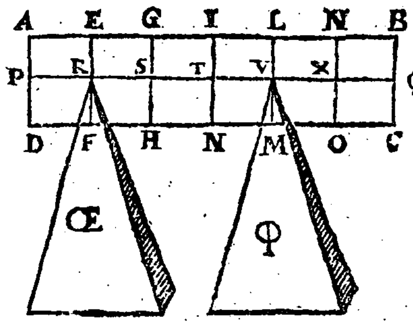
COROLLARY I.

Since by the supposition the whole prism weighs 6 lbs, of which *Æ* lifts the 4 lbs, it necessary follows than on the point *R*, that is the top of the cone *Œ*, there rest 2 lbs.

OFTE soomen an R een rechthefwicht Π vougde, inde plaets des keghels OE, als hier neuen, dat Π sal weghen 2 ℔.

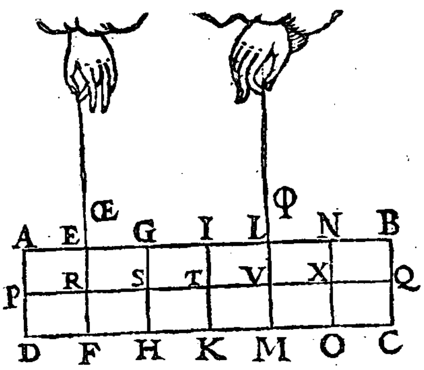


OFTE soomen an V een keghel Φ vougde, inde plaets des rechthefwichts Æ, als hier neuen, dat op den keghel OE rusten sal 2 ℔, ende op den keghel Φ 4 ℔.



OFTE soomen den pilaer ophinghe an twee * euewidighe linien OE R, ende Φ V, als hier neuen, dat ande lini OE R hanghen sal 2 ℔, ende ande lini Φ V 4 ℔.

Parallelae.

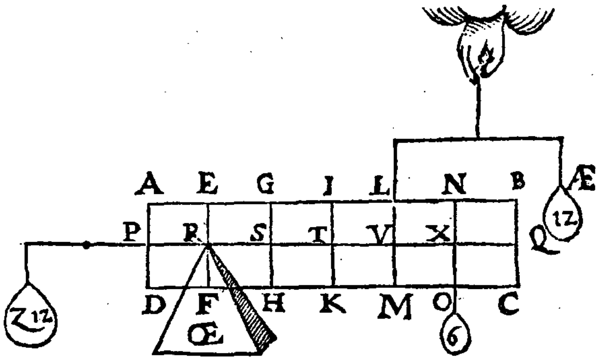


Or if a vertical lifting weight Π were attached at *R* instead of the cone *Œ*, as shown in the annexed figure, that Π will weight 2 lbs.

Or if a cone Φ were applied at *V* instead of the vertical lifting weight *Æ*, as shown in the annexed figure, that there will rest 2 lbs on the cone *Œ* and 4 lbs on the cone Φ.

Or if the prism were hung by two parallel lines *Œ R* and Φ *V*, as shown in the annexed figure, that there will hang 2 lbs by the line *Œ R* and 4 lbs by the line Φ *V*.

II VERVOLG.



SO anden pilaer (tpunt R vast sijnde als vooren) eenich gewicht ofte gewichten hingen, trechthefwicht sal oock bekent worden. Laet by voorbeelt an X hanghen 6 ℔, so sal Z moetē weghen 12 ℔ door het 3e voorstel, ende vervolghens Æ 12 ℔.

VII VERTOOCH. XV VOORSTEL.

WESENDE twee punten inden as des pilaers, t'een vast t'ander verroerlick: Trechthefwicht an tverroerlic met den pilaer euestaltwichtich, heeft sulcken reden tot den pilaer als het afstick tusschen het swaerheyts middelpunt des pilaers, ende het vastpunt, tot het afstick tusschen tvastpunt ende t'verroerlick punt.

VERCLARING.

LAET ons nemen de formen des 14 voorstels, al waer blijct dat ghelijck Æ 4 ℔, tot tghewicht des pilaers 6 ℔, alsoo T R tot R V. Maer om d'oirsaeck hier af * Wisconstelick te verclaren, soo is te weten dat ghelijck t'ghewicht Z, tottet ghewicht des pilaers, alsoo R T tot R Y door het 1e voorstel; Maer Æ is euen an Z, ende R V is euen an R Y door tghegeuen, ghelijck dan Æ tot den pilaer, alsoo T R tot R V.

Mathematicè.

TBESLVYT. Wesende dan twee punten inden as des pilaers t'een vast tander verroerlick, &c.

VIII VERTOOCH. XVI VOORSTEL.

WEsende twee punten inden as des pilaers t'een vast t'ander verroerlick: Trechthefwicht an tverroerlick dat den pilaer in een ghestalt houdt, sal hem in alle ghestalten houden.

TGHE-

COROLLARY II.

If there hung one or more weights from the prism (the point *R* being fixed, as above), the vertical lifting weight will also become known. For example, let there hang 6 lbs at *X*, then *Z*, by the 3rd proposition, will have to weigh 12 lbs, and consequently *Æ* 12 lbs.

THEOREM VII. PROPOSITION XV.

If there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point having equal apparent weight to the prism has to the prism the same ratio as the part of the axis between the centre of gravity of the prism and the fixed point to the part of the axis between the fixed point and the movable point.

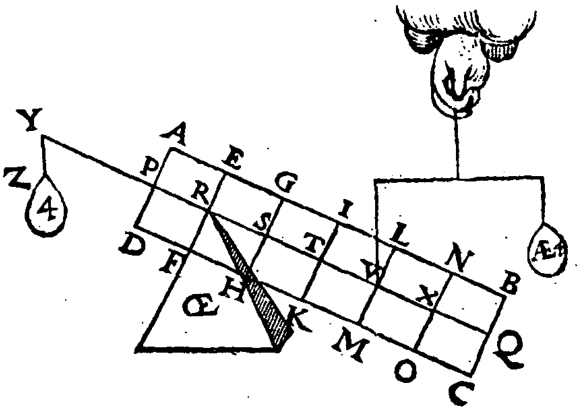
EXPLANATION.

Let us take the figures of the 14th proposition, where it is apparent that as *Æ* (4 lbs) is to the weight of the prism (6 lbs), so is *TR* to *RV*. But in order to explain the cause of this mathematically, it has to be known that as the weight *Z* is to the weight of the prism, so, by the 1st proposition, is *RT* to *RY*. Now *Æ* is equal to *Z*, and *RV* is equal to *RY* by the supposition; therefore, as *Æ* is to the prism, so is *TR* to *RV*. CONCLUSION. If therefore there are two points in the axis of the prism, one being fixed and the other movable, etc.

THEOREM VIII. PROPOSITION XVI.

If there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point which keeps the prism in one position will keep it in any position.

TGHEGHEVEN. Laet ons den pilaer met ſijn ghewichten des 14ᵉ voorſtels wat verkeeren op tvaſtpunt R, ende dat Æ 4 ℔ noch ſy rechthefwicht, alſoo dat dan alles van geſtalt ſy als hier neuen.



TBEGHEERDE. Wy moetē bewyſen dattet rechthefwicht Æ den pilaer oock in die ghegheuen gheſtalt houdt.

TBEWYS. Laet ons weeren Æ ende anhanghen Z 4 ℔, ende door het 10ᵉ voorſtel den pilaer ſal in die gheſtalt bliuen: Maer Æ doet by V ſoo grooten ghewelt anden pilaer als Z by Y door het 13ᵉ voorſtel, daerom gheweert Z, ende Æ anghehanghen, ſoo ſal Æ den pilaer oock in die gheſtalt houden. TBESLVYT. Weſende dan twee punten in den as des pilaers t'een vaſt tander verroerlick, trechthefwicht an tverroerlick, dat den pilaer in een gheſtalt houdt, ſal hem in alle gheſtalten houden, twelck wy bewyſen moeſten.

IX VERTOOCH. XVII VOORSTEL.

RVSTENDE een pilaer op twee punten inden as· Ghelijck het aſſtick tuſſchen t'ſwaerheyts middelpunt ende tſlinckerpunt, tottet aſſtick tuſſchen tſwaerheyts middelpunt ende trechterpunt, alſoo tghewicht des pilaers ruſtende op trechterpunt, tottet ghewicht ruſtende op tſlinckerpunt.

TGHEGHEVEN. Laet A B C D een pilaer ſijn weghende 6 ℔, ghedeelt als int 1ᵉ voorſtel, ruſtende met de twee punten R, V, op de punten van OE, Æ. TBEGHEERDE. Wy moeten bewyſen dat ghelijck het aſſtick T R, tottet aſſtick T V, alſo tghewicht ruſtende metter punt V op tpunt van Æ, tottet tghewicht ruſtende mettet tpunt R op tpunt van OE. TBEWYS. T R is dobbel an T V door tgheſtelde,

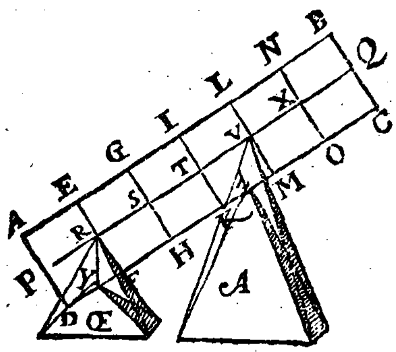
F 3 ende

SUPPOSITION. Let us turn the prism with its weights, of the 14th proposition, somewhat about the fixed point *R*, and let *Æ* (4 lbs) still be the vertical lifting weight, in such a way that the situation shall be as shown in the annexed figure. WHAT IS REQUIRED TO PROVE. We have to prove that the vertical lifting weight *Æ* also keeps the prism in that given position. PROOF. Let us take away *Æ* and attach *Z* (4 lbs); then, by the 10th proposition, the prism will remain in that position. Now, by the 13th proposition, *Æ* exerts on the prism at *V* the same force as *Z* at *Y*; therefore, *Z* being taken away and *Æ* attached again, *Æ* will also keep the prism in that position.
CONCLUSION. If therefore there are two points in the axis of the prism, one being fixed and the other movable, the vertical lifting weight at the movable point which keeps the prism in one position will keep it in any position, which we had to prove.

THEOREM IX. PROPOSITION XVII.

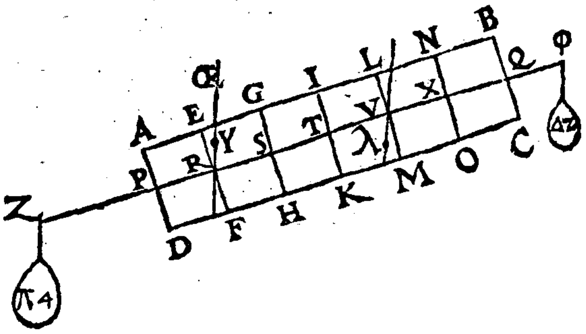
If a prism rests on two points in the axis: as the part of the axis between the centre of gravity and the lefthand point is to the part of the axis between the centre of gravity and the righthand point, so is the weight of the prism resting on the righthand point to the weight resting on the lefthand point.

SUPPOSITION. Let *ABCD* be a prism weighing 6 lbs, divided as in the 1st proposition, resting with the two points *R*, *V* on the points of *Œ*, *Æ*. WHAT IS REQUIRED TO PROVE. We have to prove that as the part of the axis *TR* is to the part of the axis *TV*, so is the weight resting with the point *V* on the point of *Æ* to the weight resting with the point *R* on the point of *Œ*. PROOF. *TR* is double of *TV* by the supposition; then on the point of *Æ* there rest 4 lbs and



ende opt tpunt van Æ rust 4 ℔, ende van Œ 2 ℔ door 1ᵉ veruolg des 14ᵉⁿ voorstels, maer 4 ℔ is tot 2 ℔ oock dobbel, ghelijck dan T R tot T V, alsoo tghewicht rustende op tpunt van Æ, tot tghewicht rustende op tpunt van Œ.

Maer om tghemeen nootsaeckelick veruolgh in allen te bewysen, laet ons voorttrecken V R tot in Z, also dat R Z euen sy an R V; aensiende daer naer R voor tvastpūt, so sal an Z moeten hanghen Π 4 ℔, om de pilaer in die ghestalt te houden door het 3ᵉ voorstel. Maer tghene an V den pilaer in die ghestalt houdt als Æ, doet daer an alsulcken gheweldt als Π, door



het 13 voorstel; An Æ dan rust een ghewicht euen an Π. Laet ons insghelicx voorttrecken, R V tot in Φ, also dat V Φ euen sy an V R, ansiende daer naer V voor vastpunt, soo sal an Φ moeten hanghen Δ 2 ℔, om den pilaer in die ghestalt te houden door het 3ᵉ voorstel, maer tghene an R den pilaer in die ghestalt houdt als Œ, doet daeran alsulcke gewelt als Δ door het 13 voorstel, An Œ dan rust een ghewicht euen an Δ. Nu anghesien Π euestaltwichtich is teghen den pilaer op tghemeen vastpunt R, so heeft den erm T R, sulcken reden tot den erm R Z, als Π tot den pilaer door 1ᵉ voorstel. Insghelijcx nemende V voor tvastpunt, soo heeft den erm T V sulcken reden tot den erm V Φ, als Δ tot den pilaer, maer R Z is altijt euen an V Φ: Wy hebben hier dan twee * eueredenheden elck van vier * palen, welcker tweede palen an malcanderen euen sijn, ende welcker laeste palen an malcanderen oock euen sijn. Maer alle twee eueredenheden elck van vier palen, welcker tweede palen an malcander

Proportiones.
Terminis.

on that of Œ 2 lbs, by the 1st corollary of the 14th proposition. But 4 lbs is also double of 2 lbs; therefore, as *TR* is to *TV*, so is the weight resting on the point of Æ to the weight resting on the point of Œ.

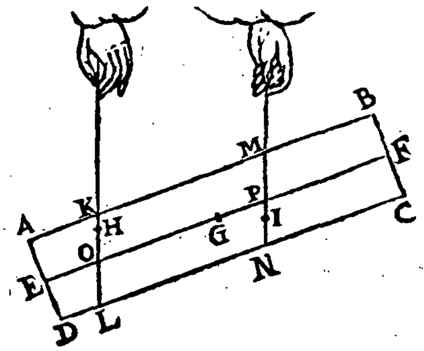
But in order to prove the general necessary consequence in all cases, let us produce *VR* to *Z*, in such a way that *RZ* shall be equal to *RV*. If we then consider *R* as the fixed point, there will have to hang at *Z* a weight Π of 4 lbs in order to keep the prism in that position, by the 3rd proposition. Now that which, acting at *V* keeps the prism in that position, as Æ, exerts on it the same force as Π by the 13th proposition. At Æ therefore rests a weight equal to Π. Let us likewise produce *RV* to Φ, in such a way that *V*Φ shall be equal to *VR*. If we then consider *V* as the fixed point, there will have to hang from Φ a weight Δ of 2 lbs in order to keep the prism in that position, by the 3rd proposition. But that which, acting at *R*, keeps the prism in that position, as Œ, exerts on it the same force as Δ, by the 13th proposition. At Œ therefore rests a weight equal to Δ. Since Π is of equal apparent weight to the prism on the common fixed point *R*, the arm *TR* has to the arm *RZ* the same ratio as Π to the prism, by the 1st proposition. Likewise, taking *V* for the fixed point, the arm *TV* has to the arm *V*Φ the same ratio as Δ to the prism. But *RZ* is always equal to *V*Φ. We therefore have two proportions, each of four terms, the second terms of which are equal to one an-cander euen sijn, ende welcker laetste palen an malcander oock euen sijn, die hebben dander palen oock eueredních, daerom ghelijck T R tot T V, alsoo Π tot Δ; maer Π is euen an tghewicht des pilaers rustende met tpunt V op tpunt van Æ, eñ tghewicht Δ is euen an t'ghewicht des pilaers rustende met tpunt R op tpunt van Œ, daerom ghelijck T R tot T V, alsō tghewicht rustende mettet tpunt V op tpunt van Æ, tottet ghewicht rustende mettet punt R op tpunt van Œ. TBESLVYT. Rustende dan een pilaer op twee punten inden as, &c.

VERVOLGH.

SOO de twee punten daer den pilaer op rust, waren inde * hanghende linien door R en V, de selue ghewichten die hier vooren op elck rustende punt waren, soudender nu oock op sijn. Laet by voorbeelt door de punten R, V, hanghende linien ghetrocken worden, ende punten inde selue ghestelt als Y λ, Ghenomen nu dat Y ende λ de punten sijn daer den pilaer op rust, tis kennelick dat op Y rusten sal 2 ℔, ende op λ 4 ℔, Waer uyt alsulcken vertooch openbaer is. *Perpendicularibus.*

10. VERTOOCH. XVIII. VOORSTEL.

RVSTENDE een pilaer op eenighe twee punten, ghelijck het asstick tusschen tswaerheyts middelpunt ende de hanghende door t'slinckerpunt, tottet asstick tusschen t'swaerheydts middelpunt ende de hanghende door trechterpunt, also tghewicht des pilaers rustende op trechterpunt, tottet ghewicht rustende op t'slinckerpunt.



TGHEGHEVEN. Laet A B C D een pilaer wesen, diens as E F, ende swaerheydts middelpunt G, ende de twee punte daer d'een pilaer op rust H I, waer duer ghetrocken sijn de hanghende linien K L, M N, sniende den as in O, P; Ick seg dat ghelijck G O tot G P, alsoo de swaerheydt rustende op tpunt I, tot de swaerheydt rustende op H, waer af tbewys openbaer is door tvervolgh des voorgaenden 17en voorstels, nochtans other and the last terms of which are also equal to one another. But if of any two proportions, each of four terms, the second terms are equal to one another and the last terms are also equal to one another, then they have the other terms also proportional. Therefore, as *TR* is to *TV*, so is Π to Δ. But Π is equal to the weight of the prism resting with the point *V* on the point of *Æ*, and the weight Δ is equal to the weight of the prism resting with the point *R* on the point of *Œ*. Therefore, as *TR* is to *TV*, so is the weight resting with the point *V* on the point of *Æ* to the weight resting with the point *R* on the point of *Œ*. CONCLUSION. If therefore a prism rests on two points in the axis, etc.

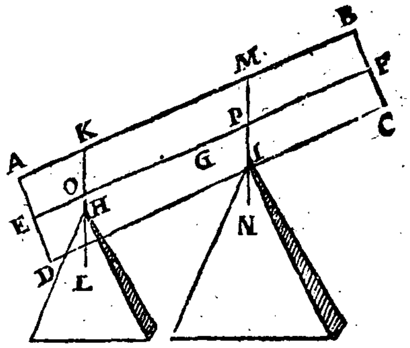
COROLLARY.

If the two points on which the prism rests were in the verticals through *R* and *V*, the same weights which were above on each point of support would then also be thereon. Let there, for example, be drawn verticals through the points *R*, *V*, and let there be marked points in them, as *Y*, Λ. Now considering *Y* and Λ to be the points on which the prism rests, it is evident that there will rest 2 lbs on *Y* and 4 lbs on Λ, from which the theorem is manifest.

THEOREM X. PROPOSITION XVIII.

If a prism rests on any two points: as the part of the axis between the centre of gravity and the vertical through the lefthand point is to the part of the axis between the centre of gravity and the vertical through the righthand point, so is the weight of the prism resting on the righthand point to the weight resting on the lefthand point.

SUPPOSITION. Let *ABCD* be a prism, its axis *EF*, the centre of gravity *G*, and the two points on which the prism rests *H*, *I*, through which are drawn the verticals *KL*, *MN*, meeting the axis in *O*, *P*. *I* say that as *GO* is to *GP*, so is the gravity resting on the point *I* to the gravity resting on *H*, from which the proof is manifest by the corollary of the 17th proposition hereinbefore. Nevertheless, in nochtans om alhier wat breeder vande nootsakelicheyt te segghen, so laet ons achten al of H ter plaets van O waer, twelck soo ghenomen tghewicht alsdan op H rustende, heeft sulcken reden tottet ghewicht op P rustende, ghelijck G P, tot G O, duer het 17ᵉ voorstel; Laet ons voort nemen dattet tpunt H vast blijuende, den pilaer in haer ghegheuen ghestalt neergheтrocken worde, soo verre als van H tot O, ende duer de 3ᵉ begheerte, de swaerheydt an H rustende blijft de selue. Sghelijcx salmen bethoonen de swaerheyt dieder op P rust, oock te rusten op I, daerom ghelijck G O tot G P, also de swaerheyt rustende op I, tot de swaerheyt rustende op H.



TBESLVYT. Rustende dan een pilaer op eenighe twee punten, &c.

VERVOLGH.

TBLIICT uyt het voorgaende dat soomen begheerde te weten de reden van tghewicht rustende op I, tottet tghewicht rustende op H, datmen trecken soude de hanghende linien K L, M N, sniende den as E F in O, P, ende de reden van G O tot G P soude de begheerde sijn waer uyt oock openbaer is, dat des pilaers swaerheyt bekent wesende, soo is oock tghewicht bekent rustende op yder punt als H ende I.

TOT HIER TOE SIIN DE GHEDAENTEN DER RECHTWICHTEN VERCLAERT: INT

volghende sullen de eyghenschappen der scheefwichten bescreuen worden, wiens ghemeene grondt dit volghende vertooch begrijpt.

XI. VERTOOCH. XIX. VOORSTEL.

Planum. *Horizontem.* WESENDE een driehouc wiens * plat rechthouckich op den * sichteinder is, met sijn grondt daer af euewidich, ende op elck der ander sijden een rollende cloot met malcanderen euewichtich: Ghelijck order to speak here somewhat more fully about the necessity, let us suppose *H* to be in the place of *O*. On this assumption, the weight then resting on *H* has to the weight resting on *P* the same ratio as *GP* to *GO*, by the 17th proposition. Let us further suppose that, the point *H* remaining fixed, the prism be pulled down in its given position as far as *H* is from *O*; then by the 3rd postulate the gravity resting on *H* remains the same. In the same way the gravity resting on *P* will be shown to rest also on *I*; therefore, as *GO* is to *GP*, so is the gravity resting on *I* to the gravity resting on *H*. CONCLUSION. If therefore a prism rests on two points, etc.

COROLLARY.

It appears from the above that if it should be required to know the ratio of the weight resting on *I* to the weight resting on *H*, the verticals *KL*, *MN*, meeting the axis *EF* in *O*, *P*, should be drawn; then the ratio of *GO* to *GP* would be the one required, from which it is also manifest that, the gravity of the prism being known, the weight resting on each of the points, as *H*, *I*, is also known.

UP TO THIS POINT

THE PROPERTIES OF VERTICAL WEIGHTS HAVE BEEN EXPLAINED;

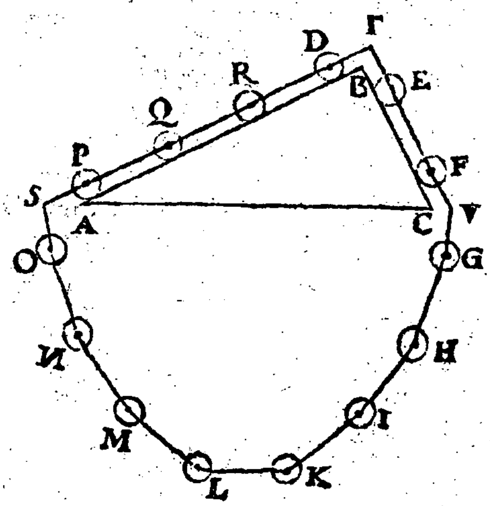
in the following pages the properties of oblique weights will be described, the common principle of which is contained in the following theorem.

THEOREM XI. PROPOSITION XIX.

Given a triangle, whose plane is at right angles to the horizon, with its base parallel thereto, while on each of the other sides there shall be a rolling sphere, of equal weight to one another: as the right side of the triangle is to the left

Ghelijck des driehoucx rechter ſijde tot de ſlincker, alſo t'ſtaltwicht des cloots opde ſlincker ſijde, tottet ſtaltwicht des cloots op de rechter ſijde.

TGHEGHEVEN. Laet A B C een driehouck weſen diens plat ſy rechthouckich op den ſichteinder, ende den grondt A C euewydich vanden ſichteinder, ende op de ſijde A B, die dobbel ſy an B C, ligghe een cloot D, ende op de ſijde B C een cloot E, euewichtich ende euegroot met den cloot D. TBEGHEERDE. Wy moeten bewyſen dat ghelijck de ſijde A B 2, tot B C 1, alſoo t'ſtaltwicht des cloots E, tottet ſtaltwicht des cloots D.



TBEREYTSEL. Laet ons maecken rondtom den driehouck A B C eenen crans van veerthien clooten, euegroot, euewichtich, ende euewijt van malcanderen, als E, F, G, H, I, K, L, M, N, O, P, Q, R, D, al gheſnoert an een lini, ſtreckende door haer * middelpunten, alſo dat ſy op die middelpunten drayen mueghen; Datter oock twee clooten paſſen op de ſijde B C, ende vier op B A, dat is ghelijck lini tot lini, alſo clooten tot clooten; laet oock an S, T, V, drie vaſtpunten ſtaen, ouer welcke de lini ofte t'ſnoer der clooten ſlieren mach, alſo dat de twee deelen des ſnoers die bouen den driehouck ſtaen, * euewydich ſijn vande ſijden A B, B C; Inder voughen dat alſmen den crans an d'een ofte d'ander ſijde neertreckt, ſoo rollen de clooten op de linien A B, B C. TBEWYS. Soo t'ſtaltwicht der vier clooten D, R, Q, P, niet euen en waer met het ſtaltwicht der twee clooten E, F, t'een of t'ander ſal ſwaerder ſijn, latet weſen (ſoot mueghelick waer) der vier D, R, Q, P; Maer de vier clooten O, N, M, L, ſijn euewichtich met de vier clooten G, H, I, K, de ſijde dan der acht clooten D, R, Q, P, O, N, M, L, is ſwaerder na de gheſtalt dan de ſijde der ſes clooten E, F, G, H, I, K: maer want het ſwaerſte altijdt het lichtſte ouerweeght, de acht clooten ſullen neerwaert rollen, ende d'ander ſes rijſen: Latet ſoo weſen, ende D ſy ghe-

Centra.

Parallela.

side, so is the apparent weight [1]) of the sphere on the left side to the apparent weight of the sphere on the right side.

SUPPOSITION. Let *ABC* be a triangle, whose plane shall be at right angles to the horizon, and the base *AC* parallel to the horizon, and on the side *AB*, which shall be double of the side *BC*, let there lie a sphere *D*, and on the side *BC* a sphere *E*, of equal weight and equal size to the sphere *D*. WHAT IS REQUIRED TO PROVE. We have to prove that as the side *AB* (2) is to *BC* (1), so is the apparent weight of the sphere *E* to the apparent weight of the sphere *D*. PRELIMINARY. Let us make about the triangle *ABC* a wreath of fourteen spheres, of equal size and equal weight, and equidistant from one another, as *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, *N*, *O*, *P*, *Q*, *R*, *D*, all of them strung on a line passing through their centres, in such a way that they can revolve about those centres; let there also fit two spheres on the side *BC* and four on *BA*, i.e. as line to line, so spheres to spheres. Let there also be three fixed points at *S*, *T*, *V*, over which the line or the string of the spheres can slide, in such a way that the two parts of the string above the triangle shall be parallel to the sides *AB*, *BC*, so that if the wreath is pulled down on one side or the other, the spheres roll on the lines *AB*, *BC*. PROOF. If the apparent weight of the four spheres *D*, *R*, *Q*, *P* were not equal to the apparent weight of the two spheres *E*, *F*, either one or the other will be the heavier. Let us suppose (if this were possible) this to be the one of the four spheres *D*, *R*, *O*, *P*. But the four spheres *O*, *N*, *M*, *L* are of equal weight to the four spheres *G*, *H*, *I*, *K*. The side therefore of the eight spheres *D*, *R*, *Q*, *P*, *O*, *N*, *M*, *L* is heavier in appearance than the side of the six spheres *E*, *F*, *G*, *H*, *I*, *K* [2]). But because that which is heavier always preponderates over that which is lighter, the eight spheres will roll downwards and the other six will rise. Let

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[1]) Here for the first time the term *staltwicht* is used, which has not been defined anywhere. It means the component of the weight which in the given situation is the only active one, i.e. the component along the inclined plane.

[2]) Of course this contention, though true in this special case, is not valid generally: if to two systems of unequal apparent weight, equal weights whose apparent weights are not equal to one another are added on both sides, the systems may become of equal apparent weight.

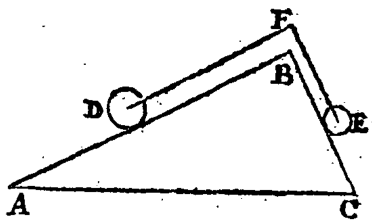
ſy gheuallen daer nu O is, ende E,F,G,H, ſullen ſijn daer nu P,Q,R,D, ende I, K, daer nu E, F, ſijn. Maer dit ſoo weſende, den crans der clooten ſal alſulcken gheſtalt hebben als ſy te vooren dede, ende ſullen om de ſelue redenende acht clooten ter ſlincker ſijde wederom ſtaltwichtigher ſijn dan de ſes clooten ter rechter, waer duer de acht clooten wederom neer ſullen rollen, ende d'ander ſes rijſen, welcke valling ter eender, ende rijſing ter ander, om dat de reden altijdt de ſelue is, altijdt ghedueren ſal, ende de clooten ſullen uyt haer ſeluen een eeuwich roerſel maken, t'welck valſch is. Het deel dan des crans D, R, Q, P, O, N, M, L, is eueſtaltwichtich met het deel E, F, G, H, I, K: Maer van ſulcke euewichtighe ghetrocken euewichtighe, de reſten ſijn euewichtich, laet ons dan van dat deel trecken de vier clooten O, N, M, L, ende van dit de vier clooten G, H, I, K, (welcke euen ſijn ande voornoemde O, N, M, L,) de reſten D, R, Q, P, ende E, F, ſullen eueſtaltwichtich ſijn, Maer weſende deſe twee eueſtaltwichtich met die vier, E ſal tweemael ſtaltſwaerder ſijn als D. Ghelijck dan de lini B A 2, tot de lini B C 1, alſo t'ſtaltwicht des cloots E, tottet ſtaltwicht des cloots D.

T'BESLVYT. Weſende dan een driehouck wiens plat, &c.

I. VERVOLGH.

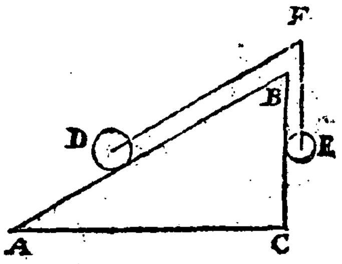
LAET A B C een driehouck ſijn als vooren, wiens ſijde A B dobbel ſy an B C, ende laet op A B ligghen een cloot D, ende op de ſijde B C een cloot E euewichtich anden helft van D, ende an F ſy een vaſtpunt daer ouer de lini D F E. (te weten uyt het *middelpunt des cloots D ouer F tot int middelpunt des cloots E) ſlieren mach, alſoo dat D F euewydich blijue van A B, ende F E van B C. Dit alſo ſijnde, anghesien de vier clooten P, Q, R, D, hier vooren, eueſtaltwichtich waren met de twee clooten E, F, ſo ſal deſen cloot D, eueſtaltwichtich ſijn teghen den cloot E: want ghelijck die P, Q, R, D, tot E, F, alſo deſe D tot E: Daerom ghelijck de lini A B, tot B C, alſoo den cloot D tot den cloot E.

*Centro.



II VERVOLGH.

LAET ons nu d'een ſijde des driehouckx als B C (ande welcke A B dobbel is) rechthouckich ſtellen op A C als hier neuen; Ende den cloot D die dobbel is an E, ſal noch met E eueſtaltwichtich ſijn, want ghelijck A B tot B C, alſo den cloot D tot den cloot E.



III VER-this be so, and let *D* have fallen where *O* is now, then *E, F, G, H* will be where *P, Q, R, D* are now, and *I, K* where *E, F* are now. But this being so, the wreath of spheres will have the same appearance as before, and on this account the eight spheres on the left side will again have greater apparent weight than the six spheres on the right side, in consequence of which the eight spheres will again roll down and the other six will rise. This descent on the one and ascent on the other side will continue for ever, because the cause is always the same, and the spheres will automatically perform a perpetual motion, which is absurd [1]). The part of the wreath *D, R, Q, P, O, N, M, L* therefore is of equal apparent weight to the part *E, F, G, H, I, K*. But if from such equal weights there are subtracted equal weights, the remainders will have equal weight [2]). Let us therefore subtract from the former part the four spheres *O, N, M, L*, and from the latter part the four spheres *G, H, I, K* (which are equal to the aforesaid *O, N, M, L*); then the remainders *D, R, Q, P* and *E, F* will be of equal apparent weight. But the two latter being of equal apparent weight to the four former, *E* will have twice the apparent weight of *D*. As therefore the line *BA* (2) is to the line *BC* (1), so is the apparent weight of the sphere *E* to the apparent weight of the sphere *D*. CONCLUSION. Given therefore a triangle, whose plane, etc.

COROLLARY I.

Let *ABC* be a triangle, as before, whose side *AB* shall be double of *BC*, and on *AB* let there lie a sphere *D* and on the side *BC* a sphere *E* being of equal weight to half of *D*. And in *F* let there be a fixed point, over which the line *DFE* (to wit, from the centre of the sphere *D* via *F* to the centre of the sphere *E*) can slide, in such a way that *DF* shall remain parallel to *AB*, and *FE* to *BC*. This being so, since the four spheres *P, Q, R, D* in the preceding case were of equal apparent weight to the two spheres *E, F*, this sphere *D* will be of equal apparent weight to the sphere *E*; for as the former *P, Q, R, D* are to *E, F*, so is the latter *D* to *E*. Therefore, as the line *AB* is to *BC*, so is the sphere *D* to the sphere *E* [3]).

COROLLARY II.

Now let us put one side of the triangle, as *BC* (*AB* being double of it) at right angles to *AC*, as in the annexed figure. Then the sphere *D*, which is double of *E*, will still be of equal apparent weight to *E*, for as *AB* is to *BC*, so is the sphere *D* to the sphere *E*.

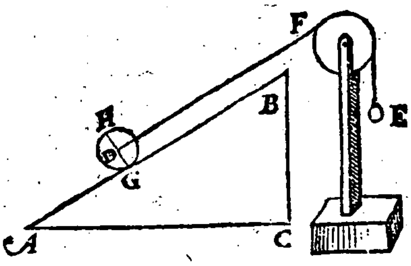
---

[1]) The conviction that a perpetual motion is impossible in physical reality is not a sufficient ground for qualifying it as absurd in the ideal sphere of rational mechanics, where friction and resistance of the air are absent.

[2]) Obviously, Stevin should here have said: but if from equal apparent weights there are subtracted equal weights, the remainders will have equal apparent weight. This again is not generally true, but it does hold in the case here considered.

[3]) Here, the so-called law of the inclined plane is finally reached.

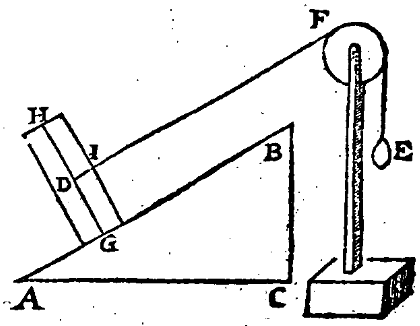
III. VERVOLGH.



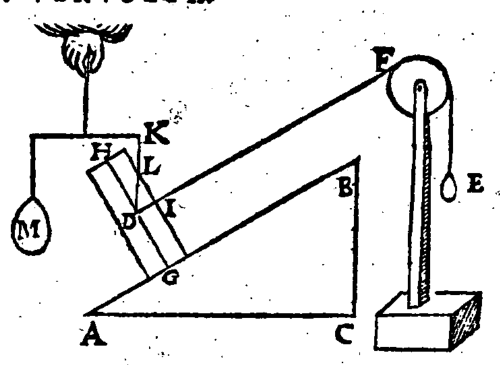
LAET ons nu inde plaets van t'punt F, stellen een caterol als hier neuen, alsô dat de scheefheflini van D naer F euewydich blijue van A B, ende inde plaets vanden cloot E sy eenich wicht van form soot valt, maer euewichtich anden cloot E: t'selue is noch euestaltwichtich met D, Daerom ghelijck A B tot B C, alsoo noch den cloot D tottet ghewicht E.

IIII VERVOLGH.

ANGHESIEN den cloot des 3en veruolgs naeſt de lini A B, in t'punt G, als vaſtpunt, ſoo ſal den as G H rechthouckich ſijn op A B; Daerom laet ons weren den cloot, ende ſtellen in die plaets den pilaer D euewichtich met den cloot, alsoo dat den as G H (dies vaſtpunt G) rechthouckich ſy op A B, eñ de ſcheefheflini tuſſchen D F noch euewydich van A B, ende ſniende de ſijde des pilaers in I, Als hier neuens. Ende is openbaer dat ghelijck A B tot B C, (dat is dobbel als vooren) alſo den pilaer D tottet t'ghewicht E. *18.v.3.BE.*



V. VERVOLGH.



LAET ons trecken de hanghende lini uyt het * middelpunt des pilaers D als D K, ſniende de ſijde des pilaers in L, t'welck ſoo ſijnde, den driehouck L D I is ghelijck an den driehouck A B C, want de houcken A C B ende L I D ſijn recht, ende L D is euewydich van B C ende D I van A B: Daerom ghelijck A B tot B C, alſoo *Centro.*

F 2 L D tot

COROLLARY III.

Let us now put in the place of the point *F* a pulley, as shown in the annexed figure, in such a way that the oblique lifting line from *D* to *F* shall remain parallel to *AB*. And in the place of the sphere *E* let there be some arbitrary weight, but which is of equal weight to the sphere *E*. This weight will still be of equal apparent weight to *D*. Therefore, as *AB* is to *BC*, so is the sphere *D* to the weight *E*.

COROLLARY IV.

Since the sphere of the 3rd corollary touches the line *AB* in the point *G* as fixed point, the axis *GH* will be at right angles to *AB*. Therefore let us take away the sphere, and put in its place the prism *D*, of equal weight to the sphere, in such a way that the axis *GH* (its fixed point being *G*) shall be at right angles to *AB*, and the oblique lifting line between *D*, *F* still parallel to *AB* and meeting the side of the prism in *I*, as shown in the annexed figure. Then it is manifest that as *AB* is to *BC* (i.e. double of it, as above), so is the prism *D* to the weight *E*.

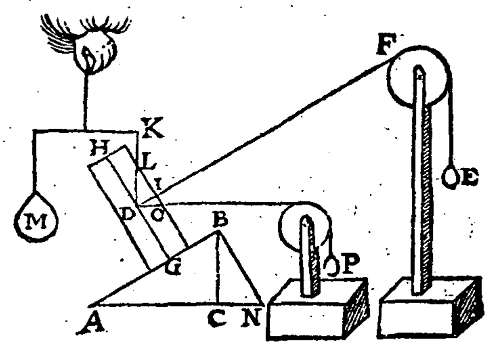
COROLLARY V.

Let us draw the vertical from the centre of the prism *D*, as *DK*, meeting the side of the prism in *L*. This being so, the triangle *LDI* is similar to the triangle *ABC*, for the angles *ACB* and *LID* are right angles, and *LD* is parallel to *BC*, and *DI* to *AB*. Therefore, as *AB* is to *BC*, so is *LD* to *DI*. But as *AB* is to *BC*, L D tot D I; Maer ghelijck A B tot B C, alſoo den Pilaer tot t'ghewicht E door het 4e veruolg, daerom ghelijck L D tot D I, alſo den pilaer tot E. Laet ons nu ande lini K D voughen trechtheſwicht M met den pilaer eueſtaltwichtich, t'ſelue ghewicht M ſal met den pilaer euewichtich ſijn door het 14e voorſtel: Daerom ghelijck L D tot D I, alſo M tot E.

VI. VERVOLGH.

LAET ons trecken B N, ſniende de voortghetrocken A C in N: Inſghelijcx D O, ſniende de voortghetrocken L I dat is de ſijde des pilaers in O, ende alſo dat den houck I D O, euen ſy anden houck C B N. Laet ons oock voughen an D O t'ſcheefheſwicht P, dat den pilaer (de ghewichten M, E gheweert ſijnde) in die ſtandthaude. Nu angheſien D L, des driehouckx D L I,* lijckſtandighe is met B A des driehouckx B A C, ende D I met B C, men beſluyt daer uyt aldus: Ghelijck B A tot B C, alſoo t'ſtaltwicht van B A tottet ſtaltwicht van B C (door het 2e vervolg) En oock ghelijck D L tot D I alſo t'ſtaltwicht van D L tot t'ſtaltwicht van D I, dat is alſoo M tot E. Maer de lijckſtandighe linien van deſe ghelijcke driehoucken A B N, L D O, ſijn B A met D L, ende B N met D O, Daerom ſegghen wy als vooren, Ghelijck B A tot B N, alſoo het ſtaltwicht van B A tot het ſtaltwicht van B N (door het 1e vervolgh) Ende oock ghelijck D L tot D O, alſo het ſtaltwicht van D L tot het ſtaltwicht van D O, dat is alſo M tot P. Maer by aldien de lini B N, ghetrocken waer van B af ouer d'ander ſijde van B C, ſo ſoude de lini D O, dan oock vallen van D ouer d'ander ſijde van D I, dat is, daer D O nu valt onder D I, ſy ſouder dan bouen vallen, ende t'voorgaende bewys ſoude oock dienen tot ſulcke gheſtalt, te weten, dat wy noch ſegghen ſouden, ghelijck B A tot B N, alſoo t'ſtaltwicht van B A, tottet ſtaltwicht van B N; Ende ghelijck D L tot D O, alſoo t'ſtaltwicht van D L, tottet ſtaltwicht van D O, dat is, alſo M tot P. Inder voughen dat deſe* eueredenheydt niet alleen en beſtaet inde voorbeelden, alwaer de heflini als D I rechthoukich is op den as, maer op allen houcken.

Homologa.



Proportio.

T'VOORGAENDE mach oock verſtaen worden van een cloot ligghende op een lini A B als hier neuens, alwaer wy ſegghen als vooren, ghelijck L D tot D O, alſoo M tot P (welverſtaende dat C L recht-so is the prism to the weight *E*, by the 4th corollary; therefore, as *LD* is to *DI*, so is the prism to *E*. Let us now attach at the line *KD* the vertical lifting weight *M* of equal apparent weight to the prism. This weight *M* will be of equal weight to the prism, by the 14th proposition. Therefore, as *LD* is to *DI*, so is *M* to *E* [1]).

COROLLARY VI.

Let us draw *BN* [2]), meeting *AC* produced in *N*; in the same way *DO*, meeting *LI* produced, that is the side of the prism, in *O*, and in such a way that the angle *IDO* shall be equal to the angle *CBN*. Let us also attach at *DO* the oblique lifting weight *P*, which shall keep the prism (the weights *M* and *E* being taken away) in that position. Now since *DL* of the triangle *DLI* is homologous to *BA* of the triangle *BAC*, and *DI* to *BC*, it may thus be concluded: As *BA* is to *BC*, so is the apparent weight of *BA* [3]) to the apparent weight of *BC* (by the 2nd corollary). And also, as *DL* is to *DI*, so is the apparent weight of *DL* [4]) to the apparent weight of *DI*, that is *M* to *E*. But the homologous lines of these similar triangles *ABN*, *LDO* are *BA* to *DL*, and *BN* to *DO*. Therefore we say, as above: as *BA* is to *BN*, so is the apparent weight of *BA* to the apparent weight of *BN* (by the 1st corollary); and also, as *DL* is to *DO*, so is the apparent weight of *DL* to the apparent weight of *DO*, that is *M* to *P* [5]). But if the line *BN* were drawn from *B* on the other side of *BC*, the line *DO* would also fall from *D* on the other side of *DI*, i.e. whereas *DO* now falls below *DI*, it would then fall above it, and the foregoing proof would also be true of such a situation, to wit that we should still say: as *BA* is to *BN*, so is the apparent weight of *BA* to the apparent weight of *BN*, and as *DL* is to *DO*, so is the apparent weight of *DL* to the apparent weight of *DO*, that is *M* to *P*. Therefore this proportion is true not only in the examples where the lifting line *DI* is at right angles to the axis, but also with any angle.

The above may also be understood of a sphere lying on a line *AB*, as shown in the annexed figure, in which case we say, as before, as *LD* is to *DO*, so is *M* to *P* (to wit: if *CL* is drawn at right angles to *AB*, i.e. parallel to the axis

[1]) As it is irrevelant that the body on the inclined plane is a prism, the following theorem may be considered to have been proved. If the vector *DL* is opposite to the weight of a body lying on the inclined plane *AB*, and *LI* is perpendicular to *AB*, *DI* represents the force along the plane which keeps the body in equilibrium. We may also say: *DL* is the resultant of the forces *DI* and *IL* (normal reaction). This is the triangle of forces for the special case that the two components are at right angles to one another.

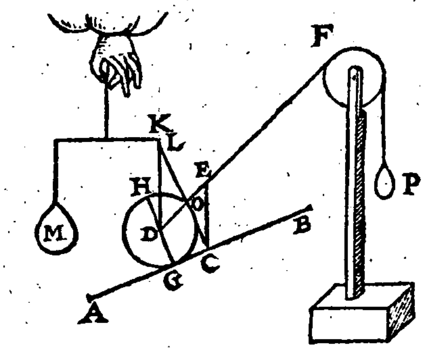
[2]) Stevin does not specify the direction of *BN*; probably it is intended to be at right angles to *AB* in accordance with the fact that the line of action of *P* is horizontal. However, this special position is irrelevant.

[3]) Evidently Stevin here takes the word ***staltwicht*** in quite a different sense from that used above, viz. proper weight of the prism on *AB*. Likewise apparent weight of *BC* now means the proper weight of a body on *BC* balancing the prism; Girard (XIII 449a) therefore has the correct expression: le poids sur *BA*, le poids sur *BC*.

[4]) Apparent weight of *DL* here means the force acting along *DL*, i.e. the weight of *M*. Likewise: apparent weight of *DI* = *E*. Girard (XIII 449a) translates: le poids appartenant à *DL*, à *DI*.

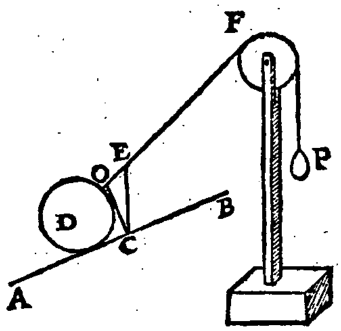
[5]) This argumentation gives the proper weight of a body *P* balancing the prism on *AB*, if it were connected with it by a string, which is first parallel to *AB* and then, after having passed over a pulley, parallel to *BN*. It is not at all clear how Stevin is able to make any inference about the force P considered by him, the line of action of which is arbitrary. It is true that his result is correct, the force *DO* (= *P*) and the normal reaction *OL* having *DL* as resultant, but the object of the proposition was to prove the triangle of forces for the case of forces not mutually perpendicular, and not to apply it.

rechthoukich ghetrocken is op A B, dat is euewydich met den as G H des cloots D) maer t'ghewicht M is euen an den cloot D, daerom ſegghen wy ghelijck L D tot D O, alſo t'ghewicht des cloots, tot P. Maer want L D ende D O binnen t'lichaem des cloots metter daet niet bequamelick en connen beſchreuen worden, ſo laet ons trecken de hanghende C E, ende ſullen dan hebben buyten t'lichaem een driehouck C E O, ghelijck anden driehouck L D O, welcker * lijckſtandighe ſijden ſijn L D met C E, ende D O met E O, daerom ghelijck L D tot D O, alſoo C E tot E O, ende veruolghens ghelijck C E tot E O, alſoo t'ghewicht des cloots, tot P.

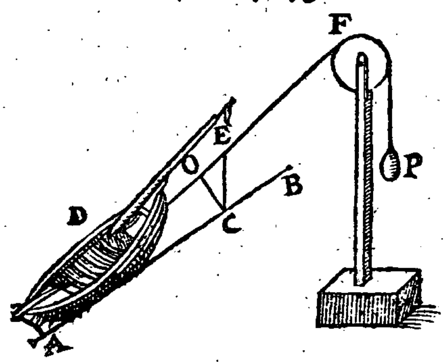


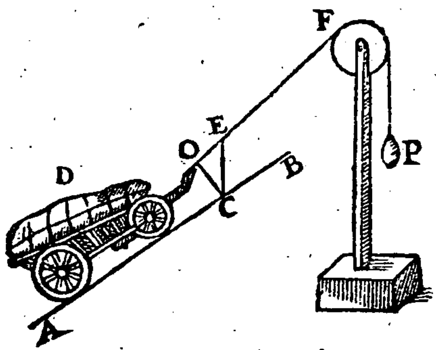
Homologa

LAET ons nu tot meerder claerheydt dit alleen ſtellen ſonder d'ander linien als hier neuen, alwaer wy ſegghen ghelijck C E tot E O, alſo t'ghewicht des cloots D tot P.



ENDE dit niet alleen van clooten maer van ander lichamen ſlierende, ofte rollende, op puntē ofte linien als hier onder (daerwy eyghentlicker





af handelen ſullen inde Weegdaet) alwaer wy noch ſegghen ghelijck C E to E O, alſo t'ghewicht des lichaems D tottet ghewicht P.

F 3 WAER

GH of the sphere *D*). But the weight *M* is equal to the sphere *D*, therefore we say: as *LD* is to *DO*, so is the weight of the sphere to *P*. But because *LD* and *DO* cannot easily be drawn in practice inside the body of the sphere, let us draw the vertical *CE*; we shall then have outside the said body a triangle *CEO* similar to the triangle *LDO*, the homologous sides of which triangles are *LD* to *CE*, and *DO* to *EO*. Therefore, as *LD* is to *DO*, so is *CE* to *EO*; and consequently: as *CE* is to *EO*, so is the weight of the sphere to *P*.

Now for the sake of greater clearness let us put this separately, without the other lines, as shown in the annexed figure, where we say: as *CE* is to *EO*, so is the weight of the sphere *D* to *P*.

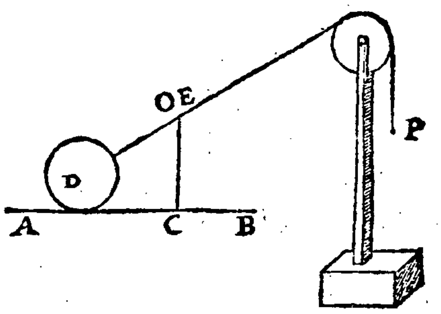
And this is true not only of spheres, but also of other solids sliding or rolling on points or lines, as shown below (a subject with which we will deal more properly in the Practice of Weighing [1])), where we still say: as *CE* is to *EO*, so is the weight of the solid *D* to the weight *P*.

[1]) See *The Practice of Weighing*, Prop. 9, Examples 3 and 4.

WAER uyt oock blijckt, dat wesende de lini A B euewydich vanden *sichteinder als hier neuens, dat C E ende C O dan in een selfde lini sullen vallen, waer duer tusschen E en O gheen langde en sal sijn, ende veruolghens C E en sal tot E O gheen reden hebben, daermen by verstaen sal dat een swaerheydt inde plaets van P hoe cleen sy mocht wesen, en sal niet euestaltwichtich connen sijn teghen t'lichaem D, maer salt (* wisconstelick verstaende) voorttrecken hoe swaer het sy: Waer uyt volght, dat alle swaerheden voortghetrocken langs den sichteinder, als schepen int water, waghens langs t'platte landt, &c. en behouuen gheen vlieghesterctens macht tot haer verroersel, meer dan de omstaende verhinderniſſen en veroirsaecken, als Water, Locht, Naecsel der assen, teghen de bussen, naecsel der rayers teghen de straet, ende dierghelijcke.

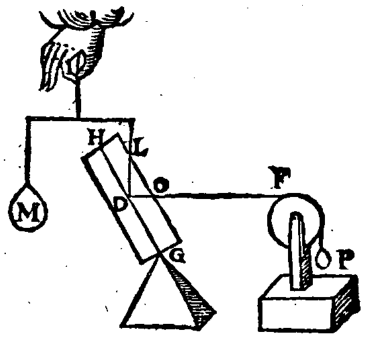
Horizonte.

Mathematicè.



MAER anghesien den driehouck A B N int 6e veruolg, tot dese *eueredenheyt niet en gheeft noch en neemt, laet ons hem weeren, ansiende G voor vastpunt des pilaers rustēde op een pin als hier neuen, ende sullen noch segghen ghelijck L D tot D O, also M tot P.

Proportionē.



VII. VERVOLGH.

MAER op dat nu blijcke dese* eueredenheydt niet alleen also te bestaen inde pilaren alwaer de rechtheflini als D L, comt uyt t'middelpunt des pilaers, ende diens vastpunt is des assens uyterste, als hier vooren G int 6e vervolg; Soo laet A B C een driehouck sijn, wiens sijde A B dobbel is an B C, ende B C sy *hanghende op A C: Ende laet D E een pilaer sijn diens as F G rechthouckich op A B, ende sniende A B in t'punt H, ende I sy eenich ander punt inden seluen as; Laet

Proportionē.

Perpendicularis.

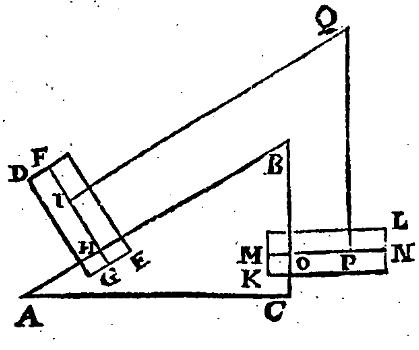
From the above it also appears that, the line *AB* being parallel to the horizon, as in the annexed figure, *CE* and *CO* will fall on the same line, so that there will be no distance between *E* and *O*; consequently *CE* will not have any ratio to *EO*, by which it is to be understood that a gravity taking the place of *P*, however small it may be, cannot be of equal apparent weight to the solid *D*, but will pull it along (mathematically speaking), however heavy it may be. From this it follows that all gravities pulled along parallel to the horizon, such as ships in the water, wagons along the level land, etc., to be moved do not require the force of a fly beyond that which is caused by the surrounding obstacles, viz. water, air, contact of the axles with the bearings, contact of the wheels with the road, and the like.

But since the triangle *ABN* in the 6th corollary is irrelevant to this proportion, let us take it away [1]), considering *G* as the fixed point of the prism resting on a peg, as in the annexed figure; then we shall still say: as *LD* is to *DO*, so is *M* to *P*.

COROLLARY VII.

But in order that it may be evident that this proportion is true not only of prisms where the vertical lifting line, as *DL*, starts from the centre of the prism, while the fixed point of the latter is the extremity of the axis, as *G* above, in the 6th corollary: let *ABC* be a triangle, whose side *AB* shall be double of *BC*, and let *BC* be vertical to *AC*. And let *DE* be a prism whose axis *FG* shall be at right angles to *AB* and meet *AB* in the point *H*, and let *I* be some other point in this

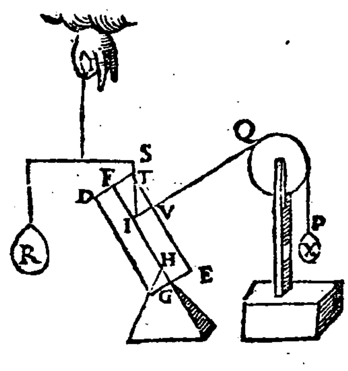
[1]) The inclined plane is now dismissed; instead of resting on it in *G*, the body is supposed to have a fixed point at *G*.



Laet oock K L een ander pilaer sijn, euen ende ghelijck anden pilaer D E, wiens as M N, ende O een punt des as naeckende B C, ende van ghelijcke gestalt in sijn pilaer, als H inden pilaer D E; Laet oock P een ander punt sijn van sulcker ghestalt inden pilaer K L, als I inden pilaer D E; Ende laet Q een vastpunt sijn daer ouer de lini I Q P slieren mach, alsoo dat de lini I Q * euewydich sy van A B, ende Q P euewydich van B C. Ende om de redenen die int 19ᵉ voorstel vande 14 clooten verclaert sijn (t'welck wy hier deur soodanighe veel slierende pilaren oock souden connen bewysen, maer want sulcx uyt t'voorgaende kennelick is, wy slaent ouer) het staltwicht des pilaers K L, sal dobbel sijn an t'staltwicht des pilaers D E. *Parallela.*

VIII. VERVOLGH.

Laet ons nu an I des 7ᵉ veruolgs voughen trechthefwicht R euestaltwichtich met den pilaer, diens rechtheflini sy I S, sniende de sijde des pilaers in T, ende I Q snie de sijde des pilaers in V, ende laet an de lini P Q hanghen een ghewicht X, inde plaets vanden pilaer K L, t'welck euen sy anden helft van t'staltwicht des selfden pilaers K L, Laet ons oock weeren den driehouck A B C, ende den pilaer D E doen rusten op t'punt H als hier neuen. Ende om de redenen als vooren, ghelijck T I tot I V, alsoo R tot X. Ende dit niet alleen als I V rechthouckich is op de as F G, maer cromhouckich soot valt, waeraf men besondet bethooch soude mueghen doen, maer tis openbaer ghenouch door het 6ᵉ veruolgh.



IX. VERVOLGH.

Wy hebben int 8ᵉ vervolgh dese * eueredenheyt verclaert, alwaer t'roerende punt I, hoogher was dan t'vastpunt H, ende alwaer de scheefheflini I V helde naer de sijde des vastpunts H; Wy moeten nu betooghen *Proportionel.*

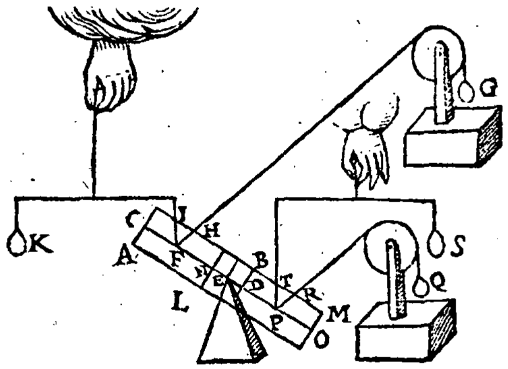
same axis. Let *LK* be another prism, equal and similar to the prism *DE*, and its axis *MN*, and *O* a point of the axis touching *BC* and similarly placed in its prism to *H* in the prism *DE*. Let *P* also be another point, similarly placed in the prism *KL* to *I* in the prism *DE*. And let *Q* be a fixed point, over which the line *IQP* can slide, in such a way that the line *IQ* shall be parallel to *AB*, and *QP* parallel to *BC*. Then, for the reasons explained in the 19th proposition with regard to the 14 spheres (which we might also prove here by means of an equal number of sliding prisms, but we omit this because it is evident from what precedes), the apparent weight of the prism *KL* will be double of the apparent weight of the prism *DE*.

COROLLARY VIII.

Let us now attach at *I* of the 7th corollary the vertical lifting weight *R* having equal apparent weight to the prism, whose vertical lifting line shall be *IS*, meeting the side of the prism in *T*, and let *IQ* meet the side of the prism in *V*. And let there hang from the line *PQ* a weight *X*, instead of the prism *KL*, which shall be equal to half the apparent weight of this same prism *KL*. Let us also take away the triangle *ABC*, and cause the prism *DE* to rest on the point *H*, as shown in the annexed figure. Then, for the same reasons as above, as *TI* is to *IV*, so is *R* to *X*. And this is true not only when *IV* is at right angles to the axis *FG*, but also when it is at any oblique angle thereto, which might be proved specifically; but it is sufficiently manifest from the 6th corollary.

COROLLARY IX.

We have explained this proportion in the 8th corollary, where the movable point *I* was higher than the fixed point *H*, and where the oblique lifting line *IV* bethooghen de selue eueredenheyt oock so te bestaen in d'ander ghestalten, ende eerst alwaer troerende punt leegher sy dan t'vastpunt, ende alwaer de scheefheflini afwyct vande sijde des vastpunts in deser voughen:



Laet A B een pilaer sijn, diens as C D, ende vastpunt E, ende t'verroerlick punt F, ende P scheefhefwicht dat hé in die ghestalt houdt sy G, diens scheefheflini F H, ende F I sy rechtheflini, diens rechthefwicht K. Laet L M oock een pilaer sijn, euen ende ghelijck an den pilaer A B, wiens as sy N O, ende vastpunt E, ende verroerlick punt P, also dat E N euen sy an E D, ende E F an E P, ende t'scheefhefwicht Q sy euen an G, ende sijn scheefheflini sy P R, *euewydich van F H, ende trechthefwicht S sy euen an K, ende sijn rechtheflini sy P T. Dit soo sijnde laet ons vergaren de twee pilaren A B ende L M, ansiende A M voor een heel pilaer, wiens *swaerheyts middelpunt ende vastpunt sal E sijn door t'ghestelde. Laet ons nu weeren de ghewichten K, G, S, Q, ende den pilaer A M sal op E alle ghestalt houden diemen hem gheeft door het 7^e^ voorstel, hy sal dan soo blijuen, ende den pilaer A B sal alsoo euewichtich blijuen teghen den pilaer L M. Laet ons nu de ghewichten Q G weder andoen, hanghende euewichtighe van ghelijcke ghestalt, an euewichtighe, ende door het 13^e^ voorstel, Q sal anden pilaer A M euen sulcken macht doen als G: Ende veruolghens Q doet sulcken macht an huer pilaer L M, als G an huer pilaer A B; maer de macht van G is A B in die ghestalt te houden door het 6 vervolg, de macht dan van Q is oock L M in die ghestalt te houden. Insghelijcx soo is oock de macht van K, den pilaer A B in die ghestalt te houden, daerom oock is de macht van S den pilaer L M in die ghestalt te houden; Nu ghelijck I F tot F H, also K tot G door het 8^e^ veruolg, Maer T P. is euen an I F, ende P R an F H, ende S an K, ende Q an G, ghelijck dan T P tot P R, alsoo S tot Q. Dese eueredenheydt dan, als wy gheseyt hebben, is so wel inde voorbeelden alwaer t'roerende punt P leegher is dan t'vastpunt E, ende alwaer de scheefheflini P R afwyct vande sijde des vastpunts E, als daert hoogher is, ende daer de scheefheflini helde naer t'vastpunt.

Parallela.

Centrum grauitatis.

X. VER-

verged towards the side of the fixed point *H*. We now have to prove that this proportion is also true in other situations, in the first place when the movable point is lower than the fixed point, and when the oblique lifting line verges away from the side of the fixed point, as follows:

Let *AB* be a prism, its axis *CD*, and the fixed point *E* and the movable point *F*. And let the oblique lifting weight keeping it in that position be *G*, its oblique lifting line *FH*; and let *FI* be the vertical lifting line, and its vertical lifting weight *K*. Let *LM* also be another prism, equal and similar to the prism *AB*, whose axis shall be *NO*, the fixed point *E* and the movable point *P*, in such a way that *EN* shall be equal to *ED*, and *EF* to *EP*, and that the oblique lifting weight *Q* shall be equal to *G*, and its oblique lifting line shall be *PR*, parallel to *FH*, while the vertical lifting weight *S* shall be equal to *K*, and its vertical lifting line shall be *PT*. This being so, let us combine the two prisms *AB* and *LM*, considering *AM* as a complete prism, whose centre of gravity and fixed point shall be *E*, by the supposition. Let us now take away the weights *K*, *G*, *S*, *Q*; then the prism *AM* will remain at rest on *E* in any position given to it, by the 7th proposition. It will therefore remain thus, and the prism *AB* shall in this way remain of equal weight [1]) to the prism *LM*. Let us now attach the weights *Q*, *G* again, hanging equal weights of the same appearance at equal weights [2]); then by the 13th proposition [3]) *Q* will exert the same force on the prism *AM* as *G*. And consequently *Q* exerts on its prism *LM* the same force as *G* on its prism *AB*. But the power of *G* is to keep *AB* in that position, by the 6th corollary; the power of *Q* therefore is also to keep *LM* in that position. In the same way the power of *K* is also to keep the prism *AB* in that position, and therefore also the power of *S* is to keep the prism *LM* in that position. Now as *IF* is to *FH*, so is *K* to *G*, by the 8th corollary. But *TP* is equal to *IF*, and *PR* to *FH*, and *S* to *K*, and *Q* to *G*; therefore, as *TP* is to *PR*, so is *S* to *Q*. As we have said, therefore, this proportion is true in the examples where the movable point *P* is lower than the fixed point *E* and where the oblique lifting line *PR* verges away from the side of the fixed point *E*, as well as in the examples where the said movable point is higher and where the oblique lifting line verged towards the fixed point.

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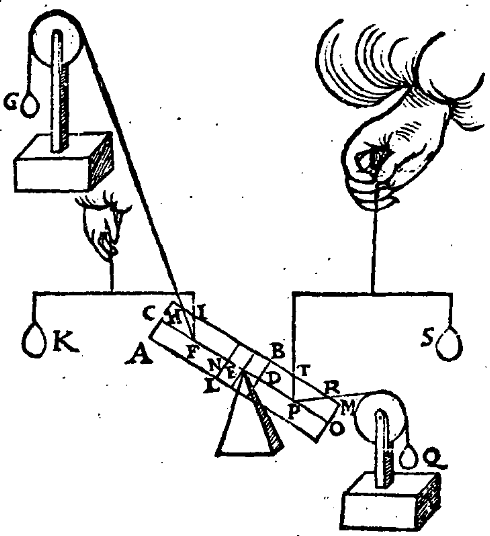
[1]) It is not relevant that *AB* and *LM* are of equal weight, but that they balance one another; they are of equal apparent weight.

[2]) The meaning of the words „of the same appearance" can be no other than that the lines of action of the forces *G* and *Q* make equal angles with *CO*, and that the points of application are equidistant from *E*. However, there is no postulate or proposition relating to this case.

[3]) It is not clear that this proposition can be applied here. Accordingly the subsequent argumentation is far from being convincing.

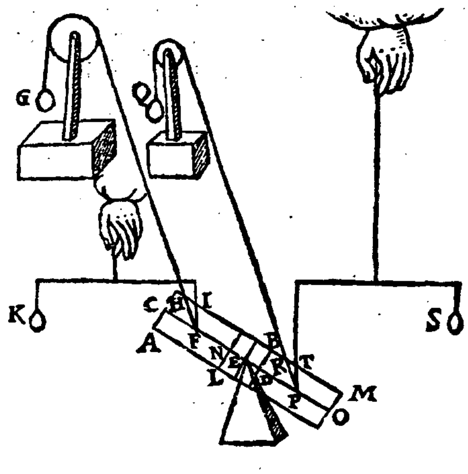
X. VERVOLGH.

LAET ons ſtellen een form ghelijck an die des 9 veruolghs, alleen daer in verſchillende dat deſe F H wyckt ouer d'ander ſijde van F I, ende dat den houck H F C, euen ſy anden houck R P O, waer duer G anden pilaer A M euen ſoo grooten ghewelt doet als Q, ende om de redenen des 9 veruolgs (die wy om cortheyt ouerſlaen) G doet euen ſulcken ghewelt anden pilaer A B, als Q anden pilaer L M; Nu ghelijck T P tot P R, alſoo S tot Q door het 9ᵉ vervolgh, maer I F is euen an T P, ende F H an P R, ende K an S, ende G an Q, daerom ghelijck I F tot F H, alſo K tot G.



XI VERVOLGH.

LAET ons ſtellen een form ghelijck an die des 10 veruolgs, alleen daer in verſchillende dat deſe P R wyckt ouer d'ander ſijde van P T, ende dat P R euewydich ſy met F H, waer deur Q anden pilaer A M, euen ſoo grooten ghewelt doet als G, ende om de redenen des 9ᵉ veruolghs, Q doet euen ſulcken gheweldt anden pilaer L M, als G anden pilaer A B; Nu ghelijck I F tot F H, alſo K tot G door het 6 veruolgh: Maer T P is euen an I F, ende P R an F H, ende S an K, ende Q an G, daerom ghelijck T P tot P R, alſo S tot Q. Ende inder ſeluer voughen ſalmen vanden anderen gheſtalten door haer contrarien altijt deſe eueredenheyt bewyſen.



COROLLARY X.

Let us take the same figure as that of the 9th corollary, with the only difference that in this case *FH* verges towards the other side of *FI*, and let the angle *HFC* be equal to the angle *RPO*, in consequence of which *G* exerts on the prism *AM* the same force as *Q*; then, for the reasons mentioned in the 9th corollary (which we omit for brevity's sake), *G* exerts on the prism *AB* the same force as *Q* on the prism *LM*. Now as *TP* is to *PR*, so is *S* to *Q*, by the 9th corollary. But *IF* is equal to *TP*, and *FH* to *PR*, and *K* to *S*, and *G* to *Q*; therefore, as *IF* is to *FH*, so is *K* to *G*.

COROLLARY XI.

Let us take the same figure as that of the 10th corollary, with the only difference that in this case *PR* verges towards the other side of *PT*, and let *PR* be parallel to *FH*, in consequence of which *Q* exerts on the prism *AM* the same force as *G*; then, for the reasons mentioned in the 9th corollary, *Q* exerts on the prism *LM* the same force as *G* on the prism *AB*. Now, as *IF* is to *FH*, so is *K* to *G*, by the 6th corollary. But *TP* is equal to *IF*, and *PR* to *FH*, and *S* to *K*, and *Q* to *G*; therefore, as *TP* is to *PR*, so is *S* to *Q*. And in the same way this proportion can always be proved to be true of the other positions, through their contraries [1]).

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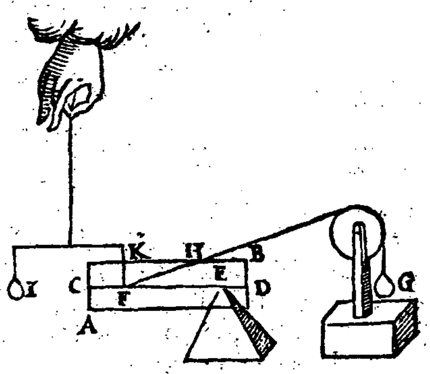
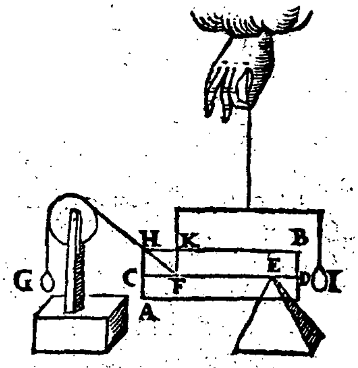
[1]) The meaning of this phrase seems to be: by taking the line of action of one of the oblique lifting weights on the other side of the vertical.

XII. VERVOLGH.

Proportio. Horizonte. MAER dat deſe * euetedenheydt oock beſtaet inde gheſtalt daer den as euewydich is vanden * ſichteinder, wort aldus bethoont: Laet A B een pilaer ſijn, diens Parallela. as C D * euewydich ſy vanden ſichteinder, ende t'vaſtpunt daer in E, ende t'roerlick punt F, ende G t'ſcheefheſwicht dat den pilaer in die geſtalt houdt, wiens ſcheefheſlini F H, ende I trechtheſwicht dat den pilaer oock in die gheſtalt houdt, wiens rechtheſlini F K; T'welck ſoo ſijnde, Laet K F tot F H een ander reden hebben (ſoot mueghelick waer) dan I tot G, By voorbeelt K F ſy tot F H, als 1 tot 2, maer I tot G, als 3 tot 7. Dit ſo ghenomen, laet ons den pilaer der eerſte form neerduwen, ofte der tweeder form oplichten, tot dat K F ſulcken reden hebbe tot F H, als 3 tot 7, ende alſdan ſal G oock eueſtaltwichtich ſijn teghen den pilaer door de voorgaende vervolghen; Inder voughen dat den pilaer hoogher ende leegher verheuen, ſal teghen G eueſtaltwichtich blijuen, t'welck openbaer Mathematicè. onmueghelick is, als oock * wiſconſtlick ſal blijcken door t'volghende 22 voorſtel. K F dan en heeft tot F H gheen ander reden dan I tot G.





UYT deſe voorgaende beſcrijuen wy een vertooch ſoodanich.

XII. VERTOOCH. XX. VOORSTEL.

WESENDE inden as des pilaers een vaſtpunt, ende een roerlick, daer an hy door een rechtheſwicht ende ſcheefheſwicht in ſeker ſtandt ghehouden wort: Ghelijck rechtheſlini tot ſcheefheſlini, alſo rechtheſwicht tot ſcheefheſwicht.

TGHEGHEVEN. Laet A B een pilaer ſijn diens as C D, ende t'vaſtpunt

COROLLARY XII.

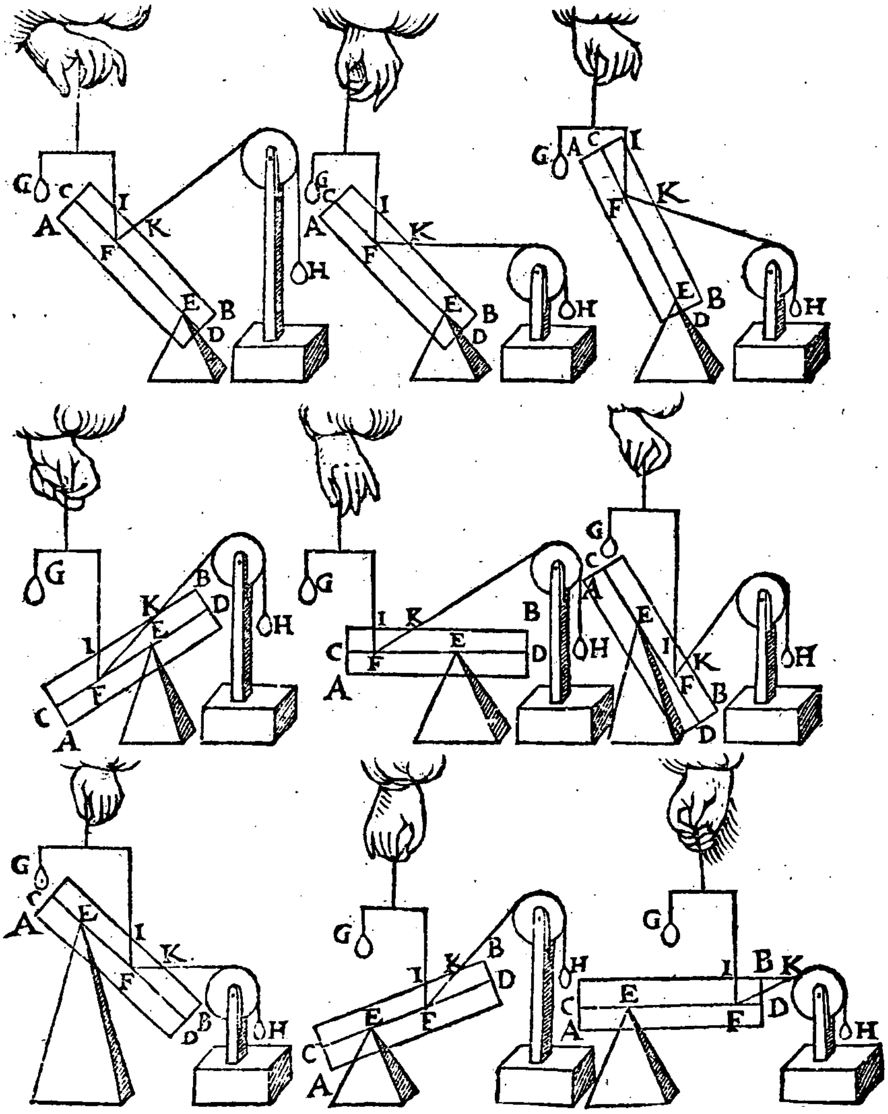
Now it is proved as follows that this proportion is also true of the position where the axis is parallel to the horizon. Let *AB* be a prism, whose axis *CD* shall be parallel to the horizon, and let the fixed point therein be *E*, and the movable point *F*, and *G* the oblique lifting weight keeping the prism in that position, whose oblique lifting line shall be *FH*, and *I* the vertical lifting weight also keeping the prism in that position, whose vertical lifting line shall be *FK*. This being so, let *KF* have to *FH* a different ratio (if this were possible) from *I* to *G*, for example let *KF* be to *FH* as 1 to 2, but *I* to *G* as 3 to 7. On this assumption let us push down the prism of the first figure or lift that of the second figure until *KF* have to *FH* the ratio of 3 to 7; then *G* will also be of equal apparent weight to the prism, by the preceding corollaries. Therefore, the prism, lifted or lowered, will remain of equal apparent weight to *G*, which is manifestly impossible, as will also be proved mathematically by the 22nd proposition hereinafter. The ratio between *KF* and *FH* therefore is not different from that between *I* and *G*.

From the preceding we derive the following theorem.

THEOREM XII. PROPOSITION XX.

If there is in the axis of the prism a fixed point and a movable point, at which it is kept in a certain position by a vertical lifting weight and an oblique lifting weight: as the vertical lifting line is to the oblique lifting line, so is the vertical lifting weight to the oblique lifting weight.

punt E, ende roerlick punt F, daeran den pilaer door t'rechthefwicht G in die ghestalt ghehouden wort, daer an oock den pilaer door t'scheefhefwicht H (welverstaende G gheweert sijnde) in die ghestalt ghehouden wort, ende de rechtheflini snie de sijde des pilaers in I, maer de scheefheflini snie de selue sijde in K: Ick seg dat ghelijck de rechtheflini I F, tot de scheefheflini F K, alsoo trechthefwicht G, tot het scheefhefwicht H. waer af t'bewys uyt de voorgaende openbaer is.



SUPPOSITION. Let *AB* be a prism, its axis *CD*, the fixed point *E*, and the movable point *F*, at which the prism is kept in that position by the vertical lifting weight *G*, at which the prism is also kept in that position by the oblique lifting weight *H* (to wit: *G* being taken away). And let the vertical lifting line meet the side of the prism in *I*, and let the oblique lifting line meet this same side in *K*. I say that as the vertical lifting line *IF* is to the oblique lifting line *FK*, so is the vertical lifting weight *G* to the oblique lifting weight *H*, the proof of which is

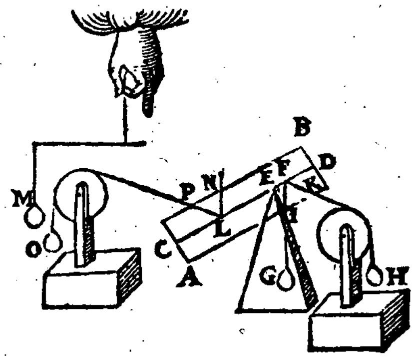
T'BESLVYT. Wesende dan inden as des pilaers een vastpunt, &c.

MERCKT. Soo eenighe der linien als I F, F K, de sijde des pilaers niet en sneen, men sal die sijde voorder trecken tot dat sy ghesneen wort, als inde voorgaende laetste form.

XIII. VERTOOCH. XXI. VOORSTEL.

WESENDE inden as des pilaers een vastpunt, ende een roerlick, daer an hy door een rechtdaelwicht eñ scheefdaelwicht in seker standt gehouden wort: Ghelijck rechtdaellini tot scheefdaellini, also rechtdaelwicht tot scheefdaelwicht.

T'GHEGHEVEN. Laet A B een pilaer sijn, diens as C D, ende vastpunt E, ende roerlick punt F, daer an den pilaer door trechtdaelwicht G in die ghestalt ghehouden wort, daer an oock den pilaer door t'scheefdaelwicht H (welverstaende G gheweert sijnde) in die ghestalt ghehouden wordt, ende de rechtdaellini snie de sijde des pilaers in I, maer de scheefdaellini snie de selue sijde in K. TBEGHEERDE. Wy moeten bewysen dat ghelijck de rechtdaellini I F tot de scheefdaellini F K, alsoo t'rechtdaelwicht G tot het scheefdaelwicht H. TBEREYTSEL. Laet ons teeckenen t'punt L, alsoo dat E L euen sy an E F, ende voughen an t'punt L t'rechthefwicht M, dat den pilaer in die ghestalt can houden, diens rechtheflini L N: Insghelijcx t'scheefhefwicht O, dat den pilaer oock in die ghestalt can houden, wiens scheefheflini L P euewydich sy met F K.



TBEWYS. Ghelijck N L tot L P, also M tot O, duer het 20e voorstel maer de macht van G is anden pilaer euen met de macht van M, eñ de macht van H met die van O duer het 13e voorstel, ende I F is euen an L N, ende F K, an L P; Daerom ghelijck de rechtdaellini I F tot de scheefdaellini F K, alsoo t'rechtdaelwicht G tottet scheefdaelwicht H, Sghelijcx sal oock t'bewys sijn van alle d'ander ghestalten als inde formen hier na volghende.

TBESLVYT

manifest from what precedes. CONCLUSION. If therefore there is in the axis of the prism a fixed point, etc.

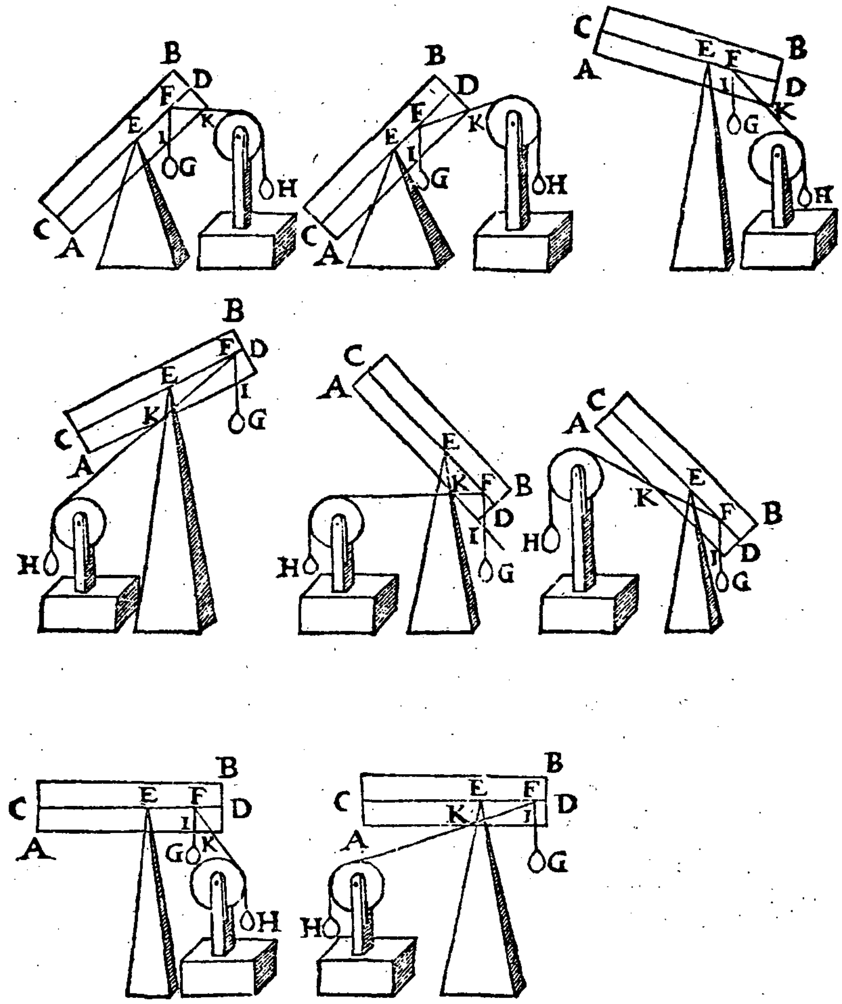
NOTE.

If any of the lines, as *IF*, *FK*, should not meet the side of the prism, this side shall be produced until such lines do meet it, as in the last of the preceding figures.

THEOREM XIII. PROPOSITION XXI.

If there is in the axis of the prism a fixed point and a movable point, at which it is kept in a certain position by a vertical lowering weight and an oblique lowering weight: as the vertical lowering line is to the oblique lowering line, so is the vertical lowering weight to the oblique lowering weight.

SUPPOSITION. Let *AB* be a prism, its axis *CD*, and the fixed point *E* and the movable point *F*, at which the prism is kept in that position by the vertical lowering weight *G*, at which the prism is also kept in that position by the oblique lowering weight *H* (to wit: *G* being taken away). And let the vertical lowering line meet the side of the prism in *I*, and let the oblique lowering line meet this same side in *K*. WHAT IS REQUIRED TO PROVE. We have to prove that as the vertical lowering line *IF* is to the oblique lowering line *FK*, so is the vertical lowering weight *G* to the oblique lowering weight *H*. PRELIMINARY. Let us mark the point *L* in such a way that *EL* shall be equal to *EF*, and let us attach at the point *L* the vertical lifting weight *M*, which can keep the prism in that position, whose vertical lifting line shall be *LN*. In the same way the oblique lifting weight *O*, which can also keep the prism in that position, whose oblique lifting line *LP* shall be parallel to *FK*. PROOF. As *NL* is to *LP*, so is *M* to *O*, by the 20th proposition, but the power of *G* on the prism is equal to the power of *M*, and the power of *H* to that of *O*, by the 13th proposition; and *IF* is equal to *LN*, and *FK* to *LP*. Therefore, as the vertical lowering line *IF* is to the oblique lowering line *FK*, so is the vertical lowering weight *G* to the oblique lowering weight *H*. A similar proof can be given with regard to all the other positions, as in the following figures.



T'BESLVYT. Wesende dan inden as des pilaers een vastpunt ende een roerlick, &c.

IX. EYSCH. XXII. VOORSTEL.

WESENDE ghegheuen een bekenden pilaer, met een vastpunt inden as, ende een roerlick punt, an t'welck eenich onbekent treckwicht den pilaer in ghegheuen ghestalt houdt: Dat treckwicht bekent te maken.

CONCLUSION. If therefore there is in the axis of the prism a fixed point and a movable point, etc.

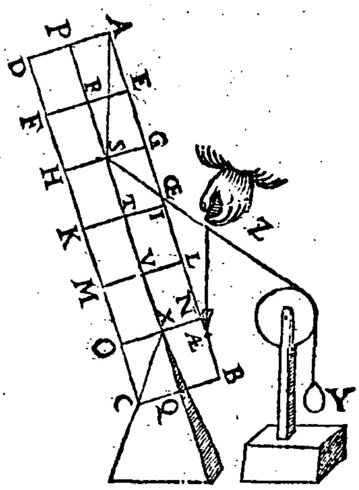
PROBLEM IX. PROPOSITION XXII.

Given a known prism, with a fixed point in the axis and a movable point at which an unknown drawing weight keeps the prism in the given position: to make known the said drawing weight.

T'GHEGHEVEN. Laet ABCD een pilaer sijn weghende 6 ℔, ende ghedeelt als int 1e voorstel, ende t'vastpunt sy X, ende het roerende punt S, an t'welck gheuoecht sy een onbekent scheefhefwicht Y, met den pilaer euestaltwichtich, ende sijn scheefheflini snie de sijde des pilaers AB in Œ. T'BEGHEERDE. Wy moeten dat onbekende scheefhefwicht Y bekent maken.

T'WERCK. Men sal sien wat rechthefwicht an S den pilaer in die ghestalt soude houden, wort beuonden door 14e voorstel, van 4 ℔, daer naer salmé ondersoucken wat reden eenighe * hangende lini als Z Æ, heeft tot Z Œ, ick neme als van 2 tot 1, daer uyt seg ick 2 gheeft 1, wat t'rechthefwicht van 4 ℔? comt voor Y 2 ℔, t'welck ick seg sijn waer ghewicht te sijne. T'BEREYTSEL. Laet ons trecken de hanghende door S welcke sy AS. T'BEWYS. Ghelijck AS tot S Œ, also t'rechthefwicht tottet scheefhefwicht Y door het 20e voorstel, maer den driehouck Œ ZB, is ghelijck anden driehouck Œ S A, welcker * lijckstandighe linien sijn Œ Z met Œ S, ende Z Æ met S A: Daerom ghelijck A S tot S Œ, alsoo Æ Z tot Z Œ, ende vervolghens ghelijck Æ Z 2, tot Z Œ 1, also t'rechthefwicht 4 ℔ tot Y, daerom Y weghende 2 ℔ is bekent ghemaect, t'welck wy bewysen moesten. Ende sghelijcx sal den voortganck sijn in allen anderen voorbeelden.

Perpendicularis.



Homologa.

T'BESLVYT. Wesende dan ghegheuen een bekenden pilaer met een vastpunt inden as, &c.

Ie MERCK.

WY *souden inde werckinq hebben mueghen segghen, A S 2, gheeft S Œ 1, wat t'rechthefwicht 4 ℔? comt voor Y 2 ℔, maer op dat sy lijckformigher soude sijn an t'ghene inde daet gheschiet (want men can binnen int lichaem qualick de linien A S, S Œ trecken) wy hebben de hanghende lini Z Æ int voorbeelt uytwendich ghenomen.*

IIe MERCK.

Inuersam & alternã proportionem. Terminorum

T I S *openbaer door de * verkeerde ende oueranderde Eueredenheydt, hoe dat elck van d'ander onbekende * palen als Rechthefwycht, Rechtheflini, scheefheflini, Pilaer, door drie bekende palen altijdt bekent sullen worden, welcker bescriuing wy om de cortheyt achterlaten.*

XIIII. VER-

SUPPOSITION. Let *ABCD* be a prism weighing 6 lbs and divided as in the 1st proposition, and let the fixed point be X and the movable point *S*, at which let there be attached an unknown oblique lifting weight *Y*, of equal apparent weight to the prism, and let its oblique lifting line meet the side of the prism *AB* in *Œ*. WHAT IS REQUIRED TO MAKE KNOWN. We have to make known that unknown oblique lifting weight *Y*. CONSTRUCTION. It shall be ascertained what vertical lifting weight at *S* would keep the prism in that position. This is found, by the 14th proposition, to be 4 lbs. Then it shall be ascertained what ratio a vertical line, as *ZÆ*, has to *ZŒ*. I take this to be 2 to 1, from which I conclude: 2 gives 1; what the vertical lifting weight of 4 lbs? *Y* becomes 2 lbs, which I say is its true weight. PRELIMINARY. Let us draw the vertical through *S*, which shall be *AS*. PROOF. As *AS* is to *SŒ*, so is the vertical lifting weight to the oblique lifting weight *Y*, by the 20th proposition. But the triangle *ŒZB* is similar to the triangle *ŒSA*, whose homologous lines are *ŒZ* to *ŒS* and *ZÆ* to *SA*. Therefore, as *AS* is to *SŒ*, so is *ÆZ* to *ZŒ*, and consequently as *ÆZ* (2) is to *ZŒ* (1), so is the vertical lifting weight (4 lbs) to *Y*. Therefore *Y* weighing 2 lbs has been made known, which we had to prove. And the procedure will be the same in all the other examples. CONCLUSION. Given therefore a known prism with a fixed point in the axis, etc.

NOTE I.

In the construction we might have said: AS 2 gives SŒ 1; what the vertical lifting weight of 4 lbs? Y becomes 2 lbs. But in order that it might be more in agreement with what happens in practice (for it is hardly possible to draw the lines AS, SŒ inside the solid), we have taken the vertical ZÆ in the example outside the solid.

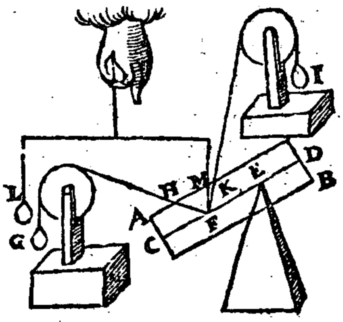
NOTE II.

It is manifest by taking the terms inversely and alternately that each of the other unknown terms, as the vertical lifting weight, the vertical lifting line, the oblique lifting line, and the prism, can always be made known from three known terms, the discussion of which we omit for brevity's sake.

XIIII. VERTOOCH. XXIII. VOORSTEL.

EVEN ghewichten der trecklinien van een selfde punt des as, ende op verscheyden sijden met den as euen houcken makende; doen anden pilaer euen ghewelden.

T'GHE. Laet A B een pilaer sijn diens as C D, ende vastpunt daer in E, en t'roerlick punt F, an t'welck een scheefhefwicht G sy, dat den pilaer in die ghestalt houde, ende diens scheefheflini F H. Laet oock an t'selue punt F gheuoucht wesen een scheefhefwicht I, ouer d'ander sijde, ende met G euewichtich, ende diens scheefheflini F K, den houck K F D euen make anden houck H F C.



T'BEGHEERDE. Wy moeten bewysen dat I anden pilaer euen sulcken ghewelt doet als G, te weten dat I (G gheweert sijnde) den pilaer oock in die ghestalt sal houden.

T'BEREYTSEL. Laet an t'punt F gheuoucht worden t'rechthefwicht L dat den pilaer oock in die ghestalt can houden, ende sijn rechtheflini sy F M.

T'BEWYS. Want de linien F H, F K, sijn tusschen de * euewydighe H K, C D, ende dat den houck H F C, euen is (door t'ghegheuen) an den houck K F D, so sijn F H ende F K euen. waer uyt volght dat ghelijck M F tot F H, alsoo M F tot F K, Maer ghelijck M F tot F H, alsoo L tot G, daerom oock ghelijck M F tot F K, also L tot G; maer I is euen an G door t'ghestelde, ghelijck dan M F tot F K, alsoo L tot I. T'welck so sijnde, I houdt den pilaer in die ghestalt door het 20 voorstel. Sghelijcx sal oock t'bewijs sijn in alle anderen voorbeelden. *Parallelae.*

T'BESLVYT. Euen ghewichten dan der trecklinien van een selfde punt des as, ende op verscheyden sijden met den as euen houcken makende; doen anden pilaer euen ghewelden, t'welck wy bewysen moesten.

XV. VERTOOCH. XXIIII. VOORSTEL.

ALS des ghewichts treckllini rechthouckich op den as is; Soo doedet anden pilaer ghegeuener ghestalt de grootste ghewelt.

T'GHE-

THEOREM XIV. PROPOSITION XXIII.

Equal weights of the drawing lines of the same point in the axis and making equal angles with the axis on different sides exert equal forces on the prism.

SUPPOSITION. Let *AB* be a prism, its axis *CD*, and the fixed point therein *E* and the movable point *F*, at which let there be an oblique lifting weight *G* which shall keep the prism in that position, and let its oblique lifting line be *FH*. Let there also be attached at this same point *F* an oblique lifting weight *I* on the other side and having equal weight to *G*, and let its oblique lifting line *FK* make the angle *KFD* equal to the angle *HFC*. WHAT IS REQUIRED TO PROVE. We have to prove that *I* exerts on the prism the same force as *G*, to wit that *I* (*G* being taken away) will also keep the prism in that position. PRELIMINARY. Let there be attached at the point *F* the vertical lifting weight *L*, which can also keep the prism in that position, and let its vertical lifting line be *FM*. PROOF. Because the lines *FH*, *FK* are contained between the parallel lines *HK*, *CD*, and the angle *HFC* is equal (by the supposition) to the angle *KFD*, *FH* and *FK* are equal. From this it follows that as *MF* is to *FH*, so is *MF* to *FK*. But as *MF* is to *FH*, so is *L* to *G*; therefore also as *MF* is to *FK*, so is *L* to *G*. But *I* is equal to *G*, by the supposition; therefore as *MF* is to *FK*, so is *L* to *I*. This being so, *I* keeps the prism in that position by the 20th proposition. A similar proof can also be given in all the other examples. CONCLUSION. Equal weights therefore of the drawing lines of the same point in the axis and making equal angles with the axis on different sides exert equal forces on the prism, which we had to prove.

THEOREM XV. PROPOSITION XXIV.

If the drawing line of the weight is at right angles to the axis, this weight exerts the greatest force on the prism in the given position.

SUPPOSITION. Let *AB* be a prism, its axis *CD*, and the fixed point *E* and the

T'GHEGHEVEN. Laet A B een pilaer ſijn diens as C D, ende vaſt-punt E, ende roerlick punt F, waer an gheuoucht is t'ſcheefheſwicht G, dat den pilaer in die gheſtalt houdt, ende alſo dat ſijn ſcheefheſlini H F rechthouckich op den as C D is; Laet oock an F gheuoucht worden t'ſcheefheſwicht I, euen an G, ende ſijn ſcheefheſlini ſy K F.

T'BEGHEERDE. Wy moeten bewyſen dat G meerder ghewelt doet anden pilaer, dan I, oock gheen meerder ghewelt daer an doen en can.

T'BEREYTSEL. Laet ons an F voughen t'rechtheſwicht L dat den pilaer in die gheſtalt houden can, diens rechtheſlini F M. T'BEWYS.

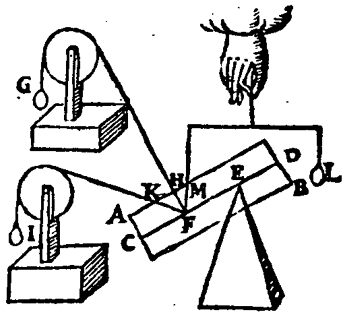
A. *Alle heſwicht dat minder reden heeft tot L, dan ſiin heſlini tot F M, is te licht om den pilaer in die gheſtalt te houden, duer het 20ᵉ voorſtel:*

I. *I is heſwicht dat minder reden heeft tot L, dan ſiin heſlini K F tot F M:*

I. *T'heſwicht I dan is te licht om den pilaer in die gheſtalt te houden.*

Syllogiſmi. DES * bewyſredens tweede voorſtel wort aldus bethoont, T'ghewicht G (t'welck den pilaer in die gheſtalt houdt) heeft ſulcken reden tot L, als H F tot F M, maer I is euen an G, ende K F is meerder dan F H, daerom I heeft minder reden tot L, dan K F tot F M, waer duer ſoo wy bouen gheſeyt hebbẽ, t'gewicht I is te licht om den pilaer in die gheſtalt te houden; maer G can der hem in houden, G dan doet anden pilaer meerder ghewelt dan I. Maer dat G daer an gheen meerder doen en can, is daer uyt openbaer, dat van F op de ſijde des pilaers gheen corter lini en can ghetrocken worden dan F H, anghesien ſy daer op rechthouckich is.

47. v. 1. b. E.



T'BESLVYT. Als dan des ghewichts trecklini rechthouckich op den as is, ſoo doedet an den pilaer ghegheuener gheſtalt de grootſte ghewelt, t'welck wy bewyſen moeſten.

VERVOLGH.

HET blijft dat hoe de houcken der trecklinien vande ghewichten, op den as den rechthouck naerder ſijn, hoe de ghewichten meerder ghewelt doen; Ende ter contrarie hoe ſy vanden rechthouck meer verſchillen, hoe de ghewichten minder ghewelt doen.

XVI. VER-movable point *F*, at which there is attached the oblique weight *G*, which keeps the prism in that position, in such a way that its oblique lifting line *HF* is at right angles to the axis *CD*. Let there also be attached at *F* the oblique lifting weight *I*, equal to *G*, and let its oblique lifting line be *KF*. WHAT IS REQUIRED TO PROVE. We have to prove that *G* exerts on the prism a greater force than does *I*, and cannot exert on it any greater force. PRELIMINARY. Let us attach at *F* the vertical lifting weight *L*, which can keep the prism in that position, and let its vertical lifting line be *FM*.

PROOF.

A [1]. *Any lifting weight which has to L a ratio less than its lifting line to FM is too light to keep the prism in that position, by the 20th proposition;*

I . *I is a lifting weight which has to L a ratio less than its lifting line KF to FM;*

I . *Therefore the lifting weight I is too light to keep the prism in that position.*

The second proposition of the syllogism is shown as follows. The weight *G* (which keeps the prism in that position) has to *L* the same ratio as *HF* to *FM*. But *I* is equal to *G*, and *KF* is greater than *FH;* therefore *I* has to *L* a ratio less than *KF* to *FM*, in consequence of which, as we have said above, the weight *I* is too light to keep the prism in that position. But *G* can keep it in that position, therefore *G* exerts on the prism a greater force than does *I*. But that *G* cannot exert on it any greater force is manifest from the fact that from *F* to the side of the prism no line shorter than *FH* can be drawn, since it is at right angles thereto. CONCLUSION. If therefore the drawing line of the weight is at right angles to the axis, this weight exerts the greatest force on the prism in the given position, which we had to prove.

COROLLARY.

It appears that according as the angles of the drawing lines of the weights with the axis are nearer to a right angle, the weights exert greater forces. And conversely: the more they differ from a right angle, the less force the weights exert.

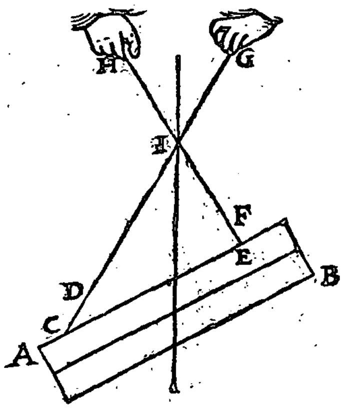
[1]) Cf. note 2) on page 143. *I* denotes a particular affirmative proposition. The mood of the syllogism is *Darii.*

XVI. VERTOOCH. XXV. VOORSTEL.

TWEE oneuewydighe linien daer een pilaer an hangt beyde oneindelick voortghetrocken, ſnien malcanderen inde ſwaerheydts middellini des pilaers.

Iᵉ VOORBEELT.

T'GHEGHEVEN. Laet A B een pilaer ſijn hanghende ande twee oneuewydighe linien C D, E F, welcke voortghetrocken ſijn tot G, H, ſniende malcanderen in I. T'BEGHEERDE. Wy moeten bewyſen dattet punt I inde ſwaerheyts middellini is des pilaers A B. TBEWYS. Den houck F E C, ofte I E C, ofte H E C, is al een ſelfden houck, alſo oock is D C E, ofte I C E, ofte G C E, daerom wat punten wy inde linien H E, ende C G voor uyterſten nemen, den pilaer houdt daer an ſijn ghegheuen ſtandt. Laet ons nemen I, ghemeen uyterſte punt van d'een ende d'ander lini, den pilaer dan houdt daer an ſijn ghegeuen ſtandt. Maer hanghēde den pilaer an t'punt I, ſo is de * hanghende door I des pilaers ſwaerheydts middellini inde welcke I is.



* *Perpendicularis.*

IIᵉ VOORBEELT.

T'GHEGHEVEN. Laet A B een pilaer ſijn hanghende ande oneuewydighe linien C D, E F, welcke voortghetrocken ſijn tot G,H, ſniende malcanderen in I. T'BEGHEERDE. Wy moeten bewijſen dattet punt I, inde ſwaerheydts middellini is des pilaers A B. TBEWYS. Laet ons D G ende F H anſien voor ſtylen ofte ſtiue linien daer den pilaer op ruſt, welcke door de 2ᵉ begheerte niet en breken noch en buyghen; der ſeluer ghewelt is euen ande ghewelt der linien C D, E F, want ghelijck deſe den pilaer in ſijn ghegheuen ſtandt houden alſoo oock die. Ende wat punten wy inde linien D G, F H voor uyterſten nemen, den pilaer

H houdt

THEOREM XVI. PROPOSITION XXV.

Two non-parallel lines, from which a prism is hanging, both of them produced indefinitely, meet in the centre line of gravity of the prism.

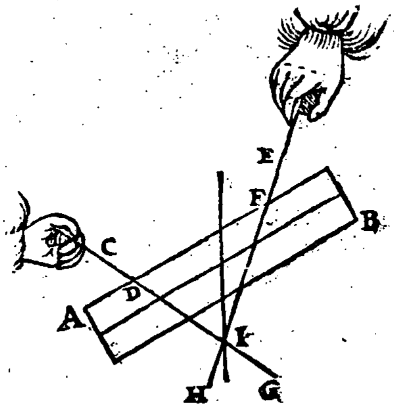
EXAMPLE I.

SUPPOSITION. Let *AB* be a prism hanging from two non-parallel lines *CD*, *EF*, which are produced to *G*, *H*, meeting in *I*. WHAT IS REQUIRED TO PROVE. We have to prove that the point *I* is in the centre line of gravity of the prism *AB*. PROOF: The angle *FEC*, or *IEC*, or *HEC* is always the same angle; similarly *DCE*, or *ICE*, or *GCE*. Therefore, whatever points in the lines *HE* and *CG* we take as their extremities, the prism will keep its given position thereon. Let us take *I*, the common extremity of both lines; the prism then keeps its given position thereon. But if the prism is hanging on the point *I*, the vertical through *I* is the centre line of gravity of the prism, in which lies *I*.

EXAMPLE II.

SUPPOSITION. Let *AB* be a prism hanging from the non-parallel lines *CD*, *EF*, which are produced to *G*, *H*, meeting in *I*. WHAT IS REQUIRED TO PROVE. We have to prove that the point *I* is in the centre line of gravity of the prism *AB*. PROOF. Let us consider *DG* and *FH* to be laths or rigid lines on which the prism rests, which by the 2nd postulate do not break or bend; the forces exerted by them are equal to the forces exerted by the lines *CD* and *EF*, for just as the latter keep the prism in its given position, so do the former. And whatever points in the lines *DG*, *FH* we take as their extremities, the prism keeps its given po-houdt daer op sijn ghegheuen standt. Laet ons nemen I, ghemeen uyterste punt van d'een en d'ander lini; den pilaer dan houdt daer op (* Wisconstlick verstaende) sijn ghegheuen standt, maer rustende den pilaer op t'punt I, soo is de hanghende door I des pilaers swaerheydts middellini, inde welcke I is.

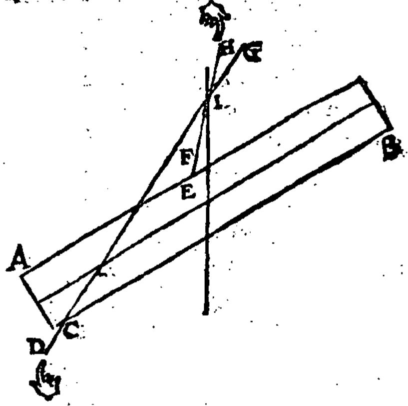
*Mathematice.



III. VOORBEELT.

T'GHEGHEVEN. Laet A B een pilaer sijn welcke in die standt ghehouden wort door de scheefdaellini C D, ende scheefheflini E F, de selue sijn voortghetrocken tot G, H, sniende malcanderen in I.

T'BEGHEERDE. Wy moeten bewysen dat I inde swaerheydts middellini is des pilaers A B. T'BEWYS. Laet ons G C ansien voor styl, ofte stijue lini ende nemen dat de macht die an D int neertrecken was, nu neersteken de sy in yder punt tusschen C en G daermen haer stelt, ende den pilaer A B, sal alsoo op allen punten diemen tusschen C, G eñ E, H voor uytersten neemt, sijn ghegheuen standt houden. Laet ons nemen I ghemeen uyterste van d'een en d'ander lini, den pilaer dan houdt daer an sijn ghegheuen standt; maer hanghende den pilaer an t'punt I, de hanghende door I is des pilaers swaerheyts middellini, inde welcke I is.



IIII. VOORBEELT.

T'GHEGHEVEN. Laet A B een pilaer sijn, welcke in die standt ghehouden wort door de scheefdaellini C D, ende de scheefheflini E F, de selue sijn voortghetrocken tot G H, sniende malcanderen in I.

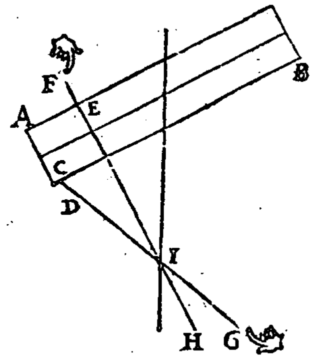
T'BEGHEERDE. Wy moeten bewysen dat I inde swaerheyts middellini sition thereon. Let us take *I*, the common extremity of both lines; the prism then (mathematically speaking) keeps its given position thereon, but if the prism rests on the point *I*, the vertical through *I* is the centre line of gravity of the prism, in which lies *I*.

EXAMPLE III.

SUPPOSITION. Let *AB* be a prism which is kept in that position by the oblique lowering line *CD* and the oblique lifting line *EF*, which are produced to *G*, *H*, meeting in *I*. WHAT IS REQUIRED TO PROVE. We have to prove that *I* is in the centre line of gravity of the prism *AB*. PROOF. Let us consider *GC* to be a lath or rigid line, and let us suppose that the force which was drawing downwardly in *D* be now pushing downwardly in any point between *C* and *G* where it is put; then the prism *AB* will keep its position in any point taken as extremity between *C*, *G*, and *E*, *H*. Let us take *I*, the common extremity of both lines; the prism then keeps its given position thereon. But if the prism hangs at the point *I*, the vertical through *I* is the centre line of gravity of the prism, in which lies *I*.

EXAMPLE IV.

SUPPOSITION. Let *AB* be a prism which is kept in that position by the oblique lowering line *CD* and the oblique lifting line *EF*, which are produced to *G*, *H*, meeting in *I*. WHAT IS REQUIRED TO PROVE. We have to prove that *I* is in dellini is des pilaers A B. T'BEWYS. Laet ons H E ansien voor stijl, ofte stiue lini, ende nemen dat de macht die an E int opheffen was, nu opstekende sy in yder punt tusschen E en H, daermen haer stelt, ende den pilaer A B sal alſo op allen punten diemen tusſchen C G ende E H voor uyterſten neemt, ſijn ghegheuen ſtandt houden. Laet ons nu nemen I ghemeen uyterſte punt van d'een en dander lini, den pilaer dan houdt daer op ſijn ghegheuen ſtandt, maer ruſtende den pilaer op t'punt I, ſoo is de hanghende door I des pilaers ſwaerheydts middellini, inde welcke I is.



T'BESLVYT. Twee oneuewydighe linien dan, daer een pilaer an hangt beyde oneyndelick voortghetrocken, ſnien malcanderen inde ſwaerheydts middellini des pilaers, t'welck wy bewyſen moeſten.

XVII. VERTOOCH. XXVI. VOORSTEL.

Soo d'eene der twee linien daer een pilaer an hangt rechthouckich op den * ſichteinder is, d'ander ſalder oock rechthouckich op ſijn: Ende ſooder d'een ſcheefhouckich op is, dander ſalder oock ſcheefhouckich op weſen: Ende ſoo deſe naer die neigt, die ſal naer deſe neighen: Maer ſo deſe van die wyckt, die ſal oock van deſe wycken. *Horizontem.*

T'GHEGHEVEN. Laet A B een pilaer ſijn hanghende an twee linien, d'een C D rechthouckich op den ſichteinder, d'ander E F (ſoot mueghelick waer) ſcheefhouckich, ende G H ſy des pilaers ſwaerheydts middellini. T'BEGHEERDE. Wy moeten bewyſen t'hinhoudt des voorſtels. T'BEREYTSEL. Laet C D ende E F voortghetrocken worden, ſniende malcander in I. T'BEWYS. Soo den pilaer in die gheſtalt blijft hanghende ande linien C D, E F, ſy ſal op alle vaſtpunten in die voortghetrocken linien de ſelue gheſtalt houden, ouermidts de houcken I C E, ende I E C, niet en veranderen: Daerom ghenomen I

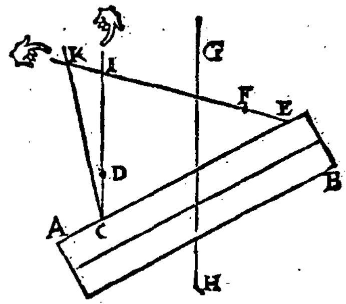
H 2 ghemeen

the centre line of gravity of the prism *AB*. PROOF. Let us consider *HE* to be a lath or rigid line, and let us suppose that the force which was lifting upwardly in *E* be now pushing upwardly in any point between *E* and *H* where it is put. Then the prism *AB* will keep its given position in any point taken as extremity between *C*, *G*, and *E*, *H*. Let us now take *I*, the common extremity of both lines; the prism then keeps its given position thereon. But if the prism rests on the point *I*, the vertical through *I* is the centre of gravity of the prism, in which lies *I*. CONCLUSION. Two non-parallel lines therefore, from which a prism is hanging, both of them produced indefinitely, meet in the centre line of gravity of the prism, which we had to prove.

THEOREM XVII. PROPOSITION XXVI.

If one of two lines from which a prism is hanging is at right angles to the horizon, the other will also be at right angles thereto; and if one is at oblique angles thereto, the other will also be at oblique angles thereto; and if the latter verges towards the former, the former will verge towards the latter, but if the latter verges away from the former, the former will also verge away from the latter.

SUPPOSITION. Let *AB* be a prism hanging from two lines, one *CD*, at right angles to the horizon, the other, *EF* (if this were possible), at oblique angles thereto, and let *GH* be the centre line of gravity of the prism. WHAT IS REQUIRED TO PROVE. We have to prove the contents of the proposition. PRELIMINARY. Let *CD* and *EF* be produced until they meet in *I*. PROOF. If the prism remains hanging in that position from the lines *CD*, *EF*, it will keep the same position on any fixed point in those lines produced, since the angles *ICE* and *IEC* do not change. Therefore, *I* being taken as the common fixed point of



ghemeen vastpunt dier twee linien, den pilaer sal daer an in sijn ghegheuen standt blijuen hanghende, ende I C sal swaerheydts middellini sijn: maer dat is onmueghelick, wanttet G H haer euewydeghe is. T'selue sal oock alsoo bethoont worden als de lini E F ouer dander sijde neigt. Wesende dan I C rechthouckich op den sichteinder, d'ander lini als E F en cander niet scheefhouckich op sijn; nootsaecklick dan rechthouckich: Ende veruolghens sooder E F scheefhouckich op is, dander moeter oock scheefhouckich op sijn.

VOORDER, anghesien E F neigt naer de sijde van A, soo sal de lini die den pilaer in die ghestalt houdt moeten neighen naer E F. Want laetse (soot mueghelick waer) daer van wycken, als C K, sniende de voortghetrocken E I in K, inder voughen dat de hanghende lini door K, sal om de redenen als bouen swaerheydts middellini wesen des pilaers, t'welck noch ongheschicter is dan doen wy die seyden door I te vallen: D'ander lini dan die den pilaer in de ghestalt can houden, en wyckt van E F niet, sy en is met haer oock gheen euewydighe als bouen bethoont is, ende ter sijden uyt te wijcken is openbaer onmueghelick, sy neigt dan nootsaecklick naer E F. Ende soo E F ouer d'ander sijde neigde, men sal insghelijcx bethoonen dat d'ander lini van haer wycken sal.

T'BESLVYT. Soo d'eene dan der twee linien, &c.

XVIII. VERTOOCH. XXVII. VOORSTEL.

HANGHENDE een pilaer eueſtaltwichtich teghen twee ſcheefheſwichten: Ghelijck ſcheefheſlini tot rechtheſlini, alſoo elck ſcheefheſwicht tot ſijn rechtheſwicht.

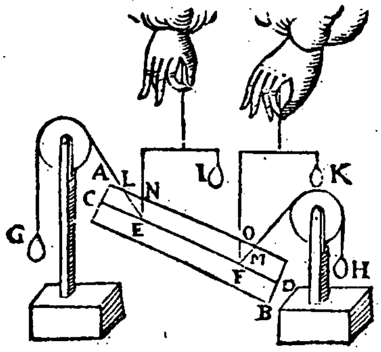
T'GHEGHEVEN. Laet A B een pilaer sijn wiens as C D, ende twee punten daer in E, F, welcker scheefhefswichten die hem in die standt houden sijn G, H, ende rechthefswichten I, K, ende scheefheflinien E L, F M, ende rechtheflinien E N, F O. T'BEGHEERDE. Wy moeten bewysen dat ghelijck L E tot E N, alsoo G tot I, ende ghelijck M F tot F O, alsoo H tot K. T'BEWYS. Laet ons F ansien voor vastpunt, ende E voor t'roerlick, daerom (door het 20e voorstel) ghelijck L E tot E N, those two lines, the prism will remain hanging thereon in its given position, and *IC* will be the centre line of gravity; but this is impossible, because it is parallel to *GH*. The same can also be shown in this way if the line *EF* verges towards the other side. Therefore, *IC* being at right angles to the horizon, the other line, as *EF*, cannot be at oblique angles thereto, so that necessarily it is at right angles thereto. And consequently, if *EF* is at oblique angles to the horizon, the other must also be at oblique angles thereto.

Further, since *EF* verges towards the side of *A*, the line keeping the prism in that position will have to verge towards *EF*. For let the said line (if this were possible) verge away therefrom, as *CK*, meeting *EI* produced in *K*; then the vertical through *K* will, for the reasons mentioned above, be the centre line of gravity of the prism, which is even more absurd than saying that it passes through *I*. The other line, therefore, which can keep the prism in that position, does not verge away from *EF*, nor is it parallel thereto, as proved above, and it is evidently impossible for it to verge sidelong; therefore it necessarily verges towards *EF*. And if *EF* verged towards the other side, it can be shown similarly that the other line will verge away therefrom. CONCLUSION. If therefore one of two lines, etc.

THEOREM XVIII. PROPOSITION XXVII.

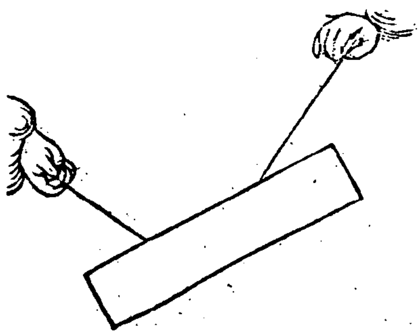
If a prism is hanging in equality of apparent weight with two oblique lifting weights: as the oblique lifting line is to the vertical lifting line, so is also each oblique lifting weight to its vertical lifting weight.

SUPPOSITION. Let *AB* be a prism, its axis *CD*, and two points therein *E*, *F*, whose oblique lifting weights keeping the prism in that position ar *G*, *H*, and the vertical lifting weights *I* and *K*, the oblique lifting lines being *EL*, *FM*, and the vertical lifting lines *EN*, *FO*. WHAT IS REQUIRED TO PROVE. We have



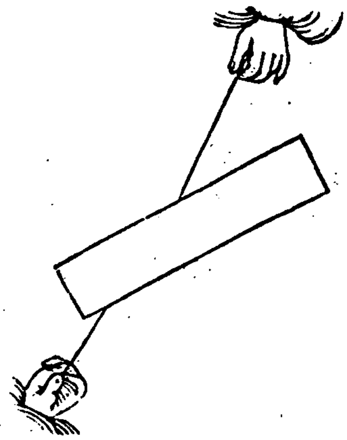
tot E N, alſoo G tot I. Laet ons ten tweeden E anſien voor vaſtpunt, ende F voor t'roerlick; Daerom (door t'voornoemde 20^e^ voorſtel) ghelijck M F tot F O, alſoo H tot K.

T'BESLVYT. Hanghende dan een pilaer eueſtaltwichtich teghen twee ſcheefhefwichten: Ghelijc ſcheefheflini tot rechtheflini, alſoo elck ſcheefhefwicht tot ſijn rechthefwicht, t'welck wy bewyſen moeſten.



VERVOLGH.

HANGHENDE een bekende pilaer an twee oneuewydighe linien als hier neuen; Tblyckt dat bekent ſal worden hoe veel ghewichts an yder lini hangt, ofte hoe veel ghewelts yder lini doet.



MERCKT.

WY hebben tot veel voorbeelden der voorſtellen deſes boucx, ghenomen den pilaer, als bequaemſte form tot de verclaring des voornemens oock vaſtpunt ende roerlickpunt gheſtelt inden as. Wy ſullen nu door dit laetſte voorſtel, bethoonen de reghelen van dien ghemeen te weſen ouer alle formen der lichamen bodanich ſy ſijn met vaſtpunt ende roerlickpunt daert valt.

H 3 XIX. VERto prove that as *LE* is to *EN*, so is *G* to *I*, and as *MF* is to *FO*, so is *H* to *K*. PROOF. Let us consider *F* to be the fixed point and *E* the movable point; therefore (by the 20th proposition), as *LE* is to *EN*, so is *G* to *I*. Let us secondly consider *E* to be the fixed point, and *F* the movable point. Therefore (by the aforesaid 20th proposition), as *MF* is to *FO*, so is *H* to *K*. CONCLUSION. If therefore a prism is hanging in equality of apparent weight with two oblique lifting weights: as the oblique lifting line is to the vertical lifting line, so is each oblique lifting weight to its vertical lifting weight, which we had to prove.

COROLLARY.

If a known prism is hanging from two non-parallel lines, as shown in the annexed figure, it appears that it will become known what weight is hanging from either line, or what force either line exerts.

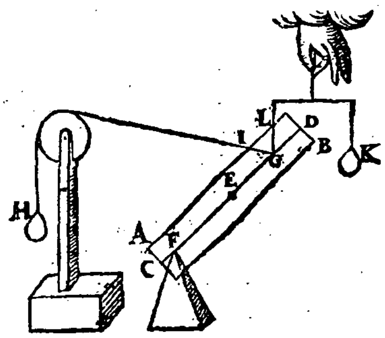
NOTE.

For many examples of the propositions of this book we have taken the prism, as being the most suitable form for explaining the meaning, and we have also put the fixed and the movable point in the axis. By the last proposition we shall now show that the respective rules are true generally of solids of whatever form, with arbitrary fixed and movable points.

Proportiones.

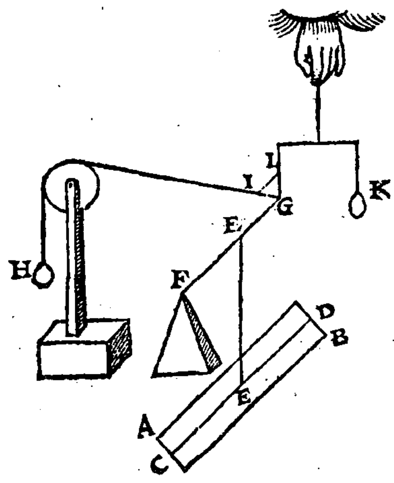
ALLE de *eueredenheden, welcke hier vooren beschreuen sijn vanden pilaer tot de ghewichten an hem hanghende, ende dier ghewichten linien: De selue te wesen van yder lichaem tot de ghewichten an hem alsoo hanghende, ende dier ghewichten linien.

T'GHEGHEVEN. Laet ons t'voorbeelt nemen der eueredenheydt des 20e voorstels aldus: Het sy een pilaer A B, diens as C D, ende swaerheyts middelpunt E, ende vastpunt daer in F, ende roerlick punt G, an t'welc gheuoucht sy een scheefhefwicht H, dat den pilaer in die ghestalt houde, dies scheefheflini G I. Daer naer trechthefwicht K, dat den pilaer oock in die ghestalt houde, diens rechtheflini G L, alwaer wy segghen, ghelijck I G tot G L, also H tot K. T'BEGHEERDE. Wy moeten bewysen dat dese eueredenheydt niet alleenlick alsoen bestaet in t'lichaem A B een pilaer sijnde, maer van sulcke form alst valt.



TBEWYS. Laet ons den pilaer A B (blijuende de linien F G ende I L op haer plaetsen) neertrecken, alsoo dat hy bliue hanghende an sijn * swaerheyts middelpunt E, wiens ghestalt dan sy als hier neuens. Ende door de 3e begheerte den pilaer en veroirsaeckt op de punten F, G, gheen ander swaerheydt dan d'eerste; ende alles blijft noch eueſtaltwichtich, ende ghelijck I G tot G L, alsoo noch H tot K.

Centrum grauitatis.

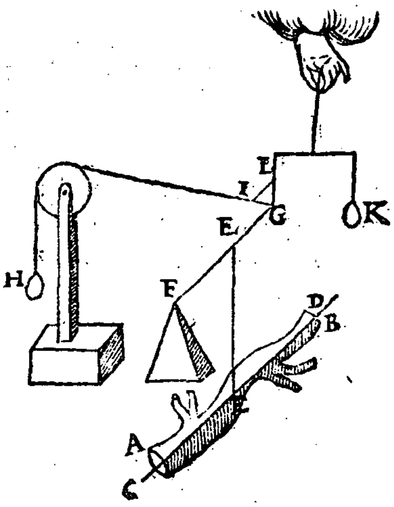


THEOREM XIX. PROPOSITION XXVIII.

Any proportions discussed above of the prism in relation to the weights hanging therefrom, and the lines of such weights, are true of any solid in relation to the weights thus hanging therefrom, and the lines of such weights.

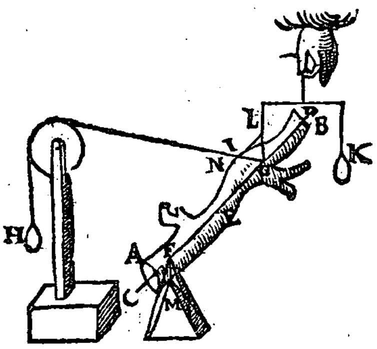
SUPPOSITION. Let us take as example the proportion of the 20th proposition, as follows. Let there be a prism *AB*, its axis *CD*, and the centre of gravity *E*, and the fixed point therein *F* and the movable point *G*, at which let there be attached an oblique lifting weight *H*, which shall keep the prism in that position, its oblique lifting line being *GI*. Then let there be the vertical lifting weight *K*, which shall also keep the prism in that position, its vertical lifting line being *GL*. We then say: as *IG* is to *GL*, so is *H* to *K*. WHAT IS REQUIRED TO PROVE. We have to prove that this proportion is true not only when the solid *AB* is a prism, but of any form whatever. PROOF. Let us pull down the prism *AB* (the lines *FG* and *IL* remaining in their places) in such a way that it shall remain hanging at its centre of gravity *E*, the situation then being as shown in the annexed figure. Then by the 3rd postulate the prism does not cause any other gravity on the points *F*, *G* than the first, and the whole still remains in equality of apparent weight; and as *IG* is to *GL*, so is *H* to *K*.

LAET nu de form des pilaers (al de stoff bliuende) verandert worden in eenighe ander ongheschickte form, als A B hier neuens, diens swaerheydts middelpunt E sy, ende een rechte lini daer deur C D (welcke vinding des swaerheyts middelpunts ende rechter linien inde Weeghdaet verclaert sal worden,* werckelick, niet Wisconstelick) ende alles blijft noch eueſtaltwichtich, ende ghelijck I G tot G L, alsoo noch H tot K.



Mechanicè non Mathematicè.

LAET nu t'lichaem A B opghetrocken worden, tot dat F G is inde lini C D, wiens ghestalt dan sy als hier neuens, ende alles blijft noch eueſtaltwichtich: want het lichaem A B hoogher ofte leegher hanghende, blijft van een selfde ghewicht door de 3e begheerte, ende veruolghens ghelijck I G tot G L, alsoo noch H tot K. De eueredenheydt dan des 20e voorstels en is niet alleenelick alsoo met den pilaer, maer met yder lichaem: Ende der ghelijcke salmen oock alsoo bethoonen van al t'ghene hier vooren in alle d'ander voorstellen vanden pilaer gheseyt is.



T BESLVYT. Alle de eueredenheden dan, welcke hier vooren bescreuen sijn vanden pilaer tot de ghewichten an hem hanghende, ende dier ghewichten linien; de selue sijn van yder lichaem tot de ghewichten an hem alsoo hanghende, ende dier ghewichten linien, t'welck wy bewysen moesten.

VERVOLGH.

Now let the form of the prism (all the material remaining) be changed into some other — irregular — form, as AB in the annexed figure, whose centre of gravity shall be E, and let there be drawn a straight line through it, as CD (which determination of the centre of gravity and of the straight lines will be explained in the Practice of Weighing [1]), not mathematically, but mechanically); then the whole still remains in equality of apparent weight, and as IG is to GL, so is H to K.

Now let the solid AB be pulled up until FG is in the line CD, the position of which shall then be as shown in the annexed figure. The whole still remains in equality of apparent weight, for the solid AB, no matter whether it hangs higher or lower, keeps the same weight, by the 3rd postulate, and consequently as IG is to GL, so is H to K. The proportion therefore of the 20th proposition is true not only of the prism, but of any solid. And the same can also be shown of all that has been said before of the prism in all the other propositions. CONCLUSION. Any proportions, therefore, discussed above of the prism in relation to the weights hanging therefrom, and the lines of such weights, are true of any solid in relation to the weights thus hanging therefrom, and the lines of such weights, which we had to prove.

[1]) See *The Practice of Weighing*, Prop. 1.

VERVOLGH.

TIS oock openbaer dat de ghegheuen punten als F, G, nietnootsaeckelick en moeten inde lini CD sijn, maer daert valt. by voorbeelt ande uyterften des lichaems M, N, Want voortghetrocken de lini I N tot inde rechte CD, t'welck ick neem te vallen in G, sghelijcx ghetrocken door M de * hanghende tot inde lini C D, welcke ick neem te vallen in F, de voornoemde eueredenheydt, te weten ghelijck I G tot G L, alsoo H tot K, bliift noch staende.

Perpendiculari.

EINDE DES EERSTEN BOVCX.

COROLLARY.

It is also manifest that the given points, as *F*, *G*, need not be in the line *CD*, but may be in any place, for example at the extremities of the solid, *M*, *N*. For the line *IN* being produced to meet the straight line *CD*, which I take to be in *G*, and likewise the vertical through *M* being drawn to meet the line *CD*, which I take to be in *F*, the aforesaid proportion, viz. as *IG* is to *GL*, so is *H* to *K*, is still true.

END OF THE FIRST BOOK.

HET TWEEDE BOVCK VANDE *BEGHINSELEN DER WEEGHCONST, DWELCK IS VANDE VINDING DER SWAERHEYDTS MIDDELPVNTEN,

Elementis.

Beſchreuen door Simon Steuin.

WY hebben in t'eerſte bouck tot het beſchriuen der wichtighe ghedaenten, ghenomen een pilaer (voldoende aldaer het voornemen) diens ſwaerheyts middelpunt door ghemeene wetenſchap bekent is, maer in veel ander lichamen en ghebueret niet alſo; wel is waer dattet door een corte ghemeene reghel in allen werckelick te vinden is, *Praxis.* ſo door t'eerſte voorſtel der * Weeghdaet blijcken ſal, maer met de* Wiſconſtighe vinding iſt anders gheſtelt; Daer af heeft eerſt gheſchreuen *Mathematica.* Archimedes in platten, ende naer hem Frederic Commandin in lichamen: Wy ſullen tottet een en t'ander (ouermits het een* afcoemſt van *Species.* beghinſelen is, byde voorgaende wel dienende, ende tottet volghende, ſo wel WATERWICHT, als WEEGHDAET, ſeer noodich) het onſe voughen, ende alles naer onſe oirden verſpreyden, daer af beſchrijuende der Beghinſelen tweede bouck.

Wat de * bepalinghen belangt vande Meetconſtighe formen, die by- *Definitiones.* gheualle hier yemandt begheeren mocht, wy nemen die * door t'gheſtel- *Per hypotheſin.* de voor bekent uyt de * Meetconſt; Alleenelick dit daer af ſegghende, *Geometria.* dat wy t'woort *Parabola*, ofte *Rectanguli coni ſectio*, beteeckenen met *Brantſne*: Ende *Conoidale Rectangulum*, met *Brander*; Reden, dat dier formen * daet voornamelicxt beſtaet int ontſteken ofte branden. *Effectus.*

EERST VANDE VINDING DER SWAERHEYTS MIDDELPVNTEN VANDE *PLATTEN.

Planis.

BY *aldien de platten eenich ghewicht hadden, ende datmen toeliete die te weſen inde reden haerder grootheden, wy ſouden eyghentlick mueghen ſpreken van haer Swaerheydt, Swaerheyts middelpunt, Swaerheyts middellini, &c. Maer*

I *anghesien*

THE SECOND BOOK

OF THE ELEMENTS OF THE ART OF WEIGHING, WHICH DEALS WITH THE FINDING OF THE CENTRES OF GRAVITY, Described by Simon Stevin

In the first book we have, for the description of the qualities of weights, taken a prism (which in that case was satisfactory for our purposes), whose centre of gravity is known by common knowledge, but in many other solids it does not so happen; it is true indeed that it can be found constructionally in all forms by means of a short common rule, as will become apparent from the first proposition of the Practice of Weighing, but with the mathematical finding it is a different matter. The first to write about this was Archimedes [1]), viz. about plane figures, and after him Frederick Commandinus [2]), about solids. We will add our own observations to both (since the subject forms a kind of elements, which have been useful for what precedes and will be highly necessary for what follows: Hydrostatics as well as the Practice of Weighing), and arrange the subject matter according to our own method, thus describing the second book of the Elements.

As to the definitions of the geometrical figures which anyone might require in this place, we assume these to be known by hypothesis from geometry, merely stating that we denote the word *Parabola,* or *Rectanguli coni sectio* [3]), by *"Brantsne"* [4]), and *Conoidale Rectangulum* [5]) by *"Brander* [6]), because the effect of these figures chiefly consists in igniting or burning.

FIRST ABOUT THE FINDING OF THE CENTRES OF GRAVITY OF PLANE FIGURES

If plane figures had any weight, and these were admitted to be proportional to their magnitudes, we might properly speak of their gravity, centre of gravity, centre line of gravity, etc., but since a plane figure has no weight, properly speak-

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[1]) Archimedes, *De Planorum Equilibriis* Libri II. Opera Omnia, ed. J. L. Heiberg, Vol. II. Leipzig, 1913, 122-213.

[2]) Federici Commandini Urbinatis *Liber de centro gravitatis solidorum.* Bononiae, 1555.

[3]) This is the current term for parabola in older Greek geometry; the curve was generated by cutting a cone of revolution the vertical angle of which is a right angle by a plane perpendicular to a generating line.

[4]) This is one of Stevin's neologisms, for which there is no English equivalent. The meaning of „brantsne" might be rendered by „burning section".

[5]) Conoidale rectangulum (ὀρθογώνιον κωνοειδές) is the Archimedean term for a paraboloid of revolution.

[6]) Another neologism. The English equivalent is „burner".

anghesien in t'plat gheen ghewicht en is, soo en isser eyghentlick sprekende gheen Swaerheydt, Swaerheydts middelpunt, noch Swaerheyts middellini in; Daerom moetmen dit alles lijckspreucklick verstaen, ende nemen als door t'ghestelde, dat der platten ghewichten inde reden haerder grootheden sijn, want* T'valsche wort toeghelaten, op datmen t'waerachtighe daer duer leere.

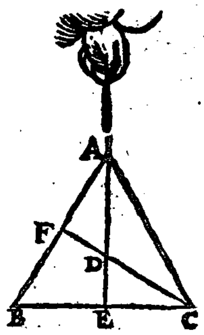
* *Metaphoricè.*

I. VERTOOCH. I. VOORSTEL.

YDER plats middelpunt der form, is oock sijn swaerheyts middelpunt.

I^e^ VOORBEELT.

T'GHEGHEVEN. Laet A B C een euesijdich driehouck wesen, diens formens middelpunt sy D. T'BEGHEERDE. Wy moeten bewysen dat D oock het swaerheyts middelpunt is des driehoucx A B C. T'BEREYTSEL. Laet ghetrocken worden van A tot int middel van B C, de lini A E, sghelijcx van C tot int middel van A B, de lini C F. T'BEWYS. Wesende de driehouck A B C opghehanghen byde lini A E, het deel A E C sal euewichtich hanghen teghen A E B, want sy sijn euen groot, ghelijck, ende van ghelijcker ghestalt; A E dan is swaerheyts middellini des driehoucx A B C, Ende om de selue reden sal F C oock des driehoucx swaerheyts middellini sijn, maer dese snien malcanderen in des formens middelpunt D, ende elck dier linien heeft in haer het swaerheyts middelpunt, tis dan D.



II^e^ VOORBEELT.

T'GHEGHEVEN. Laet A B C D een euewydich vierhouck sijn, diens formens middelpunt E. T'BEGHEERDE. Wy moeten bewysen dat E oock het swaerheyts middelpunt is. T'BEREYTSEL. Laet ghetrocken worden F G, tusschen de middelpunten van A D ende B C, insghelijcx H I, tusschen de middelpunten van A B ende D C.

T'BEWYS. Wesende den vierhouck opghehanghen byde lini H I, Het deel H I D A sal euewichtich hanghen teghen H I C B, want sy sijn euegroot ghelijck ende van ghelijcker ghestalt; H I dan is swaerheyts *ing there is no gravity, centre of gravity, nor centre line of gravity therein* [1]. *Therefore all this has to be understood metaphorically, and it has to be assumed by hypothesis that the weights of plane figures are proportional to their magnitudes, for:*

THE FALSE IS ADMITTED IN ORDER THAT THE TRUE MAY BE LEARNED THEREFROM.

THEOREM I. PROPOSITION I.

The geometrical centre of any plane figure is also its centre of gravity.

EXAMPLE I.

SUPPOSITION. Let *ABC* be an equilateral triangle, whose geometrical centre shall be *D*. WHAT IS REQUIRED TO PROVE. We have to prove that *D* is also the centre of gravity of the triangle *ABC*. PRELIMINARY. Let there be drawn from *A* to the middle point of *BC* the line *AE*; likewise from *C* to the middle point of *AB* the line *CF*. PROOF. The triangle *ABC* being suspended by the line *AE*, the part *AEC* will balance [2] *AEB*, for they are equally large, similar, and of the same form. Therefore *AE* is centre line of gravity [3] of the triangle *ABC*, and for the same reason *FC* will also be centre line of gravity of the triangle. But these lines intersect in the geometrical centre *D*, and each of these lines contains the centre of gravity; therefore the latter is *D*.

EXAMPLE II.

SUPPOSITION. Let *ABCD* be a parallelogram, whose geometrical centre shall be *E*. WHAT IS REQUIRED TO PROVE. We have to prove that *E* is also the centre of gravity. PRELIMINARY. Let *FG* be drawn, joining the middle points of *AD* and *BC*, and likewise *HI* [4], joining the middle points of *AB* and *DC*. PROOF. The quadrilateral being suspended by the line *HI*, the part *HIDA* will balance *HICB*, for they are equally large, similar, and of the same form. *HI* is

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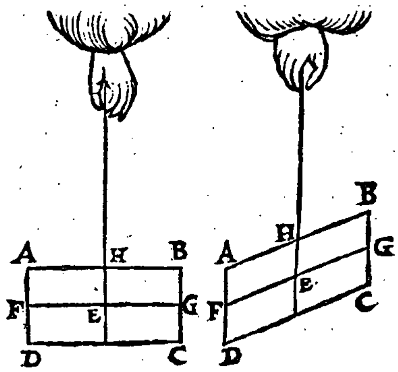
[1]) It is highly remarkable that this observation is made with regard to plane figures only, and not for solids as well, as if the latter did have weight, etc.

[2]) It is to be noted that here as elsewhere Stevin uses the term „*evewichtich*" (of equal weight) instead of „*evestaltwichtich*" (of equal apparent weight), as might have been expected.

[3]) As we remarked in our notes to Definition 5 and Proposition 6 of Book I, the term „centre line of gravity" is usually taken to mean the vertical through the point of suspension of the figure at rest. It is then assumed „by a common rule of Statics" that the centre of gravity is in the so defined centre line of gravity. This remark applies to the whole of Book II.

[4]) The letter I denoting the middle point of *DC* is lacking in the drawings.

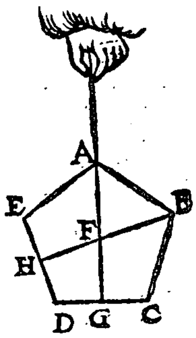
heyts middellini des vierhoucx A B C D, Ende om de ſelue reden ſal F G oock des vierhoucx ſwaerheyts middellini ſijn, maer deſe doorſnien malcanderen in E, ende elck dier linien heeft in haer het ſwaerheyts middelpunt, tis dan E.



III^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D een gheſchickt ofte inſchriuelick vijfhouck weſen, diens formens middelpunt F ſy. T'BEGHEERDE. Wy moeten bewyſen dat F oock het ſwaerheyts middelpunt is.

T'BEREYTSEL. Laet ghetrocken worden van A tot int middel van D C, de lini A G; ſghelijcx van B tot int middel van E D, de lini B H. T'BEWYS. Weſende den vijfhouck opghehanghen byde lini A G, het deel A G D E ſal euewichtich hanghen teghen het deel A G C B, want ſy ſijn euegroot, ghelijck, ende van ghelijcker ghe- ſtalt: A G dan is ſwaerheyts middellini des vijfhoucx, ende om de ſelue reden ſal B H oock des ſelfden vijfhoucx ſwaerheyts middellini weſen; maer deſe doorſnien malcanderen in des formens middelpunt F, ende elck dier linien heeft in haer het ſwaerheyts middelpunt, tis dan F. Sghelijcx ſal oock t'bewys ſijn in allen anderen hebbende een formens middelpunt als Seſhoucken, Ronden, Scheefronden, &c.



T'BESLVYT. Yder plats middelpunt der form dan, is oock ſijn ſwaerheyts middelpunt, t'welck wy bewyſen moeſten.

II. VERTOOCH. II. VOORSTEL.

YDER driehoucx ſwaerheydts middelpunt, is inde lini ghetrocken vanden houck tot int middel der ſijde.

T'GHEGHEVEN. Laet A B C een driehouck ſijn van form ſoot then centre line of gravity of the quadrilateral *ABCD*, and for the same reason *FG* will also be centre line of gravity of the quadrilateral. But these lines intersect in *E*, and each of these lines contains the centre of gravity. Therefore the latter is *E*.

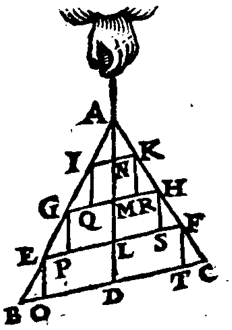
EXAMPLE III.

SUPPOSITION. Let *ABCDE* be a regular or inscribed pentagon, whose geometrical centre shall be *F*. WHAT IS REQUIRED TO PROVE. We have to prove that *F* is also the centre of gravity. PRELIMINARY. Let there be drawn from *A* to the middle point of *DC* the line *AG*; likewise from *B* to the middle point of *ED* the line *BH*. PROOF. The pentagon being suspended by the line *AG*, the part *AGDE* will balance the part *AGCB*, for they are equally large, similar, and of the same form. *AG* is therefore centre line of gravity of the pentagon, and for the same reason *BH* will also be centre line of gravity of the same pentagon. But these lines intersect in the geometrical centre *F*, and each of these lines contains the centre of gravity; therefore the latter is *F*. The same proof holds for all other figures having a geometrical centre, such as hexagons, circles, ellipses, etc. CONCLUSION. The geometrical centre of any plane figure therefore is also its centre of gravity, which we had to prove.

THEOREM II. PROPOSITION II.

The centre of gravity of any triangle is in the line drawn from the angle to the middle point of the side.

SUPPOSITION. Let *ABC* be a triangle of any form, in which from the angle valt, waer in vanden houck A tot in D middel vande ſijde B C, ghetrocken is de lini A D. T'BEGHEERDE. Wy moeten bewyſen dat des driehoucx ſwaerheyts middelpunt inde lini A D is. T'BEREYTSEL. Laet ons trecken E F, G H, I K, euewydighe van B C, ſniende A D in L, M, N, daer naer E O, G P, I Q, K R, H S, F T, euewydighe met A D.



T'BEWYS. Ouermits E F euewydighe is van B C, ende E O, F T met L D, ſoo ſal E F T O, euewydich vierhouck ſijn, wiens E L euen is met L F, oock met O D ende D T, waer deur het ſwaerheyts middelpunt des vierhoucx E F T O in D L is, door het 1e voorſtel deſes boucx. Ende om de ſelue reden ſal het ſwaerheyts middelpunt des euewydichs vierhoucx G H S P weſen in L M, ende van I K R Q in M N, ende vervolghens het ſwaerheyts middelpunt der form I K R H S F T O E P G Q ghemaeckt vande voornoemde drie vierhoucken, ſal weſen inde lini N D, ofte A D. Nu ghelijck hier in beſchreuen ſijn drie vierhoucken, alſo canmender oneindelicke ſulcke vierhoucken in beſchrijuen, ende des binneſchreuens formens ſwaerheyts middelpunt, ſal altijt ſijn (om de redenen als vooren) inde lini A D. Maer hoe datter ſulcke vierhoucken meer ſijn, hoe dat den driehouck A B C min verſchilt vande binneſchreuen form der vierhoucken; want treckende linien euewydich van B C door de middelen van A N, N M, M L, L D, t'verſchil des laetſten gheſtalts, ſal effen den helft ſijn van t'verſchil des voorgaenden gheſtalts. Wy connen dan door dat oneindelick naerderen ſulck een form binnen den driehouck ſtellen, dattet verſchil tuſſchen haer ende den driehouck, minder ſal weſen dan eenich ghegheuen plat hoe cleen het ſy: Waer uyt volght, dat ſtellende A D als ſwaerheydts middellini, ſo ſal t'ſtaltwicht des deels A D C, min verſchillen van t'ſtaltwicht des deels A D B, dan eenich plat datmen ſoude connen gheuen hoe cleen het ſy, waer uyt ick aldus ſtrie.

A. *Neuen alle verſchillende ſtaltſwaerheden, can een ſwaerheydt gheſtelt worden minder dan haer verſchil;*

O. *Neuen deſe ſtaltſwaerheden A D C ende A D B, en can gheen ſwaerheydt gheſtelt worden minder dan haer verſchil;*

O. *Deſe ſtaltſwaerheden dan A D C ende A D B en verſchillen niet.*

Daerom A D is ſwaerheyts middellini, ende vervolghens t'ſwaerheyts middelpunt des driehoucx A B C is in haer. T'BESLVYT. Yder driehoucx ſwaerheydts middelpunt dan is inde lini ghetrocken vanden houck tot int middel der ſijde, t'welck wy bewyſen moeſten.

1 EYSCH.

A to *D*, the middle point of the side *BC*, there is drawn the line *AD*. WHAT is REQUIRED TO PROVE. We have to prove that the centre of gravity of the triangle is in the line *AD*. PRELIMINARY. Let us draw *EF*, *GH*, *IK* parallel to *BC*, intersecting *AD* in *L*, *M*, *N*; after that *EO*, *GP*, *IQ*, *KR*, *HS*, *FT*, parallel to *AD*. PROOF Since *EF* is parallel to *BC*, and *EO*, *FT* to *LD*, *EFTO* will be a parallelogram, in which *EL* is equal to *LF*, also to *OD* and *DT*, in consequence of which the centre of gravity of the quadrilateral *EFTO* is in *DL*, by the 1st proposition of this book. And for the same reason the centre of gravity of the parallelogram *GHSP* will be in *LM*, and of *IKRQ* in *MN*; and consequently the centre of gravity of the figure *IKRHSFTOEPGQ*, composed of the aforesaid three quadrilaterals, will be in the line *ND* or *AD*. Now as here three quadrilaterals have been inscribed in the triangle, so an infinite number of such quadrilaterals can be inscribed therein, and the centre of gravity of the inscribed figure will always be (for the reasons mentioned above) in the line *AD*. But the more such quadrilaterals there are, the less the triangle *ABC* will differ from the inscribed figure of the quadrilaterals. For if we draw lines parallel to *BC* through the middle point of *AN*, *NM*, *ML*, *LD*, the difference of the last figure will be exactly half of the difference of the preceding figure [1]). We can therefore, by infinite approximation, place within the triangle a figure such that the difference between the latter and the triangle shall be less than any given plane figure [2]), however small. From which it follows that, taking *AD* to be centre line of gravity [3]), the apparent weight of the part *ADC* will differ less from the apparent weight of the part *ADB* than any plane figure that might be given, however small, from which I argue as follows [4]):

A. *Beside any different apparent gravities there may be placed a gravity less than their difference;*

O. *Beside the present apparent gravities ADC and ADB there cannot be placed any gravity less than their difference;*

O. *Therefore the present apparent gravities ADC and ADB do not differ.*

Therefore *AD* is centre line of gravity, and consequently the centre of gravity of the triangle *ABC* is in it. CONCLUSION. The centre of gravity of any triangle therefore is in the line drawn from the angle to the middle point of the side, which we had to prove.

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[1]) It is obviously assumed that the side *AB* is divided into *n* equal segments (in the drawing $n = 4$). The difference between the area Δ of the triangle *ABC* and that of the figure Π_n consisting of $(n - 1)$ parallelograms is $\frac{\Delta}{n}$.

[2]) Euclid X 1; porism.

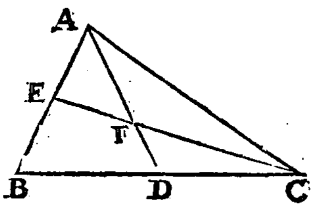
[3]) As in Prop. 6 of Book I, the term „centre line of gravity" cannot here be meant in the sense attributed to it by Defin. 5 of Book I (vertical through the centre of gravity). It is to be proved that the centre of gravity is in the line *AD*, and it would be begging the question to suppose *AD* to be centre line of gravity. Stevin's meaning may be rendered as follows: Suppose *AD* to be held in the vertical of *A*. It is then proved that the „staltwichten" of the triangles *ADB* and *ADC* relatively to *AD* are equal to one another. If now *AD* is released, it will remain in the vertical. Hence the conclusion: *AD* is the centre line of gravity (in the sense of vertical through the point of suspension of the solid in rest), and hence (by the common rule of statics, quoted in Prop. 6 of Book I) the centre of gravity is in *AD*.

[4]) See note 2 to p. 143.

I. EYSCH. III. VOORSTEL.

WESENDE ghegheuen een driehouck: Sijn ſwaerheyts middelpunt te vinden.

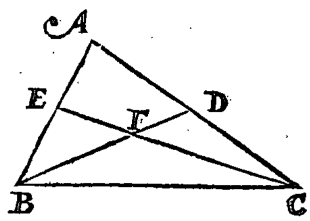
T'GHEGHEVEN. Laet A B C een driehouck weſen. T'BEGHEERDE. Wy moeten ſijn ſwaerheytds middelpunt vinden. TWERCK. Men ſal van A tot int middel van B C, trecken de lini A D, ſghelijcx van C tot int middel van A B, de lini C E, ſniende A D in F: Ick ſeg dat F t'begheerde ſwaerheydts middelpunt is. T'BEWYS. T'ſwaerheyts middelpunt des driehoucx A B C, is inde lini A D, ende oock in C E, duer het 2[e] voorſtel, tis dan F, t'welck wy bewyſen moeſten. T'BESLVYT. Weſende dan ghegheuen een driehouck: Wy hebben ſijn ſwaerheydts middelpunt gheuonden naer den eyſch.



III. VERTOOCH. IIII. VOORSTEL.

HET ſwaerheyts middelpunt eens driehoucx deelt de lini vanden houck tot int middel der ſijde alſoo, dattet ſtick naer den houck, dobbel is an t'ander.

T'GHEGHEVEN. Laet A B C een driehouck ſijn, ende vanden houck B een lini ghetrocken worden tot D int middel van A C, ſghelicx van C een lini tot E int middel van A B, ſniende B D in F voor ſwaerheyts middelpunt des driehoucx A B C. T'BEGHEERDE. Wy moeten bewyſen dat C F dobbel is an F E. T'BEWYS. Ghetrocken de reden E B 1 tot B A 2, vande reden C D 1 tot D A 1 (dat is *Reden* $\frac{1}{2}$ van *Reden* $\frac{1}{1}$) * daer reſt de reden van C F tot F E, maer treckende *Reden* $\frac{1}{2}$ van *Reden* $\frac{1}{1}$ daer blijft *Reden* $\frac{2}{1}$. C F dan is tot F E, als van 2 tot 1.



Door t'verkeerde des 12 cap. 1 lib. Almag. Ptolem.

T'BESLVYT. Het ſwaerheyts middelpunt dan eens driehoucx deelt de lini vanden houck tot int middel der ſijde alſoo, dattet ſtick naer den houck dobbel is an t'ander, t'welck wy bewyſen moeſten.

PROBLEM I. PROPOSITION III.

Given a triangle: to find its centre of gravity.

SUPPOSITION. Let *ABC* be a triangle. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. There shall be drawn from *A* to the middle point of *BC* the line *AD*, likewise from *C* to the middle point of *AB* the line *CE*, intersecting *AD* in *F*. I say that *F* is the required centre of gravity. PROOF. The centre of gravity of the triangle *ABC* is in the line *AD*, and also in *CE*, by the 2nd proposition. It is therefore *F*, which we had to prove. CONCLUSION. Given therefore a triangle, we have found its centre of gravity, as required.

THEOREM III. PROPOSITION IV.

The centre of gravity of a triangle divides the line from the angle to the middle point of the side in such a way that the segment adjacent to the angle is double of the other.

SUPPOSITION. Let *ABC* be a triangle, and let there be drawn from the angle *B* a line to *D* in the middle of *AC*, likewise from *C* a line to *E* in the middle of *AB*, intersecting *BD* in *F*, the centre of gravity of the triangle *ABC*. WHAT IS REQUIRED TO PROVE. We have to prove that *CF* is double of *FE*. PROOF [1]). The ratio of *EB* (1) to *BA* (2) being subtracted from the ratio of *CD* (1) to *DA* (1) (i.e. ratio $\frac{1}{2}$ from ratio $\frac{1}{1}$), there remains the ratio of *CF* to *FE*, but the ratio $\frac{1}{2}$ being subtracted from the ratio $\frac{1}{1}$, there remains the ratio $\frac{2}{1}$. Therefore *CF* is to *FE* as 2 to 1. CONCLUSION. The centre of gravity of a triangle therefore divides the line from the angle to the middle point of the side in such a way that the segment adjacent to the angle is double of the other, which we had to prove.

[1]) In the margin Stevin quotes the converse of Ptolemy, *Almagest* I 12. In the modern edition of the work by Heiberg (Claudii Ptolemaei *Opera quae exstant omnia*, Vol. I, *Syntaxis Mathematica* Pars I, Leipzig, 1898) this is I 13, which begins with Menelaus's Theorem. Applying this theorem to $\triangle$ *ACE* with transversal *DFB*, we obtain:

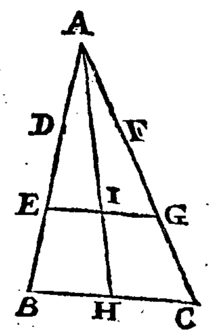
$$\frac{DA \cdot FC \cdot BE}{DC \cdot FE \cdot BA} 1 \text{ hence } \frac{CF}{FE} = \frac{\frac{DC}{DA}}{\frac{BE}{BA}} = \frac{\frac{1}{1}}{\frac{1}{2}} = 2.$$

According to the ancient terminology, which was still current in Stevin's time, the division of the ratio $DC : DA$ by the ratio $BE : BA$ was called subtraction. In Greek mathematics the theorem was enunciated in the form $\frac{DC}{DA} = \frac{CF \cdot BE}{FE \cdot BA}$. This explains perhaps why Stevin, when calculating $\frac{CF}{FE}$, refers to the converse of the theorem, the term „converse" not being taken in its ordinary sense.

IIII. VERTOOCH. V. VOORSTEL.

WESENDE twee ſijden eens driehoucx elck ghedeelt in drie euen deelen: De lini tuſſchen de twee punten der deeling naeſt de derde ſijde, ſtreckt door des driehoucx ſwaerheyts middelpunt.

T'GHEGHEVEN. Laet A B C een driehouck weſen, van t'welck yder ſijde A B ende B C ghedeelt ſy in drie euen deelen, met de punten D, E, F, G, ende tuſſchen de punten E, G, naeſt de derde ſijde B C, ſy ghetrocken de lini E G. T'BEGHEERDE. Wy moeten bewyſen dat E G duer des driehoucx A B C ſwaerheyts middelpunt ſtreckt.



T'BEREYTSEL. Laet ons trecken van A tot int middel van B C, de lini A H, ſniende E G in I. T'BEWYS. Ouermits A E ſulcken reden heeft tot E B, als A G tot G C, ſoo is E G euewydighe met B C, ende veruolghens E I is euewydighe met B H, daerom ghelijck A E tot E B, alſoo A I tot I H, maer A E is dobbel tot E B door t'ghegheuen, daerom A I is dobbel tot I H, maer weſende A I dobbel tot I H, ſoo is I t'ſwaerheyts middelpunt des driehoucx A B C door het 4ᵉ voorſtel, daerom E G ſtreckt door des ghegheuen driehoucx ſwaerheyts middelpunt. T'BESLVYT. Weſende dan twee ſyden eens driehoucx elck ghedeelt in drie euen deelen, de lini tuſſchen de twee punten der deeling naeſt de derde ſyde, ſtreckt door des driehoucx ſwaerheyts middelpunt, t'welck wy bewyſen moeſten.

2.v.6.B.E.

II. EYSCH. VI. VOORSTEL.

WESENDE ghegheuen een *rechtlinich plat: Sijn ſwaerheyts middelpunt te vinden.

Planum rectilineum.

Iᵉ VOORBEELT.

T'GHEGHEVEN. Laet A B C D een ongheſchict vierhouck weſen. T'BEGHEERDE. Wy moeten ſijn ſwaerheyts middelpunt vinden.

T'WERCK. Men ſal den vierhouck deelen in twee driehoucken met de lini A C, ende vinden het ſwaerheyts middelpunt van elck driehouck, duer het 3ᵉ voorſtel, dat van A C B ſy E, ende van A C D ſy F, ende

THEOREM IV. PROPOSITION V.

Two sides of a triangle each being divided into three equal segments, the line joining the two points of division adjacent to the third side passes through the centre of gravity of the triangle.

SUPPOSITION. Let *ABC* be a triangle, of which each of the sides *AB* and *AC* be divided into three equal segments by the points *D*, *E*, *F*, *G*, and between the points *E*, *G*, adjacent to the third side *BC*, let there be drawn the line *EG*. WHAT IS REQUIRED TO PROVE. We have to prove that *EG* passes through the centre of gravity of the triangle *ABC*. PRELIMINARY. Let us draw from *A* to the middle point of *BC* the line *AH*, intersecting *EG* in *I*. PROOF. Since *AE* has to *EB* the same ratio as *AG* to *GC*, *EG* is parallel to *BC*, and consequently *EI* is parallel to *BH*; therefore, as *AE* is to *EB*, so is *AI* to *IH*. But *AE* is double of *EB* by the supposition, therefore *AI* is double of *IH*. But *AI* being double of *IH*, *I* is the centre of gravity of the triangle *ABC* by the 4th proposition; therefore *EG* passes through the centre of gravity of the given triangle. CONCLUSION. Two sides of a triangle therefore each being divided into three equal segments, the line joining the two points of division adjacent to the third side passes through the centre of gravity of the triangle, which we had to prove.

PROBLEM II. PROPOSITION VI.

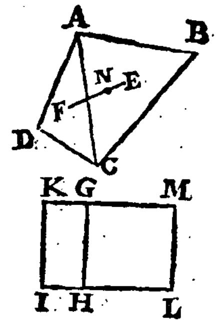
Given a rectilineal plane figure: to find its centre of gravity.

EXAMPLE I.

SUPPOSITION. Let *ABCD* be an irregular quadrilateral. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The quadrilateral shall be divided into two triangles by the line *AC*, and the centre of gravity of each triangle shall be found by the 3rd proposition. That of *ACB* shall be ende de lini E F sal balck wesen. Daer naer salmen maken twee euewydige vierhoucken van een selfde hoogde, als G H I K, euen anden driehouck A C D, ende G H L M, euen anden driehouck A C B, daer naer deelende den balck F E in N, alsoo dat den erm N E, sulcken reden hebbe tot den erm N F, als H I tot H L; Ick seg dat N t'begheerde swaerheyts middelpunt is.

Door het 45. v. 1. B. E.

Door het 10. v. 6. B. E.

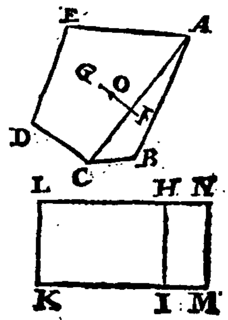


II^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D E een onghesc hickt vijfhouck sijn.

T'BEGHEERDE. Wy moeten sijn swaerheyts middelpunt vinden.

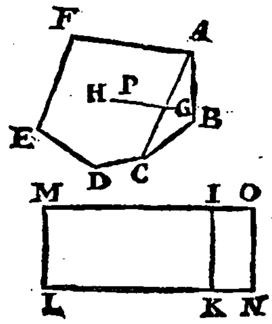
T'WERCK. Men sal trecken A C, ende vinden t'swaerheyts middelpunt des driehoucx A C B door het 3^e voorstel, t'welck F sy, en vande vierhouck A C D E duer t'voorgaende 1^e voorbeelt, t'welck G sy, ende de lini F G sal balck wesen, daer naer salmen maken twee euewydighe vierhoucken van een selfde hoochde, als H I K L euen anden vierhouck A C D E, ende H I M N euen anden driehouck A C B, deelende den balck G F in O, alsoo dat den erm O F, sulcken reden hebbe tot den erm O G, als I K tot I M; Ick seg dat O t'begheerde swaerheyts middelpunt is.



III^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D E F een onghesc hickt seshouck sijn. T'BEGHEERDE. Wy moeten sijn swaerheyts middelpunt vinden.

T'WERCK. Men sal trecken A C, ende vinden t'swaerheydts middelpunt des driehoucx A C B duer het 3^e voorstel, t'welck G sy, ende vanden vijfhouck A C D E F, duer het voorgaende 2^e voorbeelt, t'welck H sy, ende de lini G H sal balck wesen. Daer naer salmen maken twee euewydighe vierhoucken van een selfde hoochde, als I K L M, euen anden vijfhouck A C D E F, ende I K N O



euen

E, and that of *ACD* shall be *F*; then the line *EF* will be beam. After this, there shall be constructed two parallelograms of the same height, as *GHIK* equal to the triangle *ACD*, and *GHLM*, equal to the triangle *ACB*, upon which the beam *FE* shall be divided at *N* in such a way that the arm *NE* shall have to the arm *NF* the same ratio as *HI* to *HL*. I say that *N* is the required centre of gravity.

EXAMPLE II.

SUPPOSITION. Let *ABCDE* be an irregular pentagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The line *AC* shall be drawn, and the centre of gravity of the triangle *ACB* shall be found by the 3rd proposition, which shall be *F*, and that of the quadrilateral *ACDE* by the preceding 1st example, which shall be *G*. Then the line *FG* will be beam. After this, there shall be constructed two parallelograms of the same height, as *HIKL*, equal to the quadrilateral *ACDE*, and *HIMN*, equal to the triangle *ACB*, upon which the beam *GF* shall be divided at *O* in such a way that the arm *OF* shall have to the arm *OG* the same ratio as *IK* to *IM*. I say that *O* is the required centre of gravity.

EXAMPLE III.

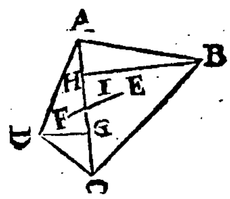
SUPPOSITION. Let *ABCDEF* be an irregular hexagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The line *AC* shall be drawn, and the centre of gravity of the triangle *ACB* shall be found by the 3rd proposition, which shall be *G*, and that of the pentagon *ACDEF* by the preceding 2nd example, which shall be *H*. Then the line *GH* will be beam. After this, there shall be constructed two parallelograms of the same height, as *IKLM*, equal to the pentagon *ACDEF*, and *IKNO*, equal to the triangle euen anden driehouck A C B, deelende den balck H G in P, alſoo dat den erm P G, ſulcken reden hebbe tot den erm P H, als de lini K M tot K N; Ick ſeg dat P t'begheerde ſwaerheydts middelpunt is. Welcke maniere van wercking in allen anderen veelſijdeghe platten ghelijck ſal ſijn ande voorgaende.

MERCKT.

Wy hebben hier bouen voorbeelden beſchreuen alwaer t'ghegheuen plat verkeert wort in euenhooghe ende euewydighe vierhoucken, wy connen t'ſelfde oock doen ſonder ſoodanighe verkeering, daer af wy verſcheyden voorbeelden beſchrijuen ſullen als volght.

IIII^e VOORBEELT.

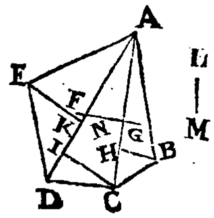
T'GHEGHEVEN. Laet A B C D een ongheſchickt vierhouck weſen. T'BEGHEERDE. Wy moeten ſijn ſwaerheydts middelpunt vinden. T'WERCK. Men ſal den vierhouck deelen in twee driehoucken, met de lini A C, ende vinden t'ſwaerheydts middelpunt van elcken driehouck door het 3^e voorſtel, dat van A C B ſy E, ende vanden driehouck A C D ſy F, de lini dan E F is balck. Daer naer ſalmen trecken D G ende B H, beyde rechthouckich op A C, deylende den balck F E en I, alſoo dat den erm I E, ſulcken reden hebbe tot den erm I F, als D G tot B H; Ick ſeg dat I t'begheerde ſwaerheyts middelpunt is.



V^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D E een ongheſchickt vijſhouck ſijn. T'BEGHEERDE. Wy moeten ſijn ſwaerheyts middelpunt vinden. T'WERCK. Men ſal den vijſhouck deelen in drie driehoucken, met eenighe linien als A D, A C, vindende daer naer het ſwaerheyts middelpunt des vierhoucx A C D E duer het 4^e voorbeelt, t'welck F ſy, ende des driehoucx A C B duer het 3^e voorſtel, t'welck G ſy, ende de lini F G, is balck, Daer naer ghetrocken B H rechthouckich op A C; Ende C I met E K rechthouckich op A D, men ſal der drie linien A D, A C, H B, vinden de vierde * euerednighe, welcke ſy L M, deelende * den balck F G in N, alſoo dat den erm N G ſulcken reden hebbe tot den erm N F, ghelijck C I met E K, tot L M; Ick ſeg dat N het begheerde ſwaerheydts middelpunt is.

Proportionalis.
Door het 12. v. 6. B. E.



VI VOOR-

ACB, upon which the beam *HG* shall be divided at *P* in such a way that the arm *PG* shall have to the arm *PH* the same ratio as the line *KM* to *KN*. I say that *P* is the required centre of gravity. This manner of construction will be identical with the preceding in all other polylateral plane figures.

NOTE.

In the above we have described examples where the given plane figure is transformed into parallelograms of the same height; we can also do the same without such transformation, of which we will describe several examples, as follows.

EXAMPLE IV.

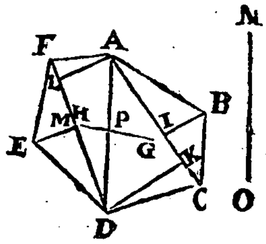
SUPPOSITION. Let *ABCD* be an irregular quadrilateral. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The quadrilateral shall be divided into two triangles by the line *AC*, and the centre of gravity of each triangle shall be found by the 3rd proposition. That of *ACB* shall be *E*, and that of the triangle *ACD* shall be *F*; the line *EF* then is beam. After this, there shall be drawn *DG* and *BH*, both at right angles to *AC*, upon which the beam *FE* shall be divided at *I*, in such a way that the arm *IE* shall have to the arm *IF* the same ratio as *DG* to *BH*. I say that *I* is the required centre of gravity.

EXAMPLE V.

SUPPOSITION. Let *ABCDE* be an irregular pentagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The pentagon shall be divided into three triangles by some lines, as *AD*, *AC*, after which the centre of gravity of the quadrilateral *ACDE* shall be found by the 4th example, which shall be *F*, and that of the triangle *ACB* by the 3rd proposition, which shall be *G*; and the line *FG* is beam. After this, *BH* being drawn at right angles to *AC*, and *CI* and *EK* at right angles to *AD*, the fourth proportional to the three lines *AD*, *AC*, *HB* shall be found, which shall be *LM*, and the beam *FG* shall be divided at *N* in such a way that the arm *NG* shall have to the arm *NF* the same ratio as *CI* with *EK* to *LM*. I say that *N* is the required centre of gravity.

VI. VOORBEELT.

T'GHEGHEVEN. Laet A B C D E F een ongheschickt seshouck sijn. T'BEGHEERDE. Wy moeten sijn swaerheydts middelpunt vinden. T'WERCK. Men sal den seshouck deelen in vier driehoucken, met eenighe linien als A C, A D, F D, vindende daer naer het swaerheyts middelpunt des vierhoucx A D C B door het 4ᵉ voorbeelt, t'welck G sy, ende des vierhoucx A D E F, t'welck H sy, ende de lini H G is balck. Daernaer ghetrocken B I ende D K rechthouckich op A C, insghelijcx A L ende E M beyde rechthouckich op F D, men sal der drie linien welcker eerste F D, de tweede A C, de derde B I met K D, vinden de vierde eueredníghe, welcke N O sy, deelende den balck H G in P, also dat den erm P G, sulcken reden hebbe tot den erm P H, ghelijck A L met E M, tot N O; Ick seg dat P het begheerde swaerheyts middelpunt is. Eñ soo salmen voort mueghen varen met ander veelhouckeghe platten.



T'BEWYS. Ghelijck int eerste voorbeelt H I tot H L, alsoo den erm N E tot den erm N F, maer ghelijck H I tot H L, alsoo den vierhouck G H I K, tot den vierhouck G H L M, ghelijck dan G H I K tot G H L M, also N E tot N F, maer G H I K is euen an den driehouck A C D, ende G H L M anden driehouck A C B door t'werck, ghelijck dan den driehouck A C D tot A C B, alsoo den erm N E tot N F. Het punt dan N is (door het 1ᵉ voorstel des 1ᵉⁿ boucx) des vierhoucx swaerheyts middelpunt. Sghelijcx sal oock bewys sijn des 2ᵉ ende 3ᵉⁿ voorbeelts. *1. v. 6. B. E.*

T'vierde voorbeelt is openbaer als wy bewesen hebben dat ghelijck D G, tot H B, alsoo den driehouck A C D, tot A C B in deser voughen: Nemende A C voor hoochde, ende D G ende H B voor gronden, soo heeft den rechthouck begrepen onder A C ende D G, sulcken reden tot den rechthouck onder A C ende H B, ghelijck D G tot H B; Maer ghelijck dien rechthouck tot desen, alsoo de driehouck A C D tot A C B, want elck driehouck is sijn rechthoucx helft, ghelijck dan D G tot H B, alsoo den driehouck A C D tot A C B. *1. v. 6. B. E.* *41. v. 1. B. E.*

Des 5ᵉⁿ voorbeelts bewys sal oock claer sijn als wy bewesen hebben, dat ghelijck E K met I C tot L M, alsoo den vierhouck A C D E tot den driehouck A C B, aldus: Anghesien L M vierde eueredníghe is der drie A D, A C, H B, de rechthouck begrepen onder A D ende L M, sal euen sijn an den rechthouck begrepen onder A C ende H B, Laet ons nu E K, I C, L M, ansien voor gronden, wiens ghemeene hoochde A D; Maer ghelijck die gronden *16. v. 6. B. E.*

K tot mal-

EXAMPLE VI.

SUPPOSITION. Let *ABCDEF* be an irregular hexagon. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The hexagon shall be divided into four triangles by some lines, as *AC*, *AD*, *FD*, after which the centre of gravity of the quadrilateral *ADCB* shall be found by the 4th example, which shall be *G*, and that of the quadrilateral *ADEF*, which shall be *H*. And the line *HG* is beam. After this, *BI* and *DK* being drawn at right angles to *AC*, likewise *AL* and *EM* both at right angles to *FD*, the fourth proportional shall be found to the three lines, of which the first is *FD*, the second *AC*, the third *BI* with *KD*; this shall be *NO*, upon which the beam *HG* shall be divided at *P* in such a way that the arm *PG* shall have to the arm *PH* the same ratio as *AL* with *EM* to *NO*. I say that *P* is the required centre of gravity. In the same way one may proceed with other polygonal plane figures. PROOF. As in the first example *HI* is to *HL*, so is the arm *EN* to the arm *NF*. But as *HI* is to *HL*, so is the quadrilateral *GHIK* to the quadrilateral *GHLM*; therefore as *GHIK* is to *GHLM*, so is *NE* to *NF*. But *GHIK* is equal to the triangle *ACD*, and *GHLM* to the triangle *ACB*, by the construction; therefore, as the triangle *ACD* is to *ACB*, so is the arm *NE* to *NF*. The point *N* therefore (by the 1st proposition of the 1st book) is the centre of gravity of the quadrilateral. The same proof holds for the 2nd and the 3rd example.

The fourth example is manifest when we have proved that as *DG* is to *HB*, so is the triangle *ACD* to *ACB*, as follows. Taking *AC* for the height, and *DG* and *HB* for bases, the rectangle contained by *AC* and *DG* has to be rectangle contained by *AC* and *HB* the same ratio as *DG* to *HB*. But as the former rectangle is to the latter, so is the triangle *ACD* to *ACB*, for each triangle is half of its rectangle. Therefore, as *DG* is to *HB*, so is the triangle *ACD* to *ACB*.

The proof of the 5th example will also be manifest when we have proved that as *EK* with *IC* is to *LM*, so is the quadrilateral *ACDE* to the triangle *ACB*, as follows. Since *LM* is fourth proportional to the three lines *AD*, *AC*, *HB*, the rectangle contained by *AD* and *LM* will be equal to the rectangle contained by *AC* and *HB*. Let us now consider *EK*, *IC*, *LM* as bases, whose common height shall be *AD*. But as these bases are to one another, so are the rectangles contained

1.v.6.B.E. tot malcanderen, alsoo de rechthoucken begrepen onder haer ende hare ghemeene hoochde, daerom oock ghelijck de twee gronden E K, I C, tot den grondt L M, alsoo dier gronden rechthoucken tot deses grondts rechthouck; maer die twee rechthoucken sijn elck het dobbel haers driehoucx; Ghelijck dan E K met I C tot L M, also het dobbel vanden vierhouck A C D E tot den rechthouck begrepen onder A D ende L M: Maer desen is euen an den rechthouck begrepen onder A C ende H B als vooren betoocht is, ende de selue rechthouck begrepen onder A C ende H B is het dobbel des driehoucx A C B, daerom ghelijck E K met I C tot L M, alsoo het dobbel des vierhoucx A C D E tot het dobbel des driehoucx A C B, ende veruolghens ghelijck E K met I C tot L M, alsoo den vierhouck A C D E tot den driehouck A C B, waer uyt de reste openbaer is. T'bewys van het 6^e voorbeelt is duer dit oock kennelick ghenouch. T'BESLVYT. Wesende dan ghegheuen een rechthouckich plat: Wy hebben sijn swaerheydts middelpunt gheuonden naer den eysch.

MERCKT.

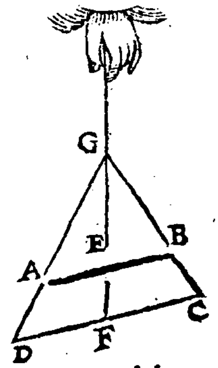
Commentarius in quadraturam paraboles. *MY is onder het drucken ter handt ghecomen, Fredric Commandins * verclaring ouer de viercanting der Brantsne van Archimedes; alwaer hy onder het 6^e voorstel de manier beschrijft, om t'swaerheyts middelpunt te vinden van yder rechtlinich plat, ende dat op een ander wijse als de twee voorgaende. So ymant tottet ouersien der selue begheerich waer, salse daer vinden.*

V. VERTOOCH. VII. VOORSTEL.

HET swaerheyts middelpunt des vierhoucx met twee * euewydighe sijden, is inde lini tusschen dier sijden middelpunten.

Parallelis.

T'GHEGHEVEN. Laet A B C D een vierhouck sijn, diens twee euewydighe sijden A B ende D C, ende de lini uyt E middel van A B, tot F middel van D C, sy E F.



T'BEGHEERDE. Wy moeten bewysen dat t'swaerheyts middelpunt des vierhoucx A B C D inde lini E F is. T'BEREYTSEL. Laet de drie linien D A, F E, C B, voortghetrocken worden, welcke om de * eueredenheyt der linien A E, E B, D F, F C, vergaren sullen in een selfde punt t'welck G sy. T'BEWYS. Laet ons den driehouck G D C ophanghen byde lini G F, ende het deel G F C sal euestaltwichtich sijn, teghen G F D door het 2^e voorstel, waer deur oock t'swaerheyts mid-

Proportionē.

delpunt

by them and their common height; therefore also, as the two bases *EK*, *IC* are to the base *LM*, so are the rectangles on the former bases to the rectangle on the latter base. But those two rectangles are each double of their triangle. Therefore, as *EK* with *IC* is to *LM*, so is the double of the quadrilateral *ACDE* to the rectangle contained by *AD* and *LM*. But the latter is equal to the rectangle contained by *AC* and *HB*, as has been argued above, and this same rectangle contained by *AC* and *HB* is double of the triangle *ACB*. Therefore, as *EK* with *IC* is to *LM*, so is the double of the quadrilateral *ACDE* to the double of the triangle *ACB*; and consequently, as *EK* with *IC* is to *LM*, so is the quadrilateral *ACDE* to the triangle *ACB*, from which the rest is manifest. The proof of the 6th example is also evident enough from this. CONCLUSION. Given therefore a rectilineal plane figure, we have found its centre of gravity, as required.

NOTE.

While this book was being printed, there came into my hands Frederick Commandinus' explanation of the quadrature of the parabola by Archimedes [1]*, where in the 6th proposition he describes the method for finding the centre of gravity of any rectilineal plane figure, such in a manner different from the preceding two. If anyone should be desirous to see this, he may find it there.*

THEOREM V. PROPOSITION VII.

The centre of gravity of the quadrilateral with two parallel sides is in the line joining the middle points of those sides.

SUPPOSITION. Let *ABCD* be a quadrilateral, whose two parallel sides shall be *AB* and *DC*, while the line from *E*, the middle point of *AB*, to *F*, the middle point of *DC*, shall be *EF*. WHAT IS REQUIRED TO PROVE. We have to prove that the centre of gravity of the quadrilateral *ABCD* is in the line *EF*. PRELIMINARY. Let the three lines *DA*, *FE*, *CB* be produced, which om account of the proportionality of the lines *AE*, *EB*, *DF*, *FC* will meet in one and the same point, which shall be *G*. Proof. Let us hang the triangle *GDC* by the line *GF* then the part *GFC* will be of equal apparent weight to *GFD* by the 2nd propo-

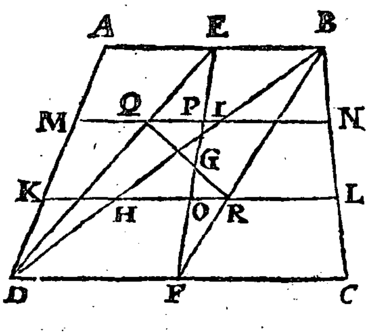
[1]) *Archimedis Opera Nonnulla a* Federico Commandino Urbinate *nuper in Latinum conversa, et commentariis illustrata.* Venetiis 1558. *Commentarii*, p. 22 v.

delpunt des driehoucx G D C inde lini G F is. Maer den driehouc G E B, is oock euestaltwichtich teghen den driehouck G E A, daerom van euestaltwychtighe ghetrocken euestaltwichtighe, de resten als de vierhoucken E F D A, E F C B, sullen noch euestaltwichtich blijuen, ende haer swaerheyts middelpunt noch inde lini G F, maer niet uyt de form in E G; Nootsaecklick dan in E F. T'BESLVYT. Het swaerheydts middelpunt dan des vierhoucx met twee euewydighe sijden, is inde lini tusschen dier sijden middelpunten, t'welck wy bewysen moesten.

VI. VERTOOCH. VIII. VOORSTEL.

HET swaerheyts middelpunt des vierhoucx met twee euewydighe sijden, deelt de lini tusschen dier euewydighens middelpunten alſo, dat het stick naer de minste sijde, tot het ander, sulcken reden heeft, als tweemael de meeste sijde met eenmael de minste, tot tweemael de minste met eenmael de meeste.

T'GHEGHEVEN. Laet A B C D een vierhouck wesen met twee euewydighe sijden A B, D C, ende de lini tusschen haer middelpunten sy E F, ende t'swaerheydts middelpunt sy G. T'BEGHEERDE. Wy moeten bewysen dat ghelijck tweemael D C met eenmael A B, tot tweemael A B met eenmael D C, alſo G E tot G F. T'BEREYTSEL. Laet ghetrocken worden D B, ende ghedeelt in drie euen deelen met de punten H, I, ende door de selue ghetrocken worden K L, ende M N, euewydich van D C, sniende E F in O en P. Daer naer de lini D E, sniende M I in Q, Ende B F sniende K L in R, Ende ten laetsten R Q.



T'BEWYS. Anghesien het swaerheydts middelpunt des driehoucx B D C, is in B F, duer het 2ᵉ voorstel, ende oock in H L duer het 5ᵉ voorstel, soo is R, sijn swaerheyts middelpunt, en om de selue reden is Q swaerheyts middelpunt des driehoucx A B D, ende Q R is dier driehoucken balck, inden welcken haer beyder, dat is des vierhoucx A B C D, swaerheyts sition, in consequence of which the centre of gravity of the triangle *GDC* is also in the line *GF*. But the triangle *GEB* is also of equal apparent weight to the triangle *GEA*. Therefore, equal apparent weights being subtracted from equal apparent weights, the remainders, viz. the quadrilaterals *EFDA*, *EFCB*, will still remain of equal apparent weight [1]), and their centres of gravity will still be in the line *GF*, but not outside the figure in *EG*; therefore it is necessarily in *EF*. CONCLUSION. The centre of gravity therefore of the quadrilateral with two parallel sides is in the line joining the middle points of those sides, which we had to prove.

THEOREM VI. PROPOSITION VIII.

The centre of gravity of the quadrilateral with two parallel sides divides the line joining the middle point of those parallel sides in such a way that the segment adjacent to the shorter side has to the other the same ratio as twice the longer side plus once the shorter to twice the shorter plus once the longer.

SUPPOSITION. Let *ABCD* be a quadrilateral with two parallel sides *AB*, *DC*, and the line joining their middle points shall be *EF*, and the centre of gravity shall be *G*. WHAT IS REQUIRED TO PROVE. We have to prove that as twice *DC* plus once *AB* is to twice *AB* plus once *DC*, so is *GE* to *GF*. PRELIMINARY. Let *DB* be drawn, and let this be divided into three equal segments by the points *H*, *I*, and let there be drawn through these points *KL* and *MN*, parallel to *DC*, intersecting *EF* in *O* and *P*. After this, let the line *DE* be drawn intersecting *MI* in *Q*, and *BF* intersecting *KL* in *R*, and finally *RQ*. PROOF Since the centre of gravity of the triangle *BDC* is in *BF*, by the 2nd proposition, and also in *HL*, by the 5th proposition, *R* is its centre of gravity. And for the same reason *Q* is the centre of gravity of the triangle *ABD*, and *QR* is the beam of these triangles, in which lies the centre of gravity of the two, that is of the quadrilateral

[1]) As has been remarked in notes 2 to p. 177 and p. 179, this inference is not generally valid.

middelpunt is, t'selue is oock in E F duer het 7^e^ voorstel, daerom G is t'swaerheyts middelpunt des vierhoucx A B C D. Maer want de twee driehoucken C D B ende A B D sijn tusschen twee euewydighe A B ende D C, so sijn sy inde reden van haer gronden, dat is ghelijck den driehouck C D B tot A B D, alsoo D C tot A B: Maer ghelijck den driehouck C D B tot A D B, also den erm G Q tot G R duer het 1^e^ voorstel des 1^en^ boucx, ghelijck dan D C tot A B, alsoo G Q tot G R; maer ghelijck G Q tot G R, alsoo P G tot G O (want sy tusschen de euewydeghe M N, K L sijn) ghelijck dan D C tot A B, alsoo G P tot P O, daerom oock ghelijck tweemael D C met eenmael A B, tot tweemael A B met eenmael D C, also tweemael G P met eenmael G O, tot tweemael G O met eenmael G P. Maer G E is euen an tweemael G P met eenmael G O, ende G F is euen an tweemael G O met eenmael G P, daerom ghelijck tweemael D C met eenmael A B, tot tweemael A B met eenmael D C, alsoo G E tot G F. T'BESLVYT. Het swaerheyts middelpunt dan des vierhoucx met twee, &c.

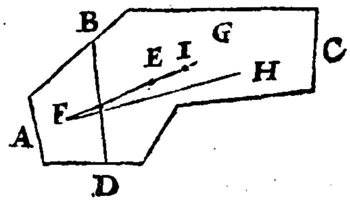
1. v. 6. B. E.

III. EYSCH. IX. VOORSTEL.

WESENDE ghegheuen t'swaerheyts middelpunt eens plats ende sijns deels, wiens reden an t'ander deel kennelick is: Het swaerheyts middelpunt van t'ander deel te vinden.

I. VOORBEELT.

T'GHEGHEVEN. Laet A B C D een rechtlinich plat wesen, diens swaerheyts middelpunt E, ende B D A deel des plats, wiens swaerheyts middelpunt F. T'BEGHEERDE. Wy moeten t'swaerheyts middelpunt vinden des ander deels B D C. T'WERCK. Men sal trecken F E tot in G, alsoo dat F E sulcken reden hebbe tot E G, als t'stick B D C tottet stick B D A: Ick seg dat G t'begheerde swaerheyts middelpunt is des ander deels B D C. T'BEWYS. Anghesien t'swaerheyts middelpunt van B D A is F, ende des heels A B C D is E, soo moet t'swaerheyts middelpunt des ander deels B D C sijn in de rechte F E oneindelick voortghetrocken. Want soot mueghelick waer, latet daer buyten wesen als H, ende laet ons trecken F H, het swaerheyts middelpunt dan des heels sal in F H sijn, maer dat is teghen * t'ghestelde, wantet E is; Ten is dan niet buyten F E oneindelick voortghetrocken maer daer in. Latet nu wesen (soot



Hypothesin.

ABCD. This centre of gravity is also in *EF* by the 7th proposition, therefore *G* is the centre of gravity of the quadrilateral *ABCD*. But because the two triangles *CDB* and *ABD* are contained between two parallel lines *AB* and *DC*, they are to one another in the ratio of their bases, that is: as the triangle *CDB* is to *ABD*, so is *DC* to *AB*. But as the triangle *CDB* is to *ADB*, so is the arm *GQ* to *GR*, by the 1st proposition of the 1st book; therefore as *DC* is to *AB*, so is *GQ* to *GR*. But as *GQ* is to *GR*, so is *PG* to *GO* (for they are contained between the parallel lines *MN*, *KL*); therefore, as *DC* is to *AB*, so is *GP* to *PO*, and therefore also as twice *DC* plus once *AB* is to twice *AB* plus once *DC*, so is twice *GP* plus once *GO* to twice *GO* plus once *GP*. But *GE* is equal to twice *GP* plus once *GO*, and *GF* is equal to twice *GO* plus once *GP*. Therefore as twice *DC* plus once *AB* is to twice *AB* plus once *DC*, so is *GE* to *GF*. CONCLUSION. The centre of gravity therefore of the quadrilateral with two etc.

PROBLEM III. PROPOSITION IX.

Given the centre of gravity of a plane figure, and of one part of it, whose ratio to the other part is known: to find the centre of gravity of the other part.

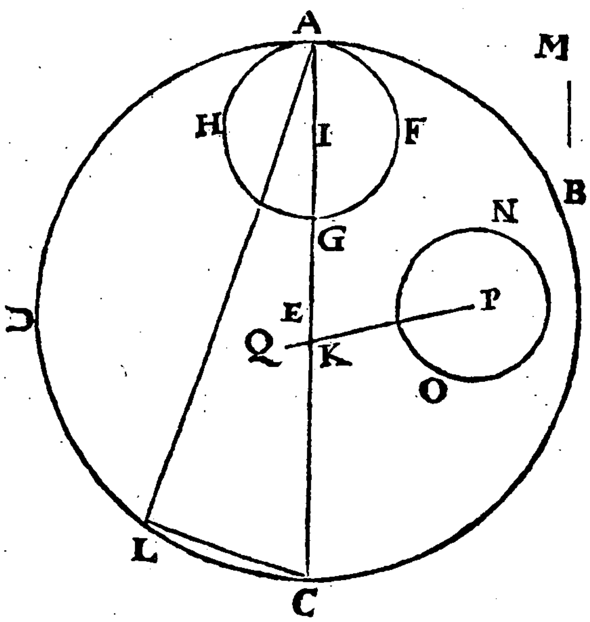
EXAMPLE I.

SUPPOSITION. Let *ABCD* be a rectilineal plane figure, whose centre of gravity shall be *E*, while *BDA* shall be a part of the figure, whose centre of gravity shall be *F*. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the other part *BDC*. CONSTRUCTION. The line *FE* shall be drawn up to *G*, in such a way that *FE* shall have to *EG* the same ratio as the part *BDC* to the part *BDA*. I say that *G* is the required centre of gravity of the other part *BDC*. PROOF. Since the centre of gravity of *BDA* is *F*, and that of the whole *ABCD* is *E*, the centre of gravity of the other part *BDC* must be in the straight line *FE* produced indefinitely. For, if it were possible, let it be outside said line, as *H*, and let us draw *FH*. The centre of gravity of the whole will then be in *FH*, but this is contrary to the supposition, because it is *E*. It is not therefore outside *FE* produced indefinitely, but in it. Now let it be (if this were possible) between the points *E* and *G*, as *I*. But then the longer arm *EF* will have to the (soot mueghelick waer) tusschen de punten E G als I; Maer den langsten erm E F sal dan meerder reden hebben tot den cortsten E I, dan de swaerste swaerheyt B D C tot de lichtste B D A, twelck teghen het 1e voorstel des 1en boucx waer. Ten is dan tusschen E G niet: Sghelijcx salmen oock bethoonen dattet bouen G niet en is. Tis dan nootsaecklick G, t'welck wy bewysen moesten.

II VOORBEELT.

T'GHEGHEVEN. Laet A B C D een rondt wesen diens * halfmiddellini E A, ende swaerheyts middelpunt E sy, ende trondt A F G H, deel des rondts A B C D, ende sijn swaerheyts middelpunt I, ende * middellini A G. T'BEGHEERDE. Wy moeten het swaerheyts middelpunt vinden des ander deels, dat is der maen A B C D H G F. *Semidiameter.* *Diameter.*

T'WERCK. Men sal I E voorttrecken tot in K, also dat I E sulcken reden hebbe tot E K, als de maen A B C D H G F tot het rondt A F G H, ende K sal t'begheerde swaerheydts middeldelpunt wesen, Daer af t'bewys ghelijck sal sijn an tvoorgaende. Maer om de reden dier maen tot dat rondt te vinden, men sal trecké C L euen met A G, daernaer A L, vindende de derde everednighe welcker eerste A L, de tweede L C, ende de derde sy M, Ende A L tot M, sal de reden sijn der maen tot het rondt A F G H. Want ouermits A L C rechthouck is (reden dat sy int half rondt staet) het ront diens middellini A L, sal euen sijn ande maen, ende A L tot M is de * ghedobbelde reden van A L tot L C, dat is van A L tot A G, daerom &c. *11.v.6.B.E.* *31.v.3.B.E.* *Duplicata ratio.*



Sghelijcx soudemen voortvaren dat int rondt A B C D meer ronden ghebraken; by voorbeelt het rondt N O, wiens middelpunt P. Want shorter *EI* a greater ratio than the heavier gravity *BDC* to the lighter *BDA*, which would be contrary to the 1st proposition of the 1st book. It is not therefore between *E* and *G*. In the same way it can also be shown not to be above *G*. It is therefore necessarily *G*, which we had to prove.

EXAMPLE II.

SUPPOSITION. Let *ABCD* be a circle, whose semi-diameter shall be *EA*, and its centre of gravity *E*, while the circle *AFGH* shall be a part of the circle *ABCD*, and its centre of gravity *I*, and its diameter *AG*. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the other part, that is of the lune *ABCDHGF*. CONSTRUCTION. *IE* shall be produced to *K*, in such a way that *IE* shall have to *EK* the same ratio as the lune *ABCDHGF* to the circle *AFGH*; then *K* will be the required centre of gravity, the proof of which will be identical with the preceding one. But in order to find the ratio of the said lune to the said circle, *CL* shall be drawn equal to *AG*; after that *AL*, upon which the third [1]) proportional shall be found, the first of which shall be *AL*, the second *LC*, and the third shall be *M*. Then *AL* to *M* will be the ratio of the lune to the circle *AFGH*. For since *ALC* is right-angled (because it is contained in a semicircle), the circle having *AL* as diameter will be equal to the lune, and *AL* to *M* is the duplicated ratio [2]) of *AL* to *LC*, that is of *AL* to *AG*; therefore, etc.

In the same way one would proceed if more circles were missing from the circle *ABCD*, for example the circle *NO*, whose centre is *P*. For *PK* being produced to *Q*, in such a way that *PK* should have to *KQ* the same ratio as the

1) M is the third proportional to AL and LC, i.e. $AL : LC = LC : M$. Now $\frac{\text{circle } (AC)}{AC^2} = \frac{\text{circle } (AG)}{LC^2} = \frac{\text{lune}}{AL^2}$.

But since $AL : M = AL^2 : LC^2$, $\frac{\text{lune}}{\text{circle } (AG)} = \frac{AL}{M}$.

2) In the ancient terminology to which we referred in note 1) to p. 233, doubling of a ratio means squaring.

P K voortghetrocken tot in Q, alſoo dat P K ſulcken reden hadde tot K Q, als het reſtende tot het rondt N O, ſo ſoude Q t'begheerde ſwaerheyts middelpunt ſijn. Ende alſoo met allen anderen formen welcker deelen reden kennelick is. T'BESLVYT. Weſende dan ghegheuen de ſwaerheyts middelpunten eens plats ende ſijns deels, wiens reden an t'ander deel kennelick is: wy hebben het ſwaerheyts middelpunt ghevonden des ander deels naer den eyſch.

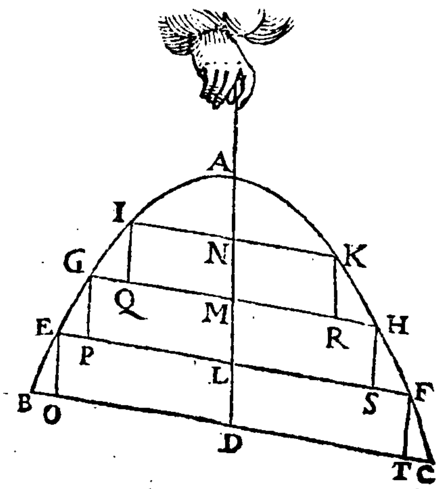
VII. VERTOOCH. X. VOORSTEL.

Parabola.

YDER * brantſnees ſwaerheyts middelpunt is in haer middellini.

T'GHEGHEVEN. Laet A B C D een brandtſne ſijn diens middellini A D. T'BEGHEERDE. Wy moeten bewyſen dat t'ſwaerheyts middelpunt inde lini A D is. T'BEREYTSEL. Laet ons trecken de linien E F, G H, I K, euewydighe van B C, ende ſniende A D in L, M, N, daer naer E O, G P, I Q, K R, H S, F T, euewydighe van A D.

T'BEWYS. Ouermidts E F euewydighe is van B C, ende E O, F T, van L D, ſoo ſal E F T O euewydich vierhouck ſijn, wiens E L euen is met L F, oock met O D ende D T, waer duer t'ſwaerheyts middelpunt van E F T O, in D L is duer het 1e voorſtel, Ende om de ſelue reden ſal t'ſwaerheyts middelpunt des euewydich vierhoucx G H S P in L M weſen, ende van I K R Q in M N, ende veruolghens t'ſwaerheyts middelpunt der form I K R H S F T O E P G Q, ghemaeckt vande voornoemde drie vierhoucken ſal inde lini N D oft A D ſijn. Maer hoe datter ſulcke vierhoucken meer gheſchreuen worden, hoe dattet verſchil des brandtſnees A B C, ende der binnenſchreuen form van die vierhoucken vergaert, minder is, wy connen dan door dat oneindelick naerderen ſulck een form binnen de brantſne ſtellen, dattet verſchil tuſſchen haer eñ de brantſne, minder ſy dan eenich ghegheuen plat hoe cleen het ſy, waer uyt volght,



dat ſtellende A D als ſwaerheyts middellini, ſo ſal t'ſtaltwicht des deels A D C, min verſchillen van t'ſtaltwicht des deel A D B, dan eenich plat datmen remainder to the circle *NO*, *Q* would be the required centre of gravity. And the same holds for all other figures the ratio of whose parts is known. CONCLUSION. Given therefore the centres of gravity of a plane figure and of a part thereof, whose ratio to the other part is known: we have found the centre of gravity of the other part, as required.

THEOREM VII. PROPOSITION X.

The centre of gravity of any parabola is in its diameter.

SUPPOSITION. Let *ABCD* be a parabola, whose diameter shall be *AD*. WHAT IS REQUIRED TO PROVE. We have to prove that the centre of gravity is in the line *AD*. PRELIMINARY. Let us draw the lines *EF*, *GH*, *IK*, parallel to *BC* and intersecting *AD* in *L*, *M*, *N*, and then *EO*, *GP*, *IQ*, *KR*, *HS*, *FT*, parallel to *AD*. PROOF Since *EF* is parallel to *BC*, and *EO*, *FT* to *LD*, *EFTO* will be a parallelogram, in which *EL* is equal to *LF*, and also to *OD* and *DT*, in consequence of which the centre of gravity of *EFTO* is in *DL*, by the 1st proposition. And for the same reason the centre of gravity of the parallelogram *GHSP* will be in *LM*, and that of *IKRQ* in *MN*; and consequently the centre of gravity of the figure *IKRHSFTOEPGQ*, composed of the aforesaid three quadrilaterals, will be in the line *ND* or *AD*. But the more of such quadrilaterals there are inscribed, the less will be the difference between the parabola *ABC* and the inscribed figure of those quadrilaterals. We can therefore, by infinite approximation, place such a figure within the parabola that the difference between said figure and the parabola shall be less than any given plane figure, however small, from which it follows that, *AD* being taken as centre line of gravity [1]), the apparent weight of the part *ADC* will differ less from the apparent weight of the

[1]) See Note 3 to p. 227.

datmen soude connen gheuen, hoe cleen het sy, waer uyt ick aldus strije:

A. *Neuen alle verschillende staltswaerheden, can een swaerheyt ghestelt worden minder dan haer verschil;*

O. *Neuen dese staltswaerheden A D C ende A D B, en can gheen swaerheyt ghestelt worden minder dan haer verschil;*

O. *Dese staltswaerheden dan A D C ende A D B en verschillen niet.*

Daerom A D is swaerheyts middellini, ende veruolghens het swaerheyts middelpunt des branders A B C is in haer. T'BESLVYT. Yder brandtsnees swaerheyts middelpunt dan, is in haer middellini, t'welck wy bewysen moesten.

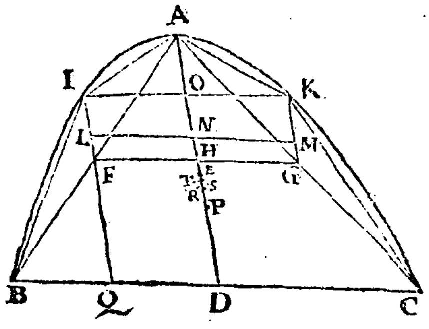
VIII. VERTOOCH. XI. VOORSTEL.

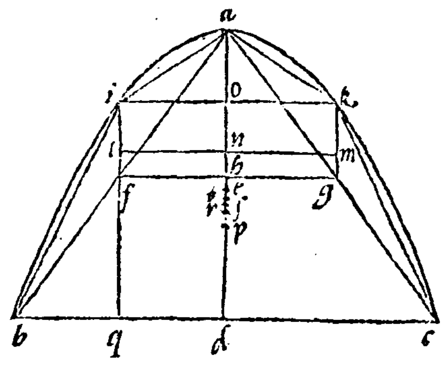
ALLER brantsneens middellinien worden van het swaerheyts middelpunt *eueredelick ghedeelt.

Proportionaliter.

T'GHEGHEVEN. Laet A B C D ende *a b c d* twee onghelijcke brantsneen sijn, diens middellinien A D, ende *a d*, ende swaerheyts middelpunten E, ende *e*. T'BEGHEERDE. Wy moeten bewysen dat ghelijck A E tot E D, alsoo *a e* tot *e d*.

T'BEREYTSEL. Laet ons trecken de linien A B, A C, die deelende in haer middelen F, G, ende trecken F G sniende A D in H, daer naer F I ende G K euewydighe van A D, ende daer naer I A, I B, K A, K C, ende laet ons stellen L in I F, alsoo dat I L dobbel sy an L F, sghelijcx M, alsoo dat K M dobbel sy an M G, ende laet ons trecken L M, sniende A D in N, ende I K, sniende A D in O, ende laet ons stellen P, alsoo dat A P dobbel sy an P D, ende laet ons I F voorttrecken tot





Q in den grondt B C. Nu anghesien A P dobbel is an P D, so is P t'swaerheyts middelpunt des driehoucx A B C, ende omme de selue reden L, M,

swaerheyts

part *ADB* than any plane figure that might be given, however small, from which I argue as follows [1]):

A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*

O. *Beside the present apparent gravities ADC and ADB there cannot be placed any gravity less than their difference;*

O. *Therefore the present apparent gravities ADC and ADB do not differ.*

Therefore *AD* is centre line of gravity, and consequently the centre of gravity of the parabola *ABC* is in this line. CONCLUSION. The centre of gravity therefore of any parabola is in its diameter, which we had to prove.

THEOREM VIII. PROPOSITION XI.

The diameters of all parabolas are proportionally divided by the centre of gravity.

SUPPOSITION. Let *ABCD* and *abcd* be two dissimilar parabolas, whose diameters shall be *AD* and *ad*, and the centres of gravity *E* and *e*. WHAT IS REQUIRED TO PROVE. We have to prove that as *AE* is to *ED*, so is *ae* to *ed*. PRELIMINARY. Let us draw the line *AB*, *AC*, dividing them in their middle points *F*, *G*, and let us draw *FG*, intersecting *AD* in *H*; after this, *FI* and *GK* parallel to *AD*, and then *IA*, *IB*, *KA*, *KC*. And let us take *L* in *IF* in such a way that *IL* shall be double of *LF*, and in the same way *M* so that *KM* shall be double of *MG*, and let us draw *LM* intersecting *AD* in *N*, and *IK* intersecting *AD* in *O*. And let us take *P* in such a way that *AP* shall be double of *PD*, and let us produce *IF* to *Q* in the base *BC*. Now since *AP* is double of *PD*, *P* is the centre of gravity of the triangle *ABC*, and for the same reason *L*, *M* are the centres of

[1]) See note 2 to p. 143.

ſwaerheyts middelpunten der twee driehoucken A B I, ende A C K, eñ veruolghens, want ſy euen ſijn, ſoo is N haer beyde ſwaerheyts middelpunt. N P dan is balck, de ſelue ghedeelt in R, alſoo dat den erm N R ſy tot R P, als den driehouck A B C tot de twee driehoucken A B I, A C K, dat is, als 4 tot 1 (want alle brantſne is tot den driehouck als A B C ghelijck 4 tot 3, duer het 24 voorſtel der viercanting des brantſnees van Archim. daerom, &c.) Laet ons nu derghelijcke linien ende punten oock beſchrijuen inde brantſne *a b c*. T'BEWYS. Ghelijck A D tot A O, alſo het viercant van D B tottet viercant van O I; Maer D Q is euen an O I, ende D Q is den helft van D B (want F is t'middel van A B, ende F Q is euewydich van A D) daerom het viercant van D B, is viervoudich an t'viercant van D Q, ofte van O I, ende veruolghens A D is viervoudich tot A O, daerom A O is $\frac{1}{4}$ van A D, ende O H oock $\frac{1}{4}$ (want A H is den helft van A D, ouermits F G ghetrocken is uyt de middelen van A B, A C) daerom doet N H $\frac{1}{12}$ van A D, daer toe ghedaen H D $\frac{1}{2}$, comt voor N D $\frac{7}{12}$, daer af ghetrocken P D $\frac{1}{3}$, reſt voor P N $\frac{1}{4}$: Maer N R is viervoudich tot R P, daerom R P doet $\frac{1}{20}$, daer toe P D $\frac{1}{3}$, doet voor R D $\frac{23}{60}$, daerom R A de reſte der lini, doet $\frac{37}{60}$, Ghelijck dan 37 tot 23, alſo A R tot R D, ende met de ſelue reden is bethoont dat *a r* tot *r* d, oock is als 37 tot 23. Deſe twee rechtſideghe formen dan ghelijckelick beſchreuen in verſcheyden brandtſneen, hebben het ſwaerheyts middelpunt in haer middellinien, alſo dat de deelen onder malcanderen * euerednich ſijn. Ende ſo wy inde brandtſnekens B I, I A, A K, K C, driehoucken beſchreuen, ſoo ghedaen is inde brantſneen A B I, A C K, vindende daer naer t'ſwaerheyts middelpunt des heels binneſcreuen rechtlinich plats, t'welck ick neem dat hier S ſoude weſen, ende daer *ſ*, wy ſouden inder ſeluer voughen als vooren bethoonen, dat ghelijck A S tot S R, alſo a *ſ* tot *ſ r*. Maer wy connen duer ſulck oneindelick inſchriuen der rechtlinighe formen oneindelick naerderen nae E, ende *e*, ende ghelijckſideghe platten ſullen altijt der middelliniens A D twee ſticken euerednich ghedeelt hebben duer haer ſwaerheyts middelpunt, ende veruolghens de heele brantſneen A B C, *a b c*, ſullen die deelen euerednich hebben. Want laet (ſoot mueghelick waer) T t'ſwaerheyts middelpunt ſijn des brantſnees A B C, ende *e* van *a b c*, ende laet ons teeckenen *t*, dat ghelijck E T tot T S, alſoo *e t* tot *t ſ*. Nu alſmen duer t'inſchrijuen veelſidegher formen in *a b c*, ſal ghecommen ſijn tot *t*, men ſal met ghelijcke veelſideghe formen in A B C, ghecomen ſijn tot T, daerom T ſal t'ſwaerheyts middelpunt ſijn der binneſchreuen form, ende oock des heelen brantſneens A B C, t'welck ongheſchickt is. T'BESLVYT. Aller brantſneens middellinien dan, worden van het ſwaerheyts middelpunt eueredelick ghedeelt, t'welck wy bewyſen moeſten.

20. v. 1. B. *Appol.*

*Proportionales.

IIII EYSCH

gravity of the triangles ABI and ACK; and consequently, because they are equal, N is the centre of gravity of both. NP therefore is beam, which shall be divided at R in such a way that the arm NR shall have to RP the same ratio as the triangle ABC to the two triangles ABI, ACK, that is 4 to 1 (for any parabola has to the triangle, as ABC, the ratio of 4 to 3, by the 24th proposition of the quadrature of the parabola of Archimedes [1]), therefore, etc.). Now let us also mark similar lines and points in the parabola *abc*. PROOF. As AD is to AO, so is the square of DB to the square of OI. But DQ is equal to OI, and DQ is equal to one-half DB (for F is the middle point of AB, and FQ is parallel to AD), therefore the square of DB is equal to four times the square of DQ or OI, and consequently AD is equal to four times AO; therefore AO is $\frac{1}{4}$ AD, and OH is also $\frac{1}{4}$ (for AH is equal to one-half AD, since FG has been drawn from the middle points of AB, AC). Therefore NH makes $\frac{1}{12}$ AD. If to this is added HD $(\frac{1}{2})$, ND becomes $\frac{7}{12}$. If from this is subtracted PD $(\frac{1}{3})$, there remains for PN $\frac{1}{4}$. But NR is equal to four times RP, therefore RP makes $\frac{1}{20}$. If to this is added PD $(\frac{1}{3})$, RD becomes $\frac{23}{60}$. Therefore RA, the remainder of the lines, makes $\frac{37}{60}$. Therefore as 37 is to 23, so is AR to RD, and in the same way it is shown that *ar* is also to *rd* as 37 to 23. These two rectilineal figures therefore, similarly inscribed in different parabolas, have the centre of gravity in their diameters, so that the segments are proportional to one another. And if we inscribed triangles in the small parabolas BI, IA, AK, KC, as has been done in the parabolas ABI, ACK, finding thereafter the centre of gravity of the whole inscribed rectilineal plane figure, which I assume would be S in the first figure and *s* in the second, we should show in the same way as above that as AS is to SR, so is *as* to *sr*. But we can, by such infinite inscription of rectilineal figures, approximate infinitely to E and *e*, and equilateral plane figures will always have the two segments of the diameters AD divided proportionally at their centre of gravity. And consequently the complete parabolas ABC, *abc* will have these segments proportional. For (if this were possible) let T be the centre of gravity of the parabola ABC, and *e* of *abc*, and let us mark *t* so that as ET is to TS, so *et* to *ts*. Now, when by inscribing polylateral figures in *abc*, the point *t* has been reached [2]), with similar polylateral figures in ABC the point T will have been reached. Therefore T will be the centre of gravity of the inscribed figure, and also of the complete parabola ABC, which is absurd. CONCLUSION The diameters therefore of all parabolas are proportionally divided by the centre of gravity, which we had to prove.

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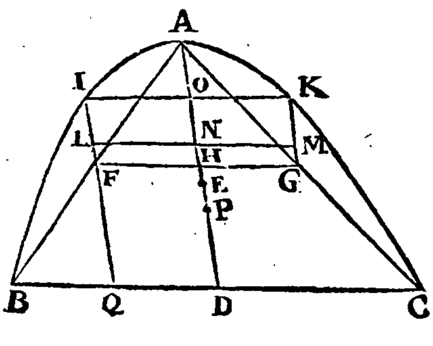
[1]) The ratio of the triangle ABC to the sum of the triangles ABI and ACK can indeed be derived from the Archimedean proposition on the ratio of a parabolic segment to its inscribed triangle, but it is not logical to do this, the said proposition being demonstrated with the aid of this ratio.

[2]) The assumption that any point of AD can be obtained as centre of gravity of an inscribed figure Π_n is, of course, unwarranted.

WESENDE ghegheuen een * brantsne: Huer swaerheyts middelpunt te vinden. *Parabola.*

T'GHEGHEVEN. Laet A B C een brandtsne sijn, diens middellini A D. T'BEGHEERDE. Wy moeten haer swaerheyts middelpunt vinden. T'WERCK. Men sal de middellini A D, deelen in E, alsoo dat A E tot E D de reden hebbe van 3 tot 2: Ick seg dat E t'begheerde swaerheyts middelpunt is. T'BEREYTSEL. Laet ghetrocken worden de rechte linien A B, ende A C, ende de selue ghedeelt in haer middelen F, G, ende ghetrocken worden F G sniende A D in H, daer naer F I ende G K euewydighe van A D, ende laet ghestelt worden t'punt L in I F, inder voughen dat I L sy tot L F, als A E tot E D: Laet oock ghestelt worden t'punt M in K G, alsoo dat M G euen sy an L F, ende laet ghetrocken worden L M sniende A D in N, ende I K sniende A D in O, ende laet I F voortghetrocken worden tot Q, inden grondt B C, ende laet ghestelt worden t'punt P, alsoo dat A P dobbel sy an P D, ende P sal swaerheyts middelpunt sijn des driehoucx A B C. ende want L, M, als swaerheyts middelpunten ghestelt sijn der brantsnekens A B I, ende A C K, soo sal N swaerheyts middelpunt sijn dier twee brandtsnekens, daerom ghedeelt den balck P N, also dat d'een erm sulcken reden hebbe tot d'ander, als den driehouck A B C tot die twee brantsnekens, wy sullen t'begheerde hebben; maer de heele brantsne heeft sulcken reden tot den driehouck A B C als 4 tot 3 (duer het 24 voorstel vande viercanting der brantsne van Archimed.) daerom den driehouck A B C heeft sulcken reden tot de twee brantsnekens, als 3 tot 1; Ghedeelt dan P N alsoo dat het opperste stick, drievoudich sy tot het onderste, wy sullen t'swaerheyts middelpunt des heels hebben. Ist dan dat wy bethoonen t'selue, te vallen in E (welcke E duer t'werck soo staet dat A E is tot E D inde reden van 3 tot 2) so is E het ware swaerheyts middelpunt.



T'BEWYS. A O en O H soo wy verclaert hebben int 11e voorstel, sijn elck $\frac{1}{4}$ van A D, Maer ghelijck 3 tot 2, alsoo A E tot E D, ende I L tot L F, ende O N tot N H, daerom ghedeelt O H $\frac{1}{4}$, in sulcken reden

L als 3

PROBLEM IV. PROPOSITION XII.

Given a parabola: to find its centre of gravity.

SUPPOSITION. Let *ABC* be a parabola, whose diameter shall be *AD*. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The diameter *AD* shall be divided at *E*, in such a way that *AE* shall have to *ED* the ratio of 3 to 2. I say that *E* is the required centre of gravity. PRELIMINARY. Let the straight lines *AB* and *AC* be drawn, and let these be divided in their middle points *F*, *G*, and let *FG* be drawn, intersecting *AD* in *H*; after that, *FI* and *GK* parallel to *AD*. And let the point *L* be taken in *IF*, in such a way that *IL* shall be to *LF* as *AE* is to *ED*. Let also the point *M* be taken in *KG*, in such a way that *MG* shall be equal to *LF*, and let *LM* be drawn, intersecting *AD* in *N*, and *IK* intersecting *AD* in *O*. And let *IF* be produced to *Q*, in the base *BC*, and let the point *P* be taken, in such a way that *AP* shall be double of *PD*. Then *P* will be centre of gravity of the triangle *ABC*, and because *L*, *M* have been taken as centres of gravity of the small parabolas *ABI* and *ACK*, *N* will be the centre of gravity of those two parabolas. Therefore, the beam *PN* being divided in such a way that one arm shall have to the other the same ratio as the triangle *ABC* to these two parabolas, we shall have the required centre of gravity. But the whole parabola has to the triangle *ABC* the ratio of 4 to 3 (by the 24th proposition of the quadrature of the parabola of Archimedes); therefore the triangle *ABC* has to the two parabolas the ratio of 3 to 1. *PN* therefore being so divided that the upper segment shall be three times the lower segment, we shall have the centre of gravity of the whole. If we then show this to be at *E* (which *E*, by the construction, is so disposed that *AE* has to *ED* the ratio of 3 to 2), *E* is the true centre of gravity. PROOF. As we have explained in the 11th proposition, *AO* and *OH* are each $\frac{1}{4}$ *AD*. But as 3 is to 2, so is *AE* to *ED*, and *IL* to *LF*, and *ON* to *NH*. Therefore, *OH* ($\frac{1}{4}$) being divided in the ratio of 3 to 2, the segment *NH* will made $\frac{1}{10}$ *AD*. If to this is added $\frac{1}{2}$ for *HD*, *ND* becomes $\frac{3}{5}$. If from this is subtracted *PD* ($\frac{1}{3}$), there remains for *NP* $\frac{4}{15}$. This is divided, by the preliminary, at *E* in such a way that *NE* is to *EP* as 3 to 1. Therefore *EP* makes $\frac{1}{15}$. If to this is added *PD* ($\frac{1}{3}$), *ED* becomes $\frac{2}{5}$ *AD*. But *ED* being $\frac{2}{5}$, *EA* will make $\frac{3}{5}$. Therefore *AE* has to *ED* the ratio of 3 to 2, and consequently *E* is the centre of gravity of the parabola *ABC*, which als 3 tot 2, so sal t'stick N H doen $\frac{1}{10}$ van A D, daer toeghedaen $\frac{1}{2}$ voor H D, doet voor N D $\frac{3}{5}$, daer af ghetrocken P D $\frac{1}{3}$ rest voor N P $\frac{4}{15}$, de selue is duer t'bereytsel ghedeelt in E, alsoo dat N E is tot E P, als 3 tot 1, daerom E P doet $\frac{1}{15}$, daer toe ghedaen P D $\frac{1}{3}$, comt voor E D $\frac{2}{5}$ van A D: Maer wesende E D $\frac{2}{5}$, so sal E A doen $\frac{3}{5}$, daerom A E heeft sulcken reden tot E D, als 3 tot 2, ende veruolghens E is t'swaerheyts middelpunt des brantsnees A B C, t'welck wy bewysen moesten.

T'BESLVYT. Wesende dan ghegheuen een brantsne: Wy hebben huer swaerheyts middelpunt gheuonden naer den eysch.

MERCKT.

HET *schijnt dat Archimedes ter kennis deses voorstels ghecommen is, duer een deser twee manieren: D'eerste dat hy lichamelicke brantsneen makende, tot het formen siinder brandtspieghels, ofte om andersins hem daer in te oefnen, beuandt duer de daet, dit deel tot dat te wesen, als 3 tot 2, souckende daer naer de sekerheyt van dien in deser vougen: Anghesien B A I ende B A C beyde brandtsneen siin, soo worden haer middellinien I F ende A D* eueredelick ghedeelt van haer swaerheyts middelpunten (soo int 11e voorstel bewesen is) daerom moet I L tot L F siin, als A E tot E D, maer O N is euen an I L, ende N H an L F, daerom moet O N sulcken reden hebben tot N H, als A E tot E D. Maer als N swaerheyts middelpunt waer der twee brantsnekens, ende P des driehoucx A B C, so moet (ouermits desen driehouck drievoudich is tot die twee brantsnekens) den erm N E drievoudich siin anden erm E P, waer uyt sulcken voorstel rijst:* Te vinden twee punten als N, E, alsoo dat de lini O N sulcken reden hebbe tot N H, als A E tot E D. *Stellende daer naer A E te doen* $\frac{3}{5}$ *van A D, ende E D de* $\frac{2}{5}$ *ende versouckende alsoo watter uyt volghen soude, heeft beuonden naer de maniere als bouen, sulcx waerachtelick te ouercommen mettet begheerde. Ofte soo hy dit aldus niet ghesocht en heeft al tastende, duer de voornoemde reden van 3 tot 2, maer duer lauter cracht der const, soo schijnet dat hy hem t'voornoemde in ghetalen voorghestelt heeft in deser vougen:* Het sijn twee ghetalen O H $\frac{1}{4}$ ende H P $\frac{1}{6}$; deelt elck alsoo, dat het minste van O H, met het meeste van H P, drievoudich sy an t'minste van H P, ende dat t'meeste van O H sulcken reden hebbe tot sijn minste, als t'meeste van H P + $\frac{1}{2}$ tot t'minste van H P + $\frac{1}{3}$.

* *Proportionaliter.*

V. EYSCH. XIII. VOORSTEL.

WESENDE ghegheuen een ghecorte brantsne: Huer swaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A B C D een ghecorte brantsne sijn (welverstaende dat A B euewydighe sy met D C) wiens middellini E F.

T'BEGHEERDE. Wy moeten haer swaerheyts middelpunt vinden.

T'WERCK

we had to prove [1]). CONCLUSION. Given therefore a parabola: we have found its centre of gravity, as required.

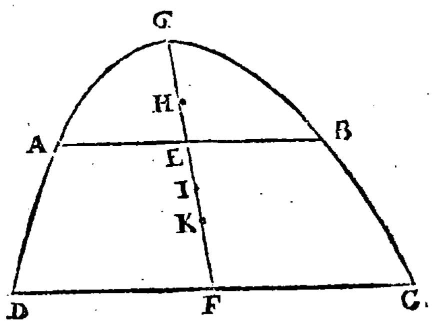
NOTE.

It seems that Archimedes discovered this proposition by one of the following two methods. The first that, making corporeal parabolas for the formation of his burning mirrors, or in order to practise the matter in some other way, he found by experience that one segment had to the other the ratio of 3 to 2, after which he sought to verify this as follows: Since BAI and BAC are both parabolas, their diameters IF and AD are proportionally divided by their centres of gravity (as has been proved in the 11th proposition); therefore IL must be to LF as AE to ED, but ON is equal to IL, and NH to LF, therefore ON must have to NH the same ratio as AE to ED. But if N were the centre of gravity of the two parabolas, and P that of the triangle ABC, then (since the latter triangle is three times the two parabolas), the arm NE must be three times the arm EP, from which arises the following proposition: To find two points, as N, E, so that the line ON shall have to NH the same ratio as AE to ED. Then taking AE to make $\frac{3}{5}$ *AD, and ED* $\frac{2}{5}$, *and trying to find what would follow from this, he found in the above manner that this is in perfect agreement with what was required to prove. But if he has not sought it thus by experience, by starting from the aforesaid ratio of 3 to 2, but purely theoretically, it is probable that he argued in numbers as follows: There are two numbers OH* ($\frac{1}{4}$) *and HP* ($\frac{1}{6}$)*; divide each of these in such a way that the lesser segment of OH plus the greater segment of HP shall be three times the lesser segment of HP, and that the greater segment of OH shall have to its lesser segment the same ratio as the greater segment of HP* $+ \frac{1}{2}$ *to the lesser segment of HP* $+ \frac{1}{3}$ [2]).

[1]) It may seem that the demonstration contains a *circulus in probando*, since it is assumed that the points K and M, which are found by dividing IF and KG respectively in the ratio 3 : 2, are centres of gravity of the parabolic segments AIB and AKC respectively. By Prop. 11 this assumption is equivalent to the supposition that E is the centre of gravity of the segment ABC. Nevertheless, the reasoning is valid; it should, however, be borne in mind that Stevin does not aim at finding out the position of the centre of gravity of a parabolic segment, but only at verifying the statement that it divides the diameter in the ratio 3 : 2. The derivation of this result might have been given by algebra. Putting the ratio $ED : AD = \lambda$ and $AD = a$, we have: $PD = \frac{1}{3}a$, $ND = \frac{1}{2}a + \lambda . \frac{1}{4}a$, $PN = \frac{1}{6}a + \frac{\lambda}{4}a$, $TE = \frac{1}{4}\left[\frac{1}{6}a + \frac{\lambda}{4}a\right]$, $DE = \left[\frac{3}{8} + \frac{\lambda}{16}\right] a$.
This gives the equation for $\lambda : \frac{3}{8} + \frac{\lambda}{16} = \lambda$,
from which: $\lambda = \frac{2}{5}$.
Stevin's procedure amounts to putting in the first member $\lambda = \frac{2}{5}$, and then calculating: $\lambda = \frac{3}{8} + \frac{2}{5} . \frac{1}{16} = \frac{2}{5}$.

[2]) Putting $AD = 1$, we have $OH = \frac{1}{4}$, $HP = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.
Now by Prop. 1 of Book I: $NE = 3\ PE$, or $NH + HE = 3\ TE$, therefore if $ED = \lambda$, $\lambda . \frac{1}{4} + (1 - \lambda)\frac{1}{6} = 3\ (\lambda - \frac{1}{3})$, from which: $\lambda = \frac{2}{5}$.
The second relation is not a condition for finding the centre of gravity, but expresses the property of this centre enunciated in Prop. 11, viz.: The diameters of all parabolas are proportionally divided by the centre of gravity.

T'WERCK. Men sal de ghecorte brantsne volmaken, daer an stellende t'ghebrekende A B G, daer naer salmen teekenen H, alsoo dat G H sy tot H E, als 3 tot 2: Insghelijcx I, alsoo dat G I sy tot I F, als 3 tot 2, daer naer K, alsoo dat I H sulcken reden hebbe tot I K, ghelijck de ghecorte brandtsne A B C D, tot de brantsne A B G; Ick seg dat K t'begheerde swaerheyts middelpunt is. T'BEWYS. I is swaerheyts middelpunt des heels, ende H des deels, ende ghelijck t'ander deel tot dit, alsoo H I tot I K, daerom K, duer het 9^e^ voorstel, is t'begheerde swaerheyts middelpunt, t'welck wy bewysen moesten. T'BESLVYT. Wesende dan ghegheuen een ghecorte brantsne, wy hebben huer swaerheyts middelpunt gheuonden naer den eysch.



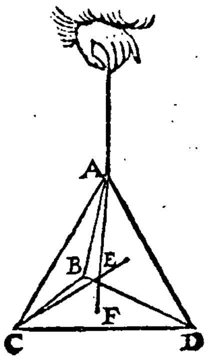
NV VANDE VINDING DER SWAERHEYTS MIDDELPVNTEN VANDE LICHAMEN.

IX. VERTOOCH. XIIII. VOORSTEL.

YDER lichaems formens middelpunt, is oock sijn swaerheyts middelpunt.

Tetraedron.

T'GHEGHEVEN. Laet A B C D een* viergrondich wesen, diens formens middelpunt E sy, ende den as van A duer E, tot in F, middelpunt des driehoucx B C D, sy A F. T'BEGHEERDE. Wy moeten bewysen dat E oock is sijn swaerheyts middelpunt. T'BEWYS. Laet ons t'lichaem ophanghen byde lini A F, maer het viergrondich bestaet uyt vier euen ende ghelijcke naelden een selfder ghestalt, wiens ghemeene sop E, daerom A F is des lichaems swaerheyts middellini, ende om de selue reden sal de lini C E oock des swaerheyts middellini sijn; E dan is oock het swaerheyts



L 2 middelpunt

PROBLEM V. PROPOSITION XIII.

Given a truncated parabola: to find its centre of gravity.

SUPPOSITION. Let ABCD be a truncated parabola (to wit, that AB shall be parallel to DC), whose diameter shall be EF. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated parabola shall be completed, by adding the missing part *ABG*. After that, *H* shall be marked so that *GH* shall be to *HE* as 3 to 2. In the same way *I*, so that *GI* shall be to *IF* as 3 to 2. After that *K*, so that *IH* shall have to *IK* the same ratio as the truncated parabola *ABCD* to the parabola *ABG*. I say that *K* is the required centre of gravity. PROOF. *I* is the centre of gravity of the whole, and *H* of the part, and as the other part is to this part, so is *HI* to *IK*. Therefore, by the 9th proposition, *K* is the required centre of gravity, which we had to prove [1]. CONCLUSION. Given therefore a truncated parabola, we have found its centre of gravity, as required.

NOW ABOUT THE FINDING OF THE CENTRES OF GRAVITY OF SOLID FIGURES [2]

THEOREM IX. PROPOSITION XIV.

The geometrical centre of any solid is also its centre of gravity [3].

SUPPOSITION. Let *ABCD* be a tetrahedron, whose geometrical centre shall be *E*, while the axis from *A* through *E* to *F*, the centre of the triangle *BCD*, shall be *AF*. WHAT IS REQUIRED TO PROVE. We have to prove that *E* is also its centre of gravity. PROOF. Let us hang the solid by the line *AF*. But the tetrahedron consists of four equal and similar pyramids of the same form [4], whose common vertex is *E*; therefore *AF* is centre line of gravity of the solid, and for the same reason the line *CE* will also be centre line of gravity. Therefore *E* is also the centre of gravity. The same proof also holds for all solids having

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[1]) This result is rather disappointing: the demonstration contains no more than the principle on which the determination of the centre of gravity of the truncated segment might be based. The determination itself had been given by Archimedes in the highly elaborate propositions 9 and 10 of the second book of the work *On the Equilibrium of Plane Figures*. The result is far from simple. If $GF = a$, $GE = b$, it is as follows:

$$\frac{IF}{EF} = \frac{3b\sqrt{b} + 6b\sqrt{a} + 4a\sqrt{b} + 2a\sqrt{a}}{5b\sqrt{b} + 10b\sqrt{a} + 10a\sqrt{b} + 5a\sqrt{a}}.$$

[2]) Most of the propositions of the second section of Book II being simply three-dimensional analogues of propositions on plane figures in the first section, they give rise to the same remarks; accordingly, we will confine ourselves to stating the correspondence.

[3]) Cf. Prop. 1.

[4]) The tetrahedron is obviously supposed to be regular.

middelpunt. Sghelijcx ſal oock t'bewys ſijn van allen lichamen hebbende middelpunten der form, ſoo wel vermeerde ende ghecorte gheſchicte lichamen, als gheſchicte, want ſoomenſe ophangt byde middellinien deur eenighen lichamelicken houck, ofte duer het middelpunt haerder gronden ende des formens middelpunt, ſoo hebben al de naelden (wiens ghemeene ſop het formens middelpunt, ende gronden de platten des lichaems ſijn) tot allen ſijden ghelijcke gheſtalt, daerom oock duer ghemeene wetenſchap, ende duer de 1e begheerte des 1en boucx, alles hangt an die lini euewichtich, ende veruolghens de ſne ſulcker twee ſwaerheyts middellinien malcander ſniende in des formens middelpunt, is ock het ſwaerheyts middelpunt. T'BESLVYT. Yder lichaems formens middelpunt dan, is oock ſijn ſwaerheyts middelpunt.

X. VERTOOCH. XV. VOORSTEL.

YDER pilaers ſwaerheyts middelpunt is int middel vanden as.

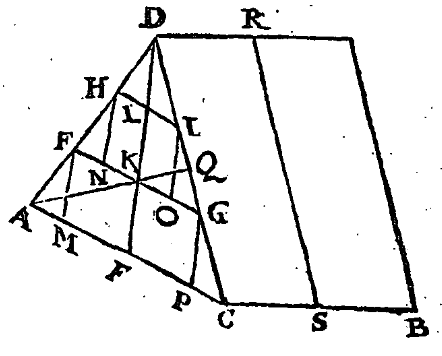
1e VOORBEELT.

T'GHEGHE. Laet A B een driehouckich pilaer ſijn diens grondt A C D. T'BEGHEERDE. Wy moeten bewyſen dat ſijn ſwaerheyts middelpunt int middel vanden as is. T'BEREYTSEL. Laet ons trecken van D tot E int middel van A C de lini D E: Daer naer F G ende H I euewydighe van A C, ſniende D E inde punten K, L, daer naer de linien F M, H N, I O, G P, euewydighe met D E, daer naer van A tot Q int middel der ſijde D C, de lini A Q: Laet ſghelijcx oock beſchreuen worden het decſel, ende laet ons den pilaer doorſnien met een *plat R S euewidich met den grondt A D C, ende S ſy int middel van C B.

Plano.

T'BEWYS. T'plat ghetrocken duer D E, ende duer haer *lijckſtandighe int deckſel, deelt den binneſchreuen pilaer uyt die twee vierhouckighe pilaren vergaert, in twee euen ende ghelijcke deelen, ende van ghelijcke gheſtalt; het doorſnijt dan dier binneſchreuen pilaers ſwaerheydts middelpunt. Maer hoe datter ſulcke vierhouckighe pilaren meer beſchreuen ſijn inden ghegheuen driehouckighen, hoe dat deſe min verſchilt van die; wy connen dan duer dat oneindelick

Homologam.



naerderen

geometrical centres, augmented and truncated regular solids as well as regular solids, for if they are hung by the centre lines through some corporeal angle, or through the centre of their bases and the geometrical centre, all the pyramids (whose common vertex is the geometrical centre, while the bases are the faces of the solid) have the same form in every direction. Therefore also, by common knowledge and by the 1st postulate of the 1st book, everything hangs balanced from that line. And consequently the point of intersection of two such centre lines of gravity, intersecting in the geometrical centre, is also the centre of gravity. CONCLUSION. The geometrical centre therefore of any solid is also its centre of gravity.

THEOREM X. PROPOSITION XV.

The centre of gravity of any prism is in the middle point of the axis [1]).

EXAMPLE I.

SUPPOSITION. Let *AB* be a triangular prism, whose base shall be *ACD*. WHAT IS REQUIRED TO PROVE. We have to prove that its centre of gravity is in the middle point of the axis. PRELIMINARY. Let us draw from *D* to *E* [2]) in the middle point of *AC* the line *DE*, after this *FG* and *HI* parallel to *AC*, intersecting *DE* in the points *K*, *L*; then the lines *FM*, *HN*, *IO*, *GP* parallel to *DE*, after this from *A* to *Q*, in the middle point of the side *DC*, the line *AQ*. Let the cover also be constructed in the same way, and let us intersect the prism by a plane *RS* parallel to the base *ADC*, and *S* shall be in the middle point of *CB*. PROOF. The plane drawn through *DE* and through its homologue in the cover divides the inscribed prism, composed of the two quadrangular prisms, into two equal and similar parts having the same form. It therefore passes through the centre of gravity of the said inscribed prism. But the more of such quadrangular prisms there are inscribed in the given triangular prism, the less the latter will differ from the former. We can therefore, by infinite approximation, inscribe

[1]) Here, as well as in subsequent propositions, axis means the line joining the centres of gravity of the two parallel faces of a prism, or the line joining the vertex of a pyramid with the centre of gravity of the base.

[2]) In the drawing this letter has been erroneously replaced by *F*.

naerderen sulck een form binnen den ghegheuen pilaer beschrijuen, dat haer verschil vande binnescreuen minder sal wesen dan eenich ghegheuen lichaem hoe cleen het sy, waer uyt volgt dat het staltwicht des deels D E C B ouer d'een sijde des plats, min verschillen sal van t'staltwicht des deels ouer d'ander sijde des plats, dan eenich lichaem datmen soude connen gheuen hoe cleen het sy, waer uyt ick aldus strie:

A. *Neuen alle verschillende staltswaerheden can een swaerheyt ghestelt worden minder dan haer verschil;*

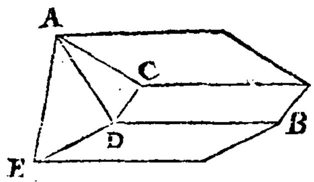
O. *Neuen dese staltswaerheden en can gheen swaerheyt ghestelt worden minder dan haer verschil;*

O. *Dese staltswaerheden dan en verschillen niet.*

Daerom t'plat duer D E ende haer * lickstandighe int decksel, lijt duer t'swaerheyts middelpunt des ghegheuen pilaers, ofte het swaerheyts middelpunt is in dat plat. Ende om de selue reden ist oock int plat duer A Q, ende haer lijckstandighe int decksel. Maer deser twee platten ghemeene sne is de rechte lini tusschen de swaerheyts middelpunten des grondts ende decsels, welcke lini den as is des ghegeuen pilaers, t'swaerheyts middelpunt dan is inden as, het is oock int plat duer R S, want t'selue deelt den pilaer in twee euen, ghelijcke, ende lijckstandighe deelen; Maer dat plat doorsnijt den as in sijn middel, het swaerheyts middelpunt dan is in des as middel. *Homologam.*

II^e VOORBEELT.

T'GHEGHEVEN. Laet A B een vierhouckich pilaer wesen, diens grondt A C D E. T'BEGHEERDE. Wy moeten bewysen dat sijn swaerheyts middelpunt int middel vanden as is. T'BERYTSEL. Laet ons trecken een plat duer A D, ende haer lijckstandighe int decksel, deelende alsoo den ghegheuen pilaer in twee driehouckighe pilaren, welcker yder het swaerheyts middelpunt int middel vanden as heeft duer het 1^e voorbeelt, daerom ghetrocken den balck tusschen die twee punten, ende den seluen ghedeelt in sijn ermen, het onderscheydt der ermen sal het swaerheyts middelpunt sijn des ghegheuen pilaers, welck punt valt in t'swaerheyts middelpunt des plats euewydich vanden grondt den pilaer in twee euen stucken deelende, ende t'selue int middel der lini tusschen de swaerheyts middelpunten des gronts ende decksels, dat is int middel vanden as; T'selue salmen oock alsoo bethoonen in yder pilaer.



T'BESLVYT. Yder pilaers swaerheyts middelpunt dan, is int middel vanden as, t'welck wy bewysen moesten.

within the given prism a figure such that its difference from the inscribed figure shall be less than any given solid, however small, from which it follows that the apparent weight of the part *DECB* to one side of the plane will differ less from the apparent weight of the part to the other side of the plane than any solid that might be given [1]), however small, from which I argue as follows [2]):

A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*

O. *Beside the present apparent gravities there cannot be placed any gravity less than their difference;*

O. *Therefore the present apparent gravities do not differ.*

Therefore the plane through *DE* and its homologue in the coves passes through the centre of gravity of the given prism, or the centre of gravity is in the said plane. And for the same reason it is also in the plane through *AQ* and its homologue in the cover. But the common intersecting line of these two planes is the straight line joining the centres of gravity of the base and the cover, which line is the axis of the given prism. The centre of gravity therefore is in the axis. It is also in the plane through *RS*, because the latter divides the prism into two equal, similar, and homologous parts. But this plane intersects the axis in its middle point. The centre of gravity therefore is in the middle point of the axis.

EXAMPLE II.

SUPPOSITION. Let *AB* be a quadrangular prism, whose base is *ACDE*. WHAT IS REQUIRED TO PROVE. We have to prove that its centre of gravity is in the middle point of the axis. PRELIMINARY. Let us draw a plane through *AD* and its homologue in the cover, thus dividing the given prism into two triangular prisms, each of which has the centre of gravity in the middle point of the axis by the 1st example. Therefore, the beam being drawn between those two points and being divided into its arms, the point of division of the arms will be the centre of gravity of the given prism, which point falls in the centre of gravity of the plane, parallel to the base, dividing the prism into two equal parts. It is then in the middle point of the line joining the centres of gravity of the base and the cover, that is in the middle point of the axis. This can also be shown of any prism. CONCLUSION. The centre of gravity of any prism therefore is in the middle point of the axis, which we had to prove.

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[1]) Cf. the note on Prop. 2.

[2]) See note 2 to p. 143

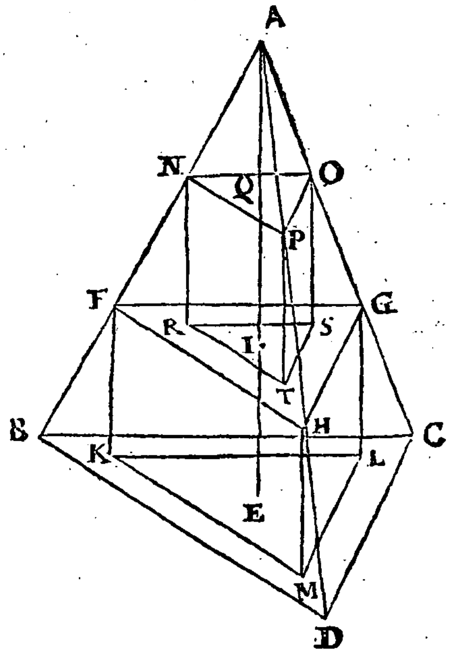
XI. VERTOOCH. XVI. VOORSTEL.

Pyramidis.

YDER* naeldens ſwaerheyts middelpunt is inden as.

T'GHEGHEVEN. Laet ABCD een naelde ſijn, diens grondt den driehouck BCD, wiens ſwaerheyts middelpunt E, ende den as AE.

T'BEGHEERDE. Wy moeten bewyſen dat des naeldens ſwaerheyts middelpunt inden as AE is. T'BEREYTSEL. Laet ons de naelde ſnien met een plat FGH euewydich met BCD, ende ſniende den as AE in I: Laet oock ghetrocken worden FK, GL, HM, euewydich vanden as AE, alſoo dat de punten K, L, M, int plat ſijn des driehoucx BCD, inder voughen dat FGHKLM een pilaer is, wiens grondt IKL euen ende ghelijck is an het deckſel FGH, ende ghelijck anden grondt BCD; Daer naer ghelijck de naelde doorſneen was met FGH, laetſe noch eenmal alſoo doorſneen ſijn met het plat NOP, ſniende den as in Q, ende daer uyt oock alſoo ghemaect den pilaer NOPRST, te weten NR, OS, PT, euewydich vanden as AE, ende de punten R, S, T, int plat FGH. T'BEWYS. Anghesien de driehoucken NOP, RST, FGH, KLM, alle ghelijck ſijn anden driehouck BCD, ende dat haer punten Q, I, E, in haer ſulcken ſtant hebben als E inde driehouck BCD, ende dat E des driehoucx BCD ſwaerheyts middelpunt is, ſoo ſijn oock die Q, I, E, haer driehouckens ſwaerheyts middelpunte, waer duer IE as is des pilaers FGHKLM, in wies middel huer ſwaerheyts middelpűt is duer het 14ᵉ voorſtel. Sghelijcx is QI oock as des pilaers NOPRST, in wiens middel huer ſwaerheyts middelpűt is, eñ vervolgens het ſwaerheyts middelpunt des lichaems uyt die twee pilaren vergaert is in QE, daerõ oock in AE; Maer hoe datter inde naelde ſulcke



pilaren meer beſchreuen worden, hoe dattet verſchil der naelde ende der binne-

THEOREM XI. PROPOSITION XVI

The centre of gravity of any pyramid is in the axis [1]).

SUPPOSITION. Let *ABCD* be a pyramid, whose base is the triangle *BCD*, the centre of gravity of the latter being *E*, and the axis being *AE*. WHAT is REQUIRED TO PROVE. We have to prove that the centre of gravity of the pyramid is in the axis *AE*. PRELIMINARY. Let us intersect the pyramid by a plane *FGH*, parallel to *BCD* and intersecting the axis *AE* in *I*. Let there also be drawn *FK*, *GL*, *HM* parallel to the axis *AE*, in such a way that the points *K*, *L*, *M* are in the plane of the triangle *BCD*, so that *FGHKLM* is a prism, whose base *IKL* is equal and similar to the cover *FGH*, and similar to the base *BCD*. After this, as the pyramid was intersected by *FGH*, let it be intersected once more in this way by the plane *NOP*, intersecting the axis in *Q*, and let there also be constructed from this the prism *NOPRST*, to wit *NR*, *OS*, *PT* parallel to the axis *AE*, and the points *R*, *S*, *T* in the plane *FGH*. PROOF. Since the triangles *NOP*, *RST*, *FGH*, *KLM* are alle similar to the triangle *BCD*, and the points *Q*, *I*, *E* therein have the same position as *E* in the triangle *BCD*, and *E* is the centre of gravity of the triangle *BCD*, those points *Q*, *I*, *E* are also the centres of gravity of their triangles, in consequence of which *IE* is the axis of the prism *FGHKLM*, in whose middle point is its centre of gravity, by the 14th proposition. In the same way *QI* is also the axis of the prism *NOPRST*, in whose middle point is its centre of gravity, and consequently the centre of gravity of the solid composed of these two prisms is in *QE*, and therefore also in *AE*. But the more of such prisms there are inscribed in the pyramid, the less will be the difference between the pyramid and

[1]) Cf. Prop. 2.

binneschreuen form van sulcke pilaren vergaert, minder is, blijuende nochtan het swaerheyts middelpunt der binneschreuen form altijt inden as A E; Wy connen dan duer dat oneindelick naerderen sulcken form binnen de naelde stellen, dattet verschil tusschen haer ende de naelde, minder sal wesen dan eenich ghegheuen lichaem hoe cleen het sy, waer uyt volght dat stellende A E voor swaerheyts middellini, der naelde, soo sal het staltwicht van d'een sijde tot d'ander, min verschillen dan eenighe swaerheyt diemen soude connen gheuen, waer uyt ick aldus strie:

A. *Neuen alle verschillende staltswaerheden can een swaerheyt ghestelt worden minder dan haer verschil;*

O. *Neuen dese staltswaerheden en can gheen swaerheyt ghestelt worden minder dan haer verschil;*

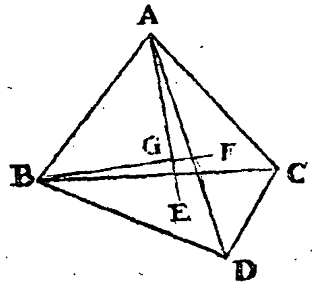
O. *Dese staltswaerheden dan en verschillen niet.*

Sghelijcx sal oock t'bewys sijn val naelden wiens gronden sijn Vierhoucken, Veelhoucken, Ronden &c. T'BESLVYT. Yder naeldens swaerheyts middelpunt dan is inden as.

VI. EYSCH. XVII. VOORSTEL.

WESENDE ghegheuen een naelde: Huer swaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A B C D een naelde wesen, diens grondt sy den driehouck B C D. T'BEGHEERDE. Wy moeten haer swaerheyt middelpunt vinden. T'WERCK. Men sal de swaerheyts middelpunten vinden van eenighe twee driehoucken, als E van B C D, ende F van A D C, treckende de linien A E, B F; welcker sne G, ick seg te wesen het begheerde swaerheyts middelpunt.



T'BEWYS. Des naldens A B C D swaerheyts middelpunt is in A E, ende oock in B F, duer het 16 voorstel, het is dan nootsaeclick G. T'BESLVYT. Wesende dan ghegheuen een naelde: Wy hebben huer swaerheyts middelpunt gheuonden naer den eysch.

XII. VERTOOCH. XVIII. VOORSTEL.

HET swaerheyts middelpunt van yder naelde deelt den as alsoo, dat het stick naer den houck drievoudich is an t'ander.

T'GHE-

the inscribed forms of such prisms, the centre of gravity of the inscribed form, however, always remaining in the axis *AE*. We can therefore, by infinite approximation, place within the pyramid a form such that the difference between the latter and the pyramid shall be less than any given solid, however small, from which it follows that, taking *AE* as centre line of gravity of the pyramid, the apparent weight of one side will differ less from the other than any gravity that might be given, from which I argue as follows [1]:

A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*

O. *Beside the present apparent gravities there cannot be placed any gravity less than their difference;*

O. *Therefore the present apparent gravities do not differ.*

The same proof also holds for pyramids whose bases are quadrilaterals, polygons, circles, etc. CONCLUSION. The centre of gravity of any pyramid therefore is in the axis.

PROBLEM VI. PROPOSITION XVII.

Given a pyramid: to find its centre of gravity [2].

SUPPOSITION. Let *ABCD* be a pyramid, whose base shall be the triangle *BCD*. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The centres of gravity shall be found of any two triangles, as *E* of *BCD* and *F* of *ADC*, and the lines *AE*, *BF* shall be drawn; whose point of intersection *G*, I say is the required centre of gravity. PROOF. The centre of gravity of the pyramid *ABCD* is in *AE*, and also in *BF*, by the 16th proposition. It is therefore necessarily *G*. CONCLUSION. Given therefore a pyramid: we have found its centre of gravity, as required.

THEOREM XII. PROPOSITION XVIII.

The centre of gravity of any pyramid divides the axis in such a way that the segment adjacent to the angle is three times the other segment [3].

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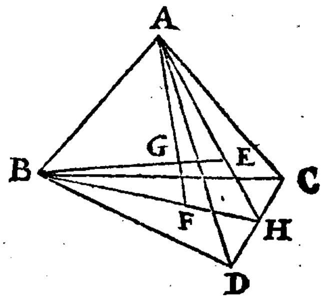
[1]) See note 2 to p. 143.
[2]) Cf. Prop. 3.
[3]) Cf. Prop. 4.

T'GHEGHEVEN. Laet A B C D een driehouckighe naelde wesen, diens sop A, ende grondt B C D, ende den as van B tot int swaerheyts middelpunt E des driehoucx A D C, sy B E, ende van A tot int swaerheyts middelpunt F des driehoucx B C D sy A F, sniende B E in G, voor t'swaerheyts middelpunt der ghegheuen naelde. T'BEGHEERDE. Wy moeten bewysen dat B G drievoudich is an G E. T'BEREYTSEL. Laet ons trecken van H middel van C D, de linien H A, H B.

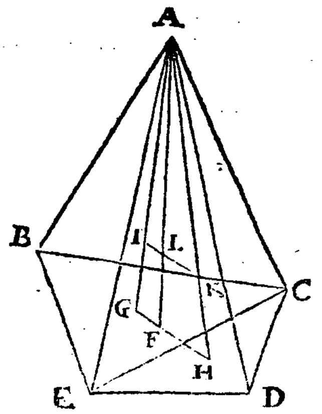
T'BEWYS. Ouermits H A ghetrocken is uyt het middel van D C tot inden houck A, ende dat E t'swaerheyts middelpunt is des driehoucx A C D, soo sal A E dobbel sijn an E H duer het 4^e^ voorstel, ende om de selue reden sal B F dobbel wesen an F H. Dit soo sijnde, ghetrocken de reden E A 2, tot A H 3, vande reden B F 2, tot F H 1 (dat is *Reden* $\frac{3}{2}$ van *Reden* $\frac{2}{1}$) * daer rest de reden van B G tot G E: Maer treckende *Reden* $\frac{2}{3}$ van *Reden* $\frac{2}{1}$ daer blijft *Reden* $\frac{3}{1}$. B G dan is tot G E, als 3 tot 1.

Door t'verkeerde des 12 cap. 1. lib. Almag. Ptol.



MAER soo des ghegheuen naeldens grondt een vierhouck waer, t'voorstel sal in die oock alsoo bewesen worden: Laet by voorbeelt A B C D E een naelde wesen, wiens grondt een vierhouck B C D E, ende as A F sy. Nu dese vierhouckighe naelde ghedeelt in twee driehouckighe, wiens gronden E C B, ende E C D, diens assen A G, ende A H, wiens swaerheyts middelpunten I, K, des heelen naeldens swaerheyts middelpunt sal inde lini I K wesen, tis oock in A F duer het 16^e^ voorstel, tis dan L: Maer want A G H een driehouck is, ende I K evewydich van G H (want I G is t'vierendeel van G A, ende H K t'vierendeel van H A daerom &c.) soo sal A L sulcken reden hebben tot L F, als A I, tot I G, dat is drieuoudich. Sghelijcx sal oock t'bewys sijn in alle naelde met veelsidighen grondt.

2. v. 6. B. E.



MAER de naelde een keghel sijnde, te weten dat den grondt waer een rondt ofte lanckrondt, t'selfde sal daerin oock alsoo bewesen worden

SUPPOSITION. Let $ABCD$ be a triangular pyramid, whose vertex shall be A and the base BCD, and the axis from B to the centre of gravity E of the triangle ADC shall be BE, and that from A to the centre of gravity F of the triangle BCD shall be AF, intersecting BE in G, which is the centre of gravity of the given pyramid. WHAT IS REQUIRED TO PROVE. We have to prove that BG is three times GE. PRELIMINARY. Let us draw from H, the middle point of CD, the lines HA, HB. PROOF. Since HA has been drawn from the middle point of DC to the angle A, and E is the centre of gravity of the triangle ACD, AE will be double of EH by the 4th proposition, and for the same reason BF will be double of FH. This being so, if the ratio of EA (2) to AH (3) is subtracted from the ratio of BF (2) to FH (1) (i.e. ratio $\frac{3}{2}$ from ratio $\frac{2}{1}$), there remains the ratio of BG to GE. But if the ratio $\frac{2}{3}$ is subtracted from the ratio $\frac{2}{1}$, there remains the ratio $\frac{3}{1}$. GB therefore is to GE as 3 to 1 [1]).

But if the base of the given pyramid be a quadrilateral, the proposition will also be proved of this in the following way. Let, for example, $ABCDE$ be a pyramid, whose base shall be a quadrilateral $BCDE$ and the axis AF. Now if this quadrangular pyramid is divided into two triangular ones, whose bases shall be ECB and ECD, the axes AG and AH, and the centres of gravity I, K, the centre of gravity of the whole pyramid will be in the line IK. It is also in AF, by the 16th proposition; it is therefore L. But because AGH is a triangle, and IK is parallel to GH (for IG is the fourth part of GA, and HK the fourth part of HA; therefore, etc.), AL will have to LF the same ratio as AI to IG, that is 3 to 1. The same proof also holds for any pyramid with a polylateral base.

But if the pyramid be a cone, to wit that the base be a circle or an ellipse, the same proposition will also be proved thereof as follows. For it is obvious from

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[1]) Cf. the note on Prop. 4 (p. 233). Menelaus's theorem is here applied to the triangle EBH with transversal GFA, giving:

$$\frac{GE \cdot FB \cdot AH}{GB \cdot FH \cdot AE} = 1; \text{ therefore } \frac{GB}{GE} = \frac{2}{1} \cdot \frac{3}{2} = \frac{3}{1}.$$

worden, want het is duer t'voorgaende kennelick, dat alle veelhouckighe naelde in haer beschreuen, t'swaerheyts middelpunt alsoo sal hebben, dattet opperste deel drievoudich is teghen het onderste. Maer hoe de naelde daer in beschreuen van meer houcken is, hoe die binneschreuen naeldens grootheyt vande ronde naelde min verschilt, daerom oock connen wy din het oneindelick naerderen, een binneschreuen setten, min verschillende vande veruatende, dan eenich ghegheuen lichaem hoe cleen het sy; Daerom oock de langde der plaets van diens swaerheyts middelpunt tot deses, corter soude moeten wesen dan eenighe langde die mueghelick is ghegheuen te worden, waer uyt ick aldus strie:

A. *Neuen alle twee punten in verscheyden plaetsen staende, connen twee punten ghestelt worden die malcander naerder sijn;*

O. *Neuen dese twee punten en connen gheen twee punten ghestelt worden die malcander naerder sijn;*

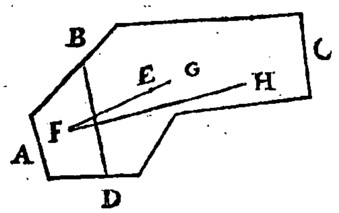
O. *Dese twee punten dan en staen in gheen verscheyden plaetsen.*

T'BESLVYT. Het swaerheyts middelpunt dan van yder naelde, deelt den as alsoo, dat het stuck naer den houck drieuoudich is an t'ander.

VII EYSCH. XIX VOORSTEL.

WESENDE ghegheuen t'swaerheyts middelpunt eens lichaems ende sijns deels, wiens reden an t'ander deel kennelick is: Het swaerheyts middelpunt des ander deels te vinden.

T'GHEGHEVEN. Laet A B C D een lichaem sijn, diens swaerheyts middelpunt E, ende B D A deel des lichaems, wiens swaerheyts middelpunt F. T'BEGHEERDE. Wy moeten t'swaerheyts middelpunt vinden des ander deels B C D. T'WERCK. Men sal trecken F E tot in G, alsoo dat F E sulcken reden hebbe tot E G, als t'stick B D C tottet stick B D A; Ick seg dat G t'begheerde swaerheyts middelpunt is, des ander sticx B D C; waer af t'bewys ghelijck sal sijn an t'bewys des 9e voorstels. Wy souden oock moghen voorbeelt setten van een heele cloot, wiens ander deel oock een cloot sy, maer sulcx is openbaer ghenouch duer het tweede voorbeelt des boueschreuen 9en voorstels in ronden. T'BESLVYT. Wesende dan ghegheuen t'swaerheydts middelpunt eens lichaems ende sijns deels, wiens reden an t'ander deel



M kennelick

the above that any polygonal pyramid inscribed therein will have the centre of gravity in such a place that the upper part shall be three times the lower part. But the more angles the pyramid described therein has, the less the magnitude of this inscribed pyramid will differ from the cone. We can therefore, by infinite approximation, inscribe a pyramid differing less from the containing cone than any given solid, however small. Therefore also the distance between the place of the centre of gravity of the former and that of the latter would have to be less than any distance that may be given, from which I argue as follows [1]:

A. *Beside any two points in different places there can be placed two points which are nearer to one another;*

O. *Beside the present two points there cannot be placed two points which are nearer to one another;*

O. *Therefore the present two points are not in different places.*

CONCLUSION. The centre of gravity therefore of any pyramid divides the axis in such a way that the segment adjacent to the angle is three times the other segment.

PROBLEM VII. PROPOSITION XIX.

Given the centre of gravity of a solid and that of a part thereof, the ratio of which to the other part is known: to find the centre of gravity of the other part [2].

SUPPOSITION. Let *ABCD* be a solid whose centre of gravity shall be *E*, and *BDA* a part of the solid, whose centre of gravity shall be *F*. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the other part *BCD*. CONSTRUCTION. *FE* shall be drawn up to *G*, in such a way that *FE* shall have to *EG* the same ratio as the part *BDC* to the part *BDA*. I say that *G* is the required centre of gravity of the other part *BDC*, the proof of which will be similar to the proof of the 9th proposition. We might also take as example a complete sphere, whose other part shall also be a sphere, but this is sufficiently manifest from the second example of the 9th proposition described above with regard to circles. CONCLUSION. Given therefore the centre of gravity of a solid

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[1] See note 2 to p. 143.
[2] Cf. Prop. 9.

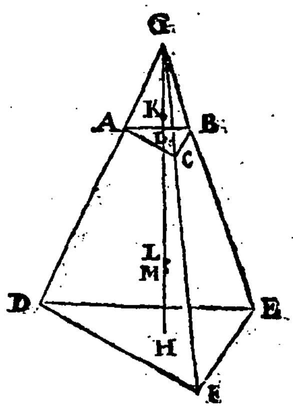
kennelick is, wy hobben t'ſwaerheyts middelpunt des ander deels gheuonden, naer den eyſch.

VIII EYSCH. XX VOORSTEL.

Pyramis truncata.

WESENDE ghegheuen een* ghecorte naelde: Huer ſwaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A B C D E F een ghecorte naelde ſijn, diens deckſel A B C, ende grondt D E F. T'BEGHEERDE. Wy moeten huer ſwaerheyts middelpunt vinden. T'WERCK. Men ſal de ghecorte naelde volmaken, daer an ſtellende het ghebrekende A B C G, vindende H ſwaerheyts middelpunt des driehoucx D E F, treckende den as G H, wiens punt inden driehouck A B C, ſy I, daernaer ſalmen teeckenen K, alſoo dat G K drievoudich ſy an K I: Inſghelijcx L, alſo dat G L drievoudich ſy an L H, teeckenende M, alſoo dat K L ſulcken reden hebbe tot L M, ghelijck de ghecorte naelde A B C D E F, tot de naelde A B C G, Ick ſeg dat M t'begheerde ſwaerheyts middelpunt is.



T'BEWYS. L is ſwaerheyts middelpunt des heels, ende K des deels, ende ghelijck t'onderſte deel totten bouenſte, alſo K L tot L M, Daerom M, door het 1ᵉ voorſtel des 1ᵉⁿ boucx is t'begheerde ſwaerheyts middelpunt, t'welck wy bewyſen moeſten. Sghelijcx ſal oock den voortganck ſijn in allen anderen ghecorte naelden. T'BESLVYT. Weſende dan ghegheuen een ghecortenaelde: Wy hebben huer ſwaerheyts middelpunt gheuonden, naer den eyſch.

IX. EYSCH. XXI. VOORSTEL.

WESENDE ghegheuen een platgrondich lichaem ſoodanigher form alſt valt: Sijn ſwaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A een ongheſchickt platgrondich lichaem ſijn, dat is omvanghen in platten ſo veel alſt ſy. T'BEGHEERDE. Wy moeten ſijn ſwaerheyts middelpunt vinden. T'WERCK. Men ſal

t'lichaem

and that of a part thereof, the ratio of which to the other part is known, we have found the centre of gravity of the other part, as required.

PROBLEM VIII. PROPOSITION XX.

Given a truncated pyramid: to find its centre of gravity.

SUPPOSITION. Let *ABCDEF* be a truncated pyramid, whose cover shall be *ABC* and the base *DEF*. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated pyramid shall be completed, adding thereto the missing part *ABCG*, finding *H*, the centre of gravity of the triangle *DEF*, drawing the axis *GH*, whose point in the triangle *ABC* shall be *I*, and after this, *K* shall be marked in such a way that *GK* shall be three times *KI*. Likewise *L* in such a way that *GL* shall be three times *LH*, marking *M* in such a way that *KL* shall have to *LM* the same ratio as the truncated pyramid *ABCDEF* to the pyramid *ABCG*. I say that *M* is the required centre of gravity. PROOF. *L* is the centre of gravity of the whole, and *K* that of the part, and as the lower part is to the upper, so is *KL* to *LM*. Therefore, by the 1st proposition of the 1st book, M is the required centre of gravity, which we had to prove. The same procedure may also be applied to any other truncated pyramid. CONCLUSION. Given therefore a truncated pyramid, we have found its centre of gravity, as required.

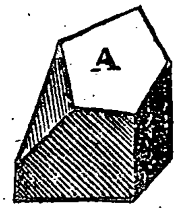
PROBLEM IX. PROPOSITION XXI

Given a polyhedron of any form whatever: to find its centre of gravity [1]).

SUPPOSITION. Let A be an irregular polyhedron, enveloped by any number of faces that may occur. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The solid shall be divided into the smallest

[1]) Cf. Prop. 6.

t'lichaem deelen inde* nalden dieder ten weynichsten ende bequamelicxt uyt vallen willen. Ten quaetsten commende men can als duer ghemeene reghel, alle platgrondich lichaem in soo veel naelden deelen alst platten heeft, stellende eenich punt int lichaem voor haer ghemeene sop. Dit soo sijnde, men sal yder naeldens swaerheyts middelpunt vinden duer het 17^{e} voorstel. Daer naer om te vinden t'ghemeene swaerheydts middelpunt van twee naelden, men sal tusschen haer swaerheyts middelpunten een balck trecken, die deelende in sulcken reden als haer twee naelden tot malcanderen sijn, weluerstaende t'cortste deel naer de swaerste naelde. Ende inder seluer vougen salmen daertoe vergaderen de derde naelde, ende alle d'ander, ende t'punt den balck alsoo ten laetsten deelende, sal t'begheerde swaerheyts middelpunt sijn, waer af t'bewys openbaer is. *Pyramidi.*



T'BESLVYT. Wesende dan ghegheuen een platgrondich lichaem soodanigher form alst valt, Wy hebben sijn swaerheyts middelpunt gheuonden, naer den heysch.

XIII. VERTOOCH. XXII. VOORSTEL.

YDER * branders swaerheyts middelpunt is inden as.

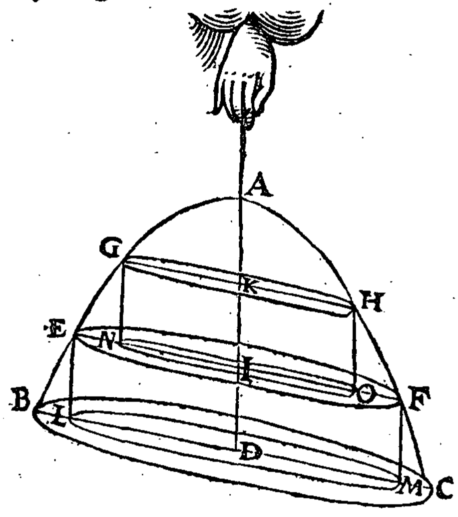
Conoidale rectangulũ.

Het swaerheyts middelpunt des rechten branders inden as te wesen is duer ghemeene wetenschap openbaer, wy sullen dan alleenelick t'voorbeelt stellen des gheens diens as opden grondt cromhouckich is.

T'GHEGHEVEN. Laet A B C een brander wesen diens grondt B C sy, ende den as A D daerop cromhouckich.

T'BEGHEERDE. Wy moetẽ bewysen dattet swaerheyts middelpunt in A D is.

T'BEREYTSEL. Laet ons den brander snien met twee platten E F, G H euewydich vanden grondt B C, welcker ghemeene sneen met den as A D, sijn I, K; Ende laet ons trecken de linien E L, F M, G N, H O:



 ende

and most suitable number of pyramids that can be made from it. In the worst case, any polyhedron can by a common rule be divided into as many pyramids as it has faces, taking some point in the solid for their common vertex. This being so, the centre of gravity of each pyramid shall be found by the 17th proposition. After this, in order to find the common centre of gravity of two pyramids, there shall be drawn between their centres of gravity a beam, which shall be divided in the same ratio as its two pyramids have to one another, that is to say: the shorter segment adjacent to the heavier pyramid. And in the same way the third pyramid shall be combined therewith, and all the others, and the final point of division of the beam will be the required centre of gravity, the proof of which is manifest. CONCLUSION. Given therefore a polyhedron of any form whatever, we have found its centre of gravity, as required.

THEOREM XIII. PROPOSITION XXII.

The centre of gravity of any paraboloid is in its axis [1]).

It is manifest by common knowledge that the centre of gravity of a right paraboloid is in its axis. We shall therefore only give the example of a paraboloid whose axis is at oblique angles to the base.
SUPPOSITION. Let *ABC* be a paraboloid, whose base shall be *BC*, and the axis *AD* at oblique angles thereto. WHAT IS REQUIRED TO PROVE. We have to prove that the centre of gravity is in *AD*. PRELIMINARY. Let us intersect the paraboloid by two planes *EF*, *GH*, parallel to the base *BC*, whose points of intersection with the axis *AD* shall be *I*, *K*. And let us draw the lines *EL*, *FM*,

[1]) Cf. Prop. 10. The reader may be reminded that paraboloid means a segment of a paraboloid of revolution. If this segment is cut off by a plane perpendicular to the axis of revolution, the segment is called „rechte brander" (right paraboloid).

Ellipses. ende LM, N O, G H, sullen*lancronden wesen ghelijck an t'lanckront B C; ende laet E M met G O pilaren sijn uyt de selue beschreuen.

Semidiameter. T'BEWYS. Want L D * halfmiddellini des lancrondts L M euen is an D M, oock an N I, ende I O, soo sal I D as sijn des pilaets E M, inde welcke diens pilaers swaerheyts middelpunt is: Ende om de selue reden sal t'swaerheyts middelpunt des pilaers G O oock wesen in K I, ende veruolghens t'swaerheyts middelpunt des lichaems uyt die twee pilaren vergaert is in K D, daerom oock in A D. Maer hoe datter sulcke pilaren indē brander meer beschreuen worden, hoe dattet verschil des branders ende der binneschreuen form van sulcke pilaren vergaert, minder is. Wy connen dan duer dat oneindelick naerderen sulcken form binnen den brander stellen, dat huer verschil minder sal wesen, dan eenich ghegheuen lichaem hoe cleen het sy; Waer uyt volght dat stellende A D voor swaerheyts middellini des branders, soo sal t'staltwicht van d'een sijde tot d'ander, min verschillen dan eenighe swaerheyt diemen soude connen gheuen, waer uyt ick aldus strie:

A. *Neuen alle verschillende staltswaerheden, can een swaerheyt ghestelt worden minder dan haer verschil;*

O. *Neuen dese staltswaerheden van d'eene en dander sijde des branders, en can gheen swaerheyt ghestelt worden minder dan haer verschil;*

O. *Dese staltswaerheden dan van d'eene ende dander sijde des branders en verschillen niet.*

Daerom A D is haer swaerheyts middellini. T'BESLVYT. Yders branders swaerheyts middelpunt dan, is inden as; t'welck wy bewysen moesten.

X. EYSCH. XXIII. VOORSTEL.

WESENDE ghegheuen een brander: Huer swaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A B C een brander wesen diens sop A, ende as A D sy. T'BEGHEERDE. Wy moeten sijn swaerheyts middelpunt vinden. T'WERCK. Men sal den as A D in E deelen, alsoo dat A E dobbel sy an E D, ende E sal t'begheerde swaerheyts middelpunt sijn; T'welck bewesen is duer Frederick Commandin int 29 voorstel, waer af den sin verclaert naer onse manier soodanich is. T'BEWYS. Laet den brander doorsneen worden met een plat F G, euewydich vanden grondt B C, ende duer t'middel des as H, ende sniende de uytersten des branders in I, K, ende laet B C G F ende I K L M twee pilaren sijn, beschreuen omme den brander, wiens middelpunten N, O, ende I K P Q een pilaer binnen den brander, wiens swaerheyts middelpunt oock O sijn sal. Nu want ghelijck A D tot A H, t'welck is als 2 tot 1, alsoo t'rondt

GN, *HO*; then *LM*, *NO*, *GH* will be ellipses similar to the ellipse *BC*. And let *EM* and *GO* be prisms described thereon. PROOF. Because *LD*, the semi-diameter of the ellipse *LM*, is equal to *DM*, also *NI* to *IO* [1]), *ID* will be the axis of the prism *EM*, in which is the centre of gravity of the said prism. And for the same reason the centre of gravity of the prism *GO* will also be in *KI*, and consequently the centre of gravity of the solid composed of those two prisms is in *KD*, and therefore also in *AD*. But the more of such prisms there are inscribed in the paraboloid, the less will be the difference between the paraboloid and the inscribed figure composed of such prisms. We can therefore, by infinite approximation, place within the paraboloid a form such that its difference with the paraboloid shall be less than any given solid, however small. From which it follows that, taking *AD* for the centre line of gravity of the paraboloid [2]), the apparent weight of one side will differ less from that of the other than any gravity that might be given, from which I argue as follows [3]):

A. *Beside any different apparent gravities there can be placed a gravity less than their difference;*
O. *Beside the present apparent gravities of one side of the paraboloid and the other there cannot be placed any gravity less than their difference;*
O. *Therefore the present apparent gravities of one side of the paraboloid and the other do not differ.*

Therefore *AD* is its centre line of gravity. CONCLUSION. The centre of gravity therefore of any paraboloid is in the axis, which we had to prove.

PROBLEM X. PROPOSITION XXIII.

Given a paraboloid: to find its centre of gravity [4]).

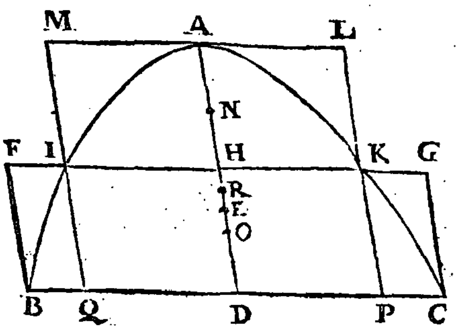
SUPPOSITION. Let *ABC* be a paraboloid, whose vertex shall be *A* and the axis *AD*. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The axis *AD* shall be divided in *E* in such a way that *AE* shall be double of *ED*; then *E* will be the required centre of gravity, which has been proved by Frederick Commandinus in the 29th proposition, the purport of which, explained in our own way, is as follows. PROOF. Let the paraboloid be intersected by a plane *FG*, parallel to the base *BC* and through the middle point of the axis, *H*, and intersecting the extremities of the paraboloid in *I*, *K*. And let *BCGF* and *IKLM* be two prisms circumscribed about the paraboloid, the centres of said prisms being *N*, *O*, and let *IKPO* be a prism within the paraboloid, the centre of gravity of this prism also being *O*. Now because as *AD* is to *AH*,

[1]) The text has „also to *NI* and *NO*", but this is obviously a mistake.
[2]) Cf. note 3 to p. 227.
[3]) See note 2 to p. 143.
[4]) It is to be noted that the reasoning here deviates from that of Props 11 and 12. A lemma corresponding to Prop. 11 is not needed here.

20 v. 1 B. vā Apollonius.

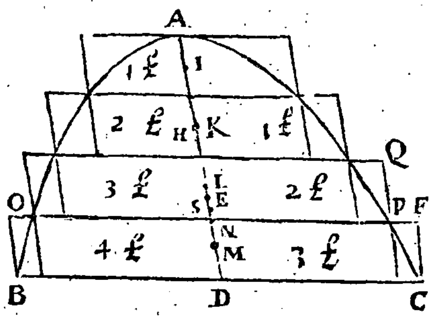
13. v. 12. B. E

t'rondt B C tottet rondt I K, soo sal den pilaer B G sulcken reden hebben tot den pilaer I L, als 2 tot 1, daerom laet B G weghen 2 ℔, ende I L 1 ℔: Maer huer swaerheydts middelpunten sijn N, O, de lini dan N O sal balck sijn de selue ghedeelt in huer ermen, dat is in R, alsoo dat N R dobbel sy an R O, soo sal R swaerheyts middelpunt sijn der twee ommeschreuen pilaren, ende O ist vande binneschreuen, ende R sal soo verre van E vallen, als O van E, te weten elck $\frac{1}{12}$ van A D: Ende sulcx sal in alle anderen der ghelijcke voorbeelden oock alsoo gheschien. Maer op dattet claerder sy, Wy sullender noch een besonder voorbeelt af beschrijuen aldus:



Laet ons den brander A B C noch eenmael snien duer de middelen van A H, ende H D, daer uyt teeckenende vier omschreuen, ende drie binneschreuen pilaren, als hier onder, alwaer A D des branders as is, ende der pilaren middelpunten sijn I, K, L, M, ende A E sy noch dobbel an E D als vooren. Nu want ghelijck A D tot A N (t'welck is als 4 tot 3) alsoo het rondt B C tottet rondt O P, soo sal den pilaer B F sulcken reden hebben tot den pilaer O Q, als 4 tot 3, ende om de selue oirsaeck sal B F sulcken reden hebben tot de derde diens middelpunt K, als 4 tot 2, ende tot den omschreuen pilaer wiens middelpunt I, als 4 tot 1: Daerom laet d'onderste der omschreuen pilaren weghen 4 ℔, d'ander 3 ℔, de volghende 2 ℔, de hoochste 1 ℔: Laet oock om de selue reden de leegste der binneschreuen pilaren weghen 3 ℔, d'ander 2 ℔, de laetste 1 ℔. T'welck soo sijnde, ende anghesien alle de swaerheyts middelpunten ende der pilaren swaerheden bekent sijn, soo ist openbaer duer het 2ᵉ voorsteldes 1ᵉⁿ boucx, dattet swaerheyts middelpunt der vier omschreuen pilaren sal vallen in L, alsoo dat L E sal doen $\frac{1}{24}$ van A D, ende der drie binneschreuen pilaren sal vallen in S, alsoo

20. v. 1. B. vā Apollonius.



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viz. 2 to 1, so is the circle BC to the circle IK, the prism BG will have to the prism IL the ratio of 2 to 1. Therefore, let BG weigh 2 lbs and IL 1 lb. Their centres of gravity are N, O; therefore the line NO will be beam, which being divided into its arms, viz. in R, in such a way that NR shall be double of RO, R will be the centre of gravity of the two circumscribed prisms, and O is the centre of gravity of the inscribed prism; then R will be the same distance from E as O from E, to wit each $\frac{1}{12}$ AD [1]). And this will be the same in all other examples of the kind. But in order to make it clearer, we will describe a special example thereof, as follows:

Let us intersect the paraboloid ABC once again through the middle points of AH and HD, drawing four circumscribed and three inscribed prisms, as shown below, where AD is the axis of the paraboloid, while the centres of the prisms are I, K, L, M, and AE shall still be double of ED, as before. Now because as AD is to AN (viz. 4 to 3), so is the circle BC to the circle OP, the prism BF will have to the prism OQ the ratio of 4 to 3, and for the same reason BF will have to the third prism, whose centre is K, the ratio of 4 to 2, and to the circumscribed prism, whose centre is I, the ratio of 4 to 1. Therefore, let the lowermost of the circumscribed prisms weigh 4 lbs, the second 3 lbs, the next 2 lbs, and the topmost 1 lb. Let also, for the same reason, the lowermost of the inscribed prisms weigh 3 lbs, the second 2 lbs, the last 1 lb. This being so, and all the centres of gravity and the gravities of the prisms being known, it is manifest by the 2nd proposition of the 1st book that the centre of gravity of the four circumscribed prisms will fall in L, in such a way that LE will make $\frac{1}{24}$ AD [2]), and that the centre of gravity of the three inscribed prisms will fall in S, in such

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[1]) This may be seen by applying Prop. 2 of Book I:
If $AD = a$, $DR = DO + \frac{1}{3} ON = \frac{1}{4} a + \frac{1}{6} a = \frac{5}{12} a$.
Therefore $RE = DR - DE = \left(\frac{5}{12} - \frac{1}{3}\right) a = \frac{1}{12} a$, whereas $EO = \frac{1}{3} a - \frac{1}{4} a = \frac{1}{12} a$.

[2]) This has probably been found by repeated application of Prop. 2 of Book I. It may be verified by using the formula for the centre of mass of a compound figure relatively to BC, which gives

for the circumscribed figure: $$DL = \frac{4 \cdot \frac{1}{8} + 3 \cdot \frac{3}{8} + 2 \cdot \frac{5}{8} + 1 \cdot \frac{7}{8}}{10} a = \frac{3}{8} a.$$

for the inscribed figure: $$DS = \frac{3 \cdot \frac{1}{8} + 2 \cdot \frac{3}{8} + 1 \cdot \frac{5}{8}}{6} a = \frac{7}{24} a.$$

Hence $LE = \frac{3}{8} a - \frac{1}{3} a = \frac{1}{24} a$ and $SE = \frac{1}{3} a - \frac{7}{24} a = \frac{1}{24} a$.

dat S E oock sal doen $\frac{1}{24}$ van A D. Dees twee punten dan L ende S vallen wederom euen verre van E.

Maer soomen om den brander schreue sulcke acht pilaren, ende seuen daer binnen, men sal sulcke punten noch euewydich vinden van E; te weten elck $\frac{1}{48}$ van A D.

Maer soomen om den brander schreue soodanighe sesthien pilaren, ende vijfthien daer binnen, men sal sulcke punten noch euewydich vinden van E, te weten elck $\frac{1}{96}$ van A D: Inder voughen dat het verschil der volghende inschrijuing, altijt den helft is der voorgaende, daer af wy naer t'nootsaecklick veruolg in allen souden trachten, ten waer wy dat lieten om de cortheyt.

Dit soo synde E is t'swaerheyts middelpunt des ghegheuen branders: want latet (soo t mueghelick waer) daer buyten sijn tusschen E L ofte E S, men sal dan duer de oneindelicke omschrijuing en binneschrijuing veler pilaren, daer toe commen, dattet swaerheyts middelpunt des omschreuen forms, leegher sal commen dan des branders: ofte der binneschreuen form, hoogher dan des branders, t'welck ommueghelick is. Ten is dan gheen ander punt dan E, t'welck wy bewysen moesten.

T'BESLVYT. Wesende dan ghegheuen een brander, wy hebben sijn swaerheyts middelpunt gheuonden, naer den eysch.

MERCKT.

ANGHESIEN des driehoucx lini vanden houck tot int middel der sijde, van t'swaerheyts middelpunt in sulcken reden ghedeelt wordt, als desen as des branders duer het 4e voorstel, soo volgt dat inden driehouck der ghelijcke ghedaenten sullen beuonden worden duer omschreuen ende binneschreuen vierhoucken, ghelijck hier vooren gheschiet is met omschreuen ende binneschreuen pilaren.

XI. EYSCH. XXIIII. VOORSTEL.

WESENDE ghegheuen een ghecorten brander: Huer swaerheyts middelpunt te vinden.

T'GHEGHEVEN. Laet A B C D een ghecorten brander sijn, diens decsel A B, ende grondt D C, ende as E F.

T'BEGHEERDE. Wy moeten huer swaerheyts middelpunt vinden.

T'WERCK. Men sal den ghecorten brander volmaken, daer an stellende t'ghebrekende A B G, Daernaer salmen teeckenen H, alsoo dat G H dobbel sy an H E, sghelijcx, I alsoo dat G I dobbel sy an I F, daernaer K, alsoo dat I H sulcken reden hebbe tot I K, als den ghecorten brander A B C D, tottet branderken A B G: Ick seg dat K t'begheerde a way that *SE* will also make $\frac{1}{24}$ *AD*. These two points *L* and *S* therefore are again at the same distance from *E*.

But if eight such prisms be circumscribed about the paraboloid and seven be inscribed therein, similar points will be found still equidistant from *E*, to wit each $\frac{1}{48}$ *AD*.

But if sixteen such prisms be circumscribed about the paraboloid and fifteen be inscribed therein, similar points will be found still equidistant from *E*, to wit each $\frac{1}{96}$ *AD*. In such a way that the distance found in the next inscription is always half that of the preceding, and in this way we might continue but that we have omitted it for brevity's sake.

This being so, *E* is the centre of gravity of the given paraboloid. For let it be (if this were possible) beyond it, between *E* and *L* or *E* and *S*; by infinite circumscription and inscription of many prisms the result will be attained that the centre of gravity of the circumscribed figure will come below that of the paraboloid, or that of the inscribed figure above that of the paraboloid, which is impossible. Therefore it is none other point but *E*, which we had to prove. CONCLUSION. Given therefore a paraboloid, we have found its centre of gravity, as required.

NOTE.

Since the line in the triangle from the angle to the middle point of the side is divided by the centre of gravity into segments having the same ratio as those of the axis of the paraboloid, by the 4th proposition, it follows that in the triangle by means of circumscribed and inscribed quadrilaterals figures analogous to those found above with circumscribed and inscribed prisms will be found.

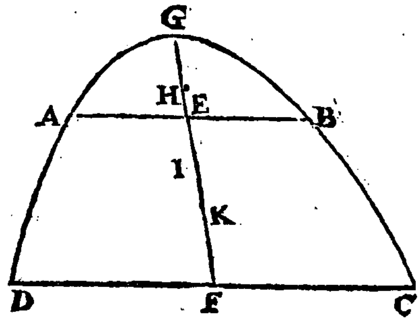
PROBLEM XI. PROPOSITION XXIV.

Given a truncated paraboloid: to find its centre of gravity [1]).

SUPPOSITION. Let *ABCD* be a truncated paraboloid, whose cover shall be *AB* and the base *DC*, and the axis *EF*. WHAT IS REQUIRED TO FIND. We have to find its centre of gravity. CONSTRUCTION. The truncated paraboloid shall be completed, adding thereto the missing part *ABG*. After this, *H* shall be marked in such a way that *GH* shall be double of *HE*, and likewise *I* in such a way that *IG* shall be double of *IF*; after this *K* in such a way that *IH* shall have to *IK* the same ratio as the truncated paraboloid *ABCD* to the small paraboloid

1) Cf. Prop. 13.

gheerde ſwaerheyts middelpunt is.



T'BEWYS. I is ſwaerheyts middelpunt des heels D C G, ende H des deels A B G, ende ghelijck t'ander deel A B C D, tot dit deel A B G, alſoo H I tot I K duer t'werck, daerom K, duer het 19ᵉ voorſtel, is t'begheerde ſwaerheyts middelpunt, t'welck wy bewyſen moeſten.

T'BESLVYT. Weſende dan ghegheuen een ghecorten brander, wy hebben huer ſwaerheyts middelpunt gheuonden naer den eyſch.

EINDE DES TWEEDEN BOVCX.

ABG. I say that *K* is the required centre of gravity. PROOF. *I* is centre of gravity of the whole *DCG*, and *H* of the part *ABG*; and as the other part *ABCD* is to the latter part *ABG*, so is *HI* to *IK* by the construction. Therefore, by the 19th proposition, *K* is the required centre of gravity, which we had to prove. CONCLUSION. Given therefore a truncated paraboloid, we have found its centre of gravity, as required.

END OF THE SECOND BOOK.

DE WEEGHDAET

THE PRACTICE OF WEIGHING

INTRODUCTION

In this work Stevin deals with the practical applications of the Art of Weighing in tools and engines. In the Preface the reader is warned that the theory developed in the preceding work does not permit of treating this subject with anything like completeness: it only leads to conditions of equilibrium, but does not provide information about the additional force required to produce motion and to maintain it.

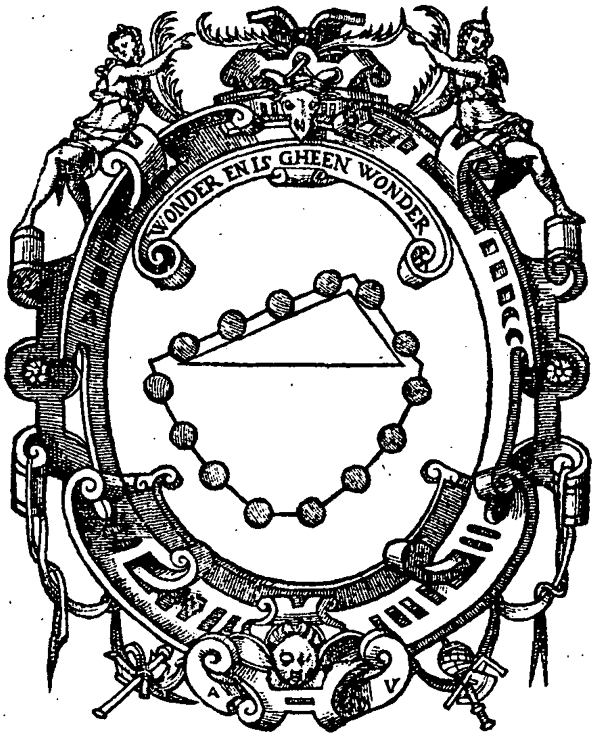
Proposition I teaches an empirical method of determining the centre of gravity of any body by suspending it in different ways and determining the point in which the verticals through the points of suspension meet. The propositions 2-4 are devoted to the ordinary balance. According to Stevin the ideal form of this instrument is that in which the equilibrium of the non-loaded balance is indifferent. In Prop. 2, instructions for its construction are given with this end in view; for practical reasons, however, a slight deviation in the direction of stable equilibrium is permitted. Prop. 3 deals with the problem what weight has to be put in one of the pans of a balance in stable equilibrium in order to keep the beam in a given position. In Prop. 4 the equilibrium of the beam is unstable, and it is asked what weight the pans must have if the equilibrium of the completed balance is to be indifferent. In Prop. 5 the construction of a steelyard is described, in Prop. 6 that of a so-called oblique balance, i.e. an instrument enabling given forces to be exerted in given directions. Prop. 7 contains the practical applications of the lever; in Prop. 8 various cases in which weights have to be carried are treated by means of the theorems on the composition of parallel or concurrent forces, proved in the Art of Weighing. In Prop. 9 the theory of the lever is applied to the windlass and to other implements for displacing heavy loads, especially ships to be hauled.

The final Prop. 10 gives a very elaborate treatment of an instrument designed by Stevin, to which he gives the name of Almighty, thereby expressing that theoretically the mechanical advantage may be increased indefinitely.

DE WEEGHDAET

Praxis artis Ponderariae.

BESCHREVEN DVER
SIMON STEVIN
van Brugghe.



TOT LEYDEN,
Inde Druckerye van Christoffel Plantijn,
By Françoys van Raphelinghen.
cIↃ. IↃ. LXXXVI.

Simon Steuin wenſcht DEN BVRGMEESTERS ENDE REGIERDERS DER STADT NVRRENBERG VEEL GHELVCX.

GHELIICT onnutte coſt vvaer, een groote ſtercke grondt te legghen, die een ſvvaer gheſticht draghen can, ſonder eintlick eenich ghebau daerop te vvillen brenghen; Alſoo is de * ſpiegheling inde be- *Theoria.* ghinſelen der conſten verloren arbeydt, daer t'einde totte*daet niet en ſtreƈt. Ghelijck oock *Effectum.* na de natuerlicke oirden, dien grondt voor t'opperghebau gaedt, alſoo deſe ſpiegheling voor huer daet. Dit ſoo ſijnde, ende vooren beſchreuen hebbende de Beghinſelen der Weeghconſt,ſoo iſt vouglick de WEEGHDAET *Praxis.* te volghen; Oock my niet onbetamelick, de ſelue an ulieder E. Heeren toe te eyghenen. ende dat om drie beſonder redenen: D'eerſte, dat haer * voorſtellen niet alleen onghehoort, *Propoſitiones.* maer oock nut ſijn. D'ander,dat haer voordering van nerghens merckelicker te vervvach-

a 2 ten en

SIMON STEVIN WISHES THE BURGOMASTERS AND RULERS OF THE CITY OF NUREMBERG MUCH HAPPINESS.

Just as it would be useless to lay large and strong foundations which can support a heavy edifice without ultimately wishing to erect any building thereon, thus in the elements of the arts theory is lost labour when the end does not tend to practice. Just as, in the natural order of things, the foundation precedes the building, thus theory precedes practice. This being so, it is appropriate that, having described the Elements of the Art of Weighing hereinbefore, I should follow this up with the Practice of Weighing. Nor is it improper for me to dedicate same to Your Worships, such for three special reasons. The first, that its propositions are not only unheard-of, but also useful. The second, that its furtherance was to be expected from nowhere more clearly than from those with whom ten en ſcheen, dan vande ghene daer de Conſten inde grootſte acht ſijn, vvaer af niet alleen en ghetuyghen de ſchriften veler gheleerden, maer oock ſelfs u onderſatens conſtighe* daden, verſpreydt in allen houcken des vveerelts. De laetſte ende voornaemſte, dat ick my duer ſulcx toecommende voorthelpers hoopte te bereyden, tot ſeker daden mijns voornemens; Met vvelcke eindelicke meining hier eindende, vvenſch V. H. alle voorſpoet ende vvelvaren. Vyt Leyden in Oogſtmaendt des 1586e. Iaers.

Effecta.

ANDEN

the arts are held in the greatest esteem, to which is borne witness not only by the writings of many scholars, but also by the ingenious works, of your subjects, scattered in all parts of the world. The last and most important, that in this way I hope to gain future collaborators, for carrying out my projects. With which final thought I here conclude, and wish Your Worships all prosperity and health. From Leyden, in Harvest Month of the year 1586.

ANDEN LESER.

VVANT in ettelicke * voorstellen der Weeghdaet ghehandelt sal worden vande roerselen der lichamen, soo heeft my goet ghedocht, eer wy tot de saeck commen, den Leser van dies wat te verclaren. Te weten dat de Weeghconst ons alleenlick leert, het roerende ter euelstaltwichticheyt brenghen mettet teroeren. Angaende t'ghewicht ofte de macht, die t'roerende bouen dien noch behouft, om het teroeren ter roerlicke daet te crygen (welck ghewicht ofte macht, ouerwinnen moet des teroerens beletsel, dat in yder teroeren * onscheydelicke ancleuing is) de Weeghconst en leert dat ghewicht ofte die macht niet * Wisconstlick vinden, d'oirsaeck is dattet een gheroerde ende sijn beletsel niet * eueredních en is mettet ander gheroerde ende sijn beletsel. Maer op dat den sin van desen duer ghelijcknis opentlicker verstaen worde, soo laet by voorbeelt een waghen bekender swaerheyt, te trecken sijn op een berch ofte hoochde bekender steylheyt; Ick seg dat de Weeghdaet leert, soo duer het 4e voorbeelt des 9en voorstels blijcken sal, hoe groote macht met die waghen euestaltwichtich, ofte euemachtich sal staen, sonder t'ansien roersel met sijn belet, a's assen teghen de bussen, raeyers teghen de straet, waghen teghen de locht, &c. welcke macht des beletsels de Weeghconst niet en leert vinden, om dat sulcke beletselen ende haer gheroerden in gheen eueredenheyt en bestaen, so wy hier souden connen bethoonen, weerleghende de * strijtredens vande ghene die in vallende swaerheden de contrarie meinen, ten waer ons voornemen is, in dese Weeghconst alleenlick met de leering voort te varen, ende d'oude dwalinghen der

Propositiones · *Inseparabile accidens.* · *Mathematicè.* · *Proportionalis.* · *Argumenta.*

TO THE READER

Because in several propositions of the Practice of Weighing the motions of bodies will be dealt with, I thought it advisable, before coming to the matter, to explain something of it to the reader. To wit, that the Art of Weighing only teaches us to bring the moving body into equality of apparent weight to the body to be moved. As to the additional weight or the force which the moving body requires in order to set in motion the body to be moved (which weight or force has to overcome the impediment of the body to be moved, which is an inseparable attribute of every body to be moved), the Art of Weighing does not teach us to find that weight or force mathematically; the cause of this is that the one moved body and its impediment are not proportional to the other moved body and its impediment. But in order that the meaning of this may be more openly understood through an example, let a wagon of known gravity have to be drawn up a mountain or height of known steepness. I say that the Practice of Weighing teaches, as will become apparent from the 4th example of the 9th proposition, what force will be of equal apparent weight or equal power to that wagon, without considering the motion with its impediment, such as axles against the bearings, wheels against the road, wagon against the air, etc.; the finding of which force of the impediment the Art of Weighing does not teach, because these impediments are not in any proportion to their moved bodies, as we might here prove, refuting the arguments of those who think the contrary to be true of falling gravities, but that it is our intention merely to continue the instruction in this Art of Weighing and to reject elsewhere the old errors concerning the *der wichtighe ghedaenten elders te verworpen. Merct oock dat dese kennis der euestaltwichticheyt tot de saeck ghenouch doet, want ligghende in elcke schael des waeghs eueveel ghewichts, ghelijck wy dan weten (hoewel de waegh oock haer belet des roersels heeft) dat tottet roersel der schalen luttel machts behouft, alsoo in allen anderen.*

Dit is van t'belet des roersels tot dien einde ghesëyt, op dat yemant duer de daet, de roerende macht altemet wat grooter beuindende, dan de gheroerde, niet en dencke sulcx t'ghebreck der const te wesen, maer nootsaeclick, ouermidts, als vooren ghesëyt is, t'roerende bouen de euestaltwichticheyt soo veel swaerder ofte machtigher moet sijn, dan het teroeren, dattet sulck belet ouerwint. Ten anderen, op dat niemant, die hem in sulcke schijn van eueredenheyt mocht betrouwen, bedroghen en worde, t'welck den ghenen alderlichtelicxt ghebuert, die t'valsche voor warachtich houden.

Argumentū.

CORTBEGRYP.

DESE Weeghdaet sal veruaten de werclicke vinding des swaerheyts middelplats, swaerheyts middellini, eñ swaerheyts middelpunts: Voort de making des alderuolmaecsten Waeghs, met verclaring van etlicke huer ghedaenten. Oock den aldervolmaecsten Onsel. Wyder, de ghedaenten der steerten daermen ghewelt me doet: De ghedaenten der ghedreghen ghewichten; Der Windassen; Der ghetrocken ghewichten; Ende des Almachtichs.

DE WEEGH-

qualities of weights[1]*. Note also that this knowledge of equality of apparent weight is sufficient for the purpose, for if the same weight lies in either pan of the balance, as we then know (though the balance also has its impediment to motion) that little force is required to move the pans, thus it is also in all other cases.*

The above has been said about the impediment to motion in order that someone, finding in practice the moving force to be perhaps slightly greater than the force moved, may not think this to be a defect of the art, but may understand it to be necessary, since, as has been said above, the moving body, over and above the equality of apparent weight, has to be so much heavier or more powerful than the body to be moved that it overcomes the impediment to motion. Secondly, in order that no one, relying on this apparent proportionality, shall be deceived, which may very easily happen to those who hold the false to be true.

ARGUMENT

This Practice of Weighing is to contain the finding by practice of the centre plane of gravity, centre line of gravity, and centre of gravity. Further the construction of the most perfect balance, with an explanation of several of its properties. Also the most perfect steelyard. Further the properties of the levers by which a force is exerted, the properties of weights that are being carried, of windlasses, of weights that are being hauled, and of the Almighty.

[1]) This promise will be fulfilled in the second chapter of the *Appendix to the Art of Weighing*. See the present volume, p. 509.

DE WEEGHDAET

Praxis artis Ponderariæ.

BESCHREVEN DVER SIMON STEVIN.

I. VOORSTEL.

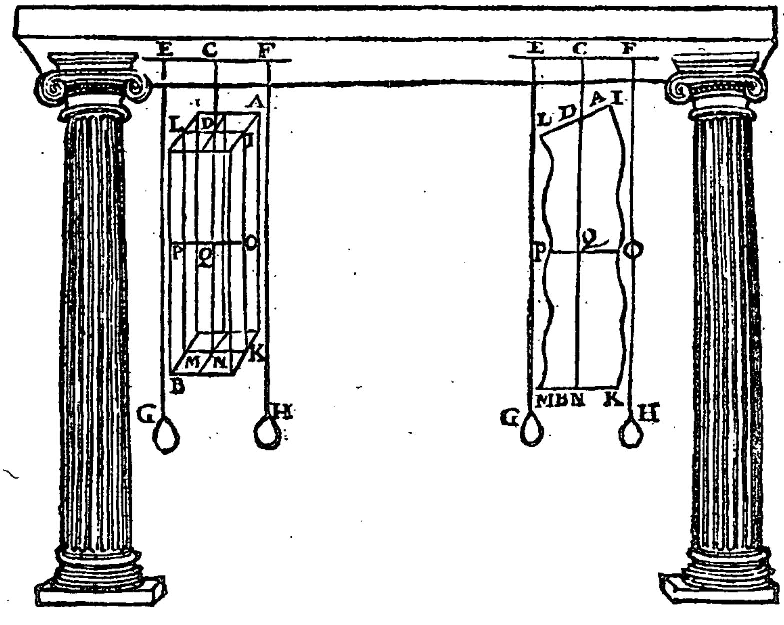
WESENDE ghegheuen een lichaem van form ſoot valt: Sijn ſwaerheyts middelplat, ſwaerheyts middellini, ende ſwaerheyts middelpunt werckelick te vinden.

I. VOORBEELT.

T'GHEGHEVEN. Laet A B een lichaem ſijn van form ſoot valt.

T'BEGHEERDE. Wy moeten ſijn ſwaerheyts middelplat, ſwaerheyts middellini, ende ſwaerheyts middelpunt werckelick vinden.

T'WERCK. Men ſal t'lichaem hanghen ande coorde C D, treckende duer t'opperſte punt C, de rechte lini E F, hanghende uyt de ſelue lini twee fijne draen met haren ghewichtkens, als E G, F H, neuens het



lichaem A B, ende t'plat veruaet tuſſchen de linien G E, F H, t'welck by ghedacht

THE PRACTICE OF WEIGHING
Described by Simon Stevin

PROPOSITION I.

Given a body of any form: to find by practice its centre plane of gravity [1]), centre line of gravity, and centre of gravity.

EXAMPLE I

SUPPOSITION. Let *AB* be a body of any form. WHAT IS REQUIRED TO FIND. We have to find by practice its centre plane of gravity, centre line of gravity, and centre of gravity. CONSTRUCTION. The body shall be hung from the cord *CD*, upon which through the highest point *C* shall be drawn the straight line *EF*, from which line shall be hung two thin threads with their small weights, as *EG*, *FH*, beside the body *AB*; then the plane contained between the lines *GE*, *FH*, which may be conceived of as passing through the body, is the centre plane

[1]) Centre plane of gravity having been defined in Def. 6 of Book I of the *Art of Weighing* as any plane through the centre of gravity of the body, it is clear that it is not a single plane, as the text seems to suggest.

ghedacht duer t'lichaem lijt,is des lichaems ſwaerheyts middelplat.Maer om ſijn uyterſte ſijden op t'lichaem te teeckenen, men mach de draen E F, G H, bekriten, die gheſpannen treckende, ende daer op teeckenende, ghelijck de Saghers haer boomen doen daer ſy doorſaeght moeten ſijn; Ick neme die linien te weſen I K, L M, teeckenende daer naer insghelijcx de linien L I, ende M K, t'plat L I K M, ſal t'begheerde ſijn.

Maer om nu de ſwaerheyts middellini te vinden, men ſal t'lichaem noch hanghende an C, een weynich draeyen ende teeckenen een ander der ghelijcke ſwaerheyts middelplat, ſniende t'voorgaende ick neem onder in N,ende bouen in D,ende haer ghemeene ſne D N ſal de begheerde ſweerheyts middellini ſijn: Maer om t'ſwaerheyts middelpunt te vinden, men ſal t'lichaem verhanghen inde dweerſde, ick neem by t'punt O, ende vinden aldaer oock des lichaems ſwaerheyts middellini alsvooren, ick neem die te weſen O P, ende daer ſy de lini D N ſniit, als in Q, is t'begheerde ſwaerheyts middelpunt.

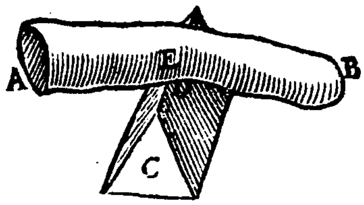
II[e] VOORBEELT.

T'GHEGHEVEN. Laet A B een lichaem ſijn van form ſootvalt.

T'BEGHEERDE. Wy moeten ſijn ſwaerheyts middelplat, ſwaerheyts middellini, ende ſwaerheyts middelpunt werckelick vinden.

T'werck. Men ſal t'lichaem A B legghen op eenighen ſcherpen cant als C D, dat vertreckende ter eender ende ander ſijde, tot datmen ſich bemercke de eueſtaltwichticheyt beyder ſijden ghetroffen te hebben,t'welc ick neem te weſen in E, daerom t'plat rechthouckich op den *ſichteinder t'lichaem door E ſniende, ſal t'begheerde ſwaerheyts middelplat ſijn. Ende een der ghelijcke plat t'voorgaende plat doorſniende, huer ghemeene ſne ſal ſwaerheyts middellini ſijn. Ende ſoodanighen derde plat ſniit die ſwaerheyts middellini in des lichaems ſwaerheyts middelpunt. Welcker bewys uyt de voorgaende openbaer is.

Horizontem.



T'BESLVYT. Weſende dan ghegheuen een lichaem van form ſoot valt,wy hebben ſijn ſwaerheyts middelplat, ſwaerheyts middellini, ende ſwaerheyts middelpunt werckelick gheuonden, naer de begheerte.

II VOORSTEL.

EEN aldervolmaeckſte waegh te maken.

T'WERCK. Men ſal eerſt int middel des balcx A B, wiens tong ter behoirlicker plaets ſy, teeckenen de lini C D onder t'middel der tong, of gravity of the body [1]. But in order to draw its boundaries on the body, we can chalk the threads *EG*, *FH*, stretching them and thus drawing them on the body, as do the sawyers with the trees where they are to be sawn. I assume these lines to be *IK*, *LM*. If thereafter the lines *LI* and *MK* are likewise drawn, the plane *LIKM* will be the required plane.

Now in order to find the centre line of gravity, the body, still hanging from *C*, shall be turned a little, upon which another, similar centre plane of gravity shall be drawn, intersecting the preceding one, say in *N* at the bottom and in *D* at the top; then its line of intersection *DN* will be the desired centre line of gravity. But in order to find the centre of gravity, the body shall be hung transversely, say at the point *O*, and the centre line of gravity of the body shall also be found in this position as above. I assume this to be *OP*, and where it intersects the line *DN*, as in *Q*, is the required centre of gravity.

EXAMPLE II.

SUPPOSITION. Let *AB* be a body of any form. WHAT IS REQUIRED TO FIND. We have to find by practice its centre plane of gravity, centre line of gravity, and centre of gravity. CONSTRUCTION. The body *AB* shall be laid on a sharp edge, as *CD*, and it shall be shifted on either side until we find we have attained the equality of apparent weight of the two sides, which I take to be in *E*. Therefore the plane at right angles to the horizon intersecting the body in *E* will be the required centre plane of gravity [2]. And a similar plane intersecting the preceding plane, their common line of intersection will be centre line of gravity. And a third such plane intersects this centre line of gravity in the centre of gravity of the body. The proof of which is manifest from the preceding. CONCLUSION. Given therefore a body of any form, we have found by practice its centre plane of gravity, centre line of gravity, and centre of gravity, as required.

PROPOSITION II.

To construct a most perfect balance.

CONSTRUCTION. In the middle of the beam *AB*, having its tongue at the appropriate place, the line *CD* shall first be drawn below the middle of the

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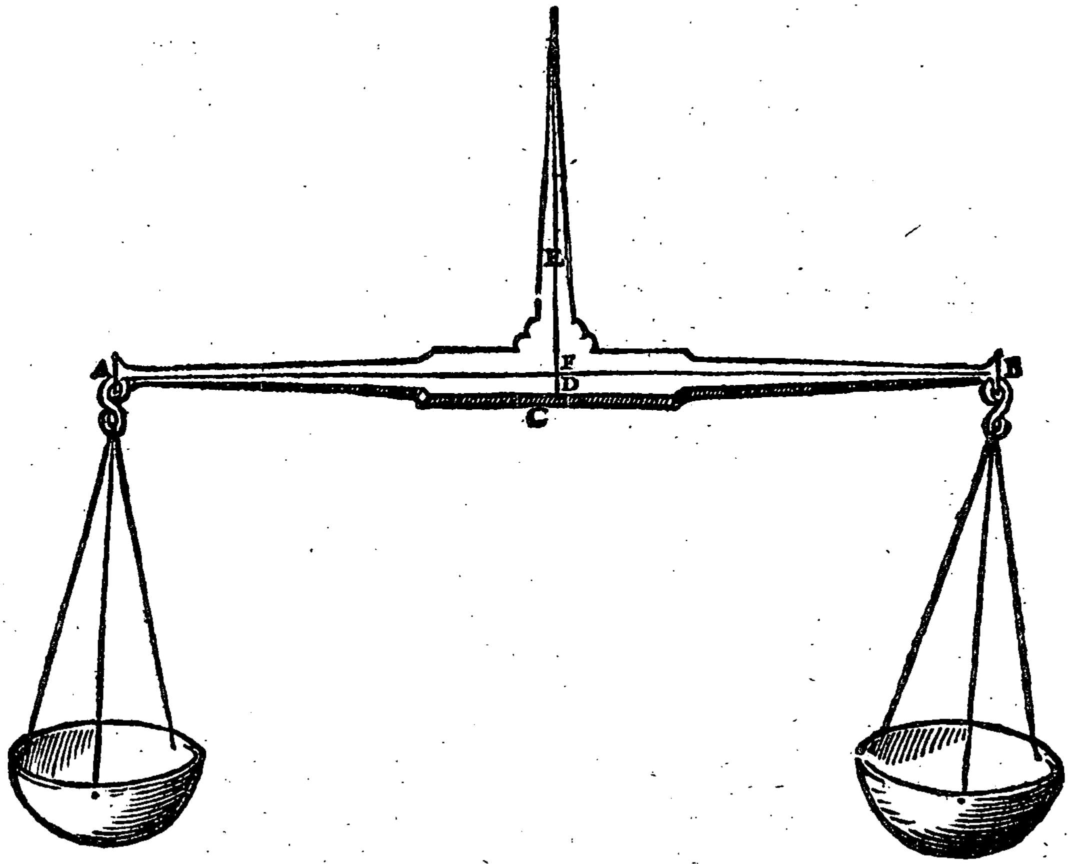
[1]) Read: a centre plane of gravity.

[2]) Read: any plane at right angles to the horizon intersecting the body in *E* will be a centre plane of gravity.

tong, rechthouckich op de canten des balcx, ende vylen ofte weeren van d'een ende d'ander sijde soo veel stof, tot dat den balck (ligghende met de lini C D op eenighen scherpen cant) ouer beyden sijden met euen ermen euewichtich beuonden wort. Daer naer salmen trecken D E oock rechthouckich op de canten, ende legghen den balck op eenen scherpen stalen punt, ghenakende inde lini D E, souckende inde selue lini D E des balcx swaerheyts middellini, te weten den balck ter eender ende ander sijde vertreckende (welverstaende dat den stalen punt altijt inde lini D E blijue) tot datmen bemerckt de euewichticheyt ghetroffen te



sijne, t'welck ic neem te wesen in F; Daernaer ghetreckent een dergheliјcke punt ouer d'ander sijde, de rechte lini door die twee punten sal de swaerheyts middellini des balcx sijn, beteeckenende t'scherp vanden dweersas, soo noem ick t'yserken daer op den balck int huysken rust.

b Daer

tongue, at right angles to the sides of the beam, and then so much material shall be filed off or taken away from either side until the beam (lying with the line *CD* on some sharp edge) is found to be in equilibrium with equal arms on either side. Thereafter, *DE* shall be drawn, also at right angles to the sides, and the beam shall be laid on a sharp steel point, touching the line *DE*, and the centre line of gravity of the beam shall be sought in this line *DE*, viz. by shifting the beam on either side (it being understood that the steel point shall always remain in the line *DE*), until equilibrium is found to be attained, which I take to be in *F*. Thereafter, a similar point being marked on the other side, the straight line passing through these two points shall be the centre line of gravity of the beam [1]), which determines the sharp edge of the transverse axis [2]) (which is the name I give to the pin on which the beam is supported in the fork). Thereafter, if

[1]) According to Def. 5 of Book I of the *Art of Weighing* this is only true if the beam is supported vertically in one of the two points mentioned.
[2]) Obviously Stevin requires the ideal balance to be in a state of indifferent equilibrium.

Daer naer soo de schalen an dien balck met haecken moeten hanghen, men sal de plaetsen der ghenaecselen des balcx ende dier haecken als an A,B, alsoo stellen, dat sy ende t'scherp vanden dweersas in een rechte lini A F B commen te staen: verstaet wel t'voornomde woort Ghenaecselen, want wy spreken vande eyghen wesentlicke ghenaecselen der haecken teghen de stof des balcx. Maer soo t'ghene daer mede de schalen anden balck hanghen yet anders waer dan haecken, men sal op haer dergheliјcke naecselen letten. T'welck ghedaen sijnde ende thuysken t'sijnder plaets ghevoucht wesende, soodanighē waegh met alle euen ghewichten diemen in haer schalen soude mueghen legghen, sal, so lang den dweersas op haer scerpte rust, alle ghestalt houden diemen haer gheeft, door het 10 voorstel des 1en boucx vande beghinselen der Weeghconst.

Maer dat alsulcken waegh de aldervolmaecste sy, is openbaer door het 1e voorbeelt van het 11e voorstel des voornomden 1en boucx, alwaer bethoont is, dat wesende E vastpunt, wat ghewicht men an D soude moeten hanghen, om den as in ghegheuen ghestalt te houden, maer sō t'vastpunt aldaer had gheweest N, te weten het swaerheyts middelpunt des ghegheuens, daer en soude gheen ghewicht so cleen connen sijn * Wisconstelick sprekende, dat an D ghehanghen, die sijde niet en soude doen gantselick neerdalen: T'selue is hier oock alsoo te verstaen, te weten dat tot d'een ofte d'ander deser euewichtigher deelen een seer cleen ghewicht gheleyt, die sijde sal stracx ten gronde sincken, daer sy van sommeghe ander waghen nau verroeren en soude.

Mathematicè.

Maer soot den Waeghmakers te moeylick viel die plaets van t'scherp des dweersas, metgaders de ghenaeckselen der haecken ende des balcx, altijt soo puntelick te treffen, sy mueghen t'ghene ghesëyt is houden als voor hun wit, dat soo naer commende als sy willen oft connen; Ende so sy van t'volmaecste yet souden begheeren te verschillen, mueghen ghedachtich sijn t'naecsel der haecken ende des balcx lieuer te stellen een haerken beneden de rechte lini A B, dan daer bouen, want daer bouen ghestelt sijnde, alles keert omme duer het 8e voorstel des 1en boucx, t'welck onbequaem is om te weghen; Ia t'ghene t'swaerste waer, soude altemet t'lichtste schijnē, voornamelick als den as duer de langde des balcx int beghin des weghens niet euewydich en waer vanden * sichteinder, ouermits alles an die sijde keert daer t'eerst beghint.

Horizonte.

Angaende dat de ermen des balcx euelanck moeten wesen, dat is kennelick, want soo d'eene een honderste deel des erms langher waer als d'ander, dat soude een bedriechlicke waegh sijn, ouermidts t'ghene euewichtich schene, soude een ten hondert verschillen; ende waer d'een een vyuentwintichste deel langher als dander, t'soude 4 ten hondert schillen, &c. Want ghelijck den langsten erm tot den corsten, alsoo dit ghewicht tot dat, duer het 1e voorstel des 1en boucx.

MERCT

the pans have to hang on the beam by means of hooks, the places of the points of contact of the beam and the hooks, as *A*, *B*, shall be so arranged that they come to fall in a straight line *AFB* with the sharp edge of the transverse axis. Do not mistake the aforesaid term "points of contact", for we are referring to the proper and real points of contact of the hooks against the material of the beam. But if the device by which the pans hang on the beam does not consist in hooks, similar points of contact shall be noted. Which being done and the fork being arranged in its proper place, such a balance with all the equal weights that might be laid in the pans will, as long as the transverse axis rests on the sharp edge, remain at rest in any position given to it, by the 10th proposition of the 1st book of the elements of the Art of Weighing.

The fact that such a balance is a most perfect one is manifest from the 1st example of the 11th proposition of the aforesaid 1st book, where it has been shown,*E* being the fixed point, what weight would have to be hung from *D* to keep the beam in its given position, but if the fixed point there had been *N*, to wit the centre of gravity of the given body, there would be no weight so small, in the mathematical sense, but would, if hung from *D*, cause that side to descend altogether. This is also to be so understood in this case, to wit that if a very small weight be added to one or the other of these balanced parts, that side will at once descend, while in some other balances it would scarcely stir.

But if the makers of balances deem it too difficult always to find exactly the place of the sharp edge of the transverse axis, and the points of contact of the hooks and the beam, let them aim at what has been said, approximating it as much as they wish to or are able to. And if they should desire to fall short of perfection a little, let them be mindful of placing the points of contact of the hooks and the beam a hair's breadth below the straight line *AB* rather than above it [1]), for if they are placed above it, everything will turn upside down by the 8th proposition of the 1st book; which makes weighing impossible. Nay, what is heaviest would sometimes appear to be lightest, especially if the axis through the length of the beam were not parallel to the horizon during the commencement of the weighing, since everything turns upside down on the side where it first begins to turn.

As to the necessity of the arms of the beam being equally long, this is obvious, for if the one were longer than the other by one-hundredth of the arm, the balance would be a deceptive one, since what seemed to be of equal weight would differ by one per cent.; and if the one were longer than the other by one-twenty-fifth, the difference would be 4 per cent., etc. For as the longer arm is to the shorter, so is the latter weight to the former, by the 1st proposition of the 1st book.

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[1]) As *A* and *B* have been introduced to denote the points of contact of the beam and the hooks, this instruction is not clear. It should read: below the horizontal plane through the sharp edge of the transverse axis.

MERCKT oock dat inde langste dunste ende lichtste balcken, t'grooste voordeel is om scherpelick te weghen. Want wesende twee euesware balcken maer d'een tweemael langher als d'ander, tis kennelick dat een once, aes oft wattet sy, ande langste tweemael meer ghewelts sal doen dan ande cortste duer t'voornoemde 1e voorstel.

T'BESLVYT. Wy hebben dan een aldervolmaeckste waegh ghemaeckt na t'voornemen.

III. VOORSTEL.

WESENDE ghegheuen een waegh diens balck euewydich blijft vanden * sichteinder: T'ghewicht te vinden dat in d'een schael gheleydt, den balck in begheerde ghestalt houde.

* *Horizonte.*

T'GHEBVERT dickmael dat d'een waegh veel stegher gaet als d'ander, sonder datmen weet waer an het liecht, want t'scherp des dweersas is van d'een soo bequaem als van d'ander, ende inde reste en openbaert hem niet ooghenschynelicx daermen de reden duer bemercken can: Daerom sullen wy d'oirsaeck beschrijuen, bethoonende wat ghewicht men in d'een schael van soodanighen waegh sal moeten legghen op dat den balck blijue in begheerde ghestalt aldus: T'GHEGHEVEN. Laet de waegh A B C D sulck sijn, dat alles vry hanghende, den balck soude eintlick euewydich vanden sichteinder rusten, ende E sy t'scherp vanden dweersas. T'BEGHEERDE. Wy moeten inde schael D eenich ghewicht legghen, sulcx, dat den balck in die ghegheuen ghestalt blijue.

T'WERCK. Men sal t'huysken ende de schalen met haren coorden ende haecken afdoen, vindende des balcx met de tong daeran swaerheyts middellini, euewydich mettet scherp vanden dweersas E, door het 1e voorstel deses boucx, t'welck ick neem F te sijne, daer naer salmen trecken een lini tusschen de plaetsen der naecselen des balcx ende der haecken vande schalen, welcke sy G H, wiens middel sy I: Daernaer salmen F I deelen, alsoo dat de stucken inde reden sijn van t'ghewicht des balcx met de tong, welcke sy 1 lb, tot de schalen met haer coorden ende haecken, welcke ick neem te weghen oock 1 lb, daerom ghedeelt F I, int middel K, soo sal K t'punt sijn daer an de ghegheuen waegh alle ghestalt soude houden diemen haer gheeft; Daernaer ghetrocken de lini K G, ende de hanghende duer E als E L, sniende K G in M; Ick seg dat een ghewicht in sulcken reden tot 2 lb (te weten 1 lb voor den balck, en 1 lb voor de schalen, t'samen 2 lb) als M K tot M G, t'begheerde sal sijn, t'welck gheleydt inde schael D, de waegh in die standt sal houden. Ghe-

Note also that the longest, thinnest, and lightest beams are those most suitable for accurate weighing. For given two beams of equal weight, but one being twice as long as the other, it is obvious that an ounce, aes [1]) or whatever it may be will exert on the longer one twice the force it exerts on the shorter one, by the aforesaid 1st proposition.

CONCLUSION. We have therefore made a most perfect balance, as intended.

PROPOSITION III.

Given a balance whose beam remains parallel to the horizon: to find the weight which, when laid in one of the pans, shall keep the beam in a required position.

It often happens that one balance moves much more slowly than another, without the reason for this being known, for the sharp edge of the transverse axis is equally good in both, and in the other parts nothing becomes apparent from which the cause can be learned. Therefore we will describe the cause, showing what weight has to be laid in one of the pans of such a balance in order that the beam shall remain in a required position. SUPPOSITION. Let the balance *ABCD* be such that, everything being freely suspended, the beam would ultimately remain at rest parallel to the horizon, and let *E* be the sharp edge of the transverse axis. WHAT IS REQUIRED TO FIND. We have to lay in the pan *D* some weight such that the beam shall remain at rest in that given position. CONSTRUCTION. The fork and the pans with their cords and hooks shall be taken away, and the centre line of gravity of the beam with the tongue shall be found, parallel to the sharp edge of the transverse axis *E*, by the 1st proposition of this book [2]), which I assume to be *F*. Thereafter a line shall be drawn between the places of the points of contact of the beam and the hooks of the pans, which shall be *GH*, whose middle point shall be *I*. Thereafter *FI* shall be so divided that the segments have the same ratio as the weight of the beam with the tongue, which shall be 1 lb, to the pans with their cords and hooks, which I take to weigh also 1 lb. Therefore, *FI* being divided in the middle point *K*, *K* will be the point on which the given balance would remain at rest in any position given to it. Thereafter the line *KG* shall be drawn, and the vertical through *E* [3]) as *EL*, intersecting *KG* in *M*. I say that a weight having to 2 lbs (i.e. 1 lb for the beam and 1 lb for the pans, together 2 lbs) the same ratio as *MK* has to *MG* will be the required weight, which, when laid in the pan *D*, will keep the balance in that position [4]). If then *MK* is assumed to be one-twenty-fifth of *MG*, one-twenty-fifth

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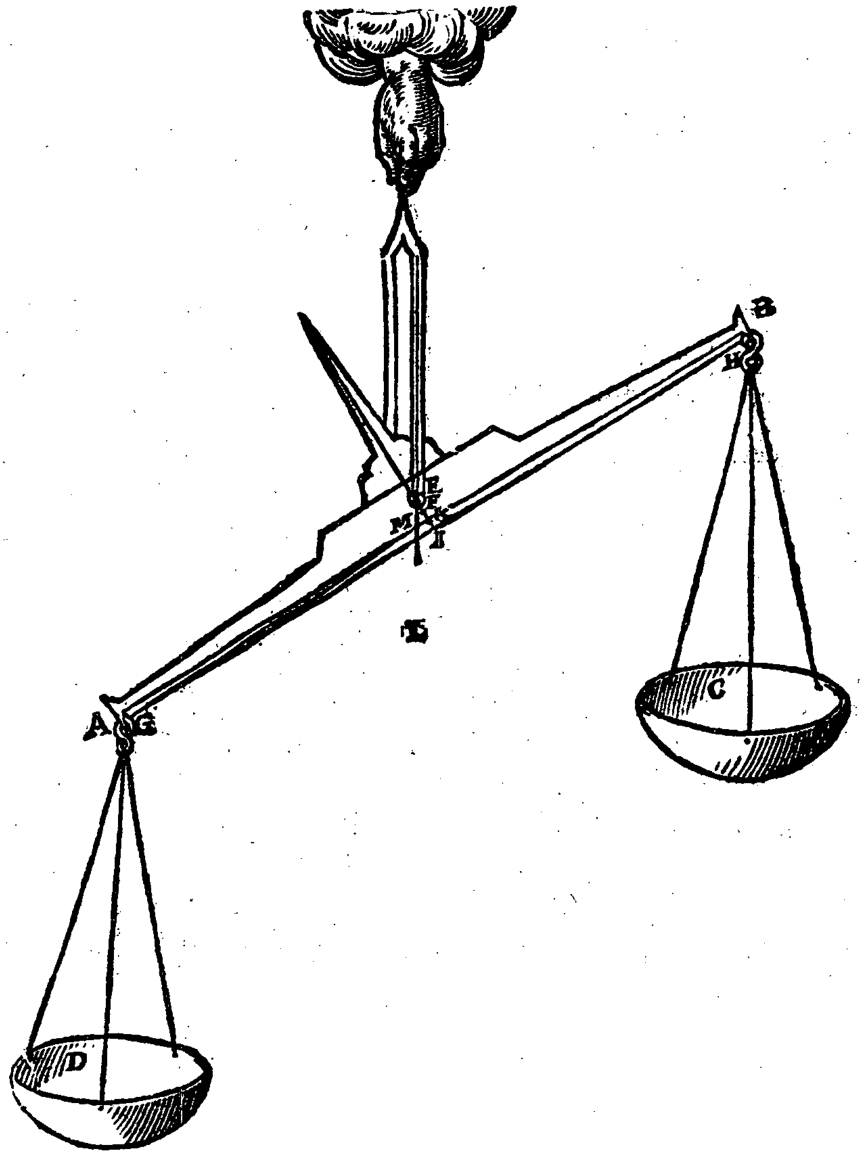
[1]) *aes* or *aas* is an old unit of weight, equivalent to 0.048 or 0.046 gramme.

[2]) According to Prop. I, in the ideal case the centre line of gravity coincides with the sharp edge of the transverse axis.

[3]) Read *F*.

[4]) Obviously the balance is first given a certain position, as shown in the figure, and then the weight is determined which has to be put in the lower pan to keep the balance in this position. The result obtained is not turned to account by Stevin in order to explain why one balance moves more slowly than the other.

nomen dan dat M K het vijuentwintichste deel waer van M G, so sal het vijuentwintichste deel van 2 ℔ de waegh in die ghestalt houden, waer-



af t'bewys openbaer is duer het 12 voorstel des 1en boucx, maer wy sullender hier om meerder claerheyt, noch een weynich af segghen.

TBEWYS. Anghesien K swaerheyts middelpunt beteeckent des ghegheuens of 2 lbs will keep the balance in that position, the proof of which is manifest from the 12th proposition of the 1st book, but we will say a little more about it here, in order to make it clearer. PROOF. Since *K* denotes the centre of gravity of the given balance, the vertical through *K* will be the centre line of gravity of ghegheuens, so sal de * hanghende duer K, des selfden swaerheyts middellini wesen, ende de hanghende duer G, is swaerheyts middellini des toegheleyden inde schael D, daerom de lini KG, tusschen die twee swaerheyts middellinien, is der seluer weeghconstighen balck; Maer sy is ghedeelt in M, also dat den erm MG, sulcken reden heeft tot den erm MK, als diens swaerheyt tot desens; De hanghende dan duer M, is swaerheyts middellini ofte handtaef des heels, ende veruolghens den balck blijft in die ghestalt, t'welck wy bewysen moesten. *Perpendicularis.*

TBESLVYT. Wesende dan ghegheuen een waegh, diens balck euewydich blijft vanden sichteinder, wy hebben t'ghewicht gheuonden, dat in d'een schael gheleyt, den balck in begheerde ghestalt houdt, na t'voornemen.

IIII^e VOORSTEL.

WESENDE ghegheuen een balck, welcke met haer schalen euewydich blijft vanden * sichteinder, maer sonder schalen op t'scherp vanden dweersas niet rusten en can: Te vinden hoe sware schalen men daer an hanghen sal, op dat den balck alle ghestalt houde diemen haer gheeft. *Horizonte.*

T'GHEBVERT sommighe balcken, dat sy sonder schalen op t'scherp van haren dweersas niet rusten en connen, maer wel de schalen daer an hanghende, welcker dinghen oirsaken wy duer de daet versoucken moeten. T'GHEGHEVEN. Laet AB een balck wesen van ghedaente deses voorstels, wiens dweersassens scherp sy C.

TBEGHEERDE. Wy moeten an desen balck twee schalen vinden (daerby men verstaen sal schalen met haer coorden en haecken) van sulck ghewicht, dat sy den balck alle ghestalt doen houden diemen haer gheeft. T'WERCK. Men sal vinden des balcx met de tong daer an swaerheyts middellini, euewydich van t'scherp des dweersas C duer het 1^e voorstel deses boucx, welcke sy D, bouen C, want in C noch onder C en salse niet vallen, ouermidts den balck op C, duer t'ghestelde niet rusten en can, noch min onder C. Daer naer salmen trecken de lini EF tusschen de plaetsen der ghenaecselen des balcx, ende de haecken der schalen, de selue sal nootsaecklick vallen onder C, want vielse daer in, of daer bouen, gheen schalen hoe swaer sy waren, en souden den balck alle ghestalt connen doen houden diemen haer gaue, ofte euewydich doen blijuen vanden sichteinder. Daer naer gheteeckent G int middel van EF, men sal trecken de rechte lini DCG, ende gheliick dan CD, tot CG, alsoo moet tghewicht der begheerde schalen HI sijn, tot

b 3 t'ghewicht

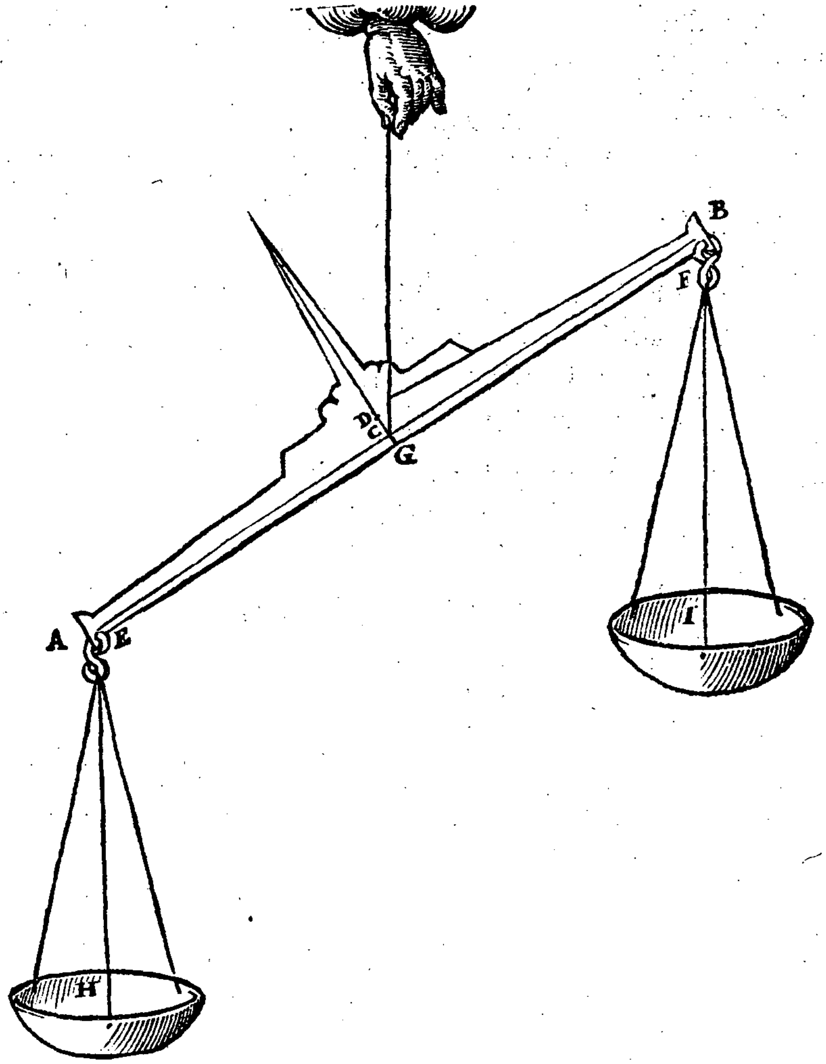
the same, and the vertical through *G* is centre line of gravity of the weight added in the pan *D*. Therefore the line *KG*, between these two centre lines of gravity, is its mathematical beam. But it is divided in *M* in such a way that the arm *MG* has to the arm *MK* the same ratio as the gravity of the former to that of the latter. The vertical through *M* therefore is centre line of gravity or handle of the whole, and consequently the beam remains at rest in that position which we had to prove. CONCLUSION. Given therefore a balance whose beam remains parallel to the horizon, we have found the weight which, when laid in one of the pans, shall keep the beam in the required position, as intended.

PROPOSITION IV.

Given a beam, which with its pans remains parallel to the horizon, but cannot remain at rest without the pans on the sharp edge of the transverse axis: to find the weight of the pans which have to be hung thereon in order that the beam shall remain at rest in any position given to it.

It happens with some beams that without the pans they cannot remain at rest on the sharp edge of their transverse axis, though they can when the pans hang thereon, the cause of which we have to investigate by practice. SUPPOSITION. Let *AB* be a beam of the kind referred to in this proposition, the sharp edge of whose transverse axis shall be *C*. WHAT IS REQUIRED TO FIND. We have to find on this beam two pans (by which are to be understood pans with their cords and hooks) of a weight such that they cause the beam to remain at rest in any position given to it. CONSTRUCTION. The centre line of gravity of the beam with the tongue shall be found, parallel to the sharp edge of the transverse axis *C*, by the 1st proposition of this book, which shall be *D*, above *C*, for it will not fall either in *C* or below *C*, since by the supposition the beam cannot remain at rest on *C*, and even less so below *C*. Thereafter the line *EF* shall be drawn between the places of the points of contact of the beam and the hooks of the pans; this will necessarily fall below *C*, for if it fell in it or above it, no pans, however heavy, could cause the beam to remain at rest in any position given to it or cause it to remain parallel to the horizon. Thereafter, *G* being marked in the middle point of *EF*, the straight line *DCG* shall be drawn, and then, as *CD* is to *CG*, so must be the weight of the required pans *H* and *I* to the weight

t'ghewicht des balcx; ick neme dat C D euen ſy an C G, t'ghewicht dan der ſchalen ſal euen moeten weſen an t'ghewicht des balcx, waer af t'be-
Mathematici.
wys * Wiſconſtlick ghedaen is int 10ᵉ voorſtel des 1ᵉⁿ boucx, daer toe wy hier tot meerder claerheyt noch een weynich ſullen ſegghen.



Perpendicularis.
TBEWYS. De * hanghende duer D, is ſwaerheyts middellini des balcx ter eender ſijden, ende de hanghende door G is ſwaerheyts middellini der ſchalen ter ander ſijde; G D dan is Weegconſtighen balck: Maer
ghelijck

of the beam. I take *CD* to be equal to *CG*. The weight of the pans will then have to be equal to the weight of the beam, the mathematical proof of which has been given in the 10th proposition of the 1st book, to which we will add a little more in order to make it clearer. PROOF. The vertical through *D* is centre line of gravity of the beam on one side and the vertical through *G* is centre line of gravity of the pans on the other side. Therefore *GD* is mathematical beam. But ghelijck den erm C D tot den erm C G, alſo deſe ſwaerheyt tot die duer t'gheſtelde',het houdt dan op C alle gheſtalt diemen hem gheeft, t'welck wy bewyſen moeſten. T'BESLVYT. Weſende dan ghegheuen een balck, welcke met haer ſchalen euewydich blijft vanden ſichteinder, maer ſonder ſchalen op t'ſcerp vanden dweerſas niet ruſten en can; wy hebben gheuonden hoe ſware ſchalen men daer an hanghen ſal, op dat de balck alle gheſtalt houde diemen haer gheeft, naer de begheerte.

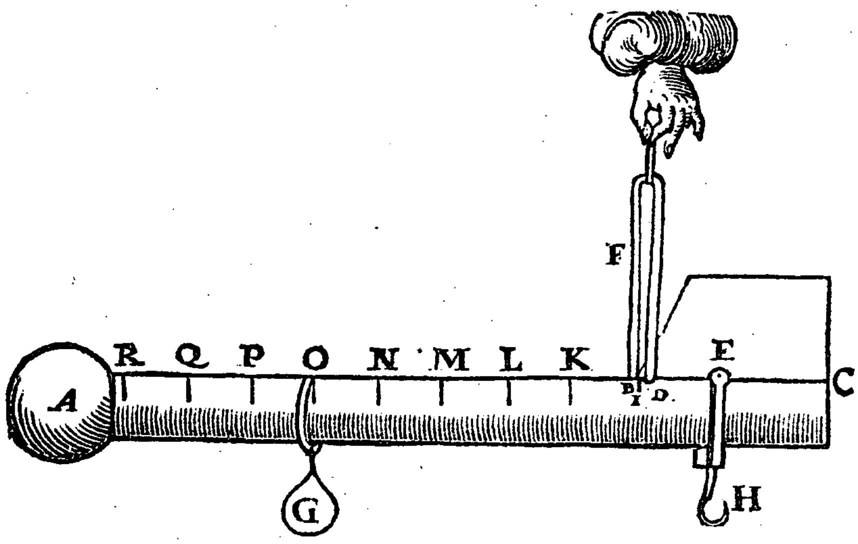
MERCKT.

Tis openbaer, dat by aldien de ſchalen yet ſwaerder waren dan bouen gheſeyt is, ofte dat in haer eenighe euen ſwaerheden gheleyt wierden, ſoo en ſoude den balck dan niet alle gheſtalt houden diemen hem gheeft, maer euewydich blijuen vanden ſichteinder, daerom en ſijn ſulcke waghen niet de volmaeckſte.

V VOORSTEL.

EEN alderuolmaecſten onſel te maken.

TWERCK. Men ſal des lichameliken balcx opperſten cant A B voorttrecken tot in C, ende laten inde lini B C de ſcherpten commen der twee dweerſaſſen D, E, weluerſtaende dat de ſcherpte van D neerwaert ſtrecke, ende van E opwaert; Daernaer ſalmen van het dickeinde des balcx naer B C, ſoo veel afvilen ofte weeren, tot dat alles int huyſken



F eueſtaltwichtich hanghe, ende dat bouen dien de ſcherpte vanden dweerſas D (t'huyſken F gheweert ſijnde) ſwaerheyts middellini blijue des as the arm *CD* is to the arm *CG*, so is the latter gravity to the former by the supposition. The beam therefore remains at rest in *C* in any position given to it, which we had to prove. CONCLUSION. Given therefore a beam which with its pans remains parallel to the horizon, but cannot remain at rest without the pans on the sharp edge of the transverse axis; we have found the weight of the pans which have to be hung thereon in order that the beam shall remain at rest in any position given to it, as required.

NOTE.

It is manifest that if the pans were somewhat heavier than stated above, or if some equal gravities were laid therein, the beam would not then remain at rest in any position given to it, but would remain parallel to the horizon; therefore such balances are not the most perfect.

PROPOSITION V.

To construct a most perfect steelyard.

CONSTRUCTION. The upper side *AB* of the corporeal beam shall be extended to *C*, and the sharp edges of the two transverse axes *D*, *E* shall be made to come in the line *BC*, it being understood that the sharp edge of *D* shall point downwards and that of *E* upwards. Thereafter so much material shall be filed off or removed from the thick end of the beam adjacent to *BC* until everything shall hang in equality of apparent weight in the fork *F*, and moreover the sharp edge of the transverse axis *D* (when the fork *F* has been removed) shall remain centre des lichamelicken balcx A C. T'welck soo sijnde ende den seluen balck int huysken F hanghende, sy sal daer in (soo lang den dweersas D op haer scherpte rust) alle ghestalt houden diemen haer gheeft. Daernaer salmen sien van wat swaerheyt t'schuyswicht G, ende den haeck H sullen sijn, diemen daer an begheert te hanghen; Ick neem G een pondt, ende H een once, dat is t'festhiende deel van G; Daerom salmen teeckenen I, alsoo dat de lini tusschen I ende t'scherp des dweersas D, euen sy an t'sestiendedeel van D E; Daernaer salmen de langde D E (dat is de lini tusschen de scherpten der twee dweersassen) teeckenen van I naer A, soo dickmael als sy daerin commen wil, t'welck ick neem te wesen in K, L, M, N, O, P, Q, R, daernaer machmen elcke langde als I K, K L, L M, &c. deelen in soo veel euen deelen alst de plaets toelaet, als in tween, oft in vieren, oft in achten, oft in sestienen, &c. ende alles sal volmaeckt sijn.

Maer oft dit soo nau passen der dweersassen den onselmaeckers te moeylick viel, sy mueghent (ghelijck int voorgaende 2e voorstel vande waegh oock gheseyt is) houden als voor hun wit, dat soo naer volghende als sy connen, ende t'scherp des dweersas D lieuer een haerken bouen de lini A C laten comen, dan daer onder.

Wat de ghebruyck belangt, als G an O hangt, ende anden haeck H een swaerheyt met de rest euestaltwichtich, die swaerheyt sal vijf pont weghen, ouermidts van I tot O vijf teeckenen staen. Maer soo elcke langde als I K, K L, L M, &c. ghedeelt waer in festhienen, elck deel soude een once beteeckenen. By voorbeelt of G hijnghe tusschen P en Q, an het vijfthiende deel van P naer Q, de swaerheyt an H soude dan sijn van 6 ℔ 15 oncen, ende alsoo metten anderen. Nu ouermits desen onsel (ghenomen t'schuyswicht niet neerwaert en sliere als d'een sijde leeghst daelt) met alle euestaltwichtighe deelen die op beyde sijden hanghen, alle ghestalt houdt diemen huer gheeft soo ist (om de redenen die wy int voorgaende voorstel vanden aldervolmaecksten waegh gheseyt hebben) den aldervolmaeckstenonsel. Angaende t'bewys, alles is openbaer door het 2e voorstel des eersten boucx. T'BESLVYT. Wy hebben dan een alderuolmacsten onsel ghemaeckt naer de begheerte.

VI. VOORSTEL.

DE scheefwaeg te maken.

WANT de ghewichten niet altemael rechtneerwaert noch rechtopwaert en roeren, maer sijdeling, ende scheef; ghelijck vooren verscheyden voorbeelden daer af beschreuen sijn, ende hier na beschreuen sullen worden, so behouwen dese een waegh van ander form dan de ghemeene, welcke wy tot onderscheyt van d'ander Scheefwaeg noemen: Hüer voornaemste line of gravity of the corporeal beam *AC*. Which being so, and the beam hanging in the fork *F*, it will remain at rest therein (as long as the transverse axis *D* rests on its sharp edge) in any position given to it. Thereafter it shall be ascertained what gravity the sliding weight *G* and the hook *H*, which are required to be hung thereon, must have. I take *G* one pound, and *H* one ounce, i.e. one-sixteenth of *G*. Therefore, *I* shall be marked in such a way that the line between *I* and the sharp edge of the transverse axis *D* shall be equal to one-sixteenth of *DE*. Thereafter the distance *DE* (i.e. the line between the sharp edges of the two transverse axes) shall be plotted from *I* to *A* as often as it will fall therein, which I take to be in *K*, *L*, *M*, *N*, *O*, *P*, *Q*, *R*. Thereafter each segment, as *IK*, *KL*, *LM*, etc., may be divided in as many equal parts as the space permits, as in two, or in four, or in eight, or in sixteen parts, etc.; then everything will be complete.

But if this accurate adjustment of the transverse axes should be too difficult for the makers of steelyards, let them (as stated also in the 2nd proposition hereinbefore with regard to the balance) aim at approximating it as much as possible, causing the sharp edge of the transverse axis *D* to come a hair's breadth above the line *AC* rather than below it.

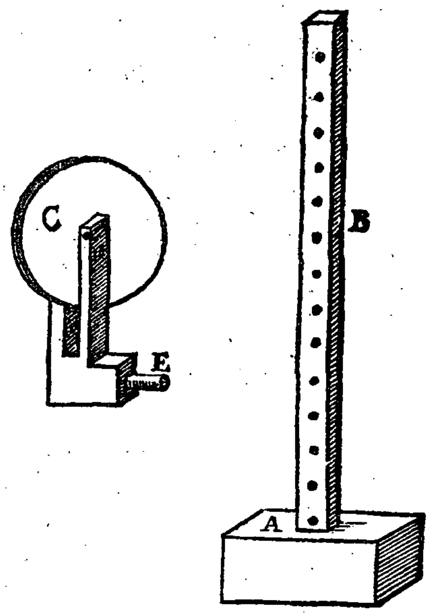
As to the use of the steelyard, if *G* hangs at *O*, and at the hook *H* hangs a gravity which is of equal apparent weight to the rest, the latter gravity will weigh five pounds, since there are five marks from *I* to *O*. But if each segment, as *IK*, *KL*, *LM*, etc., were divided into sixteen parts, each part would denote an ounce. For example, if *G* hung between *P* and *Q*, at the fifteenth part from *P* to *Q*, the gravity at *H* would then be 6 lbs 15 oz., and similarly with others. Now since this steelyard (assuming that the sliding weight shall not slide down when one side descends lowest), with all the parts in equality of apparent weight hanging on either side, remains at rest in any position given to it, it is (for the reasons we gave in the proposition hereinbefore about the perfect balance) the most perfect steelyard. As to the proof, everything is manifest from the 2nd proposition of the first book. CONCLUSION. We have therefore constructed a most perfect steelyard, as required.

PROPOSITION VI.

To construct the oblique balance.

Because weights do not all move straight downwards or upwards, but also sidelong and obliquely, of which many examples have been described hereinbefore and will be described hereinafter, these require a balance of a different form from the usual one, which we call oblique balance in order to distinguish it from naemſte einde is om duer ongenſchijnelicke eruaring te ſien, onderſoucken, ende verſtaen, de waerheydt der voorſtellen vande eueredenheydt ſoodanigher ghewichten int eerſte bouck * Wiſconſtlick beſchreuen, op datmen hem alſoo te vaſtelicker betrau in t'ghene men inde Daet tot ſ'menſchen voordering daer duer uytrechten wil. *Mathematicè.*

T'WERCK. Men ſal maken een voet als A, met een reghel daer op tot verſcheyden plaetſen duerboort als B, daer naer een caterol als C, met een grouue rondtom inden cant daer een draet in loopen mach, ende in ſijn middel ſy een as D, ruſtende met beyde haer einden in een huyſken, t'welck met het pinneken E, gheſteken mach worden inde gaetkens der reghel B, ſoo hooghe ofte leeghe alſmen wil, ende ſal volmaect ſijn. Maer t'voornaemſte daermen op letten moet (op datmen een ſcheefwaegh heb die ſcherpelick weghe) is, dat het caterol ende den as daer in al t'ſamen moeten ghedraeyt ſijn, ende t'ſelue caterol ende den as ſoo dun alſmen can, ende dat de ronde nerghens int huyſken en ghenake, latende tuſſchen de einden des dweerſas ende t'plat des caterols, eenighe dickte, wat dicker dan de einden des as. Ick heb voor my daer toe doen drayen een caterol van boſboom, wiens dickte niet meer en was dan als den rugghe van een dun mes, ende des rondts middellini van ontrent vijf duymen, ende den as (al met den anderen ghedraeyt) van yvoor, vande dickte als een cleermakers naelde, te weten ſoo dun alſt den draeybanck lijden mocht.



VII VOORSTEL.

T'ONDERSOVCKEN de ghedaenten der ſteerten daermen ghewelt mede doet.

SIENDE de menſchen datmen met langher ſteerten een merckelicker grooter ghewelt dede dan met de corter, ſy hebben veel ghemeene

 reetſchappen

the other type. Its main object is to make us see, examine, and understand through visual experience the truth of the propositions on the proportionality of such weights, described mathematically in the first book, in order that we may have all the more confidence in that which we wish to effect therewith in practice for the benefit of mankind.

CONSTRUCTION. A base shall be made, as *A*, with a ruler thereon which is perforated in several places, as *B*; thereafter a pulley, as *C*, with a groove all around the rim, through which may pass a thread, and in its centre there shall be a spindle *D*, supported with both ends in a fork, which can be put with the pin *E* into the holes of the ruler *B*, as high or low as may be desired; then the instrument will be complete. But the main point to be noted (in order to have an oblique balance weighing accurately) is that the pulley and the spindle passing through it should be turned together on the lathe, and that the pulley and the spindle should be as thin as possible, and that the circle shall not touch the fork anywhere, some space being left between the ends of the transverse axis and the surface of the pulley, a little more than the ends of the spindle. For this, I had a pulley turned of boxwood, whose thickness was no greater than the back of a thin knife, the diameter of the circle being about five inches and the axis (turned together with the other parts) being of ivory, of the thickness of a tailor's needle, to wit as thin as the lathe could produce it.

PROPOSITION VII.

To investigate the forms of the levers by which a force is exerted.

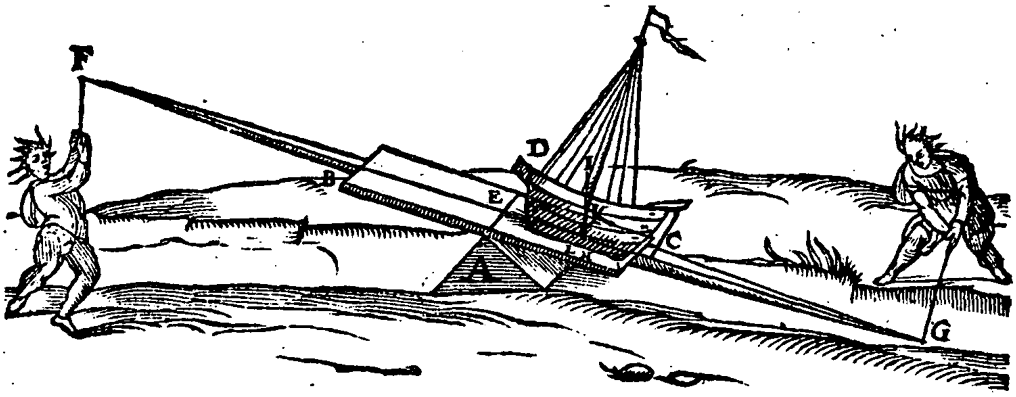
Men having noticed that considerably larger forces could be exerted with longer levers than with shorter ones, they have thereby accomplished the construction reetschappen tot hueren grooten dienste eñ voordeele daer duer ter daet ghebrocht: Maer want sulcx alleen gheschiede duer eruaringhen, ende niet duer grondelicke kennis der * eueredenheyt in huer bestaende, soo en sijn veel groote nieuwe wercken dickmael niet wel gheluckt, tot groote schade der Makers, ende verachtering des voornemens. Op datmen dan wete eermen beghint, wat de steerten int volmaeckte werck souden connen doen, wy sullen (bouen de * Wisconstighe voorstellen des eersten boucx alsulcx veruatende) eenighe daetlicke voorbeelden daer af beschrijuen. Ten eersten, want eenighe persoonen wel van meyning sijn gheweest, datmen de schepen bequamelicker ende met minder schade ouer een dam soude mueghen brenghen, duer t'behulp van langhe steerten, dan duer een windas, naer de ghemeene ghebruyck, wy sullen t'selue nemen als voorbeelt om te sien wat daer uyt volghen soude in deser voughen:

Proportionis.

Mathematicâ.

1e VOORBEELT.

T'GHEGHEVEN. Laet A een dam wesen, ende B C een plat houten bereytsel daer het schip D weghende 24000 ℔ op rusten mach (hoe t'ghewicht eens schips met al datter in is int water ligghende, bekent can worden, sal int Waterwicht sijn plaets hebben) ende dat E middel van B C passe op t'middel des dams A, ende laet B F den eenen steert sijn, ende C G (euen an B F) den anderen, ende t'schip D gheweert sijnde, so is de sijde E F euewichtich teghen E G, ende om t'schip ouer den dam te crijghen, men soude trecken an F, ofte heffen an G, ofte an beyde t'samen. Ende laet H I des schips swaerheyts middellini wesen, ende F E sy sesvoudich tot E H: Uyt het welcke men begheert te weten wat macht ofte ghewicht an F of G met het schip euestaltwichtich sal sijn.



T'WERCK. Ouermits F G is als balck eens waeghs, diens vastpunt E, ende schips swaerheyts middellini H I, ende dat F E sesvoudich is teghen

E H

of many common tools, to their great service and benefit. But because this was only done by experience, and not by thorough knowledge of the proportionality existing therein, many great new constructions often did not turn out well, to the great detriment of the makers and the delay of the work proposed. Therefore, in order that one may know before one begins what perfectly constructed levers might be capable of, we will (in addition to the mathematical propositions of the first book comprising all this) describe a few practical examples thereof. Firstly, because some people have been of opinion that ships could be hauled better and with less damage across a dam by means of long levers than with a windlass, as is the custom, we will take this as example in order to see what would result from it, in the following way:

EXAMPLE I.

SUPPOSITION. Let *A* be a dam, and *BC* a flat wooden device on which the ship *D*, weighing 24,000 lbs, is adapted to rest (the manner in which the weight of a ship with all that is in it, lying in the water, can be found is to be described in the book on Hydrostatics), and let *E*, the middle of *BC*, fit on the middle of the dam *A*, and let *BF* be one lever and *CG* (equal to *BF*) the other. Then, the ship *D* being taken away, the side *EF* balances *EG*, and in order to haul the ship across the dam one should pull at *F* or raise at *G*, or both simultaneously. And let *HI* be the ship's centre line of gravity, and let *FE* be six times *EH*. From which it is required to know what force or weight at *F* or *G* will be of equal apparent weight to the ship.

CONSTRUCTION. Since *FG* resembles the beam of a balance, whose fixed point is *E*, and the ship's centre line of gravity is *HI*, and *FE* is six times *EH*, E H, so sal t'schip sesuoudich sin teghen t'ghewicht dat an F hanghende met hem eueſtaltwichtich ſy, maer t'ſchip weeght duer t'gheſtelde 24000 ℔; An F dan ſoude moeten hanghen 4000 ℔: Daerom ſooder anhinghen 25 menſchen elck weghende 160 ℔, die ſouden teghen t'ſchip eueſtaltwichtich ſijn: Maer dit verſtaet hem op de ſtant daert nu in is, want nemende K voor ſwaerheyts middelpunt des ſchips, ende het deel E G rijſende, ſoo ſal an F min dan 4000 ℔ behouuen. Om van t'welck met voorbeelt te ſpreken, Laet ons trecken de lini K L rechthouckich op t'plat E C, inder voughen dat als t'plat E C euewydich ſal ſijn vanden ſichteinder, ſoo ſal K L des ſchips ſwaerheyts middellini ſijn. Ick neem nu dat E F ſeuevoudich ſy teghen E L, daerom t'ſeuenſte deel van 24000 ℔ als 3428 $\frac{4}{7}$ ℔, ſal t'ghewicht ſijn t'welck an F hanghende met de reſt alſdan in die ſtandt eueſtaltwichtich ſal ſijn.

MERCKT.

WY hebben hier een voorbeelt gheſtelt daermen hem in ſulcken handel ſoude naer mueghen rechten, maer tis te ghedencken dat E F ſeſvoudich ghenomen is teghen E H, t'welck wel eenen ſeer langhen ſteert ſoude moeten weſen ende ſterck naer de gheleghentheyt. Ick achte dattet in groote ſchepen (int anſien van beter) gheen goet einde en ſoude nemen; met cleyne ſchuytkens mochtet ſijn beſcheet hebben. Wel is waer, datmen an de einden F G windaſſen ſoude mueghen ſtellen, om ſoo veel volcx daer niet te behouuen, maer wy ſullen een beter manier beſchrijuen int volghende 10ᵉ voorſtel, ons hier vernoughende met de rekening van ſoodanighen voorbeelt verclaert te hebben, watmen t'ſijnen voordeele daer het te pas mocht commen, bequamelicxt ſal mueghen ghebruycken.

IIᵉ VOORBEELT.

Wy hebben in t'eerſte voorbeelt verclaert, de ghedaente der ſteerten die euelanck ende euewichtich ſijn, wy ſullen nu dit voorbeelt ſtellen van oneuen ſteerten. T'GHEGHEVEN. Laet A B C den eenen ſteert ſijn, ende A B D den anderen, ruſtende met de lini A B op de cant E; Ende de lini D C ſniende A B in F, ſy den as des heels D A C B weghende 400 ℔, ende ſijn ſwaerheyts middelpunt ſy G, (tis wel waer dattet ſwaerheyts middelplat rechthouckich opden as inde daet ghenouch ſoude doen, ſoo wel int volghende 3ᵉ ende 4ᵉ voorbeelt, als in dit, doch om eyghentlicker daer af te ſpreken, wy nemen het ſwaerheyts middelpunt) ende op het deel A B D light een ſwaerheyt H van 2000 ℔, diens ſwaerheyts middellini I K ſy, te weten K inden as D C; De vraegh is hoe ſterck men an C ſal moeten trecken, om H op te lichten.

T'WERCK. Men ſal vinden de ſwaerheyts middellini der ſwaerheyt H, ende des reetſchaps D A C B altſamen, aldus: Men ſal K G deelen in L alſoo dat G L ſulcken reden hebbe tot L K, als 2000 ℔ tot 400 ℔, dat

C 2 is als

the weight of the ship will be six times the weight which, hanging at *F*, shall be of equal apparent weight thereto. But the ship, by the supposition, weighs 24,000 lbs; therefore 4,000 lbs would have to hang at *F*. Therefore, if there hung at it 25 men, each weighing 160 lbs, these would be of equal apparent weight to the ship. But this applies to the position in which it is now, for if we take *K* to be centre of gravity of the ship and the part *EG* to be rising, less than 4,000 lbs will be required at *F*. By way of example, let us draw the line *KL* at right angles to the plane *EC* in such a way that if the plane *EC* is parallel to the horizon, *KL* will be the ship's centre line of gravity. I now take *EF* to be seven times *EL*; therefore the seventh part of 24,000 lbs, i.e. $3.428^{4}/_{7}$ lbs, will be the weight which, hanging at *F*, will then be of equal apparent weight to the rest in that position.

NOTE.

We have here given an example by which we might be guided in such a case, but it is to be borne in mind that *EF* has been taken six times *EH*, which would have to be a very long lever indeed, and strong for the occasion. I think that with big ships (as compared with better means [1])) it would not be successful, with small boats it might do. It is true that at the ends *F* and *G* windlasses might be placed, in order not to need so many men, but we will describe a better method in the 10th proposition hereinafter, contenting ourselves here with having explained the calculation of this example, which can be turned to account with profit wherever needed.

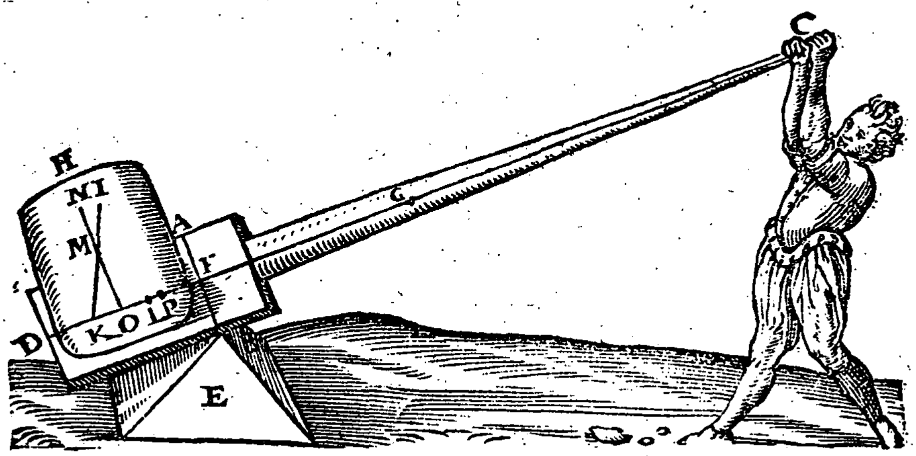
EXAMPLE II.

We have explained in the first example the forms of the levers which are equally long and of the same weight; we will now give this example of unequal levers. SUPPOSITION. Let *ABC* be one lever and *ABD* the other, resting with the line *AB* on the edge *E*, and the line *DC* intersecting *AB* in *F* shall be the axis of the whole *DACB*, weighing 400 lbs, and its centre of gravity shall be *G* (it is true indeed that in practice the centre plane of gravity at right angles to the axis would be sufficient, both in the 3rd and 4th examples hereinafter and in the present, but in order to deal with it more properly, we take the centre of gravity), and on the part *ABD* lies a gravity *H* of 2,000 lbs, whose centre line of gravity shall be *IK*, to wit *K* in the axis *DC*. The question is with what force a man will have to pull at *C* in order to lift *H*.

CONSTRUCTION. The centre line of gravity shall be found of the gravity *H* and the device *DACB* together, thus: *KG* shall be divided in *L* in such a way that *GL* shall have to *LK* the same ratio as 2,000 lbs to 400 lbs, i.e. as 5 to 1,

[1]) This somewhat obscure phrase is omitted by Girard in his French translation (XIII; iv. *Livre de la Statique*, p. 474a).

Perpendicularis. is als 5 tot 1, ende eenighe * hanghende door L sal des heels swaerheyts middellini sijn; Ick neem nu dat F C twelfvoudich beuonden sy teghen F L, daerom seg ick F C 12, gheeft F L 1, wat 2400 ℔? (te weten de somme des swaerheyts ende reetschaps) comt 200 ℔, voor t'ghene dat



an C hanghende met de rest in die gheleghentheyt euestaltwichtich sal sijn, daerom een man weghende 200 ℔, ofte so stijf treckende als 200 ℔ daer an hanghende trecken souden, sal met de reste euestaltwichtich sijn. Maer dit verstaet hem op de ghestalt daer nu in is, want nemende M voor swaerheyts middelpunt des ghewichts H, ende het deel A B D rysende, soo sal an C min dan 200 ℔ behouuen. Om t'welck opentlicker te verstaen, laet ons trecken de lini N O, duer t'punt M rechthouckich op t'plat A B D, inder voughen dat als t'plat A B D euewydich sal sijn vanden * sichteinder, soo sal N O swaerheyts middellini wesen der swaerheyt H, daerom ghedeelt O G in P, alsoo dat P G wederom vijfvoudich sy tot P O, te weten als 2000 ℔ tot 400 ℔, soo sal de hanghende duer P alsdan swaerheyts middellini wesen des heels; Ick neem nu dat F C vijfthienvoudich sy teghen F P, daerom seg ick F C 15, gheeft F P 1, wat 2400 ℔? comt 160 ℔, voor t'ghene dat an C hanghende met de rest alsdan euestaltwichtich sal sijn. *Horizonte.*

III^e VOORBEELT.

ANGHESIEN de wichtighe ghedaenten der lancien ofte dier ghelijcke, op de schauder ghedraghen, ghelijck ghenouch sijn ande ghedaenten des voorgaende tweede voorbeelts, soo sullen wy daeraf dit derde beschrijuen. T'GHEGHEVEN. Laet A een man sijn, hebbende op sijn schouder B, een lanci C D, weghende 12 ℔, wiens as sy C D, ende haer swaerheyts middelpunt sy E, ende van t'punt des naeckfels der lanci and any vertical through L shall be the centre line of gravity of the whole. I now take that FC be found twelve times FL; I therefore say: FC 12 gives FL 1, what 2,400 lbs (to wit the sum of the gravity and the device)? comes 200 lbs for that which, hanging at C, will be of equal apparent weight to the rest on this occasion [1]). Therefore a man weighing 200 lbs, or pulling as strongly as would 200 lbs hanging thereat, will be of equal apparent weight to the rest. But this applies to the position in which it is now, for if we take M to be centre of gravity of the weight H and the part ABD to be rising, less than 200 lbs will be required at C. In order to understand this more clearly, let us draw the line NO, through the point M, at right angles to the plane ABD, in such a way that if the plane ABD be parallel to the horizon, NO will be centre line of gravity of the gravity H. Therefore, OG being divided in P so that PG be again five times PO, to wit as 2,000 lbs to 400 lbs, the vertical through P will then be centre line of gravity of the whole. I now take FC to be fifteen times FP. I therefore say: FC 12 gives FP 1; what 2,400 lbs? comes 200 lbs for that which, hanging at C, will then be of equal apparent weight to the rest.

EXAMPLE III.

Since the properties of the weights of lances or the like, carried on the shoulder, are sufficiently similar to the properties of the second example hereinbefore, we will describe the third example with regard thereto. SUPPOSITION. Let A be a man having on his shoulder B a lance CD weighing 12 lbs, whose axis shall be CD and its centre of gravity E; and from the point of contact of the lance

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[1]) In this as well as other places we give a literal rendering of Stevin's formulation of the rule of three. The reader will no doubt recognize in his elliptical sentences the proportion: $FC : FL = 2{,}400 : x$; $12 : 1 = 2{,}400 : x$; $x = 200$ lbs.

lanci ende sijn schouder, sy ghetrocken de lini B F, rechthouckich op den sichteinder, sniende den as C D in G; Ende sijn handt rechtneerwaert treckende comt an t'punt H inden as, ende G H sy dobbel an G E.

T'BEGHEERDE. De vraegh is wat ghewelt de handt ande lanci doet.

T'WERCK. Ouermidts de lini G H dobbel is an G E, soo sal t'ghewicht an E, dat is der lanci, dobbel sijn an t'ghewicht an H, dat is t'ghene de handt treckt; Maer de lanci weegt 12 lb, de handt dan sal soo stijf trecken als 6 lb souden an H hanghẽde. Maer so den man A waer eẽ Snaphaen, met een ghesnapten haen I an K hanghende, weghende 3 lb, ende alsoo dat K G drievoudich waer an G H, tis kennelick dat den buyt sijn handt van 9 lb meer verswaren, ende in alles 15 lb trecken soude.

Dit is ghenomen dat de handt recht neerwaert trecke, maer als sy scheef treckt, ghelijck dan rechtdaellini tot scheefdaellini, alsoo rechtdaelwicht tot scheefdaelwicht, duer het 21e voorstel des 1en boucx der beghinselen, waer wt alles bekent wort duer het 22 voorstel des selfden boucx.

IIIIe VOORBEELT.

Wy hebben tot hier de ghedaente verclaert alwaer twee steerten sijn, ouer elcke sijde des vastpunts een; Wy sullen nu een voorbeelt gheuen vanden steert alleenelick ouer een sijde. T'GHEGHEVEN. Laet A B een steert sijn, vast an t'einde A, de rest verroerlick, weghende 400 lb, diens as A B, ende swaerheyts middellini C D, ende de steert A B sy lanck 10 voeten, waerop een ghewicht E light van 1000 lb, diens swaerheyts middellini F G. De vraegh is hoe sterck men an B sal moeten heffen om den steert met t'ghewicht E op te lichten.

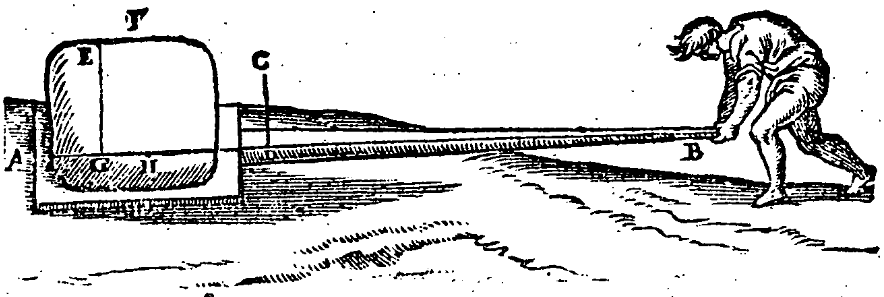
and the man's shoulder shall be drawn the line *BF* at right angles to the horizon, intersecting the axis *CD* in *G*. And the man's hand, pulling straight downwards, comes in the point *H* in the axis, and *GH* shall be double of *GE*. WHAT IS REQUIRED TO FIND. It is asked what force the hand exerts on the lance. CONSTRUCTION. Since the line *GH* is double of *GE*, the weight at *E*, i.e. that of the lance, will be double of the weight at *H*, i.e. the force exerted by the hand. But the lance weighs 12 lbs; therefore the hand will pull as strongly as would 6 lbs hanging at *H*. But if the man *A* were a poaching soldier, with a poached cock *I* hanging at *K*, weighing 3 lbs, in such a way that *KG* were three times *GH*, it is evident that the booty would weight his hand by 9 lbs more, and he would pull 15 lbs in all.

It is here assumed that the hand pulls straight downwards, but if it pulls obliquely, then as the vertical lowering line is to the oblique lowering line, so is the vertical lowering weight to the oblique lowering weight, by the 21st proposition of the 1st book of the elements, from which everything becomes manifest by the 22nd proposition of the same book.

EXAMPLE IV.

We have so far explained the form where there are two levers, one on each side of the fixed point. We will now give an example of a lever on one side only. SUPPOSITION. Let *AB* be a lever, fixed at the end *A* and the rest being movable, weighing 400 lbs, whose axis shall be *AB* and the centre line of gravity *CD*. And the lever *AB* shall be 10 feet long, on which there lies a weight *E* of 1,000 lbs, whose centre line of gravity shall be *FG* [1]. It is asked with what force a man will have to lift at *B* in order to raise the lever with the weight *E*.

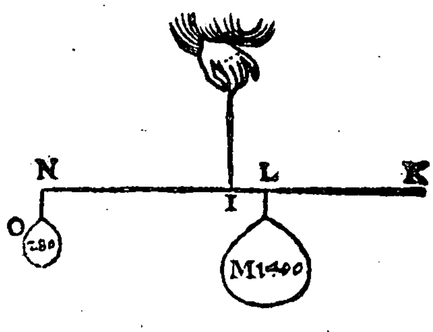
[1]) The letters *E* and *F* have been transposed in the figure.



TWERCK. Men sal vinden de swaerheyts middellini des heels,deelende eenighen balck tusschen de middellinien F G en C D, als G D, in H, alsoo dat H G sulcken reden hebbe tot H D, als 400 ℔ des steerts, tot 1000 ℔ des ghewichts F, dat is als 2 tot 5: Ick neem nu dat A H sy 2 voeten, ende seg, A B 10 voeten, gheeft A H 2 voeten, wat 1400 ℔ voor t'gheheele ghewicht des steerts ende pacx? comt 280 ℔. Men sal dan an B soo grooten ghewelt moeten doen om met de reste eueſtaltwichtich te sijn, als oftmen 280 ℔ ophielde.

Maer soo den Wegher de voornoemde rekening wilde maken door naeckter kennis des grondts, hy mach sich selfs Weeghconstighe formen beschrijuen, ghelijck den * Meter om t'versterckKen des ghedachts, hem *Meetconstighe voorstelt, aldus: Ick treck de lini I K, beteeckenende den steert A B van 10 voeten, ende ouermits A H twee voeten was, ende H swaerheyts middelpunt, ick teecken L, alsoo dat I L beteecken 2 voeten van I K 10, hanghende M 1400 ℔ an L, treckende daer naer I N euen an I K, ende houdende I voor vastpunt, ick sie wat ghewicht an N sal moeten hanghen, op dattet met M eueſtaltwichtich sy: T'selue is door het 3 voorstel des 1en boucx openbaer, maer wy sullender tot meerder claerheyt noch dit af segghen: Ouermits I L is als vijfdendeel van I N, soo moet an N (door t'voornoemde 3 voorstel des 1en boucx) t'vijfdendeel hanghen van M 1400 ℔, t'welck is voor O 280 ℔ eueſtaltwichtich teghen M; Maer O doet so veel an N dalendende, als t'selue ghewicht an K heffende, door het 13 voorstel des 1e boucx der beghinselen (want I N is euen an I K) daerom die an K heft sal moeten 280 ℔ heffen om met M eueſtaltwichtich te sijn, ende veruolghens

Geometra. *Geometricus.*



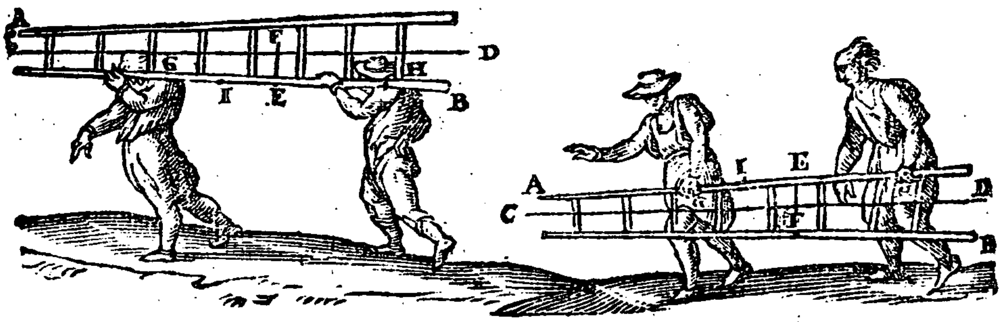
CONSTRUCTION. The centre line of gravity of the whole shall be found, which divides any beam between the centre lines *FG* and *CD*, as *GD*, in *H* in such a way that *HG* shall have to *HD* the same ratio as the 400 lbs of the lever to the 1,000 lbs of the weight *E*, i.e. 2 to 5. I now take *AH* to be 2 feet, and say: *AB* 10 feet gives *AH* 2 feet; what 1,400 lbs for the total weight of the lever and the load? comes 280 lbs. The force one has to exert at *B* in order to be of equal apparent weight to the rest will therefore be as if 280 lbs were lifted.

But if the weigher wished to make the above calculation through a nicer knowledge of the subject, let him draw for himself statical figures, just as the geometer, in order to aid his thought, imagines geometrical ones, as follows. I draw the line *IK*, representing the lever *AB* of 10 feet, and since *AH* was two feet and *H* was centre of gravity, I mark *L* in such a way that *IL* shall represent 2 feet of *IK* 10, *M* 1,400 lbs hanging at *L*. If thereafter *IN* be made equal to *IK* and *I* be taken as fixed point, I see what weight will have to hang at *N* in order that it may be of equal apparent weight to *M*. This is manifest from the 3rd proposition of the 1st book, but we will say the following about it, for the sake of greater clarity. Since *IL* is one-fifth of *IN*, there must hang at *N* (by the aforesaid 3rd proposition of the 1st book) one-fifth of *M* (1,400 lbs), which is *O* (280 lbs), of equal apparent weight to *M*. But *O* exerts at *N*, descending, the same force as the same weight at *K*, ascending, by the 13th proposition of the 1st book of the elements (for *IN* is equal to *IK*). Therefore, a man who lifts at *K* will have to lift 280 lbs to be of equal apparent weight to *M*, and consequently ghens hy moet 280 lb lichten an B, om met de rest euestaltwichtich te wesen. Der ghelijcke formen mach den Wegher in alle werckelicke voorbeelden sijn seluen altijt voorstellen, welcke hier om cortheyt achterghelaten sijn. TBESLVYT. Wy hebben dan ondersocht de ghedaenten der steerten daermen ghewelt mede doet, naer de begheerte.

VIII. VOORSTEL.

TE ondersoucken de ghedaenten der ghedreghen swaerheden.

T'GHEGHEVEN. Laet A B een leere wesen, op t'een einde swaerder als op t'ander soo sy ghemeenlick sijn, welcke ghedreghen moet worden van twee mannen, alsoo dat d'een soo veel ghewichts draghe als d'ander, dat is elck den helft, ende haer middellini C D, sal int draghen euewydich vanden * sichteinder blijuen. T'WERCK. Men sal de leere op eenighen scherpen cant legghen, die vertreckende voorwaert ende achterwaert, tot datmen bemercke de euestaltwichticheyt ghetroffen te sijne, t'welck ick neem in E te wesen, ende so sy diewils moet verdreghen sijn van d'een plaets ten anderen, men mach an E een kerfken stellen; Laet daernaer ghetrocken worden de hanghende E F, sniende C D in F, daernaer salmen teeckenen eenighe twee punten euewijt van E F, als G, H, ende die an G draecht sal euen soo veel draghen als die an H. *Horizonte.*



Maer soomen dien noch soo veel ghewichts wilde doen draghen als desen, men sal desens langde tusschen hem ende E F, dobbel maken an diens. Als H E dobbel sijnde an E I, die an I droughe soude noch soo veel ghewicht draghen als die an H. Ende alsoo salmen de reden des ghewichts vanden eenen tot den anderen, connen stellen naer de begheerte.

T'GHENE

he will have to lift 280 lbs at *B* in order to be of equal apparent weight to the rest. The weigher can always draw such figures for himself in all the real examples which have here been omitted for brevity's sake. CONCLUSION. We have therefore examined the forms of the levers by which a force is exerted, as required.

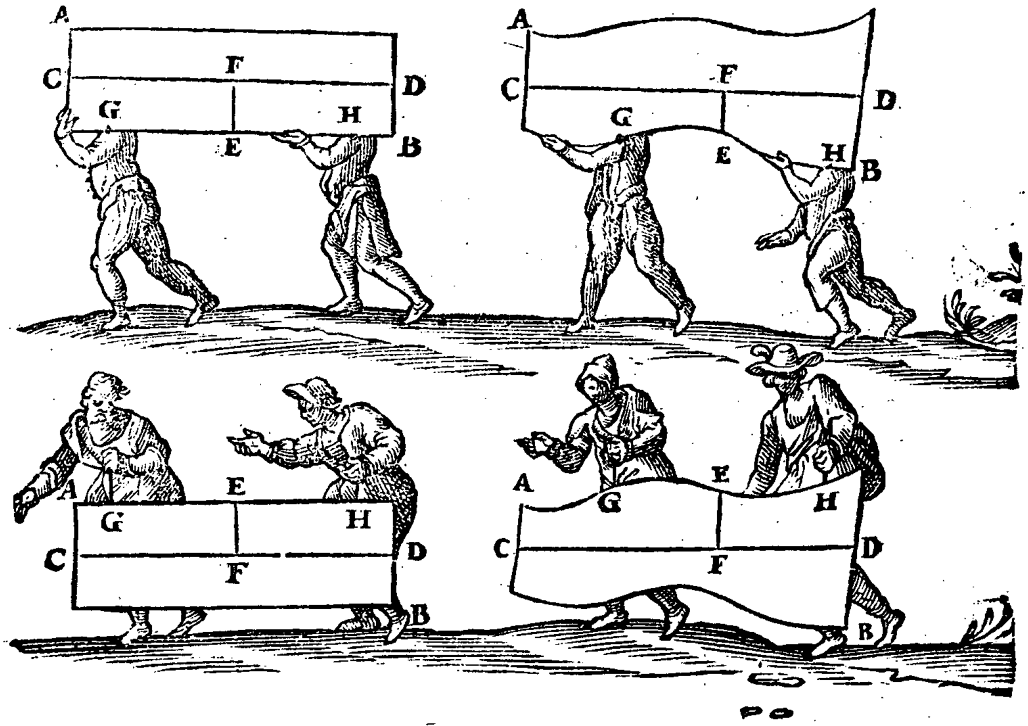
PROPOSITION VIII.

To examine the qualities of gravities that are being carried.

SUPPOSITION. Let *AB* be a ladder, heavier at one end than at the other end, as they usually are, which is to be carried by two men in such a way that one shall carry as much weight as the other, i.e. each one half; and its centre line *CD* shall remain parallel to the horizon during the carrying. CONSTRUCTION. The ladder shall be laid on a sharp edge and pulled forwards and backwards until equality of apparent weight is found to be attained, which I take to be in *E;* and if the ladder has to be often carried from one place to another, a notch may be made at *E.* Thereafter let there be drawn the vertical *EF*, intersecting *CD* in *F*, and then two points shall be marked at equal distances from *EF*, as *G* and *H*; then the man carrying the ladder at *G* will carry the same weight as the one at *H.*

But if it be desired to have that one carry twice the weight of this one, the distance between the latter and *EF* shall be made double of that between the former and *EF.* If *HE* were double of *EI,* the man carrying at *I* would carry twice the weight of the man at *H.* And thus the ratio of the weight of the one to that of the other can be fixed according to requirement.

TGHENE bouen gheſeyt is vande leere ſal hem alſoo verſtaen op yder lichaem, als by voorbeelt, de form hier onder, ghedenckende dat der ongheſchicter lichamen linien door haer ſwaerheyts middelpunt lijdende als C D, gheuonden worden door het 1e voorſtel deſes boucx, oock dat de hangende linien door G en H, eueuerre ſijn vande linien E F.

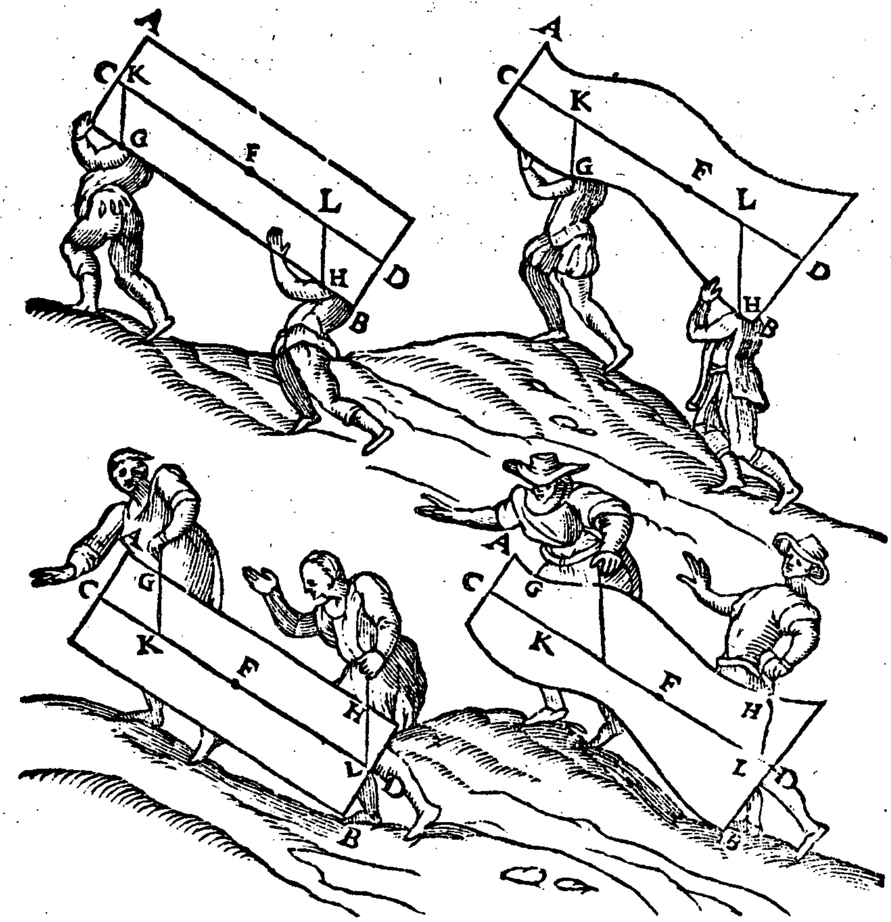


WY hebben hier voorbeelden gheſtelt alwaer de lini C D ghenomen is euewydich vanden ſichteinder, maer ſoo ſy daer af onevewydich waer, ende dat de ſelue mannen eenen berch ofte hoochde opſteghen, de reden vande ghewichten ſoude veranderen, doch bekent blijuen. Laet tot meerder claerheyt de voornoemde mannen een hoochde opgaen als hier onder, die an G vooren gaende d'ander achter.

Nv ghe-

What has been said above of the ladder will also apply to any body whatever, as for example the figure shown below, bearing in mind that with irregular bodies the lines passing through their centres of gravity, as *CD*, are found by means of the 1st proposition of the present book, and also that the verticals through *G* and *H* are equidistant from the lines *EF*.

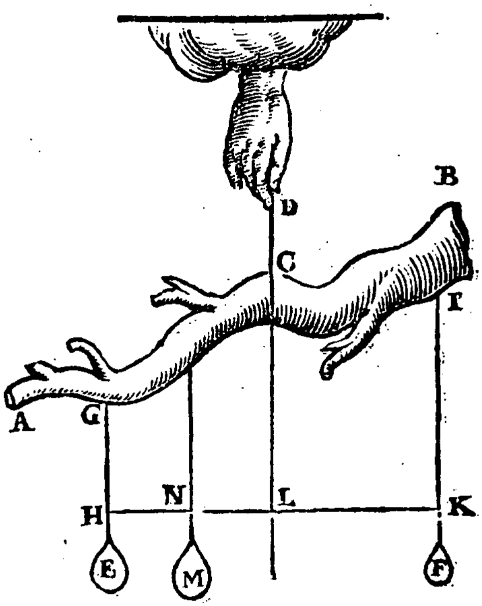
We have here given examples in which the line *CD* is taken parallel to the horizon, but if it were non-parallel thereto, and the men were ascending a mountain or height, the ratio of the weights would change, but remain known. For the sake of greater clarity, let the aforesaid men ascend a height, as shown below, the man at *G* going in front and the other behind.



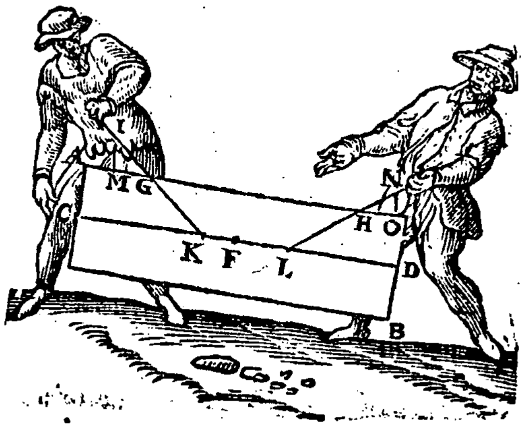
Nv ghetrocken * hanghende linien door de punten G, H, snijende C D in K en L, so en sal dan elck niet euen veel draghen als in d'eerste ghestalt, want F K inde twee opperste formē is meerder dan F L, eñ inde onderste formen minder: Ende ghelijck F K tot F L, alsoo t'ghewicht des draghers an H, tot het ghewicht des draghers an G. Alwaer oock blijckt dat als de vastpunten G, H, onder de lini C D sijn, soo draecht den voorsten minst, maer die vastpunten bouen de lini C D wesende, soo draecht den voorsten meest. Tis oock kennelick dat de vastpunten G, H, inde lini C D sijnde, dat alsdan elck oueral altijt sijn selfde ghewicht sal draghen, soo wel een berch opstighende, als langs den sichteinder. van alle welcke de bewysen openbaer sijn door de 14[e] 15[e] 16[e] 17[e] 18[e] 27[e] 28[e] voorstellen des 1[en] boucx. Maer want veler wercklieden gheleghentheyt niet en is die voorstellen te leeren, noch hemlieden daer in te oeffenen, *Perpendiculares.*

d ende

The verticals through the points *G*, *H* now being drawn, which intersect *CD* in *K* and *L*, each of the men will not then carry as much as in the first figure, for *FK* in the two figures at the top is greater than *FL*, and in the figures at the bottom less. And as *FK* is to *FL*, so is the weight of the man carrying at *H* to the weight of the man carrying at *G*. From which it also appears that if the fixed points *G*, *H* are below the line *CD*, the man in front carries least, but if these fixed points are above the line *CD*, the man in front carries most. It is also evident that if the fixed points *G*, *H* are in the line *CD*, each of the men will then always carry the same weight everywhere, both when ascending a mountain and when walking parallel to the horizon, the proofs of all of which are manifest from the 14th, 15th, 16th, 17th, 18th, 27th, 28th propositions of the 1st book. But because many workmen have no opportunity to learn these propositions or to ende nochtans gheerne wat ooghenſchijnelicx ſaghen, waer duer ſijt ghe-loofden, die mueghen nemen een rechten gheſchickten, ofte crommen ongheſchickten ſtock, ſoot valt, als A B, hem hanghende tot eenigher plaets als C, an een coorde C D. Daer naer hanghende anden ſtock euen ghewichten als E, F, alſoo dat haer coorden G H, I K, eueverre ſijn vande lini C D neerwaert ghetrocken, te weten dat H L euen ſy an L K, den ſtock ſal haer eerſte ſtant houden, t'ſelue ſal ſy oock doen ſoomen E weerde, ende datmen anhinghe t'ghewicht M, dobbel an F, ende alſo dat L K oock dobbel ſy an L N, ende ſoo met allen anderen, waer uyt ſy de nootſaeclicheyt van t'ghene bouen gheſeyt is, lichtelick gheuoelen ſullen.



DE linien daer mede de mannen inde voorgaende formen t'lichaem draghen, ſijn rechthouckich op den ſichteinder gheſtelt, maer ſoo ſy daer op ſcheefhouckich waren, als hier neuen, ſy ſullen t'ſamen meerder ghewelt moeten doen, dan de eyghen ſwaerheydt des lichaems is. Maer om te weten hoe veel yeghelick draecht, men ſal trecken de* hanghende linien I M, ende N O, ſegghende



Perpendiculares.

exercise themselves therein, and yet would like to see some evidence of it from which they can believe it, let them take any straight, regular or crooked, irregular stick, as *AB*, suspending it in some place, as *C*, by a cord *CD*. Thereafter, equal weights, as *E* and *F*, hanging at the stick in such a way that their cords *GH*, *IK* are equidistant from the line *CD* drawn downwards, to wit that *HL* shall be equal to *LK*, the stick will keep its first position. It will do the same, if *E* be taken away and the weight *M*, double of *F*, be suspended in such a way that *LK* shall also be double of *LN*, and thus with all the others, from which they will easily understand the necessity of what has been said above.

The lines by which the men in the preceding figures carry the body are placed at right angles to the horizon, but if they are at oblique angles thereto, as in the figure opposite, they will have to exert together a greater force than the gravity of the body itself. But in order to know how much each of them is carrying, the verticals *IM* and *NO* shall be drawn, and it shall be said: as *MI* is to *IG*, ſegghende, ghelijck M I tot I G, alſoo diens rechtheſwicht tot t'ghewicht dat den man an G treckt, wederom ghelijck O N tot N H, alſoo diens rechtheſwicht tot t'ghewicht dat den man an H treckt, duer het 27ᵉ voorſtel des 1ᵉⁿ boucx der beghinſelen, ende yders macht wort bekent door het 22ᵉ voorſtel des ſeluen boucx.

Wy ſouden meer verſcheyden voorbeelden vande wichtighe ghedaenten der ghedreghen lichamen mueghen beſchrijuen, maer wy latent eensdeels om de cortheyt, ten anderen dat ſy duer t'voorgaende kennelick ghenouch ſchijnen.

IX. VOORSTEL.

TE onderſoucken de ghedaenten der windaſſen, ende der ghetrocken ſwaerheden.

HET treckendwicht ende ghetrockenwicht des windas, ſijn* euerednich met de * halfmiddellini des as, ende de halfmiddellini des radts, maer om alles oirdentlicker te beſchrijuen, wy ſullender een * Vertooch af maken ſoodanich.

Proportionales.
Semidiameter.
Theorema.

VERTOOCH.

WESENDE *een Windas an diens as een ghewicht hangt, eueſtaltwichtich teghen t'ghewicht an t'einde des radts middellini die euewydich is vanden * ſichteinder: Ghelijck dan de halfmiddellini des radts, tot de halfmiddellini des rondts vanden as, alſoo t'ghewicht anden as, tottet ghewicht an t'radt.*

Horizonti.

T'GHEGHEVEN. Laet A B C D E F G een windas ſijn, diens as E F G, wiens rondts middellini E F, ende middelpunt H ſy, ende I een ghewicht anden as hanghende, ende A B C D ſy het radt, diens middellini euewydich vanden ſichteinder, ſy A C, an wiens einde A een ghewicht K hangt, eueſtaltwichtich teghen I, ende L ſy t'onderſte ghenaecſel des as teghen t'ghene daer ſy op ruſt.

T'BEGHEERDE. Wy moeten bewyſen dat ghelijck H A tot H F, alſoo I tot K.

T'BEWYS. Laet ons t'radt A B C D anſien als voor balck eens waeghs, diens hanthaef L B, inder voughen dat de ſijde des radts B D A, de ghewichten K, I, gheweert ſijnde, euewichtich hangt teghen de ſijde B D C. Laet ons nu nemen dattet ghewicht I hanghe an t'punt F (want het daer vande ſelue macht ſoude ſijn, diet t'ſijnder plaets is) ende K t'ſijnder plaets an A. Dit ſo weſende, ghelijck den langſten erm H A,

d 2 tot den

so is the vertical lifting weight along *MI* to the weight pulled by the man at *G*, and again as *ON* is to *NH*, so is the vertical lifting weight along *ON* to the weight pulled by the man at *H*, by the 27 proposition of the 1st book of the elements, and the force exerted by each becomes known from the 22nd proposition of the same book.

We might describe several more examples of the qualities of the weights of bodies that are being carried, but we omit it firstly for brevity's sake, and secondly because they seem to be sufficiently clear from the foregoing.

PROPOSITION IX.

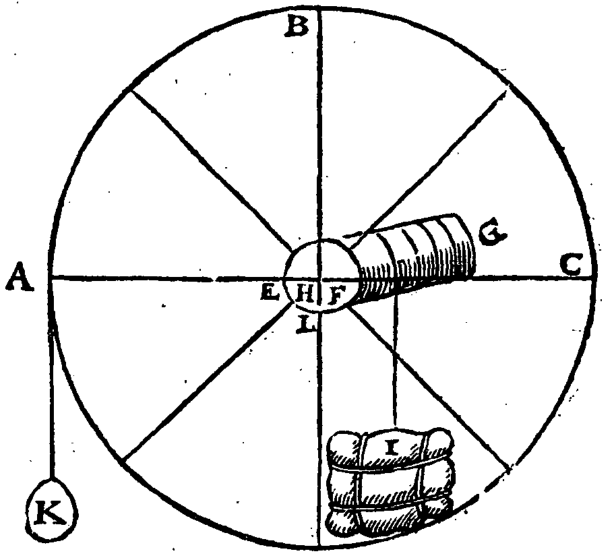
To examine the qualities of windlasses and of gravities that are being hauled.

The drawing weight and the drawn weight of the windlass are proportional to the semi-diameter of the axle and the semi-diameter of the wheel, but in order to describe everything more systematically, we will make of it a theorem, as follows.

THEOREM.

Given a windlass on whose axle hangs a weight of equal apparent weight to the weight at the end of the wheel's diameter which is parallel to the horizon: as the semi-diameter of the wheel is to the semi-diameter of the circle of the axle, so is the weight af the axle to the weight at the wheel.

SUPPOSITION. Let *ABCDEFG* be a windlass, whose axle shall be *EFG*, the diameter of the latter's circle being *EF* and its centre *H*, and *I* a weight hanging at the axle; and *ABCD* shall be the wheel, whose diameter parallel to the horizon shall be *AC*, at whose end *A* hangs a weight *K*, of equal apparent weight to *I*, and *L* shall be the lowest point of contact of the axle with that on which it rests. WHAT IS REQUIRED TO PROVE. We have to prove that as *HA* is to *HF*, so is *I* to *K*. PROOF. Let us consider the wheel *ABCD* as the beam of a balance, whose handle be *LB*, in such a way that the side of the wheel *BDA*, the weights *K* and *I* being taken away, balances the side *BDC*. Let us now take the weight *I* to hang at the point *F* (for it would there exert the same force as in its own place), and *K* in its tot den cortsten H F, alsoo de swaertste swaerheyt I, tot de lichtste K, duer het 1ᵉ voorstel des 1ᵉⁿ boucx der beghinselen. Daerom by aldien H A sesvoudich waer tegen H F, soo sal I sesvoudich wesen teghen K, dat is, weghende I ses hondert pont, K salder hondert weghen, daerom een man treckende an A, so stijf alshōdert ponden neerdrucken, die soude teghen I 600 lb eueſtaltwichtich sijn, eñ om I te doen rijsen soude (om t'ghenaecsel des as, &c.) wat stijuer moeten trecken dan 100 lb neerdrucken.



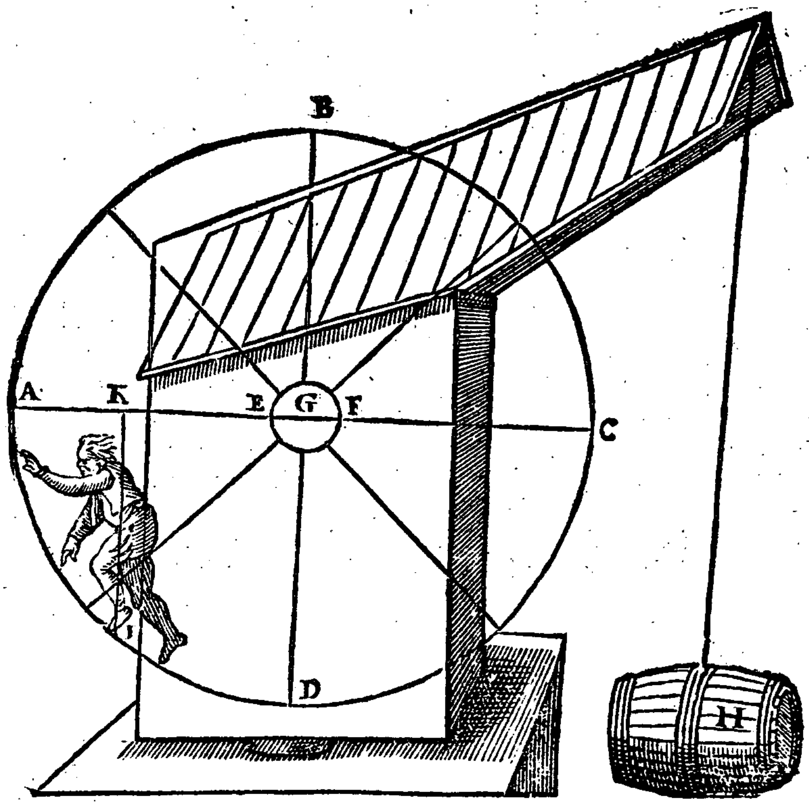
II VOORBEELT.

De ghedaenten der cranen ende der ghelijcke raeyers daer menschen in gaen sijn duer t'voorgaende oock openbaer. Laet tot voorbeelt
Diameter. A B C D een radt wesen, diens * middellini A C, euewydich sy vanden
Horizonte. * sichteinder, ende t'rondt des as sy E F, wiens middelpunt G, ende t'ghewicht anden as sy H, ende I sy een man in tradt eueſtaltwichtich teghen H, diens swaerheyts middellini rechthouckich op A C sy I K. Ende is kennelick dat ghelijck G K tot G F, alsoo t'ghewicht H tot het ghewicht des mans I, ghenomen dan dat G K viervoudich sy teghen G F, so sal t'ghewicht H viervoudich sijn teghen t'ghewicht des mans, daerom soo den man woughe 150 lb, soo sal H weghen 600 lb. Oock en sal den man op die plaets de swaerheyt H niet connen opwinden, ouermidts hy aldaer maer eueſtaltwichtich teghen H en staet; Maer by aldien hy voortgaet naer A, soo sal H rijsen, want de reden vande lini G K tot G F, soude dan grooter wesen dan sy nu is. Maer alsser meer menschen int radt gaen dan een, die naest A sijn doen t'meeste ghewelt, ende

place at *A*. This being so, as the longer arm *HA* is to the shorter arm *HF*, so is the heavier gravity *I* to the lighter *K*, by the 1st proposition of the 1st book of the elements. Therefore if *HA* be six times *HF*, *I* will be six times *K*, i.e. if *I* weighs six hundred pounds, *K* will weigh one hundred pounds. Therefore, a man pulling at *H* with the same force as one hundred pounds pressing downwards would be of equal apparent weight to *I* (600 lbs), and in order to raise *I* (because of contact of the axle, etc.) he would have to pull a little more strongly than 100 lbs pressing downwards.

EXAMPLE II.

The qualities of cranes and similar wheels in which go human beings are also manifest from the foregoing. By way of example, let *ABCD* be a wheel, whose diameter *AC* shall be parallel to the horizon, and the circle of the axle shall be *EF*, whose centre shall be *G*, and the weight at the axle shall be *H*, and *I* shall be a man in the wheel, of equal apparent weight to *H*, whose centre line of gravity at right angles to *AC* shall be *IK*. Then it is evident that as *GK* is to *GF*, so is the weight *H* to the weight of the man *I*. Taking therefore *GK* to be four times *GF*, the weight *H* will be four times the weight of the man; therefore, if the man weighs 150 lbs, *H* will weigh 600 lbs. Although in this place the man will not be able to hoist the gravity *H*, since he is there only of equal apparent weight to *H*, if he proceed to *A*, *H* will rise, for the ratio of the line *GK* to *GF* will then be greater than it is now. But if more men than one go in the wheel, those who



ende de reden van haer altſamen ende van yder int beſonder tot t'ghewicht H, is openbaer duer het 5^{e} voorſtel des 1^{en} boucx.

III. VOORBEELT.

DIT heeft hem alſoo met de ghewichten die recht op ghetrocken worden, als packen ende vaten diemen duer cranen uyt ſchepen windt, ende dier ghelijcke; Maer de ghewichten die ſcheef opwaert commen, als onder anderen de ſchepen diemen in Neerlandt tot veel plaetſen ouer dammen windt, tis met de *eueredenheyt van dien wat anders gheſtelt. *Proportione.* Laet tot voorbeelt A een dam weſen, ende B een ſchuyt die daer ouer ghetrocken moet worden, ende C D het radt, wiens middellini euewydich vanden *ſichteinder ſy C D; ende daer in een man teghen de ſchuyt B eueſtaltwichtich, wiens ſwaerheyts middellini F E, ende de coorde ſy G H, ende des aſſens rondt ſy I K, ende haer *middelpunt L: Laet oock ghetrocken ſijn N M, rechthouckich op t'plat des dams, ende

Horizonte.

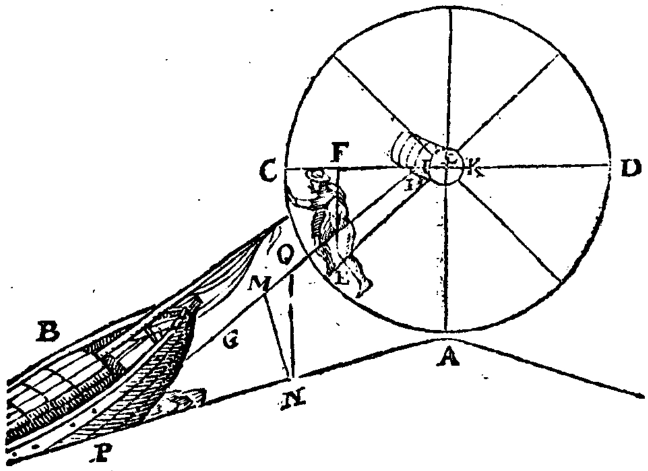
Centrum.

are nearest to A will exert the greatest force, and the ratio of all of them together and of everyone in particular to the weight H is manifest from the 3rd proposition of the 1st book.

EXAMPLE III.

This applies to the weights which are drawn up vertically, such as bales and barrels which are hoisted from ships by means of cranes and the like. But as to the weights which are raised obliquely, such as the ships which in Holland are hauled across dams in many places, the proportion of these is somewhat different. For example, let A be a dam and B a barge which has to be hauled across it, and CD the wheel, whose diameter parallel to the horizon shall be CD, and therein a man being of equal apparent weight to the barge B, whose centre line of gravity shall be FE, and the rope shall be GH, and the circle of the axle shall be IK and its centre L. Let there also be drawn NM, at right angles to the plane of the dam, *Perpendicularis.*

M inde coorde G H; Daernaer de * hanghende O N; Laet nu L F ſesvoudich ſijn tot L K, ende N O drievoudich tot O M, ende den man weghen 150 ℔. Dit ſoo ſijnde ghelijck L F tot L K, alſoo t'ghewicht dat ande coorde H G rechtneer ſoude hanghen, tot t'ghewicht des mans van 150 ℔, duer t'voorgaende vertooch, maer L F is duer t'ghestelde ſesvoudich an L K, t'ghewicht dan dat ande coorde H G rechtneer hinghe, ſoude ſesvoudich ſijn an 150 ℔, dat is 900 ℔; den man dan doet in t'radt ſoo veel ghewelts ande ſchuyt B, als ofter met de ſcheefwacgh 900 ℔ an hinghen. T'welck ſo ſijnde t'ghewicht der ſchuyt B, heeft ſulcken reden tot die 900 ℔, als N O tot O M duer het 20e voorſtel des



1en boucx; Maer N O is drievoudich an O M duer t'gheſtelde, de ſchuyt dan weeght driemael 900 ℔, dat is 2700 ℔, dat is achtienmael den man. T'welck hem ſoo verſtaet weſende de ſchuyt in die gheſtalt, maer als ſy hoogher comt, ſoo ſal de coorde G H ſteylder ſijn (ten waer men die ande ſchuyt verſette) ende veruolghens de lini als M O ſal wat meerder reden hebben tot O N, dan ſy nu doet, waer duer oock het eveſtaltwicht teghen de ſchuyt alſdan meerder ſoude ſijn dan 900 ℔; Daerom yemant willende een radt ende as van pas bauwen, niet te groot noch te cleen, mach ſijn rekening maken naer de gheſtalt daer in een der ſwaerſte ſchuyten ofte ſchepen de meeſte ghewelt behouft.

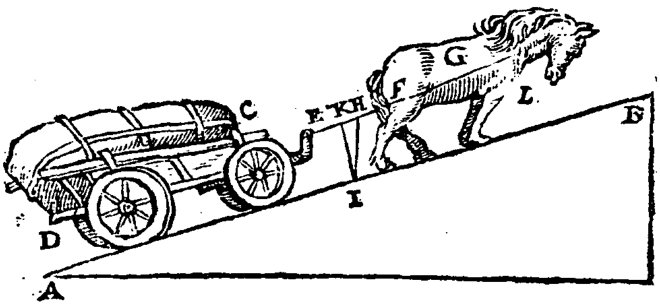
Tis oock te ghedencken dat den man E in tradt de meeſte ghewelt doet, als de coorde G H evewydich is van t'plat des dams P N, duer het 24e voorſtel des 1en boucx der beghinſelen; want dan is H G rechthouckich op den as (op dat ickſe ſoo noem) der ſchuyt, dat is op de lini duer t'ſwaer-with *M* in the rope *GH*, and thereafter the vertical *ON*. Now let *LF* be six times *LK*, and *NO* three times *OM*, and let the man weigh 150 lbs. This being so, as *LF* is to *LK*, so is the weight which would hang down vertically at the rope *HG* to the weight of the man (150 lbs), by the foregoing theorem. But *LF*, by the supposition, is six times *LK*; therefore, the weight which would hang down vertically at the rope *HG* would be six times 150 lbs, i.e. 900 lbs. Therefore, the man exerts in the wheel as much force on the barge *B* as if 900 lbs were hanging at it with the oblique balance. Which being so, the weight of the barge *B* has to these 900 lbs the same ratio as *NO* to *OM*, by the 20th proposition of the 1st book. But *NO* is three times *OM* by the supposition; therefore the barge weighs three times 900 lbs, i.e. 2,700 lbs, that is eighteen times the weight of the man. This applies to the case when the barge is in that position, but when it reaches a higher point, the rope *GH* will be steeper (unless it be displaced at the barge), and consequently the line *MO* will have a somewhat greater ratio to *ON* than it has now, as a result of which that which is of equal apparent weight to the barge will then also be more than 900 lbs. Therefore, if anyone wishes to construct a suitable wheel and axle, neither too large nor too small, let him make his calculations according to the position in which one of the heaviest barges or ships requires the greatest force.

It should also be borne in mind that the man *E* in the wheel exerts the greatest force when the rope *GH* is parallel to the plane of the dam *PN*, by the 24th proposition of the 1st book of the elements. For then *HG* is at right angles to the axis (if I may so call it) of the barge, i.e. to the line through the centre of t'swaerheyts middelpunt der schuyt, ende rechthouckich op t'plat P N: Daerom hoe dat G H ende P N de euewydicheyt naerder sijn, hoe lichter werck, ende hoe verder, hoe swaerder.

IIII^e VOORBEELT.

UYT het voorgaende is oock kennelick, hoe veel ghewichts een peert in een waghen ghespannen, meer treckt een hoochde op styghende, dan opt plat landt. Laet by voorbeelt A B t'plat sijn eens berghs, ende C D een waghen, weghende met datter op is al tsamen 2000 ℔, ende E F (inde plaets der strijnghen) sy de coorde, ende G sy t'peert, eveſtaltwichtich teghen den waghen. Laet oock ghetrocken sijn de * hanghende H I, ende I K, rechthouckich op t'plat A B, ende laet I H viervoudich sijn tot H K, ende is kennelick duer het 20 voorstel des 1^en boucx der beghinselen, dat ghelijck K H tot H I, alsoo t'ghewicht der scheefwaegh sooder een waer (in diens plaets nu t'peert is) tottet ghewicht des waghens, maer K H is t'vierendeel van H I duer t'ghestelde; des scheefwaegs wicht dan soude van 500 ℔ sijn, te weten t'vierendeel van t'ghewicht des waghens; Daerom t'gareel oft riem oft sulcx alst waer, druckt t'peert soo stijf voor den borst L, als een pack van 500 ℔ op sijn rugghe duwen soude, ende dat (wel verstaende alst voortgaet) bouen het duytsel dattet lijdt op t'plat landt treckende.

Perpendicularis.



TIS oock openbaer duer het 24^e voorstel des 1^e boucx, ende duer t'ghene wy hier vooren vande schuyt gheseyt hebben, dat als de strijngen euewydich sijn vande wech daer de waghen ouer vaert, dat de peerden dan de meesten ghewelt ande waghē doen, welverstaende op eenen harden gantsch effenen wech, maer op eenen oneffenen hobbelighen eñ sandighen, so voorderet de strijnghen achter wat leegher te doen dan vooren. T'welck den Hollantschen voerlien duer deruaring niet onbekent en is, diens waghens daer naer ghemackt sijnde, doen de strijnghen, langs t'zeestrant varende, ende in dergelijcke euen harde weghen, achter hoogher gravity of the barge and at right angles to the plane *PN*. Therefore, the nearer *GH* and *PN* are to being parallel, the lighter will be the work, and the further they are from being so, the heavier will be the work.

EXAMPLE IV.

From the foregoing it is also evident with how much greater weight a horse harnessed to a wagon draws when ascending a height than on level land. For example, let *AB* be the surface of a mountain and *CD* a wagon weighing, together with all that is on it, 2,000 lbs; and *EF* (in the place of the traces) shall be the rope, and *G* shall be the horse, of equal apparent weight to the wagon. Let there also be drawn the vertical *HI*, and *IK* at right angles to the plane *AB*, and let *IH* be four times *HK*; then it is evident by the 20th proposition of the 1st book of the elements that as *KH* is to *HI*, so is the weight of the oblique balance if there were one (whose place is now taken by the horse) to the weight of the wagon. But *KH* is one-fourth of *HI* by the supposition; therefore the weight of the oblique balance would be 500 lbs, to wit one-fourth of the weight of the wagon. Therefore the breastharness or strap or whatever it is presses as strongly against the horse's breast *L* as a bale of 500 lbs would press on its back, such (provided it is proceeding) over and above the pressure it would bear when pulling on level land.

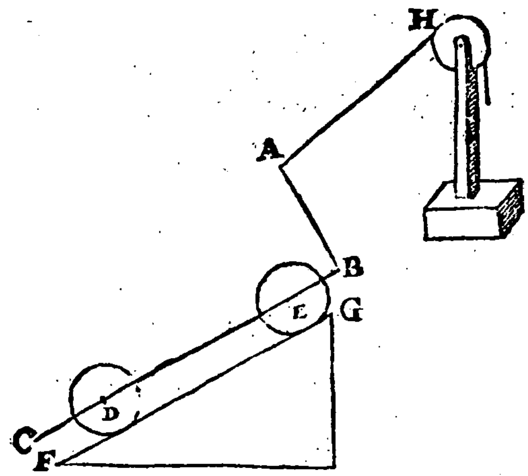
It is also manifest from the 24th proposition of the 1st book and from what we have said above of the ship that if the traces are parallel to the road on which the wagon moves, the horses then exert the greatest force on the wagon, that is to say on a hard, perfectly smooth road, but on a rough, bumpy, and sandy road it is of advantage to attach the traces somewhat lower at the back than in front; which through experience is not unfamiliar to the Dutch carriers, whose wagons, being adapted thereto, have the traces higher at the back when travelling along the hoogher, dan inde oneuen ende ſandighe. Reden is deſe, dat weſende de ſtrijnghen euewydich vanden ſichteinder, ſo en ſijnſe niet euewydich met die oneuen verheffelen, t'welck int ouertrecken nootſakelick meerder laſt anbrengt, dan als de ſtrijnghen achter leegher ſijn, ouermidts ſy dan de euewydicheyt met die verheffelen naerder ſijn. Inde ſandighe daer de waghen diep insinckt, daer drucken de raeyers dieper ende moeylicker duer t'ſant, weſende de ſtrynghen euewydich vanden ſichteinder, dan als ſy achter wat leegher ſijn.

MERCKT.

YEMANT mocht ons nu twee ſaken voorworpen; Ten eerſten, warom wy hier bouen gheſeyt hebben: *Ghelijck K H tot H I, alſoo t'ghewicht der ſcheefwaegh ſoo der een Waer (in diens plaets nu t'peert is) tottet ghewicht des Waghens*, Achtende datmen niet en behoort te ſegghen *tottet ghewicht des Waghens*, maer, *tottet rechthefwicht van t'ghewicht des waghens.*

Ten tweeden waerom wy gheen onderſcheyt beſchreuen en hebben vande plaets der coorde E F, t'wyſelende dat de ſelue voortghetrocken ende lydende duer t'ſwaerheyts middelpunt des waghens, een ander ghewicht voor t'peert mocht veroirſaken, dan als ſy daer bouen of daer onder comt. Om op t'welck te verantwoorden, ende * Wiſconſtlick te bewyſen dat de bouēſchreuen eueredenheyt volcommen is: ſo laet A B C een waghen ſijn, al van wiſconſtighe linien ghemaeckt, wiens raeyers ſijn D, E, ende den wech daer hy op ruſt ſy F G, ende de coorde des toecōmenden ſcheefſhewichts ſy A H.

Mathematicè.



LAET ons nu op deſen waghen legghen een pilaer I K als hier onder, alſoo dat H A voortghetrocken comme in des pilaers ſwaerheyts middelpunt L, eñ laet het ſcheefhefwicht M teghen den pilaer eueſtaltwichtich ſijn; Laet oock an L gheuoucht worden t'rechthefwicht N, met beach and similar smooth and hard roads than on rough and sandy roads. The reason is that if the traces are parallel to the horizon, they are not parallel to those unevennesses, which in being passed across must needs produce a greater load than if the traces are lower at the back, since they are then nearer to being parallel to those unevennesses. On sandy roads, into which the wagon sinks deeply, the wheels force themselves deeper and with more difficulty through the sand if the traces are parallel to the horizon than if they are somewhat lower at the back.

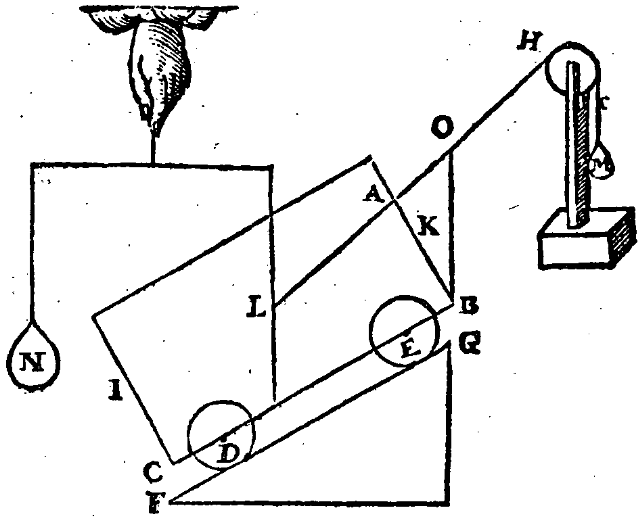
NOTE.

Someone might now raise the following two objections: Firstly, why we have said above: *As KH is to HI, so is the weight of the oblique balance if there were one (whose place is now taken by the horse) to the weight of the wagon;* considering that we ought not to say *to the weight of the wagon,* but *to the vertical lifting weight of the weight of the wagon* [1]).

Secondly, why we have not made any distinction as to the place of the rope *EF*, wondering whether this rope, when produced and passing through the centre of gravity of the wagon, would not cause a weight for the horse other than if it comes above or below it. In order to account for this and to prove mathematically that the proportion described above is perfect: let *ABC* be a wagon, wholly made up of mathematical lines, whose wheels be *D* and *E*, and the road on which it rests shall be *FG*, and the rope of the oblique lifting weight applied shall be *AH*.

Now let us place on this wagon a prism *IK*, as shown below, in such a way that *HA* produced shall come in the centre of gravity *L* of the prism, and let the oblique lifting weight *M* be of equal apparent weight to the prism. Let there also be applied to *L* the vertical lifting weight *N*, of equal apparent weight to the

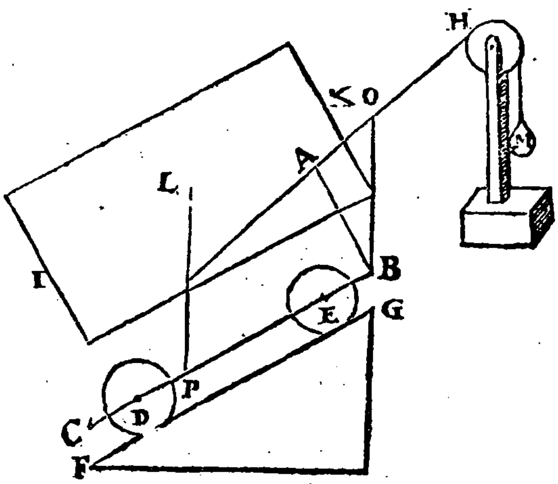
[1]) This means: to the force directed vertically upwards which keeps the weight of the wagon in equilibrium.



met den pilaer eueſtaltwichtich: Laet ons oock trecken de * hanghende B O, ſniende A H in O, t'welck ſo ſijnde, wy ſegghen duer het 20ᵉ voorſtel des 1ᵉⁿ boucx, dat ghelijck A O tot O B, alſou M tot t'rechthefwicht N; Maer anghesien N gheuoucht is an t'ſwaerheyts middelpunt L, des pilaers I K, ſoo ſal N euewichtich ſijn met den pilaer duer het 14 voorſtel des 1ᵉⁿ boucx; Daerom mueghen wy ſegghen ghelijck A O tot O B, alſo M tot den pilaer, waer uyt d'eerſte voorgheworpen ſaeck openbaer is als A H comt uyt het ſwaerheyts middelpunt L.

Perpendicularem.

MAER om nu het tweede voorgeworpen te bewyſen, dat is de ſelue * euerеdenheydt dus ooc te beſtaen al en cōt A H niet uyt het ſwaerheyts middelpunt L, ſo laet ons den pilaer I K recht-opwaert uyt dē waghen treccken, ruſtende op de * hanghende lini L P, als hier neuen: Ende duer de 3 Begheerte, ſy en



Proportionel.

Perpendicularem.

e

prism. Let us also draw the vertical *BO*, intersecting *AH* in *O*, which being so, we say, by the 20th proposition of the 1st book, that as *AO* is to *OB*, so is *M* to the vertical lifting weight *N*. But since *N* is applied to the centre of gravity *L* of the prism *IK*, *N* will balance the prism by the 14th proposition of the 1st book. Therefore we can say: as *AO* is to *OB*, so is *M* to the prism, by which the first objection in settled, when *AH* proceeds from the centre of gravity *L*.

But in order to settle the second objection, i.e. that the proportion in question also holds even if *AH* does not proceed from the centre of gravity *L*, let us pull the prism *IK* vertically up from the wagon, resting on the vertical *LP*, as shown opposite. Then according to the 3rd postulate it will not apply to the wagon ſy en brengt op den waghen A B C, gheen meerder ſwaerheyt dan in d'eerſte gheſtalt, eñ vervolghens M en heeſt niet meer te trecken dan ſy te vooren en dede: Maer H A voortghetrocken comt nu onder t'ſwaerheyts middelpunt L. Commende dan de voortghetrocken H A onder t'ſwaerheyts middelpunt L, ſoo treckt M t'ſelfde ghewicht dat ſy track doen H A in t'ſwaerheyts middelpunt quam. T'ſelue ſalmen oock alſoo bethoonen commende de voortghetrocken lini H A bouen t'ſwaerheyts middelpunt L, dat is treckende den pilaer I K rechtneerwaert onder den waghen. Vyt het welcke t'voornemen openbaer ende beweſen is.

x. VOORSTEL.

T'MAECKSEL ende de eyghenſchappen des Almachtichts te verclaren.

PLVTARCHVS ende ander, ſchrijuen dat Hiero Koninck van Sicilien, dede bauwen een ſchip van uytnemender grootheyt ende conſtigher form, om te ſchencken an Ptolemeus Koninck van Egypten; T'welck, doent volmaeckt was, de burgheren van Syracuſa om ſijn ſwaerheyt in zee niet crijghen en conden, maer doen Archimedes daer an gheſtelt had ſijn reetſchap die de Griecken Chariſtion noemen, Hiero heeft het daer duer ſelfs alleen metter handt vertrocken. Deſe Chariſtion (naer de form die Iacques Beſſon daer af heeft laten uytgaen, gheuonden inde* bouckamer des Kuenincx van Vranckrijck) had aſſen met vijſen, draeyende inde canten van ettelicke raeyers: Een werck voorwaer weerdich ſijn eeuwighe ghedachtnis, ende ſoudent hier beſchriuen ouermidts wy tot ſulcke ſtof ghecommen ſijn, ten waer wy in die plaets ſtelden TALMACHTICH (reetſchap die wy om ſijne ouergroote macht dien naem gheuen) tot ſulcke daer bequamer, te weten Stercker gheduerigher werck; Van minder coſt; Duer t'welckmen op corter tijt meer afveerdicht; Ende (ghelijck de Chariſtion) van oneindelicke cracht,* machtelick welverſtaende, niet daetlick. T'maeckſel daer af is ſoodanich:

Bibliotheca.

Potentia non actu.

Men ſal nemen een boom ofte balck als A B, ſterck ende groot naer de cracht dieder duer ghedaen moet ſijn: Daer naer ſalmen maken een yſer ſterreken als C, ick neem dat ſijn* middellini van drie duymen ſy, ende dattet ſes tanden heb, ende in ſijn middel ſtekende een yſeren as C D, an d'einden C D viercantich, ende tuſſchen beyden rondt, daer naer de ſterre E, ick neem met 18 tanden, ende t'ſterreken F met 6 tanden, daer in ſtekende een yſeren as E F, euen ende ghelijck met den as C D, te weten ande einden viercantich, ende tuſſchen beyden rondt

Diameter.

ABC any greater gravity than in the first position, and consequently *M* does not have to draw any greater weight than before. But *HA* produced now comes below the centre of gravity *L*. Therefore, *HA* produced coming below the centre of gravity *L, M* draws the same weight it did when *HA* came in the centre of gravity. The same can also be shown if the line *HA* produced comes above the centre of gravity *L*, i.e. when the prism *IK* is pulled vertically down below the wagon. By which the objection is settled.

PROPOSITION X.

To explain the construction and the properties of the Almighty.

Plutarch [1]) and others [2] report that Hiero, King of Sicily, had a ship built, exceptionally large and of ingenious form, in order to present it to Ptolemy, King of Egypt. However, when it was completed, the citizens of Syracuse could not get it into the sea because of its heaviness, but when Archimedes had applied to it his device called Charistion [3]) by the Greeks, Hiero was able to move the ship by himself, by hand. This Charistion (judging from the drawing which Jacques Besson [4]) made of it, found at the library of the King of France) had shafts with screws, rotating in the sides of several wheels, a device truly worthy to be eternally remembered. We should describe it here, since we have now come to this subject matter, but that we are dealing instead with the Almighty (a device we so call on account of its exceptional power), which is more suited to such work, for the following reasons: sturdier and more durable construction; of lower cost; by which more is done in shorter time; and (like the Charistion) of infinite power, that is to say: potentially, not actually. The construction of this device is as follows:

There shall be taken a tree or beam, as *AB*, sufficiently strong and big for the force which has to be exerted therewith. Thereafter a small iron gear wheel, as *C*, shall be made; I assume that its diameter is three inches and that it has six teeth, and in its centre an iron shaft, as *CD*, of square cross-section at the ends *C* and *D*, and round in between. Thereafter the gear wheel *E*, I take with 18 teeth, and the

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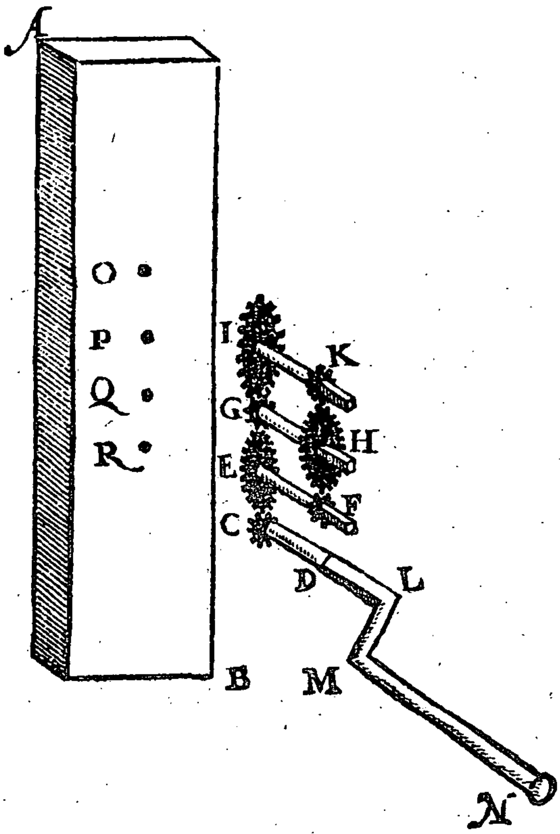
1) Plutarchus, *Vita Marcelli* XIV, 7.

2) An elaborate description of the great ship Surakosia (later Alexandris), which Hiero had had built for King Ptolemy of Egypt, is given by Athenaeus in his *Deipnosophistae*. *Athenaei Naucratitiae Dipnosophistarum Libri XV*; rec. G. Kaibel (Leipzig 1887) V, 40—44.

3) Not all Greek authors who tell the story of the ship use the name Charistion for the device applied by Archimedes. It is also called a polyspaston (Plutarchus, *Vita Marcelli* XIV, 8) or trispaston (Tzetzes, *Chiliades* II. Hist. 35, 107). The name Charistion is given by Simplicius (*In Aristotelis Physicarum libros Commentaria*; ed. H. Diels, Berlin 1895, p. 1110) to the device Archimedes would have used for moving the earth, if he had been able to find a fixed point outside it. He says that it was a kind of balance, and if the Charistion may be identified with the Charasto mentioned by Gerard of Cremona in a Latin translation of a work by Tabit ibn Qurra (*Liber Charastonis*), this was also the meaning attached to it in the Middle Ages. Tzetzes (*Chiliades* II. Hist. 35, 130) also mentions the Charistion in this connection, without, however, explaining the name. Obviously Stevin and his source, Besson, identify the Charistion with the Baroulkos described by Heron (*Mechanicorum Fragmenta*, ed. G. Schmidt, Leipzig 1900, p. 256; *Dioptra*, ed. H. Schöne, Leipzig 1903, p. 306). In this device a windlass is turned by a system of toothed wheels, the last of which is put in motion by means of an endless screw.

4) Jacques Besson, *Théatre des instruments mathématiques et méchaniques*, Lyon. Or: *Theatrum instrumentorum et machinarum*, Lyon, 1582.

den rondt: ende ghelijck E F is, ſoo ſalmen oock maken G H, ende I K, dat is G ende K elck met ſes tanden, ende H ende I met 18 tanden. Maer want de bouenſte ſterren de meeſte laſt ſullen lijden, als hier naer blijcken ſal, ſo ſullen ſy ſtercker ende grooter ſijn dan d'onderſte, daer uyt ooc volghen ſal dat de aſſen euewydich van malcanderen weſende, de ſterre H ſal connen ghenaken an F, ende niet an K, ende de ſterre G an I, ende niet an E, t'welck ſoo weſen moet.



Daer naer ſalmen maken de kruck L M N, wiens viercantich gat des viercantighen cokers L, paſſe op alle de viercantighe einden der aſſen, als D, F, H, K, ende L M ſy lanck een voet, ſo dickmael ſulcke langden in crucken van ſlijpſteenen ende dierghelijcke ſijn, ende M N ſoo lanck als hier naer gheſeyt ſal worden. Daer naer ſalmen inden boom A B vier gaten booren, van malcander in ſulcker wyde als de vier aſſen ſtaen, ghelijck de gaten O, P, Q, R, van achter duer commende, daer de vier aſſen I K, G H, E F, C D in paſſen mueghen, ende de langde der aſſen tuſſchen de ſterren, ſal euen ſijn ande dickte des booms, ende der aſſen vierhouckighe einden an K, H, F, D ſullen al ontrent de drie ofte vier duymen buyten de ſterren ſteken; Daer naer afstreckende de ſterre I, men ſal den as I K ſteken, int gat O, ende inſghelijcx den as G H in t'gat P, ende E F in Q, ende C D in R, ſtellende wederom elcke ſterre van achter an huer as, alſo

e 2 dat de

small gear wheel *F* with 6 teeth, in which there is an iron shaft *EF*, equal and similar to the shaft *CD*, to wit of square cross-section at the ends and round in between. And as *EF* is, in the same way *GH* and *IK* shall also be made, that is *G* and *K* each with six teeth, and *H* and *I* with 18 teeth. But because the upper wheels will have to bear the greatest load, as will be found hereinafter, they must be stronger and larger [1]) than the lower, from which it also follows that, the shafts being parallel to each other, the wheel H will be able to touch *F* and not *K*, and the wheel *G* will be able to touch *I* and not *E*, which has got to be.

Thereafter the crank *LMN* shall be made, the square hole of whose square tube *L* shall fit all the square ends of the shafts, as *D*, *F*, *H*, *K*, and *LM* shall be one foot long, as is often the length of the cranks of grindstones and the like, and *MN* shall have the length to be specified hereinafter. Thereafter there shall be drilled in the tree *AB* four holes, at the same distances from one another as the four shafts, as the holes *O*, *P*, *Q*, *R*, extending at the back of the tree, in which the four shafts *IK*, *GH*, *EF*, *CD* may fit; and the length of the shafts between the wheels shall be equal to the thickness of the tree, and the square ends of the shafts at *K*, *H*, *F*, *D* shall all extend about three or four inches beyond the wheels. Thereafter, when the wheel *I* has been drawn off, the shaft *IK* shall be put into the hole *O*, and similarly the shaft *GH* into the hole *P*, and *EF* into *Q*, and *CD* into *R*, upon which each wheel shall again be mounted on its shaft at

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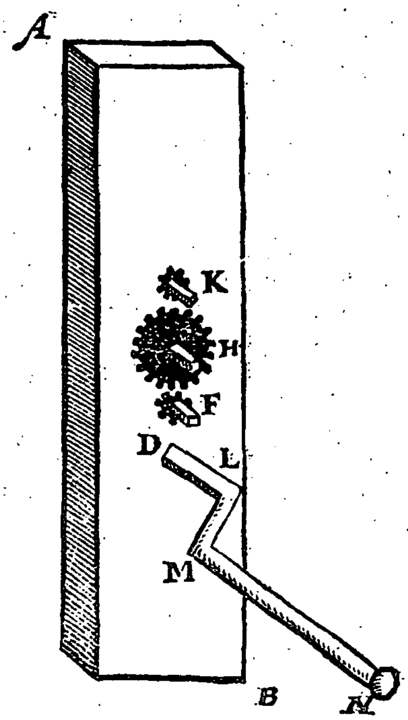
[1]) This entails that the diameters of the wheels should also be enlarged; the drawing however, shows no signs of this.

dat de tanden der ſterre F ande voorſte ſijde, mueghen doen draeyen de ſterre H, ende dat de tanden der ſterre C ande achterſte ſijde, mueghen doen drayen de ſterre E, ende dat de tanden van G, doen draeyen I, ende haer ghe-ſtalt voor volmaeckt Almachtich ſal dan ſijn als hier neuens.



Nu ghelijck wy t'voorbeelt hier ghegheuen hebben van vier aſ-ſen, ſoo machmender meer ofte min ſtellen: Ende de 18 tanden der groote ſterren welcke drievou-dich ſijn tot de ſes tanden der kleene ſterren, die machmen in meerder ofte minder reden ſtel-len, naer gheleghentheydt van t'ghene daermen T'Almachtich toe maeckt.

VAN TGHEBRVYCK ENDE ANDER

ANCLEVING DES ALMACHTICHS.

MAER om de ghebruyck deſes Almachtichs te verclaren, wy ſul-len een voorbeelt gheuen daer alle d'ander ghenouch duer ſullen bekent ſijn, te weten van ſchepen daer mede ouer dammen of dijcken te trecken, want dat den cleynſten dienſt niet en ſchijnt, die deſe landen hier in ghedaen mach worden, voornamelick Hollandt. Laet A B t'bouenſchreuen Almachtich ſijn, met de ſterren K, H, F, ouer dees ſijde des booms A B, ende de ſterren I, G, E, C, ouer d'ander ſijde, eñ L M N ſy de cruck, ende S den as, diens middellini van 1 $\frac{1}{2}$ voet ſy, commende duer den boom met een yſer ſterre an t'einde als T, wiens middellini ick neem te ſijn van 2 voeten (ſy moet ten minſten ſoo veel langher ſijn dan de middellini van t'rondt des as S, dat de ſterre I den as S niet en ghe-raecke) ende te hebben 36 tanden, ende V ſy den dam, wiens hoochde bouen t'onderſte des ſcheps vrielick int water ligghende (dat is int anſien der * hanghende lini van t'ſop des dams totter plat euewydich vanden ſicht-

Perpendicu-laris.

the back, in such a way that the teeth of the wheel *F* in front can rotate the wheel *H*, and that the teeth of the wheel *C* at the back can rotate the wheel *E*, and that the teeth of *G* rotate *I*; then the form of the completed Almighty will be as shown opposite.

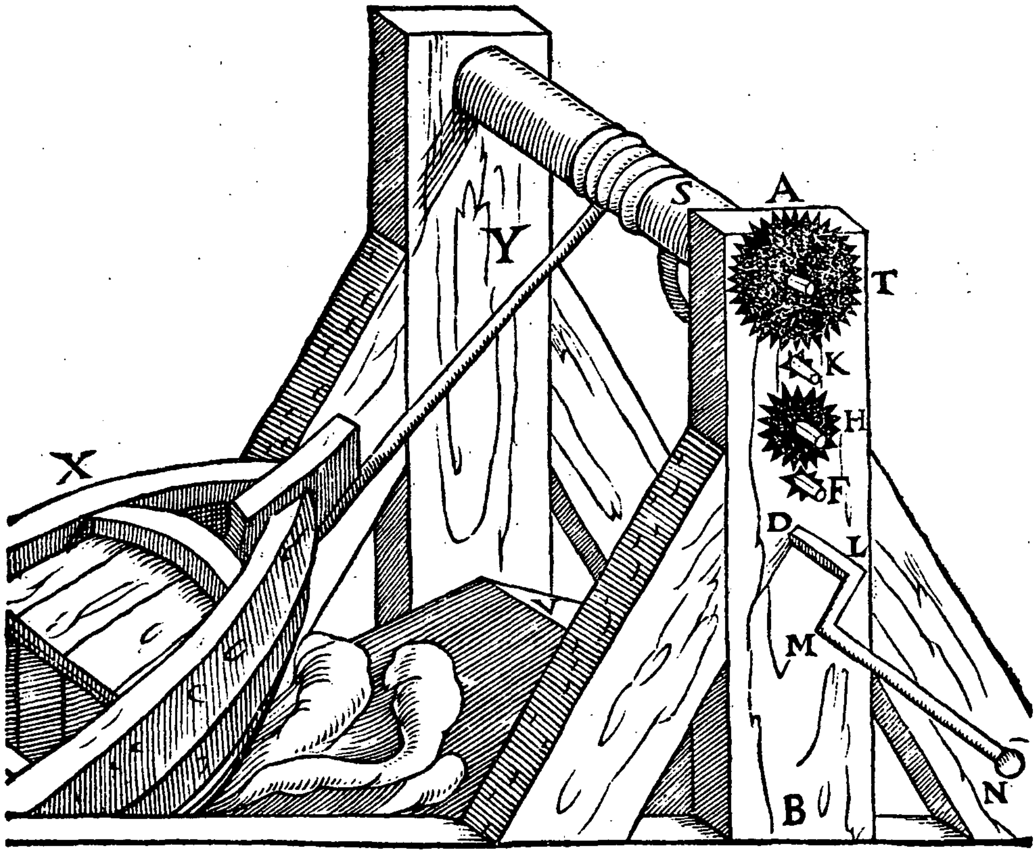
Now just as we have here given an example of four shafts, there may also be more or fewer. And the 18 teeth of the large wheels, which are three times the six teeth of the small wheels, may be taken in a greater or lesser ratio, according to the occasion for which the Almighty is made.

OF THE USE AND OTHER ATTRIBUTES OF THE ALMIGHTY

In order to explain the use of this Almighty we will give an example from which all the others will be sufficiently clear, to wit of hauling ships therewith across dams or dykes, for this does not appear to be the least service it may render to these countries, chiefly Holland. Let *AB* be the Almighty described above, with the wheels *K*, *H*, *F* on this side of the tree *AB* and the wheels *I*, *G*, *E*, *C* on the other side, and let *LMN* be the crank, and *S* the axle, whose diameter shall be $1\frac{1}{2}$ feet, extending through the tree with an iron wheel at the end, as *T*, whose diameter I take to be 2 feet (it should be at least so much longer than the diameter of the circle of the shaft *S* that the wheel *I* cannot touch the axle *S*) and which I take to have 36 teeth [1]), and *V* shall be the dam, whose height above the bottom of the ship lying freely in the water (i.e. measured along the vertical from the top of the dam to the plane parallel to the horizon through the bottom of the ship)

[1]) Girard (XIII; iv, 482) here finds a difficulty. If the teeth of the wheel *T* are to be equal in size to those of the wheels *C* . . . *K*, *T* cannot have 36 teeth as well as a diameter of 2 feet. The former number calls for a diameter of 1.5 feet, the second for a number of teeth of 48. The solution of this riddle may perhaps be found in the gradual increase of the diameters mentioned in note 1 to page 357. On the strength of this it is possible that both values given by Stevin are correct. Cf. note 1 to page 361.

*sichteinder duer t'onderste des schips) sy vier voeten, ends X sy t'schip. *Horizonte.*
Nu om t'selue ouer te winden, men sal draeyen ande kruck L M N, daerom sal d'hanthaef M N so lanck sijn, datter al de ghene diemender toe bruycken wil, ouer beyden sijden bequamelick an staen mueghen.



REDEN DIEDER IS VANDE KEEREN DES KRVCX TOT DE KEEREN DES AS.

WANT de kruck L M N driemael omdraeyt teghen F eenmael, so sal sy 9 mael omdraeyen teghen H eenmael, ende 27 mael teghen K eenmael, ende 162 mael teghen T ofte den as S eenmael. Tis oock kennelick dat de kruck ghestelt an t'einde des as F, sy sal 54 mael omdraeyen teghen S eenmael, ende ghestelt an K, sal ses mael omdraeyen teghen S eenmael, ende ghestelt an T, sal so dickmael omdraeyen als S.

e 3 Maer

shall be four feet, and X shall be the ship. Now in order to haul the latter across, one has to turn the crank LMN. Therefore the handle MN must be so long that all the men who are to be employed for it can suitably stand on either side of it.

THE RATIO BETWEEN THE REVOLUTIONS OF THE CRANK AND THE REVOLUTIONS OF THE AXLE

Because the crank LMN revolves three times against F once, it will revolve 9 times against H once, and 27 times against K once, and 162 times against T or the axle S once. It is also evident that the crank, when mounted at the end of the shaft F, will revolve 54 times against S once, and when mounted at K, six times against S once, and when mounted at T, as often as S. But if a man should turn at a shaft higher than D, for example at K, in order that all the lower wheels shall not rapidly revolve too, which would cause unnecessary gravity, he must displace somewhat on its shaft the next in order of the lower ones, which would here be G, until its teeth are free of the teeth of I, and then all the lower wheels will stand still.

RATIO BETWEEN THE FORCE APPLIED BY THE TURNER TO THE CRANK AND THE WEIGHT THAT IS BEING HAULED, AS THE SHIP X

Because LM, one foot long by the supposition, is eight times the semi-diameter of the wheel C, the weight caused by the wheel E on C will be to the gravity or force at MN which is of equal apparent weight to it as 8 to 1, and for the same reason the weight caused by H on F as 24 to 1, and by I on G as 72 to 1, and by T on K as 216 to 1. But the circle of the axle S is potentially equal to the circle of T (we say potentially, for actually the diameter of the circle of S is $1\frac{1}{2}$ feet as against T two feet, by the supposition, but because the teeth of T are six times the teeth of C, its diameter will potentially be six times the diameter of G, being 3 inches, that of T then being 18 inches, i.e. $1\frac{1}{2}$ feet, like the diameter of the axle S); therefore the weight hanging vertically down at the axle S will have to its equal apparent weight or force at MN the ratio of 216 to 1 [1]). We might

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[1]) The mechanical advantage of the Almighty may be determined as follows: Put $ML = r$, the radii of the wheels $C, F, G, K = r_1$, the radius of $T = r_2$, that of $S = \varrho$; the number of teeth of the wheels $C, F, G, K = n$, that of $E, H, I = n_1$, that of $T = n_2$. In one complete revolution of M, S performs

$$\left(\frac{n}{n_1}\right)^3 \cdot \frac{n}{n_2} \text{ revolutions.}$$

If now the force applied at M is P, and the force exerted by the rope of S, Q, we have

$$2\pi r . P = \left(\frac{n}{n_1}\right)^3 \cdot \frac{n}{n_2} \cdot 2\pi\varrho . Q;$$

consequently the mechanical advantage.

$$(1)\ \frac{Q}{P} = \frac{r}{\varrho} \cdot \left(\frac{n_1}{n}\right)^3 \cdot \frac{n_2}{n}$$

Taking $r = 1$ ft., $\varrho = 3/4$ ft., $n_1 = 18$, $n = 6$, $n_2 = 36$, we find

$$\frac{Q}{P} = \frac{4}{3} \cdot 27 \cdot 6 = 216.$$

This is also Stevin's value; however, he arrives at it in a different way. He first determines the force exerted by the teeth of C against those of E, which is

$$\frac{r}{r_1} \cdot P.$$

Maer als yemant draeyt an een hoogher dan D, by ghelijcknis an K, op dat dan alle de onderste sterren niet snellick mede omdraeyen, t'welck onnoodighe swaerheyt soude anbrenghen, soo salmen d'eerstvolghende der onderste, t'welck hier G soude wesen, op sijn as wat verschuyuen, tot dat huer tanden buyten de tanden van I sijn, ende dan sullen alle d'onderste stil staen.

REDEN VANDE CRACHT DES DRAEYERS ANDE CRVC, TOT HET GHETROCkenwicht als t'schip X.

WANT L M lanck een voet door t'ghestelde, achtvoudich is teghen de halue middellini vande sterre C, so sal t'ghewicht veroirsaeckt uyt de sterre E op C, teghen sijn euestaltwichtighe swaerheyt ofte macht an M N, wesen als van 8 tot 1, ende om de selue reden duer t'veroirsaeckte van H op F, als van 24 tot 1, ende van I op G, als 72 tot 1, ende van T op K, als 216 tot 1: Maer trondt des as S

Potentia. is * machtelick euen an trondt T (wy segghen machtelick, want * daet-

Actu. lick, de middellini des rondts van S doet $1\frac{1}{2}$ voet van T twee voeten duer t'ghestelde, maer want de tanden van T sesvoudich sijn an de tanden van C, daerom sal sijn middellini machtelick sesvoudich sijn teghen de middellini van G, doende 3 duymen, die van T dan 18 duymen, dat is $1\frac{1}{2}$ voet, als de middellini des as S) t'ghewicht dan anden as S rechtneerhangende, sal sulcken reden hebben tot sijn euestaltwicht, ofte macht an M N, als van 216 tot 1. Wy souden ooc connen vanden as S neerwaert de rekening maken, ghelijck sy hier van onderen opwaert ghedaen is.

Wy connen t'voornomde oock aldus verclaren: Anghesien M N 162 mael omdraeyt, teghen den as S eenmael (als bouen bewesen is) ende dat de middellini des radts beschreuen duer den keer van M N, sulcken reden heeft tot de middellini des rondts vanden as S, als 4 tot 3 (want

Semidiameter. L M is een voet, ende de * halfmiddellini des rondts vanden as S is $\frac{3}{4}$ voets) soo sal de langde der omtrecken vande 162 ronden beschreuen duer de keeren van M N, sulcken reden hebben tot de langde vanden omtreck des rondts der as S, als 216 tot 1, de selfde reden sullen oock hebben de 216 halfmiddellinien van dat rondt, tot de eenighe halfmiddellini van dit rondt; Daerom oock, duer het 1e voorstel des 1en boucx, sal t'ghewicht an die, sulcken reden hebben an t'ghewicht ofte de macht an dese, als van 216 tot 1 ghelijck vooren. Waer uyt volght dat wesende an M N een ghedurighe macht soo groot als 25 lb souden neertrecken, t'welck ick de macht schat van een man, ende grooter als hy wil (wel is waer dat een man ter noot onghelijck veel grooter macht doen can, maer

wy nemen

also make our calculations from the axle S downwards, at is has here been done upwards from the bottom.

We can also explain the foregoing as follows: Since MN revolves 162 times against the axle S once (as has been proved above), and the diameter of the circle described by the revolution of MN has to the diameter of the circle of the axle S the ratio of 4 to 3 (for LM is one foot, and the semi-diameter of the circle of the axle S is $\frac{3}{4}$ foot), the length of the circumferences of the 162 circles described by the revolutions of MN will have to the length of the circumference of the circle of the axle S the ratio of 216 to 1; the same will also be the ratio of the 216 semi-diameters of the former circle to the sole semi-diameter of the latter circle. Therefore also, by the 1st proposition of the 1st book, the weight at the former will have to the weight or force at the latter the ratio of 216 to 1, as before. From which it follows that if there is exerted at MN a constant force as great as 25 lbs would pull downwards, which I estimate to be the force that can be exerted by one man — and greater if he cares (it is true that if need be a man can exert a much greater force than this, but we take this by way of example) —,

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Consequently the force of F against H is $\frac{r}{r_1} . P . \frac{n_1}{n}$

the force of G against I $\frac{r}{r_1} . P . \left(\frac{n_1}{n}\right)^2$

the force U of K against T $\frac{r}{r_1} . P . \left(\frac{n_1}{n}\right)^3$.

Now putting $n_1 = 18$, $n = 6$, $r = 1$ ft., $r_1 = 1/8$ ft., we find

$$U = 3^3 . 8 . P = 216\,P$$

This result, however, is given as representing the force exerted by the rope of S. However, continuing this line of reasoning and considering the windlass formed by T and S, we should expect

$$(2)\quad Q = \frac{r_2}{\varrho} . \left(\frac{n_1}{n}\right)^3 . \frac{r}{r_1} . P = \frac{4}{3} . 216.P \text{ or } \frac{Q}{P} = 288.$$

To explain this discrepancy we have to take into account Stevin's remark that T and S are potentially equivalent, which can only mean that the multiplication by $\frac{r_2}{\varrho}$ is not necessary. To prove this, he remarks that $r_2 : r_1 = n_2 : n$.

Now (2) takes the form $\frac{Q}{P} = \frac{r}{\varrho} . \left(\frac{n_1}{n}\right)^3 . \frac{n_2}{n}$,

which is identical with (1).

But the demonstration naturally is not valid if $r_2 = 1$ ft., $r_1 = 1/8$ ft., $n_2 = 36$, $n = 6$. It is, however, possible to arrive at Stevin's result in a legitimate manner when we suppose, in accordance with note 1 to page 357, that the radii of the wheels increase continually from C to K. Put the radius of $F = \lambda r_1$, that of $G = \lambda^2 r_1$, that of $K = \lambda^3 r_1$. It is then necessary that

$$\frac{n_2}{n} . \lambda^3 r_1 = r_2;$$

consequently $\lambda^3 = \frac{r_2}{r_1} . \frac{n}{n_2}$.

Now the force exerted by the teeth of K against those of T is found to be

$$\left(\frac{r}{r_1}\right)\left(\frac{3}{\lambda}\right)^3 . P,$$

and the mechanical advantage $\frac{Q}{P} = \frac{r_2}{\varrho} . \frac{r}{r_1}\left(\frac{3}{\lambda}\right)^3 = \frac{r}{\varrho} . \lambda^3 \frac{n_2}{n} . \frac{3^3}{\lambda^3} = \frac{4}{3} . 6 . 3^3 = 216.$

wy nemen dit voorbeeltsche wyse) de selue macht sal euestaltwichtich sijn teghen 5400 ℔ (dat is 216 mael 25 ℔) rechtneerhanghende anden as S: Ghenomen nu dat het schip X sesvoudich sy, teghen dat sijn euestaltwicht an den as S rechtneerhanghende, soo sal t'schip X weghende 32400 (dat is 9 last ghewichts rekenende 3600 ℔ voor t'last) euestaltwichtich sijn teghen t'ghewicht, ofte die gheduerighe macht van 25 ℔ an M N.

VANDE MENICHTE DER KEEREN

DES CRVCX OM T'SCHIP OVER DEN DAM TE *Winden: Ende vanden tijdt die de aerbeyders behouuen.*

MAER wesende duer t'ghestelde t'schip sesvoudich teghen t'ghewicht anden as S hanghende, so sal de langde van t'sop des dams scheefneerwaert, oock sesvoudich sijn teghen de hoochde diet schip moet verheuen worden (duer het 19e voorstel des 1en boucx) welcke duer t'ghestelde is 4 voeten, de selue dan ses mael maeckt 24 voeten, voor de voornomde langde; Laet ons nu nemen dat dese 24 voeten ghewonden moeten worden op den as S, om t'swaerheyts middelpunt des schips ouer t'middel des dams te krijghen; Soo wy dan als vooren, den omtreck van t'rondt des as stellen als drievoudich (die reden is in desen gheualle naer ghenouch) teghen sijn middellini $1\frac{1}{2}$ voet, sy sal $4\frac{1}{2}$ voeten wesen, de selue sijn inde voornomde 24 voeten $5\frac{1}{3}$ mael, den as S dan, sal $5\frac{1}{3}$ keeren moeten omdraeyen, maer elcke keer van die behouft 162 keeren van M N als vooren bewesen is, daer sullen dan in als behouuen 864 keeren van M N.

Wy souden oock mueghen aldus segghen: Elcken keer van M N veruoert 25 ℔ ses voeten verre, dat is, hanghende een ghewicht an den as S van 5400 ℔, elcken keer van M N doet soo veel, als oft het van dien telckemael 25 ℔ 6 voeten hoogh trocke, ende veruolghens als oftet sesmael 25 ℔, dats 150 ℔ des scheeps, trocke 6 voeten verre, daerom ghedeelt 32400 ℔ duer 150 ℔, comt 216. waer duer t'schip met elcke 216 keeren van M N ses voeten voort commen sal, maer t'moet viermael 6 voeten commen, t'moet dan hebben viermael 216 keeren, dat is als vooren 864 keeren. Ofte andersins (anghesien t'scip in als 4 voeten hooch moet commen) men mach aldus segghen, met eenen keer trecktmen 25 ℔ ses voeten hooch, met hoe veel keeren salmen 32400 ℔ trecken 4 voeten hooch? comt als voren met 864 keeren.

Maer wy achten datter een man 1000 can doen op een vierendeel uyrs, ghenomen dan dat hem alles soo heb als ghesyt is, hy sal t'schip met datter in is t'samen 9 last weghende, alleen ouerwinden op min dan een vierendeel this force will be of equal apparent weight to 5,400 lbs (i.e. 216 times 25 lbs), hanging down vertically at the axle *S*. Now assuming that the ship *X* be six times its equal apparent weight hanging down vertically at the axle *S*, the ship *X* weighing 32,400 lbs (that is 9 lasts, taking a last at 3,600 lbs) will be of equal apparent weight to the weight, or the constant force of 25 lbs at *M*.

OF THE NUMBER OF REVOLUTIONS OF THE CRANK NECESSARY TO HAUL THE SHIP ACROSS THE DAM, AND OF THE TIME THE WORKMEN NEED FOR IT

But if, by the supposition, the ship be six times the weight hanging at the axle *S*, the distance from the top of the dam obliquely downwards will also be six times the height the ship has to be raised (by the 19th proposition of the 1st book), which by the supposition is 4 feet; this, multiplied by six, makes 24 feet for the aforesaid distance. Let us now assume that these 24 feet have to be wound on to the axle *S* in order to raise the ship's centre of gravity above the centre of the dam. If then, as before [1]), we put the circumference of the circle of the axle at three times (this ratio being near enough in this case) its diameter of $1\frac{1}{2}$ feet, it will be $4\frac{1}{2}$ feet; these are contained $5^1/_3$ times in the aforesaid 24 feet, therefore the axle *S* will have to make $5^1/_3$ revolutions. But each of the revolutions of the axle requires 162 revolutions of *MN*, as proved above, to that 864 [2]) revolutions of MN in all will be required.

We might also say as follows: Each revolution of *MN* moves 25 lbs six feet further, i.e., a weight of 5,400 lbs hanging at the axle *S*, each revolution of *MN* does as much as if it hauled each time 25 lbs thereof 6 feet high, and consequently as if it hauled six times 25 lbs, i.e. 150 lbs of the ship, 6 feet further. Therefore, 32,400 lbs divided by 150 lbs makes 216, so that with every 216 revolutions of *MN* the ship will be moved six feet further. But it has to be moved four times 6 feet therefore it requires four times 216 revolutions, i.e. 864 revolutions, as stated before. Or otherwise (since the ship has to be raised 4 feet in all) it may be said as follows: with one revolution 25 lbs are hauled six feet up, with how many revolutions will 32,400 lbs be hauled 4 feet up? As before, this works out at 864 revolutions.

Now we consider that a man can perform 1,000 revolutions in a quarter of an hour. Therefore, taking everything to be as said, he will by himself haul the ship with all that is in it, weighing together 9 lasts, across in less than a quarter of an hour. But if there be three men, they may put the crank at *F*, and then they will haul the ship across in one-third of a quarter of an hour, that is in $^1/_{12}$ hour [3]). And if there be nine men, they may put the crank at *H* and will haul it across in $^1/_{36}$ hour. It is also possible to provide an Almighty at the other tree *Y*, as at the tree *AB*, and place the men on either side.

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1) We do not remember Stevin having used this approximation $\pi = 3$ before.

2) The exact value is $\frac{24}{1.5\,\pi} \cdot 162 = \frac{2592}{\pi}$ revolutions.

3) One man, putting the crank at *F*, would have to exert a threefold force; if he were able to exert this force while at the same time keeping up the same number of revolutions per unit of time, he would be able to perform the work in one third of the time, because the number of revolutions required for a given displacement is now one-third of the original number. However, he alone will not be able to achieve this, but three men working together and each exerting the original force will succeed in doing so.

vierendeel uyrs. Maer sooder drie mannen toe waren, sy mueghen de kruck an F steken, ende sullent dan ouertrecken op t'derdendeel van een vierendeel uyrs, dat is op $\frac{1}{12}$ uyrs: Ende sooder neghen mannen toe waren, sy mochten de kruck an H steken, ende sullent in $\frac{1}{36}$ uyrs ouerwinden. Men soude oock mueghen anden anderen boom Y een Almachtich maken als an den boom A B, eñ bedeelen de menschen op beyden sijden.

MERCKT.

Wy hebben hier een voorbeelt ghestelt al of t'schip in t'overwinden voor den aerbeyders altijdt van eenvaerdigher swaerheyt waer, welcke nochtans merckelick verandert naer de form ende ghestalt van t'voorghesette, want swaerder gadet int laetste dan int beghin, om de redenen die int 3e voorbeelt des 9e voorstels deses boucx van der ghelijcke ghesеyt sijn; Daerom salmen t'voorgaende achten als voorbeelt verclarende hoemen in yder voorghestelde ofte begheerde form sijn rekening maken sal.

Angaende de sterren die in t'Almachtich recht bouen malcanderen ghestelt sijn, men soudese oock mueghen neuen den anderen voughen, ofte met paren, daert de gheleghentheyt hiessche.

VERCLARING VAN TGHENE VOOREN BELOOFT IS.

Wy hebben hier vooren int t'beghin deses voorstels beloofT, dattet Almachtich soude sijn stercker werck; Ende van minder cost dan den Charistion; Ende duer t'welckmen op corter tijdt meer afveerdicht; Ende van oneindelicke cracht.

Angaende de sterckte des wercx, ick achte die openbaer (daerbeneuen een beter nummermeer versmaende) want wat soudemen tot sulcken daet vromer wenschen, dan een stercke boom so hy ghewassen is, wiens stof vaster in een houdt dan eenich ghereck van verscheyden stucken vergaert. De cleynen cost is oock kennelick.

Wat den corteren tijt belangt, die volght daer uyt, datmen de kruck mach steken an sulcken as der sterren alsmen wil, naer gheleghentheydt vande menichte der arbeydende menschen, ende het tetreckenwicht, te weten voor de lichter ghewichten de kruck hoogher, ende voor de swaerder leegher te steken, alsoo datmen duer eenen behoirlicken arbeydt, het tetreckenwicht hoe swaer het sy, altijt gaende houdt, sonder stil staen, t'welck inde Charistion noch ander windassen soo niet gheschien en can, want om een cleyne lichte schuyt, ghebruycktmen duer windassen, t'ghene een veel grooter cracht vermach, t'welck den tijdt langher doet anloopen. Maer is het tetreckenwicht swaerder dan daer duer bequamelick can ghedaen worden, soo moetmen daer toe nemen groote

menichte

NOTE.

We have here given an example as if the ship were always of uniform gravity during the hauling by the workmen, though this gravity alters considerably in accordance with the form and shape of the given vessel, for the work will be more difficult at the end than at the beginning, for the reasons given in the 3rd example of the 9th proposition of this book about a similar case. Therefore the foregoing is to be looked upon as an example explaining how to make one's calculations with any form proposed or desired.

As to the wheels placed vertically above each other in the Almighty, they might also be placed side by side, or in pairs, as the occasion demands.

EXPLANATION OF THAT WHICH HAS BEEN PROMISED HEREINBEFORE

We have promised hereinbefore, at the beginning of this proposition, that the Almighty would be of sturdier construction and of lower cost than the Charistion; by which more is done in shorter time, and of infinite power.

As regards the sturdiness of construction, I consider this to be manifest (though a better one is by no means to be despised), for what better could one wish for, with a view to such work, but a strong tree such as it has grown, whose substance coheres better than any device made up of several pieces. The low cost is also evident.

As regards the shorter time, this follows from the fact that the crank may be put on the shaft of any desired wheel, in accordance with the number of workmen employed and the weight to be hauled, to wit for lighter weights the crank may be put higher and for heavier lower, in such a way that with a suitable effort the weight to be hauled, however heavy, is kept moving, without stopping, which is not possible either with the Charistion or with other windlasses, for in order to haul a small, light ship use is made in windlasses of that which is capable of exerting a much greater force, and this lengthens the time required. But if the weight to be hauled is heavier than can easily be hauled by them, it is necessary to use a great many men or horses, working hard at one time and stopping at another, thus lengthening the time. Nay, in addition they greatly damage the ships, for one of the biggest hauled across the Leyden Dam, weighing thirteen or fourteen lasts requires twenty men going in the wheels, [1]) who will often descend all together to the same position of rest, and do severe damage to the ships by the violent shock, which is not possible with the Almighty, since the ship always moves on uniformly and gently.

But in order to speak of its infinite power, let it be known that with the crank above at *D* as great a force may be exerted as with a windlass the diameter of whose wheel should be 324 feet, which is shown as follows. Let there be a wheel whose diameter shall be 324 feet, and its axle shall be *S*, the diameter of whose circle shall be $1^1/_2$ feet, in consequence of which the semi-diameter of

[1]) Prop. IX, Ex. II.

menichte van menschen ofte peerden, welcke met grooten arbeydt altemet voortgaen, altemet stilstaen, ende daer duer den tijt verlanghen; Ia bouen dien de schepen seer beschadighen, want een der grootste die ouer den Leydtschen Dam ghetrocken worden van derthien oft veerthien last, behouft twintich menschen die inde raeyers gaen, welcke dickwils naer eenen stillestandt alt samen neerdalen, ende met eenen grooten gheweldighen hurt de schepen seer quetsen, t'welck duer t'Almachtich niet gheschien en can, ouermidts t'schip altijdt eenvaerdelick ende sachtkens voortcomt.

Maer om vande oneindelicke cracht te segghen, het is te weten datmen met de kruc hier bouen an D, soo veel vermach alsmen soude met een windas diens radts middellini van 324 voeten waer, t'welck aldus betoocht wort: Laet wesen een radt diens middellini 324 voeten, ende sijn as sy S, wiens rondts middellini sy van 1 $\frac{1}{2}$ voet, waer duer de halfmiddellini des radts sulcken reden sal hebben tot de halfmiddellini des as, als 216 tot 1, daerom oock t'ghewicht ofte de macht anden as, sal sulcken reden hebben tot sijn eueſtaltwicht an t'uyterste des radts, als 216 tot 1 duer het 9 voorstel deses boucx, de selue reden isser oock van t'ghewicht an den as S, tot sijn eueſtaltwicht an M N, daerom so wy gheseyt hebben, wesende de kruck an D, men sal duer haer anden as S soo veel vermueghen, als duer een radt diens middellini lanck waer 324 voeten. Maer de meeste diemen maeckt en schijnen de 30 voeten niet te bereycken, waer uyt opentlick blijckt hoe veel t'Almachtich meer vermach dan de windassen, wel is waer dat eenen gaende int radt eens windas, sijn ghewelt met minder aerbeydt doet, maer ghemerckt de voorgaende omstaende, ten is niet het nutste. Doch soo yemandt sulcken voordeel duer den ganck in tradt begheerde daert noodich viel, hy soude an eenighen as der assen D, F, H, K, T, mueghen steken een schijfloop, inde plaets vande kruck, stellende tanden an t'uyterste van eenich radt eens windas, die in die schijfloop draeyen mochten, maer t'voordeel en soude dickmael de oncosten niet weerdich sijn.

Maer soo dees voornomde reetschap niet gheweldich ghenouch beuonden wierde, om daer mede t'voornemen te volbrenghen, daer en is verloren cost, noch onnoodighen aerbeyt ghedaen, want stellende alleenelick onder D noch een as als d'ander, drievoudighende de voorgaende 216, ende daer an de kruck stellende, 1 soude ande selue eueſtaltwichtich sijn teghen 648 anden as S. duer welck middel men tot de begheerde ghewelt commen sal.

Maer datmen alsoo maeckte een Almachtich met 30 assen, wiens tanden vande grootste sterren thienvoudich waren teghen de tanden vande cleynste sterren, ende het deel des crucx als L M, euen ande halfmiddellini der grootste sterre, ende t'rondt des as als S, euen an t'rondt der

f cleenste

the wheel shall have to the semi-diameter of the axle the ratio of 216 to 1; therefore also the weight or the force exerted at the axle will have to its equal apparent weight at the rim of the wheel the ratio of 216 to 1, by the 9th proposition of this book. The same is also the ratio of the weight at the axle *S* to its equal apparent weight at *MN*. Therefore, as we have said, the crank being at *D*, the same force can be applied therewith to the axle *S* as with a wheel whose diameter should be 324 feet. But most wheels that are made do not seem to attain 30 feet, from which it is manifest how much more powerful than windlasses is the Almighty. It is true indeed that a man going in the wheel of a windlass exerts his force with less effort, but considering the circumstances mentioned above this is not the most profitable thing. But if anyone should desire this advantage of going in the wheel where it is necessary, he might put at one of the shafts *D*, *F*, *H*, *K*, *T* a wallower [1]) instead of the crank, providing teeth at the rim of a wheel of a windlass, which can engage with that wallower, but the advantage frequently would not be worth the expense.

But if the aforesaid device were to be found not sufficiently powerful to perform therewith the proposed work, no cost has been lost and no unnecessary work has been done, for by merely putting below *D* another such shaft triplicating the foregoing 216, and putting the crank there, 1 would thus be of equal apparent weight to 648 at the axle *S*, by which means the desired force would be attained.

But if in this way an Almighty with 30 shafts were made, the teeth of whose largest wheels should be ten times the teeth of the smallest wheels, and the part of the crank, as *LM*, equal to the semi-diameter of the largest wheel, and the circle of the axle, as *S*, equal to the circle of the smallest wheel (which would not be such a prodigious work), the weight hanging at such an axle would have to its equal apparent weight at the crank the ratio of
1,000,000,000,000,000,000,000,000,000,000 to 1 [2]). Taking therefore the circumference of the circle of the smallest wheel to be 1 foot, the circumference of the wheel of a windlass (the circumference of the circle of whose axle should also be one foot), in order to exert the same force therewith, would have to be
1,000,000,000,000,000,000,000,000,000,000 feet [3]). But the circumference of the greatest circle of the earth (taking the degree at 480 stades, and each stade at 125 geometrical strides, and each geometrical stride at 5 feet) is only 108,000,000

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[1]) A wallower or lantern wheel is a cylindrical device in which pins are mounted between two circular flanges. The cogs of the wheel of the windlass engage with the pins of the wallower, the (horizontal) axis of which is attached to one of the shafts of the Almighty.

[2]) Using the notation of note 1 to page 361, we have to put $n_1 = 10n$, $n_2 = n$, $\varrho = r_1$, $r = 10r_1$
and to replace 3 by 29. This gives $\frac{Q}{P} = \frac{10r_1}{r_1} \cdot 10^{29} = 10^{30}$.

[3]) 10^{30}.

cleenste sterre (t'welck alsoo wonderlicken grooten werck niet en waer) t'ghewicht an sodanighen as hangende, soude sulcken reden hebben tot sijn eueftaltwicht ande kruc, als 10000000000000000000000000000 tot 1: Ghenomen dan dat den omtreck van t'rondt der cleenste sterre waer 1 voet, soo soude den omtreck van t'radt eens windas (diens assens rondts omtreck oock een voet) om de selue cracht daermede te doen, moeten sijn van 10000000000000000000000000000 voeten: Maer den omtreck van t'grootste rondt des eertrijcx (rekenende den * trap op 480 stadien, ende elcke stadie op 125 Meetconstighe stappen, ende elcken Meetconstighen stap op 5 voeten) en is maer 108000000 voeten; Siet dan hoe menich hondertduysentmael grooter dan t'grootste rondt des eertbodems, dat het rondt eens radts van een windas soude moeten wesen, om soo grooten ghewelt mede te doen als met sulcken slechten Almachtich. Laet ons nu ande kruck (stekende anden leeghsten as van soodanighen Almachtich) een kindeken stellen, wat meer macht daer an doende dan een hanghende pondt, t'selue soude op den hoochsten as een ghewicht winden van 10000000000000000000000000000 ℔ (maer ghedenckt dat dien hoochsten as op den eersten dach gheen heelen keeren doen en soude) dat is een ghewicht swaerder dan vierduysent mael t'eertrijck met al datter in is, t'welck aldus bewesen wort: Den omtreck van t'grootste rondt des eertrijcx is van 108000000 voeten, als bouen gheseyt is, daerom t'plat des selfden rondts is minder dan 1000000000000000 voeten, daerom oock is t'vlack des weerelts cloot minder dan 4000000000000000 voeten, ende t'sestendeel der middellini is corter dan 6000000 voeten, daermede vermenichvuldicht de voornoemde 4000000000000000, soo is t'eertrijck minder dan 24000000000000000000000 teerlijnckscke voeten; Laet elcken voet 1000 ℔ weghen (sy en is op veel na so swaer niet) t'gheheele eertrijck dan is lichter als 24000000000000000000000000 ℔, t'selfde is meer dan vierduysent mael in 10000000000000000000000000000 ℔ soo wy bethoonen wilden.

Gradum.

Wy segghen dan met reden dat dese reetschap machtelick van oneindelicke cracht is: Daerom doen Archimedes seyde, soomen hem een vaste plaets leuerde buyten t'eertrijck daer hy sijn Charistion mocht stellen, hy soude t'eertrijck uyt sijn plaets trecken, hoe vreemt het luyt, tis nochtans de reden lijckformich, want by aldien het soo niet en waer, de swaerste swaerheyt en soude niet sulcken reden hebben tot de lichtste, als den langsten erm tot den cortsten, t'welck duer het 1ᵉ voorstel des 1ᵉⁿ boucx onmueghelick is. Maer om by voorbeelt hier af te spreken; Ghenomen dat de Charistion ofte t'Almachtich op soodanighen plaets stonde, ende dat het eertrijck woughe alsvooren gheseyt is 24000000000000000000000000 ℔, ende dat een man met yder keer

der

feet [1]). Thus you may see how many hundred thousands of times greater than the greatest circle of the earth the circle of a wheel of a windlass would have to be in order to exert therewith the same force as with this simple Almighty. Let us now place at the crank (put at the lowest shaft of this Almighty) a little child, applying at it a slightly greater force than one pound hanging thereat, this child would wind on to the highest shaft a weight of 100,000,000,000,000,000,000,000,000,000,000 lbs [2]) (but it should be borne in mind that this highest shaft would not perform a complete revolution on the first day), i.e. a weight heavier than four thousand times the earth with all that is in it, which is proved as follows: The circumference of the greatest circle of the earth is 108,000,000 feet, as stated above; therefore the area of this circle is less than 1,000,000,000,000,000 feet [3]), and therefore also the area of the terrestrial globe is less than 4,000,000,000,000,000 feet, and the sixth part of the diameter is shorter than 6,000,000 feet. If the aforesaid 4,000,000,000,000,000 be multiplied thereby, the earth is less than 24,000,000,000,000,000,000,000 cubic feet [4]). Let each foot weigh 1,000 lbs (it is not so heavy by far), then the whole earth is lighter than 240,000,000,000,000,000,000,000,000 lbs [5]); this is contained more than four thousand times in 1,000,000,000,000,000,000,000,000,000,000 lbs [6]), which we wished to show.

We therefore justly say that this device is potentially of infinite power. Therefore, when Archimedes said that if he were given a fixed place outside the earth where he could put his Charistion [7]), he would move the earth out of its place, however strange it may sound, yet it is in accordance with reason. For if it were not so, the heavier gravity would not have to the lighter the same ratio as the longer arm to the shorter, which is impossible on account of the 1st proposition of the 1st book [8]). But to give an example: Assuming that the Charistion or the Almighty were to stand in such a place and the earth weighed — as stated before — 240,000,000,000,000,000,000,000,000 lbs, and that a man hauled 100

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[1]) Taking 1 ft. = 0.3 m, this amounts to the value 32,400 km for the earth's circumference.

[2]) 10^{32}, obviously a mistake for 10^{30}.

[3]) As a rule Stevin, when dealing with areas and volumes, does not use the correct terms (square feet and cubic feet), but merely writes 'feet'.

[4]) The calculation is equivalent to applying the formula $V = \frac{P^3}{6\pi^2}$ (V = volume of a sphere the greatest circle of which has the circumference P).
For $P = 108 \,.\, 10^6$ we find

$$V = \frac{108^3 \,.\, 10^{18}}{6\pi^2} < \frac{108^3 \,.\, 10^{18}}{6 \,.\, 9} = 2 \,.\, 108^2 \,.\, 10^{18} < 24 \,.\, 10^{21}.$$

[5]) $24 \,.\, 10^{25}$. It is even lighter than $24 \,.\, 10^{24}$ lbs.

[6]) $$\frac{10^{30}}{24 \,.\, 10^{25}} = \frac{10^5}{24} > 4{,}000.$$

[7]) See note 3 to page 355.

[8]) Here Stevin seems to take the Charistion to be a lever, in accordance with Simplicius.

der kruck 100 ℔ drie voeten hoogh trocke, ende op yder uyr 4000 keeren dede, ende dat gheduerende thien iaren lanck, rekenende t'iaer op 365 daghen, tis openbaer dat hy t'eertrijck op dien tijdt het $\frac{10512}{24000000000000000000}$ eens voets, dats bycans $\frac{1}{2400000000000000}$ voets, uyt sijn plaets vertrecken soude, t'welck wel is waer een onsienlicke langde is, maer wy moesten de oneindelicke cracht verclaren, die machtelick hier in bestaet.

Nu hoe alsulcke Almachtighen tot verscheyden wercken sullen mueghen ghevoucht worden, als dat een schip in hem sal connen hebben een seer cleen reetschap van gheringhen cost, nochtans seer crachtich, hem dienende voor Craen om te laden ende ontladen: Om groote anckers op te winden: Voort om perssen te maken, als lakenperssen ende dierghelijcke, gheweldelicker druckende dan noyt perssen en dructen: Om in groote ghebauwen sware steenen op te trecken, ende meer ander daermen groote ghewelt behouft, van alle dese en gheuen wy gheen besonder voorbeelden, ouermits yder uyt het voorschreuen Almachtich an sijn werck een derghelijcke, naer gheleghentheyt sal mueghen voughen, beter dan wy segghen connen; Tis voor ons ghenouch sijn ghedaente alhier beschreuen te hebben.

T'EINDE DER WEEGHDAET.

lbs three feet up with each revolution of the crank and performed every hour 4,000 revolutions, such during ten years, taking the year at 365 days, it is manifest that in that time he would move the earth $\frac{10,512}{24,000,000,000,000,000,000}$ foot, i.e. nearly $\frac{1}{2,400,000,000,000,000}$ foot from its place [1]); which is indeed an invisible distance, but we had to explain the infinite power potentially inherent therein.

Now as to the construction of such Almighties for various purposes, e.g. that a ship may have on board a very small device of slight cost and yet very powerful, serving as a crane for loading and unloading; for winding up big anchors; further to make presses, such as cloth presses and the like, pressing more powerfully than presses ever did; in order to haul up heavy stones in big buildings, and in many other cases in which great force is required; of all these we are not giving any special examples, since anyone will be able on the basis of the Almighty described above to apply a similar one to his own work, according to the occasion, more effectively than we can tell. For us it suffices that we have here described its form.

THE END OF THE PRACTICE OF WEIGHING.

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1) In 10 years the work done would be $10 \cdot 365 \cdot 24 \cdot 4{,}000 \cdot 100 \cdot 3 = 10{,}512 \cdot 10^7$ ft.-pnd. and the displacement of the earth

$$\frac{10{,}512 \cdot 10^7}{24 \cdot 10^{25}} = \frac{10{,}512}{24 \cdot 10^{18}} \backsim \frac{10^4}{24 \cdot 10^{18}} = \frac{1}{24 \cdot 10^{14}} \text{ ft.}$$

DE BEGHINSELEN DES WATERWICHTS

THE ELEMENTS OF HYDROSTATICS

INTRODUCTION

Stevin's work *Beghinselen des Waterwichts (Elements of Hydrostatics)* which is reproduced hereafter, has the same significance in the history of hydrostatics as his *Beghinselen der Weeghconst (Elements of the Art of Weighing)* in the history of statics: it was the first systematic treatise on the subject after the Archimedean work *On Floating Bodies* (3rd century B.C.) and not only contained a new and ingenious treatment of its fundamental theorems, but also extended it beyond the phase of development attained by the Syracusan and never since equalled.

The contents of the work may be summarized as follows:

In the preface Stevin returns to his favourite topic, the excellency of the Dutch language for scientific purposes, which is here illustrated by means of a comparison of the enunciation of two propositions in the *Conica* of Apollonius in Latin and in Dutch respectively.

The definitions 1—5 amount to an introduction of the concept of specific gravity, which is, however, denoted by the term "gravity" alone. An important definition is No 7, in which the concept of "vlackvat" is defined, i.e. the geometrical boundary surface of a physical body from which the material contents are conceived to have been removed; we render this term in English by "surface vessel". The definitions 10 and 11 show the distinction between *ydel* (a vacuum, i.e. containing nothing at all) and *ledich* (empty, i.e. containing only air).

There are 7 postulates, but most of them are never explicitly used. Postulate 3, however, plays a vital part in the demonstration of some fundamental theorems.

The propositions may be classed in five groups:

I. Props 1—9 deal with the behaviour of bodies submerged in a liquid. The principle of Archimedes is demonstrated in a very simple and ingenious way in Prop. 8.

II. In Prop. 10 it is demonstrated that the pressure exerted by a liquid on a horizontal surface immersed therein depends only on the area of this surface and the depth of immersion, and not on the volume of the liquid in the vessel (hydrostatic paradox).

III. Props 11—17 contain theorems and problems concerning the pressure exerted on non-horizontal surfaces.

IV. In Props 18—20 the position of the centre of pressure for non-horizontal surfaces is determined.

V. Props 21 and 22 are simple problems relating to specific gravity.

Just as the *Art of Weighing* is followed by the *Practice of Weighing,* the *Elements of Hydrostatics* was to have been supplemented with a *Practice of Hydrostatics.* Of this work, however, only three propositions are extant, the first of which is a problem about the depth of immersion of a ship, while the second gives experimental illustrations of the hydrostatic paradox, and the third is devoted to the ancient problem why a diver is not crushed by the weight of the incumbent water.

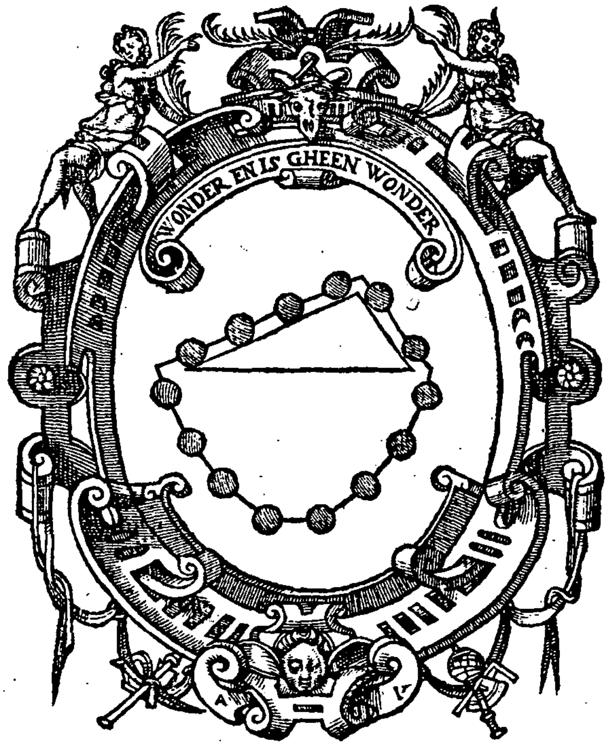
DE BEGHINSELEN DES WATERWICHTS

Elementa.

BESCHREVEN DVER

SIMON STEVIN

van Brugghe.



TOT LEYDEN,

Inde Druckerye van Christoffel Plantijn,

By Françoys van Raphelinghen.

cIↃ. IↃ. LXXXVI.

Simon Steuin Wenscht

DEN STATEN DER VEREENICHDE NEERLANDEN VEEL GHELVCX.

ANGHESIEN kennelick ghenouch is, E. Heeren, de gheduerighe oefning die dese landen mettet vvater hebben, meer als ander; vvaer in oock blijckelick is, vvat grooter voordeel hun de oirsaeclicke kennis der vvichtighe ghedaenten des vvaters doen can; ghemerckt daerbeneuen dat onse Weeghconst die duer d'uyterste beghinselen openbaert: Soo sende ick V. H. de beschrijuing der seluer, ghelijck sy, vvel is vvaer, eertijts int vvater bestonden, maer vele van dien (t'vvelck ick te vrielicker seg, om dat my docht t'volghende sulcx ghenouch te beprouuen, ten anderen op dat ick reden gheef, vvat my vervoordert an V. H te schrijuen) gheen der sterflicke voor ons bekent.

Aa 2 Waer

SIMON STEVIN WISHES THE STATES OF THE UNITED NETHERLANDS MUCH HAPPINESS

Since it is sufficiently known, Your Worships, what continual practice these countries have with the water, more so than any others; from which it also appears of what greater profit the knowledge about the causes of the statical properties of water may be to them; considering further that our Art of Weighing reveals these through the fundamental elements; I am sending Your Worships the description thereof, as indeed they formerly were present in water, but many of which (which I say the more frankly because I thought the following treatise proves this sufficiently, secondly in order to give the reason inducing me to write to Your Worships) were not known to any mortals before us [1]). From which

[1]) The remark that the newly discovered properties already existed before their discovery may appear somewhat superfluous. It was, however, also made by Archimedes in the preface to his work *De Sphaera et Cylindro*. Archimedis Opera Omnia, ed. J. L. Heiberg I, p. 4, ll. 9-13. Leipzig 1910.

4

Effecta. Waer uyt oock * daden sullen volghen, byden voorighen niet ghesien noch ghevveten. vvelcke, ouermidts sy tot grooten voordeele des Landts strecken, de voordering van V. H. niet onbilichlick vervvachten. Vaert daerentusschen vvel, in vermeerdering ende alle voorspoet. Vyt Leyden in Oogstmaent des 1586ᵉ Iaers.

ANDEN

there will also ensue effects not seen or known among our predecessors; which, since they are to the great benefit of the country, not improperly expect to be furthered by Your Worships. Meanwhile may you thrive in progress and all prosperity. From Leyden, in harvest month of the year 1586.

ANDEN LESER.

VVAT beweeghlicke oirsaeck Archimedes had, om te schrijuen t'ghene by ons in t'Bouck vande dinghen die int water ghedreghen worden, naghelaten heeft; daer hy de natuer heerlick begon te treffen, en weet ick niet; maer wel dit, dat hy de myne gheweest is, dat beken ick gheern, in sulcke stof ter form te brenghen die wy haer ghegheuen hebben. Belijde oock daerby, dat icker een beter helpende oirsaeck toe ghehadt heb dan Archimedes, namelick de spraeck, welcke DVYTSCH was, de sijne maer Griecx. Want dit moet ghy weten, dat de sprakens goetheyt niet alleen voorderlick en is om de Consten bequaemlick daer duer te leeren, maer oock den* Vinders in haer soucking. Om van t'welck met reden te spreken, so merct dat ghelijck inde* Beghinselen der Meetconst, t'punt ghenomen moet worden sonder langde, de lini sonder breede,* t'vlack sonder dicte, alsoo ist inde Beghinselen des Waterwichts noodich, om* Wisconstlick daer in te handelen, vaten te stellen sonder lichamelicke grootheyt, ende sonder ghewicht; sulc noemden wy na hun ghedaenten (want nieuwe Consten brenghen nieuwe woorden me) Vlacvat, ouermits sijn stof uyt vlacken bestaet, soo inde volghende 7 bepaling gheseyt sal worden. Ende om der ghelijcke redenen moesten wy segghen van Stofswaerheyt, Stoflichtheyt, Euestofswaer, en dierghelijcke, daer t'volghende vol af is; welcke woorden de Griecken soo cort, ende by haren yderman soo verstaenlic, oock so eyghentlick haer grondt beteeckenende, noyt en hebben connen segghen, nu niet en connen, noch, dat kennelick ghenouch is, inder eewicheyt niet connen en sullen. Want datter niet in en is en cander niet uytghe-

Lib. de iis quæ vehuntur in aqua.

* *Inuentorib.*

* *Elementis Geometriæ.*

* *Superficies.*

* *Mathematicè.*

Aa 3 trocken

TO THE READER

What was the cause that moved Archimedes to write that which he left to us in the Book of the things supported in the water, *where he began to hit off Nature wonderfully, I do not know. But I do know, and gladly confess, that he was the cause which induced me to cast this matter into the form we have given it. I also avow that I had a better aiding cause therefore than Archimedes, viz. the language, which was Dutch, his only being Greek* [1]*. For you must know that the excellence of language is conducive not only to learning the arts well through it, but also to the search of the inventors. In order to discuss this with good reason, it is to be noted that as in the Elements of Geometry the point is to be taken without length, the line without breadth, the plane without thickness, in the same way it is necessary in the Elements of Hydrostatics, in order to deal therewith geometrically, to assume vessels without any corporeal magnitude and without any weight. These we have called, in accordance with their properties (for new arts call for new words) surface vessels, since their material consists of surfaces, as will be said in the 7th definition hereinafter. And for similar reasons we had to speak of specific gravity, specific levity, being of equal specific gravity, and the like, in which the following abounds; which words the Greeks never were, are not now, and never to all eternity will be able to say so shortly, and so universally intelligibly to everyone of them, and also describing its nature so aptly, as is sufficiently obvious. For what is not in it* [2]*, cannot be extracted from it. Its property is to make short, clear, intelligible propositions,*

[1]) See the *Uytspraeck van de Weerdicheyt der Duytsche Tael.* Present volume, p. 58.
[2]) sc. in a language.

*trocken worden. Haer eyghenschap is te maken corte clare verstaenlicke * voorstellen, niet alleen voor den leerlinghen, maer oock self den Vinders, om opentlick t'veruolg van t'een uyt het ander te bemercken. Begheerdi hier af bouen het teghenwoordighe bouck een ander voorbeelt, soo neemt onder sommighe voorstellen des eersten der * Keghelsche bouken van Appollonius het 11e, t'welck duer Fredericus Commandinus (diens naem ick met eerbieding gheern ghedenck, als van een sterre onder de * Wisconstnaers t'sijnder tijdt, oock duer wiens neersticheyt veel saken die int Griecx verborghen laghen, anden dach ghebrocht sijn) uytet Griecx int Latijn aldus ouergheset is:*

Propositiones

Conicorum.

Mathematicos.

Si conus plano per axem secetur, secetur autem & altero plano secante basim coni secundũ rectam lineam, quæ ad basim trianguli per axem sit perpendicularis: & sit diameter sectionis vni laterum trianguli per axem æquidistans: recta linea, quæ à sectione coni ducitur æquidistans cõmuni sectioni plani secantis, & basis coni, vsque ad sectionis diametrum, poterit spacium æquale contento linea, quæ ex diametro abscissa inter ipsam & verticem sectionis interiicitur, & alia quadam, quæ ad lineam inter coni angulum, & verticem sectionis interiectam, eam proportionem habeat, quam quadratum basis trianguli per axem, ad id quod reliquis duobus trianguli lateribus continetur. dicatur autem huiusmodi sectio parabole.

Daer vooren sullen wy, int eerste dier keghelsche boucken dat wy dencken int Duytsch te laten uytgaen, veel corter en claerder aldus segghen:

T'viercant vande oirdentlicke der brantsne, is euen *not only for the pupils, but also for the inventors themselves, in order to see clearly how one thing follows from another. If you desire some other example in addition to the present book, take among some propositions of the first of the books on Conics by Apollonius the 11th, which has been translated as follows from Greek into Latin by Fredericus Commandinus* [1] *(whose name I mention with respect as that of a star among the mathematicians of his time, through whose zeal also many matters which lay hidden in Greek have become revealed):*

Si conus plano per axem secetur; secetur autem & altero plano secante basim coni secundum rectam lineam, quae ad basim trianguli per axem sit perpendicularis: & sit diameter sectionis uni laterum trianguli per axem aequidistans: recta linea, quae à sectione coni ducitur aequidistans communi sectioni plani secantis, & basis coni, usque ad sectionis diametrum; poterit spacium aequale contento linea, quae ex diametro abscissa inter ipsam & verticem sectionis interiicitur, & alia quadam, quae ad lineam inter coni angulum, & verticem sectionis interiectam, eam proportionem habeat, quam quadratum basis trianguli per axem, ad id quod reliquis duobus trianguli lateribus continetur. dicatur autem huiusmodi sectio parabole.

Instead of this we will say much more shortly and clearly in the first of those books on Conics which we intend to publish in Dutch [2]:

The square of the ordinate of the parabola is equal to the rectangle compre-

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[1]) *Apollonii Pergaei Conicorum libri quattuor ... Quae omnia nuper* F. Commandino *illustravit.* Bononiae 1566.

[2]) This translation never appeared. In this as well as the next example it has escaped Stevin's notice that the cause of the condensation is not the use of Dutch, but the introduction of special mathematical terms, such as *latus rectum*, which the Greek mathematicians were wont to circumscribe.

euen anden rechthouck begrepen onder haer middelliniens hoochste deel, ende des brantsnees redelicke lini.

Begheerdy hier by t'volghende 12e voorstel des bouesschreuen 1en boucx, soo siet noch langher en duysterder stof, in corte clare verkeert.

SI conus plano per axem secetur; secetur autem & altero plano secante basim coni secundum rectam lineam, quæ ad basim trianguli per axem sit perpendicularis: & sectionis diameter producta cum vno latere trianguli per axem, extra verticem coni conueniat: recta linea, quæ à sectione ducitur æquidistans communi sectioni plani secantis, & basis coni vsque ad sectionis diametrum, poterit spatium adiacens lineæ, ad quam ea, quæ in directum constituitur diametro sectionis, subtenditurque angulo extra triangulum, eandem proportionem habet, quam quadratum lineæ, quæ diametro æquidistans à vertice sectionis vsque ad basim trianguli ducitur, ad rectangulum basis partibus, quæ ab ea fiunt, contentum: latitudinem habens lineam, quæ ex diametro abscinditur, inter ipsam & verticem sectionis interiectam; excedensque figura simili, & similiter posita ei, quæ continetur linea angulo extra triangulum subtensa, & ea, iuxta quam possunt quæ ad diametrum applicantur. vocetur autẽ huiusmodi sectio hyperbole.

D'ouersetting daer af is soodanich.

Tviercant vande oirdentlicke der wassendesne, is euen anden rechthouck begrepen onder haer middelhended by the upper part of its diameter and the latus rectum of the parabola [1]).

If you desire in addition the following 12th proposition of the above-mentioned 1st book, you will see even longer and more abstruse matter made short and clear.

Si conus plano per axem secetur; secetur autem & altero plano secante basim coni secundum rectam lineam, quae ad basim trianguli per axem sit perpendicularis: & sectionis diameter producta cum uno latere trianguli per axem, extra verticem coni conveniat: recta linea, quae a sectione ducitur aequidistans communi sectioni plani secantis, & basis coni usque ad sectionis diametrum, poterit spatium adiacens lineae, ad quam ea, quae in directum constituitur diametro sectionis, subtenditurque angulo extra triangulum, eandem proportionem habet, quam quadratum lineae, quae diametro aequidistans à vertice sectionis usque ad basim, trianguli ducitur, ad rectangulum basis partibus, quae ab ea fiunt, contentum: latitudinem habens lineam, quae ex diametro abscinditur, inter ipsam & verticem sectionis interiectam, excedensque figura simili, & similiter posita ei, quae continetur linea angulo extra triangulum subtensa, & ea, iuxta quam possunt quae ad diametrum applicantur. vocetur autem huiusmodi sectio hyperbole.

The translation of this is as follows::

The square of the ordinate of the hyperbola is equal to the rectangle comprehend-

[1]) $y^2 = px$, where p denotes the *latus rectum.*

middelliniens hoochste deel, ende de lini in sulcken reden tot haer redelicke, ghelijck t'hoochste deel met de opstaende, tot de opstaende.

Sulcken helpende oirsaec (als voorghenomen was te verclaren) hebben wy ghehadt; Soo ist mettet Duytsch ghestelt, ende diets hem niet en verstaet, bidt hem, beminde leser, dat hijt leere, lieuer dan als een dwaes van het Duytsch dwaeslick te oirdeelen. Angaende v yemandt voortbrenghen mocht, dat vele met dese nieuwe costlicheyt der Duytsche tael, daer wy elders breeder afghesseyt hebben, haer spot sullen houden, wanneer sijder af hooren, daer en stoot v niet an, want sulcx is den loop des weerelts; maer denct in v seluen, dat haer ydel woorden, ghetuych van haer verworpen ydelheyt, licht vertreden sullen worden duer v vulle saken, oircondt van v looflicke vulheyt, daerentusschen ghenietende dat sy deruen moeten.

CORTBEGRYP.

Argumentũ.

Definitiones. WY sullen ten eersten beschriuen de * bepalinghen van d'eyghen woorden deser Const, metgaders de begheerten. Daer naer de voorstellen, welcker neghen eerste verclaren sullen, ettelicke wichtighe eyghenschappen der lichamen int water. Het 10^e, 11^e, 12^e, 13^e, 14^e, 15^e, voorstel sal sijn vande macht der drucking des waters teghen bodems. Het 16^e ende 17^e voorstel, vande noodighe langden der sijden des bodems om begheerde drucking des waters daer teghen te hebben. Het 18^e, 19^e, eñ 20^e voorstel, vande swaerheyts middelpunten der gheprangselen des waters in bodems vergaert. Het 20^e voorstel, om duer t'ghewicht des waters sijn grootheyt te vinden. Het 21^e ende laetste voorstel, vande *Proportionibus.* * eueredenheden bestaende tusschen der lichamen grootheyt, stofswaerheyt, ende ghewicht. Achter t'bouesschreuen sal noch volghen den Anvang der Waterwichtdaet.

BEGHIN-ed by the upper part of its diameter and the line having to its latus rectum the same ratio as the upper part plus the transverse side to the transverse side [1]).

Such was the aiding cause (as was intended to be explained) that we had. That is what Dutch is like, and when anyone does not understand it, beg him, dear reader, to learn it rather than judge foolishly of Dutch like a fool. If anyone should argue before you that many people will jest about this new costliness of the Dutch language — about which we have spoken more fully elsewhere [2]) — when they hear of it, do not be offended, for such is the way of the world, but think to yourself that their empty words, testifying to their reprehensible emptiness, will easily be trodden down by your full things, the manifestation of your laudable fullness, while you enjoy meanwhile what they must do without.

THE ARGUMENT

We will first describe the definitions of the proper terms of this Art, and also the postulates. Thereafter the propositions, the first nine of which are to explain several statical properties of bodies in water. The 10th, 11th, 12th, 13th, 14th, 15th propositions are to deal with the force of the pressure of the water against surfaces. The 16th and the 17th proposition with the length of the sides of the surface required to have the desired pressure of the water against it. The 18th, the 19th, and the 20th proposition with the centres of gravity of the total pressures of the water on surfaces. The 20th proposition serves to find from the weight of the water its volume. The 21st and last proposition is to deal with the proportions between the volumes, specific gravities, and weights of bodies. The above is to be followed by the Preamble of the Practice of Hydrostatics.

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[1]) The equation is $y^2 = x\left(\frac{x+q}{q}\right)p$, which follows from $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $p =$ *latus rectum* $= 2\frac{b^2}{a}$ and $q =$ transverse side $= 2a$, and the origin is chosen in one of the vertices.

[2]) See the *Uytspraeck van de Weerdicheyt der Duytsche Tael*. Present volume p. 58.

DE BEGHINSELEN DES WATERWICHTS BESCHREVEN DVER SIMON STEVIN.

EERST DE BEPALINGHEN. *Definitiones*

I. BEPALING.

BEKENDE ſwaerheyt noemen wy hier, diens bekende grootheyt duer bekent ghewicht gheuytet wort.

II. BEPALING.

EVESTOFSWARE lichamen, diens euegrootheden inde locht euewichtich ſijn.

III. BEPALING.

MAER Stofſwaerſte lichaem, dat der euegrooten t'ſwaerſte is.

IIII. BEPALING.

ENDE Stoflichtſte lichaem, dat dier euegrooten t'lichtſte is.

V. BEPALING.

ENDE ſoo menichmael t'ſwaerſte der euegrooten ſwaerder is dan t'lichtſte, ſo menichmael ſtofſwaerder ſegghen wy dat als dit.

VI BEPALING.

STIIFLICHAEM is, diens ſtof niet en vliet, duer t'welck oock water noch locht en dringt.

VII. BEPALING.

VLACVAT is t'gheheel *Meetconſtich vlack eens lichaems, duer t'gedacht daer af ſcheydelick. *Geometrica ſuperficies.*

Bb VIII BE-

THE ELEMENTS OF HYDROSTATICS,
Described by Simon Stevin

FIRST THE DEFINITIONS

DEFINITION I.

A known gravity we here call one whose known volume is expressed by a known weight [1]).

DEFINITION II.

Bodies of equal specific gravity we call those bodies, equal volumes of which are equally heavy in air.

DEFINITION III.

But body of greatest specific gravity we call that which is the heaviest of those of equal volume.

DEFINITION IV.

And body of greatest specific levity we call that which is the lightest of those of equal volume.

DEFINITION V.

And as many times as the heaviest of the bodies of equal volume is heavier than the lightest, so many times the latter is called of greater specific gravity than the former [2]).

DEFINITION VI.

Solid body is one whose matter does not flow, and through which penetrates neither water nor air.

DEFINITION VII.

Surface vessel is the complete geometrical surface of a body, conceived as separable therefrom.

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[1]) *Gravity* here means *specific gravity*. The specific gravity of a substance is known when we know the weight of a known volume of the substance. It will be seen that in the following treatise *gravity* may signify both *weight* and *specific gravity*. In all cases the meaning is clear from the context.

[2]) It may here be remarked that the definitions 1-5 do not yet amount to the definition of specific gravity expressed by the relation

$$S = \frac{G}{V},$$

where G denotes the weight and V the volume of a quantity of a substance with specific gravity S. The low stage of development of symbolic algebra in the 16th century, together with the difficulties inherent in the application of mathematics to physics, prevented this simple formulation.

VIII BEPALING.

BODEM is alle vlack daer eenich water teghen ruſt.

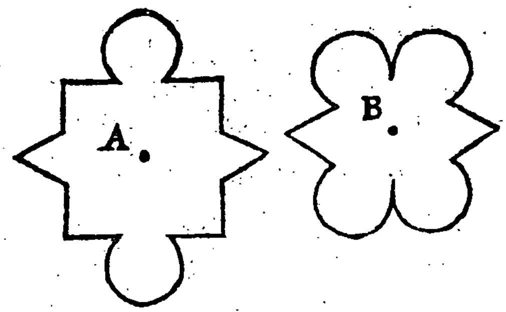
IX BEPALING.

GHESCHICKT bodem noemen wy yder *plat, t'welck met alle rechte lini duer ſijn *middelpunt, in twee euen deelen ghedeelt wort.

Planum. *Centrum.*

VERCLARING.

Als ronden, ſcheefronden, euewydighe vierhoucken, ende alle gheſchickte veelhoucken in t'rondt beſchrijuelick, diens menichte der ſijden effental is, eñ allen anderen van wat form ſy ſouden mueghen weſen, als A, B, ende dierghelijcke, welcke duer haer middelpunt met alle rechte lini in twee euen deelen connen ghedeelt worden, noemen wy Gheſchickte bodems, tot onderſcheydt der ghene die met alle rechte lini duer haer middelpunt niet in twee euen deelen ghedeelt en worden, welcke duer t'verkeerde deſer bepaling al ongheſchickte bodems heeten, als driehoucken, ende veelhoucken met oneuen menichte der ſijden, ende dierghelijcke. D'oirſaeck der bepaling deſes Gheſchickts bodems is (ſo in t'volghende blijcken ſal) dat den pilaer diens grondt een gheſchickt bodem is, in twee euen deelen ghedeelt wort, met alle plat duer twee *lijckſtandighe punten ſchoens teghen ouer malcander ſtaende inde omtrecken des grondts ende deckſels.



Homologa.

X BEBALING.

YDEL noemen wy een plaets daer gheen lichaem in en is.

XI BEPALING.

LEDICH daer niet dan locht in en is.

BEGHEER-

DEFINITION VIII.

Bottom is any plane against which rests any water [1]).

DEFINITION IX.

Regular surface we call any plane which is divided into two equal parts by any straight line through its centre.

EXPLANATION.

As circles, ellipses, parallelograms, and all regular polygonal figures that can be inscribed in a circle, the number of whose sides is even, and any others, of whatever form they may be, as A, B, and the like, which can be divided into two equal parts by any straight line through their centre, we call regular surfaces, to distinguish them from those which cannot be divided into two equal parts by any straight line through their centre, which by the inverse of this definition are all called irregular surfaces, as triangles, and polygons with an odd number of sides, and the like. The cause of the definition of this regular surface is (as will become apparent in the following) that the prism whose base is a regular surface is divided into two equal parts by any plane through two homologous points diametrically opposite to each other in the circumferences of the base and of the cover.

DEFINITION X.

Vacuum we call any place in which no body is present.

DEFINITION XI.

Empty we call any place in which there is only air.

[1]) The sense in which the Dutch word „bodem" is used by Stevin deviates from general usage in that it denotes not only the bottom of a vessel, but also any surface exposed to the pressure of a liquid.

BEGHEERTEN. *Postulata.*

1e BEGHEERTE.

DER lichamen ghewicht inde locht eyghen ghenoemt te worden, maer in t'water naer de gheſtalt.

II BEGHEERTE.

T'VOORGHESTELDE water oueral eenvaerdigher ſwaerheyt te ſijn.

III BEGHEERTE.

TGHEWICHT dat een vat ondieper doet ſincken, lichter te weſen, maer dieper, ſwaerder, ende euediep, eueſwaer te ſijn.

IIII BEGHEERTE.

T'VLACKVAT te connen water ende ander ſtof houden ſonder breken of form te veranderen.

V BEGHEERTE.

T'VLACKVAT vol waters uytghegoten ſijnde, ledich te blijuen.

VERCLARING.

Ledich te blijuen, dat is niet ydel, want anders t'ghewicht des lochts ſouder ghebreken.

VI BEGHEERTE.

YDER waters oppervlack * plat te weſen, euewydich vanden ſichteinder. *Eſſe planum parallelũ cũ Horizonte.*

VERCLARING.

T'WELCK int anſien dattet deel des clootvlacx ofte weereltvlacx is (weereltvlack noemen wy alle clootvlack diens middelpunt des weerelts middelpunt is) oock in een droppel erghẽs op ligghende ofte anhangende, ofte inwater daer eenich lichaem me beſtreken mocht weſen, ſo niet en is, maer in ſoo cleyne menichvuldicheyt waters als deſe, noch in ſoo groote als daer t'ghinſte in merckelick is, en verkeerẽ de volgende* voorſtellen *Propoſitiones*

THE POSTULATES OF HYDROSTATICS

POSTULATE I.

The weights of bodies in air to be called their proper weights, but in water apparent weights.

POSTULATE II.

The water under consideration to be of uniform gravity throughout.

POSTULATE III.

The weight causing a vessel to sink less deep to be lighter, but the weight causing it to sink deeper to be heavier, and that causing it to sink to the same depth, equally heavy.

POSTULATE IV.

The surface vessel to be capable of holding water and other matter without breaking or being transformed.

POSTULATE V.

The surface vessel full of water, the latter being poured out, to be left empty.

EXPLANATION.

To be left empty, that is not a vacuum, for otherwise the weight of the air would be absent.

POSTULATE VI.

Any water's upper surface to be plane, parallel to the horizon.

EXPLANATION.

Since the water's upper surface forms part of the spherical surface or world surface (world surface we call any spherical surface which has its centre in the centre of the world), in reality this is not the case; nor is it with a drop lying on or adhering to something, or with water with which a body has been moistened. The propositions hereinafter, however, relate to quantities of water neither as ſtellen niet. Wel is waer dat wy des waters oppervlack ſouden mueghen nemen voor deel des weereltvlacks,ende de volghende beſchrijuing daer na rechten, maer wanttet moeylicker waer, ende tottet einde, dat is de Waterwichtdaet,niet voorderlicker, ſoo worter begheert datmen toelate, yder waters oppervlack plat te weſen,euewydich vanden ſichteinder.

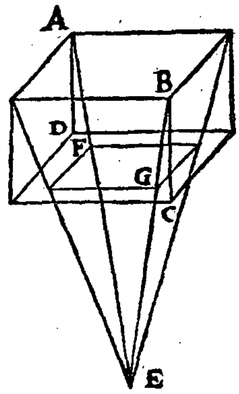
VII BEGHEERTE.

WESENDE den grondt ende deckſel eens pilaers euewydich vanden ſichteinder, eñ de rechte linien tuſſchen* lijckſtandighe punten der ſeluer rechthouckich opden ſichteinder: Dat die linien voortghetrocken in t'weerelts middelpunt vergaren; oock ſulcke grondt ende deckſel deelen van weereltvlacken te ſijn.

Homologa.

VERCLARING.

Laet A B C D een pilaer weſen diens deckſel A B, ende grondt D C euewydich ſijn vanden ſichteinder, eñ B C ſy een rechte lini rechthouckich opden ſichteinder tuſſchen twee lijckſtandighe punten C, B, maer E ſy t'weerelts middelpunt, laet nu ghetrocken worden de linien A E, eñ B E, naeckende den grondt D C inde punten F, G, tuſſchen welcke beſchreuen ſy den grondt F G ghelijck met D C. Dit ſo weſende, t'blyckt dat de linien B C ende A D voortghetrocken, niet en vergaren in E, want dieder in vergaren ſijn A F, ende B G, oock en ſijn de platten A B ende D C gheen deelen van weereltvlacken, nochtan begheeren wy toeghelaten te worden, dat B C ende A D voortghetrocken, daer in verſamen, ende dat die platten A B, D C deelen van weereltvlacken ſijn, reden dat in al t'ghene ons inde Waterwichdaet ontmoet, ſulck verſchil onbemerckelick is, ſoot oock is tuſſchen den pilaer A B C D ende * t'naeldenſdeel A B G F, ſchoon ghenomẽ dat A B ende F G deelen van weereltvlacken waren. Tis wel ſoo, dat wy inde plaets des pilaers A B C D, ſouden mueghen nemen ſoodanich lichaem A B G F, ende de volghende voorſtellen daer naer rechten, maer om ſulcke redenen als onder de 6e begheerte gheſeyt ſijn, ſo iſt beter ghelaten, want ghelijckt inde * Sterconſt ſlichheyt waer, niet toe te laten t'eertrijck voor des weerelts middelpunt ghenomen te worden, alſoo dat oock hier.



Pars Pyramidis.

Astrologia.

NV DE

small as the latter, nor as large as the former. It is true that we might consider the water's upper surface as part of the world surface, and adjust the following description accordingly, but since this would be more difficult and not more conducive to the end in view, viz. the Practice of Hydrostatics, it is postulated that any water's upper surface is plane and parallel to the horizon.

POSTULATE VII.

The base and the cover of a prism being parallel to the horizon, and the straight lines joining homologous points thereof being at right angles to the horizon: that those lines produced meet in the centre of the world; also that such base and cover are parts of world surfaces.

EXPLANATION [1]).

Let *ABCD* be a prism, whose cover *AB* and base *DC* shall be parallel to the horizon, and *BC* shall be a straight line at right angles to the horizon, joining two homologous points *C*, *B*, but *E* shall be the centre of the world. Let there now be drawn the lines *AE* and *BE*, touching the base *DC* in the points *F*, *G*, between which let there be described the base *FG*, similar to *DC*. This being so, it is apparent that the lines *BC* and *AD* produced do not meet in *E*, for those meeting therein are *AF* and *BG*. Nor are the surfaces *AB* and *DC* parts of world surfaces. Nevertheless we postulate that *BC* and *AD* produced meet therein, and that those surfaces *AB* and *DC* are parts of world surfaces, because in all the cases we shall meet with in the Practice of Hydrostatics this difference is imperceptible, just as it is between the prism *ABCD* and the part of a pyramid *ABGF*, even if it is assumed that *AB* and *FG* are parts of world surfaces. It is true that instead of the prism *ABCD* we might take such a body *ABGF*, and adjust the following propositions accordingly, but for the reasons mentioned in the 6th postulate it is better not to do so, for just as in astronomy it would be stupid not to grant that the earth be taken for the centre of the world [2]), so it is here, too.

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[1]) Here as well as in the fifth postulate of the *Art of Weighing* Stevin scruples to consider all verticals as parallel. There is, however, one notable difference. In the *Art of Weighing* we were asked to grant that they are parallel; here it is postulated that vertical lines, which are parallel, (viz. the vertical sides of a prism) meet in the centre of the world. As far as we have been able to ascertain, the postulate has not been used on any occasion.

[2]) This is not to be understood as a rejection of the Copernican system of the world, in which the earth is no longer at the centre of the universe. Indeed, we know from the *Hemelloop* (XI; i, 3) that Stevin was an ardent supporter of this system. The statement only means that the earth is to be considered as a point in regard to the dimensions of the world, and consequently may be called centre, if we take the geocentric view. In a similar way Stevin here assumes the centre of the earth to be at infinity.

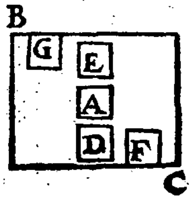
NV DE VOORSTELLEN.

Propositiones

I. VERTOOCH. I. VOORSTEL.

T'GHESTELDE water houdt alle plaets diemen hem binnen water gheeft.

T'HEGHEVEN. Laet het water in t'vlackvat A t'ghestelde water sijn in t'water B C. T'BEGHEERDE. Wy moeten bewysen dattet water A in die plaets sal blijuen. T'BEWYS. En latet water A (soot mueghelick waer) sijn plaets niet houden, maer het sy ghedaelt daer D is; Dit so toeghelaten, t'water dat daer naer inde plats van A ghecommen is, sal om de selue oirsaeck oock ter plaets van D dalen, t'welck daer naer oock een derghelijcke ander doen sal, inder voughen dat dit water (om dat de reden altijdt de selue is) een eewich roersel sal maken, t'welck ongheschickt is. Sghelijcx sal oock bethoont worden dat A niet rijsen, ofte naer eenighe ander sijden hem begheuen en can. T'blijckt oock dat soomen A stelde binnen t'water ter plaets van D, E, F, of G, dattet om de voornoemde redenen, op yder van die plaetsen, ende oueral daerment in B C set, blijuen sal.

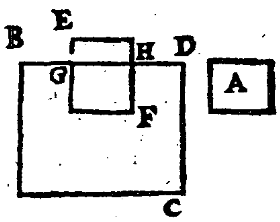


T'BESLVYT. T'ghestelde water dan, houdt alle plaets diemen hem binnen water gheeft, t'welck wy bewysen moesten.

II. VERTOOCH. II. VOORSTEL.

EEN styflichaem stoflichter dan water, en sijnckt niet heel daer onder, maer een deel blijfter uyt stekende.

T'GHEGHEVEN. Laet het styflichaem A, stoflichter sijn dan t'water B C, diens oppervlack B D. T'BEGHEERDE. Wy moeten bewysen dat A, gheleyt in t'water B C, niet heel daer onder sincken en sal, maer datter een deel buyten t'water sal blijuen steken.



T'BEREYTSEL. Laet E F een vlackvat sijn, wiens deel dat binnen t'water ende met water ghevult is, sy G F, euegroot ende ghelijck an A, ende sijn oppervlack G H sal in t'vlack B D sijn, ouermidts t'vlackvat E F licht noch swaer en is.

T'BEWYS. Anghesien A stoflichter is duer t'ghegheuen dan t'water

Bb 3 G F, ende

NOW THE PROPOSITIONS

THEOREM I. PROPOSITION I.

The water under consideration keeps any place given to it in water.

SUPPOSITION. Let the water in the surface vessel *A* be the water under consideration in the water *BC*. WHAT IS REQUIRED TO PROVE. We have to prove that the water *A* will remain in that place. PROOF. Let the water *A* (if it were possible) not remain in that place, but be descended where *D* is. This being granted, the water which has thereafter reached the place of *A* will for the same reason also descend to *D*, which a similar other quantity of water will then also do, in such a way that this water (because the reason is always the same) will perform a perpetual motion, which is absurd. In the same way it can also be shown that *A* cannot rise or move towards any other side. It also appears that if *A* be placed within the water in the place of *D*, *E*, *F* or *G*, it will, for the aforesaid reasons, remain in each of those places, and wherever it is placed in *BC*.
CONCLUSION. The water under consideration therefore keeps any place given to it in water, which we had to prove.

THEOREM II. PROPOSITION II.

A solid body of greater specific levity than water does not sink completely below the upper surface, but a part continues to stick out of it.

SUPPOSITION. Let the solid body *A* be of greater specific levity than the water *BC*, whose upper surface shall be *BD*. WHAT IS REQUIRED TO PROVE. We have to prove that *A*, when laid in the water *BC*, will not sink completely below the upper surface, but that a part will continue to stick out of the water. PRELIMINARY. Let *EF* be a surface vessel, whose part which is in the water and filled with water shall be *GF*, equal and similar to *A*, and its upper surface *GH* shall be in the surface *BD*, since the surface vessel *EF* is neither light nor heavy. PROOF. Since by the supposition *A* is of greater specific levity than the water

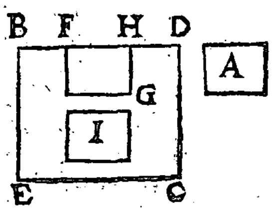
G F, ende dat G F euegroot is an A, soo is G F swaerder dan A. Laet ons nu t'water G F dat in t'vlackvat E F is, uytghieten, ende legghen daerin t'lichaem A, t'welck die plaets effen vullen sal, ouermits A duer t'bereytsel ghelijck eñ euegroot is an G F; Maer als vooren ghesëyt is t'lichaem A is lichter dan t'uytghegoten water; T'vlackvat dan E F en sal van A soo diep niet sincken alst van t'water G F dede, duer de 3ᵉ begheerte; Maer soo veel t'vlackvat E F ondieper sinckt, soo veel moetet lichaem A nootsakelick buyten t'water steken. T'BESLVYT. Een stijflichaem dan stoflichter als water, en sinckt niet heel daer onder, maer een deel blijfter uytstekende; t'welck wy bewysen moesten.

III. VERTOOCH. III. VOORSTEL.

EEN stijflichaem stofswaerder dan water sinckt tot den grondt.

T'GHEGHEVEN. Laet A een stijflichaem wesen stofswaerder dan t'water B C, diens oppervlack B D, ende grondt E C sijn.

T'BEGHEERDE. Wy moeten bewysen dat A gheleyt in t'water B C, sincken sal tot den grondt E C. T'BEREYTSEL. Laet F G een vlackvat sijn met water ghevult, euegroot ende ghelijck an A, wiens oppervlack F H in t'vlack B D sy.



T'BEWYS. Anghesien A stofswaerder is duer ghestelde dan t'water F G, ende dat F G euegroot is an A, soo is A swaerder dan F G. Laet ons nu t'water F G dat in t'vlackvat F G is, uytghieten, ende legghen daerin t'lichaem A, t'welck die plaets effen vullen sal, ouermits A duer t'ghestelde ghelijck en euegroot is an F G; Maer soo wy vooren ghesëyt hebben, A is swaerder dan het uytghegoten water; T'vlackvat dan F G sal van A dieper sincken alst van t'water F G dede duer de 3ᵉ begheerte. Wy hebben dan bethoont dattet lichaem A sincken sal. Daer rest noch bewesen te worden dattet oock sincken sal tot op den grondt E C, aldus: En latet (soot mueghelick waer) niet sincken tot E C, maer opden wech tusschen beyden blijuen als daer I is, ende laet ons t'stijflichaem datter in t'vlackvat I steeckt, weeren, ende vollen dat met water, t'selue sal duer het 1ᵉ voorstel op die plaets blijuen: Maer dit water is lichter als dat lichaem, een swaerder dan ende een lichter, sullen op een selfde plaets blijuen, t'welck ongheschickt ende teghen de 3ᵉ begheerte is. T'lichaem A dan, en can tusschen t'oppervlack B D ende den grondt E C niet blijuen, t'moet dan nootsakelick sincken tot dattet op den grondt

GF, and *GF* is equal to *A*, *GF* is heavier than *A*. Let us now pour out the water *GF* that is in the surface vessel *EF*, and lay therein the body *A*, which will just fill that place, since by the preliminary *A* is similar and equal to *GF*. But, as has been said above, the body *A* is lighter than the water poured out. The surface vessel *EF* therefore will not sink as deep under the influence of *A* as it did under the influence of the water *GF*, by the 3rd postulate. But by as much as the surface vessel *EF* sinks less deep, by so much must the body *A* necessarily stick out of the water. CONCLUSION. A solid body of greater specific levity than water therefore does not sink completely below the upper surface, but a part continues to stick out of it, which we had to prove.

THEOREM III. PROPOSITION III.

A solid body of greater specific gravity than water sinks to the bottom.

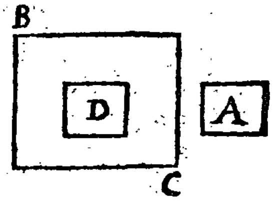
SUPPOSITION. Let *A* be a solid body of greater specific gravity than the water *BC*, whose upper surface is *BD* and its base *EC*. WHAT IS REQUIRED TO PROVE. We have to prove that *A*, when laid in the water *BC*, will sink to the base *EC*. PRELIMINARY. Let *FG* be a surface vessel filled with water, equal and similar to *A*, whose upper surface *FH* shall be in the plane *BD*. PROOF. Since by the supposition, *A* is of greater specific gravity than the water *FG*, and *FG* is equal to *A*, *A* is heavier than *FG*. Let us now pour out the water *FG* which is in the surface vessel *FG*, and lay therein the body *A*, which will just fill that place, since by the supposition *A* is similar and equal to *FG*. But as we have said before, *A* is heavier than the water poured out. The surface vessel *FG* therefore will sink deeper under the influence of *A* than under the influence of the water *FG*, by the 3rd postulate. We have therefore shown that the body *A* will sink. It remains to be proved that it will also sink to the base *EC*, thus: Let it (if this were possible) not sink to *EC*, but remain somewhere between the two, as where *I* is, and let us take away the solid body in the surface vessel *I*, and fill it with water; it will remain in that place by the 1st proposition. But this water is lighter than that body. A heavier and a lighter body will therefore remain in the same place, which is absurd, and contrary to the 3rd postulate. The body *A* therefore cannot remain between the upper surface *BD* and the base *EC*; it must therefore necessarily sink gront E C rust. T'BESLVYT. Een stijflichaem dan stofswaerder als water, sinckt tot den grondt, t'welck wy bewysen moesten.

IIII VERTOOCH. IIII VOORSTEL.

EEN stijflichaem euestofswaer an water, houdt alle plaets diemen hem binnen water gheeft.

T'GHEGHEVEN. Laet het stijflichaem A, euestofswaer sijn mettet t'water B C. T'BEGHEERDE. Wy moeten bewysen dat A in t'water B C gheleydt, alle plaets houdt diemen hem daer gheeft.

T'BEREYTSEL. Laet D een vlackvat vol waters sijn, euegroot ende ghelijck an A. T'BEWYS. Anghesien A euestofswaer is door t'ghegheuen an t'water D, ende dat D euegroot is met A, soo is D oock euesvvaer met A; Laet ons nu t'water D dat in t'vlackvat D is, uytghieten, ende legghen daerin sijn euewichtich lichaem A, t'welck die plaets effen vullen sal, ouermits A door t'ghestelde ghelijck en euegroot is an G F; T'vlackvat dan D en sal van A niet dieper sincken noch hooger rijsen dan van t'water D, duer de 3e begheerte: Maer t'water D hielt in B C alle plaets diemen hem gaf duer het 1e voorstel, t'stijflichaem A dan, houdt in t'water B C alle plaets diemen hem gheeft.



T'BESLVYT. Een stijflichaem dan euestofswaer an water, houdt alle plaets diemen hem binnen water gheeft, t'welck wy bewysen moesten.

V VERTOOCH. V VOORSTEL.

EEN stijflichaem stoflichter dan water daert inlight, is euewichtich an t'water euegroot met sijn deel dat binnen t'water is.

T'GHEGHEVEN. Laet A B een stoflichter stijflichaem sijn dan t'water C D daert in ligt, ende sijn vlackvat sy A B, ende sijn deel binnen t'water sy E B. T'BEGHEERDE. Wy moeten bewysen dat het stijflichaem A B, euewichtich is an t'water dat euegroot is met het deel E B dat binnen t'water C D is. T'BEWYS. Laet ons t'stijflichaem A B trecken uyt het vlackvat A B, ende vullen t'vlackvat weder met water, tot dattet soo diep inghesoncken is alst eerst mettet lichaem was. T'welck soo sijnde, t'water E B datter in t'vlackvat A B is, sal (want t'oppervlack van alle water des vlackvats met een deel buyten t'water stekende until it rests on the base *EC*. CONCLUSION. A solid body of greater specific gravity than water therefore sinks to the bottom, which we had to prove.

THEOREM IV. PROPOSITION IV.

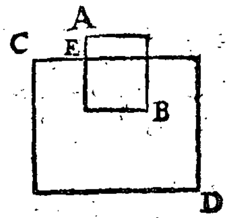
A solid body of equal specific gravity to water keeps any place given to it in water.

SUPPOSITION. Let the solid body *A* be of equal specific gravity to the water *BC*. WHAT IS REQUIRED TO PROVE. We have to prove that *A*, when laid in the water *BC*, keeps any place given to it therein. PRELIMINARY. Let *D* be a surface vessel full of water, equal and similar to *A*. PROOF. Since, by the supposition, *A* is of equal specific gravity to the water *D*, and *D* is equal to *A*, *D* is also of equal gravity to *A*. Let us now pour out the water *D* which is in the surface vessel *D*, and lay therein the body *A* having the same weight, which will just fill that place, since by the supposition *A* is similar and equal to *GF*. The surface vessel *D* therefore will neither sink deeper nor rise higher under the influence of *A* than under the influence of the water *D*, by the 3rd postulate. But the water *D* kept in *BC* any place given to it, by the 1st proposition. The solid body *A* therefore keeps in the water *BC* any place given to it. CONCLUSION. A solid body of equal specific gravity to water therefore keeps any place given to it in water, which we had to prove.

THEOREM V. PROPOSITION V.

A solid body of greater specific levity than the water in which it lies is of equal weight to the water having the same volume as its part in the water.

SUPPOSITION. Let *AB* be a solid body of greater specific levity than the water *CD* in which it lies, and its surface vessel shall be *AB*, and its part in the water shall be *EB*. WHAT IS REQUIRED TO PROVE. We have to prove that the solid body *AB* is of equal weight to the water having the same volume as the part *EB* which is in the water *CD*. PROOF. Let us pull the solid body *AB* from the surface vessel *AB*, and fill the surface vessel again with water until it has sunk to the same depth at which it was first when filled with the body. This being so, the water *EB* which is in the surface vessel *AB* will (because the upper surface of any water in the surface vessel, a part of which is sticking out of the kende, is altijdt in t'oppervlack des omuanghenden waters, ouermidts t'vlackvat niet en weeght) euewichtich sijn an t'ghegheuen lichaem A B, Reden, dat twee ghewichten die een vat euediep doen sincken oock eueswaer sijn, duer de 3e begheerte. T'BESLVYT. Een stijflichaem dan stoflichter als water daert in light, is euewichtich an t'water euegroot met sijn deel dat binnen t'water is, t'welck wy bewysen moesten.



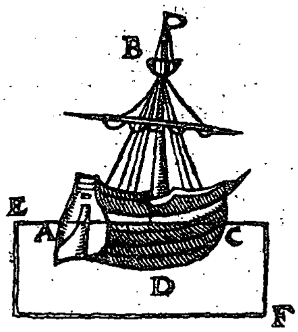
I EYSCH. VI VOORSTEL.

LIGGHENDE t'een deel des stijflichaems bekender grootheyt, in water bekender swaerheyt, ende t'ander deel daer buyten: Te vinden t'ghewicht des heelen lichaems.

T'GHEGHEVEN. Laet A B C D een stijflichaem wesen van form soot valt ende E F een water van t'welck een teerlincksche voet weeght 65 ℔ (soo veel weeght naer d'eruaring een Delfsche voet Delfs water, ende daerop sullen wyse inde volghende voorbeelden altijdt schatten) ende des lichaems deel binnen t'water sy A C D, wiens grootheyt sy van 10000 teerlijncsche voeten. T'BEGHEERDE. Wy moeten vinden hoe swaer t'heel lichaem A B C D sy, met al datter in ende op is.

T'WERCK. Men sal 10000 menichvuldighen met de 65 ℔ come 650000 ℔ voor t'begheerde.

T'BEWYS. Het heel lichaem A B C D is euewichtich an t'water euegroot met A C D duer het 5e voorstel, maer t'water euegroot an A C D weeght 650000 ℔, het heel lichaem dan A B C D weeght 650000 ℔, t welc wy bewysen moesten.



T'BESLVYT. Ligghende dan t'een deel des stijflichaems bekender grootheyt, in water bekender swaerheyt, ende t'ander deel daer buyten; wy hebben t'ghewicht des heelen lichaems ghevonden naer den eysch.

VI VER-water, is always in the upper surface of the surrounding water, since the surface vessel has no weight) be of equal weight to the given body *AB*, because two weights which cause a vessel to sink to the same depth are also of equal gravity, by the 3rd postulate. CONCLUSION. A solid body of greater specific levity than the water in which it lies is therefore of equal weight to the water having the same volume as its part in the water, which we had to prove.

PROBLEM I. PROPOSITION VI.

One part of a solid body of known volume lying in water of known gravity, and the other outside it: to find the weight of the whole body.

SUPPOSITION. Let *ABCD* be a solid body of any form, and *EF* a water one cubic foot [1]) of which weighs 65 lbs (that is, by experience, the weight of a Delft foot of Delft water, and this weight we will always assume for it in the following examples), and the part of the body in the water shall be *ACD*, whose volume shall be 10,000 cubic feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the whole body *ABCD*, with all that is in or on it. CONSTRUCTION. Multiply 10,000 by the 65 lbs, then the required weight will be 650,000 lbs. PROOF. The whole body *ABCD* is of equal weight to the water having the same volume as *ACD*, by the 5th proposition, but the water having the same volume as *ACD* weighs 650,000 lbs; the whole body *ABCD* therefore weighs 650,000 lbs, which we had to prove. CONCLUSION. One part of a solid body of known volume therefore lying in water of known gravity, and the other part outside it: we have found the weight of the whole body, as required.

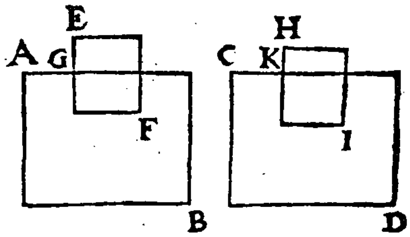
[1]) It is only here that Stevin uses the correct term „cubic foot". Elsewhere „foot" means the unit of length as well as the corresponding units of area (square foot) and volume (cubic foot).

VI VERTOOCH. VII VOORSTEL.

WESENDE twee oneueſtoffware wateren, ende een ſtijflichaem ſtoflichter dan eenich van dien: Ghelijck de ſtoffwaerheyt des ſwaerſten waters, tot de ſtoffwaerheyt der lichtſten, alſo de grootheyt diens ſtijflichaems binnen t'water in t'lichtſte water gheleyt, tot ſijn grootheyt binnen t'water in t'ſwaerſte gheleyt.

T'GHEGHEVEN. Laet A B een water ſijn, ſtofſwaerder dan t'water C D, ende E F ſy een ſtijflichaem ſtoflichter dan eenich dier twee wateren, t'welck eerſt gheleyt in t'water A B, ſoo daelter onder t'water het deel G F, maer t'ſelue lichaem E F gheleyt in t'water C D, t'welck daer ſy H I, ſoo ſinckter onder t'water het deel K I. T'BEGHEERDE. Wy moeten bewyſen dat ghelijck de ſtofſwaerheyt des waters A B, tot de ſtofſwaerheyt des waters C D; alſoo de grootheyt K I, tot G F.

T'BEWYS. T'water des waters A B euegroot an G F, is eueſwaer mettet lichaem E F, ende t'water des waters C D euegroot an K I, is eueſwaer mettet lichaem H I duer het 5ᵉ voorſtel, maer t'lichaem E F ofte H I is al een ſelfde lichaem duer t'ghegheuen, daerom t'water des waters A B euegroot met G F, is eueſwaer an t'water des waters C D euegroot met K I; Maer weſende twee eueſware waterẽ, ghelijc haer grootheyt tot grootheyt, alſo ouerandert haer ſtofſwaerheyt tot ſtofſwaerheyt, als nootſakelick volght uyt de toeghelatẽ 5ᵉ bepaling, daerom ghelijck de ſtofſwaerheyt des waters A B, tot de ſtofſwaerheyt des waters C D, alſoo de grootheyt K I, tot de grootheyt G F.



T'BESLVYT. Weſende dan twee oneueſtofſware wateren ende een ſtijflichaem, &c.

VII VERTOOCH. VIII VOORSTEL.

YDER ſtijflichaems ſwaerheyt is ſo veel lichter in t'water dan inde locht, als de ſwaerheyt des vvaters met hem euegroot.

T'GHEGHEVEN. Laet A een ſtijflichaem ſijn, ende B C een water.

THEOREM VI. PROPOSITION VII.

Given two waters of unequal specific gravity, and a solid body of greater specific levity than either of these: as is the specific gravity of the heavier water to the specific gravity of the lighter, so is the volume of that solid body within the water when laid in the lighter water to its volume within the water when laid in the heavier.

SUPPOSITION. Let *AB* be a water of greater specific gravity than the water *CD*, and *EF* shall be a solid body of greater specific levity than either of those two waters; when this body is first laid in the water *AB*, the part *GF* will sink below the water, but when the same body *EF*, which now shall be *HI*, is laid in the water *CD*, the part *KI* will sink below the water. WHAT IS REQUIRED TO PROVE. We have to prove that as the specific gravity of the water *AB* is to the specific gravity of the water *CD*, so is the volume *KI* to *GF*. PROOF. The water of the water *AB* which has the same volume as *GF* is of equal gravity to the body *EF*, and the water of the water *CD* which has the same volume as *KI* is of equal gravity to the body *HI*, by the 5th proposition, but the body *EF* or *HI* is one and the same body, by the supposition. Therefore the water of the water *AB* which has the same volume as *GF* is of equal gravity to the water of the water *CD* which has the same volume as *KI*. But given two bodies of water of equal gravity, as their volumes are to each other, so are their specific gravities inversely to each other, as follows necessarily from the 5th definition that has been granted. Therefore, as the specific gravity of the water *AB* is to the specific gravity of the water *CD*, so is the volume *KI* to the volume *GF*.
CONCLUSION. Given therefore two waters of unequal specific gravity, and a solid body, etc.

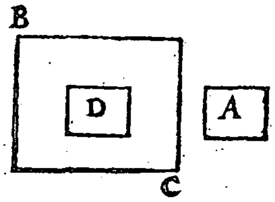
THEOREM VII. PROPOSITION VIII.

The gravity of any solid body is as much lighter in water than in air as is the gravity of the water having the same volume [1]).

SUPPOSITION. Let *A* be a solid body, and *BC* a water. WHAT IS REQUIRED

[1]) This proposition contains the famous principle of Archimedes *(On Floating Bodies* I, Props 6 and 7). Here, just as in his demonstration of the law of the inclined plane (*Art of Weighing* I, Prop. 19), Stevin shows his remarkable gift for demonstrations which immediately appeal to common sense and hence may be understood without any previous knowledge.

T'BEGHEERDE. Wy moeten bewysen dat A in t'water B C gheleyt, aldaer soo veel lichter sal sijn dan inde locht, als de swaerheyt des waters met hem euegroot. T'BEREYTSEL. Laet D een vlackvat vol waters sijn, euen ende ghelijck an A. T'BEWYS. T'vlackvat D vol waters, en is in t'water B C licht noch swaer, want het daer in alle ghestalt houdt diemen hem gheeft, duer het 1e voorstel, daerom t'water D uytghegoten, t'vlackvat sal t'ghewicht des waters lichter sijn dant in sijn eerste ghedaente was, dat is, van soo veel volcommentlick licht: Laet ons nu daer in legghen t'lichaem A, t'selue sal daerin effen passen, om dat sy euen ende ghelijck sijn duer t'ghestelde. Ende t'vlackvat mettet lichaem A alsoo daer in, sal weghen t'ghewicht van A met sijn voornoemde lichticheyt, dat is t'ghewicht van A min t'ghewicht des waters datter eerst uytghegoten was, maer dat water is euegroot an A. Daerom A in t'water B C gheleyt, is daer in soo veel lichter dan inde locht, als de swaerheyt des waters met hē euegroot.



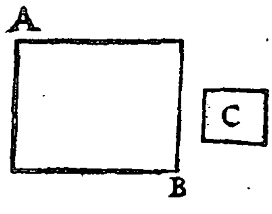
T'BESLVYT. Yder styflichaems swaerheyt dan, is soo veel lichter in t'water dan inde locht, als de swaerheyt des waters met hem euegroot, t'welck wy bewysen moesten.

II EYSCH. IX VOORSTEL.

WESENDE ghegheuen de reden der stofsvvaerheyt des vvaters, ende eens stijflichaems, ende des stijflichaems svvaerheyt: Sijn staltvvicht in t'vvater te vinden.

1e VOORBEELT *alwaer t'stijflichaem stoflichter is dan water.*

T'GHEGHEVEN. Laet A B een water sijn, ende C een stijflichaem weghende 2 ℔, ende de stofswaerheyt des waters, tot de stofswaerheyt des styflichaems sy als van 5 tot 1. T'BEGHEERDE. Wy moeten des styflichaems C staltwicht in t'water A B vinden. T'WERCK. Men sal sien hoe veel een lichaem waters euegroot met C, weghen soude, wort beuonden 5 mael 2 ℔, dat is 10 ℔, de selue ghetrocken van 2 ℔ des styflichaems C, rest min 8 ℔, dat is licht ofte rysendwicht 8 ℔ voor C in t'water A B.



Om t'welck

TO PROVE. We have to prove that *A*, when laid in the water *BC*, will there be as much lighter than in air as is the gravity of the water having the same volume. PRELIMINARY. Let *D* be a surface vessel full of water, equal and similar to *A*. PROOF. The surface vessel *D* full of water is in the water *BC* neither light nor heavy, for it there keeps any place given to it, by the 1st proposition. Therefore, the water *D* being poured out, the surface vessel will be lighter by the weight of the water than it was in the first position, i.e. it will be so light absolutely. Let us now lay therein the body *A*; this will just fit therein, because they are equal and similar by the supposition. Then the surface vessel with the body *A* therein will weigh as much as the weight of *A* with its aforesaid levity, i.e. the weight of *A* minus the weight of the water that was first poured out of it, but this water has the same volume as *A*. Therefore *A*, when laid in the water *BC,* is therein as much lighter than in air as is the gravity of the water having the same volume.

CONCLUSION. The gravity of any solid body therefore is as much lighter in water than in air as is the gravity of the water having the same volume, which we had to prove.

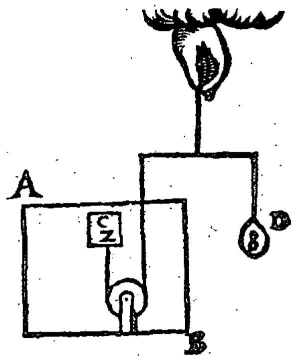
PROBLEM II. PROPOSITION IX.

Given the ratio between the specific gravity of the water and that of a solid body, and the gravity of the solid body: to find its apparent weight in the water.

EXAMPLE I, where the solid body is of greater specific levity than water.

SUPPOSITION. Let *AB* be a water, and *C* a solid body weighing 2 lbs, and the specific gravity of the water shall be to the specific gravity of the solid body as 5 to 1. WHAT IS REQUIRED TO FIND. We have to find the apparent weight of the solid body *C* in the water *AB*. CONSTRUCTION. It shall be ascertained what would be the weight of a body of water having the same volume as *C*; this is found to be 5 times 2 lbs, that is 10 lbs. This being subtracted from the 2 lbs of the solid body *C*, there is left minus 8 lbs, that is light or rising weight of 8 lbs for *C* in the water *AB*.

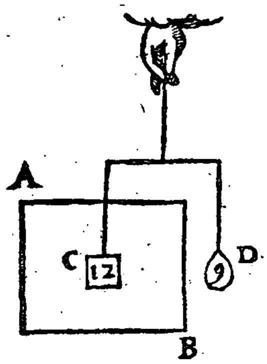
Om t'welck opentlicker te verclaren, soo neemt dat C in t'water A B ghesteken sy, ende daerteghen ghehanghen t'ghewicht D van 8 ℔, als hier neuens, ende D sal met C euestalwichtich sijn.



11ᵉ VOORBEELT, *alwaer t'styflichaem stofswaerder is dan t'water, diens werckingh ghelijck is ande voorgaende.*

T'GHEGHEVEN. Laet de reden der stofswaerheyt des waters A B hier bouen, tot het styflichaem C, nu sijn als van 1 tot 4, ende laet C weghen 12 ℔. T'BEGHEERDE. Wy moeten des styflichaems staltwicht in t'water A B vinden. T'WERCK. Men sal sien hoe veel een lichaem waters euegroot met C weghen soude; wort beuonden het $\frac{1}{4}$ van C 12 ℔, dat is 3 ℔, de selue ghetrocken van 12 ℔ des styflichaems C, rest 9 ℔ voor t'ghewicht van C in t'water A B.

Om t'welck breeder te verclaren, so neemt dat C in t'water A B ghesteken sy, ende daer teghen hanghe t'ghewicht D van 9 ℔, als hier neuens, ende D sal met C euestaltwichtich sijn.



Wy souden oock mueghen een derde voorbeelt setten, alwaer de reden der stofswaerheyt des waters ende styflichaems euen waer; maer t'is blijckelick dat (oock volghende de reghel der voorgaender wercking) sulcken styflichaem in t'water licht noch swaer sijn en sal, van alle welcke t'bewys openbaer is duer t'bouesschreuen 8ᵉ voorstel.

T'BESLVYT. Wesende dan ghegheuen de reden der stofswaerheyt des waters, ende eens styflichaems, ende des styflichaems swaerheyt: Wy hebben sijn staltwicht in t'water ghevonden naer den eysch.

In order to explain this more clearly, assume that *C* be put in the water *AB*, and that against this there be suspended the weight *D* of 8 lbs, as in the figure opposite; then *D* will be of equal apparent weight to *C*.

EXAMPLE II, where the solid body is of greater specific gravity than the water, the procedure of which is similar to the preceding.

SUPPOSITION. Now let the ratio of the specific gravity of the water *AB* above to that of the solid body *C* be as 1 to 4, and let *C* weigh 12 lbs. WHAT IS REQUIRED TO FIND. We have to find the apparent weight of the solid body in the water *AB*. CONSTRUCTION. It shall be ascertained what would be the weight of a body of water having the same volume as *C*. This is found to be ¼ of C (12 lbs), that is 3 lbs. The latter being subtracted from the 12 lbs of the solid body *C*, there is left 9 lbs for the weight of *C* in the water *AB*.

In order to explain this more fully, assume that *C* be put in the water *AB*, and that against this there be suspended the weight *D* of 9 lbs, as in the figure opposite. Then *D* will be of equal apparent weight to *C*.

We might also give a third example, where the ratio between the specific gravity of the water and that of the solid body should be equal; but it is evident that (also according to the rule of the preceding procedure) such a solid body will be neither light nor heavy in water, of all of which the proof is manifest from the 8th proposition described above.

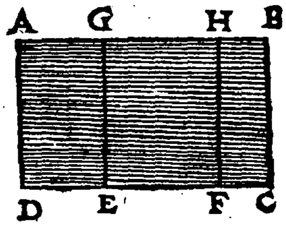
CONCLUSION. Given therefore the ratio of the specific gravity of water to that of a solid body, and the gravity of the solid body: we have found its apparent weight in water, as required.

VIII VERTOOCH. X VOORSTEL.

Parallelũ cũ Horizonte. OP yder bodem des vvaters * euevvydich sijnde vanden sichteinder, rust een ghevvicht euen ande svvaerheyt vvaters die euegroot is met den pilaer, vviens grondt dien bodem is, ende hoochde, de * hanghende lini van * t'plat duer t'vvaters oppervlack tot den grondt.

Perpendicularis. *Plano.*

T'GHEGHEVEN. Laet A B C D een water sijn, van form een lichamelick rechthouck, diens oppervlack A B is, ende eenighen bodem daer in E F, euewydich vanden sichteinder; Laet oock G E de hanghende lini sijn van t'plat duer t'waters oppervlack totten grondt E F, ende den pilaer begrepen onder den bodem E F eñ hoochde E G, sy G H F E.



T'BEGHEERDE. Wy moeten bewysen dat opden bodem E F, rust het ghewicht euen ande swaerheyt waters des pilaers C H F E. T'BEWYS. Soo opden bodem E F meer ghewicht rust dan des waters G H F E, dat sal moeten commen van weghen t'neuenstaende water; Laetet sijn soot mueghelick waer, van t'water A G E D ende H B C F; Maer dat soo ghenomen, daer sal op den bodem D E, van weghen t'water G H F E, om dat de reden de selue is, oock meer ghewichts rusten dan des waters A G E D; ende op den bodem F C, oock meer ghewichts dan des waters H B C F, ende veruolghens op den heelen bodem D C sal meer ghewichts rusten, dan des heelen waters A B C D, t'welck (ghemerckt A B C D een lichamelick rechthouck is) ongheschickt waer. S'ghelijcx salmen oock bethoonen dat opden bodem E F niet min en rust dan t'water G H F E, daer rust dan nootsakelick op t'ghewicht euen ande swaerheyt waters des pilaers G H F E.

I VERVOLGH.

Laet ons nu in t'water A B C D des 10en voorstels, legghen een stijflichaem I K L M, stoflichter dan water, dat is driuende op t'water, mettet deel N O L M daer binnen, ende mettet deel N O K I daer buyten, welcker ghestalt dan sy als hier onder. Dit soo sijnde, t'stijflichaem I K L M is euewichtich metter water euegroot an N O L M duer het 5e voorstel, waer duer t'lichaem I K L M, met de rest des waters rondom hem, euewichtich is an een lichaem waters euegroot an A B C D: daerom segghen wy noch naer luyt des voorstels, dat teghend en bodem E F een ghe-

THEOREM VIII. PROPOSITION X.

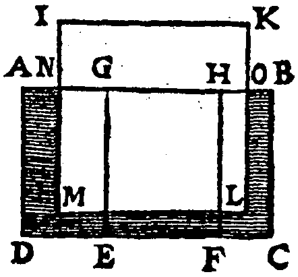
On any bottom of the water being parallel to the horizon there rests a weight equal to the gravity of the water the volume of which is equal to that of the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the base.

SUPPOSITION. Let *ABCD* be a water, whose form be a corporeal rectangle, whose upper surface be *AB* and a bottom therein *EF*, parallel to the horizon. Let also *GE* be the vertical from the plane through the water's upper surface to the bottom *EF*, and the prism comprehended by the bottom *EF* and the height *EG* shall be *GHFE*. WHAT IS REQUIRED TO PROVE. We have to prove that on the bottom *EF* there rests a weight equal to the gravity of the water of the prism *GHFE*. PROOF. If there rests on the bottom *EF* more weight than that of the water *GHFE*, this will have to be due to the water beside it. Let this, if it were possible, be due to the water *AGED* and *HBCF*. But this being assumed, there will also rest on the bottom *DE*, owing to the water *GHFE*, because the reason is the same, more weight than that of the water *AGED*; and on the bottom *FC* also more weight than that of the water *HBCF*; and consequently on the entire bottom *DC* there will rest more weight than that of the whole water *ABCD*, which (in view of *ABCD* being a corporeal rectangle) would be absurd. In the same way it can also be shown that on the bottom *EF* there does not rest less than the water *GHFE*. Therefore, on it there necessarily rests a weight equal to the gravity of the water of the prism *GHFE*.

COROLLARY I.

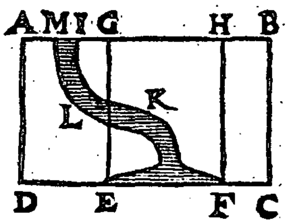
Let us now lay in the water *ABCD* of the 10th proposition a solid body *IKLM*, of greater specific levity than water, i.e. floating on the water, with the part *NOLM* within the water and with the part *NOKI* above it, the position then being as shown below. This being so, the solid body *IKLM* is of equal weight to the water having the same volume as *NOLM*, by the 5th proposition, owing to which the body *IKLM*, with the remainder of the water surrounding it, is of equal weight to a body of water having the same volume as *ABCD*. Therefore we still say, according to the proposition, that against the bottom *EF* there rests a weight een ghewicht rust, euen ande swaerheyt waters die euegroot is met den pilaer, diens grondt E F is, ende hoochde de hanghende lini G E, van * t'plat A B, duer t'waters oppervlack A N totten grondt E F: Waer uyt blijckt dat eenighe drijuende stof in t'water gheleyt, sy en verswaert noch en verlicht (welverstaende als t'water inde selfde hoochde blijft) den grondt niet.

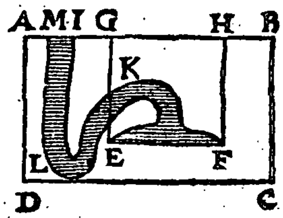
* Plano.



II. VERVOLGH.

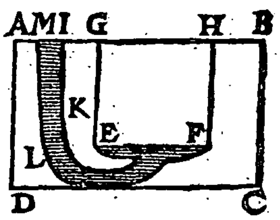
Laet andermael int water A B C D, legghen een styflichaem, ofte verscheyden styflichamen euestofswaer mettet water, ick neem alsoo, datter maer water en blijft als t'begrepen binnen I K F E L M; T'welck so sijnde, dese lichamen en beswaren noch en verlichten den grondt E F niet meer dan t'water eerst en dede: Daerom segghen wy noch naer luyt des voorstels, dat teghē den bodē E F, een ghewicht rust euen ande swaerheydt waters die euegroot is metten pilaer, wies grondt E F is, eñ hoochde de hanghende lini G E, van t'plat A B duer t'waters oppervlack M I, totten grondt E F.





III VERVOLCH.

Laet wederom A B C D t'eenemael water sijn, ende E F een bodem daer in euewydich vanden sichteinder. T'welck soo wesende, t'water onder den bodem E F, stoot euen soo stijf daer teghen opwaert, als t'water bouen den bodem E F, daer teghen neerwaert stoot: Want by aldient soo niet en waer, t'cranckste soude voor t'sterckste wycken, t'welck niet en ghebuert, want yder houdt sijn ghegheuen plaets duer het 1e voorstel. Laet nu eenighe styflichamen euestofswaer mettet water, alsoo gheleyt worden, dattet water I K E F L M, van onder anstoot teghen E F, als hier neuens. Dit soo sijnde, t'water onder den bodem E F, stoot nu so stijf teghen E F, dat is teghen t'stijflichaem,



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equal to the gravity of the water having the same volume as the prism whose base is *EF* and whose height is the vertical *GE*, from the plane *AB* through the water's upper surface *AN* to the base *EF*. From which it appears that if any floating substance is laid in the water, it does not weight or lighten the bottom (provided the water remain at the same level).

COROLLARY II.

Let there again be put in the water *ABCD* a solid body, or several solid bodies of equal specific gravity to the water. I take this to be done in such a way that the only water left is that enclosed by *IKFELM*. This being so, these bodies do not weight or lighten the base *EF* any more than the water first did. Therefore we still say, according to the proposition, that against the bottom *EF* there rests a weight equal to the gravity of the water having the same volume as the prism whose base is *EF* and whose height is the vertical *GE*, from the plane *AB* through the water's upper surface *MI* to the base *EF*.

COROLLARY III.

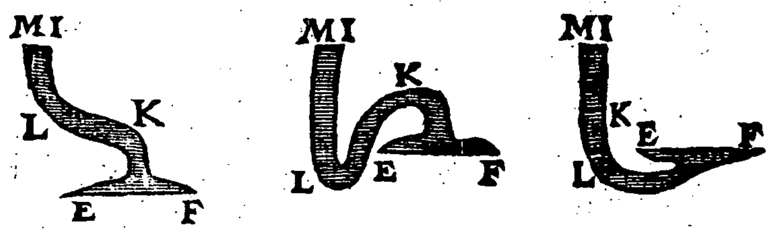
Let again *ABCD* be a water, and *EF* a bottom therein, parallel to the horizon. This being so, the water below the bottom *EF* exerts an upward thrust against it as great as the downward thrust which the water above the bottom *EF* exerts against it. For if this were not so, the weakest would give way to the strongest, which does not happen, for each keeps its appointed place, by the 1st proposition. Now let a number of solid bodies of equal specific gravity to the water be laid therein in such a way that the water *IKEFLM* thrusts against *EF* from below, as in the figure opposite. This being so, the water below the bottom *EF* exerts the same thrust against *EF*, i.e. against the solid body, as it did before against the alst te vooren teghen t'water dede; maer t'stac daer teghen soo stijf als t'bouenste teghen E F stiet, soo vooren gheseyt is, ende t'bouenste stiet teghen E F naer luydt deses voorstels, daerom t'onderste stoot oock teghen E F naer luydt deses voorstels, dat is soo wy bouen gheseyt hebben, dat teghen den bodem E F noch een ghewicht rust, euen ande swaerheyt waters die euegroot is metten pilaer, diens grondt E F is, ende hoochde de hanghende lini G E, van t'plat A B duer t'waters oppervlack M I totten grondt E F.

IIII VERVOLGH.

Laet ons nu de styflichamen des 2en ende 3en vervolghs tot haer plaets hechten, ende t'water uytghieten, ende daer sal een ledighe plaets I K F E L M blijuen, ende den grondt E F en sal gheen ghewicht draghen; waer uyt blijckt, dat met die cleyne ledighe plaets weder vol waters te ghieten, so salmen den grondt E F euen so seer beswaren, als of t'gheheele vat A B C D (de ingheleyde styflichamen gheweert sijnde) vol waters waer.

V VERVOLGH.

Maer anghesien de ingheleyde styflichamen des 2en ende 3en veruolghs t'haerder plaets ghehecht sijn, soo en gheeft noch en neemt haer uyterste stof tot de beswaring ofte verlichting des grondts E F, daerom laet ons de stof der seluer rondtom afcorten, alsoo datter blijuen de inwendighe ongheschickte formen oft vaten met water ghevult M I K F E L, als hier onder.



Ende sullen noch segghen naer luyt des voorstels, dat teghen den bodem E F een ghewicht rust, euen ande swaerheyt waters die euegroot is metten pilaer, wiens grondt E F is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack M I, totten grondt E F. Ende dit alsoo om de selue reden van alle ander formen diens bodems in een plat sijn euewydich vanden sichteinder. TBESLVYT. Op yder bodem dan des waters euewydich sijnde, &c.

Leest d'eruaringhen hier af breeder inden Anuang der Waterwichtdaet.

MERCKT

water. But it exerted against the latter the same thrust as the upper part against *EF*, as has been said above, and the upper part exerted a thrust against *EF* according to the present proposition. Therefore the lower part also exerts a thrust against *EF* according to the present proposition, that is, as we have said above, that against the bottom *EF* there still rests a weight equal to the gravity of the water having the same volume as the prism whose base is *EF* and whose height is the vertical *GE*, from the plane *AB* through the water's upper surface *MI* to the base *EF*.

COROLLARY IV.

Let us now put the solid bodies of the 2nd and the 3rd corollary in their places, and pour out the water. Then there will be left an empty space *IKFELM*, and the base *EF* will not bear any weight; from which is is apparent that by pouring that small empty space full of water again, the base *EF* will be weighted as much as if the whole vessel *ABCD* (the solid bodies laid therein being taken away) were full of water.

COROLLARY V.

But since the solid bodies of the 2nd and the 3rd corollaries are put in their places, their outward matter neither adds to nor subtracts from the weighting or lightening of the base *EF*. Therefore let us cut away the matter thereof all round, in such a way that the interior irregular forms or vessels filled with water, MIKFEL, are left, as shown below.

And we shall still say, according to the proposition, that against the bottom *EF* there rests a weight equal to the gravity of the water having the same volume as the prism, whose base is *EF* and whose height is the vertical from the plane through the water's upper surface *MI* to the base *EF*. And this applies for the same reason to any other forms whose bottoms are in a plane parallel to the horizon. CONCLUSION. On any bottom of the water therefore being parallel, etc.

Read the experiences hereof more amply in the Preamble of the Practice of Hydrostatics.

MERCKT. Wy souden t'boveschreuen 10e voorstel eyghentlicker aldus uytghesproken hebben:

OP yder bodem des waters in een weereltvlack sijnde, rust een ghewicht euen ande swaerheyt waters die euegroot is mettet clootsdeel begrepen tusschen den bodem ende tweereltvlack duer t'waters hoochste punt, ende t'vlack tusschen die twee vlacken, beschreuen met de oneindelicke rechte lini vast in t'weerelts middelpunt, ende ghedraeyt duer des bodems * omtreck. *Circumferentiam.*

Daer af bewysende sulcx als bouen bewesen is, maer om de redenen onder de 7e begheerte verclaert, soo ist beter ghelaten.

IX. VERTOOCH. XI. VOORSTEL.

WESENDE een ghefchickt bodem diens hoochste punt in t'waters oppervlack is: T'ghewicht daer teghen rustende is euen anden helft des pilaers waters, diens grondt euen an dien bodem is, ende hoochde, de * hanghende lini van des bodems hoochste punt, tot het * plat euewydich vandē * sichteinder duer des bodems leeghste punt. *Perpendicularis.* *Plano.* *Horizonte.*

1e VOORBEELT.

T'GHEGHEVEN. Laet A B een vat waters wesen, ende den bodem A C D E sy ten eersten een euewydich vierhouck, oneuewydich vanden sichteinder, daer op rechthouckich, diens hoochste sijde A C in t'waters oppervlack A C F G is, ende A E sy de hanghende lini van des bodems hoochste punt, tot het plat euewydich vanden sichteinder duer des bodems leeghste punt, dat is duer E D, ende A G sy so lanck alst valt. Laet oock de lini D B euewydich sijn vanden sichteinder, ende daer in gheteeckent H, alsoo dat D H euen sy an D G, oock ghetrocken worden C H, ende met A C H D E sy beteeckent den helft des pilaers diens grondt A C D E, ende hoochde D H euen an A E.

T'BEGHEERDE. Wy moeten bewysen dattet ghewicht waters teghen den bodem A C D E rustende, euen is anden voornoemden haluen pilaer A C H D E; Dat is (om t'selue opentlicker te verclaren) ghenomen dat

NOTE.

It would have been more appropriate to have worded the above 10th proposition as follows:

On any bottom of the water being in a world surface there rests a weight equal to the gravity of the water having the same volume as the part of a sphere comprehended by the bottom in question and the world surface through the highest point of the water, and the surface between these two surfaces, described by the infinite straight line fixed in the centre of the world, and revolved through the circumference of the bottom in question.

We might prove this as above, but for the reasons explained in the 7th postulate it is better to omit it.

THEOREM IX. PROPOSITION XI.

Given a regular bottom whose highest point is in the water's upper surface: the weight resting against it is equal to the half of the prism of water whose base is equal to that bottom and whose height is the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

EXAMPLE I.

SUPPOSITION. Let *AB* be a vessel with water, and the bottom *ACDE* shall first be a parallelogram [1]), not parallel to the horizon, but at right angles thereto, whose highest side *AC* is in the water's upper surface *ACFG,* and *AE* shall be the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, i.e. through *ED*, and *AG* shall have any given length. Let also the line *DB* be parallel to the horizon [2]), and let *H* be marked therein in such a way that *DH* shall be equal to *DC*, and let *CH* also be drawn, and *ACHDE* shall denote the half of the prism whose base is *ACDE* and whose height *DH* is equal to *AE.* WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom *ACDE* is equal to the aforesaid half prism *ACHDE;* i.e. (to explain it more clearly), assuming that *I* be an oblique weight [3]) of equal gravity to *ACHDE,* whose drawing line *KL* is parallel to *DH,* and that *K*

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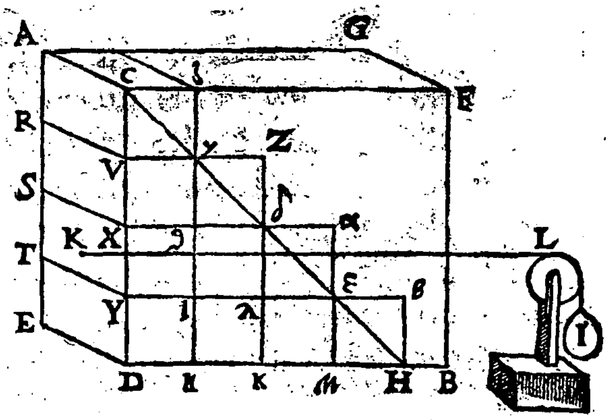
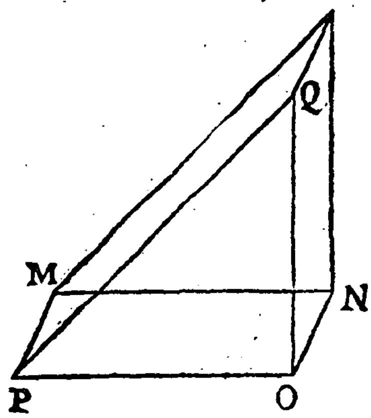
[1]) The subsequent reasoning is valid only if *ACDE* is a rectangle.
[2]) and at right angles to *AC*.
[3]) oblique, because the drawing line *KL* is not vertical.

men dat I een ſcheefwicht ſy, eueſwaer met A C H D E, diens trecklini K L, euewydich is met D H, ende dat K ſwaerheyts middelpunt ſy vande macht des ghe-prangs vergaert indé bodé(wiens middelputs vinding duer t'volghende 18e voorſtel bekent wört) t'ghewicht I ſtaet tegen t'gheprang des waters euewichtich, houdende dé bodem A C D E(ghenomen datſe beweeghlick waer) in die ſtandt.



Ofte tot meerder claerheyt, laet M N O P een bodem ſijn, euen ende ghelick an A C D E, te weten de ſijde M P * lijckſtandighe met A C, ende M N met A E, op welcken bodem M N O P, light een ſtijflichaem M N O P Q, euen, ghelijck, ende eueſwaer met den haluen pilaer A C H D E, ende de lini Q O euen an D H, ſy rechthouckich opden ſichteinder. Ick ſeg dat alſulcken gheprang als dat ſtijflichaem M N O P Q, doet teghen den bodem M N O P, te weten meer pranghende naer N O dan naer M P, om dattet aldaer dicker en ſwaerder is dan alhier, euen ſoodanighen gheprang doet t'water A B, oock teghen den bodem A C D E, meer pranghende naer E D dan naer A C.

Homologa.



T'BEREYTSEL. Laet de ſijde A E ghedeelt worden in vier euen deelen, met de punten R, S, T, eñ daer uyt ghetrocken worden R V, S X, T Y, euewydighe met A C; Laet oock ghetrocken worden V Z, χ α, Y ϛ euewydighe met D H, ende ſniende C H inde punten γ, δ, ε, ende alſo, dat yder der linien γ Z, δ α, ε ϛ, euen ſy an Y γ; Laet daernaer duer t'punt γ ghetrocken worden de lini ζ η, euewydighe met C D, ſniende χ α in τ, ende Y β in ι, ſghelijcx de lini Z κ duer δ, ſniende Y β in λ, ſghelijcx de lini α μ duer ε, ende ten laetſten ϛ H.

T'BEWYS. Teghen den bodem A C V R ruſt meer ghewichts dan niet, be centre of gravity of the force of the total pressure on the surface (the finding of whose centre of gravity becomes known from the 18th proposition hereinafter), the weight *I* is in equilibrium with the pressure of the water, keeping the bottom *ACDE* (assuming it to be movable) in that position.

Or, to make it clearer, let *MNOP* be a bottom, equal and similar to *ACDE*, i.e. the side *MP* being homologous to *AC*, and *MN* to *AE*, on which bottom *MNOP* lies a solid body *MNOPQ*, equal, similar, and of equal gravity to the half prism *ACHDE*, and the line *QO*, equal to *DH*, shall be at right angles to the horizon. I say that the same pressure as that exerted by the solid body *MNOPQ* against the bottom *MNOP*, to wit thrusting more heavily adjacent to *NO* than adjacent to *MP*, because it is thicker and heavier in the former place than in the latter, is also exerted by the water *AB* against the bottom *ACDE*, thrusting more heavily adjacent to *ED* than adjacent to *AC*. PRELIMINARY. Let the side *AE* be divided into four equal parts by the points *R*, *S*, *T*, and from these let there be drawn *RV*, *SX*, *TY* parallel to *AC*. Let there also be drawn *VZ*, $X\alpha$, $Y\beta$, parallel to *DH* and intersecting *CH* in the points γ, δ, ε and in such a way that each of the lines γZ, $\delta\alpha$, $\varepsilon\beta$ be equal to $V\gamma$. Thereafter let there be drawn through the point γ the line $\zeta\eta$, parallel to *CD*, intersecting $X\alpha$ in ϑ and $Y\beta$ in ι. In the same way the line $Z\kappa$ through δ, intersecting $Y\beta$ in λ, in the same way the line $\alpha\mu$ through ε, and finally βH. PROOF. Against the bottom *ACVR* there rests more weight than nothing, for if that bottom were in the niet, want waer dien bodem in t'waters oppervlack, soo souder niet teghen rusten, maer sy comt nu leegher, daer rust dan meer teghen als niet: Ten anderen seg ick datter min teghen rust dan t'lichaem waters A C ζ γ V R, want waer sy euewydich vanden sichteinder duer R V, so souder dat lichaem A C ζ γ V R teghen rusten, duer het 10^e voorstel, maer sy comt nu hoogher, daer rust dan min teghen. S'ghelijcx seg ick dat teghen den bodem R V X S, meer ghewichts rust dan des lichaems A C ζ γ V R, want waer dien bodem euewydich vanden sichteinder duer R V, daer soude dat lichaem teghen rusten duer het 10^e voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem R V γ ϑ X S is euen an t'lichaem A C ζ γ V R, daerom teghen den bodem R V X S, rust meer ghewichts dan des lichaems R V γ ϑ X S. Ten anderen seg ick datter min teghen rust dan t'lichaem A C ζ ϑ X S, want waer dien bodem euewydich vanden sichteinder duer S X, soo souder dat lichaem A C ζ τ X S teghen rusten duer het 10^e voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem R V Z δ X S is euen an t'lichaem A C ζ ϑ X S, daerom rust teghen den bodem R V X S min als t'lichaem R V Z δ X S. S'ghelijcx seg ick dat teghen den bodem S X Y T meer ghewichts rust dan des lichaems A C ζ ϑ X S, want waer dien bodem euewydich vanden sichteinder duer S X, daer soude dat lichaem teghen rusten duer het 10^e voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem S X δ λ Y T is euen an t'lichaem A C ζ ϑ X S, daerom teghen den bodem S X Y T rust meer ghewichts dan des lichaems S X δ λ Y T. Ten anderen seg ick datter min teghen rust dan t'lichaem A C ζ ι Y T, want waer dien bodem euewydich vanden sichteinder duer T Y, soo souder dat lichaem A C ζ ι Y T teghen rusten, duer het 10^e voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem S X α ε Y T is euen an t'lichaem A C ζ ι Y T, daerom rust teghen den bodem S X Y T min als t'lichaem S X α ε Y T. S'ghelijcx seg ick dat teghen den bodem T Y D E, meer ghewichts rust dan des lichaems A C ζ ι Y T, want waer dien bodem euewydich vanden sichteinder duer T Y, daer soude dat lichaem teghen rusten duer het 10^e voorstel, maer sy comt nu leegher, daer rust dan meer teghen, maer t'lichaem T Y ε μ D E is euen an t'lichaem A C ζ ι Y T, daerom teghen den bodem T Y D E rust meer ghewichts dan des lichaems T Y ε μ D E. Ten anderen seg ick datter min teghen rust dan t'lichaem A C ζ η D E, want waer dien bodem euewydich vanden sichteinder duer E D, so souder dat lichaem A C ζ η D E teghen rusten, duer het 10^e voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer t'lichaem T Y ϵ H D E, is euen an t'lichaem A C ζ η D E, daerom rust teghen den bodem T Y D E min als t'lichaem T Y ϵ H D E. Nu anghesien als vooren bewesen is, dat teghen den bodem A C V R meer rust dan niet, ende teghen

Dd

water's upper surface, nothing would rest against it; but it comes lower, so there rests more than nothing against it. On the other hand I say that there rests against it less than the body of water $AC\zeta\gamma VR$, for if it were parallel to the horizon through RV, the body $AC\zeta\gamma VR$ would rest against it, by the 10th proposition; but it comes higher, so there rests less against it. In the same way I say that there rests against the bottom $RVXS$ more weight than that of he body $AC\zeta\gamma VR$, for if that bottom were parallel to the horizon through RV, that body would rest against it by the 10th proposition; but it comes lower, so there rests more against it. But the body $RV\gamma\vartheta XS$ is equal to the body $AC\zeta\gamma VR$, so there rests against the bottom $RVXS$ more weight than that of the body $RV\gamma\vartheta XS$. On the other hand I say that there rests against it less than the body $AC\zeta\vartheta XS$, for if that bottom were parallel to the horizon through SX, that body $AC\zeta\vartheta XS$ would rest against it by the 10th proposition; but it comes higher, so there rests less against it. But the body $RVZ\delta XS$ is equal to the body $AC\zeta\vartheta XS$, so there rests against the bottom $RVXS$ less than the body $RVZ\delta XS$. In the same way I say that there rests against the bottom $SXYT$ more weight than that of the body $AC\zeta\vartheta XS$, for if that bottom were parallel to the horizon through SX, that body would rest against it by the 10th proposition. But it comes lower, so there rests more against it. But the body $SX\delta\lambda YT$ is equal to the body $AC\zeta\vartheta XS$, so there rests against the bottom $SXYT$ more weight than that of the body $SX\delta\lambda YT$. On the other hand I say that there rests less against it than the body $AC\zeta\iota YT$, for if that bottom were parallel to the horizon through TY, that body $AC\zeta\iota YT$ would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But the body $SX\alpha\varepsilon YT$ is equal to the body $AC\zeta\iota YT$, so there rests against the bottom $SXYT$ less than the body $SX\alpha\varepsilon YT$. In the same way I say that there rests against the bottom $TYDE$ more weight than that of the body $AC\zeta\iota YT$, for if that bottom were parallel to the horizon through TY, that body would rest against it, by the 10th proposition. But it comes lower, so there rests more against it. But the body $TY\varepsilon\mu DE$ is equal to the body $AC\zeta\iota YT$, so there rests against the bottom $TYDE$ more weight than that of the body $TY\varepsilon\mu DE$. On the other hand I say that there rests less against it than the body $AC\zeta\eta DE$, for if that bottom were parallel to the horizon through ED, that body $AC\zeta\eta DE$ would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But the body $TY\beta HDE$ is equal to the body $AC\zeta\eta DE$, so there rests against the bottom $TYDE$ less than the body $TY\beta HDE$. Now since, as has been proved above, against the bottom ACVR there rests more than nothing, and against the bottom $RVXS$ more than the body ghen den bodem R V X S meer als t'lichaem R V γ ϑ X S, ende teghen den bodem S X Y T meer dan t'lichaem S X δ λ Y T, ende teghen den bodem T Y D E meer als t'lichaem T Y ε μ D E, soo rust teghen den heelen bodem A C D E meer dan t'ghewicht van alle die lichamen t'samen, t'welck is t'binneschreuen lichaem R V γ ϑ δ λ ε μ D E inden haluen pilaer A C H D E: Tis oock bewesen dat teghen den bodem A C V R min rust dan t'lichaem A C ζ γ V R, ende teghen den bodem R V X S min als tlichaem R V Z δ X S, ende teghen den bodem S X Y T min dan t'lichaem S X α ε Y T, ende teghen den bodem T Y D E min als t'lichaem T Y ϛ H D E, daerom rust teghen den heelen bodem A C D E min dan t'ghewicht van alle die lichamen t'samen, dat is t'omschreuen lichaem A C ζ γ Z δ α ε ϛ H D E. Maer datmen nu den bodem A C D E welcke hier bouen ghedeelt is in vier euen deelen, alsoo deelde in acht euen deelen, tis kennelick dat het binneschreuen lichaem inden haluen pilaer A C H D E, ende het omschreuen, alsdan van dien haluen pilaer maer den helft soo veel verschillen en souden als sy nu doen, tis dan openbaer duer sulcke oneindelicke deeling des bodems, datter gheen ghewicht soo cleen ghegheuen en can worden, oft men sal bethoonen dattet verschil (sooder eenich waer) des ghewichts teghen den bodem A C D E rustende, tot het ghewicht des haluen pilaers A C H D E noch minder is, waer uyt ick aldus strije:

A. *Alle swaerheyt die min verschilt van t'ghewicht teghen den bodem A C D E rustende dan ghegheuen can worden, is euen mettet ghewicht teghen den bodem A C D E rustende;*

I. *T'ghewicht des haluen pilaers A C H D E, is een swaerheyt die min verschilt van t'ghewicht teghen den bodem A C D E rustende dan ghegheuen can worden;*

I. *T'ghewicht dan des haluen pilaers A C H D E, is euen mettet ghewicht teghen den bodem A C D E rustende.*

II^e VOORBEELT.

T'GHEGHEVEN. Laet A B andermael een vat waters wesen, ende den bodem A C D E sy een euewydich vierhouck des selfden, oneuewydich vanden sichteinder, ende daerop scheefhouckich, diens hoochste sijde A C in t'waters oppervlack A C F G is; T'selue water ende bodem sy alsoo ghedeelt ende gheteeckent als t'water des 1^en voorbeelts, ende A ν sy hanghende lini van des bodems hoochste sijde, tot het plat euewydich vanden sichteinder duer des bodems leegste sijde E D.

T'BEGHEERDE. Wy moeten bewysen dattet ghewicht waters teghen den bodem A C D E rustende, euen is anden helft des pilaers diens bodem A C D E, ende hoochde A ν.

T'BERBYT-

RV$\gamma\vartheta$*XS*, and against the bottom *SXYT* more than the body *SX*$\delta\lambda$*YT*, and against the bottom *TYDE* more than the body *TY*$\varepsilon\mu$*DE*, there rests against the whole bottom *ACDE* more than the weight of all those bodies together, which is the inscribed body *RV*$\gamma\vartheta\delta\lambda\varepsilon\mu$*DE* in the half prism *ACHDE*. It has also been proved that there rests against the bottom *ACVR* less than the body *AC*$\zeta\gamma$*VR*, and against the bottom *RVXS* less than the body *RVZ*δ*XS*, and against the bottom *SXYT* less than the body *SX*$\alpha\varepsilon$*YT*, and against the bottom *TYDE* less than the body *TY*β*HDE*. So there rests against the whole bottom *ACDE* less than the weight of all those bodies together, i.e. the circumscribed body *AC*$\zeta\gamma$*Z*$\delta\alpha\varepsilon\beta$*HDE*. But if the bottom *ACDE*, which is divided above into four equal parts, were thus divided into eight equal parts, it is evident that the inscribed body in the half prism *ACHDE*, and the circumscribed body, would differ from that half prism by only half as much as they do now. It is therefore manifest [1]), through this infinite division of the bottom, that no weight so small can be given but it can be shown that the difference (if there were any) between the weight resting against the bottom *ACDE* and the weight of the half prism *ACHDE* is even less, from which I argue as follows: [2])

A. *Any gravity which differs less from the weight resting against the bottom ACDE than can be given is equal to the weight resting against the bottom ACDE;*

I. *The weight of the half prism ACHDE is a gravity which differs less from the weight resting against the bottom ACDE than can be given;*

I. *Therefore the weight of the half prism ACHDE is equal to the weight resting against the bottom ACDE.*

EXAMPLE II.

SUPPOSITION. Let *AB* again be a vessel with water, and the bottom *ACDE* shall be a parallelogram [3]) therein, not parallel to the horizon, and at oblique angles thereto, whose highest side *AC* is in the water's upper surface *ACFG*. This water and bottom shall be divided and marked in the same way as the water of the 1st example, and *A*ν shall be the vertical from the highest side of the bottom to the plane parallel to the horizon through the lowest side *ED* of the bottom. WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom *ACDE* is equal to the half of the prism, whose base is *ACDE* and whose height is *A*ν. PRELIMINARY. Let the side *A*ν be divided

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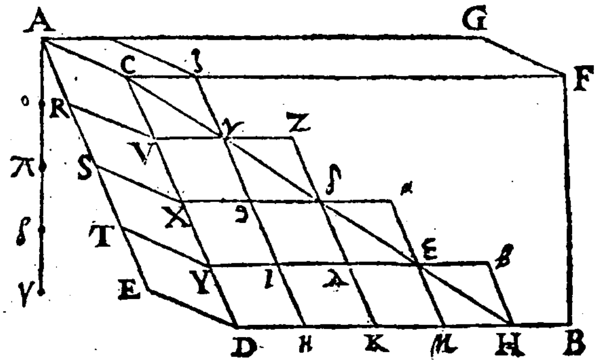
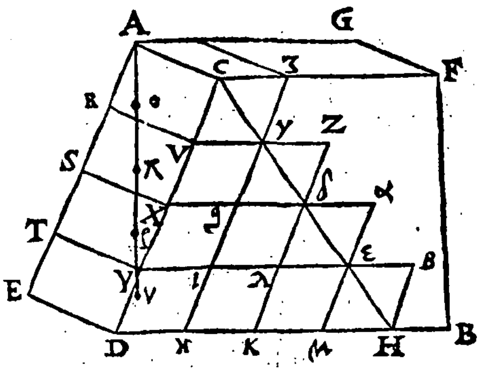
1) Euclid X 1, porism.
2) See note 2 to p. 143.
3) Again the parallelogram is supposed to be a rectangle.

T'BEREYTSEL. Laet de sijde A ν ghedeelt worden in vier euen deelen met de punten o,π,ρ. T'BEWYS. Teghen den bodem A C V R, rust meer ghewichts dã niet, want waer dien bodem in t'waters oppervlac, soo souder niet teghen rusten, maer sy comt nu leeger, daer rust dan meer teghen als niet: Ten anderen seg ick datter min teghẽ rust



dan dẽ pilaer diens gront A C V R is, ende hoochde A o, want waer sy euewydich vanden sichteinder duer R V, soo souder dien pilaer teghen rusten duer het 10e voorstel, maer sy comt nu hoogher, daer rust dan min teghen, maer A C ζ γ V R is euen an dien pilaer, daerom teghen dẽ bodem A C V R rust min ghewicht dan



31.v.11.B.E

des pilaers A C ζ γ V R. S'ghelijcx salmen oock al de rest bethoonen euen soo sy in t'eerste voorbeelt bewesen was, waer uyt besloten sal worden dattet ghewicht teghen den bodem A C D E rustende, euen is an t'lichaem A C H D E, maer dat lichaem is euen anden helft des pilaers diens bodem A C D E, ende hoochde A ν, (want A ν is euen ande hanghende van H rechthouckich op t'plat duer A C D E) Daerom t'ghewicht teghen den bodem A C D E rustende, is euen anden helft des pilaers waters diens grondt euen is an A C D E, ende hoochde A ν.

III. VOORBEELT.

T'GHEGHEVEN. Laet A B eenich gheschickt bodem sijn; Ick neem een * scheefrondt, diens hoochste punt A in t'waters oppervlack is, ende B sy t'leeghste punt, ende A C de hanghende lini van t'hoochste *Ellipsim.*

Dd 2 punt

into four equal parts by the points o, π, ρ. PROOF. Against the bottom *ACVR* there rests more weight than nothing, for if that bottom were in the water's upper surface, nothing would rest against it. But it comes lower, so there rests more than nothing against it. On the other hand I say that there rests less against it than the prism whose base is *ACVR* and whose height is Ao, for if it were parallel to the horizon through *RV*, that prism would rest against it, by the 10th proposition. But it comes higher, so there rests less against it. But $AC\zeta\gamma VR$ is equal to that prism, so there rests against the bottom *ACVR* less weight than that of the prism $AC\zeta\gamma VR$. In the same way all the rest shall be proved just as it was in the first example, from which it shall be concluded that the weight resting against the bottom *ACDE* is equal to the body *ACHDE*. But that body is equal to the half of the prism whose base is *ACDE* and whose height is $A\nu$ (for $A\nu$ is equal to the vertical [1]) from *H* at right angles to the plane through *ACDE*). Therefore the weight resting against the bottom *ACDE* is equal to the half of the prism of water whose base is equal to *ACDE* and whose height is $A\nu$.

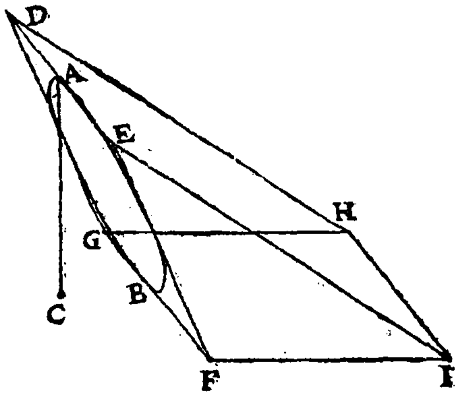
EXAMPLE III.

SUPPOSITION. Let *AB* be any regular bottom. I take it to be an ellipse, whose highest point *A* is in the water's upper surface, and *B* shall be the lowest point, and *AC* the vertical from the highest point *A* to the plane parallel to the horizon

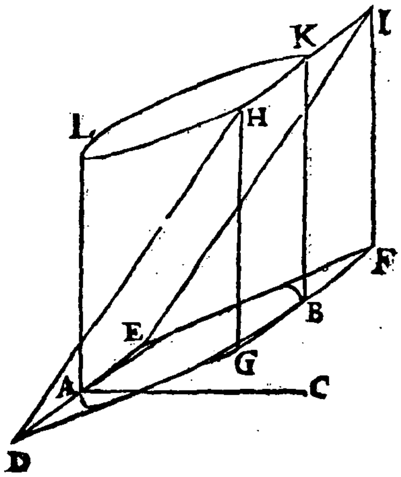
[1]) The use of the term ***vertical*** (hanghende) is perhaps to be explained by considering *ACDE* as horizontal.

punt A, tot het plat euewydich vanden sichteinder duer B.

T'BEGHEERDE. Wy moeten bewysen dattet ghewicht waters teghen den bodem A B rustende, euen is anden helft des pilaers diens grondt den bodem A B is, ende hoochde A C.



T'BEREYTSEL. Laet ghetrocken sijn een euewydich vierhouck D E F G, in wiens plat begrepē sy t'scheefrondt A B, alsoo dat D E in t'waters oppervlack sijnde, naecke an t'punt A, ende dat G F naecke an t'punt B; Laet daernaer ghetrocken worden F I euen an A F ende rechthouckich op F G maer euewydich vanden sichteinde, ende uyt G F ende F I sy beschreuen den rechthouck F G H I, voorts de linien E I ende D H.



Laet daer naer een ander form ghestelt sijn euen, ghelijck, ende euesvvaer met de voorgaende, maer alsoo dat F I rechthouckich sy opden sichteinder ghelijck hier neuen. Ende laet in dese tweede form t'lichaem D E F G H I een stijflichaem wesen, rustende opden bodem D E F G.

T'BEWYS. Alsulcken drucksel als t'stijflichaem D E F G H I der tweeder form, veroirsaeckt teghen den bodem D E F G, euen soodanigen drucksel veroirsaeckt het water des eersten forms teghen sijn bodem D E F G, soo bouen bewesen is, ende veruolghens alsulcken drucksel allser valt teghen t'scheefrondt A B der tweede form, euen soodanighen drucksel valter oock teghen t'scheefrondt A B der eerste form, maer het drucksel op t'scheefrondt der tweede form is den helft des pilaers (soo wy hier onder verclaren sullen) diens grondt dat scheefrondt is, ende hoochde euen an A C (want ghetrocken een hanghende lini van K rechthouckich op t'plat duer t'scheefrondt A B, sy is euen through *B*. WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom *AB* is equal to the half of the prism whose base is the bottom *AB* and whose height is *AC*. PRELIMINARY. Let there be drawn a parallelogram [1]) *DEFG*, in whose plane shall be comprehended the ellipse *AB* in such a way that *DE*, being in the water's upper surface, shall touch at the point *A* and *GF* shall touch at the point *B*. Thereafter let there be drawn *FI*, equal to *EF* and at right angles to *FG*, but parallel to the horizon, and from *GF* and *FI* there shall be drawn the rectangle *FGHI*, further the lines *EI* and *DH*.

Thereafter let there be drawn another figure, equal, similar, and of equal gravity to the preceding one, but such that *FI* shall be at right angles to the horizon, as shown opposite. And in this second form let the body *DEFGHI* be a solid body resting on the bottom *DEFG*. PROOF. The same pressure as is exerted by the solid body *DEFGHI* of the second figure against the bottom *DEFG* is exerted by the water of the first figure against its bottom *DEFG*, as has been proved above, and consequently the same pressure as is exerted against the ellipse *AB* of the second figure is also exerted against the ellipse *AB* of the first figure. But the pressure on the ellipse of the second figure is the half of the prism (as we shall explain below) whose base is that ellipse and whose height is equal to *AC* (for if a vertical [2]) be drawn from *K* at right angles to the plane through

[1]) rectangle.
[2]) See note 1 to p. 429.

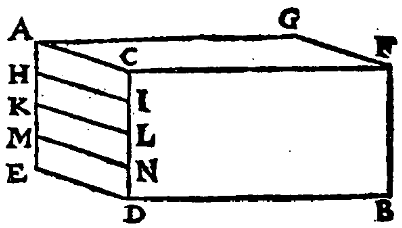
euen an A C) daerom het druckfel des waters teghen t'fcheefrondt A B der eerfte form, is euen anden helft des pilaers wiens grondt dat fcheefrondt is, ende hoochde A C.

Maer dattet ghewicht ruftende in defe tweede form teghen t'fcheefrondt A B, euen is anden helft des pilaers diens grondt dat fcheefrondt is, ende hoochde euen an A C, wort aldus bethoont: Laet ghetrocken worden de lini B K euen ende euewydighe met F I; Laet nu het onderfte B der feluer lini B K, ghedraeyt worden inden omtreck des fcheefrondts A B, tot dat fy weder ter plaets comt daer fy begon te roeren, ende blijuende int roeren altijdt euewydich van F I, de felue fal tuffchen de twee bodems een pilaer A B K L befchrijuen, welcke mettet plat D E I H, ghefneen wort duer twee lijckftandighe punten A, K, fchuens teghen ouer malcander ftaende inde omtrecken der bodems; Maer alle pilaer diens grondt een ghefchickt bodem fijnde, ghefneen wort met een plat duer twee lijckftandighe punten inde omtrecken der bodems fchuens teghen ouer malcander ftaende, die pilaer wort van dat plat in twee euen deelen ghedeelt, daerom het deel diens pilaers onder t'plat D E I H, is den helft des heelen pilaers A B K L ruftende op t'fcheefrondt A B, maer dat den pilaer A B K L euen is anden pilaer diens grondt A B ende hoochde A C, blijckt daer an, dat fijn hoochde euen is an A C, daerom t'ghewicht ruftende teghen t'fcheefrondt A B, is euen anden helft des pilaers diens grondt dat fcheefrondt is, ende hoochde euen an A C.

IIII^e VOORBEELT.

Wy heben hier bouen drie voorbeelden ghegheuen met * Wifconftich bewys, t'welck, wel is waer, den grondt volcommentlicker verclaert als ander; doch anghefien t'bewys duer ghetalen tot opentlicker kennis van alles niet en verachtert, fullen dit 4^e voorbeelt duer ghetalen ftellen. *Mathematica demonstratione.*

TGHEGHEVEN. Laet A B een vat waters fijn, diens bodem wy nemen te wefen een rechthouckich vierhouck, rechthouckich opden fichteinder, ende d'hoochfte fijde A C doende een voet, fy int waters oppervlack A C F G, ende A E doe oock een voet, maer A G fy foo lanck alft valt. TBEGHEERDE. Wy moeten duer ghetalen bewyfen, dattet ghewicht waters ruftende teghen den bodem A C D E, euen is anden helft des pilaers waters, wiens grondt euen is an dien bodem, ende hoochde de hanghende lini A E: Maer dien pilaer is een teerlinck doende een voet, wy moeten dan bethoonen dat teghen den bodem A C D E ruft



the ellipse *AB*, it is equal to *AC*). Therefore the pressure of the water against the ellipse *AB* of the first figure is equal to the half of the prism whose base is that ellipse and whose height is *AC*.

But that the weight resting in this second figure against the ellipse *AB* is equal to the half of the prism whose base is that ellipse and whose height is equal to *AC*, is shown as follows: Let there be drawn the line *BK*, equal and parallel to *FI*. Now let the lowest point *B* of this line *BK* be revolved in the circumference of the ellipse *AB* until it reaches again the place from which its motion started, and if during the motion it always remain parallel to *FI*, it will describe between the two surfaces a prism *ABKL*, which is cut by the plane *DEIH* in two homologous points *A*, *K*, diametrically opposite to each other in the circumferences of the bases. But if any prism whose base is a regular surface is cut by a plane in two homologous points in the circumferences of the bases, diametrically opposite to each other, it is divided by that plane into two equal parts. Therefore the part of that prism below the plane *DEIH* is the half of the whole prism *ABKL* resting on the ellipse *AB*. But that the prism *ABKL* is equal to the prism whose base is *AB* and whose height is *AC*, is evident from the fact that its height is equal to *AC*. Therefore the weight resting against the ellipse *AB* is equal to the half of the prism whose base is that ellipse and whose height is equal to *AC*.

EXAMPLE IV.

We have given above three examples with a mathematical proof, which indeed explains the cause more perfectly than any other, but since there is no harm in giving the proof by means of numbers in order to make everything clearer, we shall give this 4th example by means of numbers.

SUPPOSITION. Let *AB* be a vessel of water, the bottom of which we take to be a right-angled quadrilateral, at right angles to the horizon, and the highest side *AC*, being one foot, shall be in the water's upper surface *ACFG*, and *AE* shall also be one foot, but *AG* shall have any length. WHAT IS REQUIRED TO PROVE. We have to prove by means of numbers that the weight of the water resting against the bottom *ACDE* is equal to the half of the prism of water whose base is equal to that bottom and whose height is the vertical *AE*. But that prism is a cube of one foot. We therefore have to show that against the bottom het ghewicht van een halue voet waters. T'BEREYTSEL. Laet duer den bodem ghetrocken worden drie euewydighe linien met A C als H I, K L, M N, alſoo dat A H euen ſy an H K, ende an K M, ende an M E.

T'BEWYS. Tis blijckelick dat op den bodem A I meer ruſt dan o, want alwaer ſulcken bodem duer A C euewydich vanden ſichteinder, ſo ſouder o, op ruſten, maer ſy comt nu leegher daer ruſt dan meer op als o. Ten anderen ſeg ick datter min op ruſt dan $\frac{1}{16}$ voets, want al waer ſulcken bodem duer H I euewydich vanden ſichteinder, ſoo ſouder $\frac{1}{16}$ voets op ruſten, maer ſy comt nu hoogher, daer ruſt dan min op als $\frac{1}{16}$, ende om der ghelijcke reden iſt oock openbaer, dat opden bodem H L meer ruſt dan $\frac{1}{16}$, ende min als $\frac{2}{16}$, ende op den bodem K N meer dan $\frac{2}{16}$, ende min als $\frac{3}{16}$; maer op den bodem M D meer dan $\frac{3}{16}$ ende min als $\frac{4}{16}$. Nu dan vergaert de vier ghewichten (ghenomen dat o ghewicht waer) die lichter ſijn d'ander op elcken bodem ruſt, als o. $\frac{1}{16}$. $\frac{2}{16}$. $\frac{3}{16}$. maken t'ſamen $\frac{6}{16}$: Inſghelijcx vergaert de vier ghewichten die ſwaerder ſijn d'ander op elcken bodem ruſt, als $\frac{1}{16}$. $\frac{2}{16}$. $\frac{3}{16}$. $\frac{4}{16}$. maken t'ſamen $\frac{10}{16}$: Tis dan openbaer dat opden heelen bodem A C D E meer ruſt dan $\frac{6}{16}$ voets, ende min als $\frac{10}{16}$ voets, tuſſchen welcke twee den $\frac{1}{2}$ voet is, die wy noch bewyſen moeten opden bodem A C D E te ruſten.

Nu ghelijck den bodem hier bouen duer de drie euewydighe linien ghedeelt is in vieren, alſo mueghen wyſe deelen in ſoo veel deelen alſt ons belieſt, latet ſijn in thienen, eñ om de voorgaende redenen, de thien ghewichten die lichter ſijn d'ander op elcken bodem ruſt, ſullen ſijn o. $\frac{1}{100}$. $\frac{2}{100}$. $\frac{3}{100}$. $\frac{4}{100}$. $\frac{5}{100}$. $\frac{6}{100}$. $\frac{7}{100}$. $\frac{8}{100}$. $\frac{9}{100}$. t'ſamen $\frac{45}{100}$. ſ'ghelijcx de thien ghewichten die ſwaerder ſijn d'ander op elcken bodem ruſt, als $\frac{1}{100}$. $\frac{2}{100}$. $\frac{3}{100}$. $\frac{4}{100}$. $\frac{5}{100}$. $\frac{6}{100}$. $\frac{7}{100}$. $\frac{8}{100}$. $\frac{9}{100}$. $\frac{10}{100}$. maken t'ſamen $\frac{55}{100}$; tis dan kennelick dat opdẽ bodem A C D E meer ruſt dan $\frac{45}{100}$ voets, eñ min als $\frac{55}{100}$ voets, tuſſchen welcke twee den haluen voet is die wy noch
Termini. bewyſen moeten opden bodem A C D E te ruſten: Maer deſe twee *palen ſijn naerder den haluen voet dan d'eerſte twee, want min verſchilt $\frac{45}{100}$ van $\frac{1}{2}$, dan $\frac{6}{16}$, alſoo oock verſchilt $\frac{55}{100}$ min van $\frac{1}{2}$, dan $\frac{10}{16}$; Waer uyt blijckt dat hoe wy den bodem A C D E in meer ſulcke euen deelen ſnien, hoe dat wy den haluen voet altijdt naerder commen.

T'welck ſo verſtaen ſijnde, laet opden bodem A C D E min of meer ruſten $\frac{1}{1000}$ voets (waert mueghelick) dan een halue voet, ende laet ons de waerheyt daeraf onderſoucken, deelende den boden duer de ghedacht in 1000 euen deelen alſvooren. Ende om de voorgaende redenen, de duyſent ghewichten die lichter ſijn dander op elcken bodem ruſt, ſullen ſijn o, $\frac{1}{1000000}$. $\frac{2}{1000000}$ ende ſo voorts tot het laetſte, dat ſijn ſal van $\frac{999}{1000000}$, alle welcke ghetalen t'ſamen, ſullen maken (wiens corte manier om te vergaren wy hier onder verhalen ſullen) $\frac{499500}{1000000}$. *Sghelijcx* de duyſent ghewichten die ſwaerder ſijn d'ander op elcken bodem ruſt

als

ACDE there rests the weight of half a foot of water. PRELIMINARY. Let there be drawn through the bottom three lines parallel to *AC*, as *HI*, *KL*, *MN*, in such a way that *AH* be equal to *HK*, and to *KM*, and to *ME*. PROOF. It is evident that on the bottom *AI* there rests more than 0, for if this bottom were parallel to the horizon through *AC*, 0 would rest thereon; but it comes lower, so there rests more than 0 on it. On the other hand I say that there rests on it less than $\frac{1}{16}$ foot, for if this bottom were parallel to the horizon through *HI*, $\frac{1}{16}$ foot would rest on it. But it comes higher, so there rests less than $\frac{1}{16}$ on it. And for similar reasons it is also manifest than on the bottom *HL* there rests more than $\frac{1}{16}$ and less than $\frac{2}{16}$, and on the bottom *KN* more than $\frac{2}{16}$ and less than $\frac{3}{16}$; but on the bottom *MD* more than $\frac{3}{16}$ and less than $\frac{4}{16}$. Now adding together the four weights (assuming 0 to be a weight) which are lighter than what rests on each bottom, viz. 0, $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, this makes $\frac{6}{16}$. Adding together likewise the four weights which are heavier than what rests on each bottom, viz. $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$, $\frac{4}{16}$, this makes $\frac{10}{16}$. It is therefore manifest than on the whole bottom *ACDE* there rests more than $\frac{6}{16}$ foot, and less than $\frac{10}{16}$ foot, between which two is the $\frac{1}{2}$ foot which we still have to prove rests on the bottom *ACDE*.

Now just as the bottom above is divided into four parts by the three parallel lines, we may divide it into as many parts as we like, say into ten parts. Then, for the above reasons, the ten weights which are lighter than what rests on each bottom will be 0, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, together $\frac{45}{100}$. In the same way, the ten weights which are heavier than what rests on each bottom, viz. $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{10}{100}$, making together $\frac{55}{100}$. It is therefore evident that on the bottom *ACDE* there rests more than $\frac{45}{100}$ foot and less than $\frac{55}{100}$ foot, between which two is the half foot which we still have to prove rests on the bottom *ACDE*. But these two terms are nearer to the half foot than the first two, for $\frac{45}{100}$ differs less from $\frac{1}{2}$ than $\frac{6}{16}$, and likewise $\frac{55}{100}$ differs less from $\frac{1}{2}$ than $\frac{10}{16}$. From which it is evident that the greater the number of such equal parts into which we divide the bottom *ACDE*, the more we shall approximate to the half foot.

This being grasped, let there rest on the bottom *ACDE* $\frac{1}{1000}$ foot (if this were possible) less or more than one half foot, and let us examine the truth thereof, dividing the bottom in thought into 1000 equal parts as before. Then, for the above reasons, the thousand weights which are lighter than what rests on each bottom will be 0, $\frac{1}{1,000,000}$, $\frac{2}{1,000,000}$, and so on to the last, which will be $\frac{999}{1,000,000}$, all of which numbers together will make (the short method for adding up such numbers is to be related below) $\frac{499,500}{1,000,000}$. In the same way the thousand weights als $\frac{1}{1000000}$, $\frac{2}{1000000}$, $\frac{3}{1000000}$, ende ſo voorts tot het laetſte, dat ſijn ſal van $\frac{1000}{1000000}$, maken t'ſamen $\frac{500500}{1000000}$, daer ruſt dan meer opden bodē als $\frac{499500}{1000000}$ voets, ende min dan $\frac{500500}{1000000}$ voets; Maer $\frac{499500}{1000000}$ en is maer $\frac{1}{2000}$ minder dan $\frac{1}{2}$, daer en ruſt dan gheen $\frac{1}{1000}$ voets min opden bodem dan $\frac{1}{2}$ voet. Alſoo en is $\frac{500500}{1000000}$ maer $\frac{1}{2000}$ meerder dan $\frac{1}{2}$. daer en ruſt dan gheen $\frac{1}{1000}$ meer op den bodem dan $\frac{1}{2}$ voet. Ende alſoo ſalmen dierghelijcke bethoonen ouer alle gheſtelt deel hoe cleen het ſy. Het blijckt dan, dat het verſchil (ſooder eenich waer) tuſſchen t'water opden bodem A C D E ruſtende, ende een halue voet waters, minder ſoude moeten ſijn dan mueghelick is gheſtelt te worden, waer uyt ick aldus ſtrije:

A. *Neuen yder ghewicht dat met een halue voet waters verſchil heeft, can een ghewicht gheſtelt worden daer af min verſchillende;*

O. *Neuen t'ghewicht waters opden bodem A C D E ruſtende, en can gheen ghewicht gheſtelt worden van een halue voet waters min verſchillende;*

O. *T'ghewicht waters dan opden bodem A C D E ruſtende, een heeft met een halue voet waters gheen verſchil.*

TBESLVYT. Weſende dan een gheſchict bodem diens hoochſte punt int waters, &c.

DE reden waerom het half hier bouen, altijt blijft tuſſchen de twee ghetalen, welcke an het half oneindelick naerderen, maer nummermeer daer toe en gheraken, is begrepen in ſulcken * vertooch: *Theoremate.*

WESENDE een voortganck van ghetalen malcanderen in eenheydt te bouen gaende, ende beghinnende vande eenheyt. Den helft des viercants van t'laetſte, is meerder dan de ſomme van al de ghetalen, maer minder dan de ſomme van al de ghetalen min t'laetſte.

MAER om te verclaren (als bouen belooft is) de manier om duer cortheyt te vergaren die groote menichte der ghetalen; Soo is ten eerſten kennelick, dat haer noemers al euen ſijn, waerduer wy alleenelick op der ghetalen telders te letten hebben, de ſelue ſijn in oirdentlicke * voortganck beghinnende vande * eenheyt, ende met eenheyt malcanderen te bouengaende, daerom vermenichvuldicht t'laetſte met ſijn helft, ende an t'uytbreng noch ghevoucht ſijn helft, gheeft de begheerde ſomme. By voorbeelt ick wil weten hoe veel de ſomme is van 1, 2, 3, 4, 5, 6; Ick ſeg 6 mael 3 is 18, met 3 maeckt 21 voor de begheerde ſomme *Progreſſione.* *Vnitate.*

which are heavier than what rests on each bottom, viz. $\frac{1}{1,000,000}$, $\frac{2}{1,000,000}$, $\frac{3}{1,000,000}$, and so on to the last, which will be $\frac{1,000}{1,000,000}$, make together $\frac{500,500}{1,000,000}$. So there rests on the bottom more than $\frac{499,500}{1,000,000}$ foot and less than $\frac{500,500}{1,000,000}$ foot. But $\frac{499,500}{1,000,000}$ is only $\frac{1}{2,000}$ less than $\frac{1}{2}$; so there does not rest $\frac{1}{1,000}$ foot less than $\frac{1}{2}$ foot on the bottom. In the same way $\frac{500,500}{1,000,000}$ is only $\frac{1}{2,000}$ more than $\frac{1}{2}$, so there does not rest $\frac{1}{1,000}$ more than $\frac{1}{2}$ foot on the bottom. And in the same way such a thing can be proved with regard to any given part, however small. It therefore appears that the difference (if there were any) between the water resting on the bottom *ACDE* and half a foot of water would have to be less than is possible to be assumed, from which I argue as follows: 1)

A. *Beside any weight differing from half a foot of water there can be placed a weight differing less therefrom;*

O. *Beside the weight of the water resting on the bottom ACDE there cannot be placed any weight differing less from half a foot of water;*

O. *Therefore the weight of the water resting on the bottom ACDE does not differ from half a foot of water.*

CONCLUSION. Given therefore a regular bottom whose highest point is in the water's upper surface, etc.

The reason why the half referred to above always remains between the two numbers infinitely approximating to the half without ever reaching it, is given in the following theorem:

Given a progression of numbers exceeding one another by unity and starting from unity: the half of the square of the last number is less than the sum of all the numbers, but more than the sum of all the numbers minus the last.

Now in order to explain (as promised above) the short method for adding up that great multitude of numbers, it is firstly evident that their denominators are all equal, so that we only have to heed the numerators of the numbers; these are in a regular progression, starting from unity and exceeding one another by unity, therefore when the last number is multiplied by its half, and to the product is added the half thereof, the required sum is obtained. For example, I wish to know what is the sum of 1, 2, 3, 4, 5, 6. I say: 6 times 3 are 18, plus 3 makes 21

1) See note 2 to p. 143.

ſomme. Laet het laetſte nu oneuen ghetal ſijn, als 1, 2, 3, 4, 5, 6, 7; Ick ſeg 7 mael $3\frac{1}{2}$ is $24\frac{1}{2}$, met $3\frac{1}{2}$ maeckt 28, voor de begheerde ſomme. Maer als t'laetſte aldus oneuen is, ſoo vallet lichter om duer gheen ghebroken te wercken, datmen t'laeſte menichvuldicht duer den helft der ſomme van t'laetſte met 1, als andermael willende weten de ſomme van 1, 2, 3, 4, 5, 6, 7; Ick doe 1 tot 7, maeckt 8, ſijn helft is 4, die vermenichuuldicht duer 7, comt alſbouen 28, voor de begheerde ſomme, ende alſoo met allen anderen.

MERCKT.

Angheſien de boueſchreuen helft des pilaers euen is anden heelen pilaer diens grondt den ghegeuen boden is, ende hoochde den helft der hanghende lini van des bodems hoochſte punt, tottet plat euewydich vanden ſichteinder duer des bodems leeghſte punt, men ſoude t'boueſchreuen 11e voorſtel oock mueghen aldus uyten:

WESENDE een gheſchickt bodem diens hoochſte punt in t'waters oppervlack is: T'ghewicht daer tegheu ruſtende is euen anden pilaer waters diens grondt euen an dien bodem is, ende hoochde den helft der hanghende lini van des bodems hoochſte punt, tottet plat euewydich vanden ſichteinder duer des bodems leeghſte punt.

Ende na ſulcke wyſe ſullen wy t'laeſte deel deſes 12en voorſtels formen.

X. VERTOOCH. XII. VOORSTEL.

WESENDE een gheſchickt bodem diens hoochſte punt onder t'waters oppervlack is: T'ghewicht daer teghen ruſtende is euen anden pilaer waters diens grondt euen is an dien bodem, ende hoochde de * hanghende lini van * t'plat duer t'waters oppervlack, tot des bodems hoochſte punt, ende bouen dien den helft der hanghende lini van des bodems hoochſte punt, tottet plat euewydich vanden * ſichteinder duer des bodems leeghſte punt.

Perpendicularis. *Plano.* *Horizonte.*

I VOORBEELT.

T'GHEGHEVEN. Laet A B C D een gheſchickt bodem ſijn, als ten eerſten for the required sum. Now let the last be an odd number, viz. 1, 2, 3, 4, 5, 6, 7. I say: 7 times $3\frac{1}{2}$ are $24\frac{1}{2}$, plus $3\frac{1}{2}$ makes 28 for the required sum. But if the last is thus an odd number, it is easier, in order not to operate with fractions, to multiply the last by the half of the sum of the last and 1. Thus, if again I wish to know the sum of 1, 2, 3, 4, 5, 6, 7, I add 1 to 7, which makes 8; the half of that is 4, which when multiplied by 7 makes 28, as above, for the required sum. And the same with all others [1].

NOTE.

Since the above-mentioned half of the prism is equal to the complete prism whose base is the given bottom and whose height is the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, the 11th proposition described above might also be worded as follows:

Given a regular bottom whose highest point is in the water's upper surface: the weight resting against it is equal to the prism of water whose base is equal to that bottom and whose height is the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

And in this way we will word the last part of the 12th proposition.

THEOREM X. PROPOSITION XII.

Given a regular bottom whose highest point is below the water's upper surface: the weight resting against it is equal to the prism of water whose base is equal to that bottom and whose height is the vertical from the plane through the water's upper surface to the highest point of the bottom, and in addition thereto the half of the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom.

EXAMPLE I.

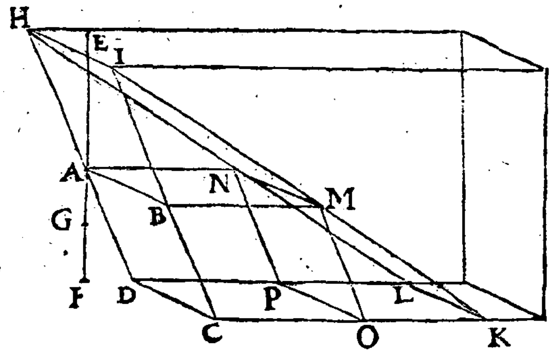
SUPPOSITION. Let *ABCD* be a regular bottom, viz. first a parallelogram [2])

[1]) Obviously Stevin here applies one of the numerous rules of arithmetic, which were communicated as recipes, without any demonstration.

[2]) rectangle.

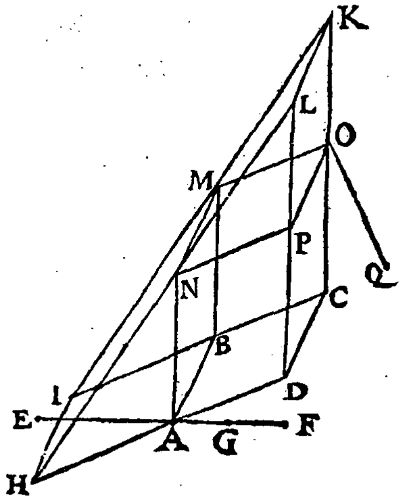
eersten een euewydich vierhouck diens hoochste sijde A B onder t'waters oppervlack is, euewydich neem ick, vanden sichteinder, ende E A sy de hanghende lini van t'waters oppervlack, tot des bodem hoochste punt A, ende A F de hanghende lini van A, tot het plat euewydich Vanden sichteinder duer D C, ende A G sy den helft van A F.

T'BEGHEERDE. Wy moeten bewysen dattet ghewicht waters teghen den bodem A B C D rustende, euen is anden pilaer diens grondt dien bodem is eñ hoochde G E.



T'BEREYTSEL. Laet D A ende C B voortghetrockẽ worden tot H ende I, beyde in t'waters oppervlack; laet oock ghetrocken sijn H I, daer naer C K euewydich vanden sichteinder, ende euen an C I, maer rechthouckich op D C, sghelijcx D L euen ende euewydighe met C K, voort L K, daer naer I K ende H L, voort B M euewydighe met C K, ende also dat M inde lini I K sy, daer naer A N euen ende euewydighe met B M, voort M O ende N P beyde euen ende euewydighe met B C.

Laet daer na een ander form ghestelt worden, euen, ghelijck, eñ euewichtich mettet water euen ande voorgaende C D H I K L, maer alſo dat C K rechthouckich sy opdẽ sichteinder als hier neuen.



T'BEWYS. Alsulcken druckſel als t'stijflichaem C D H I K L der tweede form, veroirsaeckt teghen den bodem C D H I, euen ſoodanigen druckſel veroirsaeckt t'water des eersten forms teghen sijn bodem C D H I ſo bewesen is int 11ᵉ voorstel ende veruolghens ſulcken druckſel alsser valt teghen het deel A B C D der tweede

Ee form,

whose highest side *AB* is below the water's upper surface, parallel — I assume — to the horizon, and *EA* shall be the vertical from the water's upper surface to the highest point *A* of the bottom, and *AF* the vertical from *A* to the plane parallel to the horizon through *DC*, and *AG* shall be the half of *AF*. WHAT IS REQUIRED TO PROVE. We have to prove that the weight of the water resting against the bottom *ABCD* is equal to the prism whose base is that bottom and whose height is *GE*. PRELIMINARY. Let *DA* and *CB* be produced to *H* and *I*, both in the water's upper surface. Let there also be drawn *HI*, thereafter *CK* parallel to the horizon and equal to *CI*, but at right angles to *DC*; in the same way *DL* equal and parallel to *CK*, further *LK*; thereafter *IK* and *HL*, further *BM* parallel to *CK* and in such a way that *M* shall be on the line *IK*. Thereafter *AN* equal and parallel to *BM*, further *MO* and *NP* both equal and parallel to *BC*.

Thereafter let there be drawn another figure, equal, similar, and of equal weight to the water having the same volume as the above *CDHIKL*, but in such a way that *CK* be at right angles to the horizon, as shown opposite. PROOF. The same pressure as is exerted by the solid body *CDHIKL* of the second figure against the bottom *CDHI* is also exerted by the water of the first figure against its bottom *CDHI*, as has been proved in the 11th proposition. And consequently the same pressure as is exerted against the part *ABCD* of the second figure is also exerted form, euen soodanighen druckſel valter oock teghen het deel A B C D der eerſte form, maer het druckſel teghen A B C D der tweede form is t'lichaem A B C D L K M N, t'welck euen is anden pilaer diens bodem A B C D ende hoochde G E, ghelijck wy terſtont ſegghen ſullen, daerom t'ghewicht des waters ruſtende teghen A B C D der eerſte form, is euen anden pilaer diens grondt A B C D, ende hoochde G E. Maer dattet lichaem A B C D L K N M euen is anden pilaer diens bodē A B C D ende hoochde G E blijckt aldus: Ghetrocken O Q rechthouckich op t'plat duer A B C D, de ſelfde O Q is d'hoochde des pilaers A B C D P O M N, daerom dat lichaem is euen anden pilaer diens grondt A B C D ende hoochde O Q: Maer anghesien A H euen is an O C, ende den houck H A E euen anden houck C O Q, ende dat A E rechthouckich is op t'plat duer de punten H, E, ſghelijcx O Q rechthouckich op t'plat duer de punten C, Q, ſoo is A E euen an O Q, daerom t'lichaem A B C D P O M N is euen anden pilaer diens grondt A B C D, ende hoochde A E. Maer t'lichaem M N P O K L is euen anden pilaer diens grondt A B C D ende hoochde A G duer t'vervolg des 11^{en} voorſtels, daerom die twee lichaemen makende t'ſamen t'lichaem A B C D L K N M, ſijn euen anden pilaer diens bodem A B C D ende hoochde G E.

ANDER BEWYS.

Ghenomen datter in t'water der eerſte form hier bouen een bodem ſy, euen ende ghelijck met A B C D, maer euewydich vanden ſichteinder in t'plat daer A B in is: Teghen den ſeluen bodem ſal ruſten t'ghewicht euen anden pilaer waters diens grondt euen is an A B C D, ende hoochde A E duer het 10^{e} voorſtel, t'ſelue ghewicht ruſt oock teghen alle bodem die euen is an dien bodem ende leegher; Daer ruſt dan voor al teghen A B C D, een pilaer diens grondt euen is an A B C D, ende hoochde A E: Nu gheweert al t'water datter bouen den voornomden bodem is, die wy euen ſtelden an A B C D, alſoo dat A B in t'waters oppervlack ſy ſoo ruſter teghen A B C D duer t'veruolg des 11^{en} voorſtels, den pilaer diens grondt euen is an A B C D, ende hoochde A G, welcke twee pilaren maken t'ſamen den pilaer diens grondt A B C D, ende hoochde E G, voor t'ghewicht ruſtende teghen den bodem A B C D als vooren.

II VOORBEELT.

Laet A B eenich gheſchickt bodem weſen, diens hoochſte punt onder t'waters oppervlack ſijnde, is A, ende t'leeghſte B, ende de hanghende lini van t'waters oppervlack tot des bodems hoochſte punt ſy C A, ende van des bodems hoochſte punt tot het plat euewydich vandē ſichteinder duer des bodems leeghſte punt B, ſy A D, diens helft A E. Ick ſeg dattet ghewicht against the part *ABCD* of the first figure. But the pressure against *ABCD* of the second figure is the body *ABCDLKMN*, which is equal to the prism whose base is *ABCD* and whose height is *GE*, as we shall say presently. Therefore the weight of the water resting against *ABCD* of the first figure is equal to the prism whose base is *ABCD* and whose height is *GE*. But that the body *ABCDLKNM* is equal to the prism whose base is *ABCD* and whose height is *GE* becomes evident as follows: *OQ* being drawn at right angles to the plane through *ABCD*, this *OQ* is the height of the prism *ABCDPOMN*; therefore that body is equal to the prism whose base is *ABCD* and whose height is *OQ*. But since *AH* is equal to *OC*, and the angle *HAE* is equal to the angle *COQ*, and *AE* is at right angles to the plane through the points *H, E*, [1]) and likewise *OQ* at right angles to the plane through the points *C, Q*, [2]) *AE* is equal to *OQ*, Therefore the body *ABCDPOMN* is equal to the prism whose base is *ABCD* and whose height is *AE*. But the body *MNPOKL* is equal to the prism whose base is *ABCD* and whose height is *AG*, by the corollary of the 11th proposition [3]). Therefore those two bodies, together making up the body *ABCDLKNM*, are equal to the prism whose base is *ABCD* and whose height is *GE*.

OTHER PROOF.

Assuming that there be in the water of the first figure above a bottom, equal and similar to *ABCD*, but parallel to the horizon in the plane in which is *AB*: against this bottom there will rest the weight that is equal to the prism of water whose base is equal to *ABCD* and whose height is *AE*, by the 10th proposition. This weight also rests against any bottom that is equal to that bottom and lower. So in any case there rests against *ABCD* a prism whose base is equal to *ABCD* and whose height is *AE*. Now if all the water is taken away which is above the aforesaid bottom, which we assumed to be equal to *ABCD*, in such a way that *AB* be in the water's upper surface, there rests against *ABCD*, by the corollary of the 11th proposition, the prism whose base is equal to *ABCD* and whose height is *AG*. Which two prisms together make the prism whose base is *ABCD* and whose height is *EG*, for the weight resting against the bottom *ABCD* as before.

EXAMPLE II.

Let *AB* be any regular bottom, whose highest point being below the water's upper surface is *A* and the lowest *B*, and the vertical from the water's upper surface to the highest point of the bottom shall be *CA*, and from the highest point of the bottom to the plane parallel to the horizon through the lowest point *B* of the bottom shall be *AD*, the half of which is *AE*. I say that the weight of the

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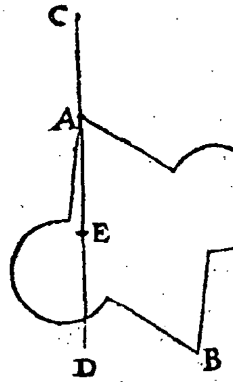
[1]) the plane *HEI*.
[2]) the plane of *ABCD*.
[3]) There is no corollary to Prop. 11. Stevin evidently refers to the Note, and applies the proposition in the form given there, assuming *AB* to be in the water's upper surface.

ghewicht waters teghen den bodem A B rustende, euen is anden pilaer waters diens bodem euen is ande selue A B, ende hoochde E C, waer af t'bewys sijn sal als van t'voorgaende.

T'BESLVYT. Wesende dan een gheschict bodem diens hoochste punt, &c.



MERCKT.

Wy hebben hier bouen bethoont t'ghewicht teghen een gheschickt bodem rustende, mettet behulp der hanghende lini duer des bodems hoochste punt; Maer alst een onghe schickt bodem is, soo en wort dat ghewicht duer die hanghende lini niet bekent: Tis wel waer, datter altijt vooral op rust t'ghewicht euen an den pilaer waters diens grondt den bodem is, ende hoochde de hanghende lini van t'waters oppervlack tot des bodems hoochste punt, maer t'ander deel en is niet euen anden helft des pilaers wiens grondt dien bodem is, ende hoochde de hanghende lini van des bodems hoochste punt, tottet plat euewydich vanden sichteinder duer des bodems leeghste punt, waer af d'oirsaeck is, dat den pilaer met een onghe schickt bodem niet nootsakelick in twee euen deelen (ghelijck den pilaer met een gheschickt bodem) ghedeelt en wort, met een plat, duer twee lijckstandighe punten schoens teghen ouer malcander staende inde omtrecken der bodems. Maer op dat wy t'ghewicht teghen alle ongeschickt plat bodem oock bekent maken, sullen daer af soodanighen eysch beschrijuen.

III EYSCH. XIII VOORSTEL.

WESENDE in t'water een platte bodem van form soot valt: Te vinden een lichaem waters euesvaer an t'ghewicht teghen dien bodem rustende.

T'GHEGHEVEN. Laet A B een platte bodem in t'water sijn, gheschickt ofte onghe schickt soot valt. T'BEGHEERDE. Wy moeten een lichaem waters vinden euesvaer an t'ghewicht rustende teghen A B.

T'WERCK. Ick treck het plat A B ouer allen sijden oneindelick voort, diens ghemeen sne mettet waters oppervlack sy C, uyt de selue sne C trek ick een lini duer t plat A B als C D, maer alsoo dattet plat rechthouckich opden sichteinder duer C D, oock rechthouckich sy op t'oneindelick plat duer den ghegheuen bodem; Daernaer treck ick de lini D E, euen ande lini D C, maer euewydich vanden sichteinder, ende

water resting against the bottom *AB* is equal to the prism of water, whose base is equal to this *AB* and whose height is *EC*, the proof of which will be the same as that of the preceding example.
CONCLUSION. Given therefore a regular bottom whose highest point, etc.

NOTE.

We have shown above the weight resting against a regular bottom, with the aid of the vertical through the highest point of the bottom. But if it is an irregular bottom, that weight does not become known through that vertical. It is true indeed that in any case there always rests on it the weight that is equal to the prism of water whose base is the bottom and whose height is the vertical from the water's upper surface to the highest point of the bottom, but the other part is not equal to the half of the prism whose base is that bottom and whose height is the vertical from the highest point of the bottom to the plane parallel to the horizon through the lowest point of the bottom, the cause of which is that the prism with an irregular base is not necessarily divided into two equal parts (like the prism with a regular base) by a plane through two homologous points diametrically opposite to each other in the circumferences of the bases. But in order to make also known the weight against any irregular plane bottom, we shall describe a problem with regard to this.

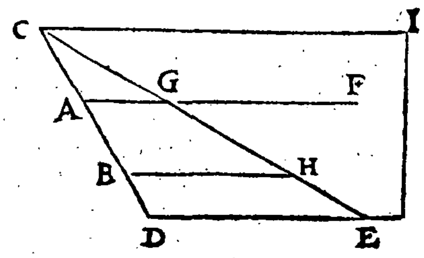
PROBLEM III. PROPOSITION XIII.

Given in the water a plane bottom of any form: to find a body of water of equal weight to the weight resting against that bottom.

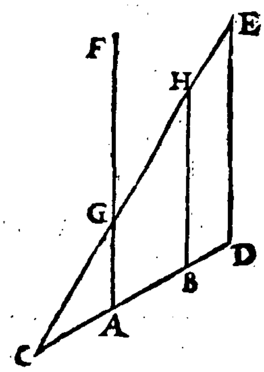
SUPPOSITION. Let *AB* be a plane bottom in the water, regular or irregular, as the case may be. WHAT IS REQUIRED TO FIND. We have to find a body of water of equal weight to the weight resting against *AB*. CONSTRUCTION. I produce the plane *AB* indefinitely on all sides, whose common intersection with the water's upper surface shall be *C*. From this intersection *C* I draw a line through the plane *AB*, as *CD*, but in such a way that the plane at right angles to the horizon through *CD* be also at right angles to the infinite plane through the given bottom [1]). Thereafter I draw the line *DE*, equal to the line *CD*, but parallel to the horizon and at right angles to the line which in the infinite plane

[1]) This means that the plane of the drawing is perpendicular to the line of intersection of the plane of the surface under consideration with a horizontal plane.

rechthouckich op de lini die in t'oneindelick plat duer D oock euewydich vanden sichteinder is; Daer naer treck ick duer de ghemeen sne C, ende duer E, een oueraloneindelick plat C E; voort, uyt eenich punt vanden omtreck des ghegheuen bodems als uyt A, een oneindelicke lini A F, draeyende de selue mettet punt A in des bodems A B omtreckt, tot datse weder ter plaets comt daerse begon te roeren, maer alsoo datse int roeren altijt euewydich blijft met de lini D E, beschrijuende alsoo een lichaem begrepen tusschen de twee oneindelicke platten, ende t'vlack van die roerlicke lini beschreuen, als t'lichaem A G H B. Ick seg dat een lichaem waters euegroot an t'lichaem A G H B, eueswaer is an t'ghewicht rustende teghen den ghegheuen bodem.



T'BEREYTSEL. Laet beschreuen sijn dese tweede form euen ende ghelijck an d'eerste, ende eueswaer an water, maer also dat de lini D E rechthouckich sy opden sichteinder. T'BEWYS. Alsulcken ghewicht alsser rust teghen den bodem A B der tweede form, euen soodanighen ruster oock teghen den bodem A B van d'eerste, als vooren bewesen is, maer teghen A B der tweede form, rust het ghewicht des lichaems A G H B, daerom teghen den bodem A B der eerste form, rust oock een ghewicht eue an t'lichaem waters A G H B, t'welck wy bewysen moesten.



T'BESLVYT. Wesende dan int water een platte bodem van form soot valt, wy hebben een lichaem waters gheuonden eueswaer an t'ghewicht teghen dien bodem rustende, naer den eysch.

XI. VERTOOCH. XIIII. VOORSTEL.

WESENDE twee euewydighe vierhouckighe bodems van euen breeden, ende euediep int water, ende haer hoochste sijden int waters oppervlack: Ghelijck der bodems langde tot langde, alsoo haer gheprang des waters, tot gheprang des waters.

T'GHE-

through *D* [1]) is also parallel to the horizon. Thereafter I draw through the common intersection *C* and through *E* a plane *CE* which is infinite in every direction; further, from some point of the boundary of the given bottom, as from *A*, an infinite line *AF*, revolving this with the point *A* in the boundary of the bottom *AB* until it again reaches the place where it started its motion, but in such a way that during its motion it always remains parallel to the line *DE*, thus describing a body comprehended by the two infinite surfaces and the plane described by that moving line, viz. the body *AGHB*. I say that a body of water having the same volume as the body *AGHB* is of equal weight to the weight resting against the given bottom.

PRELIMINARY. Let there be described this second figure, equal and similar to the first and being of equal weight to water, but in such a way that the line *DE* be at right angles to the horizon. PROOF. The same weight as rests against the bottom *AB* of the second figure also rests against the bottom *AB* of the first figure, as has been proved before, but against *AB* of the second figure rests the weight of the body *AGHB*; therefore against the bottom *AB* of the first figure rests also a weight equal to the body of water *AGHB*, which we had to prove.

CONCLUSION. Given therefore in the water a plane bottom of any form, we have found a body of water of equal weight to the weight resting against that bottom, as required.

THEOREM XI. PROPOSITION XIV.

Given two bottoms in the form of parallelograms [2]), of equal breadths and at equal depths in the water, their highest sides being in the water's upper surface: as the length of one bottom to that of the other, so is the pressure of the water against one to that against the other.

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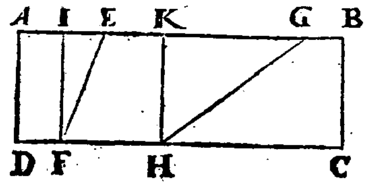
[1]) the plane of the surface.
[2]) rectangles.

T'GHEGHEVEN. Laet A B C D een water sijn, daer in twee euewydighe vierhouckighe bodems wesende, E F ende G H, van euen breeden, ende euediep int water, te weten, dat de * hanghende I F, euen sy ande hanghende K H, ende haer hoochste sijden E en G, sijn int waters oppervlack. T'BEGHEERDE. Wy moeten bewysen dat ghelijck de langde E F, tot de langde G H, alsoo t'gheprang des waters teghen den bodem E F, tottet gheprang des waters teghen den bodem G H. *Perpendicularu.*



T'BEWYS. T'ghewicht des waters teghen den bodem E F rustende, is euen anden helft des pilaers waters diens hoochde I F, ende grondt het plat E F, duer het 11e voorstel; Sghelijcx is t'ghewicht des waters teghen den bodem G H rustende, euen anden helft des pilaers waters diens hoochde K H, ende grondt het plat G H: maer dit sijn twee pilaren met euen hoochden * daerom sijnse inde reden haerder gronden; maer ghelijck de langde E F, totte langde G H, alsoo den grondt E F, totten grondt G H, want sy * duer t'ghestelde van euen breeden sijn, daerom ghelijck de langde E F totte langde G H, alsoo diens pilaer tot desens pilaer, ende wyder alsoo diens haluen pilaer tot desens haluen pilaer, ende veruolghens alsoo diens ghewicht des waters teghen haer rustende, tot desens ghewicht des waters teghen haer rustende. *32. v. 11. B. E* *Per hypothesin.*

T'BESLVYT. Wesende dan twee euewydighe vierhouckighe bodems, van euen breeden, ende euediep int water, ende haer hoochste sijden int waters oppervlack: Ghelijck der bodems langde tot langde, alsoo haer gheprang des waters, tot gheprang des waters, t'welck wy bewysen moesten.

IIII EYSCH. XV VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck oneuewydich vanden * sichteinder, met sijn bekende hoochste sijde in t'waters oppervlack, ende bekent wesende de lini vande hoochste sijde rechthouckich op de voortghetrocken leeghste, oock de * hangende vande hoochste sijde tot het * plat euewydich vanden sichteinder duer de leeghste sijde: Te vinden t'ghewicht waters daer teghen rustende. *Horizonte.* *Perpendicularis.* *Plano.*

Ee 3 MERCKT.

SUPPOSITION. Let *ABCD* be a water, in which there be two bottoms in the form of parallelograms, *EF* and *GH*, of equal breadths and at equal depths in the water, to wit that the vertical *IF* be equal to the vertical *KH*, and their highest sides *E* and *G* be in the water's upper surface. WHAT IS REQUIRED TO PROVE. We have to prove that as the length *EF* is to the length *GH*, so is the pressure of the water against the bottom *EF* to the pressure of the water against the bottom *GH*. PROOF. The weight of the water resting against the bottom *EF* is equal to the half of the prism of water whose height is *IF* and whose base is the plane *EF*, by the 11th proposition. In the same way the weight of the water resting against the bottom *GH* is equal to the half of the prism of water whose height is *KH* and whose base is the plane *GH*. But these are two prisms with equal heights; therefore they are in the same ratio as their bases. But as the length *EF* is to the length *GH*, so is the base *EF* to the base *GH*, for by the supposition they have equal breadths. Therefore, as the length *EF* is to the length *GH*, so is the former's prism to the latter's prism, and further so is the former's half prism to the latter's half prism, and consequently so is the weight of the water of the former resting against it to the weight of the water of the latter resting against it.
CONCLUSION. Given therefore two bottoms in the form of parallelograms, of equal breadths and at equal depths in the water, their highest sides being in the water's upper surface: as the length of one bottom is to that of the other, so is the pressure of the water against one to that against the other, which we had to prove.

PROBLEM IV. PROPOSITION XV.

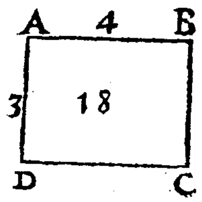
The bottom in the water being a parallelogram non-parallel to the horizon, with its known highest side in the water's upper surface, and the line from the highest side at right angles to the lowest side produced being known, as also the vertical from the highest side to the plane parallel to the horizon through the lowest side: to find the weight of the water resting against it.

MERCKT.

ALLE *euewydich vierhouck euewydich vanden ſichteinder met ſiin hoochſte ſiide int Waters oppervlack, is of rechthouckich of ſcheefhouckich, ende elck van deſen is op den ſichteinder rechthouckich oft ſcheefhouckich, daerom vallender vier verſcheyden ghestalten, daer Wy ſo Wel inde volghende twee voorſtellen als indit, vier voorbeelden af beſchrijuen ſullen: T'eerste van een rechthouc opden ſichteinder rechthouckich, Wiens drie linien, als de ſiide oneuewydich vanden ſichteinder, ende de lini uyt het uyterſte vande hoochſte ſiide rechthouckich opde voortghetrocken leeghſte ſiide, ende de hanghende uyt het uyterste vande hoochſte ſiide tottet plat euewydich vanden ſichteinder duer de leeghste ſiide, al een ſelfde lini ſiin: Het tweede voorbeelt ſal ſiin van een euewydich ſcheefhouckich vierhouck opden ſichteinder rechthouckich; diens twee linien als de lini vande hoochſte ſiide rechthouckich opde leeghſte ſiide, ende de hanghende vande hoochste ſiide tottet plat euewydich vanden ſichteinder duer de leeghſte ſiide, beyde een ſelue ſiin: Het derde voorbeelt ſal ſiin van een rechthouck ſcheefhouckich opden ſichteinder, diens twee linien, als de ſiide oneuewydich vanden ſichteinder, ende de lini van t'uyterſte der hoochſte ſiide rechthouckich op de leeghſte ſiide, beyde een ſelue ſiin: T'vierde voorbeelt van een euewydich ſcheefhouckich vierhouck opden ſichteinder ſcheefhouckich, diens voornoemde drie linien al verſcheyden ſiin.*

I^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D een rechthouck weſen rechthouckich opden ſichteinder, diens ſijde A B in t'waters oppervlack doe 4 voeten, ende A D 3 voeten. T'BEGHEERDE. Wy moeten t'ghewicht waters vinden ruſtende teghen A B C D.



T'WERCK. Ick menichvuldighe 3 van A D duer 4 van A B, maeckt 12, die andermael ghemenichvuldicht duer 3 van A D comt 36 voeten, diens helft voor t'begheerde 18 voeten. Ofte anderſins ick menichvuldighe t'vircant der 3 van A D, duer den helft der 4 van A B, comt als vooren 18 voeten. nu ghenomen den voet te weghen 65 ℔, ſoo ruſter 1170 ℔ teghen.

II^e VOORBEELT.

T'GHEGHEVEN. Laet A B C D een euewydich ſcheefhouckich vierhouck weſen, rechthouckich op den ſichteinder, diens ſijde A B in t'waters oppervlack doe 4 voeten, ende A E hanghende lini vande hoochtſte ſijde A B, tot inde voortghetrocken C D, ſy van 3 voeten.

T'BEGHEERDE. Wy moeten t'ghewicht waters vinden ruſtende teghen A B C D.

T'WERCK

NOTE.

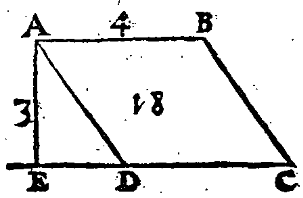
Any parallelogram non-parallel to the horizon with its highest side in the water's upper surface is either right-angled or oblique-angled, and each of them is at right angles or oblique angles to the horizon. Therefore there are four different cases, of which we shall describe four examples both in the following two propositions and in the present: the first of a rectangle at right angles to the horizon, three lines of which, viz. the side non-parallel to the horizon, and the line from the end of the highest side at right angles to the lowest side produced, and the vertical from the end of the highest side to the plane parallel to the horizon through the lowest side, are all one and the same line. The second example is to be of an oblique-angled parallelogram at right angles to the horizon, two lines of which, viz. the line from the highest side at right angles to the lowest side, and the vertical from the highest side to the plane parallel to the horizon through the lowest side, are one and the same line. The third example is to be of a rectangle at oblique angles to the horizon, two lines of which, viz. the side non-parallel to the horizon, and the line from the end of the highest side at right angles to the lowest side, are one and the same line. The fourth example is to be of an oblique-angled parallelogram at oblique angles to the horizon, the aforesaid three lines of which are all different.

EXAMPLE I.

SUPPOSITION. Let *ABCD* be a rectangle at right angles to the horizon, whose side *AB* in the water's upper surface be 4 feet, and *AD* 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against *ABCD*. CONSTRUCTION. I multiply 3 of *AD* by 4 of *AB*, which makes 12, which being multiplied again by 3 of *AD* makes 36 feet, whose half gives what was required: 18 feet. Or else I multiply the square of the 3 of *AD* by the half of the 4 of *AB*, which works out at 18 feet, as above. If I now take one foot to weigh 65 lbs, there rest against it 1,170 lbs.

EXAMPLE II.

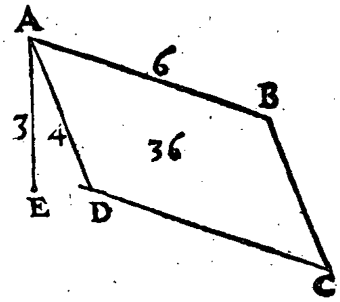
SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram, at right angles to the horizon, whose side *AB* in the water's upper surface be 4 feet, and *AE*, the vertical from the highest side *AB* to *CD* produced, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against



TWERCK. Ick menichvuldighe 3 van A E duer 4 van A B, maeckt 12, die andermael ghemenichvuldicht duer 3 van A E comt 36 voeten, diens helft voor t'begheerde 18 voeten. Oft andersins ick menichvuldighe als bouen t'viercant der 3 met den helft der 4 van A B.

III. VOORBEELT.

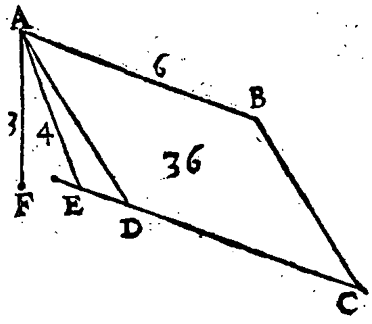
TGHEGHEVEN. Laet ABCD een rechthouck wesen scheefhouckich op den sichteinder, diens sijde A B in t'waters oppervlack sijnde doet 6 voeten, ende AD 4 voeten, maer A E hanghende van A tot in t'plat euewydich vanden sichteinder duer D C doe 3 voeten.



TBEGHEERDE. Wy moeten t'ghewicht waters vinden teghen ABCD rustende. TWERCK. Ick menichvuldighe 4 duer 6 comt 24, de selue duer 3 maeckt 72 voeten, diens helft voor t'begheerde 36 voeten. Ofte andersins ick menichvuldighe den uytbreng van 3 met 4, duer den helft van 6, comt als vooren 36 voeten.

IIII^e VOORBEELT.

TGHEGHEVEN. Laet ABCD een euewydich scheefhouckich vierhouck sijn, scheefhouckich opden sichteinder, diens sijde A B in t'waters oppervlack sijnde doet 6 voeten, ende A E rechthouckich op de voortghetrocken C D doet 4 voeten, ende A F hanghende van A tot het plat euewydich vanden sichteinder duer D C doet 3 voeten.



TBEGHEERDE. Wy moeten t'ghewicht waters vinden teghen A B C D rustende. TWERCK. Ick menichvuldighe 4 van A E, met 6 van A B comt 24, t'selue met 3 van A F, comt 72 voeten, diens helft voor t'begheerde 36 voeten. Ofte andersins, ick menichvuldighe, als vooren, den uytbreng van 3 met 4, duer den helft van 6, comt oock 36 voeten.

TBEWYS.

ABCD. CONSTRUCTION. I multiply 3 of *AE* by 4 of *AB*, which makes 12, which being multiplied again by 3 of *AE* makes 36 feet, whose half gives what was required: 18 feet. Or else I multiply, as above, the square of 3 by the half of the 4 of *AB* [1]).

EXAMPLE III.

SUPPOSITION. Let *ABCD* be a rectangle at oblique angles to the horizon, whose side *AB* being in the water's upper surface be 6 feet, and *AD* 4 feet, but *AE*, the vertical from *A* to the plane parallel to the horizon through *DC*, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against *ABCD*. CONSTRUCTION. I multiply 4 by 6, which makes 24; this being multiplied by 3 makes 72 feet, whose half gives what was required: 36 feet. Or else I multiply the product of 3 and 4 by the half of 6, which makes 36 feet, as before.

EXAMPLE IV.

SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram, at oblique angles to the horizon, whose side *AB* being in the water's upper surface be 6 feet, and *AE*, at right angles to *CD* produced, be 4 feet, and *AF*, the vertical from *A* to the plane parallel to the horizon through *DC*, be 3 feet. WHAT IS REQUIRED TO FIND. We have to find the weight of the water resting against *ABCD*. CONSTRUCTION. I multiply 4 of *AE* by 6 of *AB*, which makes 26; this, being multiplied by 3 of *AF*, makes 72 feet, whose half gives what was required: 36 feet. Or else I multiply, as before, the product of 3 and 4 by the half of 6, which

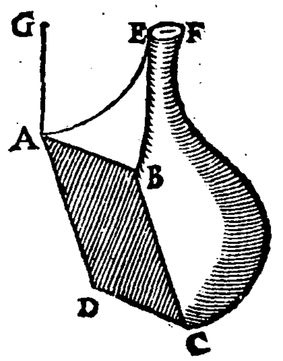
[1]) Here and in Example 4, the 11th proposition is applied to an oblique-angled parallelogram, though the demonstration given applied only to a rectangle.

T'BEWYS. Wesende een pilaer diens grondt 12 voeten, ende hoochde 3 voeten, den helft van dien doet 18 voeten; maer sulcken lichaem rustet teghen den bodem A B C D des 1en voorbeelts, duer het 11e voorstel, daer rust dan t'ghewicht van 18 voeten waters teghen. Sghelijcx sal oock t'bewys sijn van d'ander voorbeelden. T'BESLVYT. Wesende dan den bodem des waters een euewydich vierhouck, &c.

I VERVOLGH.

Uyt het bouesschreuen is blijckelick, hoemen vinden sal t'ghewicht waters teghen een euewydich vierhouck rustende, wesende d'hoochste sijde des ghegheuen vierhoucx onder t'waters oppervlack, want tot het ghewicht gheuonden alsvooren, noch vergaert den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack tot de hoochste sijde des bodems, de somme sal t'begheerde sijn.

Laet by voorbeelt A B C D een euewydich vierhouck sijn oneuewydich vanden sichteinder, diens hoochste sijde A B onder t'waters oppervlack E F is, ende G A doende drie voeten sy de hanghende lini van t'plat duer E F tot de sijde A B, ende t'plat A B C D sy groot 20 voeten, ende als A B in t'waters oppervlack waer, soo souder op rusten (t'welck ick neem gheuonden te sijne duer de voorgaende leering) 40 voeten waters: Vraegh hoe veel datter nu op rusten? Ick menichvuldighe 20 des plats van A B C D, duer 3 van G A, comt een pilaer van 60 voeten, die tot de 40 maeckt 100 voeten dieder teghen A B C D rusten.



II. VERVOLGH.

Soo den ghegheuen platten bodem onghesschickt waer, men sal vinden een lichaem waters euesswaer an t'ghewicht teghen dien bodem rustende duer het 13e voorstel, t'selue lichaem ghemeten sal de begheerde swaerheyt bekent maken.

V EYSCH. XVI VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck, oneuewydich vanden * sichteinder, met sijn hoochste sijde int waters oppervlack, ende bekent sijnde t'ghewicht daer teghen rustende,

Horizonte.

also makes 36 feet. PROOF. Given a prism, whose base is 12 feet and whose height is 3 feet; the half of that makes 18 feet. But such a body rests against the bottom *ABCD* of the 1st example, by the 11th proposition; so there rests against it the weight of 18 feet of water. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram, etc.

COROLLARY I.

From the above it appears how one is to find the weight of the water resting against a parallelogram, if the highest side of the given quadrilateral is below the water's upper surface, for if to the weight found as before there be added the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the highest side of the bottom, the sum will be the required weight.

For example, let *ABCD* be a parallelogram non-parallel to the horizon, whose highest side *AB* is below the water's upper surface *EF*, and *GA*, being three feet, shall be the vertical from the plane through *EF* to the side *AB*, and the plane *ABCD* shall be 20 feet. If *AB* were in the water's upper surface, there would rest on it 40 feet of water (which I take to have been found by means of the preceding theory). It is asked how much there now rests on it. I multiply 20 of the area of *ABCD* by 3 of *GA*. The product is a prism of 60 feet; these being added to the 40 feet, there will rest against *ABCD* 100 feet.

COROLLARY II.

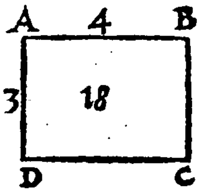
If the given plane bottom is irregular, a body of water shall be found which is of equal weight to the weight resting against that bottom by the 13th proposition. When this body is measured, the required gravity will be known.

PROBLEM V. PROPOSITION XVI.

The bottom in the water being a parallelogram, non-parallel to the horizon, with its highest side in the water's upper surface, and the weight resting against it rustende, oock de lini vande hoochste sijde rechthouckich opde voortghetrocken leeghste sijde, mette *hanghende vande hoochste sijde, tottet *plat euewydich vandẽ sichteinder duer deleeghste sijde: D'hoochste sijde bekent te maken. *Perpendiculari.* *Planum.*

I VOORBEELT.

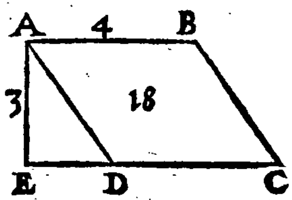
T'GHEGHEVEN. Laet A B C D een rechthouck wesen rechthouckich opden sichteinder, daer teghen rustende t'ghewicht van 18 voeten waters, ende d'hoochste sijde A B in t'waters oppervlack sy onbekent, maer A D doet 3 voeten. T'BEGHEERDE. Wy moeten de sijde A B bekent maken.



T'WERCK. Ick deel de 18 duer t'viercant der 3 van A D comt 2 voeten, diens dobbel voor A B 4 voeten.

II VOORBEELT.

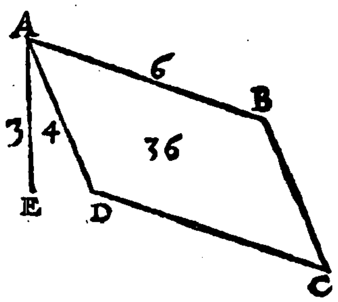
T'GHEGHEVEN. Laet A B C D een euewydich scheefhouckich vierhouck wesen, rechthouckich opden sichteinder, daer teghen rustende t'ghewicht van 18 voeten waters, ende de hoochste sijde A B in t'waters oppervlack si onbekent, maer de lini A E vande hoochste sijde rechthouckich opde voortghetrocken leeghste sijde doet 3 voeten.



T'BEGHEERDE. Wy moeten de sijde A B bekent maken. T'WERCK. Ick deel de 18 duer t'viercant der 3 van A E comt 2 voeten, diens dobbel voor A B 4 voeten.

III VOORBEELT.

T'GHE. Laet A B C D een rechthouck wesen scheefhouckich opden sichteinder, daer teghen rustende t'ghewicht van 36 voeten waters, ende d'hoochste sijde A B in t'waters oppervlack sy onbekent, maer de lini A D doet 4 voeten, ende A E hanghende vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghste sijde doet 3 voeten.



T'BEGHEERDE. Wy moeten de

being known, as also the line from the highest side at right angles to the lowest side produced, with the vertical from the highest side to the plane parallel to the horizon through the lowest side: to make known the highest side.

EXAMPLE I.

SUPPOSITION. Let *ABCD* be a rectangle at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side *AB* in the water's upper surface shall be unknown, but *AD* is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side *AB*. CONSTRUCTION. I divide the 18 by the square of the 3 of *AD*, which makes 2 feet; the double of this for *AB* is 4 feet.

EXAMPLE II.

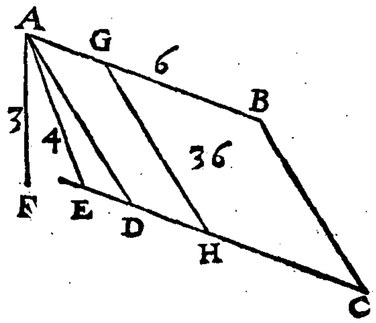
SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side *AB* in the water's upper surface shall be unknown, but the line *AE* from the highest side at right angles to the lowest side produced is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side *AB*. CONSTRUCTION. I divide the 18 by the square of the 3 of *AE*, which makes 2 feet; the double of this for *AB* is 4 feet.

EXAMPLE III.

SUPPOSITION. Let *ABCD* be a rectangle at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side *AB* in the water's upper surface shall be unknown, but the line *AD* is 4 feet, and *AE*, the vertical from the highest side to the plane parallel to the horizon through the lowest side, is 3 feet. WHAT IS REQUIRED TO FIND. We have to make ſijde A B bekent maken. T'WERCK. Ick menichvuldighe 3 van A E duer 4 van A D comt 12, daer duer ghedeelt de 36 comt 3 voeten, diens dobbel voor A B 6 voeten.

IIII^e VOORBEELT.

T'GHE. Laet A B C D een euewydich ſcheefhouckich vierhouck ſijn ſcheefhouckich opden ſichteinder, daer teghen ruſtende t'ghewicht van 36 voeten waters, ende de hoochſte ſijde A B in t'waters oppervlack ſy onbekent, maer A E lini vande hoochſte ſijde rechthouckich op de voortghetrocken leeghſte ſijde C D, doet 4 voeten, ende A F hanghende vande hoochſte ſijde tot het plat euewydich vanden ſichteinder duer de leeghſte ſijde, doet 3 voeten.



T'BEGHEERDE. Wy moeten de ſijde A B bekent maken.

T'WERCK. Ick menichvuldighe 3 van A F, met 4 van A E, comt 12, daer duer ghedeelt de 36, comt 3 voeten, diens dobbel voor A B 6 voeten. T'BEWYS. Soo A B des 1^en voorbeelts langher of corter waer als 4 voeten, t'ghewicht waters teghen den bodem ruſtende ſoude moeten meerder of minder ſijn dan 18 voeten, t'welck teghen t'gheſtelde waer, daerom A B is van 4 voeten. Sghelijcx ſal oock t'bewys ſijn van d'ander voorbeelden.

T'BESLVYT. Weſende dan den bodem des waters een euewydich vierhouck oneuewydich, &c.

I VERVOLGH.

Uyt het voorgaende is blijckelick, hoemen d'hoochſte ſijde bekent ſal maken, als ſy onder t'waters oppervlack is, want van t'gheheel ghewicht waters teghen den bodem ruſtende, ghetrocken den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack tot de hoochſte ſijde des bodems, daer ſal reſten t'ghewicht waters opden bodem ruſtende als haer hoochſte ſijde in t'waters oppervlack is, waer duer ſy alſdan bekent ſal worden als vooren gheleert is.

II VERVOLGH.

Soomen inden bodem een lini wilde trecken euewydich met de ſijde die vanden ſichteinder oneuewydich is, de noodighe langde der hoochſte ſijde can bekent worden. Laet by voorbeelt inde form des boueſchreuen 4^en voorbeelts, te trecken ſijn een lini als G H, euewydich met A D, known the side *AB*. CONSTRUCTION. I multiply 3 of *AE* by 4 of *AD*, which makes 12. The 36, divided by this, makes 3 feet; the double of this for *AB* is 6 feet.

EXAMPLE IV.

SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side *AB* in the water's upper surface shall be unknown, but *AE*, the line from the highest side at right angles to the lowest side *CD* produced, is 4 feet, and *AF*, the vertical from the highest side to the plane parallel to the horizon through the lowest side, is 3 feet. WHAT IS REQUIRED TO FIND. We have to make known the side *AB*. CONSTRUCTION. I multiply 3 of *AF* by 4 of *AE*, which makes 12. The 36, divided by this, makes 3 feet; the double of this for *AB* is 6 feet. PROOF. If *AB* of the 1st example were longer or shorter than 4 feet, the weight of the water resting against the bottom would have to be more or less than 18 feet, which would be contrary to the supposition. Therefore *AB* is 4 feet. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel, etc.

COROLLARY I.

From the foregoing it appears how one is to make known the highest side when it is below the water's upper surface, for if from the complete weight of the water resting against the bottom there be subtracted the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the highest side of the bottom, there will be left the weight of the water resting on the bottom when its highest side is in the water's upper surface, from which it will then become known, as has been taught before.

COROLLARY II.

If a line were to be drawn in the bottom, parallel to the side which is non-parallel to the horizon, the necessary length of the highest side can become known. For example, in the figure of the 4th example described above let there be drawn a line, as *GH*, parallel to *AD*, in such a way that there shall rest on *AGHD* the

A D, alsoo dat op A G H D ruste t'ghewicht van 12 voeten waters. Ick sie wat deel dese 12 sijn vande 36 dieder teghen rusten, wort beuonden het derdedeel, daerom oock sal A G $\frac{1}{3}$ wesen van A B dat sijn 2 voeten.

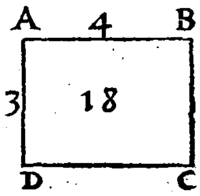
VI. EYSCH. XVII. VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck oneuewydich vanden * sichteinder, met sijn bekende hoochste sijde in t'waters oppervlack, ende bekent sijnde t'ghewicht daer teghen rustende, oock * de hanghende lini vande hoochste sijde tot het * plat euewydich vanden sichteinder duer de leeghste sijde: De lini vande hoochste sijde rechthouckich op de voortghetrocken leeghste sijde bekent te maken.

Horizonte. *Perpendicularis.* *Planum.*

I VOORBEELT.

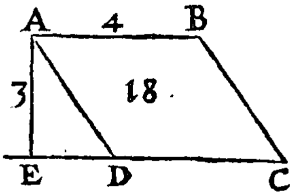
T'GHEGHEVEN. Laet A B C D een rechthouck wesen, rechthouckich opden sichteinder, daer teghen rustende t'ghewicht van 18 voeten waters, ende d'hoochste sijde A B in t'waters oppervlack sijnde, doet 4 voeten.



T'BEGHEERDE. Wy moeten de sijde A D bekent maken. T'WERCK. Ick deel de 18 duer 2, helft van A B, comt 9, diens viercantighe sijde voor A D doet 3 voeten.

II VOORBEELT.

T'GHEGHEVEN. Laet A B C D een euewydich scheefhouckich vierhouck wesen, rechthouckich opden sichteinder, daer teghen rustende t'ghewicht van 18 voeten waters, ende d'hoochste sijde A B in t'waters oppervlack sijnde doet vier voeten.



TBEGHEERDE. Wy moeten de lini A E bekent maken.

T'WERCK. Ick deel de 18 duer 2 helft van A B, comt 9, diens viercantighe sijde voor A E is 3 voeten.

III VOORBEELT.

T'GHEGHEVEN. Laet A B C D een rechthouck wesen scheefhoucweight of 12 feet of water. I ascertain what part these 12 are of the 36 resting against it. This is found to be one-third. Therefore also *AG* will be $\frac{1}{3}$ of *AB*, that is 2 feet.

PROBLEM VI. PROPOSITION XVII.

The bottom in the water being a parallelogram non-parallel to the horizon, with its known highest side in the water's upper surface, and the weight resting against it being known, as also the vertical from the highest side to the plane parallel to the horizon through the lowest side: to make known the line from the highest side at right angles to the lowest side produced.

EXAMPLE I.

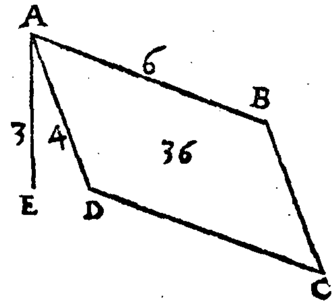
SUPPOSITION. Let *ABCD* be a rectangle, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side *AB*, being in the water's upper surface, is 4 feet. WHAT IS REQUIRED TO FIND. We have to make known the side *AD*. CONSTRUCTION. I divide the 18 by 2, the half of *AB*, which makes 9, being the square of the side *AD*, which is thus 3 feet.

EXAMPLE II.

SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram, at right angles to the horizon, against which there rests the weight of 18 feet of water, and the highest side *AB*, being in the water's upper surface, is four feet. WHAT IS REQUIRED TO FIND. We have to make known the line *AE*. CONSTRUCTION. I divide the 18 by 2, the half of *AB*, which makes 9, being the square of the side *AE*, which is thus 3 feet.

EXAMPLE III.

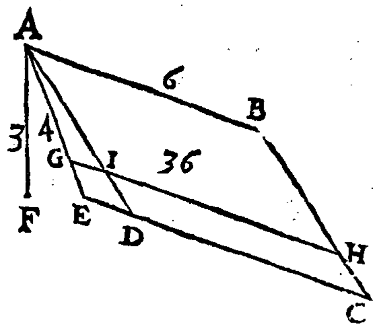
SUPPOSITION. Let *ABCD* be a rectangle at oblique angles to the horizon, against kich opden sichteinder, daer teghen rustende t'ghewicht van 36 voeten waters, ende de hoochste sijde A B in t'waters oppervlack sijnde doet 6 voeten, ende A E doende 3 voeten, sy de hanghende lini vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghste sijde.



T'BEGHEERDE. Wy moeten A D bekent maken. T'WERCK. Ick deel de 36 duer 3 helft van A B, comt 12, de selue ghedeelt duer 3 van A E, comt 4 voeten voor A D.

IIII VOORBEELT.

T'GHEGHEVEN. Laet A B C D een euewydich scheefhouckich vierhouck sijn scheefhouckich opden sichteinder, daerop rustende t'ghewicht van 36 voeten waters, ende de hoochste sijde A B in t'waters oppervlack sijnde doet 6 voeten, ende A E sy de lini van d'hoochste sijde rechthouckich opde voortghetrocken leeghste sijde, ende A F doende 3 voeten, is de hanghende vande hoochste sijde tot het plat euewydich vanden sichteinder duer de leeghste sijde. T'BEGHEERDE. Wy moeten de lini A E bekent maken. T'WERCK. Ick deel de 36 duer 3 helft der 6 van A B, comt 12, de selue duer 3 van A F comt 4 voeten, voor A E. T'BEWYS. So A D des 1en voorstels langher of corter waer als 3 voeten, t'ghewicht waters teghen den bodem rustende soude moeten meerder of minder sijn dan 18 voeten, t'welck teghen ghestelde is, A D dan is van 3 voeten. Sghelijcx sal oock t'bewys sijn van d'ander voorbeelden. T'BESLVYT. Wesende dan den bodem des waters een euewydich vierhouck oneuewydich vanden sichteinder, &c.



I VERVOLGH.

Uyt het voorgaende is blijckelick hoemen de lini vande hoochste sijde rechthouckich opde leeghste sijde, bekent sal maken, als de hoochste sijde onder t'waters oppervlack is, want van t'gheheel ghewicht waters teghen den bodem rustende, ghetrocken den pilaer diens grondt dien bodem is, ende hoochde de hanghende lini van t'plat duer t'waters oppervlack, which there rests the weight of 36 feet of water, and the highest side *AB*, being in the water's upper surface, is 6 feet, and *AE*, which is 3 feet, shall be the vertical from the highest side to the plane parallel to the horizon through the lowest side. WHAT IS REQUIRED TO FIND. We have to make known *AD*. CONSTRUCTION. I divide the 36 by 3, the half of *AB*, which makes 12; this being divided by 3 of *AE* makes 4 feet for *AD*.

EXAMPLE IV.

SUPPOSITION. Let *ABCD* be an oblique-angled parallelogram at oblique angles to the horizon, against which there rests the weight of 36 feet of water, and the highest side *AB*, being in the water's upper surface, is 6 feet, and *AE* shall be the line from the highest side at right angles to the lowest side produced, and *AF*, which is 3 feet, is the vertical from the highest side to the plane parallel to the horizon through the lowest side. WHAT IS REQUIRED TO FIND. We have to make known the line *AE*. CONSTRUCTION. I divide the 36 by 3, the half of the 6 of *AB*, which makes 12; this being divided by 3 of *AF* makes 4 feet for *AE*. PROOF. If *AD* of the 1st proposition were longer or shorter than 3 feet, the weight of the water resting against the bottom would have to be more or less than 18 feet, which is contrary to the supposition; *AD* therefore is 3 feet. The same proof will also be true of the other examples. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel to the horizon, etc.

COROLLARY I.

From the foregoing it appears how one is to make known the line from the highest side at right angles to the lowest side when the highest side is below the water's upper surface, for if from the complete weight of the water resting against the bottom there be subtracted the prism whose base is that bottom and whose height is the vertical from the plane through the water's upper surface to the oppervlack, tot de hoochste sijde des bodems, daer sal resten t'ghewicht waters opden bodem rustende als haer hoochste sijde in t'waters oppervlack is, waer duer sy alsdan bekent sal worden als voren gheleert is.

II VERVOLGH.

Soomen inde ghegheuen bodem een lini wilde trecken euewydich met de hoochste sijde, alsoo datse affne een deel des bodems daer een begheert ghewicht teghen ruste, de noodighe langde der lini vande hoochste sijde rechthouckich opde voortghetrocken leeghste sijde can bekent worden. Laet by voorbeelt inde form des bouescrhreuen 4ᵉ voorbeelts, te trecken sijn een lini als G H, sniende A D in I, euewydich met A B, alsoo dat op A B H I rust t'ghewicht van 24 voeten waters; Ick deel die 24 duer 3, helft van A B, comt 8, daer naer vinde ick twee ghetalen tot malcanderen in sulcken reden als 3 van A F, tot 4 van A E, ende dat haer uytbreng de voornomde 8 make, die ghetalen sijn $\sqrt{6}$ ende $\sqrt{10\frac{2}{3}}$, t'laetste is voor A G, want uyt G ghetrocken G H euewydighe met A B, daer sal teghen A B H I rusten t'ghewicht van 24 voeten waters duer het 15 voorstel.

MERCKT.

Wy moeten nu naer luyt des Cortbegrijps, inde volghende 18ᵉ, 19ᵉ, 20ᵉ, voorstellen, schrijuen vande swaerheyts middelpunten der gheprangselen des waters in bodems vergaert; alwaer niet onbillichlick eerst soude mueghen gheseyt worden, vande bodems euewydich siinde vanden sichteinder, maer ouermidts der seluer swaerheyts middelpunten (welcke gheuonden worden na de leering des 2ᵉⁿ boucx vande beghinselen der Weeghconst) oock de swaerheyts middelpunten siin der voornoemde haer gheprangselen, soo en beschrijuen wy daer af om cortheyts wil, gheen besonder voorstel. Sullen dan beghinnen ande bodems oneuewydich vanden sichteinder als volght.

XII VERTOOCH. XVIII VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck oneuewydich vanden * sichteinder, diens hoochste sijde in t'waters oppervlack is, uyt welcke sijdens middel een lini ghetrocken is, tot in t'middel vande leeghste sijde: * T'swaerheyts middelpunt des gheprangs inden bodem vergaert, deelt die lini alsoo, dat haer opperste stuck dobbel is an t'onderste.

Horizonte.

Centrum grauitatis.

highest side of the bottom, there will be left the weight of the water resting against the bottom when its highest side is in the water's upper surface, from which it will then become known, as has been taught before.

COROLLARY II.

If a line were to be drawn in the given bottom, parallel to the highest side, in such a way that it cut off a part of the bottom against which there should rest a desired weight, the necessary length of the line from the highest side at right angles to the lowest side produced can become known. For example, in the figure of the 4th example described above let there be drawn a line, as *GH*, intersecting *AD* in I, parallel to *AB*, in such a way that there rests on *ABHI* the weight of 24 feet of water. I divide those 24 by 3, the half of *AB*, which makes 8. Thereafter I find two numbers in the proportion 3 (of *AF*) to 4 (of *AE*) and so that their product be the aforesaid 8. Those numbers are $\sqrt{6}$ and $\sqrt{10\frac{2}{3}}$; the latter is *AG*, for when from *G* there be drawn *GH*, parallel to *AB*, there will rest against *ABHI* the weight of 24 feet of water, by the 15th proposition.

NOTE.

As announced in the Argument, we now have to write, in the 18th, 19th, and 20th propositions, about the centres of gravity of the total pressure of the water on bottoms; here it would not be inappropriate to speak first of the bottom being parallel to the horizon, but since the latter's centres of gravity (which are found by the theory of the 2nd book of the elements of the Art of Weighing) are also the centres of gravity of their aforesaid pressure, we will, for brevity's sake, not describe any separate proposition about this. We shall therefore start with the bottoms which are non-parallel to the horizon, as follows.

THEOREM XII. PROPOSITION XVIII.

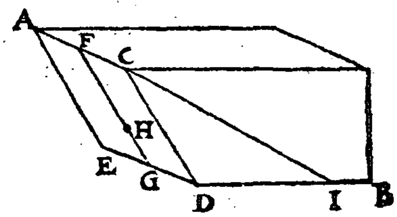
The bottom in the water being a parallelogram non-parallel to the horizon, whose highest side is in the water's upper surface, from the middle point of which side is drawn a line to the middle point of the lowest side: the centre of gravity of the total pressure on the bottom so divides that line that its upper part is double of the lower.

I VOORBEELT.

T'GHEGHEVEN. Laet A B een water sijn, ende den bodem A C D E sy een euewydich vierhouck oneuewydich vanden sichteinder, diens hoochste sijde A C in t'waters oppervlack is, ende F sy t'middel van A C, ende G t'middel van E D, ende tusschen de punten F G sy ghetrocken de lini F G, welcke in H alsoo ghedeelt is, dat F H dobbel is tot H G.

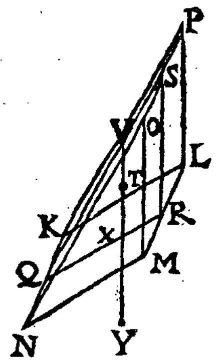
T'BEGHEERDE. Wy moeten bewysen dat H t'swaerheyts middelpunt is des gheprangs inden bodem vergaert.

T'BEREYTSEL. Laet ghetrocken worden de lini C I, alsoo dat D I euen sy an D C, ende mettet lichaem A C I D E sy beteeckent den helft des pilaers diens grondt A C D E, ende hoochde de hanghende van A tot in t'plat euewydich vanden sichteinder duer E D.



Laet daer naer ghetrocken worden t'stijflichaem K L M N O P euen ende ghelijck ende eueswaer an t'licham A C I D E, te weten K L M N * lijckstandich plat met A C D E, ende M O rechthouckich opden sichteinder, sy lijckstandighe lini met D I ende Q R sy lijckstandighe lini met F G, ende van S in t'middel van O P, sy ghetrocken de lini S Q, ende S R, ende des driehoucx Q S R swaerheyts middelpunt sy T, duer t'welck ghetrocken is de lini V X rechthouckich opden sichteinder.

Homologum planum.



T'BEWYS. Alsulcken gheprang als t'lichaem K L M N O P doet teghen den bodem K L M N, euen sulcken gheprang doet t'water A B teghen dē bodem A C D E duer het 11ᵉ voorstel, daerom ghelijck t'swaerheyts middelpunt des gheprangs inden bodem K L M N valt, alsoo salt oock vallen inden bodem A C D E. Om dan tottet bewys te commen, soo is ten eersten blijcklick dat T, welcke duer * t'ghestelde swaerheyts middelpunt is des driehoucx Q S R, oock swaerheyts middelpunt is (duer het 15 voorstel des 2ᵉⁿ boucx der beghinselen vande Weeghconst) des lichaems K L M N O P, maer V X is duer T rechthouckich opden sichteinder, V X dan is des lichaems swaerheyts middellini, daerom soo wy de

Hypothesin.

EXAMPLE I.

SUPPOSITION. Let *AB* be a water, and the bottom ACDE shall be a parallelogram non-parallel to the horizon, whose highest side *AC* is in the water's upper surface, and *F* shall be the middle point of *AC*, and *G* the middle point of *ED*, and between the points *F* and *G* there shall be drawn the line *FG*, which is so divided in *H* that *FH* is double of *HG*. WHAT IS REQUIRED TO PROVE. We have to prove that *H* is the centre of gravity of the total pressure on the bottom. PRELIMINARY. Let there be drawn the line *CI*, in such a way that *DI* shall be equal to *DC*, and by the body *ACIDE* there shall be denoted the half of the prism whose base is *ACDE*, and whose height is the vertical from *A* to the plane parallel to the horizon through *ED*.

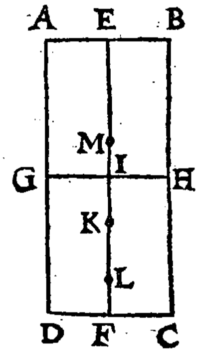
Thereafter let there be drawn the solid body *KLMNOP*, equal, similar, and of equal weight to the body *ACIDE*, to wit *KLMN* being a plane homologous to *ACDE*, and *MO*, at right angles to the horizon, shall be a line homologous to *DI*, and *QR* shall be a line homologous to *FG*, and from *S* in the middle point of *OP* there shall be drawn the line *SQ*, and *SR*, and the centre of gravity of the triangle *QSR* shall be *T*, through which is drawn the line *VX* at right angles to the horizon. PROOF. The same pressure as is exerted by the body *KLMNOP* against the bottom *KLMN* is also exerted by the water *AB* against the bottom *ACDE*, by the 11th proposition. Therefore, just as the centre of gravity of the pressure falls in the bottom *KLMN*, so it will also fall in the bottom *ACDE*. Now to arrive at the proof, firstly it is evident that *T*, which by the supposition is centre of gravity of the triangle *QSR*, is also centre of gravity (by the 15th proposition of the 2nd book of the elements of the Art of Weighing) of the body *KLMNOP*. But *VX* is through *T* at right angles to the horizon, therefore *VX* is the centre line of gravity of the body. If therefore we produce the line *XY* downwards, the body *KLMNOP* will, with the point *X* on the line *XY*, keep its given position in the mathematical sense; therefore *X* is centre of gravity of the total pressure of the body on the bottom *KLMN*. But *VX* is through the centre of gravity *T* at right angles to the horizon, and thus also parallel to *SR*. And consequently it intersects *QR* (by the 5th proposition of the 2nd book of the elements of the Art of Weighing), in such a way that *QX* is double of *XR*. But as has been said above, the centre of gravity falls in the bottom *ACDE* in the same way as it does in the bottom *KLMN*; therefore it falls in it in such a way that the upper part of the line *FG* is double of the lower. But that is in *H*; there fore *H* is the centre of gravity of the total pressure of the water on the bottom *ACDE*.

wy de lini X Y neerwaert trecken, t'lichaem K L M N O P sal mettet punt X op de lini X Y, * Wisconstlick verstaen, sijn ghegheuen standt houden, daerom X is swaerheyts middelpunt van des lichaems ghe-prang, vergaert inden bodem K L M N, maer V X is duer t'swaerheyts middelpunt T rechthouckich opden sichteinder, daerom oock euewydich met S R, ende vervolghens sy snijt Q R (duer het 5e voorstel des 2e boucx vande beghinselen der Weeghconst) alsoo dat Q X dobbel is an X R; Maer so bouen gheseyt is, t'swaerheyts middelpunt valt inden bodem A C D E, in sulcken ghestalt ghelijct inden bodem K L M N doet, het valter dan alsoo in, dattet bouenste deel der lini F G, dobbel is an t'onderste, maer dat is in H, daerom H is t'swaerheyts middelpunt van t'gheprangh des waters inden bodem A C D E vergaert. *Mathematicè.*

II VOORBEELT.

OM alsulcke redenen als int 4e voorbeelt des 11en voorstels gheseyt sijn, sullen wy hier bouen t'voorgaende * Wisconstich bewys, noch een voorbeelt duer ghetalen stellen, aldus: *Mathematicam demonstrationem.*

Laet A B C D een bodem sijn, daer in ghetrocken is de lini E F, tusschen de middelen van A B, ende D C, deelende dien bodem in ettelicke euen deelen (die wy maten noemen) met linien euewydich van A B, ick neem ten eersten in tween, mette lini G H, sniende E F in I, ende t'punt K sy alsoo, dat E K dobbel is an K F, welcke K wy bewysen moeten t'swaerheyts middelpunt des gheprangs te sijn aldus: Ghenomen dat teghen A B H G, ruste 1 pondt, ofte ghewicht waters, soo salder teghen G H C D sucke 3 ghewichten rusten: Dit so sijnde, ick acht ten eersten al oft t'swaerheyts middelpunt des gheprangs van A B H G, waer in I, ende van G H C D in F (tis seker dat sy hoogher sijn) so sal I F balck wesen, welcke ghedeelt in haer ermen tot malcanderen in sulcken reden als de voornoemde ghewichten van 3 tot 1, t'welck in t'punt L valle, so sal F L doen $\frac{1}{4}$ eender maet, dat is $\frac{1}{4}$ van I F. Ten tweeden so achtick, al of t'swaerheyts middelpunt des gheprangs van A B H G waer in E, ende van G H C D in I (tis seker dat sy leegher sijn) soo sal haer ghemeen swaerhaeyts middelpunt vallen een maet bouen L als in M. Tis dan blijckelick dattet ware begheerde swaerheyts middelpunt is tusschen M ende L. Maer ghelijck wy den bodem hier bouen ghedeelt hebben in tween, alsoo canmense deelen in oneindelicke stucken, daer af vindende twee swaerheyts middelpunten als bouen, tusschen de welcke altijdt is, het ware begheerde swaerheyts middelpunt. Wy connen dan duer



sulcke

EXAMPLE II.

For all such reasons as mentioned in the 4th example of the 11th proposition, we will, in addition to the foregoing mathematical proof, also give an example by means of numbers, as follows:

Let *ABCD* be a bottom in which there is drawn the line *EF*, joining the middle points of *AB* and *DC*, dividing that bottom into several equal parts (which we call measures) by lines parallel to *AB*. I take the bottom first to be divided in two, by the line *GH*, intersecting *EF* in I, and the point *K* shall be such that *EK* is double of *KF*, which *K* we have to prove to be the centre of gravity of the pressure, as follows. Assuming that there rest against *ABHG* 1 pound or one weight of water, there will rest 3 such weights against *GHCD*. This being so, I first imagine the centre of gravity of the pressure of *ABHG* to be in *I* and that of *GHCD* in *F* (it is certain that they are higher); then *IF* will be beam, and if this is divided into its arms having to each other the same ratio as the aforesaid weights, i.e. 3 to 1, which point of division shall fall in *L*, *FL* will be $\frac{1}{4}$ of a measure, i.e. $\frac{1}{4}$ of *IF*. Secondly I imagine the centre of gravity of the pressure of *ABHG* to be in *E*, and that of *GHCD* in *I* (it is certain that they are lower); then their common centre of gravity will fall one measure above *L*, viz. in *M*. It is therefore evident that the true required centre of gravity is between *M* and *L*. But just as above we divided the bottom in two, it is also possible to divide it into an infinite number of parts, and find two centres of gravity thereof [1]), as above, between which is always the true required centre of gravity. We can therefore, by this means, always approximate infinitely. If therefore we find by this experience that the point *L* never reaches *K*, but remains very near to it and always below it; in the same way that the point *M* never reaches *K*, but always remains above it, we conclude from this that *K* is the true required centre of gravity. But because it would be a difficult calculation to find in this way the common centre of gravity of all those bottoms, we shall explain a short method for doing this, as follows. I write down a progression, as 1.3.5.7.9 and so on, always ascending by 2, for in such a progression and proportion are the pressures of the equal parts of a bottom *ABCD* by the 15th proposition. Thereafter I place $\frac{1}{4}$ (which has been found above for *FL*) above the second number 3, as below:

$$\begin{array}{cccccc} & \frac{1}{4} & & & & \\ 1\ . & 3\ . & 5\ . & 7\ . & 9\ . & 11 \end{array}$$

Thereafter I add up 4, the denominator of $\frac{1}{4}$, and 5 (the third term of the progression), which makes 9; I place that as denominator above the 5, and above the 9 I place 5, i.e. the sum of the denominator and the numerator of $\frac{1}{4}$, so that the scheme is then as follows:

$$\begin{array}{cccccc} & \frac{1}{4} & \frac{5}{9} & & & \\ 1\ . & 3\ . & 5\ . & 7\ . & 9\ . & 11 \end{array}$$

In the same way I also find all the others, for in order to have the number that is to be placed above 7, I add up the denominator 9 and 7, which makes 16. Above this I place the sum of 9 and 5 (which are the denominator and the

[1]) To wit one centre of gravity, when the centre of each strip is taken to be in its lowest side, and one, when it is taken to be in the highest side.

ſulcke middel altijt oneindelick naerderen, daerom als wy duer deſe eruaring beuinden, dattet punt als L nummermeer tot K en comt, maer ſeer by ende altijt daer onder blijft; Sghelijcx dattet punt als M nummermeer tot K en comt, maer altijt daer bouen blijft, wy beſluyten uyt ſulcx, dat K het ware begheerde ſwaerheyt middelpunt is. Maer want het moylicke rekeninghe ſoude ſijn t'ghemeene ſwaerheyts middelpunt van alle die bodems alſo te vinden, wy ſullen daer af een corte manier verclaren aldus, ick ſchrijf een * voortganck als 1. 3. 5. 7. 9. ende ſo voort altijt met tween opclimmende, want in ſulcken voortganck ende reden ſijn de prangſelen der euen deelen eens bodems A B C D duer het 15e voorſtel, daer naer ſtel ick $\frac{1}{4}$, (twelck hier bouen beuonden is voor F L) bouen het tweede ghetal 3, als hier onder:

*Progreßionē.

$$\begin{matrix} & \frac{1}{4} & & & & \\ 1. & 3. & 5. & 7. & 9. & 11. \end{matrix}$$

Daer naer vergaer ick 4, noemer van $\frac{1}{4}$, met de 5 derde in d'oirden, comt 9, die ſtel ick als noemer bouen de 5, ende bouen de 9 ſet ick 5, dat is de ſomme des noemers en telders van het $\frac{1}{4}$ welcker gheſtalt dan aldus is:

$$\begin{matrix} & \frac{1}{4} & \frac{5}{9} & & & \\ 1. & 3. & 5. & 7. & 9. & 11. \end{matrix}$$

Sghelijcx vinde ick oock alle d'ander, want om t'ghetal te hebben dat bouen 7 comen ſal, ick vergaer den telder 9 ende 7, maeckt 16, daer bouen ſtelick de ſomme van 9 ende 5 (die noemer ende telder ſijn vande $\frac{5}{9}$) maeckt 14. Inder voughen dat bouen de 7 comen ſal $\frac{14}{16}$, wiens gheſtalt dan aldus ſijn ſal:

$$\begin{matrix} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & & \\ 1. & 3. & 5. & 7. & 9. & 11. \end{matrix}$$

Ende ſoo voortgaende, bouen de 9 ende 11 ſullen ghetalen comen als hier onder:

$$\begin{matrix} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & \frac{30}{25} & \frac{55}{36} \\ 1. & 3. & 5. & 7. & 9. & 11. \end{matrix}$$

Dit ſoo verſtaen ſijnde, men wilt weten neem ick, waer t'punt als L vallen ſal, wanneer den bodem ghedeelt is in vijf euen deelen: Ick ſien wat ghetal datter bouen t'vijfde in d'oirden ſtaet, dat is bouen de 9, ende beuinde $\frac{30}{25}$ diens eerſte ghebroken doet $\frac{6}{5}$, daer uyt beſluyt ick dat de lini als L F van ſulcken bodem in vijuen ghedeelt, ſijn ſal van $\frac{6}{5}$ eender maet, der maten daer den bodem in ghedeelt is, maer dat die min ſijn dan $\frac{1}{3}$ van E F, ende dat haer uyterſte als L vallen ſal onder K, wort aldus bethoont: De $\frac{6}{5}$ eender maet der maten daer den bodem in ghedeelt is, dat is $\frac{6}{5}$ van $\frac{1}{5}$ doen $\frac{6}{25}$, vande heele lini als E F, welcke $\frac{6}{25}$ minder ſijn als numerator of $\frac{5}{9}$), which makes 14. In such a way that above the 7 there shall be placed $\frac{14}{16}$, so that the scheme will then be as follows:

$$\begin{matrix} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & & \\ 1 . & 3 . & 5 . & 7 . & 9 . & 11 \end{matrix}$$

And proceeding in this way, above the 9 and 11 there will come the numbers shown below:

$$\begin{matrix} & \frac{1}{4} & \frac{5}{9} & \frac{14}{16} & \frac{30}{25} & \frac{55}{36} \\ 1 . & 3 . & 5 . & 7 . & 9 . & 11 \end{matrix}$$

This being understood, I assume that it is desired to know where the point *L* will fall when the bottom is divided into five equal parts. I ascertain what number is above the fifth term of the progression, i.e. above the 9, and find $\frac{30}{25}$, which in its lowest terms is $\frac{6}{5}$; from this I conclude that the line *LF* of this bottom divided into five parts will be $\frac{6}{5}$ of a measure, of the measures into which the bottom is divided. But that this is less than $\frac{1}{3}$ of *EF*, and that its extremity, viz. *L*, will fall below *K*, is proved as follows: The $\frac{6}{5}$ of a measure of the measures into which the bottom is divided, i.e. $\frac{6}{5}$ of $\frac{1}{5}$, make $\frac{6}{25}$ of the complete line *EF*, which $\frac{6}{25}$ is less than $\frac{1}{3}$ *FK*, for if $\frac{6}{25}$ be subtracted from $\frac{1}{3}$, there is left $\frac{7}{75}$ of the line *EF*, and the point *L* will be at this distance from *K*. But in order to find the point *M*, I add one measure to the $\frac{6}{5}$ measures, which makes $\frac{11}{5}$ of a measure. This is $\frac{11}{25}$ of the complete line *EF*, which $\frac{11}{25}$ is more than $\frac{1}{3}$ of *FK*. For if $\frac{1}{3}$ be subtracted from $\frac{11}{25}$, there is left $\frac{8}{75}$ of the line *EF*, and the point *M* will be at this distance from *K*, that is $\frac{1}{75}$ further away from it than *L*. And the same with all the others, for if the bottom *ABCD* were divided into 40 equal parts, the line *FL* would be found to be $\frac{20,550}{1,600}$ of a measure, that is of one fortieth part of the line *EF*, through which the points *L* and *M* would be found to be much nearer than above, though they will never reach it, the necessity of which has been proved mathematically in the 1st example described above. The ground of the above short method for finding the common centre of gravity of the various pressures will be easily understood by those who seek for it at full length according to the theory of 2nd proposition of the 1st book of the elements of the Art of Weighing [1]). CON-

1) The procedure may be explained as follows. Dividing the surface into n strips, putting the pressure on the highest strip k_n, and the height of each strip p_n, we have: Pressures on successive strips (starting from the highest):

$$k,\ 3k,\ 5k\ \ldots\ldots\ (2n - 1)k$$

Distances from the centres of pressure, supposed to be in the lowest sides of the strips, to *F*:

$$(n-1)p_n,\ (n-2)p_n\ \ldots\ldots\ p_n,\ 0$$

Moments of the pressures with respect to *F*:

$$(n-1)p_n\,k_n\ \ 3(n-2)p_n\,k_n,\ \ldots\ldots\ (2n-3)p_n\,k_n.$$

Total pressure

$$(1 + 3 + 5 \ldots + 2n - 1)k_n = n^2 . k_n$$

We now put the total moment $p_n\,k_n\,[(n-1) + 3(n-2) + \ldots (2n-3)] = p_n\,k_n\,T_n$

ſijn als $\frac{1}{3}$ F K, want ghetrocken $\frac{6}{25}$ van $\frac{1}{3}$, blijft $\frac{7}{75}$ der lini E F, ende ſoo verre ſal dan t'punt als L van K vallen. Maer om t'punt als M te vinden, ick doe een maet tot de $\frac{6}{5}$ maets, comt $\frac{11}{5}$ eender maet, de ſelue doen $\frac{11}{25}$ vande heele lini E F, welcke $\frac{11}{25}$ meerder ſijn dan $\frac{1}{3}$ van F K, want ghetrocken $\frac{1}{3}$ van $\frac{11}{25}$ blijft $\frac{8}{75}$ der lini E F, ende ſo verre ſal dan t'punt als M van K vallen, dat is $\frac{1}{75}$ verder dander L afviel, ende alſoo met allen anderen, want ſoomen den bodem A B C D deelde in 40 euen deelen, de lini als F L ſoude beuonden worden van $\frac{20550}{1600}$ eender maet, dat is eens veertichſtendeels der lini E F, duer t'welcke men de punten als L M veel naerder ſoude beuinden dan bouen, maer nummermeer daer toe comen, waer af de nootſakelicheyt int bouenſchreuen 1ᵉ voorbeelt Wiſconſtelick betoocht is. De reden vande boueſchreuen corte manier der vindingh des ghemeen ſwaerheyts middelpunts van die verſcheyden prangſelen, ſal den ghenen lichtelick connen bemercken, dieſe in t'langhe ſouckt naer de leering des 2ᵉⁿ voorſtels van het 1ᵉ bouck der beghinſelen vande Weeghconſt. T'BESLVYT. Weſende dan den bodem des waters een euewydich vierhouck oneuewydich, &c.

XIII VERTOOCH. XIX VOORSTEL.

WESENDE den bodem des waters een euewydich vierhouck oneuewydich vanden* ſichteinder, diens hoochſte ſijde onder t'waters oppervlack is, maer euewydich vanden ſichteinder, uyt welcke ſijdens middel een lini ghetrocken is, tot in t'middel vande leeghſte ſijde: T'ſwaerheyts middelpunt des gheprangs inden bodem vergaert, is inde lini tuſſchen t'middelpunt des bodems, ende t'punt dat het onderſte derdendeel dier lini afſnijt; ende tuſſchen die twee punten in ſoodanighen punt, t'welck t'onderſte deel alſoo afſnijt, dattet ſulcken reden heeft tottet bouenſte, ghelijck de* hanghende lini van t'waters oppervlack in des bodems leeghſte ſijde, tot den helft der hanghende lini van des bodems hoochſte ſijde, tottet *plat euewydich vanden ſichteinder duer des bodems leeghſte ſijde.

Horizonte.

Perpendicularis.

Planum.

Gg T'GHE-

CLUSION. The bottom in the water therefore being a parallelogram non-parallel, etc.

THEOREM XIII. PROPOSITION XIX.

The bottom in the water being a parallelogram non-parallel to the horizon, whose highest side is below the water's upper surface, but parallel to the horizon, from the middle point of which side there is drawn a line to the middle point of the lowest side: the centre of gravity of the total pressure on the bottom is in the line joining the middle point of the bottom and the point cutting off the lower third part of that line, and between those two points in a point such as cuts off the lower part in such a way that it has to the upper part the same ratio as the vertical from the water's upper surface in the highest 2) side of the bottom to the half of the vertical from the highest side of the bottom to the plane parallel to the horizon through the lowest side of the bottom.

The distance from the resulting centre of pressure to F is

$$\frac{p_n k_n . T_n}{n^2 . k_n} = \frac{p_n}{n^2} T_n.$$

If we now replace n by $(n + 1)$, the total pressure becomes $(n + 1)^2 . k_{n+1}$.
The sum of the moments is now:

$$n . p_{n+1} . k_{n+1} + 3(n - 1) p_{n+1} . k_{n+1} + \ldots (2n - 1) p_{n+1} . k_{n+1} =$$
$$p_{n+1} . k_{n+1} [T_n + \{ 1 + 3 + \ldots (2n - 1) \}] = p_{n+1} . k_{n+1} [T_n + n^2] =$$
$$p_{n+1} . k_{n+1} . T_{n+1}.$$

The distance $LF = \frac{T_{n+1}}{(n+1)^2} . p_{n+1}$, where $T_{n+1} = T_n + n^2$.
This recurrent relation, together with the initial value 0 for $n = 1$, determines the successive values of the coefficient of p_n. This gives indeed Stevin's series for $\frac{T_n}{n^2}$

$\frac{0}{1} \frac{1}{4} \frac{5}{9} \frac{14}{16} \frac{30}{25}$

For $n = 5$ we find with Stevin for the distance LF, if $EF = l$, $\frac{30}{25} \cdot \frac{l}{5} = \frac{6}{25} l$ and so for LK $(\frac{1}{3} - \frac{6}{25}) l = \frac{7}{75} l$.
If the centres are all taken in the highest sides of the strips, the recurrent relation is

$$T_{n+1} = T_n + (n + 1)^2,$$

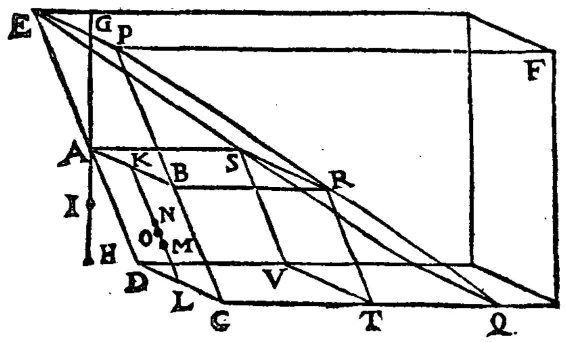
and the series of coefficients becomes $\frac{1}{1} \frac{5}{4} \frac{14}{9} \frac{30}{16} \frac{55}{25}$.
For the distance MK from the resultant centre to K we now find with Stevin $(\frac{55}{25} \cdot \frac{1}{5} - \frac{1}{3}) l = \frac{8}{75} l$.

The general expression for FL proves to be: $FL = \frac{1}{6} \cdot \frac{n(n - 1)(2n - 1)}{n^2} \cdot \frac{l}{n}$, which gives for $n = 40$

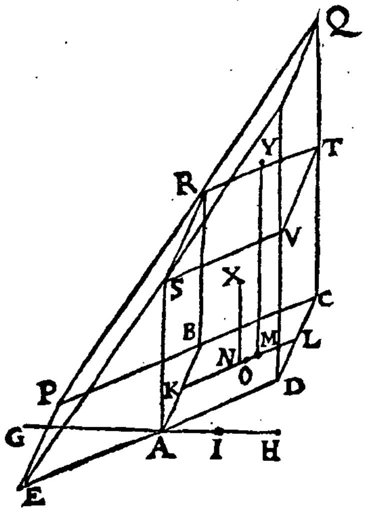
$FL = \frac{1}{6} \cdot \frac{40.30.79}{1{,}600} \cdot \frac{l}{40} = \frac{20{,}540}{1{,}600} \cdot \frac{l}{40}$ and which for $n \to \infty$ converges towards $\frac{l}{3}$.
Stevin finds 20,550 for the nominator.

2) Stevin's text has „leeghste", which obviously is an error.

T'GHEGHEVEN. Laet A B C D een bodem sijn oneuewydich vanden sichteinder diens hoochste sijde A B onder t'waters oppervlack E F is, maer euewydich vanden sichteinder, ende G A sy de hanghende lini van t'waters oppervlack tot de hoochste sijde A B, ende A H de hanghende lini van A, tottet plat euewydich vanden sichteinder duer D C, ende A I sy den helft van A H, ende K L sy de lini ghetrocken tusschen de middelen van A B ende D C, ende L M sy het derdendeel vande lini L K, ende N t'middelpunt des bodems A B C D, ende O een punt tusschen M ende N, alsoo dat O M sulcken reden heeft tot O N, ghelijck A G tot A I. T'BEGHEERDE. Wy moeten bewysen dat O t'swaerheyts middelpunt is van t'gheprang des waters inden bodem A B C D vergaert. T'BEREYTSEL. Laet C B ende D A voortghetrocken worden tot in t'waters oppervlack, als tot P eñ E, daer naer C Q euen an C P, maer euewydich vanden sichteinder, ende rechthouckich op C D, daer naer B R euewydighe met C Q, wesende R inde lini P Q: Sghelijcx A S euen ende euewydighe met B R, voort R T, eñ S V euen ende euewydighe met B C.



Laet daer naer een ander form ghestelt worden, euen ghelijck ende euewichtich ande voorgaende E P C D Q, maer also dat C Q rechthouckich sy opdē sichteinder, ende X sy swaerheydts middelpunt des pilaers A B C D R S V T, ende Y swaerheyts middelpunt des lichaems



SUPPOSITION. Let *ABCD* be a bottom non-parallel to the horizon, whose highest side *AB* is below the water's upper surface *EF*, but parallel to the horizon, and *GA* shall be the vertical from the water's upper surface to the highest side *AB*, and *AH* the vertical from *A* to the plane parallel to the horizon through *DC*, and *AI* shall be the half of *AH*, and *KL* shall be the line joining the middle points of *AB* and *DC*, and *LM* shall be the third part of the line *LK*; and *N* the centre of the bottom *ABCD*, and *O* a point between *M* and *N*, in such a way that *OM* has to *ON* the same ratio as *AG* to *AI*. WHAT IS REQUIRED TO PROVE. We have to prove that *O* is the centre of gravity of the total pressure of the water on the bottom *ABCD*.

PRELIMINARY. Let *CB* and *DA* be produced to the water's upper surface, viz. to *P* and *E*; thereafter let *CQ* be made equal to *CP*, but parallel to the horizon and at right angles to *CD*; thereafter *BR* parallel to *CQ*, *R* being in the line *PQ*. In the same way *AS* equal and parallel to *BR*; further *RT* and *SV* equal and parallel to *BC*.

Thereafter let there be drawn another figure, equal, similar, and of equal weight to the preceding *EPCDQ*, but in such a way that *CQ* be at right angles to the horizon, and *X* shall be centre of gravity of the prism *ABCDRSVT* and chaems R S V T Q, Laet oock ghetrocken worden de linien X N ende Y M. T'BEWYS. Anghesien in dese tweede form, X swaerheyts middelpunt is des pilaers A B C D R S V T, ende N swaerheyts middelpunt haers grondts A B C D, ende dat C T rechthouckich is opden sichteinder, soo is X N haer euewydighe, oock rechthouckich opden sichteinder, ende veruolghens huer swaerheyts middellini, daerom oock is N swaerheyts middelpunt des gheprangs diens pilaers; Maer M swaerheyts middelpunt te wesen des gheprangs van t'lichaem S R T V Q dat is int 18e voorstel betoocht: T'welck so sijnde, M N is Weeghconstighen balck, die in O alsoo ghedeelt is, dat ghelijck A G tot A I, alsoo O M tot O N duer t'ghegheuen, maer ghelijck A G tot A I, alsoo den pilaer A B C D R S V T, tottet lichaem S R T V Q, daerom ghelijck den pilaer A B C D R S V T, tottet lichaem S R T V Q, alsoo O M tot O N, waer duer O t'swaerheyts middelpunt is deser tweede form, duer het 1e voorstel des eersten boucx vande beghinselen der Weeghconst, maer t'swaerheyts middelpunt van d'eerste form, om de redenen alsvooren, valt aldaer ghelijck in de tweede, O dan der eerste form, is t'begheerde swaerheyts middelpunt. T'BESLVYT. Wesende dan den bodem des waters een euewydich vierhouck oneuewydich vanden sichteinder, &c.

VII EYSCH. XX VOORSTEL.

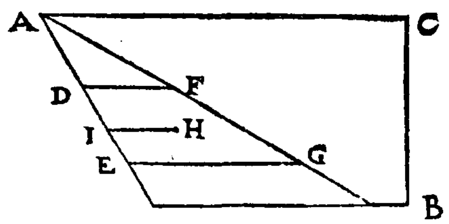
WESENDE den bodem in t'water een rechtlinich* plat van form soot valt: Te vinden het swaerheyts middelpunt des gheprangs inden bodem vergaert.

*Planum.

T'GHEGHEVEN. Laet A B een water wesen, diens oppervlack A C, ende D E een bodem, welcke een rechtlinich plat sy.

T'BEGHEERDE. Wy moeten het swaerheyts middelpunt vinden van t'gheprang des waters in dien bodem vergaert.

T'WERCK. Men sal eerst vinden een lichaem waters euesw aer an t'gheprang teghen den bodem D E, naer de leering des 13e voorstels, t'selue sy D E F G, vindende daer naer sijn swaerheyts middelpunt duer het 21e voorstel des tweeden boucx vande beghinselen der Weeghconst,



t'welck H sy, daer naer ghetrocken H I euewydighe met G E, diens

Y centre of gravity of the body *RSVTQ*. Let there also be drawn the lines *XN* and *YM*. PROOF. Since in this second figure X is centre of gravity of the prism *ABCDRSVT*, and *N* centre of gravity of its base *ABCD*, and *CT* is at right angles to the horizon, *XN* — which is parallel to it — is also at right angles to the horizon, and consequently also its centre line of gravity. Therefore *N* is centre of gravity of the pressure of that prism. But that *M* is centre of gravity of the pressure of the body *SRTVQ* has been shown in the 18th proposition. Which being so, *MN* is a mathematical beam, which is so divided in *O* that as *AG* is to *AI*, so is *OM* to *ON* by the supposition. But as *AG* is to *AI*, so is the prism *ABCDRSVT* to the body *SRTVQ*; therefore, as the prism *ABCDRSVT* is to the body *SRTVQ*, so is *OM* to *ON*, owing to which *O* is the centre of gravity of this second figure, by the 1st proposition of the first book of the elements of the Art of Weighing. But, for the reasons mentioned above, the centre of gravity of the first figure falls there as in the second. *O* of the first figure therefore is the required centre of gravity. CONCLUSION. The bottom in the water therefore being a parallelogram non-parallel to the horizon, etc.

PROBLEM VII. PROPOSITION XX.

The bottom in the water being a rectilinear plane figure of any form: to find the centre of gravity of the total pressure on the bottom.

SUPPOSITION.Let *AB* be a water, whose upper surface is *AC*, and *DE* a bottom which shall be a rectilinear plane figure. WHAT IS REQUIRED TO FIND. We have to find the centre of gravity of the total pressure of the water on that bottom. CONSTRUCTION. There shall first be found a body of water of equal weight to the pressure against the bottom *DE*, according to the theory of the 13th proposition. This shall be *DEFG*. Thereafter its centre of gravity shall be found, by the 21st proposition of the second book of the elements of the Art of Weighing, which shall be *H*, and if then *HI* be drawn parallel to *GE*, whose extremity *I* uyterste punt I inden bodem D E sy; Ick seg t'selue punt I te wesen t'begheerde swaerheyts middelpunt, waer af t'bewys ghelijck sal sijn ande bewysen des voorgaenden 18^en^ ende 19^en^ voorstels.

T'BESLVYT. Wesende dan den bodem int water een rechtlinich plat, &c.

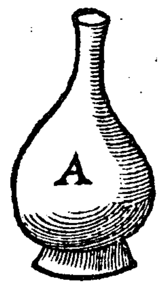
VIII EYSCH. XXI VOORSTEL.

WESENDE ghegheuen een water onbekender grootheyt, maer bekender swaerheyt: Sijn grootheyt duer sijn eyghenwicht te vinden.

Geometricè. MERCKT. Men soude des waters grootheyt mueghen*Meetconstlick vinden naer de ghemeene reghel van dien, maer want het in cleyne menichvuldicheyt, Weeghconstlick ghereeder ende sekerder wercking is, voornamelick inde onghe schickte formen, wy sullense daer duer beschrijuen.

T'GHEGHEVEN. Laet A een water sijn diens grootheyt onbekent is, maer tis bekender swaerheyt, dat is (duer de 1^e^ bepaling deses boucx) dat sijn bekende grootheyt duer bekent ghewicht can gheuytet worden; ick neem dat een voet des selfden weghe 65 ℔.

T'BEGHEERDE. Wy moeten de grootheyt van A duer haer eyghenwicht vinden.



T'WERCK. Men sal t'water A weghen, t'welck ick neem beuonden te worden van 5 ℔, die ghedeelt duer de voornomde 65 ℔, comt $\frac{1}{13}$; dat is $\frac{1}{13}$, voets voor de begheerte grootte van A.

T'BEWYS. Anghesien t'water A 5 ℔ weeght, ende dat een voet des selfden weeght 65 ℔, ende dattet oueral eenvaerdigher swaerheyt is duer de 2^e^ begheerte, soo heeft sijn ghewicht sulcken reden tot 65 ℔, als sijn grootheyt tot een voet, maer 5 ℔ heeft tot 65 ℔, de reden van 1 tot 13, daerom sijn grootheyt heeft sulcken reden tot 1 voet, als 1 tot 13, de grootheyt dan des waters A is $\frac{1}{13}$ voets, t'welck wy bewysen moesten.

T'BESLVYT. Wesende dan ghegheuen een water onbekender grootheyt maer bekender swaerheyt, wy hebben sijn grootheyt duer sijn eyghenwicht gheuonden, naer den eysch.

IX EYSCH. XXII VOORSTEL.

WESENDE ghegheuen tweer lichamen redenen der grootheyt, eñ stofswaerheyt, eñ t'ghewicht vã t'een lichaem: T'ghewicht van t'ander te vinden.

T'GHE-

shall be in the bottom *DE*, I say that this point *I* is the required centre of gravity, the proof of which will be identical with those of the 18th and 19th propositions hereinbefore. CONCLUSION. The bottom in the water therefore being a rectilinear plane figure, etc.

PROBLEM VIII. PROPOSITION XXI.

Given a water of unknown volume, but known gravity [1]): to find its volume from its proper weight.

NOTE.

One might find the water's volume geometrically according to the common rule about this, but because with a small quantity the weighing method is quicker and surer, especially with irregular forms, we shall describe them by this latter method.

SUPPOSITION. Let *A* be a water whose volume is unknown, but whose gravity is known, i.e. (by the 1st definition of this book) its known volume can be expressed by the known weight. I assume that one foot of it weighs 65 lbs. WHAT IS REQUIRED TO FIND. We have to find the volume of *A* from its proper weight. CONSTRUCTION. The water *A* shall be weighed, which I take to be found 5 lbs; the latter, divided by the aforesaid 65 lbs, makes $\frac{1}{13}$, i.e. the required volume of *A* is $\frac{1}{13}$ foot. PROOF. Since the water *A* weighs 5 lbs, and one foot of it weighs 65 lbs, while it has uniform gravity throughout, by the 2nd postulate, its weight has to 65 lbs the same ratio as its volume to one foot. But 5 lbs has to 65 lbs the ratio of 1 to 13, therefore its volume has to 1 foot the ratio of 1 to 13. The volume of the water *A* therefore is $\frac{1}{13}$ foot, which we had to prove. CONCLUSION. Given therefore a water of unknown volume, but known gravity, we have found its volume from its proper weight, as required.

PROBLEM IX. PROPOSITION XXII.

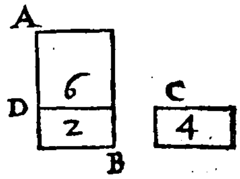
Given the ratio of the volumes and that of the specific gravities of two bodies, and the weight of the one body: to find the weight of the other.

[1]) This naturally means: specific gravity.

T'GHEGHEVEN. Laet A B t'een lichaem wesen, ende C t'ander, ende de reden der grootheyt van A B tot C, sy van 3 tot 1 ende der stofswaerheyt van 1 tot 2, ende A B weghe 6 ℔.

T'BEGHEERDE. Wy moeten t'ghewicht des lichaems C vinden.

T'WERCK. Ick teecken D B euegroot met C, de selue D B dan is het derdendeel van A B 6 ℔, daerom D B weeght 2 ℔, maer de stofswaerheyt van D B tot C, is als van 1 tot 2, daerom soo weeght C 4 ℔.



T'BEWYS. Laet C (soot mueghelick waer) meer dan 4 ℔ weghen; T'welck soo ghenomen huer swaerheyt sal meerder dan dobbel reden hebben tot de swaerheyt van D B, want D B weeght 2 ℔, ende veruolghens de stofswaerheyt van C (anghesien C ende D B euen groot sijn) sal in meerder dan dobbel reden sijn tot D B, t'welck teghen t'ghestelde is, daerom en weeght C niet meer dan 4 ℔. Sghelijcx salmen oock bethoonen dat sy niet min en weeght, sy weeght dan nootsakelick 4 ℔ t'welck wy bewysen moesten.

T'BESLVYT. Wesende dan ghegheuen t'weer lichamen redenen der grootheyt, ende stofswaerheyt, ende t'ghewicht van t'een lichaem, wy hebben t'ghewicht van t'ander lichaem gheuonden na den eysch.

VERVOLGH. Tis uyt het voorgaende openbaer dat,

Ghetrocken reden der grootheyt, van reden des ghewichts, rest reden der stofswaerheyt.

Ghetrocken reden der stofswaerheyt, van reden des ghewichts, rest reden der grootheyt.

Vergaert reden der stofswaerheyt, tot reden der grootheyt, comt reden des ghewichts.

WAER uyt blijckt dat een ghebrekende * pael der ses, duer de vijf ghegheuen palen altijt bekent can worden. Maer om t'selue by voorbeelt te verclaren, laet A weghen 6 ℔, ende groot sijn 5 voeten; ende t'ghewicht van B sy onbekent, maer huer grootheyt is van 2 voeten, ende de reden der stofswaerheyt van A tot B, sy van 4 tot 7. Nu om t'onbekende ghewicht van B te vinden, ick vergaer redẽ der stofswaerheyt, dat is *Reden* $\frac{4}{7}$ tot reden der grootheyt, dat is *Reden* $\frac{5}{2}$, comt reden des ghewichts *Reden* $\frac{10}{7}$, t'ghewicht dan van A heeft sulcken reden tottet ghewicht van B, als 10 tot 7

* *Terminus.*

Ghewichten.	6 ℔.	$4\frac{1}{3}$ ℔
Grootheden.	5 voet.	2 voet
Stofswaerheden.	4	7

SUPPOSITION. Let *AB* be the one body and *C* the other, and the ratio of the volume of *AB* to that of *C* shall be 3 to 1, and that of the specific gravities 1 to 2, and *AB* shall weigh 6 lbs. WHAT IS REQUIRED TO FIND. We have to find the weight of the body *C*. CONSTRUCTION. I draw *DB* as having the same volume as *C*; this *DB* is therefore one-third of *AB* (6 lbs), so *DB* weighs 2 lbs. But the specific gravity of *DB* to that of *C* is 1 to 2, therefore *C* weighs 4 lbs. PROOF. Let *C* (if this were possible) weigh more than 4 lbs. This being assumed, its gravity will be more than double of the gravity of *DB*, for *DB* weighs 2 lbs. And consequently the specific gravity of *C* (seeing that *C* and *DB* have the same volume) will be more than double of *DB*, which is contrary to the supposition. Therefore *C* does not weigh more than 4 lbs. In the same way it can also be shown that it does not weigh less. Therefore it necessarily weighs 4 lbs, which we had to prove.

CONCLUSION. Given therefore the ratio of the volumes and of the specific gravities of two bodies, and the weight of the one body, we have found the weight of the other body, as required.

COROLLARY.

It is manifest from the foregoing that

The ratio of the volumes being subtracted [1] *from the ratio of the weights, there remains the ratio of the specific gravities. The ratio of the specific gravities being subtracted from the ratio of the weights, there remains the ratio of the volumes. The ratio of the specific gravities being added to the ratio of the volumes, there comes the ratio of the weights.*

From this it is clear that if one of the six terms is unknown, it can always be made known from the five given terms. But in order to explain this with an example, let *A* weigh 6 lbs and be 5 feet, and the weight of *B* shall be unknown, but its volume is 2 feet, and the ratio of the specific gravities of *A* to *B* shall be 4 to 7. Now in order to find the unknown weight of *B*, I add the ratio of the specific gravities, i.e. ratio $\frac{4}{7}$, to the ratio of the volumes, i.e. ratio $\frac{5}{2}$, which yields the ratio of the weights: ratio $\frac{10}{7}$. The weight of *A* therefore has to the weight of *B* the ratio 10 to 7. But *A* weighs 6 lbs. Therefore I say: 10 gives 7, what 6 lbs? the weight of *B* becomes $4\frac{1}{5}$ lbs.

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[1] In the theory of ratios current in Stevin's days, subtraction of ratios meant division of the corresponding fractions, addition of ratios meant multiplication of the fractions. By subtracting the ratio $a : b$ from the ratio $c : d$ we therefore obtain the ratio $cb : ad$; by adding it to $c : d$, we get $ac : bd$.

tot 7, maer A weeght 6 ℔, daerom ſeg ick 10 gheeft 7, wat 6 ℔? comt voor t'ghewicht van B $4\frac{1}{5}$ ℔.

LAET ten tweeden de grootheyt van B onbekent ſijn, welcke wy duer d'ander vijf palen vinden willen. Ick treck reden der ſtofſwaerheyt, dat is *Reden* $\frac{4}{7}$, van reden des ghewichts, dat is *Reden* $\frac{10}{7}$, reſt reden der grootheyt *Reden* $\frac{5}{2}$; de grootheyt dan van A, heeft ſulcken reden tot de grootheyt van B, als 5 tot 2, maer A is groot 5 voeten, daerom ſeg ick 5 gheeft 2 wat 5 voeten? comt voor B 2 voeten.

LAET ten laetſten de reden der ſtofſwaerheyt onbekent ſijn, welcke wy door d'ander twe ghegheuen redenen bekent willen maken. Ick treck reden der grootheyt, dat is *Reden* $\frac{5}{2}$, van reden des ghewichts, dat is *Reden* $\frac{10}{7}$, reſt reden der ſtofſwaerheyt van 4 tot 7.

Dit voorſtel is ghemeen ouer alle ſtoffen, doch ſchijnt ſijn grootſte ghebruyck in * waterſche verſchillen te beſtaen.

Aquaticis quaestionibus.

T'EINDE DER BEGHINSELEN DES WATERWICHTS.

Secondly, let the volume of B be unknown, which we wish to find from the other five terms. I subtract the ratio of the specific gravities, i.e. ratio $\frac{4}{7}$, from the ratio of the weights, i.e. ratio $\frac{10}{7}$; the remainder is the ratio of the volumes: ratio $\frac{5}{2}$. The volume of A therefore has to the volume of B the ratio 5 to 2. But A is 5 feet. Therefore I say: 5 gives 2, what 5 feet? B becomes 2 feet.

Lastly, let the ratio of the specific gravities be unknown, which we wish to make known from the other two given ratios. I subtract the ratio of the volumes, i.e. ratio $\frac{5}{2}$, from the ratio of the weights, i.e. ratio $\frac{10}{7}$; the remainder is the ratio of the specific gravities: 4 to 7.

This proposition is common to all substances, but it seems to be used most in questions relating to waters.

THE END OF THE ELEMENTS OF HYDROSTATICS.

ANVANG DER WATERWICHTDAET,

BESCHREVEN DVER SIMON STEVIN van Brugghe.

ANDEN LESER.

NADIEN *hier vooren beschreuen sijn de Beghinselen des Waterwichts, soo soudet betamelick sijn, dat beken ick, de Waterwichtdaet te volghen, van sulcx als wy daer af connen verclaren; maer hebben om seker redenen gheschict, dat voor t'eerste niet schriftelick, maer werckelick te laten gheschien: Alleenlick sullen hier drie voorstellen setten, die opentlick uyt het voorgaende volghen, welcke ons niet weerdich dunckende den naem van Waterwichtdaet te verstrecken, doch ghemeenschap daer mede hebbende, wy noemense Anuang van dien. De selue beminde Leser belieue v int goede te nemen, ende de rest t'sijnder tijdt te verwachten.*

HOE

PREAMBLE OF THE PRACTICE OF HYDROSTATICS

Described by Simon Stevin of Brugghe.

TO THE READER.

Since in the foregoing there have been described the Elements of Hydrostatics, it would, I confess, be appropriate for the Practice of Hydrostatics to follow, in as far as we can explain it. But for certain reasons we have arranged this not to be done in writing for the present, but in actual fact. We shall only give three propositions which follow manifestly from the foregoing, and because we do not deem them worthy of the name of Practice of Hydrostatics, though they are connected therewith, we call them Preamble thereof. Dear reader, do not take this amiss, and expect the rest in due time.

HOE t'ghewicht van een schip met al datter in ende op is, oft van eenich lichaem int water driuende, bekent wort duer de bekende grootheyt des deels int water liggende, sulcx is uyt het 6e voorstel openbaer ghenouch, daerom sullen wy dat ouerslaen, ende wat segghen van t'ghene uyt het 7e volght, aldus.

I VOORSTEL.

TE vinden hoe veel een selfde lichaem dat stoflichter is als water, in t'een water dieper sijncken sal als int ander dat stofswaerder is.

LAET by voorbeelt een schip ligghen inden Rhijn te Leyen, ende men wil weten hoe veel dattet daerin dieper sincken sal dan in See voor Catwyck. Men sal ondersoucken de reden der stofswaerheyt van dat water tot dit, welcke sy als van 42 tot 43, soo heb ickse in Hoymaent duer d'eruaring beuonden, want nemende twee euegroote lichamen, dat vanden Rhijn wouch 4260 azen maer t'Seewater 4362 azen, t'welck na ghenouch is als van 42 tot 43.

Daerom salmen segghen, de grootheyt des deels van dat schip onder water in den Rhijn is tot de grootheyt van sulck deel onder water in See voor Catwyck, als van 43 tot 42, waer uyt den * Meter naer gheleghentheyt der form des voorghestelden schips, dese diepte tot die sal connen oirdeelen. waer af de nootsaecklicheyt * Wisconstlick blijckt int 7e voorstel der beghinselen des Waterwichts.

Geometra. *Mathematicè.*

II VOORSTEL.

DVER daetlicke voorbeelden te verclaren het 10e voorstel der beghinselen des Waterwichts.

WY hebben int 10e voorstel der beghinselen des Waterwichts, int 5e vervolgh Wisconstlick bewesen, dat den bodem des waters aldaer E F, duer een grooter water (d'hoochde de selfde blijuende) niet meer beswaert en wort dan duer een cleinder, ende weder verkeert, datse duer een cleinder water soo seer beswaert wort, als duer een grooter: Maer want den menighen dat voor onnatuerlick mocht achten, sullen bouen t'voorgaende Wisconstich bewys, daeraf vijf daetlicke voorbeelden beschrijuen, welcke yghelijck versoucken, ende ooghenschijnlick sien mach.

Ie VOORBEELT.

LAET den bodem A B euen ende ghelijck sijn anden bodem C D, ende de hoochde des waters op A B als E F, sy euen ande hoochde des waters

How the weight of a ship with all that is in or on it, or of any body floating in the water, becomes known from the known volume of the part lying in the water, this is sufficiently manifest from the 6th proposition. We will therefore omit that, and say something about what follows from the 7th, as follows.

PROPOSITION I.

To find how much deeper the same body, which is of greater specific levity than water, will sink in one kind of water than in another, which is of greater specific gravity.

For example, let a ship lie in the Rhine at Leyden, and it is required to know how much deeper it will sink therein than it does in the sea off Katwijk. The ratio of the specific gravity of the former water to the latter shall be ascertained, which shall be 42 to 43; this is the ratio I have found in Hay Month [1]) by experience, for taking two bodies of equal volume, that of the Rhine weighed 4,260 azen [2]), but the seawater 4,362 azen, which is substantially 42 to 43.

Therefore it shall be said that the volume of the part of the ship under water in the Rhine is to the volume of such part under water in the sea off Katwijk as 43 to 42. From this the geometer will be able to judge, according to the form of the ship in question, the ratio of the latter depth to the former, the necessity of which is proved mathematically in the 7th proposition of the elements of Hydrostatics.

PROPOSITION II.

To explain by practical examples the 10th proposition of the elements of Hydrostatics.

We have proved mathematically in the 10th proposition of the elements of Hydrostatics, in the 5th corollary, that the bottom in the water there, *EF*, is not subject to any heavier pressure from a larger quantity of water (the height remaining the same) than from a smaller, and also the reverse: that it is subject to the same pressure from a smaller quantity of water as from a larger. But because many people may consider this unnatural, we will, in addition to the foregoing mathematical proof, describe five practical examples thereof, which anyone may test and see with his own eyes.

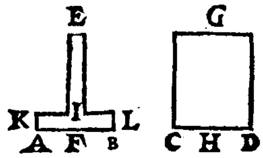
EXAMPLE I.

Let the bottom *AB* be equal and similar to the bottom *CD*, and the height of the water on *AB*, viz. *EF*, shall be equal to the height of the water on *CD*, viz.

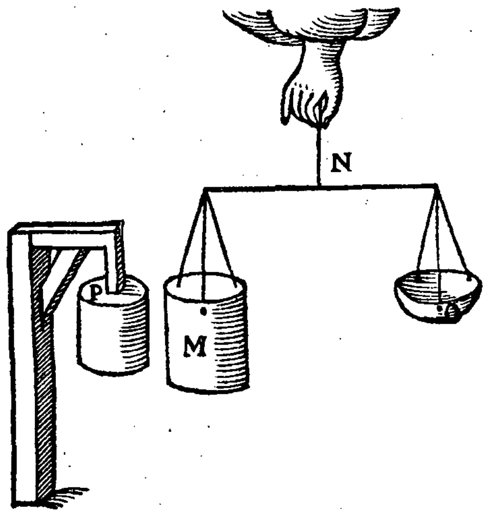
[1]) Haymonth is August.

[2]) The „aas" is an ancient unit of weight. The word is derived from Latin *as*, unit of a system. The value of an „aas" is 46 or 48 mg. See K. M. C. Zevenboom and Dr D. A. Wittop Koning, *Nederlandse gewichten. Stelsels, ijkwezen, vormen, makers en merken*. Leiden 1953, p. 147.

waters op C D, als G H; maer het deel I E bouen t'water K L B A ſtaende, ſy cleender dan alſulck deel des lichaems G C D, ende t'water van E A B weghe 1 ℔, ende van G C D 10 ℔, ende de form van G C D ſy een rond: pilaer, ende t'water G C D ſal thienmael grooter en ſwaerder ſijn dan t'water E A B, nochtans ſegghen wy t'ghewicht des waters E A B, euen ſoo ſtijf te drucken opden grondt A B, als t'ghewicht des waters G C D opden bodem C D. T'welck aldus daetlick beweſen wort:



Laet M N O een waegh ſijn, diens ſchalen M, O, welcker ſchalen M vande form eens pilaers ſy, euen ende ghelijck an t'vat hier bouen G C D, ende ſal houden 10 ℔ waters; Laet oock P een houten lichaem weſen, vaſt ſtaende als hier neuen, eñ lijckformich ande ſchael M, maer ſoo veel cleender datment daerin ſteken can ſonder erghens ande ſchael te ghenaken.



Laet nu t'lichaem P ghesteken worden inde ſchael M, als in dees tweede form, ende inde ſchael O ſy gheleyt t'ghewicht Q van 10 ℔, ende den bodem der ſchael M ſal ſoo ſtijf ghenaken teghen t'onderſte des lichaems P, als de 10 ℔ van Q veroirſaken. Ick neem nu dat de ledighe plaets tuſſchen t'lichaem P ende de ſchael M, ghevult can worden met 1 ℔ waters, dat is met een lichaẽ waters euegroot an t'lichaem E A B; Daerom 1 ℔ waters in die ledighe plaets ghegoten, ſal de ſchael M doen dalen, eñ O doen rijſen, ſo deruaring dat betuyghen ſal, ende ghelijck de redenen daer af oock openbaer ſijn duer t'boueſchreuen 10ᵉ voorſtel. Dat 1 ℔ waters dan in die ſchael M gheleyt, ſal daer in ſo grooten macht doen, als 10 ℔ ghewichts van loot yſer ofte eenighe ander ſtijue ſtof ande ſchael M ghehecht. Ende om de ſelue reden ſal 1 ℔ waters, alſo connen meer ghewelts doen dan duyſent ponden ander ghewicht. Dit ſoo ſijnde daer is water tuſſchen den bodẽ der ſchael M eñ t'onderſte des lichaẽs P,

 teghen

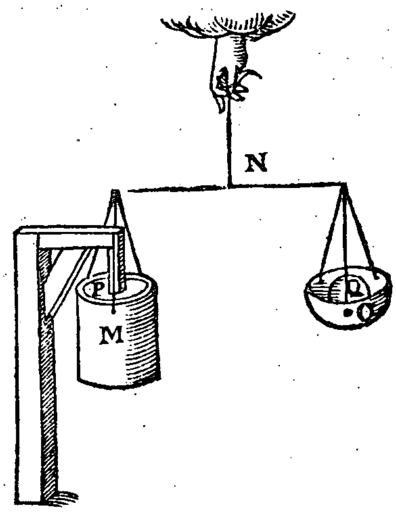
GH. But the part *EI* standing above the water *KLBA* shall be smaller than such part of the body *GCD*, and the water of *EAB* shall weigh 1 lb and that of *GCD* 10 lbs, and the form of *GCD* shall be a circular prism, and the water *GCD* shall be ten times greater and heavier than the water *EAB*. Nevertheless we say that the weight of the water *EAB* exerts the same pressure on the bottom *AB* as the weight of the water *GCD* on the bottom *CD*. This is proved in practice as follows:

Let *MNO* be a balance, whose pans are *M* and *O*, of which pans *M* shall have the form of a prism, equal and similar to the vessel *GCD* above, and it shall contain 10 lbs of water. Let *P* also be a wooden body, fixed as shown opposite and similar to the pan *M*, but so much smaller that it can be put therein without touching the pan anywhere.

Now let the body *P* be put in the pan *M*, as shown in the second figure, and in the pan *O* there shall be laid the weight *Q* of 10 lbs, and the bottom of the pan *M* shall touch the bottom of the body *P* as strongly as is caused by the 10 lbs of *Q*. I now assume that the empty space between the body *P* and the pan *M* can be filled with 1 lb of water, i.e. with a body of water having the same volume as the body *EAB*. Therefore, if 1 lb of water be poured into that empty space, this will cause the pan *M* to descend and *O* to ascend [1]), as experience will show, and as the reasons thereof are also manifest from the 10th proposition described above. The 1 lb of water put in the pan *M* will therefore exert thereon the same force as 10 lbs of lead, iron or some other solid material attached to the pan *M*. And for the same reason 1 lb of water will thus be able to exert a greater force than one thousand pounds of weight in another form. This being so, there is water between the bottom of the pan *M* and the bottom of the body *P*, against

[1]) It is evident that if the bottom of *M* is completely covered with water, one pound of water in *M* will balance 10 pounds in *Q*, but not that *M* will descend and *O* ascend.

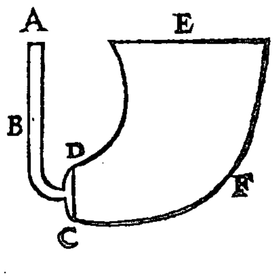
teghen welck water den bodem van M nu soo stijf druck, als sy eerst teghen t'onderste des lichaems P stack, want t'selue ghewicht Q ligt noch in d'ander schael O; Maer sy stack eerst soo stijf daer teghen als 10 ℔ van Q veroirsaeckten, daerom den bodem van M steeckt soo stijf teghen t'water als de 10 ℔ van Q veroirsaken, ende, weder verkeert, t'water steeckt soo stijf teghen den bodem M, als die 10 ℔ van Q veroirsaecken. Laet ons nu nemen dattet water opden bodem M ligghende, euegroot sy an t'water K L B A, ende de rest rondtom t'licham P staende, euegroot mettet water I E; t'water dan E A B, druckt euen soo stijf teghen den grondt A B, als dit water tegen den grondt M, maer dit druckt soo stijf als 10 ℔, soo bouen bethoont is, dat water E A B dan, druckt oock soo stijf



teghen den grondt A B als 10 ℔, ende so stijf druck oock t'water G C D teghen den grondt C D: Daerom soo wy voorghenomen hadden daetlick te bewysen, t'water E A B weghende 1 ℔, druckt euen soo stijf teghen sijn grondt A B, als t'water G C D weghende 10 ℔, teghen sijn grondt C D. Ende ghelijck wy hier bewesen hebben 1 ℔ so stijf te drucken als 10 ℔, alsoo salmen oock bewysen 1 ℔ stijuer te connen drucken als duysent ponden.

II VOORBEELT.

Laet A B C D een cleen dun buysken sijn, eñ C D E F een groot dick vat afghesondert van t'buysken, met een ghemeene bodem C D, ende beyde vol waters, alsoo dat der wateren oppervlacken in een selfde weereltvlack sijn. Nu dat het groot water des vats C D E F, niet stijuer en druckt teghen den bodem C D, dan t'cleyne water der cleyne buys, blijckt daetlick aldus: Laet gheweert worden den bodem D C, eñ t'groot water sal op die plaets teghen t'cleynste stooten: Nu soo t'water C D E F, van te vooren stijuer ghestooten had teghen den bodem



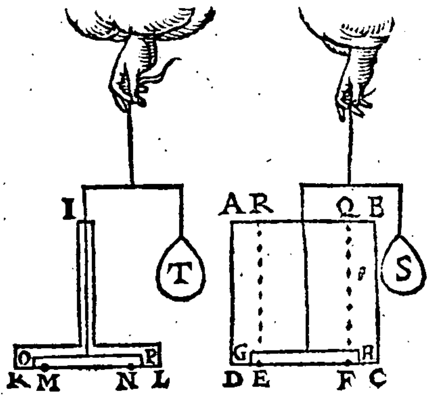
D C, dan t'water A B C D, soo salt nu oock stijuer stooten teghen dat which water the bottom of *M* now exerts as strong a pressure as it first did against the bottom of the body *P*, for the same weight *Q* still lies in the other pan *O*. But it first exerted against it a pressure such as was caused by 10 lbs of *Q*; therefore the bottom of *M* exerts against the water a pressure such as is caused by the 10 lbs of *Q*, and also the reverse: the water exerts against the bottom *M* a pressure such as is caused by the 10 lbs of *Q*. Let us now assume that the water lying on the bottom *M* be equal in volume to the water *KLBA*, and the remainder surrounding the body *P* be equal in volume to the water *IE*. The water *EAB* therefore exerts against the bottom *AB* the same pressure as this water against the bottom *M*. But the latter exerts a pressure of 10 lbs, as has been shown above. The water *EAB* therefore also exerts a pressure of 10 lbs against the bottom *AB*, and the water *GCD* exerts the same pressure against the bottom *CD*. Therefore, as we had proposed to prove by practical examples, the water *EAB* weighing 1 lb exerts against its bottom *AB* the same pressure as the water *GCD* weighing 10 lbs against its bottom *CD*. And just as we have here proved 1 lb to exert the same pressure as 10 lbs, it can also be proved that 1 lb can exert a greater pressure than a thousand pounds.

EXAMPLE II.

Let *ABCD* be a small, thin tube, and *CDEF* a large, wide vessel, separated from the tube, with a common bottom *CD* and both full of water, in such a way that the upper surfaces of the waters are in the same world surface. Now that the large water of the vessel *CDEF* does not exert any greater pressure against the bottom *CD* than does the small water of the small tube, appears in practice as follows: Let the bottom *DC* be taken away; then the large water will thrust against the smaller in that place. Now if the water *CDEF* had previously exerted a greater pressure against the bottom *DC* than the water *ABCD*, it will now also dat water dan dat teghen dit: waer duer t'cranckste voor t'sterckste sal moeten wycken, dat is, t'water A B C D sal rijsen, ende van C D E F sal dalen; Maer dit so wesende, haer oppervlacken en sullen niet euen hooch sijn t'welck opentlick teghen d'eruaring is. Daerom t'cleinste water A B C D druckt euen soo stijf teghen den bodem C D, als t'grootste water C D E F.

III VOORBEELT.

Laet A B C D een vat vol waters sijn, in wiens bodem D C euewydich ligghende vanden sichteinder, een rondt gat E F is waer op ligt een ronde houten schijf G H, stoflichter dan water, ende dat gat E F bedeckende, eñ rondtom dicht sluytende teghen den bodem D C. Laet oock I K L een ander vat vol waters sijn, euenhooch mettet vat A B C D, maer cleinder, in wiens bodem K L oock een rondt gat M N sy, euen an t'gat E F, waer op light een schijf O P, euegroot ende eueswaer ande schijf G H: T'welck soo wesende, d'eruaring sal bethoonen dat de schijf G H niet rijsen en sal, naer de ghemeenen aert des hauts in t'water, maer sal so stijf op t'gat E F drucken, als een ghewicht eueswaer an t'water dat euegroot is anden pilaer E F Q R, min t'verschil des ghewichts der houte schijf G H, tot het ghewicht des waters an die schijf euegroot. Maer om sulcx duer de daet oock te sien, men mach ande schijf G H een waegh voughen, diens ghewicht S eueswaer sy an dat voornomde ghewicht, ende de schijf G H sal daer teghen euewichtich blijuen. Laet nu insghelijcx ande schijf O P oock een waegh voughen. diens ghewicht T eueswaer sy an S, ende de schijf O P sal daer teghen oock euewichtich blijuen. Maer soomen S ende T yet swaerder maeckt, sy sullen haer schijuen doen rijsen, inder voughen dat de schijuen G H, O P, duer sulcke euewichten beuonden worden euestijf teghen haer bodems te drucken, waer uyt het voornemen blijckt, te weten het cleinder water I K L, euen so stijf teghen sijn grondt te drucken, als t'grooter A B C D.



MERCKT.

Tis kennelick dat so t'verschil des ghewichts der schijf als G H, tot het ghewicht des waters an haer euegroot, meerder waer dan t'ghewicht des exert a greater pressure against the former water than that against this, as a result of which that which is weaker will have to yield to that which is stronger, i.e. the water *ABCD* will ascend and that of *CDEF* will descend. But this being so, their upper surfaces will not be on a level, which is manifestly contrary to experience. The smaller water *ABCD* therefore exerts the same pressure against the bottom *CD* as the larger water *CDEF*.

EXAMPLE III.

Let *ABCD* be a vessel full of water, in whose bottom *DC*, which is parallel to the horizon, there is a round hole *EF*, on which there lies a round wooden disc *GH* of greater specific levity than water and covering that hole *EF* and closely fitting all round the bottom *DC*. Let also *IKL* be another vessel full of water, of the same height as the vessel *ABCD*, but smaller, in whose bottom *KL* there be also a round hole *MN*, equal to the hole *EF*, on which there lies a disc *OP* of the same volume and weight as the disc *GH*. This being so, experience will show that the disc *GH* will not rise, in accordance with the common nature of wood in water, but will exert on the hole *EF* the same pressure as a weight of equal gravity to the water which is equal in volume to the prism *EFQR*, minus the difference between the weight of the wooden disc *GH* and the weight of the water having the same volume as that disc. But in order to see this also in practice, there can be applied to the disc *GH* a balance whose weight *S* shall be of equal gravity to the aforesaid weight, and the disc *GH* will be in equilibrium therewith. Let there now, in the same way, also be applied to the disc *OP* a balance whose weight *T* shall be of equal gravity to *S*, and the disc *OP* will also be in equilibrium therewith. But if *S* and *T* be made a little heavier, they will cause their discs to rise, in such a way that the discs *GH*, *OP* are found by such equilibria to exert the same pressure against their bottoms, from which the fact intended to be proved is evident, to wit that the smaller water *KL* exerts the same pressure against its bottom as the larger *ABCD*.

NOTE.

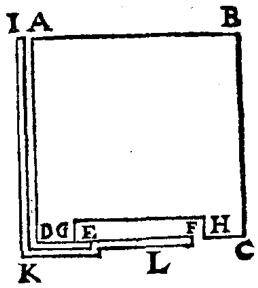
It is evident that if the difference between the weight of the disc as *GH* and the weight of an equal volume of water were greater than the weight of the waters euegroot anden pilaer, als E F Q R, sulcken schijf en soude op t'gat als E F niet connen rusten, maer soude noodsakelick oprijsen.

T'is oock blijckelick dat soo de schijf al G H stofswaerder waer dan water, als van loot, yser, &c. datse dan op t'gat E F soo stijf drucken soude, als een ghewicht des waters euegroot anden pilaer E F Q R, meer t'verschil des ghewichts der schijf, tottet ghewicht des waters an haer euegroot.

Maer waer de schijf G H euestofswaer an t'water, tis openbaer datse dan effen so stijf op t'gat E F drucken soude, als een ghewicht des waters euegroot anden pilaer E F Q R.

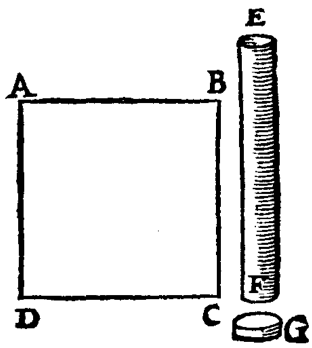
IIII^e VOORBEELT.

Laet A B C D een vat vol waters sijn, met een gat E F inden grondt C D, daerop een schijf G H light, stoflichter dan t'water, de selue sal op t'gat E F so stijf pranghen als vooren bewesen is. Laet oock, I K L een cleen dun buysken wesen, diens opperste gat I inde selfde hoochde van A B sy, ende t'onderste gat sy E F: Daer naer dit buysken vol waters ghegoten, dat cleen water sal soo groot ghewelt doen teghen den grondt des schijfs G H, als al t'water dat in t'vat A B C D is, want de schijf G H sal rijsen. Inder voughen dat 1 ℔ waters (duer t'welck ick neem de buys I K L te mueghen ghevult worden) meer ghewelts sal connen doen teghen de schijf G H, dan hondert duysent ponden als S hier vooren, t'welckmen der naturen verborghenheyt soude mueghen noemen dat d'oirsaken onbekent waren.



V VOORBEELT.

Om nu oock werckelick betooch te gheuen ouer de voorbeelden alwaer t'water opwaert teghen den bodem steeckt, als int 3^e veruolgh des voornomden 10^en voorstels, so laet A B C D een water sijn, ende E F een dichte buys, ende G een schijf stofswaerder dan water, ick neem van loot, als in dese eerste form.



Laet

water having the same volume as the prism *EFQR*, this disc could not rest on the hole *EF*, but would of necessity ascend.

It is also evident that if the disc *GH* were of greater specific gravity than water, for example if it were lead, iron, etc., it would then exert on the hole *EF* the same pressure as the weight of the water having the same volume as the prism *EFQR*, plus the difference between the weight of the disc and the weight of the water having the same volume.

But if the disc *GH* were of equal specific gravity to the water, it is manifest that it would then exert on the hole *EF* the same pressure as the weight of the water having the same volume as the prism *EFQR*.

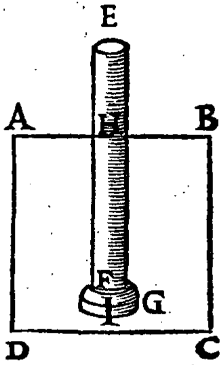
EXAMPLE IV.

Let *ABCD* be a vessel full of water, with a hole *EF* in the bottom *CD*, on which there lies a disc *GH*, of greater specific levity than the water; this will exert on the hole *EF* the pressure that has been proved before. Let also *IKL* be a small, thin tube, whose upper opening *I* shall be on a level with *AB*, and the lower opening shall be *EF*. If thereafter this tube is filled with water, the small water will exert the same pressure on the base of the disc *GH* as all the water that is in the vessel *ABCD*, for the disc *GH* will ascend. In such a way that 1 lb of water (with which I take that the tube *IKL* can be filled) will be able to exert a greater pressure against the disc *GH* than a hundred thousand pounds, as *S* above, which might be called one of the secrets of Nature, if the cause were unknown.

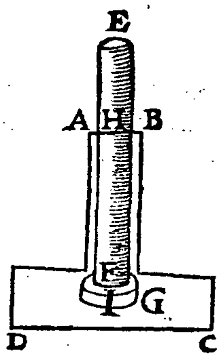
EXAMPLE V.

In order to give a practical explanation about the examples in which the water exerts an upward thrust against the bottom, as in the 3rd corollary of the aforesaid 10th proposition, let *ABCD* be a water, and *EF* a closed tube, and *G* a disc of greater specific gravity than water, say of lead, as in the first figure.

Laet deſe ſchijf G gheleyt worden teghen t'gat F, alſoo datſe dicht daer op pas, ende de buys met de ſchijf dan alſoo t'ſamen in t'water A B C D gheſteken, ick neem tot H toe, als hier onder, de ſchijf G en ſal naer den ghemeenen aert des loots, in t'water niet ſincken, maer ande buys blijuen hanghen, ende daer teghen ſoo ſtijf drucken, als een ghewicht eueſwaer an t'water dat euegroot is anden pilaer, diens grondt de groote des gats F, ende hoochde H I is, min t'verſchil des ghewichts der ſchijf G, tot t'ghewicht des waters an die ſchijf euegroot. Maer ſoo de ſchijf G niet dicht ghenouch teghen de buys en ſlote, ende datter eenich water indrong, ſoo ſal de ſchijf G daer ſoo langhe anhanghen, tot dat ſulck inghedronghen water t'voornomde ghewicht ouerwint.



Maer want nu yemandt dencken mocht, het groot ſwaer water rondom de buys ſtaende, ſtijuer drucking der ſchijf teghen de buys te veroirſaken, dan een cleender water van de ſelfde hoochde, ſoo laet ons t'water neuen de buys rondom afcorten, dat is, dat de reſte water ſy in een vat van form als hier neuen, ende d'eruaring ſal bewyſen (verſouckende de macht des gheprangs in t'een en t'ander water, duer euewichten inde buys op G ruſtende) dit cleen water euen ſoo ſtijf te drucken als t'voornomde grooter, waer af de reden bouen grondelick beſchreuen is.



T'BESLVYT. Wy hebben dan duer daetlicke voorbeelden het 10ᵉ voorſtel der beghinſelen des Waterwichts verclaert, naer t'voornemen.

MERCKT.

Wat het 11ᵉ voorſtel belangt, daer uyt is onder anderen kennelick, wat ghewicht waters datter druct, teghen elcke ſijde der duer van een ſluys, ende dierghelijcke: Oock dattet water ouer d'een ſijde alleenlick een ſtroobreet, daer teghen ſoo ſtijf prangt als t'water diens breede de Zee van Oceane ouer d'ander ſijde; Welverſtaende als ſy euenhooghe ſijn. Van welcke dinghen wy om haer voornoemde claerheyt hier gheen beſonder voorſtellen en maken.

Hh 3   111 VOOR-

Let this disc *G* be laid against the hole *F*, in such a way that it fits closely thereto, and if then the tube together with the disc be put in the water *ABCD* — I take as far as *H*, as shown below —, the disc *G* will not, in accordance with the common nature of lead, sink into the water, but cling to the tube and exert against it the same pressure as a weight of equal gravity to the water having the same volume as the prism whose base has the size of the hole *F* and whose height is *HI*, minus the difference between the weight of the disc *G* and the weight of the water having the same volume as that disc. But if the disc *G* should not fit closely enough against the tube, and some water should enter into it, the disc *G* will cling to it until this entering water shall overcome the aforesaid weight.

But because someone might suppose that the large, heavy water surrounding the tube would cause a greater pressure of the disc against the tube than a smaller water of the same height, let us take away the water all around the tube, i.e. so that the remainder of the water be in a vessel of a form as shown opposite. Then experience will prove (testing the force of the pressure in either water by means of equal weights in the tube resting on *G*) that this small water exerts the same pressure as the aforesaid larger water, the cause of which has been thoroughly described above. CONCLUSION. We have thus explained, by means of practical examples, the 10th proposition of the elements of Hydrostatics, as intended.

NOTE.

As regards the 11th proposition, from that it is evident, among other things, what is the weight of the water pressing against either side of the gate of a lock and the like. Also that the water on one side, even if it were only the width of a straw, exerts the same pressure against it as the water having the breadth of the Ocean on the other side, provided they are on the same level. Of these matters we do not draw up any special propositions, in view of their aforesaid clearness.

III VOORSTEL.

D'OIRSAECK te verclaren waerom een mensch diep onder t'water swemmende, niet doot gheprangt en wort, van t'groot ghewicht des waters op hem ligghende.

Laet een mensch 20 voeten diep onder water ligghen, weghende elcke voet waters 65 ℔, ende t'gheheel vlack sijns lichaems sy groot 10 voeten. Dit soo wesende, daer sal teghen sijn lijf perssen byde 13000 ponden ghewichts, duer het 10ᵉ ofte 11ᵉ voorstel vande beghinselen des Waterwichts. T'welck soo sijnde, hoe ist mueghelick, sal ymant segghen, dat sulcken ghewicht den mensch niet dooten druct? D'antwoort is daerop soodanich:

A. *Alle duwing die t'lichaem weedom andoet, verset eenich deel des lichaems uyt sijn natuerlicke plaets;*

O. *Dese duwing des waters en verset gheen deel des lichaems uyt sijn natuerlicke plaets;*

O. *Dese duwing des waters dan, en doet het lichaem gheen weedom an.*

Syllogismi minor. Des * bewysredens tweede voorstel is openbaer duer de daet, waer af de reden dese is: Soo eenich deel als vleesch, bloet, vochticheyt, wattet sy, uyt sijn natuerlicke plaets verset wierde, t'soude moeten plaets hebben daert in ghinghe, die plaets en is buyten t'lichaem niet, ouermidts t'water oueral euestijf anstoot (Angaende t'onderste deel een weynich stijuer gheprangt wort dan t'opperste, duer het 11ᵉ voorstel der Beghinselen des Waterwichts, dat en is in desen gheualle van gheender acht, want sulck verschil gheen deel uyt sijn natuerlicke plaets versetten en can) sy en is oock binnen t'lichaem niet, wanttet daer soo vol lichamelicheyts is als daer buyten, waer duer yder dit deel, soo stijf stoot teghen yder dat deel, als yder dat, teghen yder dit, ouermits t'water rondom t'lichaem tot allen sijden met een selue reden staet. Die plaets dan en is buyten t'lichaem niet, noch daer binnen, daerom nerghens, waerduer het onmeughelick is, dat eenich deel uyt sijn natuerlicke plaets ghebrocht worde, ende vervolghens t'lichaem en can daeraf gheen weedom ontfaen.

Maer om t'selue metter daet noch merckelicker te bewysen, laet A B C D een water sijn, hebbende inden grondt D C een gat, ghesloten met den tap E, ende opden seluen grondt ligghe een man F, met sijn rug op E: T'welck soo sijnde, daer en can van weghen t'ghewicht des waters op hem ligghende, gheen deel des lichaems uyt sijn natuerlicke plaets verset worden,

PROPOSITION III.

To explain the cause why a man, swimming deep below the water, is not crushed to death by the great weight of the water lying on him.

Let a man lie 20 feet below the water's surface, each foot of water weighing 65 lbs, and the whole area of his body shall be 10 feet. This being so, there will be exerted against his body a pressure of about 13,000 pounds, by the 10th or 11th proposition of the elements of Hydrostatics. Which being so, how is it possible, it may be said, that this weight does not crush the man to death? The answer to this as follows [1]):

A. *Any thrust which hurts the body moves some part of the body from its natural place;*

O. *This thrust of the water does not move any part of the body from its natural place;*

O. *Therefore this thrust of the water does not hurt the body.*

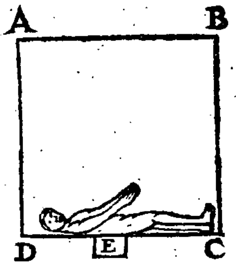
The second proposition of the syllogism is clear from practice, the reason of which is as follows: If any part, such as flesh, blood, fluid or whatever it be were moved from its natural place, it would have to find a place into which it might enter. That place is not outside the body, since the water exerts the same pressure against it on all sides (as to the fact that the lower part is subject to a somewhat greater pressure than the upper part, by the 11th proposition of the Elements of Hydrostatics, that is of no account in this case, for this difference cannot move any part from its natural place); nor is that place inside the body, for there it is as full of corporeity as outside, so that one part exerts the same pressure against the other as that against this, since the water round the body has the same position on all sides. That place therefore is neither outside the body nor inside it; therefore it is nowhere, owing to which it is impossible that any part should be moved from its natural place, and consequently the body cannot be hurt by it.

But in order to prove this even more clearly by experience, let *ABCD* be a water, having in the bottom *DC* a hole closed with the plug *E*, and on this bottom there shall lie a man *F* with his back on *E*. Which being so, it is not possible for any part of the body to be moved from its natural place by the weight of the water

[1]) See note 2 to p. 143.

worden, ouermits t'water an allen sijden euestijf anstoot, als vooren ghefeyt is.

Maer wildi nu daetlick sien dit de waerachtighe oirsaeck te wesen, so treckt den tap E uyt, ende dan en sal teghen sijn rug an E gheen stootsel sijn, als an alle d'ander plaetsen sijns lichaems, daerom oock sal het lichaem daer prangsel lijden, ende dat soo stijf als int derde voorbeelt des 2en voorstels van desen betoocht is; te weten soo stijf, als veroirsaeckt wort duer t'ghewicht des pilaers waters, diens gront het gat E is, ende hoochde, de hoochde des waters bouen hem, waermede t'voornemen opentlick bewesen is.



T'EINDE DES ANVANGS DER WATERWICHTDAET.

lying on him, since the water exerts the same pressure on all sides, as has been said before.

But if you desire to see by experience that this is the true cause, pull out the plug *E*, then there will be no pressure against his back at *E* such as there is at all the other places of his body. Therefore the body will there be subjected to pressure, and this as strongly as has been shown in the third example of the 2nd proposition, i.e. to a pressure such as is caused by the weight of the prism of water whose base is the hole *E* and whose height is the height of the water above him, with which the fact intended to be proved is clearly evinced [1]).

THE END OF THE PREAMBLE OF THE PRACTICE OF HYDROSTATICS

[1]) This experiment calls forth the same sceptical comment that was made by Boyle on similar experiments of Pascal: more ingenious than practicable. R. Boyle, *Hydrostatical Paradoxes Made Out by New Experiments.* Works. London 1772. II 732-797.

ANHANG VAN DE WEEGHCONST

APPENDIX TO THE ART OF WEIGHING

INTRODUCTION

It is one of Stevin's firm principles never to mingle his scientific argument with polemics. On the other hand there are several controversial questions on which he likes to express his opinion. Some of these are dealt with in the following *Appendix* to the *Art of Weighing*.

In the first chapter he combats the so-called principle of virtual displacements, which enables statics to be founded on dynamical principles. In the second he refutes various erroneous opinions on the motion of falling bodies in resistant media. In the third it is contended that the Art of Weighing is entitled to be considered an independent branch of mathematics on the same footing with Arithmetic and Geometry. In the fourth chapter certain objections to the use of numbers in proofs of statical propositions are refuted, and the final chapter contains some further elucidations on Prop. VIII of the *Elements of Hydrostatics* (Archimedes' principle).

Appendix.

ANHANG,

INDE WELCKE ONDER ANDEREN WEERLEYDT WORDEN ETLICKE DWALINGHEN DER wichtighe ghedaenten.

ANDEN LESER.

Argumentis. *MY ghedenckende van t'mishaghen dat ick somwylen ghehadt heb inde * strijtredens ettelicker schrijuers, welcke ghedreuen van haer ghemoet, ander persoonens dwalinghen in consten so verachtelick berispten, dat sy daermede een ghetuych gauen, van haer veel slimmer dwalinghen inde seden; ende dat my daer beneuen ouervloedighe* stof ontmoet was, om te connen weerlegghen veel dolinghen vande wichtighe ghedaenten duer sommighe beschreuen: Heb ghevreest int verclaren der seluer, den Leser van my een vermoeden te mueghen gheuen, van sulcx als my in anderen misuiel. Nochtans achtende hier beneuen, dattet gantschelick verswyghen (want wy met voorset daer af inde voorgaende boucken niet en hebben willen r eren, om de leering met gheen * strijding te verduysteren) den sommighen eenich misverstant ende achterdeel mocht veroirsaken, heb my ghepoocht naer t'middel te trachten, ende inde plaets van veel besonder dwalinghen, alleen haer ghemeene oirspronck duer de twee eerstvolghende * hoofstlicken te verclaren niet tot vermindering des naems van so weerdigen schrijuers, maer veel eer om die met danckbaerheyt te helpen vermeerderen, als van beweghende oirsaken haerder nacommers, sonder welcke veel besonderheden dickmael ongheroert souden ghebleuen hebben.*

Materia. *Argumentatione.* *Capita.*

1 HOOFT-

APPENDIX

IN WHICH, AMONG OTHER THINGS, MANY ERRORS ABOUT THE QUALITIES OF WEIGHTS ARE REFUTED.

TO THE READER.

Bearing in mind the displeasure I have sometimes had in the arguments of many writers who, moved by their feelings, censured the scientific errors of other persons so contemptuously that by this they testified to their own — much worse — errors in manners, and further having found abundant material to refute many errors about the qualities of weights described by some writers, I feared lest in pointing them out I might give the Reader cause to suspect me of the same thing that had displeased me in others. Further considering nevertheless that complete silence (for we intentionally refrained from referring thereto in the preceding books, in order not to obscure the theory with argumentations) might cause misunderstanding and disadvantage to some people, I have tried to steer a middle course, and instead of many particular errors point out only their common origin in the two following chapters, not in order to throw discredit upon the reputation of such worthy writers, but rather in order to aggrandize it by gratitude, because they are the moving causes for their successors, without which many particulars might often not have been referred to at all.

CHAPTER I, that the cause of bodies being of equal apparent weight does not reside in the circles described by the extremities of the arms.

The reasons why equal gravities at equal arms are of equal apparent weight are known by common knowledge, but not so the cause of the equality of apparent weight of unequal gravities at unequal arms proportional [1]) thereto, which cause the Ancients, when they inquired into it, considered to reside in the circles de-

[1] Read: inversely proportional.

Ie HOOFTSTIC, DAT DER EVESTALTWICHTIGHEN OIRSAECK NIET EN SCHVYLT *onder de ronden beschreuen met d'uytersten der ermen.* *Caput 1.*

DE redenen waerom euen swaerheden an euen ermen eueſtaltwichtich ſijn, is duer ghemeene wetenſchap bekent, maer niet alſo d'oirſaeck der eueſtaltwichticheyt van oneuen ſwaerheden an oneuen ermen met haer * eueredenich, welcke oirſaeck d'ouden onderſouckende, hebben die gheacht te ſchuylen onder de ronden beſchreuen duer d'uyterſten der ermen, als blijckt by Ariſtoteles *in Mechanicis* met ſijn nauolghers; T'welck wy ontkennen ende reden daer af aldus gheuen: *Proportionales.*

E. *Dat ſtil hangt en beſchrijft gheen rondt;*
A. *Twee eueſtaltwichtighe hanghen ſtil;*
E. *Twee eueſtaltwichtighe dan en beſchrijuen gheen rondt.*

Ende veruolghens ſoo en iſſer gheen rondt; Maer alwaer gheen rondt en is daer en can t'rondt het ghene niet weſen daer eenige oirſaeck onder ſchuylt, waer duer de ronden hier t'ghene niet en ſijn, daer d'oirſaeck der eueſtaltwichticheyt onder beſtaet. Angaende (op dat wy des * Bewyſredens tweede voorſtel verclaren) t'roerſel ofte te beſchrijuing der ronden welcke haer ooghenſchijnlick mach vertooghen, die en is niet eyghen der eueſtaltwichtighen, maer by gheualle, als duer windt, hurting, oft eenighe ander beweghing, met welcke niet alleen deſe, maer oock d'oneueſtaltwichtighe ronden connen beſchrijuen. Tis dan openbaer dat deſe oirſaeck in gheen ronden en beſtaet, maer onder t'ghene int 1e voorſtel des 1en boucx vande beghinſelen der Weeghconſt, daer af * Wiſconſtlick bethoocht is. Daerom die ſulcke dwaling voor ſeker grondt namen, ten is gheen wonder dat ſy, ſonder te comen tot kennis der oirſaecken, oock ſonder te crijghen form van Weeghconſt, hemlien in veel valſche voorſtellen oeffenden, die wy hier int beſonder ſouden connen weerlegghen, maer ſulcx laten om de redenen hier bouen verhaelt, te meer dat ſy duer haer contrari, als t'voornoemde warachtighe, ghenouch bekent ſijn. *Syllogiſmi minorem.* *Mathematicè.*

Men ſoude hier oock mueghen weerlegghen ettelicke voorſtellen van ſcheefwichten, beſchreuen duer Cardanus *lib. 5. De proportionib.* daer hy ſe raemt uyt ſeker houcken van linien oft platten, maer dat de ſelue ghemiſt ſijn, is openbaer ghenouch duer het Wiſconſtich bewys van ander * eueredenheyt, int 19e voorſtel des 1en boucx vande beghinſelen der Weeghconſt. *Proportione.*

IIe HOOFTSTICK, DAT DE GHEROERDEN MET HAER BELETSELEN IN *gheen * eueredenheyt en beſtaen.* *Proportione.*

WY hebben inde voorreden der Weeghdaet anden Leſer, gheſeyt, dat de gheroerden met haer beletſelen niet eueredenich en ſijn, oock

Ii aldaer

scribed by the extremities of the arms, as appears in Aristotle's *In Mechanicis* [1] and his successors [2].

This we deny, and we give the following reason therefor [3])

E. *That which hangs still does not describe a circle;*

A. *Two gravities of equal apparent weight hang still;*

E. *Therefore two gravities of equal apparent weight do not describe circles.*

And consequently there is no circle. But where there is no circle, the circle cannot be that in which resides any cause, so that the circles are not here that in which resides the cause of the equality of apparent weight. As regards (in order to explain the second proposition of the syllogism) the motion or the description of the circles which may appear to present itself, this is not peculiar to gravities of equal apparent weight, but accidental, as by wind, pushing or some other movement by which not only these, but also gravities of unequal apparent weight can describe circles. It is therefore manifest that this cause does not reside in circles, but in that which has been proved about it mathematically in the 1st proposition of the 1st book of the elements of the Art of Weighing. Therefore it is no wonder that those who took this error as a sure foundation, without becoming acquainted with the causes and also without attaining some sort of Art of Weighing, practised many false propositions, which we might refute here in particular but that we refrain from doing so, for the reasons mentioned above, the more so because they are sufficiently known from their contraries, being the aforesaid truth [4].

It would also be possible to refute here many propositions on oblique weights, described by Cardanus lib. 5, *De proportionib.*, where he estimates them from certain angles of lines or planes, but that these are wrong is sufficiently manifest from the mathematical proof of another proportion, in the 19th proposition of the 1st book of the elements of the Art of Weighing [5].

CHAPTER II, that bodies in motion are not proportional to their impediments.

We have said in the preface to the Practice of Weighing to the Reader that bodies in motion are not proportional to their impediments, and also promised there to give a more appropriate proof thereof elsewhere, which we have arranged to do here, where are to be refuted the arguments of those who are of the opposite opinion, as follows. Aristotle — in the 4th book of Physics in the chapter on vacuum [6] — and his followers hold that if two similar bodies fall

[1] Stevin refers to the pseudo-Aristotelian treatise *Quaestiones Mechanicae.*

[2] Abundant documentation on these successors is to be found in Ernest A. Moody and Marshall Clagett, *The Medieval Science of Weights.* Madison 1952.

[3] See Note 2 to p. 143.

[4] The above passage refutes the assertion not uncommon in books on the history of science that Stevin was the initiator of the principle of virtual displacements or of virtual work. This principle, which originates in the pseudo-Aristotelian work *Quaestiones Mechanicae*, had been applied in the Middle Ages by Jordanus Nemorarius and his followers.

[5] Cardanus, *Opus novum de proportionibus* etc. Basileae 1578, V, 72. Cardano contends, that the force required to hold a body at rest on an inclined plane is proportional to the angle of inclination.

[6] Stevin obviously means *Physica* IV 8; 215a-216a. It may here be remarked that Aristotle considers separately the influence of the density of the medium acting as an impediment (215a 29-215b 12) and that of the gravity which furthers the motion (216a 12-21). He nowhere asserts the proportionality of gravity and impediment.

aldaer belooft elders van dies eyghentlicker bewys te doen, t'welck wy
Argumenta. hier veroirdent hebben, alwaer weerleyt sullen worden de * strijtredens vande ghene die de contrarie meynen, aldus: Aristoteles int 4e bouck der Natuer int hooftstuc des ydels met sijn nauolghers, willen, dat vallende twee lijckformighe lichamen duer de locht, ghelijck de swaerheyt van t'een tot die van t'ander, also diens tijt des duerlijdens tot desens, dat is, ghelijck swaerheyt tot swaerheyt, also belet tot belet. T'welck sijn meyning soo te wesen hy in verscheyden boucken opentlicker verclaert als *lib.* 6. *Phys.* oock *lib.* 1. 2. 3. 4. *de Cælo*, tot veel plaetsen. Hier in heeft teghen Aristoteles ghescreuen *Ioannes Taisnier Hannonius*, willende oock eueredenheyt, doch so, dat die twee voornomde lichamen in euen tijden duer euen langden des lochts vallen; In welcke meyning Cardanus oock is *lib.* 5. *de Proportionib. prop.* 110. Maer d'een noch d'ander en heeft de saeck ghetroffen, t'welck wy eerst met daetlicke eruaring verclaren sullen, ende daer naer d'oirsaeck bethoonen. D'eruaring teghen Aristoteles is dese: Laet nemen (soo den hoochgheleerden H. IAN CORNETS DE GROOT vlietichste ondersoucker der Naturens verborghentheden, ende ick ghedaen hebben) twee loyen clooten d'een thienmael grooter en swaerder als d'ander, die laet t'samen vallen van 30 voeten hooch, op een bart oft yet daer sy merckelick gheluyt tegen gheuen, ende sal blijcken, dat de lichste gheen thienmael langher op wech en blijft dan de swaerste, maer datse t'samen so ghelijck opt bart vallen, dat haer beyde gheluyden een selue clop schijnt te wesen. S'ghelijcx beuint hem daetlick oock also, met twee euegroote lichamen in thienvoudighe reden der swaerheyt, daerom Aristoteles voornomde eueredenheyt is onrecht. D'eruaring teghen Taisnier is dusdanich: Neemt een cleen ynckel cort haerken boomwols, en een paxken des selfden stijf in een ghebonden, weghende een pondt, ende van ghelijcke form mettet haerken; dese laet t'samen neeruallen van vijf ofte ses voeten hooch, ende d'eruaring sal betoogen dattet haerken (niet teghenstaende sijn stof veel dichter in een ghesloten is, dan des pacx, waer in veel ledighe plaets ofte locht is) wel vijuentwintich mael langher op wech blijft dan t'paxken. daerom sy en vallen na sijn meyning op gheen euen tijden duer euen langden des lochts. Ander eruaring blijct oock teghen Taisnier int rijsendwicht, als in een lanck claer glas vol waters, t'welck gheroert, also datter veel lochtclooten ofte lochtbellen in commen, en daerna stil ghehouden, de grootste bellen sullen snellick in een ooghenblick opcommen, de cleender niet soo ras, maer de minste als sandekens, soo traechlick als een slecke cruypt; t'welck verre is van euen tijden. Dit is van d'eruaring ghesyt. Daer rest nu noch d'oirsaeck te verclaren, waerom hier gheen eueredenheyt en is, aldus: Yder roerende lichaem heeft eenich belet sijns roersels,
Superficiei. dat van een vallende lichaem duer de locht, is t'ghenaecsel sijns * vlacx teghen de

through the air, as the gravity of the one is to that of the other, so is the time of passage of the latter to that of the former, i.e. as gravity is to gravity, so is impediment to impediment [1]. Which in several books he more openly declares to be his opinion, as in lib. 6 *Phys* [2], also lib. 1.2.3.4. *de Caelo,* in many places. In this, Johannes Taisnier Hannonius [3] has written against Aristotle, also maintaining proportionality, but in such a way that the two aforesaid bodies fall in equal times through equal distances of the air. Which opinion is also held by Cardanus, lib. 5 *de Proportionib. prop* 110 [4]. But neither the one nor the other has hit on the truth, which we will first expound by means of practical experience, after which we will set forth the cause. The experience against Aristotle is the following: Let us take (as the very learned Mr. Jan Cornets de Groot, most industrious investigator of the secrets of Nature, and myself have done) two spheres of lead, the one ten times larger and heavier than the other, and drop them together from a height of 30 feet on to a board or something on which they give a perceptible sound. Then it will be found that the lighter will not be ten times longer on its way than the heavier, but that they fall together on to the board so simultaneously that their two sounds seem to be one and the same rap [5]. The same is found also to happen in practice with two equally large bodies whose gravities are in the ratio of one to ten; therefore Aristotle's aforesaid proportion is incorrect. The experience against Taisnier is as follows: Take a small, single, short hair of cotton and a packet of the same, tightly tied together, weighing one pound and of similar form to the hair; drop these together from a height of five or six feet, and experience will show that the hair (in spite of the fact that its material is much more compact than that of the packet, in which there is much empty space or air) is at least twenty-five times longer on its way than the packet; therefore they do not, as was his opinion, fall in equal times through equal distances of the air. Another experience also appears to be against Taisnier, with ascending weight; for example, in a high, clear glass of water, which is stirred in such a way that many spheres of air or air bubbles are produced therein, and thereafter kept still, the largest bubbles will ascend rapidly, in a moment, the smaller ones not so rapidly, but the smallest, like grains of sand, as slowly as a snail creeps; which is far from equal times. This is what is said about the experience. It now remains to set forth the cause why there is no proportionality here, as follows. Every body in motion has some impediment to its motion, that of a body falling through the air is the friction of its surface against the air.

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[1]) It is by no means clear, how this inference is drawn.

[2]) This must be a mistake for 7. The reference is to *Physica* VII 5; 250 a.

[3]) *Opusculum perpetua memoria dignissimum de natura magnetis, et eius effectibus. Item De motu continuo. Demonstratio proportionum motuum localium contra Aristotelem et alios Philosophos. De motu celerrimo, hactenus incognito* Authore Ioanne Taisnerio Hannonio. Coloniae 1562. Jean Taisnier was born at Ath (Hainault) in 1509; the year of his death is unknown. After various peregrinations he became master of the archiepiscopal church choir at Cologne. The name of his birth place accounts for the adjective Athensis which is sometimes added to his name. For further information the reader may consult: L. Thorndike, *A History of Magic and Experimental Science.* V 580-588.

[4]) Hieronymi Cardani Mediolanensis ... *Opus Novum de Proportionibus* .. Basileae 1578. Prop. 110. p. 104.

[5]) This experiment is usually ascribed to Galileo, who is said to have performed it during his professorship at Pisa (1589-1592). The story is rather suspect. At all events he was anticipated by Stevin and De Groot by more than three years.

teghen de locht, daerom ontfangt t'meeste der ghelijcke lichamen wel t'meeste beletsel, maer ouermidts der lichamen grootheden met haer vlacken selfs niet eueredtnich en sijn (want twee teerlinghen in achtvoudighe reden, hebben haer vlacken alleen in viervoudighe) so en connen sy met die beletselen niet eueredtnich wesen, ende daerom ist dat de minste lichamen meer belet ontfanghen, int ansien der eueredenheyt, dan de meeste, waerduer sy oock traechlicker neervallen.

Ende of de vlacken schoon inde reden haerder grootheden waren, so is t'middel daer de lichamen duer vallen, alleen oock een oirsaeck die sulcke eueredenheyt weert, t'welck opentlick blijct in twee lichamen, t'een int water sinckende, t'ander daer op driuende, wiens beletselen der vlacken eenighe reden hebben, maer de tijden gheen, daerom en sijnse niet eueredtnich. Ymant sal hier toe mueghen segghen t'ghemeen woort *D'ander parich, dat is, hem sulcx alleen te verstaen van lichamen die beyde duer t'water sincken. Ick seg dat de voornomde eueredenheyt in sulcke oock niet en bestaet: Om t'welck te bewysen so laet twee lichamen sijn, A t'swaerste, B t'lichste, die beyde int water sincken, ende tusschen hun besta de voornomde eueredenheyt. Dit soo wesende, tis kennelick datmen neuen A, ander oneindelicke menichte van lichamen voughen can, t'een lichter als t'ander, ende elck lichter als B, die alle daer in sincken. Nu yder van dese verleken met A, men sal allencx naerderen t'ghene bouen ghesyt is gheen eueredenheyt te wesen, dat is men sal naken de verlijcking eens lichaems dat sinct, met een dat niet en sinct: Maer dit soo naerderende, ende in A, B, de begheerde eueredenheyt bestaende, seker gheen dier oneindelicke menichte der lichamen met A verleken, en sullen die eueredenheyt hebben; want sooser in waer, sy en souden niet naerderen, t'welck teghen t'ghestelde is. Daerom soo wy voorghenomen hadden te verclaren, t'middel daer de lichamen duer vallen, is oock een oirsaeck die de voornomde eueredenheyt weert.

Caeteris paribus.

Maer hier aldus bethoont hebbende, gheen eueredenheyt te bestaen tusschen de gheroerden met haer beletselen inde alderghelschicste voorbeelden, alwaer maer een eenvoudich strijcsel der vlacken en is teghen de locht, oft teghen t'water, soo en salder uyt noch stercker reden, gheen eueredenheyt wesen in ongheschicter voorbeelden van verscheyden stoffen, als in reetschappen van haut, ijser, en dier ghelijcke, want dit wort beolijt, dat besmeert, t'een can met een vochtich weer opswellen, t'ander verroesten, alle welcke (ick laet veel ander varen) de roerselen der reetschappen verlichten of beswaren. Daerom soo ghesyt is inde bouescreuen voorreden der Weeghdaet, men sal hem op dese schijn van eueredenheyt niet verlaten, maer t'ghene Cardanus *lib. 5. de Proportionibus* in veel verscheyden voorstellen, met meer ander Schrijuers daer af besluyten, voor dwalinghen houden, sich vernoughende met de Wis-

Therefore the largest of such bodies indeed meets with the greatest impediment, but since the volumes of bodies are not proportional to their surfaces (for two cubes in the ratio of one to eight have their surfaces only in the ratio of one to four), they cannot be proportional to those impediments, and that is why the smaller bodies meet with a relatively greater impediment than the greater, owing to which they also fall more slowly.

And even if the surfaces were proportional to their volumes, the medium alone through which the bodies fall is also a cause preventing such proportionality, which is clearly apparent with two bodies, one sinking in the water and the other floating thereon, the impediments of whose surfaces have a certain ratio, but the times have not; therefore they are not proportional. Someone might here add the common term: other things being equal, i.e. that this applies only to bodies which both sink in the water. I say that the aforesaid proportionality does not exist between such bodies either. In order to prove this, let there be two bodies, *A* the heavier, *B* the lighter [1]), which both sink in the water, and let them be in the aforesaid ratio. This being so, it is evident that besides *A* there can be added an infinite number of bodies, one lighter than the other, and each lighter than *B*, which all sink therein. Now if each of these is compared with *A*, we shall gradually approach the situation in which, as has been said before, there is no proportion, i.e. we shall approximate to a comparison of a body that sinks with one that does not sink. But with this approximation, and the desired proportionality existing between *A*, *B*, certainly none from that infinite number of bodies compared with *A* will have that proportionality; for if it were there, they would not approximate, which is against the supposition. Therefore, as we intended to set forth, the medium through which the bodies fall is also a cause preventing the aforesaid proportionality.

But since we have thus proved that there is no proportionality between bodies in motion and their impediments in the most obvious examples, where there is only simple friction of the surfaces against the air or against the water, there will *a fortiori* be no proportionality in less obvious examples of several materials, such as tools of wood, iron, and the like, for the former is oiled, the latter greased; the one can swell in moist weather, the other rust; all of which things (I omit many others) lighten or weight the motions of the tools. Therefore, as has been said in the above-mentioned preface to the Practice of Weighing, we should not rely on this appearance of proportionality, but consider the conclusions reached by Cardanus, lib. 5 *de Proportionibus* in a great many different propositions, and by other writers, to be errors, being satisfied with the mathematical knowledge of

[1]) It is obvious that „heavier" and „lighter" here refer to specific gravity. The weights of *A* and *B* are supposed to be equal. Stevin considers a series of bodies of this same weight *G* with a gradually decreasing specific gravity *S*. If the resistance *R* exerted by the water were proportional to the volume, *R* would be inversely proportional to *S*, and consequently *S* would be proportional to the time required for the motion through a given distance. If *S* approaches the specific gravity S_1 of the water, the time tends to a certain limit, whereas, on the other hand, if $S = S_1$, there is no motion at all.

constighe kennis der euestaltwichticheyt van t'roerende ende het te roeren, als tottet voornemen ghenouch doende.

III^e. HOOFTSTICK, DAT DE WEEGHCONST EEN BESONDER VRIE * WISCONST IS.

Ars Mathematica.

Quaestiones.

Materia.

Arithmetica

Geometria.

Accidentia.

TIS wel waer, dat van der dinghen namen die de saeck niet en verduysteren, dickwils onnoodighe * verschillen sijn, onder de welcke ick niet en ontken dit derde hoofstick te mueghen gherekent worden, doch anghesien wy de Weeghconst daert te pas ghecommen heeft, een vrye Wisconst ghenoemt hebben, soo moeten wy met corte woorden daer af wat redens gheuen, aldus: Ouermits de * stof des ghetals al een ander is dan die der grootheyt, soo sijn de leeringhen haerder eyghenschappen te recht vanden anderen ghescheyden, ende elck voor een besonder Const ghehouden, als * Telconst ende * Meetconst, op dat elcke alsoo oirdentlicker, eyghentlicker, ende verstaenlicker soude mueghen beschreuen worden. Ten anderen, want haer diepsinnighe * ancleuinghen ons niet uyter natuer bekent en sijn, maer dat wy die leeren uyt de vergaerde schriften der ghene die duer besonder vliet hun daer in gheoeffent hebben, ia dickmael by gheualle ter kennis van yet besonders ghe-rocht sijn, ende dat haer wetenschap den menschen daerenbouen seer nut is, soo wordense met recht vrye consten ghenoemt. Ten derden, nadien de sekerheyt in haer bestaende, de ghewisheyt van d'ander Consten verre te bouen gaet, soo wordense billichlick daerbeneuen Wisconsten gheheeten. T'selue is om der ghelijcke redenen vande Weeghconst oock te oirdeelen; want anghesien haer stof, te weten swaerheyt, al een ander is dan ghetal ofte grootheyt; oock dat de nutte eyghenschappen van dese, in diepsinnicheyt an d'eyghenschappen van die niet en wycken (t'welck daerin blijct, dat sy om sulcx laetst tot smenschen kennis ghecomen sijn, ende of sy v schoon licht dochten, dat muecht ghy d'onbegrijpelicke volmaectheyt der Duytsche spraeck dancken) Voorts dat sy duer haer uyterste beghinselen, in sulcken ghewisheyt bestaet als die, soo sal sy om haer ghemeene reden, een besonder vrye Wisconst ghenoemt worden.

Species.

Theoremata.

Yemant sal hier teghen mueghen segghen, dat de Meetconst tot haer bewysen dickmael ghebruyct wort, ende die daerom als * afcomst der Meetconst stellen. Ick anwoord de Telconst sulcx oock te ghebueren, nochtans een besonder vrye Wisconst blijuende. Want wat voornamelicke * Vertoogen heeftse, diens grondelicke kennis duer de Meetconstighe formen niet vercreghen en wort? Ia die Meetconst seluer hoe soudese sonder ghetalen bestaen? Siet haer beghinselen als die van Euclides, hoe dickmael d'een form des anders dobbel, dese drie platten euen an die twee ghesëyt worden. T'blijct dan die voorstellen sonder t'behulp van ghetalen the equality of apparent weight of the moving body and the body to be moved as being sufficient for the purpose.

CHAPTER III, that the Art of Weighing is a distinct, free branch of mathematics.

It is true that there are often unnecessary points of controversy about the names of things, which do not give rise to obscurity, among which I cannot deny but this third chapter may be reckoned, but since we have, where relevant, called the Art of Weighing a free branch of mathematics, we briefly have to account for this, as follows. Since the subject matter of number is quite different from that of magnitude, the theories of its properties are justly dissociated from the other, and each is considered a distinct art, viz. arithmetic and geometry, so that each might thus be described in a more orderly, appropriate, and comprehensible way. Secondly, because their profound attributes are not known to us by nature, but are learned by us from the collected writings of those who have studied them with special zeal, nay, have often quite accidentally become acquainted with some special feature, and because their knowledge is moreover very useful to mankind, they are rightly termed free arts. Thirdly, since the certainty residing in them far exceeds that of the other arts, they are also on that account rightly termed "Wisconsten" [1]. The same can for similar reasons also be said of the Art of Weighing. For since its subject matter, to wit gravity, is quite different from number or magnitude; also because the useful properties of the latter are not inferior in profundity to the properties of the former (which is evident from the fact that for this reason they were the last to come to man's knowledge, and though they may seem easy to you, you owe that to the incomprehensible perfection of the Dutch language); further because in its fundamental principles it is of equal certainty to the former, it is, on account of this common reason, to be termed a distinct, free branch of mathematics.

Someone may object to this that geometry is often used for its proofs, and therefore may call the Art of Weighing a species of geometry. I reply that this also happens with arithmetic, which nevertheless remains a distinct, free branch of mathematics. For what important theorems does it have, thorough knowledge of which is not acquired by means of geometrical figures? Nay, how could even geometry itself exist without numbers? Consider its elements, for example, those of Euclid, how often one figure is said to be double of another, these three plane surfaces are said to be equal to those two. It is therefore found that those propositions cannot be proved without the aid of numbers, without, however, one

[1] „Wisconst" literally means a sure, certain art. The pun could not be preserved in the translation.

ghetalen onbewyslick te wesen, nochtans d'een des anders afcomst niet sijnde, ende alsoo oock met de Weeghconst.

Angaende dat de * Duersichtighe ende * Spieghelconst voor gheen besonder vrye Wisconsten, maer als afcomsten der Meetconst gheacht sijn, by welcke yemandt de Weeghconst mocht willen verlijcken; hun redens sijn seer verscheyden, ouermidts de stof van dese, te weten swaerheyt, sulcx is, dat sy ghelijck de grootheyt, bestaet in alle voorghestelde wesentlicke saeck, met s'menschen groote nutbaerheyt; maer niet also de stof van die. Wy besluyten dan te recht, de Weeghconst een besonder vrye Wisconst te sijne, ghelijck ons voornemen was te bethoonen. *Perspectiua* *Catoptrica*

IIII^e. HOOFTSTICK, DAT SOMMIGHE VOORGAENDE BEWYSEN DVER T'BEHVLP *der ghetalen* * *Wisconstich sijn.*

Mathematica.

DE gheleerden maken onderscheyt tusschen Wisconstich ende * Werckelick bewys: T'welck niet sonder reden en is, want dat is ghemeen ouer allen, oock grondelick d'oirsaeck verclarende, dit besonder alleenlick op t'ghegheuen, sonder kennis der reden waerom dat also gheschiet. Als by voorbeelt, yemant om te bewysen dattet viercant der langste sijde eens rechthouckich driehoucx, euen is ande twee viercanten van d'ander sijden, neemt een driehouck, diens cortste sijde van 3 voeten, d'ander van 4, de derde van 5 voeten is, de selue wort rechthouckich beuonden; Bethoont daer mede dattet viercant der langste sijde 25, euen is ande viercanten van d'ander twee sijden, als 16 ende 9. Maer dit is alleenlick bewys van dat voorghestelde; waer uyt niet en volght sulcx ouer alle rechthouckighe driehoucken soo te moeten wesen, oock en sietmen daer duer de oirsaeck niet, waerom dat also ghebuert. ende ouermits dit aldus werckelick gheschiet, so wordet daerom oock werckelick bewys gheheeten. Maer t'betooch van sulcx duer Euclides ghedaen int 47^e voorstel des 1^en boucx, is ghemeen ouer allen, anwysende duer d'uyterste beghinselen, de reden waerom dat so is, ende niet anders sijn en can; t'selue wort om sulcke ghewisheyt Wisconstich ghenoemt, t'welck de * Wisconstnaers om de redenen hier vooren verhaelt, lieuer ghebruycken dant werckelick duer ghetalen. Yemandt mocht nu segghen; Dit so sijnde, waerom hebdy dan de bewysen der 4^e, 11^e, 12^e, 18^e, voorstellen des 2^en boucx vande beginselē der Weeghconst, duer ghetalen ghedaen? D'antwoort valt daer op, dat de ghetalen der bewysen ons op tweederley manieren ontmoeten, d'eene die als * palen alleenlick de * redenen ende * eueredenheden der deelen des voorghestelden forms verclaren, d'ander de * menichvuldicheyt; T'bewys duer die is Wisconstich, wanttet hem op alle

Mechanicam *Mathematicis* *Termini.* *Rationes.* *Proportiones.* *Quantitati.*

being a species of the other; and the same is also true of the Art of Weighing. As to the fact that perspective and catoptrics are considered not to be distinct, free branches of mathematics, but species of geometry, to which someone might wish to compare the Art of Weighing, the reasons are quite different, since the subject matter of the latter, to wit gravity, is such that, like magnitude, it resides in every real thing under consideration, to the great advantage of man; but not so the subject matter of the former. We therefore rightly conclude that the Art of Weighing is a distinct, free branch of mathematics, as we intended to prove.

CHAPTER IV, that some of the preceding proofs in which numbers were used are mathematical.

Scholars make a distinction between a mathematical and a practical proof; which is not without reason, for the former applies to all cases and also thoroughly sets forth the cause, and the latter only applies to the particular given case, without the cause why it thus happens being known. For example, in order to prove that the square of the longest side of a rightangled triangle is equal to the two squares of the other sides, a man takes a triangle whose shortest side is 3 feet, the second 4, and the third 5 feet; this is found to be rightangled. Therewith he proves that the square of the longest side, 25, is equal to the squares of the other two sides, viz. 16 and 9. But this is only a proof of the case under consideration, from which it does not follow that this must be so with all rightangled triangles. Nor do we thus see the cause why it happens in this way, and since this is done by practical means, it is called a practical proof. But the proof thereof given by Euclid in the 47th proposition of the 1st book applies to all cases, and shows, by means of the fundamental principles, the reason why this is so and cannot be otherwise; because of such certainty, this proof is called "Wisconstich", which mathematicians prefer, for the reasons given above, to the practical proof by means of numbers. Now someone might say: if this is so, why then have you given the proofs of the 4th, 11th, 12th, 18th propositions of the 2nd book of the elements of the Art of Weighing by means of numbers? The reply to this is that we meet with numbers in the proofs in two different ways, one in which as terms they only set forth the ratios and proportions of the parts of the figure under consideration, the other the quantity [1]). The proof by means of the former is mathematical, for it applies to all species of the figure under consideration and sets forth the causes, but the proof by means of the latter is not, for the opposite reason. Which Eutocius, the commentator of Apollonius, in the 11th proposition of the 1st book

[1]) The meaning of this passage is: In the propositions quoted, the line segments that occur therein have certain definite ratios to each other, which are expressed by numbers, e.g. in a triangle the centre of gravity divides a median into parts which are in the ratio 1 : 2. This way of using numbers is quite different from the one where we suppose the median to be of a given length. It appears that in Stevin's day the explicit mention of numbers gave rise to objections, and that a distinction was made between this and the Greek method of expressing ratios in words. This difference is, of course, quite inessential.

Species. alle * afcomſten des voorgheſtelden forms verſtaet, ende d'oirſaken verclaert, maer deſe niet om de contrarie redenen. T'welck Eutochius *Cõmentator.* * uytleggher van Appollonius int 11ᵉ voorſtel des 1ᵉⁿ bouck alſoo mede verſtaet ſegghende: *Niemant en beroer hem hier in dat dit duer de ghetalen bethoont is, want d'ouden pleghen ſulcke bewyſinghen te ghebruycken, ſo jy doch beter Wiſconſtich ſijn dan Telconſtich, om de eueredenheyts wil; Merct oock dattet beghenrde Telconſtich is, want de eueredenheden, ende de menichvuldicheden der eueredenheden, oock de *menichvuldighinghen, ſijn ten eerſten inghetalen, ten anderen duer ghetalen inde grootheden, na t'oirdeel van hem die aldus* *Multiplicationes.* *gheſchreuen heeſt:* ταῦτα γὰρ τὰ μαθήματα δοκοῦντι εἶμεν ἀδελφά. *dat is, deſe Conſten alle een moers kinderen te ſchijnen.* Nu ſoude ymant mueghen voortbrenghen, dat Ptolæmeus, Archimedes, Appollonius, Commandinus, Regiomontanus, ende meer ander in der ghelijcke voorſtellen, alle palen met gheen ſo uytghedruckte ghetalen en beteeckenen, als hier ghedaen is. Daerop antwoord ick, dat met alſulcke reden als ſy ſegghen van der palen tweevoudicheyt, drievoudicheyt, mette ſelue ſalmen oock mueghen ſpreken daert te pas comt, van haer t'welfvoudicheyt, als A D tot R D int voornomde 23ᵉ voorſtel, ende van haer reden als 37 tot 23, ghelijck A R tot R D des bouéſchreuen 11ᵉ voorſtels, ende alſo met allen anderen, want ſulcke linien in yder voorgheſtelde form dier afcomſt, gheen ander reden en hebben. Nu anghesien datmen int onderſoucken der eyghenſchap ſulcker formen, deſe ghetalen ghebruyct, die ons als ſeker anwys, met lichticheyt tot claer verſtant der ſaeck brenghen, ſoo iſt nut int beſchrijuen der ſeluer, die ghetalen daer oock by te ſetten, om *Inuentoribus* voor anderen niet duyſter na te laten, t'ghene den* Vinders ſelfs licht en openbaer was. Want ſulcx is t'recht Wiſconſtich bewys, t'voorgheſtelde duer d'oirſaken verclarende; T'welck ons voornemen was te bethoonen.

Angaende ſommighe bewyſen des eerſten boucx vande beghinſelen der Weeghconſt, oock des Waterwichts, inde welcke de ſwaerheden duer ghetal en bekent ghewicht, als ponden, beteeckent ſijn, t'welck yemant voor gheen Wiſconſtighe, maer voor werckelicke handeling mocht achten; die ſal weten, dat beneuen ſoodanighe, oock mede gheſtelt ſijn haer Wiſconſtighe bewyſen, als int 1ᵉ voorbeelt des eerſten voorſtels van t'eerſte bouck, alwaer duer ghetalen ende bekent ghewicht, des voorſtels inhoudt bethoont is, maer int tweede voorbeelt, is t'ſelue oock Wiſconſtelick beweſen, ende alſo met d'ander. Inder voughen dattet werckelick bewys tot meerder claerheyt ſomwylen by t'Wiſconſtich ghevoucht is.

Vᵉ. HOOFTSTICK, WELCK VERCLARING IS OP HET VIIIᵉ VOORSTEL DER *beghinſelen des Waterwichts.*

DAER is int bouéſchreuen 8ᵉ voorſtel aldus gheſeyt: *Yder ſtyflichaems ſwaerheyt is ſo veel lichter int water dan inde locht, als de ſwaerheyt des waters* also understands in this way saying [2]): *Let not one become agitated because this has been proved by means of numbers, for the Ancients are accustomed to use such proofs, since they are better versed in mathematics than in arithmetic, for the sake of the proportionality. Note, too, that what is required to prove is of an arithmetical nature, for ratios, and quantities of ratios* [1]), *and multiplications firstly reside in numbers, and secondly through numbers in magnitudes, according to the opinion of him who has written:* [2]) ταῦτα γὰρ τὰ μαθήματα δοκοῦντι εἶμεν ἀδελφά: [3]), i.e. all these arts seem to be children of one mother. Now someone might advance that Ptolemy, Archimedes, Apollonius, Commandinus, Regiomontanus, and many others do not designate in such propositions all the terms with such explicit numbers as has been done here. To this I reply that with the same right with which they speak of the double and the treble of the terms, we may also speak of their twelvefold, wherever it is relevant, as *AD* to *RD* in the aforesaid 23rd proposition [4]), and of their ratio 37 to 23, as *AR* to *RD* in the above-mentioned 11th proposition [5]), and in this way with all others, for such lines do not have any different ratio in each figure of this species. Now since in an examination of the property of such figures these numbers are used, which are a certain indication easily leading to a clear understanding of the matter, it is useful in a description thereof to add also those numbers, in order not to leave obscure for others what was light and manifest for the inventors. For such is the true mathematical proof, explaining the matter under consideration by means of the causes, as we intended to show.

As to some proofs of the first book of the elements of the Art of Weighing, and also of Hydrostatics, in which the gravities are designated by numbers and known weights, such as pounds: if someone should hold these not to be mathematical, but practical proofs, he should know that side by side with these are also given the mathematical proofs, as in the 1st example of the first proposition of the first book, where the contents of the proposition have been shown by means of numbers and known weights, but in the second example it has also been proved mathematically, and similarly with the others. In such a way that the practical proof has sometimes been added to the mathematical one, for the sake of greater clarity.

CHAPTER V, which is a commentary on Proposition VIII of the elements of Hydrostatics.

It has been said as follows in the above-mentioned 8th proposition: *The gravity of any solid body is as much lighter in water than in air as is the gravity of*

[1]) The translation here follows the Greek text, not Stevin's rendering, which is defective. The quantity of a ratio (ἡ πηλικότης) is the number after which the ratio is called (οὗ παρώνυμος ἐστιν ὁ λόγος).

[2]) *Apollonii Pergaei quae graece exstant cum commentariis antiquis*, ed. J. L. Heiberg. Leipzig 1891-1893. II 220. The reference is not quite to the point in that Eutocius does not speak of cases where a certain numerical ratio occurs, but of the consideration of ratios, whether rational or irrational, as numbers. After giving a demonstration in which ratios are dealt with in this way, he refutes, in the words quoted by Stevin, in advance any possible objections to this method, which indeed is at variance with the style of classical Greek mathematics.

[3]) Archytas. Diels, *Fragmente der Vorsokratiker*, 35 B.

[4]) Prop. 23 of Book II of the *Art of Weighing*. For AD : RD read AD : RE.

[5]) Prop. 11 of Book II of the *Art of Weighing*.

Waters met hem euegroot. Waer uyt ymant sulcken veruolgh mocht willen maken: *T'der stijflichaems swaerheyt is so veel lichter int quicsiluer dan int Water, als de swaerheyt des quicsiluers met hem euegroot.* Ofte aldus: *T'der stijflichaems swaerheyt is soo veel lichter int Water dan inde olie, als de swaerheyt des Waters met hem euegroot.* ende soo met dierghelicke. Welck nootlick veruolgh, de saeck eenuoudichlick anghesien, d'eruaring teghen schijnt, want een pont loot en sal na de ghemeene ghebruyck van wegen, int water niet so veel lichter sijn dan in de olie, als de swaerheyt des waters met hem euegroot, maer alleenlick soo veel lichter, als t'verschil tweer lichamen van water en olie met dat voornomde loot euegroot. Doch den grondt dieper inghesien, ende * d'ander parich ghestelt, so bestaet alles *Cateris paribus.* in d'uyterste volmaectheyt. Om t'welck te bewysen, so is t'anmercken, dat inde 1e begheerte der beghinselen des Waterwichts versocht is, *Der lichamen ghewicht inde locht eyghen ghenoemt te Worden*, ende inde 5e begheerte, *T'vlacvat vol Waters uytghegoten siinde, ledich te blijuen*, dat is vol lochts te wesen duer de 11e bepaling, daerom ghenomen dat de twee middelen quicsiluer en water sijn, alwaer nu water inde plaets des lochts ghestelt is, ende datmen hier sghelijcx begheere, *Der lichamen ghewicht int Water eyghen ghenoemt te Worden.* Oock, *T'vlacvat vol quicsiluers uytghegoten siinde, vol Waters te blijuen*, so is t'voornomde voorstel (*T'der stijflichaems swaerheyt is soo veel lichter int quicsiluer dan int Water, als de swaerheyt des quicsiluers met hem euegroot*) waerachtich. Om t'welck duer ghelijcknis noch opentlicker te verclaren, so neemt dat een man gantsch onder t'water sy, aldaer by hem hebbende quicsiluer, gout, met een waegh, eñ houdende t'water als voor locht: Ick seg dattet gaudt aldaer so veel lichter sal sijn int quicsiluer dan int water, als de swaerheyt des quicsiluers mettet gaudt euegroot; t'welck openbaer is. Tis wel waer dat soomen naem, *Der lichamen ghewicht int ydel eyghen ghenoemt te Worden*, soot in eenvoudich ansien oock is, men soude naer sulcke eyghenheyt mueghen segghen, *T'der stijflichaems swaerheyt is soo veel lichter int Water dan int ydel, als de swaerheyt des Waters met hem euegroot.* Maer anghemerct d'omstaende, te weten dat ons ghemeene daetlicke weghinghen (naer welcke de *Spiegheling altijt opsicht behoort te nemen) niet int ydel en gheschien, maer *Theoria.* inde locht, soo ist beter na d'eerste wyse, der lichamen eyghenwicht inde locht te stellen; Int ansien van welcken, t'bouesch reuen 8e voorstel met dieder uytvolghen, in haer uyterste volcommenheyt sijn, soo wy voorghenomen hadden te verclaren.

T'EINDE DES ANHANGS.

the water having the same volume. From which someone might wish to make the following corollary: *The gravity of any solid body is as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume.* Or as follows: *The gravity of any solid body is as much lighter in water than in oil as is the gravity of the water having the same volume,* and thus with similar cases. Which necessary corollary, when looking at the matter in a simple way, seems to be against experience, for a pound of lead will not, according to the common weighing practice, be as much lighter in water than in oil as is the gravity of the water having the same volume, but only as much lighter as is the difference between two bodies of water and oil having the same volume as the aforesaid lead. But when we look more deeply into the matter, and other things are taken to be equal, everything is quite perfect. In order to prove this, it is to be noted than in the 1st postulate of the elements of Hydrostatics it has been asked to grant *The weights of bodies in air to be called their proper weights,* and in the 5th postulate, *The surface vessel full of water, the latter being poured out, to be left empty,* i.e. to be full of air, according to the 11th definition; therefore, taking the two media to be quicksilver and water, where now water is substituted for air, and postulating similarly *The weights of bodies in water to be called their proper weights;* also *The surface vessel full of quicksilver, the latter being poured out, to be left full of water,* the aforesaid proposition (*The gravity of any solid body is as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume* [1])) is true. In order to explain this even more clearly by means of comparison, assume a man to be completely under water, having there with him quicksilver, gold, with a balance, and let the water be taken for air: I say that the gold will there be as much lighter in quicksilver than in water as is the gravity of the quicksilver having the same volume as the gold; which is manifest. It is indeed true that if we took *The weights of bodies in a vacuum to be called their proper weights,* as is the case when looking at it in a simple way, it might be said in accordance therewith: *The gravity of any solid body is as much lighter in water than in a vacuum as is the gravity of the water having the same volume* [2]). But considering the circumstances, to wit that our common practical weighings (at which the theory should always be directed) do not take place in a vacuum, but in air, it is better to postulate in the first manner the proper weights of bodies in air; in view of which the above-mentioned 8th proposition with those following therefrom is quite perfect, as we intended to set forth.

THE END OF THE APPENDIX

[1]) When weighed in water.
[2]) When weighed in a vacuum.

BYVOUGH DER WEEGHCONST

SUPPLEMENT TO THE ART OF WEIGHING

INTRODUCTION TO THE SUPPLEMENT TO THE ART OF WEIGHING

The second edition of the *Art of Weighing*, which forms part of the *Wisconstighe Ghedachtenissen* (Work XI) contains a Supplement consisting of four treatises out of six that had been planned for it.

The first, which bears the title *Of the Cord Weight*, deals with *Spartostatics*, i.e. statical problems relating to systems of stretched cords carrying weights. The solutions of these problems are based on Theorem 27 of Book I of the *Art of Weighing*, which is equivalent to the parallelogram (or triangle) of forces.

In the second treatise, *Of the Pulley Weight*, the movable pulley and the block-and-tackle are discussed. The cords carrying the lowest pulley, and consequently the weight to be raised, are first supposed to be vertical, afterwards oblique. The mechanical advantage is found in the first case by considering the number of ropes bearing the weight, and in the second case by applying the above-mentioned Prop. 27.

The third treatise, *Of the Floating Top-heaviness*, is based on a practical military problem. For the assault of a town or fortress use was occasionally made of ladders on boats, to be ascended by the soldiers carrying out the assault. In order to avoid the risk of the boats capsizing, a test ascension would be made. Stevin now endeavours to render this superfluous by means of calculation, without, however, succeeding in the enterprise.

The *Supplement* is concluded with a treatise entitled *Of the Pressure of the Bridle* and dealing with practical matters of horsemanship. The principal object is to understand the action of the bridle on the basis of the statical principles exposed in the *Art of Weighing*. Present-day experts on equestrian mechanics no longer accept Stevin's method. It is nevertheless a remarkable symptom of the scientific attitude which he and his princely pupil took towards the affairs of practical life that they felt impelled to study from a mechanical point of view a device which was in general use. Moreover, we learn from this treatise that they constructed an adjustable test bridle in order to verify their theoretical conclusions.

The Summary of the *Supplement* mentions two more titles: *Of the Drawing of Water* and *Of the Weight of the Air*. We do not know whether the first was to have been concerned with dredging problems, water wheels or marsh mills. In the latter case it may perhaps have been identical with the treatise on mills to be published in our Volume V.

The treatise *Of the Weight of the Air*, if written at all, appears to be irretrievably lost, and we can do no more than guess at its possible contents. The title suggests that Stevin attributed weight to the air, which is in agreement with some passages of his *Hydrostatics* (Defs X and XI; Postulate I) and the *Appendix to the Art of Weighing* (Ch. V). In how far he would have dealt with aerostatics on the same principles as hydrostatics, thus again forestalling Pascal, it is, however, impossible to decide.

Argumentum.

CORTBEGRYP

deses byvoughs der VVeeghconst.

Theoria quàm praxi.

MY sijn na d'eerste beschrijving der Weeghconst, verscheyden stoffen der wichtighe ghedaenten voorghecommen, soo in * spiegheling als daet, diemen in dese tweede druck elck t'haerder plaets van d'eerste oirden soude hebben meugen schicken, om daer af een verknocht lichaem te maken: Maer insiende dattet gheschapen staet, na dese meer ander te connen volghen, die om de selve reden dan dergelijcke schicking souden vereysschen, sulcx datter elckemael een verandering van t'voorgaende mocht vallen, so en souder des veranderens geen eynde sijn: En hoe wel dat in sijn selven best mocht wesen, nochtans belet van ander noodigher dinghen en latet my niet toe: Inder voughen dat ick d'eerste beschrijving der Weeghconst (veranderende alleen de veranderlicke) in haer form ghelaten heb, daer by voughende de voorschreven na ghecommen stoffen, die ick int gheheel BYVOUGH noem, inhoudende ses deelen:

Het eerste van het Tauwicht.

Het tweede van het Catrolwicht.

Het derde vande vlietende Topswaerheyt.

Het vierde vande Toomprang.

Het vijfde vande Watertrecking.

Het seste vant Lochtwicht.

SUPPLEMENT TO THE ART OF WEIGHING

ARGUMENT
OF THIS SUPPLEMENT TO THE ART OF WEIGHING

After the original description of the Art of Weighing there have occurred to me several matters concerning static properties, both in theory and in practice, which in this second edition [1]) might each have been arranged in its place in the first edition, so as to make a whole of it. But seeing that the position is such that, after these, others may follow, which for the same reason would then require to be similarly arranged, so that each time the preceding edition would be changed, there would be no end of such changes. And though this might in itself quite well be done, I am prevented by more necessary things from doing so, so that I have left the original description of the Art of Weighing as it was (only changing the things that had to be changed), adding thereto the above-mentioned matters that afterwards occurred to me, the total of which I call SUPPLEMENT, containing six parts:

The first of the Cord Weight.
The second of the Pulley Weight.
The third of the Floating Top-heaviness.
The fourth of the Pressure of the Bridle.
The fifth of the Drawing of Water.
The sixth of the Weight of the Air [2]).

[1]) This *Supplement* first appeared in the *Wisconstighe Ghedachtenissen* (XI), in which the *Art of Weighing* (VI) was reprinted.
[2]) As has been remarked in the Introduction, the last-mentioned two treatises are missing.

EERSTE DEEL
DES BYVOVGHS
DER WEEGHCONST,
VAN HET
TAVWICHT.

FIRST PART
OF THE SUPPLEMENT
TO THE ART OF WEIGHING,
OF THE CORD WEIGHT

Argumentum.

CORTBEGRYP
DES TAVWICHTS.

WY hebben inde drie laetste voorstellen des 1 boucx der VVeeghconst, beschreven de vvichtige ghedaenten van svvaerheden hangende an tvvee linien, gehecht ant lichaem tot tvvee verscheyden plaetsen. Maer vvant de svvaerheden op meer ander vvysen an linien connen hanghen, vvaer afmen oock begheert te vveten vvat ghevvelt op yder lini ancomt, soo hebben vvy daer af dese besonder handel ghemaeckt: Ende om dat in plaets van sulcke linien metter daet onder ander stoffen meest tauvven gebruyct vvorden, so noemen vvy dit na t'gemeenste gebruyck TAVWICHT; *vvaermen by verstaen mach, een handel verclarende vvat ghevvelt datter ancomt op yder tau, van verscheyden tauvven daer een bekende svvaerheyt anhangt. De somme des inhouts is dusdanich: Int 27 voorstel des 1 boucx der VVeeghconst, is bevvesen dat hangende een pylaer evestaltvvichtich teghen tvvee scheefhefvvichten: Gelijck alsdan scheefheflijn tot rechtheflijn, also scheefhefvvicht tot sijn rechthefvvicht: Hier uyt sullen vvy in dit 1 deel verscheyden vervolghen trecken, in vviens plaets men vvel soude hebben meughen maken gheformde* voorstellen, doch is dat ghelaten, eensdeels om cortheyt, ten anderen dat dese vervolghen uyt het voorgaende aldus claer ghenouch schijnen.*

*Propositiones.

1 VER-

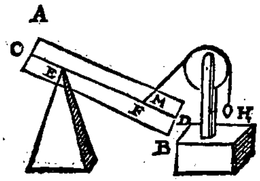
ARGUMENT OF THE CORD WEIGHT

In the three last propositions of the 1st book of the Art of Weighing we have described the static properties of gravities hanging on two lines, attached to the body in two different places. But because gravities can hang on lines in several other ways, with regard to which it is also desired to know what force acts on each line, we have made thereof the following special treatise. And because instead of such lines, among other things, cords are mostly used in practice, according to the most common usage we call this cord weight; by which is to be understood a treatise setting forth what force acts on each cord, among several cords on which a known gravity is hanging. The gist of the contents is as follows: In the 27th proposition of the 1st book of the Art of Weighing it has been proved that if a prism is hanging in equality of apparent weight with two oblique lifting weights, as the oblique lifting line then is to the vertical lifting line, so is the oblique lifting weight to its vertical lifting weight. From this we will in this 1st part draw different corollaries, instead of which regular propositions might have been made, but this has been omitted, in the first place for brevity's sake, in the second place because these corollaries appear thus to be clear enough from what precedes.

EERSTE VERVOLGH

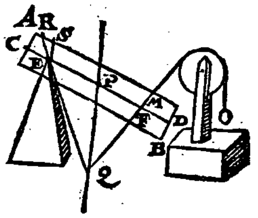
des 27 voorstels vant 1 bouck der VVeeghconst.

SOOMEN inde form des 27 voorstels vant 1 bouck, an t'punt E, in plaets van het scheefwicht G, vervoughde een vastpunt als hier nevens, tis kennelick dat teghen dit vastpunt een persing soude sijn, even an t'ghewicht G, en dat met sulcken scheefheyt teghen t'selve punt E ancommende, als de scheeflijn L E anwijst.



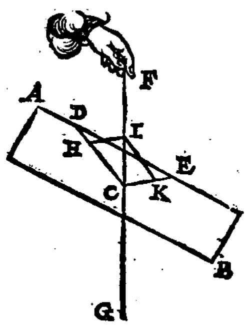
2 VERVOLGH.

Soomen int boveschreven 27 voorstel de twee scheeflinien L E, M F, voorttreckt tot datse versamen, tis kennelick deur het 25 voorstel, dattet punt der saming comt inde hanghende swaerheyts middellijn des lichaems: Daerom somen wilde weten wat scheve persing datter ancomt, opt vastpunt E des 1 vervolghs, men sal aldus meughen doen: Ick treck deur des pylaers swaerheyts middelpunt als P hier nevens, de oneyndelicke swaerheyts middellijn, welcke vande voortgetrocken M F, ontmoet wort in Q; daer na van Q deur E de lini Q R, vallende R in A M. T'welck soo sijnde, de persing op E ancommende, is als van R na E, en om te weten hoe groot, men ghebruyckt int werck E R voor scheefheflijn, en E S voor rechtheflijn, waer me men openbaerlick tottet begheerde comt.



3 VERVOLGH.

Maer om nu te commen tot verclaring vande ghedaenten der gewichten an tauwen hanghende, soo laet A B een pylaer sijn, diens middelpunt C, en hanghende ande twee vastpunten D, E, met twee linien C D, C E, commende uyt het swaerheyts middelpunt C; de selve C D en C E sijn swaerheyts middellijnen des pylaers deur de 5 bepaling: Daerom tusschen D C en C F, ghetrocken H I evewijdeghe met C E, soo is deur de 13 bepaling C I rechtheflijn, C H scheefheflijn, waer me wy segghen, dat ghelijck C I tot C H, also diens rechthefwicht, tot desens scheefhefwicht: Maer t'rechthefwicht van C I, is even ant ghewicht des heelen pylaers: Daerom ghelijck C I tot C H, alsoo t'ghewicht des heelen pylaers, tottet ghewicht op D ancommende: Ende inder selver voughen vintmen oock t'gewicht op E ancommende, midts te trecken van I tot in C E, de lini I K, evewijdeghe met D C, en meughen dan segghen, ghelijck rechtheflijn C I, tot scheefheflijn C K, also t'gewicht des heelen pylaers, tottet ghewicht op E ancommende.



Q 3 Maer

1st COROLLARY

of the 27th proposition of the 1st book of the Art of Weighing

If in the figure of the 27th proposition of the 1st book there were attached at the point *E*, instead of the oblique weight *G*, a fixed point as shown opposite, it is obvious that against this fixed point a pressure equal to the weight *G* would be exerted, this pressure acting on the said point *E* with such obliqueness as is indicated by the oblique line *LE* [1]).

2nd COROLLARY

If in the above-mentioned 27th proposition the two oblique lines *LE, MF* are produced until they meet, it is obvious by the 25th proposition that the meeting point comes in the vertical centre line of gravity of the body. Therefore, if one should wish to know what oblique pressure acts on the fixed point *E* of the 1st corollary, one can do as follows: I draw through the prism's centre of gravity, viz. *P* opposite, the infinite vertical centre line of gravity, which *MF* produced meets in *Q*; thereafter from *Q* through *E* the line *QR*, *R* falling in *AM*. This being so, the pressure acting on *E* is as from *R* to *E* [2]), and in order to know how great it is, *ER* is used as oblique lifting line in the construction, and *ES* as vertical lifting line, by means of which the required quantity is manifestly found [3]).

3rd COROLLARY

But in order to set forth the properties of weights hanging on cords, let *AB* be a prism, whose centre be *C* and which be hanging in the two fixed points *D, E*, with two lines *CD, CE* coming from the centre of gravity *C*; these lines *CD* and *CE* are centre lines of gravity of the prism by the 5th definition [4]). Therefore, if *HI* be drawn between *DC* and *CF* [5]), parallel to *CE*, by the 13th definition *CI* is vertical lifting line, *CH* oblique lifting line, with which we say that as *CI* is to *CH*, so is the former's vertical lifting weight to the latter's oblique

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1) The letter *L* only occurs in the figure of Prop. 27 of Book 1 of the *Art of Weighing*.

2) This means that the pressure on *E* is directed along *RE*.

3) With the aid of Prop. 17 of Book 1 of the *Art of Weighing* the required vertical force is found, from which by means of Prop. 20 the oblique force may be derived.

4) The reader is reminded that, as has been said in note 3 to p. 101 of the present volume, the meaning of „centre line of gravity" in the second edition of the *Art of Weighing* is not the same as in the first. In the second edition, and consequently also in this *Supplement*, centre line of gravity is any line through the centre of gravity.

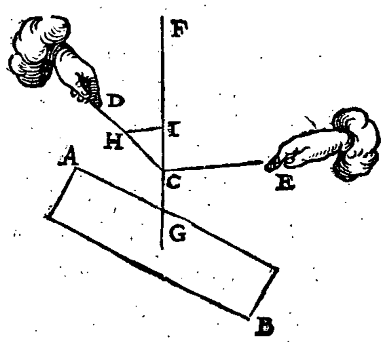
5) *CF* is the vertical through *C*.

Maer C K valt altijt even an H I, daerom en ist niet noodich te trecken dese laetste lini I K, maer hebben alle noodighe bekende palen inde drie sijden des driehoucx H I C, met welcke wy meughen aldus segghen:

Ghelijck C I tot C H, alsoo t'ghewicht des pylaers, tottet ghewicht op D ancommende. Voort ghelijck C I tot I H, alsoo t'ghewicht des pylaers, tottet ghewicht op E ancommende. Weerom ghelijck C H tot H I, alsoo t'ghewicht op D ancommende, tottet ghewicht op E ancommende.

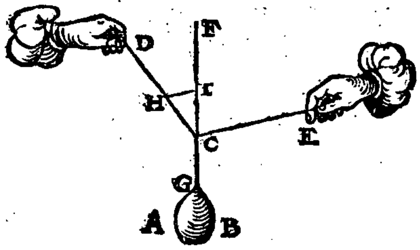
4 VERVOLGH.

Maer op dat wy ons voorghenomen verclaring der ghedaente van ghewichten an tauwen hanghende noch naerder commen, so laet dē pylaer A B neerwaert ghetrocken worden, alsoo datse nu sy ter plaets als hier onder, en deur de 3 begeerte, soo en veroirsaeckse an t'ghene daerse an hangt, gheen ander ghewicht danse eerst en dede hoogher hanghende: Daerom de voorgaende everedenheyt des 3 vervolghs is noch inde form des 4 vervolghs.



5 VERVOLGH.

Laet ons nu in plaets des pylaers A B int 4 vervolgh, hanghen een ander lichaem der selfde swaerte, maer van form en stofswaerheyt soot valt, als A B in dit 5 vervolgh. Ende is noch openbaer dat ghelijck C I tot C H, also t'ghewicht A B, tottet ghewicht op D ancommende. Voort gelijck C I tot I H, also t'gewicht A B, tottet gewicht op E ancommende. Weerom ghelijck C H tot H I, alsoo t'ghewicht op D ancommende, tottet gewicht op E ancommende.



Hier uyt is openbaer, dat sooder aen een lini D C E als coorde, hinghe een bekent ghewicht A B, en de houcken F C D, F C E, oock bekent sijnde, datmen can segghen hoe veel gewelt elck deel D C, C E te draghen heeft.

6 VERVOLGH.

Maer by aldien an een lini alsoo hinghen twee of meer ghewichten, als inde volghende form de lini A B C D E F, diens uyterste vastpunten sijn A en F, an welcke lini hanghen vier bekende ghewichten, als G, H, I, K: Tis openbaer datmen can segghen hoe veel gewelt datter comt an elck der vijf linien A B, B C, C D, D E, E F: Want treckende, by voorbeelt ghesteyt, de lini G B voorwaert, als tot L, daer na M N evewijdeghe met B C: Ick segh B N gheeft B M, wat t'gewicht G? T'ghene daer uyt volght is voor t'ghewelt op A B ancommende.

Weder

lifting weight. But the vertical lifting weight of *CI* is equal to the weight of the whole prism. Therefore as *CI* is to *CH*, so is the weight of the whole prism to the weight acting on *D*. And in the same way the weight acting on *E* is also found, provided there be drawn from *I* to *CE* the line *IK*, parallel to *DC*; we can then say: as the vertical lifting line *CI* is to the oblique lifting line *CK*, so is the weight of the whole prism to the weight acting on *E*.

But *CK* is always equal to *HI*, therefore it is not necessary to draw this latter line *IK*, but we have all the requisite known terms in the three sides of the triangle *HIC*, so that we can say as follows:

As *CI* is to *CH*, so is the weight of the prism to the weight acting on *D*. Further as *CI* is to *IH*, so is the weight of the prism to the weight acting on *E*. Again, as *CH* is to *HI*, so is the weight acting on *D* to the weight acting on *E* [1]).

4th COROLLARY

But in order that we may make our proposed explanation of the property of weights hanging on cords even clearer, let the prism *AB* be pulled down in such a way that it be now in the place shown below [2]); then by the 3rd postulate it does not cause on that from which it is hanging any different weight from that it first did, when hanging higher. Therefore the foregoing proportion of the 3rd corollary still holds in the figure of the 4th corollary.

5th COROLLARY

Now let us hang instead of the prism *AB* in the 4th corollary another body of equal gravity, but of any form and specific gravity, viz. *AB* in this 5th corollary. Then it is also manifest that as *CI* is to *CH*, so is the weight *AB* to the weight acting on *D*. Further, as *CI* is to *IH*, so is the weight *AB* to the weight acting on *E*. Again, as *CH* is to *HI*, so is the weight acting on *D* to the weight acting on *E*.

From this it is manifest that if from a line *DCE* as cord there were hanging a known weight *AB*, and if the angles *FCD*, *FCE* were also known, it can be said how much weight each part *DC*, *CE* has to carry.

6th COROLLARY

But if there were thus hanging on a line two or more weights, as in the following figure the line *ABCDEF*, whose extreme fixed points are *A* and *F*, on which line there are hanging four known weights, viz. *G*, *H*, *I*, *K*, it is manifest that it can be said how much force acts on each of the five lines *AB*, *BC*, *CD*, *DE*, *EF*. For if, for example, the line *GB* be produced to *L*, and *MN* be then drawn parallel to *BC*: I say *BN* gives *BM*, what the weight *G*? What follows therefrom is the force acting on *AB* [3]).

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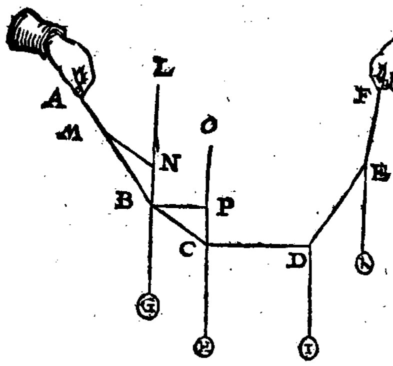
[1]) Here, once again, it is seen that Stevin was fully acquainted with the parallelogram (or triangle) of forces.

[2]) Actually this figure is found opposite these words.

[3]) Up to this point Stevin has always spoken of the weight to be carried by a fixed point or of the force acting on that point. Here there is some doubt whether „force acting on *AB*" means „force on *A* acting along *AB*" or „force on *B* acting along *BA*" It is probable that the first meaning was intended, but Stevin is fully aware of the fact that the second force is equal and opposite to the first.

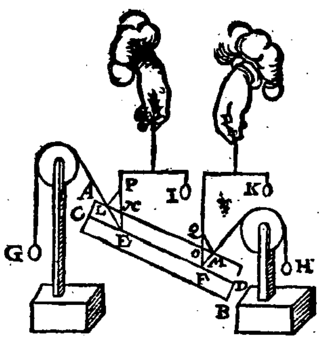


Weerom B N gheeft M N, wat T'ghewicht G? T'ghene daer uyt volght is voor t'ghewelt op B C ancommende.

Laet andermael H C voorwaert ghetrocken worden tot O, en B P evewijdeghe met C D: Ick segh C P gheeft C B, wat t'ghewicht H? T'gene daer uyt comt is voor t'ghewelt op B C ancommende. Waer uyt blijckt datmen alsdan t'selve sal moeten vinden, datmen te vooren van B C vant: En soo voort met al d'ander. Hier af en van meer ander heeft sijn VORSTELICKE GHENADE dadelicke proef ghedaen, en bevonden die gantschelick t'overcommen mette *spiegheling.

*Theoria.

De everedenheyt des 27 voorstels can deur ander manier uyt gesproken worden dan daer gedaen is, waer uyt lichter wercking volght. Om t'welck by voorbeelt te verclaren, laet hier andermael gestelt wordẽ de form des selven 27 voorstels, alwaermen seght, ghelijck scheefhefwicht tot rechthefwicht, also elck scheefhefwicht tot sijn rechthefwicht. Maer om dit deur ander manier uyt te spreken, waer uyt lichter wercking volght; ick treck tusschen rechtheflijn en scheefheflijn, een lini als L P evewijdighe met F M: T'welck soo wesende, ick segh nu, ghelijck rechtheflijn tot scheefheflijn, alsoo t'ghewicht des heelen pylaers, tot haer scheefhefwicht, dat is, ghelijck E P tot E L, alsoo t'ghewicht des pylaers tot G. Wederom ghelijck E P tot P L, alsoo t'ghewicht des pylaers tot H: Na welcke manier het vinden der onbekende palen openbaerlick corter valt, als na d'ander. Merckt noch datmen in plaets van L P, oock soude hebben meugen trecken een lini tusschen d'ander rechtheflijn en haer scheefheflijn, als hier M Q evewijdeghe met E L, waer me men dergelijcke soude meugen doen als met L F gedaen is, en tot een selve besluyt commen: Want ghelijck P E tot E L, alsoo Q F tot F M, uyt oirsaeck dat den driehouck F M Q, even en gelijck is met L P E, deur dien Q F evewijdeghe is met P E, en M F met L P.



Q 4 VER-

Again *BN* gives *MN*, what the weight *G*? What follows therefrom is the force acting on *BC*.

Let *HC* again be produced to *O*, and *BP* drawn parallel to *CD*. I say: *CP* gives *CB*, what the weight *H*? What follows therefrom is the force acting on *BC*. From which it is evident that the same will then have to be found that was previously found of *BC*. And so on with all the others. This and several other things his PRINCELY GRACE tested in practice and found to be entirely in agreement with the theory.

The proportion of the 27th proposition can be expressed differently from the way in which it has there been done, through which the construction is easier. In order to explain this by means of an example, let there be taken again the figure of this 27th proposition, where it is said: as the oblique lifting weight is to the vertical lifting weight, so is each oblique lifting weight to its vertical lifting weight [1]). But in order to express this differently, through which the construction is easier, I draw between the vertical lifting line and the oblique lifting line a line, viz. *LP*, parallel to *FM*. This being so, I now say: as the vertical lifting line is to the oblique lifting line, so is the weight of the whole prism to its oblique lifting weight, that is: as *EP* is to *EL*, so is the weight of the prism to *G*. Again, as *EP* is to *PL*, so is the weight of the prism to *H* [2]). By which method the finding of the unknown terms is obviously shorter than by the other. It should also be noted that instead of *LP* one might also have drawn a line between the other vertical lifting line and its oblique lifting line, such as here *MQ* parallel to *EL*, with which one might do the same as has been done with *LF*, and come to the same conclusion. For as *PE* is to *EL*, so is *QF* to *FM*, because the triangle *FMQ* is equal and similar [3]) to *LPE*, since *QF* is parallel to *PE*, and *MF* to *LP*.

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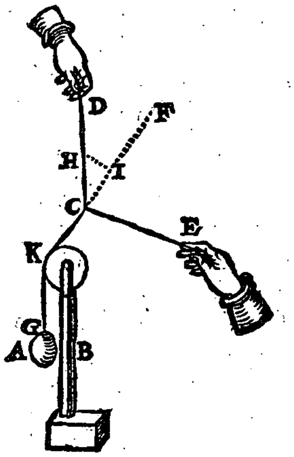
[1]) Stevin naturally means to say: as the oblique lifting *line* is to the vertical lifting *line*, etc.

[2]) This of course is true, but it is quite a different thing from what was taught in Prop. 27. The object of the latter was to find the ratio of the vertical and the oblique forces which have to act on a given point in order to keep a body in equilibrium, if at another point a vertical or an oblique force is also acting on it. Stevin here determines the ratio of each of the aforesaid forces to the weight of the whole prism. It is no longer the ratio $NE : LE$ which matters, but $PE : LE$, where PL is parallel to FM.

[3]) Only similar.

8 VERVOLGH.

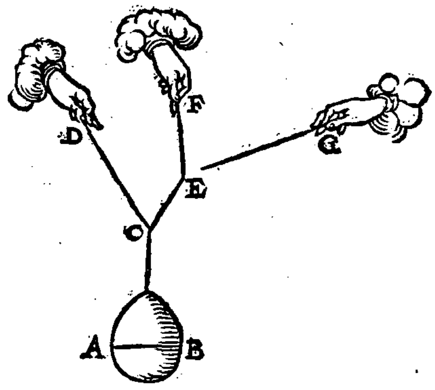
Tot hier toe is ghefeyt van ghewichten hanghende an twee linien: Int volghende willen wy handelen van ghewichten an meer dan twee linien hangende: Tot desen eynde fegh ick aldus: Laet ons andermael nemen de form des 5 vervolghs, welcke sy de onderschreven deses 8 vervolghs, alleenelick daer in verschillende, dat de lini CG hier comt over een catrol an K, sulcx dat hoewel KCF een rechte lini is, nochtans comtse scheefhouckich op den sichteinder: Voort sy dit ghewicht AB t'selve, en de twee houcken DCF, FCE oock de selve. Dit soo wesende, tis kennelick dat wy hier meughen seggen als int 5 vervolgh, ghelijck CI, tot CH, alsoo t'ghewicht AB tottet ghewicht op D ancommende: Voort ghelijck CI, tot IH, alsoo t'ghewicht AB tottet ghewicht op E ancommende: Weerom ghelijck CH, tot HI, alsoo t'ghewicht op D ancommende, tottet ghewicht op E ancommende.



Hier uyt is openbaer dat sooder an een lini DCE als coorde, hinghe een ghewicht AB, datmen can segghen hoe veel ghewelt elck deel DC, CE, te doen heeft.

9 VERVOLGH.

Soo een ghewicht hinghe an drie linien, als hier onder, t'ghewicht AB hanghende ande twee linien CD, CE, maer de selve CE ande twee linien EF, FG, sulcx dattet ghewicht AB hangt ande drie linien CD, EF, EG, men can weten hoe veel ghewelt datter op elcke der selve drie linien ancomt. Want deur het 5 vervolgh is openbaer watter op CD en CE ancomt: Maer bekent wesende wat ghewelt op CE ancomt, soo wort deur het 8 vervolgh gheweten watter op elcke der twee linien EF, EG ancomt.



Maer sooder ande lini CD hier boven ghehecht waren sulcke twee treckende linien als an CE, gelijck hier onder DH, DI, tis openbaer dattet ghewicht an yder dier twee linien, alsoo bekent soude worden ghelijck over d'ander sijde, en vervolghens dat bekent soude sijn hoe veel ghewelt op yder der vier linien EF, EG, DH, DI ancomt, t'sy oock dat de linien

8th COROLLARY

So far weights hanging on two lines have been discussed. In the following we will deal with weights hanging on more than two lines. To this end I say as follows: Let us take once more the figure of the 5th corollary, which shall be the one below, of the 8th corollary, differing only in that the line *CG* here passes across a pulley at *K*, in such a way that though *KCF* is a straight line, it comes at oblique angles to the horizon. Further this weight *AB* shall be the same, and the two angles *DCF*, *FCE* shall also be the same. This being so, it is obvious that we can here say, as in the 5th corollary: as *CI* is to *CH*, so is the weight *AB* to the weight acting on *D*. Further, as *CI* is to *IH*, so is the weight *AB* to the weight acting on *E*. Again, as *CH* is to *HI*, so is the weight acting on *D* to the weight acting on *E*.

From this it is manifest that if on a line *DCE* as cord there were hanging a weight *AB*, it can be said how much force each part *DC*, *CE* has to carry.

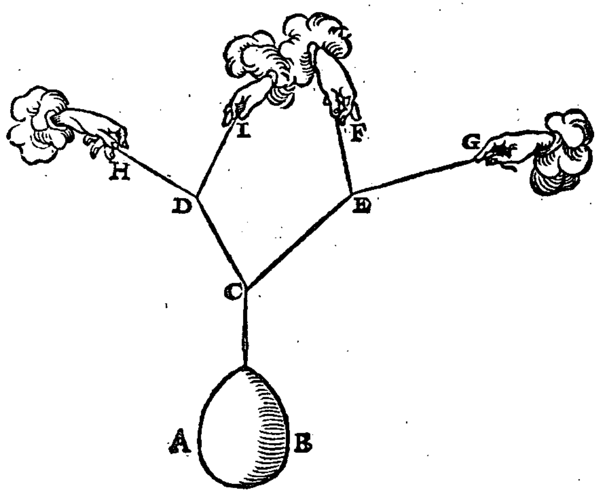
9th COROLLARY

If a weight were hanging on three lines, as below, the weight *AB* hanging on the two lines *CD*, *CE*, but this *CE* on the two lines *EF*, *FG*, in such a way that the weight *AB* is hanging on the three lines *CD*, *EF*, *EG*, it can be known how much force acts on each of these three lines. For by the 5th corollary it is manifest what force acts on *CD* and *CE*. But if it is known what force acts on *CE*, it is known by the 8th corollary what force acts on each of the two lines *EF*, *EG*.

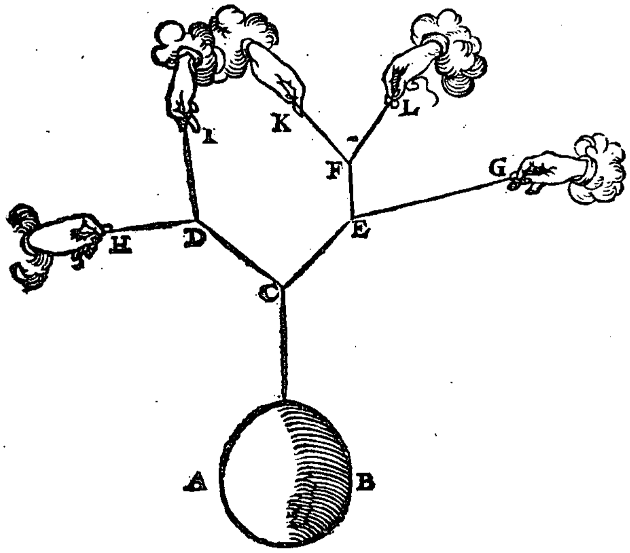
But if to the line *CD* above there were attached two such drawing lines as at *CE*, as below *DH*, *DI*, it is manifest that the weight on each of those two lines would become known in the same way as on the other side, and that consequently it would be known how much force acts on each of the four lines *EF*, *EG*, *DH*, *DI*, no matter whether the lines as *DH* and *EF* and the like come in the same plane or not.

linien als D H en E F met dierghelijcke, commen in een ſelve plat of niet.

Merckt noch openbaer te ſijn dat de linien als C E G, C E F en dierghelijcke, niet recht en connen weſen, maer moeten een houck hebben an E, want de lini



E F eenighe ghewelt doende deur t'gheſtelde, moet de lini C E G nootſakelick eenighe cromte gheven an E, alſoo oock moet de lini E G ande lini C E F.



Maer ſo ande lini E F hier boven, ghehecht waren ſulcke twee treckende linien als F K, F L hier onder, men can weten om de voorgaende redenen hoe veel ghewelt datter ancomt op elck der twee linien F K, F L: En vervolgens hoe veel an elck der vijf linien D H, D I, F K, F L, E G. En ſoo voort int oneyndelick met allen anderen dierghelijcke.

10 VER-

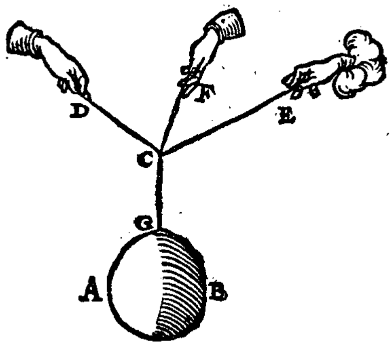
It should also be noted that it is manifest that the lines as *CEG*, *CEF* and the like cannot be straight, but must have an angle at *E*, for if the line *EF* exerts some force, by the supposition, it must necessarily impart some curvature to the line *CEG* at *E*; the same must also be done by the line *EG* to the line *CEF*.

But if on the line *EF* above there were attached two such drawing lines as *FK*, *FL* below, it can be known for the above reasons how much force acts on each of the two lines *FK*, *FL*. And consequently, how much force acts on each of the five lines *DH*, *DI*, *FK*, *FL*, *EG*. And so on *ad infinitum* with all other similar cases.

10 VERVOLGH.

Tot hier toe is ghefeyt van een ghewicht als A B, hanghende an een lini die ftreckt tot C, en commende vande felve C twee ander linien C D, C E. Maer foder van die C fulcke drie linien quamen, de * fpiegelingen vallen anders. Om hier af met onderfcheyt te fpreken ick fegh aldus: De voorfchreven drie linien fijn of in een felve plat, of niet: In een felve wefende, het voorftel en heeft geen eenich feker befluyt. Laet tot voorbeelt A B een ghewicht fijn, en de drie linien

Theorie.



daert an hangt fijn C D, C E, C F: De lini van C tottet ghewicht fy C G: Laet daer na de lini C F deurfneen of ghebroken worden, fulcx dattet ghewicht A B blijve hanghen ande twee linien C D, C E; t'welck foo fijnde, t'ghewicht A B blijft op fijn felve plaets, en de twee houcken D C G, E C G blijven oock de felve fonder verandering; hoewel nochtans op de twee linien C D, C E, nu meer ghewelt ancomt dan eer de lini C F deurfneen was, wantfe d'ander twee fo veel verlichte, als heur ghewelt veroirfaeckte: Maer de ghewelt can an C F ghe-ftelt worden van oneyndelicke verfcheydenheden, d'een grooter als d'ander, waer uyt openbaerlick blijckt fulck voorftel gheen eenich feker befluyt te hebben, ghelijck het voornemen was te verclaren.

11 VERVOLGH.

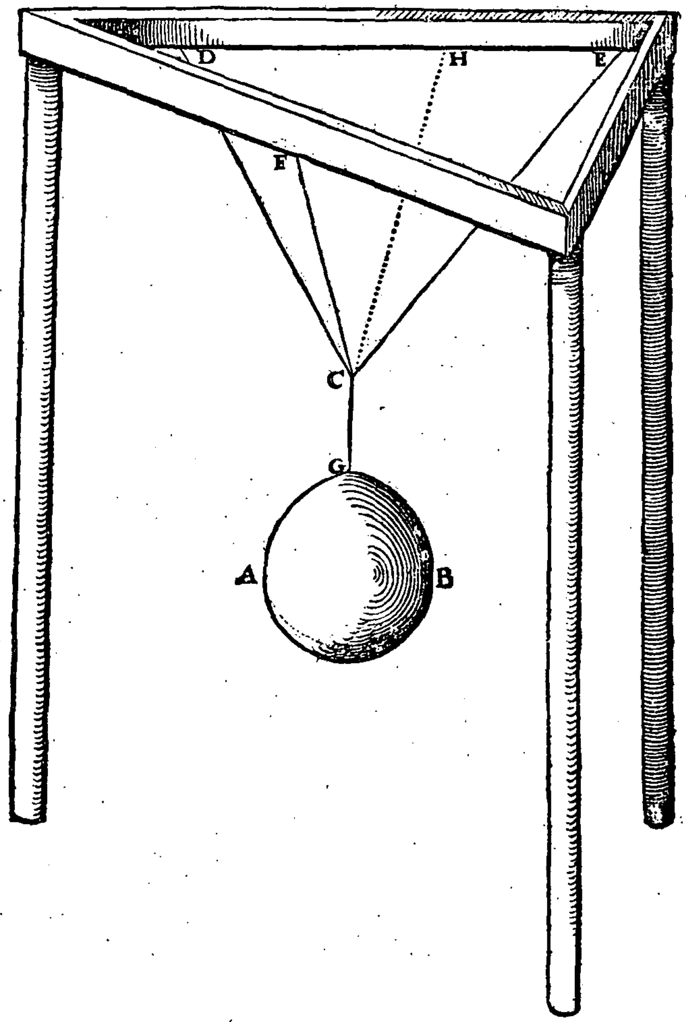
Maer foo de bovefchreven drie linien in twee verfcheyden platten waren, het voorftel en heeft maer een befluyt, en dat bekent. Laet by voorbeelt t'gewicht A B hier onder genome worden te hangen ande drie linien C D, C E, C F. Maer foo datfe nu niet al in een felve plat en fijn, voort is C G de lini van C tottet ge-wicht. Om nu te vinden t'ghewicht op een der drie linien ancommende, alsop C F, ick neem de ghemeene fne des plats daer C D, C E in fijn, en des plats daer G C, C F in fijn, welcke ghemeene fne fy de lini C H: De felve ghenomen voor lini daer t'ghewicht A B an hangt, en d'ander twee C D, C E doorfneen, of ghe-broken fijnde, fulcx dattet alleenelick blijft hanghen ande twee linien C H, C F, tis kennelick dat den houck G C F de felve blijft, diefe was voor de deurfnijding der twee linien C D, C E; en de ghewelt die eerft op C F an quam, blijft na de doorfnijding oock de felve: Daerom ghenomen t'ghewicht A B te hangen ande voorfchreven twee linien C F, C H, foo is deur het 5 vervolgh bekent wat ghe-welt

10th COROLLARY

So far a weight has been discussed, such as *AB*, hanging on a line extending to *C*, with two other lines *CD*, *CE* coming from this *C*. But if from this *C* there came three such lines, the theory is different. In order to make a distinction, I say as follows: The above-mentioned three lines are either in the same plane or not. If they are in the same plane, the proposition does not have any single definite conclusion. By way of example, let *AB* be a weight, and let the three lines on which it is hanging be *CD*, *CE*, *CF*. The line from *C* to the weight shall be *CG*. Thereafter let the line *CF* be intersected or broken, in such a way that the weight *AB* continue to hang on the two lines *CD*, *CE*. This being so, the weight *AB* remains in the same place, and the two angles *DCG*, *ECG* also remain the same, without any change, though nevertheless there now acts more force on the two lines *CD*, *CE* than before the line *CF* was intersected, because it relieved the other as much as was caused by its own force. But the force on *CF* can be taken of infinite variety, one greater than the other, from which it is manifest that this proposition does not have any single definite conclusion, as was intended to be set forth.

11th COROLLARY

But if the above-mentioned three lines are in two different planes, the proposition has only one conclusion, and this is known. For example, let the weight *AB* below be taken to be hanging on the three lines *CD*, *CE*, *CF*, but in such a way that now they are not all in the same plane. Further *CG* is the line from *C* to the weight. Now in order to find the weight acting on one of the three lines, viz. on *CF*, I take the common intersection of the plane in which are *CD*, *CE*, and the plane in which are *GC*, *CF*, which common intersection shall be the line *CH*. If this is taken for the line on which the weight *AB* is hanging, and the other two *CD*, *CE* are intersected or broken, in such a way that it continues to hang only on the two lines *CH*, *CF*, it is obvious that the angle *GCF* remains the same that it was before the intersection of the two lines *CD*, *CE*, and the force which first acted on *CF* also remains the same after the intersection. If therefore the weight *AB* is taken to be hanging on the above-mentioned two lines *CF*, *CH*, welt op C F ancomt. En alſoo ſal oock bekent worden wat ghewelt op elck der twee ander linien C D, C E ancomt.



Tis oock openbaer dat by aldien an eenige, of an elcke deſer drie treckende linien noch ander treckende linien quamen, na de manier des 9 vervolghs, dat bekent ſoude worden wat ghewelt op yder ancomt.

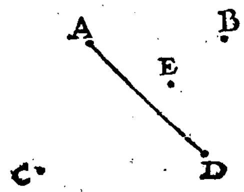
12 VERVOLGH.

By aldien een gnewicht hinghe an ſulcke vier linien, ghelijckt int 11 vervolgh an drie hangt, t'voorſtel en heeft gheen ſeker eenich beſluyt. Laet tot voorbeelt A, B, C, D, als in gronteyckening, ſijn vier uyterſte bovenſte punten der vier linien daer an deur t'ghedacht het ghewicht hangt: De hanghende ſwaerheyts middellijn des ſelfden ſal commen of inde lini A D, of daer buyten binnen den driehouck A D B, of binnen den driehouck A D C. (want buyten den vierhouck A B C D, of in ſijn omtreck te vallen is onmeugelick) Maer inde lini it is known by the 5th corollary what force acts on *CF*. And in this way it will also become known what force acts on each of the two other lines *CD*, *CE*.

It is also manifest that if to any or each of these three drawing lines there were added more drawing lines, in the manner of the 9th corollary, it would become known what force acts on each.

12th COROLLARY

If a weight be hanging on four such lines, as it is hanging on three lines in the 11th corollary, the proposition does not have any single definite conclusion. For example, let *A*, *B*, *C*, *D*, as in the plan, be four upper extremes of the four lines on which the weight is conceived to be hanging. The vertical centre line of gravity of the latter will come either in the line *AD*, or outside it within the triangle *ADB*, or within the triangle *ADC* (for it is impossible that it should fall outside the quadrilateral *ABCD* or in its perimeter). But if it falls in the line lini A D valſende, tis kennelick dat de ghewelt der twee linien onder B en C commende, wel meughen verlichten de ghewelt der twee linien onder A en D commende, maer de gheſtalt des driehoucx dier twee linien, te weten de twee onder A en D, mette derde A D, en crijcht gheen verandering: En daerom meugen oneyndelicke verſcheyden grooter en cleender ghewelden ande linien onder C, B, vervought worden, die de ghewelden op A en D ancommende veranderen, blijvende nochtans de form van t'ghegheven de ſelve, ſulcx datter gheen ſeker eenich



beſluyt en is. Maer vallende de hanghende ſwaerheyts middellijn in een der driehoucken, ick neem inden driehouck A D B an t'punt E, tis kennelick dat alſdan de ghewelt opt punt C ancommende, gheen verandering en geeft ande gheſtalt der drie linien commende onder A, B, D, waer uyt het ſelve alſvooren volght, te weten ſulck voorſtel gheen ſeker eenich beſluyt te hebben.

Noch valt hier dit te bedencken: Angheſien t'voorſtel met een gewicht hanghende an vier linien als in dit 12 vervolgh, gheen ſeker eenich beſluyt en heeft, ſoo en ſal uyt noch ſtercker reden, t'voorſtel met meer dan vier linien gheen ſeker eenich beſluyt hebben. Voort angheſien een ghewicht hanghende an drie linien die in een ſelve plat ſijn, als int 10 vervolgh, gheen ſeker eenich beſluyt en hebben, ſoo en ſal uyt noch ſtercker reden een ghewicht hanghende an vier of meer linien die in een ſelve plat ſijn, gheen ſeker eenich beſluyt hebben.

MERCKT.

Een lichaem can noch hanghen an drie linien op een ander wijſe dan de voorgaende des 11 vervolghs, te weten ſoo dat de linien ant lichaem ſelf tot verſcheyden plaetſen ghehecht ſijn, in ſulcker voughen datſe voortgetrocken nerghens in een ſelve punt en vergaren, ghelijckt nootſakelick gebeurt alſt lichaem alleenelick an twee linien hangt deur het 25 voorſtel des 1 boucx. Maer hoe gevonden ſal worden t'ghewicht op yder van ſulcke drie linien ancommende, daer heb ick op gedacht, maer int beſchrijven van deſen en is t'begeerde my niet verſchenen, watter een ander mael of commen wil, of wat ymant anders daer in ſal doen of niet, dat wert den tijt leeren.

13 VERVOLGH.

Tot hier toe is gheſeyt van ghewichten hanghende an een lini, uyt een punt van welcke twee of drie ander linien na verſcheyden oirten ſtrecken: Waer deur openbaer ſijn dergelijcke wichtighe ghedaenten, van ſwaerheden hanghende an twee of drie linien, die ande ſelve ſwaerheyt ghehecht en opwaert voortſtreckende, vergaren inde hanghende ſwaerheyts middellijn in een ſelve punt. Laet by voorbeelt A B een ſwaerheyt ſijn, hanghende ande twee linien D C, E C, verſamende in C, en hanghende ande ſwaerheyts middellijn C F. Om hier af te vinden de ghewelt op elck der twee linien D C, E C ancommende, men treckt F C voorwaert na G, en uyt eenich punt in D C, ick neem H, een lini tot in C G, als H I, evewijdich met C E. T'welck ſoo ſijnde, ick ſegh dat ghelijck

CI

AD, it is obvious that the forces acting on the two lines represented by *B* and *C* can indeed relieve the force acting on the two lines represented by *A* and *D*, but the form of the triangle of those two lines, to wit the two represented by *A* and *D*, with the third *AD*, is not changed. And therefore an infinite variety of larger and smaller forces can be applied to the lines represented by *C*, *B*, which alter the forces acting on *A* and *D*, the form of the given figure nevertheless remaining the same, so that there is no single definite conclusion. But if the vertical centre line of gravity falls within one of the triangles—I assume in the triangle *ADB* in the point *E*—it is obvious that in this case the force acting on the point *C* does not change the position of the three lines represented by *A*, *B*, *D*, from which follows the same as before, to wit that this proposition does not have any single definite conclusion.

In addition the following should be borne in mind: Since the proposition with a weight hanging on four lines, as in this 12th corollary, does not have any single definite conclusion, *a fortiori* the proposition with more than four lines will not have any single definite conclusion. Further, since a weight hanging on three lines which are in the same plane, as in the 10th corollary, does not have any single definite conclusion, *a fortiori* a weight hanging on four or more lines which are in the same plane will not have any single definite conclusion.

NOTE

A body can also be hanging on three lines in a different way from the foregoing one of the 11th corollary, to wit in such a way that the lines are attached to the body itself in different places, so that, when produced, they never meet in the same point, as necessarily happens when the body is hanging only on two lines, by the 25th proposition of the 1st book. But as to how the weight acting on each of such three lines is to be found: I have thought about this, but I have not been able to find the required construction; time will show whether I shall succeed another time, or what someone else will find in this matter or not [1]).

13th COROLLARY

So far weights have been discussed which are hanging on a line, from a point from which two or three other lines extend in different directions. From which are manifest such static properties of gravities hanging on two or three lines which, attached to this gravity and extending upwardly, meet in the vertical centre line of gravity in one and the same point. For example, let *AB* be a gravity hanging on the two lines *DC*, *EC*, meeting in C, and hanging on the centre line of gravity *CF*. In order to find from this the force acting on each of the two lines *DC* and *EC*, *FC* is produced to *G*, and from some point in *DC*—I take *H*—a line is drawn to *CG*, viz. *HI*, parallel to *CE*. This being so, I say that

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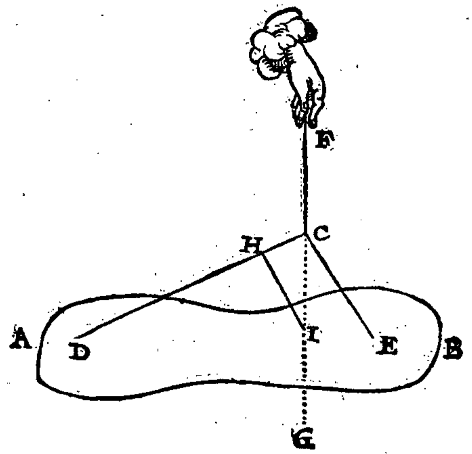
[1]) Stevin here comes up against the problem how to resolve a system of forces acting on a rigid body along skew lines, a problem which was not adequately solved until the beginning of the 19th century.

CI tot CH, alsoo t'ghewicht AB tottet ghewicht op D ancommende. Voort ghelijck CI tot IH, alsoo ghewicht AB, tottet ghewicht op E ancommende.



Wederom ghelijck CH tot HI, alsoo t'ghewicht op D ancommende, tot het ghewicht op E ancommende, waer aft'bewijs blijckt int 5 vervolgh.

Tis oock openbaer dat sulcke eyghenschappen als ghesseyt sijn te vallen inde formen van ghedaente des 9, 10, 11 en 12 vervolghs, dergheliicke eyghenschappen oock te vallen in dergheliicke formen van ghedaente deses 13 vervolghs.

TAVWICHTS
EYNDE.

R

as *CI* is to *CH*, so is the weight *AB* to the weight acting on *D*. Further, as *CI* is to *IH*, so is the weight *AB* to the weight acting on *E*. Again, as *CH* is to *HI*, so is the weight acting on *D* to the weight acting on *E*, the proof of which appears from the 5th corollary.

It is also manifest that such properties as have been said to be present in the figures of the 9th, 10th, 11th, and 12th corollaries also exist in similar figures of the 13th corollary.

END OF THE CORD WEIGHT

TWEEDE DEEL
DES BYVOVGHS
DER WEEGHCONST,
VANT
CATROLWICHT.

SECOND PART OF THE SUPPLEMENT TO THE ART OF WEIGHING, OF THE PULLEY WEIGHT

Argumentum.

CORTBEGRYP

DES CATROLWICHTS.

ALſoo ſijn VORSTELICKE GHENADE deurſien hadde het bouck Delle fortificationi di Buonaiuto Lorini, en daer in overleſen een handel van catrollen, vvaer in gheſeyt vvort van ghevvichten alleenlick recht opgaende, deur treckende crachten recht neervvaert ſtreckende: En dat nochtans metter daet dicvvils de ſelve niet recht op en neer en gaen, ſo is hy begheerich gevveeſt oock te verſtaen de crachten, reden en oirſaken der ſcheeve, om alſoo van deſen handel volcommen kennis te hebben, vvelcke gheneghentheyt oock in genouchſaem reden ghegront ſchijnt, ghemerct catrollen dadelick ſeer ghebruyckt vvorden, tot optrecking van groote ghevvichten, en dattet ſomvvijlen oirboir can ſijn, van te vooren te vveten vvat macht datter behouft om een voorgheſtelde ſvvaerheyt op te trecken. Nu alſoo hy hem gheoeffent hadde inde voorgaende VVeeghconſt, mettet eerſte deel des Byvoughs, vvaer deur de vvichtighe ghedaenten des Catrolvvichts grondelick connen verſtaen vvorden, en dat hy hem dadelick daer toe begaf, ſoo heb ick t'ghene daer af ghedaen vviert onder ſijn vviſconſtighe ghedachteniſſen vervought, als volght.

VOOR-

ARGUMENT OF THE PULLEY WEIGHT

As his PRINCELY GRACE had looked through the book *Delle Fortificationi di Buonaiuto Lorini* [1]), and had read therein a treatise about pulleys, in which weights moving only upwards through drawing forces tending straight downwards are discussed, and because nevertheless in practice they frequently do not move straight up and down, he was anxious to understand also the forces, reasons, and causes of the oblique weights, in order thus to have perfect knowledge of this matter, a desire which also seems to be founded on sufficient reasons, seeing that pulleys are frequently used in practice for pulling up large weights, and that it may sometimes be useful to know beforehand what force is required to pull up a given gravity. Now having exercised himself in the preceding Art of Weighing, with the first part of the Supplement, through which the static properties of the pulley weight can be thoroughly understood, and because he applied himself to it in practice, I have included the matter referring thereto among his mathematical memoirs, as follows.

1) *Delle fortificationi di Buonaiuto Lorini, Nobile Fiorentino, Libri Cinque.* Venetia 1592, 1597, 1609. We do not know which of these editions it was that Maurice consulted.

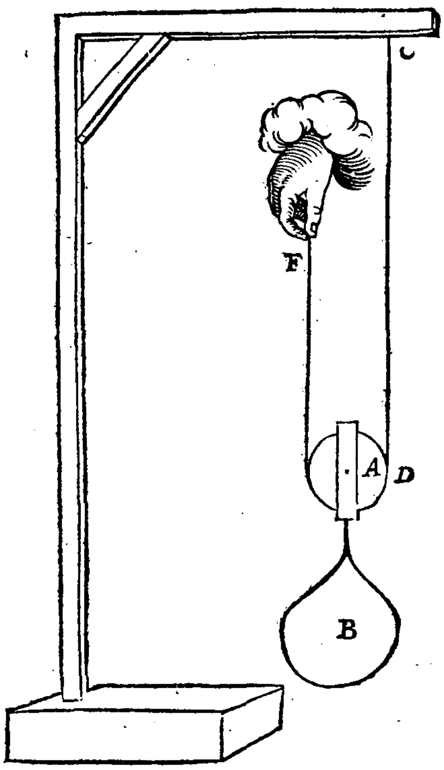
VOORSTEL.

T'onderſoucken de ghedaente der ghevvichten opghetrocken met catrollen.

Eer wy totte ſaeck commen ſullen int ghemeen dit ſegghen: Als wy ſpreken van een ghegeven ghewicht, men mach ſich int ghedacht beelden, om vande ſaeck met volcommenheyt claerlicker te handelen, dattet ghewicht des onderſten catrols, mettet ghewicht daer an hangende, t'ſamen maken t'ghegeven gewicht; voort dattet verſchil der ſwaerheyt veroirſaeckt deur de tau, hier voor gheen verſchil ghenomen en wort.

1 Voorbeelt met rechtvvichticheyt.

Laet in dees eerſte form A een catrol ſijn, hanghende daer an t'ghewicht B, de tau ſy C D E F, wiens twee deelen C D, F E, evewijt van malcander ſijn, of beyde rechthouckich op den * ſichteinder. Dit aldus weſende, en het heel ghewicht B alſoo hanghende ande twee deelen C D, F E, en op yder deel eveveel ghewelts ancommende, ſoo hangt om de draeyende beweeghlickheyt der ſchijf *Horizontem.*



an yder deel den helft van B: Daerom ſoo ymant ſijn hant ſtelde ant punt F, houdende t'ghewicht in die ſtandt, op ſijn handt ſoude commen den helft der

R 3 ſwaer-

PROPOSITION

To investigate the properties of weights pulled up by means of pulleys.

Before coming to the point, we shall say this in general: When we speak of a given weight, in order to deal more clearly and completely with the matter it is to be imagined that the weight of the lowest pulley together with the weight hanging thereon constitutes the given weight; further that the difference in gravity caused by the cord is here taken to be no difference.

1st Example, with vertical weights

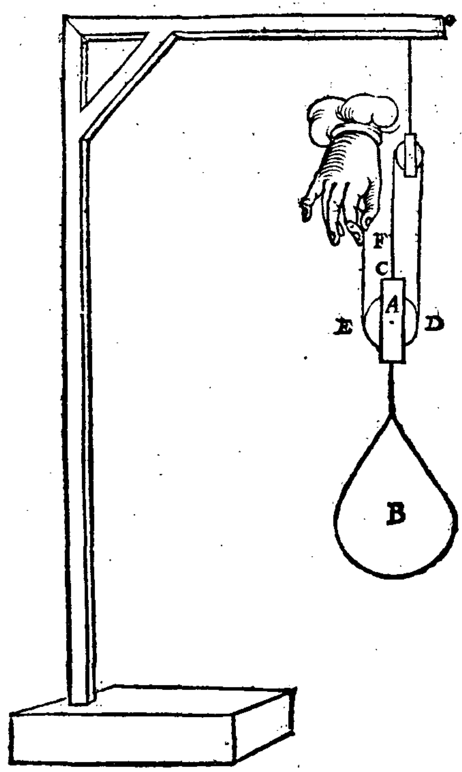
In this first figure let *A* be a pulley, on which be hanging the weight *B*, the cord be *CDEF,* whose two parts *CD, FE,* are equidistant from each other or both at right angles to the horizon. This being so, and the whole weight *B* thus hanging on the two parts *CD*, *FE*, and an equal force acting on each part, there hangs, owing to the rotatability of the disc, on each part the half of *B*. Therefore, if a man applied his hand at the point *F*, keeping the weight in that position, his hand ſwaerheyt van B, waer uyt de oirſaeck blijckt, waerom de ghewichten alſoo met een catrol lichter opgetrocken worden dan ſonder catrol. Merckt nochdanmen hier ſiet plaets te houden deſe ghemeene weeghconſtighe reghel:

Ghelijck wech des doenders, tot wech des lijders,

Alſoo ghewelt des lijders, tot ghewelt des doenders.

Want de hant an F, welcke hier doender is, opgaende 2 voeten, t'ghewicht B, dats hier lijder, en gaet maer op 1 voet, en dat om bekende oirſaken.

Deur t'ghene tot hier toe verclaert is vande eerſte form, alwaer t'ghewicht opghetrocken wort over een ſchijf, canmen verſtaen dergheli jcke ghedaente wanneerment treckt over twee ſchijven, als in dees tweede form, alwaer C



weerom tander uyterſte der tau beteyckent: Want het ghewicht B dan hangende an drie tauwen, die elck een derdendeel draghen, ſoo en heeft de hant an F dan maer de ghewelt te doen van een derdendeel des ghegheven ghewichts.

Ende

would have to carry the half of the gravity of *B*, from which appears the cause why weights are thus pulled up more easily with a pulley than without a pulley. It is also to be noted that the following common rule of statics is here seen to hold:

As is the path of the doer to the path of the sufferer,
So is the force of the sufferer to the force of the doer [1]).

For the hand at *F*, which here is the doer, rising 2 feet, the weight *B*, which here is the sufferer, rises only 1 foot, such through well-known causes.

From that which has so far been set forth about the first figure, where the weight is pulled up over one disc, similar properties can be understood when the weight is pulled over two discs, as in this second figure, where *C* again designates the other end of the cord. For the weight *B* then hanging on three cords, each of which carries one-third, the hand at *F* then need only exert the force of one-third of the given weight.

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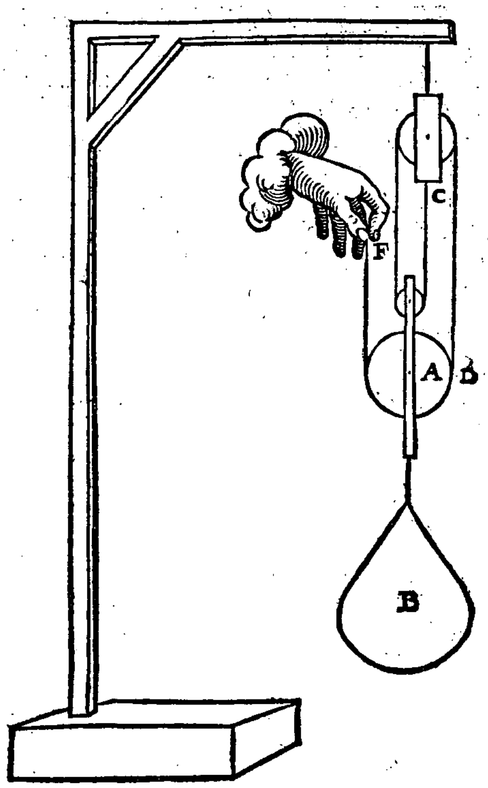
[1]) This is a literal translation of Stevin's Dutch rendering of the rule which in Latin reads as follows:

Ut spatium agentis ad spatium patientis
Sic potentia patientis ad potentiam agentis.

Stevin translates *agens* by *doender* (doer) and *patiens* by *lijder* (sufferer).

Ende over noch een schijf meer loopende als in dees 3 form, want het ghewicht B dan hanghende an vier tauwen die elck een vierendeel draghen van B, soo en heeft de hant an F dan maer een vierendeel des ghewichts B ghewelt te



doen. Waer me bekent is de ghemeene reghel van ghewichten over meer schijven ghetrocken sijnde.

Hier staet noch te dedencken datmen metter daet selden alsoo an F opwaert treckt, ghelijck wy om claerder bewijs wille inde bovesehreven drie formen by voorbeelt ghestelt hebben, maer men doet ghemeenelick de tau loopen over noch een schijf meer, om van boven neerwaert te trecken als in dese 4 form: Doch soo is te weten dat sulcke vierde oft laetste schijf, ande hant F gheen verlichting noch verandering desghewichts en brengt, om dattet gewicht B maer an vier tauwen en hangt ghelijck inde 3 form, want dese laetste tau een vijfde tau schijnende, en is eyghentlick mette vierde al maer een selve. Waer by te verstaen is, dat al liepe die tau over noch hondert sulcke catrollen, dat den trecker daer me gheen verlichting en crijcht.

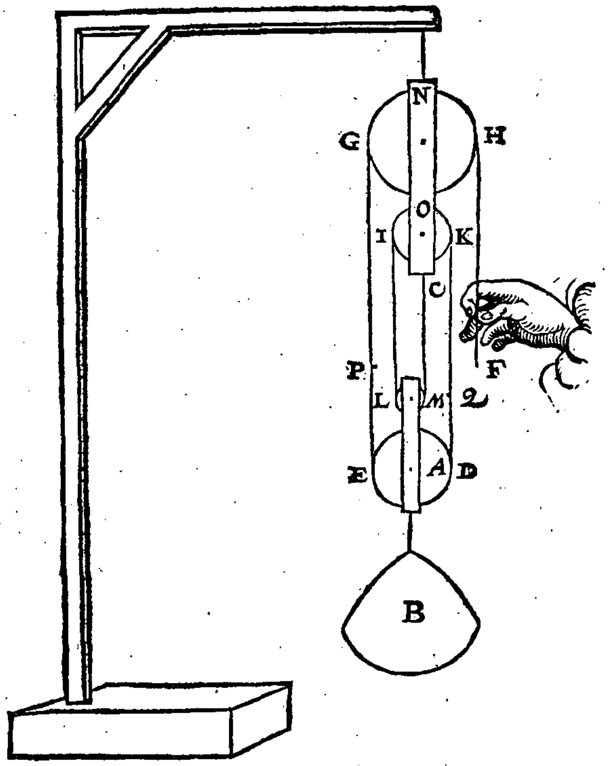
Maer soomen van t'voornomde dadelicke proef wilde sien, men sal an F deser vierde form, in plaets des hants hanghen een ghewicht als doender, wesende t'vierendeel van het optreckelick ghewicht, en sullen teghen malcander soo int werck gheen faute en is, evestaltwichtich bevonden worden. Maer om dat optreckelick ghewicht heel volcommelick uyt te spreken, het is de somme deser drie, te weten t'ghewicht B, t'onderste catrol A, en t'ghewicht veroirsaeckt deur de swaerheyt der tau. Maer om de selve swaerheyt der tau breeder te verclaren, soo laet D en E sijn de uyterste gheraeckselen der tau teghen de schijf A, en G H

K 4 de uyter-

And when the cord passes over one more disc, as in this 3rd figure, because the weight *B* then hangs on four cords, each of which carries one-fourth of *B*, the hand at *F* then need only exert the force of one-fourth of the weight *B*. With which is known the common rule of weights pulled over several discs.

It should be borne in mind that in practice one will seldom pull upwards at *F* in this way, as we have assumed by way of example in the above three figures, for the sake of a clearer proof, but the cord is usually passed over one more disc, in order to pull downwards from above, as in this 4th figure. But it should be known that such fourth or last disc does not cause any relief or change of the weight on the hand at *F*, because the weight *B* only hangs on four cords, as in the 3rd figure, for this last cord, which seems to be a fifth cord, is in reality one and the same with the fourth. By this it is understood that even if that cord ran over a hundred more such pulleys, the puller would not receive any relief therefrom.

But if it were desired to see a practical proof of the foregoing, at *F* of this fourth figure there shall be hung instead of the hand a weight as doer, being one-fourth of the weight to be pulled up, and if there is no error in the construction, they will be found to be of equal apparent weight with each other. But to describe that weight to be pulled up quite completely: it is the sum of these three, to wit the weight B, the lower pulley A, and the weight caused by the gravity of the cord. But in order to set forth this gravity of the cord more fully, let *D* and *E* be the extreme points of contact of the cord against the disc *A*, and *G* and *H* the extreme points of contact of the cord against the upper disc of the de uyterste gheraeckselen der tau teghen de bovenste schijf des bovenste catrols, L M de uyterste gheraeckselen der tau teghen de bovenste schijf des ondersten catrols; voort sy N t'middelste punt der tau tusschen G en H, en O t'middelste



punt der tau tusschen I en K, en C t'ander uyterste der tau: Laet voort gheteyckent worden in G E t'punt P, alsoo dat G P even sy met H F: Daer na in K D t'punt Q, alsoo dat K Q even sy met I L. Dit so wesende, N G P is even en evewichtich met N H F, en O I L met O K Q: Maer C M en brengt lichticheyt noch swaerheyt by. Sulcx dattet ghegeven gewicht mettet catrol, noch beswaert worden, so veel als veroirsaken de drie sticken taus, te weten des halfronts L M, des halfronts D E, en het recht stick Q D.

Merckt noch dat alsmen met catrollen dadelick yet optreckt, alsoo dattet eynde der voortghetrocken tau inde locht blijft hanghen, sonder vloer te gheraken, soo veel dat voortghetrocken deel taus weeght, soo veel sal openbaerlick den trecker min ghewelt behouven te doen.

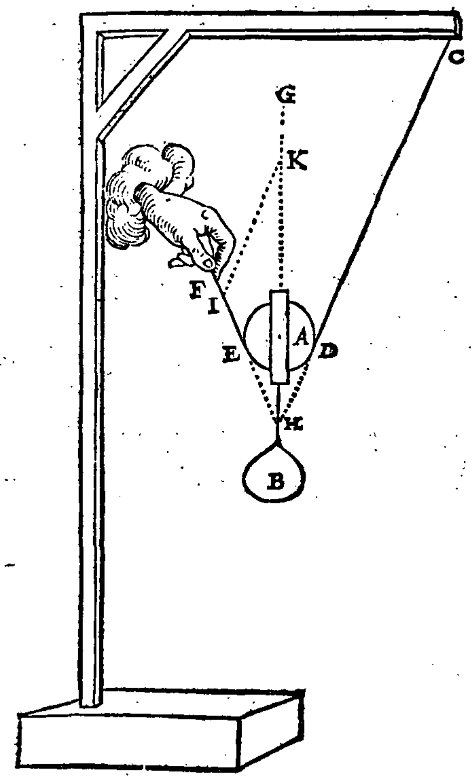
2 *Voorbeelt met scheefvvichticheyt.*

Laet dese eerste form sijn alsins ghelijck d'eerste des eersten voorbeelts, uytghenomen dat de hant hier an F niet recht op en treckt, maer scheefter sijdewaert uyt, t'welck soo sijnde, t'ghewicht op elcke tau ancommende, wort bekent deur het 5 vervolgh des 1 deels deses byvoughs der Weeghconst. Maer om daer af met een wat verclaring te doen; ick treck de lini daer t'ghewicht B an hangt opwaert tot G, als B G, en F E voorwaert, tot datse de oneyndelicke door B G ontmoet, t'welck sy in H: Daer na uyt eenich punt der lini H F als I, een lini upper pulley, L and M the extreme points of contact of the cord against the upper disc of the lower pulley; further N shall be the middle point of the cord between G and H, and O the middle point of the cord between I and K, and C the other end of the cord. Further let there be marked in GE the point P so that GP be equal to HF; thereafter in KD the point Q so that KQ be equal to IL. This being so, NGP is equal and of equal weight to NHF, and OIL to OKQ. But CM adds neither levity nor gravity, so that the given weight with the pulley is weighted by the weight of the three pieces of cord, to wit that of the semicircle LM, that of the semicircle DE, and the straight piece QD.

It is also to be noted that if in practice something is pulled up by means of pulleys, so that the end of the cord that is being pulled hangs in the air, without touching the floor, it is manifest that the puller will need to exert so much less force as is the weight of the piece of cord that is being pulled.

2nd Example, with oblique weights

Let this first figure be in every respect identical with the first of the first example, except that the hand here does not pull vertically upwards at F, but obliquely sidelong, which being so, the weight acting on each cord becomes known by the 5th corollary of the 1st part of this Supplement to the Art of Weighing. But in order to give at once some explanation of this: I produce the line on which the weight B is hanging upwards to G, viz. BG, and FE forwards until it meets the vertical through BG, which shall be in H. Thereafter I draw from some point of the line HF, viz. I, a line meeting BG in K, viz. IK parallel to DC. This being lini gherakende B G in K, als I K evewijdeghe met D C. T'welck soo sijnde, ick segh ghelijck I K tot K H, alsoo t'ghewicht deur de hant F ghetrocken, tottet ghegheven ghewicht B: Voort ghelijck H I tot I K (die in voorbeelden met een schijf als dit altijt evelanck moeten sijn, want C D voortghetrocken wesende moet commen in H, en den houck G H I, valt om bekende redenen altijt even anden houck G H C) alsoo t'ghewelt op de hant F ancommende, tottet ghewelt op C ancommende, welcke twee machten in voorbeelden met een schijf als dit,



altijt even moeten sijn, doende elck den helft eens ghewichts, dat in sulcken reden is tottet ghegheven ghewicht, als H K tot H I deur het voorschreven 5 vervolgh des 1 deels deses Byvoughs der Weeghconst.

Maer by aldien de scheefstreckende tauwe liepe over twee of meer schijven, alles wort oock bekent. Laet by voorbeelt dese tweede form sijn alsins ghelijck de tweede des eersten voorbeelts, uytghenomen dat de hant hier an F niet recht op en treckt, maer scheef ter sijdewaert uyt, t'welck soo sijnde, t'ghewicht op elcke tau ancommende, wort oock bekent deur het boveschreven 5 vervolgh. Maer om daer af met een wat verclaring te doen, ick treck de lini daer t'gewicht an hangt opwaert tot G, als B G, en F E voorwaert tot datse de oneyndelicke door B G ontmoet, t'welck sy in H, teyckenende daer na t'bovenste punt daert bovenste catrol an hangt met I, en treck H I, daer na wt eenich punt der lini H F als K, een lini gherakende H G in L, als K L evewijdighe met H I: T'welck soo sijnde, ick segh ghelijck K H tot L H, alsō t'ghewelt op de hant ancommende, tottet gegeven ghewicht: Maer K H is in alle voorbeelden met twee schijven als dit, altijt even an den helft van K L, daerom t'ghewelt op F ancommende, is den so, I say: as IH is to KH, so is the weight pulled by the hand F to the given weight B. Further, as HI is to IK (which in examples with a disc like this one should always be equally long, because CD being produced must come in H, and for known reasons [1]) the angle GHI is always equal to the angle GHC), so is the force acting on the hand F to the force acting on C, which two forces in examples with a disc like this one should always be equal, each being the half of a weight which has the same ratio to the given weight as HK to HI [2]), by the above-mentioned 5th corollary of the 1st part of this Supplement to the Art of Weighing.

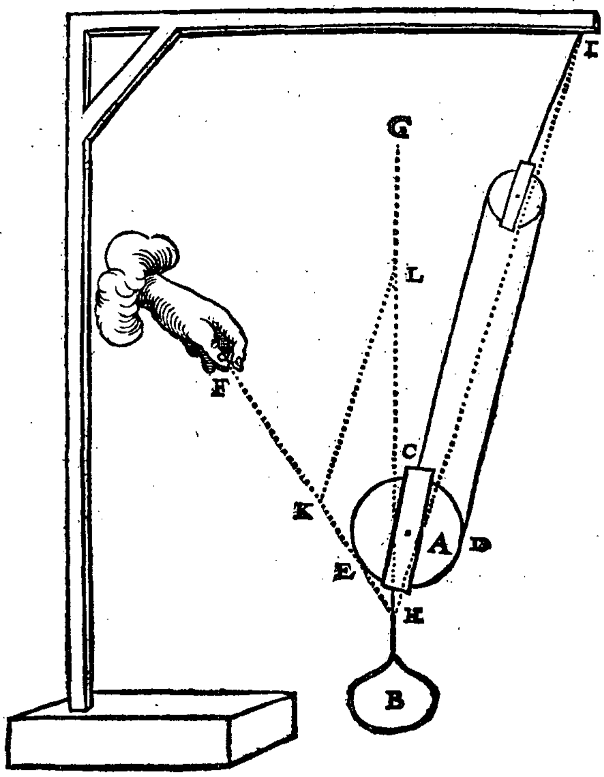
But if the oblique cords pass over two or more discs, everything also becomes known. For example, let this second figure be in every respect identical with the second of the first example, except that the hand here does not pull vertically upwards at F, but obliquely sidelong, which being so, the weight acting on each cord also becomes known by the above-mentioned 5th corollary. But in order to give at once some explanation of this, I produce the line on which the weight is hanging upwards to G, viz. BG, and I draw FE forwards until it meets the infinite vertical through BG, which shall be in H, marking thereafter the highest point on which the upper pulley is hanging by I, and I draw HI, thereafter from some point of the line HF, viz. K, a line meeting HG in L, viz. KL parallel to HI. This being so, I say: as KH is to LH, so is the force acting on the hand to the given weight. But KH is, in all examples with two discs like this one, always equal to the half of KL; therefore the force acting on F is the half of the force acting

[1]) The angle EHD being bisected by AH.

[2]) We are at a loss to understand this. If we take this passage as it stands, we find Stevin asserting that the force X acting along HF is the half of a force Y determined by the proportion $\frac{Y}{G} = \frac{HK}{HI}$.

Now we also have the proportion $\frac{G}{X} = \frac{HK}{HI}$; hence $G^2 = X.Y = 2X^2$, and since $HI = IK$, the triangle KHI would have to be isosceles and rectangular. However, not only does the figure show nothing of the kind, but also: why should the angle FHK have to be 45^0?

den helft des gewelts op I ancommende, waer deur op elck der drie tauwen eveveel ghewelts comt, te weten het derdendeel eens ghewichts, dat in sulcken reden is tottet ghegheven ghewicht, als L H tot H K, daerom segghende in alle



sulcke voorbeelden, K H gheeft H L, wat t'ghegheven ghewicht? het derdendendeel van t'ghene daer uyt comt is voor de ghewelt op de hant F ancommende, en oock op elcke van d'ander twee tauwen.

Maer alsser alsoo drie schijven sijn, soo ist kennelick datmen dan moet nemen het vierendeel van dat uytcommende ghewicht, en soo voort met allen anderen.

De reden waerom K L hier boven meer evewijdeghe moest sijn met H I, dan met eenighe der tauwen, is kennelick deur t'ghene wy van dergheliicke ghesejt hebben int 2 en 3 vervolgh vant 1 deel des Byvoughs der Weeghconst, want de hanghende swaerheyts middellijn des gheheels, streckt deur t'punt H, van welck punt openbaerlick de twee linien moeten commen daer wy ons rekening op maken. TBESLVYT. Wy hebben dan ondersocht de ghedaente der ghewichten opghetrocken met catrollen, na den eysch.

CATROLWICHTS
EYNDE.

on I, as a result of which an equal force acts on each of the three cords, to wit one-third of a weight which has the same ratio to the given weight as LH to HK. Therefore we say in all such examples: KH gives HL, what the given weight? the third part of what follows therefrom is the force acting on the hand F, and also on each of the other two cords [1]).

But if in this way there are three discs, it is obvious that the fourth part of the resulting weight should then be taken, and so on with all others [2]).

The reason why KL above had to be parallel to HI rather than to anyone of the cords is obvious from what we have said about similar things in the 2nd and the 3rd corollary of the 1st part of the Supplement to the Art of Weighing, for the vertical centre line of gravity of the whole passes through the point H, from which point must manifestly proceed the two lines which are used in the calculation. CONCLUSION. We have therefore examined the properties of weights pulled up by means of pulleys, as required.

END OF THE PULLEY WEIGHT

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[1]) Here the same difficulty arises as was pointed out in Note 2.

If really $X = 1/3\ Y$, and $\frac{Y}{G} = \frac{LH}{KH}$,

while at the same time $\frac{G}{X} = \frac{LH}{KH}$,

then $G^2 = 3\ X^2$ or $HL^2 = 3\ KH^2$. Now according to Stevin, $KH = 1/2\ KL$, thence $KL^2 = 4\ KH^2 = HL^2 + KH^2$, so that the angle HKL would have to be a right angle, and consequently KH a horizontal line.

[2]) When we follow up Stevin's reasoning, we here again arrive at impossible results.

DERDE DEEL
DES BYVOUGHS
DER WEEGHCONST,
VANDE
VLIETENDE TOP-
SWAERHEYT.

THIRD PART OF THE SUPPLEMENT TO THE ART OF WEIGHING, OF THE FLOATING TOP-HEAVINESS

Argumentum.

CORTBEGRYP

der vlietende Topsvvaerheyt.

T Is ghebeurt datmen vvilde bereyden seker schuyten, met leeren daer in overeynde staende, ontrent 20 voeten hooch, om crijchsvolck daer op te gaen: Maer alsoot in tvvijfel stont oft niet te groote topsvvaerheyt by en soude brengen, sulcx dat de schuyt mocht ommeslaen, en t'volck int vvater vallen, men bereyde, om versekerder te sijn, een schuyte met haer leere en toebehoorten; daer na versochtment dadelick. Dit veroirsaeckte my te overdencken, oft niet meughelick en soude sijn sulcx te vveten deur vveeghconstighe rekeninghen op ghestelde formen en svvaerheden, sonder de saeck eerst int groot te moeten maken, en daer na dadelick te versoucken. Tot dien eynde vonden en beschreven vvy het volghende vertooch: T'vvelck alsment een onderscheyden naem vvilde gheven, naghelegentheyt vant voornaemste eynde daert toe streckt, men soudet meugen heeten* Vertooch der vlietende Topsvvaerheyt, *dat is van topsvvaerheyt der stoffen die opt vvater vlieten, of drijven, vvant van ander topsvvaerheyt der lichamen opt vast lant, die omvallen als des lichaems svvaerheyts middelpunt is buyten de hanghende svvaerheyts middellijn, en is ons voornemen niet hier te handelen.*

* Theorema.

VER-

ARGUMENT OF THE FLOATING TOP-HEAVINESS

It has sometimes happened that it was desired to make certain vessels, with ladders standing upright therein, about 20 feet high, for soldiers to ascend them. But since it was doubted whether this would not cause too great top-heaviness, so that the vessel might capsize and the soldiers fall into the water, a vessel was made, in order to be surer, with its ladder and accessories; thereafter it was tested in practice. This set me thinking whether it would not be possible to know this through static calculations of assumed forms and gravities, without first having to make the thing on a large scale and then testing it in practice. To this end we found and described the following theorem: which, if one wished to give it a distinct name, might be called, because of the chief end it serves, *Theorem of the Floating Top-heaviness,* i.e. of the top-heaviness of materials floating on water, for we do not intend to deal here with other top-heaviness, of bodies on firm land, which turn over when the body's centre of gravity is outside the vertical centre line of gravity [1]).

[1]) Since the vertical centre line of gravity is defined as the vertical through the centre of gravity of the body, this situation cannot arise. Stevin probably means to say: when the vertical centre line of gravity meets the floor outside the perimeter of the base.

VERTOOCH.

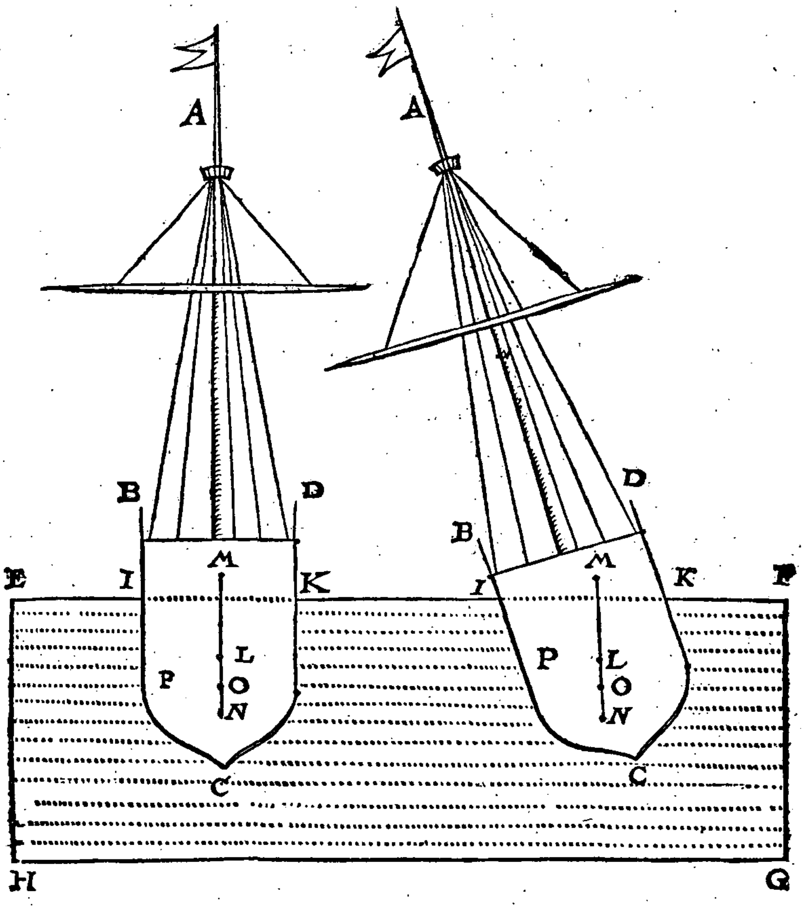
Een lichaem vlietende opt vvater, neemt daer in altijt sulcken ghestalt, dat sijn svvaerheyts middelpunt, is in des vvaterhols hanghende svvaerheyts middellijn.

TGHEGHEVEN. Laet A B C D een lichaem sijn, drijvende opt water E F G H, diens oppervlack E F, en steeckt daer in tot I K toe, sulcx dat I C K het waterhol beteyckent, diens swaerheyts middelpunt L, sijn hanghende swaerheyts middellijn M L N, en des lichaems A B C D swaerheyts middelpunt O.

TBEGEERDE. Wy moeten bewijsen dattet lichaems A B C D swaerheyts middelpunt O, is in des waterhols I C K hangende swaerheyts middellijn M N.

TBEWYS.

Laet ons t'lichaem A B C D uyt het water trecken, en nemen door t'gedacht dattet waterhol I C K in sijn selve form blijve: En tot noch meerder claerheyt,



dattet selve waterhol sy een vlackvat, na de wijse der 7 bepaling des waterwichts. Dit vlackvat aldus gheledicht wesende van sijn lichaem, latet vol waters ghegoten worden: En want water in water alle ghestalt hout diemen hem gheeft, deur het 1 voorstel des waterwichts, so sal t'vlackvat in die ghestalt blijven, sulcx

S dat

THEOREM

A body floating on the water always takes therein such a position that its centre of gravity is in the vertical centre line of gravity of the body of displaced fluid.

SUPPOSITION. Let *ABCD* be a body, floating on the water *EFGH*, whose upper surface is *EF*, and it is submerged therein as far as *IK*, so that *ICK* designates the body of displaced fluid, whose centre of gravity is *L*, its vertical centre line of gravity being *MLN*, while the centre of gravity of the body *ABCD* is *O*.
WHAT IS REQUIRED TO PROVE. We have to prove that the centre of gravity *O* of the body *ABCD* is in the centre line of gravity *MN* of the body of displaced fluid *ICK*.

THE PROOF

Let us pull the body *ABCD* out of the water and conceive that the body of displaced fluid *ICK* keeps the same form, and for greater clarity, that this body of displaced fluid be a surface vessel, in the manner of the 7th definition of hydrostatics. This surface vessel thus being emptied of its body, let it be poured full of water. And because water keeps in water any place given to it, by the 1st proposition of hydrostatics, the surface vessel will keep that place, so that it keeps the same place, whether it be filled with water or with the body *ABCD*. But the centre of gravity of this water poured in is also the centre of gravity of the body of displaced fluid or surface vessel, to wit *L*; and therefore the centre of gravity of the body *ABCD* must be in the vertical centre line of gravity *MN* of the surface vessel. For let it, if it were possible, be outside it, I assume in the point *P*. But this cannot happen without change of the form of the body of displaced fluid *ICK*, for since it had this form when the centre of gravity of the body was in *O* by the supposition [1]), through displacement of the material of the body in such a way that the centre of gravity should come in *P*, *B* would then have to sink, *D* to rise, and *C* to turn towards *K*, which would be contrary to the supposition, and the body of displaced fluid would be another than was assumed. There-

[1]) The argument is far from being convincing. It has to be proved that the centre of gravity of the floating body, viz. *O*, is in the vertical through *L*, the centre of gravity of the displaced fluid. Here, however, *O* is said to be in this vertical by the supposition. It is to be noted that Stevin nowhere gives an equivalent of the statement that the upthrust experienced by the floating body acts along the vertical through the centre of gravity *L* of the displaced fluid, and that *O* therefore has to be in the vertical of *L* in order that the upthrust may balance the weight of the body.

dattet so wel met water ghelaen, als mettet lichaem A B C D, al een selve ghestalt hout: Maer dit inghegoten waters swaerheyts middelpunt is oock des waterhols of vlackvats swaerheyts middelpunt, te weten L; en daerom moet des lichaems ABCD swaerheyts middelpunt sijn in des vlackvats hangende swaerheyts middellijn M N: Want latet soot meughelick waer daer buyten wesen, ick neem ant punt P: Maer dat en can niet gheschien sonder verandering vande form des waterhols I C K, want nadient dese gestalt hadde wesende des lichaems swaerheyts middelpunt an O deur t'ghestelde, so soude deur verlegging der stof des lichaems, sulcx dattet swaerheyts middelpunt quaem an P, alsdan B moeten dalen, D oprijsen, en C keeren na K toe, t'welck teghen t'gestelde waer, en een ander waterhol soude sijn dan daer verschil af is: Daerom des lichaems swaerheyts middelpunt is in M N, te weten of onder des waterhols swaerheyts middelpunt L, of daer boven, of daer in. TBESLVYT. Een lichaem dan drijvende opt water, neemt daer in altijt sulcken ghestalt, dat sijn swaerheyts middelpunt is in des waterhols hanghende swaerheyts middellijn, t'welck wy bewijsen moesten.

1 VERVOLGH.

Tis kennelick dat als des lichaems swaerheyts middelpunt, is boven des waterhols swaerheyts middelpunt, so heeftet sulcken topswaerheyt dat alles omkeert, (midts welverstaende dattet niet onderhouden en worde) tot dat des lichaems swaerheyt middellijn, is in des waterhols hanghende swaerheyts middellijn, onder des waterhols swaerheyts middelpunt. Als by voorbeelt een cromme stock opt water vlietende, sy hout daer in een seker ghestalt, sulcx dat al keertmen opwaert t'gene onder was, ten wil so niet blijven, maer neemt weerom d'eerste gestalt, uyt oirsaec dat des stocx swaerheyts middelpunt, dan niet en is in des waterhols hangende swaerheyts middellijn, onder des selfden swaerheyt middelpunt.

2 VERVOLGH.

Tis kennelick dat eenich gewicht in een schip of ander vat verleyt sijnde, sulcx dat de form des waterhols verandert, dat daer me oock verandert de plaets van des waterhols swaerheyts middelpunt.

3 VERVOLGH.

Tis openbaer dat alle ghewicht geleyt onder des waterhols swaerheyts middelplat, dat evewijdich is metten sichteinder, streckt tot vaster ganck des schips de topswaerheyt min onderworpen sijnde: Maer alle ghewicht datmen daer boven leght, streckt tot meerder topswaerheyt.

MERCKT.

Soo de twee swaerheyts middelpunten, te weten des waterhols en des schips, met al de lichamelicke stof dieder in en op is, licht om vinden waer, tis kennelick datmen soude connen segghen deur weeghconstige wercking sonder dadelicke ervaring te doen, wat scheefheyt of ghestalt een verdocht gheladen schip int water nemen sal; en of t'water over de canten soude commen of niet, gelijck mijn voornemen was te willen weten: Maer want die soucking der swaerheyts middelpunten van soo veel verscheyden stoffen als ghemeenlick in een schip sijn te moeyelick soude vallen, soo en dienet niet om in sulck voorbeelt hem daer me te behelpen. Nochtans insiende dat kennis der oirsaken van topswaerheyt, en der ghestalt eens vlietende lichaems int water elders can te pas commen: Oock me dat de ghene die moeyte mocht doen van dat te soucken, hier me geholpen can worden, soo heb ick dit by ghedachtnis ghestelt alsboven.

DER VLIETENDE TOPSWAERHEYTS

fore the body's centre of gravity is in *MN*, to wit either below the centre of gravity *L* of the body of displaced fluid, or above it, or in it. CONCLUSION. A body therefore, floating on the water, always takes therein such a place that its centre of gravity is in the vertical centre line of gravity of the body of displaced fluid, which we had to prove.

1st COROLLARY

It is obvious that if the body's centre of gravity is above the centre of gravity of the body of displaced fluid, it has such top-heaviness that everything turns over [1]) (provided, however, it be not supported) until the body's centre line of gravity [2]) is in the vertical centre line of gravity of the body of displaced fluid, below the centre of gravity of the body of displaced fluid. For example, if a crooked stick floats on the water, it keeps therein a certain place, in such a way that even if it is turned upside down, it will not remain thus, but resumes its first place, because the stick's centre of gravity is then not in the vertical centre line of gravity of the body of displaced fluid, below the latter's centre of gravity.

2nd COROLLARY

It is obvious that if some weight in a ship or other vessel is displaced, in such a way that the form of the body of displaced fluid changes, the position of the centre of gravity of the body of displaced fluid also changes accordingly.

3rd COROLLARY

It is manifest that any weight laid below the centre plane of gravity of the body of displaced fluid, which is parallel to the horizon, tends to steady the course of the ship, which is then less subject to top-heaviness. But any weight laid above the said plane tends to increase the top-heaviness.

NOTE

If it were easy to find the two centres of gravity, to wit of the body of displaced fluid and of the ship with all the physical material that is in and on it, it is obvious that it could be told by static construction without practical experience what obliqueness or place an imaginary loaded ship will assume in the water; and whether the water would wash over the sides or not, as I wished to know. But because it would be too difficult to find the centres of gravity of the many varied materials that are usually present in a ship, it is no use managing therewith in this example. Realizing, howver, that knowledge of the causes of top-heaviness and of the place of a floating body in the water may be convenient elsewhere, and also that this may be of use to the man who should make an attempt to find it, I have written the above *pro memoria.*

END OF THE FLOATING TOP-HEAVINESS

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[1]) It is now common knowledge that this is not generally true. The condition for stable equilibrium is that the centre of gravity of the body shall be below the metacentre. The conception of metacentre, however, was not introduced until Bouguer (1698-1758)

[2]) Read: centre of gravity.

VIERDE DEEL
DES BYVOUGHS
DER WEEGHCONST,
VANDE
TOOMPRANG.

FOURTH PART OF THE SUPPLEMENT TO THE ART OF WEIGHING, OF THE PRESSURE OF THE BRIDLE

Argumentum.

CORTBEGRYP

DES TOOMPRANGS.

HEbbende ſijn VORSTELICKE GHENADE van kintſche daghen af tot noch toe, hem gheduerlick met grooten yver ſeer vlietich gheoeffent inde Ruyterconſt, (ſoo vvort der Italianen Cavallarizzo, in Duytſch ghenoemt, deur den Schryver L.B. C. Stalmeeſter des Keyſers) en benevens mondelicke t'ſaemſpraeck mette ervarenſte die hem in deſe ſtof ontmoeteden, noch deurleſen veel verſcheyden Schrÿvers daer af handelende, ſoo vvel nieu uytcommende als ouden: En heeft nochtans deur vvoorden noch ſchriften, noyt connen geraken tot grondelicke kennis der reden van t'geprang der toomen, t'vvelc deur cleyne vercorting, verlanging en cromming der toomdeelen, haeſt groote onſeker veranderinghen crycht int regieren des peerts. Sulcx dat onder anderen oock dit, hem ſeer begheerich maeckte te verſtaen de voorgaende VVeeghconſt, verhopende daer deur tot grondelicke kennis dier ſaeck te commen: T'vvelck tot ſijn vernougen oock ghebeurde, ſulcx dat hy nu toomen doet maken, niet onſekerlick taſtende ghelijck te vooren, maer met kennis der reden. Al t'vvelck op * vviſconſtighen gront gebout ſijnde, my heeft behoirlick ghedocht t'ſelve (dat hier om de voorgaende redenen int gheméen TOOMPRANG ghenoemt vvort) by ſijn vviſconſtighe ghedachteniſſen te vervoughen: Te meer dat anderen dit ter hant commende, noch meer daer in fuller meughen mercken tot voordering deſer ſtof ſtreckende.

Subiecto Mathematico.

BEPA

ARGUMENT OF THE PRESSURE OF THE BRIDLE

His PRINCELY GRACE having from early childhood to this day continually practised the Art of Riding (that is how the Italians' *Cavallarizzo* is called in Dutch by the writer L. B. C., the Emperor's master of the horse [1])) with great zeal and diligence, and having, besides oral conversations with the greatest experts he has met with in this field, also read through many different writers dealing therewith, both newly appearing and old, nevertheless he never succeeded in gaining, either through words or writings, thorough knowledge of the reason of the pressure of bridles, which through slight shortening, lengthening, and twisting of the parts of the bridle may soon bring about great uncertain changes in managing the horse. So that this, among other things, also made him very anxious to understand the foregoing Art of Weighing, hoping thereby to gain thorough knowledge of that matter. Which to his pleasure happened, so that he now causes bridles to be made, not groping uncertainly, as before, but with knowledge of the reason. All this being founded on a mathematical basis, it seemed appropriate to me to include this (which, for the above reasons, is here generally called *Pressure of the Bridle)* among his mathematical memoris. The more so that others, when it comes to their notice, may discover more about it, which will tend to advance this matter.

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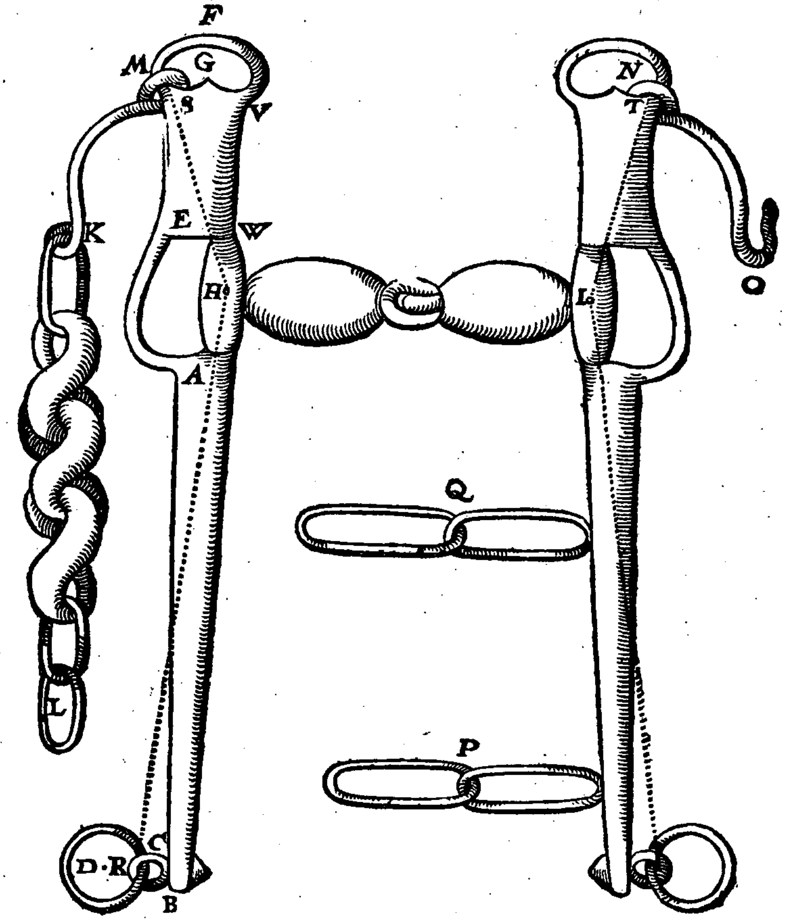
[1]) We do not know who L.B.C. was. Probably he was a German, but *Ruyterconst* in Stevin's text is a Dutch word. Here again it is evident that Stevin does not distinguish sharply between the two languages.

BEPALINGHEN.

DE ghewoonlicke namen vande deelen des tooms tot dit voornemen noodich, worden deur de byghestelde form verclaert als volght.



1 BEPALING.

AB Stang.

2 BEPALING.

C Stangbout.

3 BEPALING.

D Teughelrinck.

4 BEPALING.

EF Strangs bovedeel.

S 3 5 BEPA-

DEFINITIONS

The usual names of the parts of the bridle, required for this purpose, are set forth as follows by the accompanying figure.

1st DEFINITION

AB Cheek

2nd DEFINITION

C Bit bolt.

3rd DEFINITION

D Bit ring.

4th DEFINITION

EF Upper cheek.

5 BEPALING.

G Oogh.

6 BEPALING.

HI Montſtick.

7 BEPALING.

KL Kinketen.

8 BEPALING.

KM De es.

9 BEPALING.

NO Kinketenhaeck.

10 BEPALING.

P,Q. Tvvee tuſſcheketens.

11 BEPALING.

Wree toom, of vvree deelen der ſelve, ſijn die t'montſtick ſtijf teghen het onderſte tantvlees en de kinketen tegen de kin doen drucken. Slappe, die ter ſacht tegen doen drucken.

VERCLARING.

Hoe wel een ghetrocken toom verſcheyden druckingen veroirſaeckt, als beneven de boveſchreven teghen het tantvlees, en kin noch vande tuſſcheketen teghen de borſt: En vanden teughelrinck teghen de ſtangbout: Nochtans ſoo verſtaetmen mettet woort wreetheyt, alleenelick de ſtijve drucking des montſticx teghen het onderſte tantvlees en des kinketens teghen de kin, als weſende de drucking daer t'peert deur beweeght wort, en die hem wee doet, ſulcx dattet om die weedom te verſachten, de kin na ſijn borſt brengt, en den hals cromt: Want ghenomen dat de kin deur de tueghel een palm verre na t'peert ghetrocken worde, het can deur de buyging vanden hals, maken dat de drucking onvermeerdert blijve. Tis oock deſe perſing die hem doet achterwaert deyſen, meynende de ſelve alſoo t'ontcommen of verminderen, en vreeſende deur voorwaert te gaen die te vermeeren. Dit dan wreetheyt ſijnde, ſoo worden die toomen of deelen der ſelve, welcke alſo het montſtick ſtijf of ſacht teghen het tantvlees en kinketen teghen de kin doen drucken, gheſeyt wreet, of ſlap te ſijn, als wree toom, ſlappe toom, wree ſtang, ſlappe ſtang, wree bovedeel, ſlap bovedeel.

12 BEPA-

5th DEFINITION

G Eye.

6th DEFINITION

HI Bit.

7th DEFINITION

KL Curb chain.

8th DEFINITION

KM The S [1]).

9th DEFINITION

NO Curb hook.

10th DEFINITION

P, Q Two intermediate chains.

11th DEFINITION

Severe bridle, or severe parts thereof, are those parts which force the bit tightly against the lower gums and the curb chain against the chin. Gentle are those parts which press them gently against the gums and the chin.

EXPLANATION

Although a bridle that is being pulled causes different pressures, viz. besides those described above against the gums and the chin also that of the intermediate chain against the breast, and of the bit ring against the bit bolt, nevertheless by the word severity is only meant the tight pressure of the bit against the lower gums and of the curb chain against the chin, this being the pressure by which the horse is constrained and which hurts it, so that in order to relieve this pain it approaches its chin to its breast and bends its neck. For if it is assumed that the chin is pulled a palm further towards the horse by the rein, it can, by bending its neck, cause the pressure to remain unchanged. It is also this pressure which makes it start back, thinking that it can thus escape from it or decrease it and fearing that by going forward it may increase it. This therefore being severity, those bridles or parts thereof which thus force the bit tightly or gently against the gums and the curb chain against the chin are said to be severe or gentle, viz. severe bridle, gentle bridle, severe cheek, gentle cheek, severe upper cheek, gentle upper cheek.

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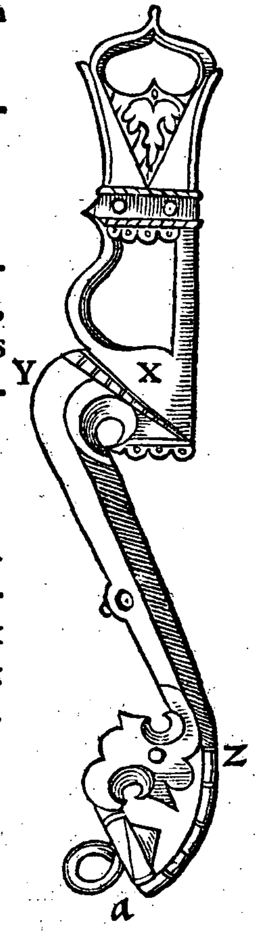
[1]) Stevin writes *es*, and since the part *KM* actually is shaped like the letter *S* in its long form, it is probable that *es* merely stands for the pronunciation of this letter. Hence it may be rendered in English by *es* as well.

12 BEPALING.

De cromme bochten der ſtanghen vvorden * keeren ghenoemt. *Int hoochduyts wronghen. Int Françoys coudes.*

VERCLARING.

De ſtanghen worden recht en crom ghemaeckt, recht als in d'eerſte form, crom als in deſe tweede, met een bocht keerende van X na Y, van Y na Z, en van Z na *a*, welcke men daerom deſes ſtangs keeren noemt.



DE VOLGHENDE BEPALINGHEN SYN NIEV.

13 BEPALING.

T'middelſte punt R des raeckſels vanden teughelrinck D teghen den bout C, als t'peert ghetoomt ſijnde de teughels gheſpannen ſtaen, noemen vvy Teugelrijncx raeckpunt.

14 BEPALING.

T'middelſte punt S des raeckſels van de es teghen het oogh, oock het middelſte punt T des raeckſels vanden haeck teghen het oogh als t'paert ghetoomt ſijnde de teughels geſpannen ſtaen, noemen vvy ooghraeckpunt.

15 BEPALING.

Het punt H vanden as des montſticx int middel vande olive commende daer den as in draeyt, noemen vvy Montſticx aſpunt.

16 BEPALING.

Den houck R H S begrepen tuſſchen tvvee linien, d'eene van des teughelrincx raeckpunt R, tot des montſticx aſpunt H; d'ander vant montſticx aſpunt H, tottet ooghraeckpunt S, noemen vvy Raeckpunthouck.

S 4 17 BEPA-

12th DEFINITION

The curved bends of the cheeks are called twists.

EXPLANATION

The cheeks are made straight and curved, straight as in the first figure, curved as in this second figure, with a bend twisting from *X* to *Y*, from *Y* to *Z*, and from *Z* to *a*, which are therefore called the twists of this cheek.

THE FOLLOWING DEFINITIONS ARE NEW.

13th DEFINITION

The middle point *R* of the area of contact of the bit ring *D* against the bolt *C* when, the horse being bridled, the reins are tight, we call the point of contact of the bit ring.

14th DEFINITION

The middle point *S* of the area of contact of the *S* against the eye, also the middle point *T* of the area of contact of the hook against the eye when, the horse being bridled, the reins are tight, we call point of contact of the eye.

15th DEFINITION

The point *H* of the axis of the bit, coming in the middle of the olive in which the axis turns, we call axial point of the bit.

16th DEFINITION

The angle *RHS* contained between two lines, one from the point of contact *R* of the bit ring to the axial point *H* of the bit, and the other from the axial point *H* of the bit to the point of contact *S* of the eye, we call angle at the point of contact.

17 BEPALING.

Prouftoom noem ick, een toom dienende om an alle peerden te prouven wat ghebruyckelicke toom hun bequaemst sal sijn, en die met sekerheyt ten eersten wvelpassende te maken.

Vande form en omstandighen deses prouftooms sal int volghende t'sijnder plaets ghesèyt worden.

1 VOORSTEL.

De keeren an een stang meerder noch minder wvreetheyt te veroirsaken.

Sijn VORSTELICKE GHENADE voorseker wetende, dattet ghemeen ghevoelen van velen onrecht is, gheloovende de keeren der stang tot wreetheyt of slapheyt te helpen, blijvende nochtans de drie punten als R, H, S, t'haerder plaets, seght daer teghen aldus: Laet op de rechte stang A B hier vooren, gheschrouft of ghehecht worden yser stucken, die de stang een form gheven als met groote keeren ghemaeckt te sijn: Soomen nu seght uyt die anhechting eenighe verandering der wreetheyt te volghen, het is soo veel al ofmen seyde dat de selve aenghehechte ysers eenighe verborghen treckende of stekende cracht in haer hadden, ghelijck de seylsteen heeft, of dierghelijcke: T'welck ongeschickt waer. Belanghende sy segghen verandering metter daet te blijcken, dat wort weerleyt met te seggen dat sulcx metter daet niet en blijckt. Angaende Pyqueurs, toommakers, en ander met desen handel dadelick omgaende, sullen voortbrenghen de ghemeene spreuck, *Men moet yghelick in sijn const ghelooven*: Daer wort op gheantwoort sulcx teghen hemlien te strijden, om dat sy oirdeelen vande wichtighe ghedaenten sonder in Weeghconst ervaren te wesen, waer inmen verstaet datter verandering gheschien can deur verandering der boveschreven drie punten R, H, S: Maer die blijvende, en vervolghens oock de twee verdochte linien R H, H S, metten houck R H S, soo blijft de wreetheyt oock de selve, uytghenomen, om heel eyghentlick te spreken, t'verschil dattet ghewicht des bygevoughden ysers mocht veroirsaken, t'welck tot dese saeck niet en ghelt: En alsmender immers op letten wilde, t'can soo wel tot achterdeel strecken van t'ghene sy drijven, als tot voordeel.

Merckt noch wijder, dat de lini des bovedeels der stang als hier vooren V W, tot gheen seker ghemeene gront en can verstrecken om daer uyt de bocht der stang te veroirdenen, ghelijck gemeenlick ghedaen wort, maer wel de lini H S, want d'een stangs bovedeel een breeder oogh hebbende als d'ander, t'gheeft verandering en onsekerheyt inde saeck. TBESLVYT. De keeren dan en veroirsaken meerder noch minder wreetheyt an een stang, t'welck wy bewijsen moesten.

2 VOORSTEL.

De cortste stanghen de vvreetste te sijn.

De reden is hier af tweederley: D'eene, dat met eveveel optrecking der teu-

ghels

17th DEFINITION

Test bridle I call a bridle serving to test on all horses what actual bridle will be most suitable for them, and to make it first of all fit with certainty.

The form and conditions of this test bridle will be discussed in the following in its appropriate place.

1st PROPOSITION

The twists in a cheek cause neither more nor less severity.

His PRINCELY GRACE, knowing positively that the common opinion of many people is wrong, who believe that the twists of the cheek are conducive to severity or gentleness, the three points *R*, *H*, *S* remaining nevertheless in their places, argues against it as follows:

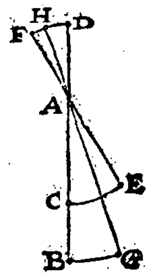
Let there be screwed or fastened on the straight cheek *AB* hereinbefore some iron pieces which give to the cheek a form as if it were made with large twists. If one should now say that from this addition there follows a change of severity, this is as if one should say that these added irons had some hidden attractive or repellent force in them, as the magnet has, or something of the kind; which would be absurd. As regards their saying that the change becomes manifest in practice, this is refuted by saying that this does not become manifest in practice. As to the fact that riding-masters, bridle-makers, and other people practically engaged in these matters will advance the common saying: *Everyone is to be trusted in his own art,* to this it is replied that this argues against them, because they judge of the properties of weights without being versed in the Art of Weighing, in which it is understood that a change may be brought about by a change of the above-mentioned three points *R*, *H*, *S*. But if the latter remain, and consequently also the two imaginary lines *RH*, *HS*, with the angle *RHS*, the severity also remains the same, except—to speak quite accurately—for the difference which the weight of the added iron may cause, which is of no account in this matter. And even if it were to be taken into account, it may be to the detriment as well as the advantage of that which they argue.

It is further to be noted that the line of the upper cheek hereinbefore *VW* cannot serve as a certain common basis on which to prescribe the bend of the cheek, as is usually done, but rather the line *HS*, for if one upper cheek has a wider eye than the other, this produces change and uncertainty in the matter. CONCLUSION. The twists therefore cause neither more nor less severity in a cheek, which we had to prove.

2nd PROPOSITION

The shortest cheeks are the most severe.

The reason hereof is twofold. One is that with the same amount of pulling of the reins the curb chain is moved more by short than by long cheeks. In order to explain this, let *AB* signify a long cheek, *AC* a shorter one, having the same upper cheek *AD*, whose point of contact of the eye is *D*; further, through the pulling of the reins, the point of contact *C* of the bit ring of the shorter cheek *AC* shall have reached *E*, having described the arc *CE*. And the point of contact ghels, meerder beweeghnis des kinketens ghemaeckt wort deur corte stanghen dan deur lange. Om van t'welck verclaring te doen; Laet A B een langhe stang beteyckenen, A C een corter, hebbende een selfbovedeel der stang A D, diens ooghraeckpunt D is, voort sy deur optrecking der teughels, des teughelrincx raeckpunt C vande corste stang A C gecommen tot E, beschreven hebbende de booch C E: En het ooghraeckpunt D sal ghecommen wesen tot F, beschreven hebbende de booch D F: Laet daer na deur der teughels even soo veel optrecking als d'eerste, des teughelrincx raeckpunt B vande langste stang, ghecommen sijn tot G, te weten dat de booch B G, even sy ande booch C E, en het ooghraeckpunt D, sal ghecommen wesen tot H, beschreven hebbende de booch D H. Maer de booch D F is meerder dan D H, en daer teghen in sulcken reden als de langste stang A B, totte cortste A C: Daerom de kinketen ant oogh vast sijnde, crijcht met eveveel optrecking der teughels, meerder beweeghnis deur corte stanghen dan deur langhe. Maer de meeste beweging of opganck des kinketens druckt stijver teghen de kin, en veroirsaeckt oock de stijfste drucking des montsticx teghen het tantvlees: Daerom de corter stangen veroirsaken de meeste wreetheyt, en vervolghens sijn daerom de wreetste.



D'ander reden is de bochtighe form van t'peerts hals, welcke maeckt dat de tusscheketen der cortste stang, verder vande borst staet dan vande langher, waer uyt volght datmen de tueghels van een corte stang, verder can voorttrecken eer de tusscheketen de borst gheraeckt, dan de tuegels van een langhe stang, t'welck soo ghebeurt openbaerlick oock meerder wreetheyt mebrengt.

MERCKT.

Ymant mocht nu twijfelen, en dencken hoe dit overcomt mette weeghconstighe reghelen, die leeren dat de langste steerten de grootste gewelt doen, want ansiende B D voor stock die de timmerlien waegh noemen, wiens langste steert daer den * Doender an treckt A B is, en A vastpunt, soo schijnt hier t'verkeerde besloten te worden: Men antwoort hier op aldus: De vraegh en is niet na de gewelt die den rijder metter hant int trecken doet, want hy an een corter stang, om het ooghraeckpunt eveveel beweging te gheven, stijver moet trecken dan an een langher: Maer stijf ghenouch ghetrocken wesende, men vraecht welcke trecking alsdan de meeste wreetheyt mebrengt. TBESLVYT. De cortste stanghen dan sijn de wreetste, t'welck wy bewijsen moesten. *Efficiens.*

3 VOORSTEL.

De langste bovedeelen der stang de vvreetste te sijn.

De reden is dat met eveveel optrecking der teughels, meerder beweeghnis des kinketens ghemaeckt wort deur langhe bovedeelen der stang dan deur corte: Om van t'welck verclaring te doen; Laet A B een lanck bovedeel beteyckenen, diens ooghraeckpunt B, en A C een corter, diens ooghraeckpunt C, en hebbende beyde een selve stang A D. Voort sy deur optrecking der teugels, des teughelrincx raeckpunt D, gheeommen tot E, en het ooghraeckpunt B sal ghecommen sijn tot F, beschreven hebbende den booch B F: Maer het ooghraeckpunt C tot G, beschreven hebbende de booch C G, cleender dan B F, want ghelijck

D of the eye shall have reached *F*, having described the arc *DF*. Thereafter, by pulling the reins to the same extent as in the first case, let the point of contact *B* of the bit ring of the longer cheek have reached *G*, to wit that the arc *BG* be equal to the arc *CE*, and the point of contact *D* of the eye shall have reached *H*, having described the arc *DH*. But the arc *DF* is greater than *DH* and has thereto the same ratio as the longer cheek *AB* to the shorter *AC*. Therefore the curb chain, if attached to the eye, by the same amount of pulling of the reins is moved more by short than by long cheeks. But the greatest movement or rise of the curb chain forces it more tightly against the chin and also causes the tightest pressure of the bit against the gums. Therefore shorter cheeks cause the greatest severity, and consequently are the most severe [1]).

The other reason is the curved form of the horse's neck, which causes the intermediate chain of the shorter cheek to be further away from the breast than that of the longer cheek, from which it follows that the reins of a short cheek can be pulled further before the intermediate chain touches the breast than the reins of a long cheek, which when it thus happens manifestly also involves greater severity.

NOTE

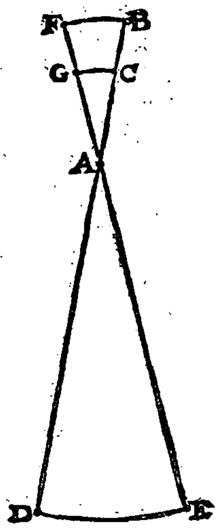
Someone might now be in doubt and think how this is in accordance with the rules of statics which teach that the longest levers exert the greatest force, for if we look upon *BD* as the stick which the carpenters call "waegh", whose longest lever, at which the doer pulls, is *AB* and *A* the fixed point, it seems that the opposite is concluded here. To this the following reply is given: The question is not what is the force which the rider exerts with his hand in pulling, for in order to give the same movement to the point of contact of the eye, he has to pull more firmly at a shorter than at a longer cheek. But when the pulling is firm enough, it is asked which pulling then involves the greatest severity. CONCLUSION. The shortest cheeks therefore are the most severe, which we had to prove.

3rd PROPOSITION

The longest upper cheeks are the most severe.

The reason is that with the same amount of pulling of the reins the curb chain is moved more by long than by short upper cheeks. In order to explain this, let *AB* signify a long upper cheek, whose point of contact of the eye is *B*, and *AC*

[1]) The gist of Stevin's reasoning consists in the assumption that the cheek *AB* and the upper cheek *EF* may be considered as the two arms of a lever of the first kind, the fulcrum being in *A*. This assumption, however, is untenable. The device indeed constitutes a lever, but it is one of the second kind (load between force and fulcrum), in which the pressure to be exerted on the gums acts as load, the point of contact of the eye is the fulcrum, and the force is applied at the bit ring. According to J. H. Anderhub, who pointed out Stevin's error in a paper *Hier irrt Simon Stevin* (Deutsche Mathematik 7, 2-3, 1943; p. 299-304) the first to interpret the cheek as a lever of the second kind was G. O. d'Aquino, *Disciplina del Cavallo*, Udini 1636; p. 204. His words are: „...già che la guardia tutta altro non è, che una leva, la cui forza mediante le redini è posta nel pedicino e il sostegno nell'altra estremità dell'occhio dove và il porta morso, e il cui peso è l'incastro dove opera l'imboccatura.

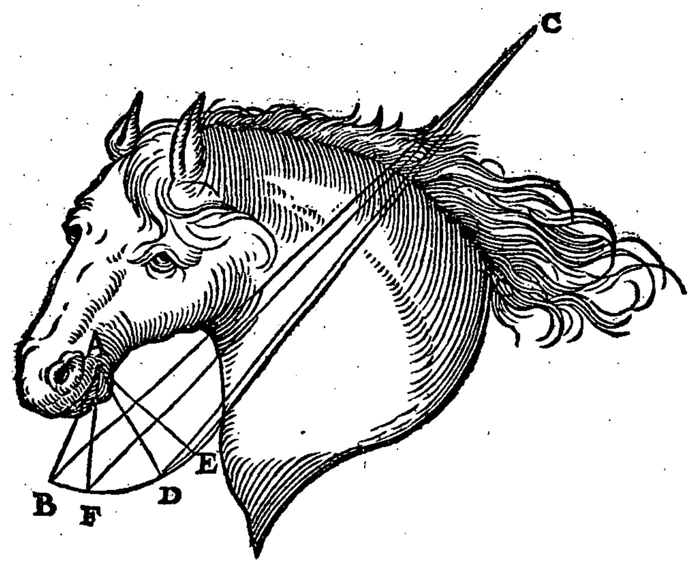


lijck A C tot A B, alſo C G tot B F: Daerom de kinketen ant oogh B des langſte bovedeels der ſtang vaſt ſijnde, crijcht met eveveel optrecking der teughels, meerder beweeghnis dan ant oogh C des cortſte bovedeels vaſt ſijnde: Maer de meeſte beweging of opganck der kinketen druckt ſtijver teghen de kin, en veroirſaeckt oock de ſtijfſte drucking des montſticx teghen het tantvlees, daerom de langſte bovedeelen ſijn de wreetſte. Angaende ymant twijfelen mocht waerom den Doender an D, meer ghewelt doet op des waeghs langer eynde A B, dan op het corter A C, ſchijnende teghen de Weeghconſtighe reghelen te ſtrijden: De reden daer af machmen verſtaen deur t'ghene van derghelijcke gheſeyt is int Merck des 2 voorſtels. TBESLVYT. Langhe bovedeelen dan ſijn de wreetſte, t'welck wy bewijſen moeſten.

4 VOORSTEL.

Teughelrincx raeckpunt verder vant peerts borſt, geeft meerder vvreetheyt.

TGHEGHEVEN. Laet A den as des montſticx beteyckenen, A B een ſtang, B C den teughel, B des teughelrincx raeckpunt, A D een ander ſtang even an A B, en D C ſijn teughel, D des teughelrincx raeckpunt: Ende het teughelrincx



raeckpunt B, ſy verder vant peerts borſt dan het teughelrincx raeckpunt D.

TBEGHEERDE. Wy moeten bewijſen dattet teughelrincx raeckpunt B, meerder wreetheyt geeft dan D. TBEREYTSEL. Laet opt punt A als middelpunt, mette halfmiddellijn A B, beſchreven worden de booch B D E: Daer na ſy a shorter, whose point of contact of the eye is *C*, both having the same cheek *AD*. Further, through pulling of the reins, the point of contact *D* of the bit ring shall have reached *E*, and the point of contact *B* of the eye shall have reached *F*, having described the arc *BF*. But the point of contact *C* of the eye shall have reached *G*, having described the arc *CG*, smaller than *BF*, for as *AC* is to *AB*, so is *CG* to *BF*. Therefore, if the curb chain is attached to the eye *B* of the longest upper cheek, by the same amount of pulling of the reins the curb chain is moved more than if it is attached to the eye *C* of the shortest upper cheek. But the greatest movement or rise of the curb chain forces it more tightly against the chin and also causes the tightest pressure of the bit against the gums, therefore the longest upper cheeks are the most severe. If anyone should doubt why the doer at *D* exerts more force on the longer end *AB* of the "waegh" than on the shorter *AC*, which seems to be contrary to the rules of statics: The reason thereof can be understood from what has been said about a similar point in the Note to the 2nd proposition. CONCLUSION. Long upper cheeks therefore are the most severe, which we had to prove.

4th PROPOSITION

When the point of contact of the bit ring is further away from the horse's breast, this causes greater severity.

SUPPOSITION. Let *A* signify the axis of the bit, *AB* a cheek, *BC* the rein, *B* the point of contact of the bit ring, *AD* another cheek, equal to *AB*, and *DC* its rein, *D* the point of contact of the bit ring; and the point of contact *B* of the bit ring shall be further away from the horse's breast than the point of contact *D* of the bit ring. WHAT IS REQUIRED TO PROVE. We have to prove that the point of contact *B* of the bit ring causes greater severity than *D*. PRELIMINARY. Let there be described on the point *A* as centre, with the semi-diameter *AB*, the arc *BDF*. Thereafter the point of contact *B* of the bit ring shall, through pulling na ſy des teughelrincx raeckpunt B, deur optrecking des teughels ghecommen tot F, en des teughelrincx raeckpunt D tot E, ſulcx dat de booch D E, even ſy an de booch B F.

TBEWYS.

Tis daer voor te houden, dat ſoo veel de lini B C langher is dan F C, ſoo veel heeft de treckende hant by C, hoogher moeten ſijn weſende des teughelrincx raeckpunt an F, dan doent was an B. S'ghelijcx dat ſoo veel de lini D C langer is dan E C, ſo veel heeft de treckende hant by C, hoogher moeten ſijn weſende des teughelrincx raeckpunt an E dan doent was an D: Maer E C verſchilt meer van D C, dan F C van B C: En daerom ſoo veel t'verſchil dier twee verſchillen bedraecht, ſoo veel gaet de hant hoogher mettet roerſel des teughelrincx raeckpunt van D tot E, dan mettet roerſel van B tot F: Maer t'roerſel of de booch B F, is even an t'roerſel of de booch D E deur t'bereytſel, daerom de hant an C, gaet op evegroote roerſels van B en D, hoogher mettet roerſel van D, dan mettet roerſel van B: En vervolgens by aldien de hant an d'een en d'ander even hooch ginghe, ſoo ſoude t'roerſel van B na F, grooter moeten ſijn dan t'roerſel van D na E: Maer t'grooter roerſel van B na F, veroirſaeckt oock grooter roerſel des ooghs, en vervolghens des kinketens, dan het cleender roerſel van D na E: Daerom de hant an d'een en d'ander even hooch ghegaen hebbende, ſoo ſal t'roerſel des kinketens veroirſaeckt deur trecking van B na F, grooter ſijn dan deur t'roerſel des kinketens veroirſaeckt deur trecking van D na E: Maer t'grooter roerſel of grooter opganck des kinketens, druckt ſtijver teghen des peerts kin, ende vervolghens doedet montſtick ſtijver drucken teghen het tantvlees dan een cleender opganck des kinketens: Daerom met evenhooghe trecking des hants an C, doetmen het peert meer weedom, weſende des teughelrincx raeckpunt an B der ſtang A B, dan an D der ſtang A D: En vervolghens het teughelrincx raeckpunt B verder vant peerts borſt, geeft meerder wreetheyt dan D.

1 MERCK.

Angheſien den houck A D C, naerder den rechthouck is dan den houck A B C, die veel ſcherper is, ſoo doet de macht des hants by C, meerder ghewelt ande ſtang A D, dan de ſelve macht des hants by C, ande ſtang A B deur t'vervolgh des 24 voorſtels vant 1 bouck der Weeghconſt. Maer want ymant dencken mocht dit te ſtrijden teghen t'voorgaende bewijs, ſoo ſegghen wy daer op ghelijck int merck des 2 voorſtels gheantwoort wiert, te weten dat de vraegh niet en is wat macht de hant an C doet, maer de hant opden houck A B C, ſoo veel ſtijver treckende dan op den houck A D C, datſe op d'een en d'ander eveveel verhoocht, men vraeght welcke trecking alſdan de meeſte wreetheyt mebrengt.

2 MERCK.

Beneffens de voorgaende oirſaeck der wreetheyt, vervought heur ſomwijlen noch een tweede, in deſer voughen: Hoe het teughelrincx raeckpunt naerder des peerts borſt comt, hoe de tuſſcheketen oock meer de borſt naerdert, volgende de ghemeene manier diemen int toommaken ghebruyckt: Maer die tuſſcheketen ſoo na commende, datſe int trecken des tooms de borſt gheraeckt, ſoo is de wreetheyt daer ten eynde; want al treckmen dan veel ſtijver, dat comt al opt peerts of the reins, have reached *F*, and the point of contact *D* of the bit ring shall have reached *E*, so that the arc *DE* shall be equal to the arc *BF*.

PROOF

It is to be assumed that by so much as the line *BC* is longer than *FC*, by so much the pulling hand at *C* had to be higher when the point of contact of the bit ring was in *F* than when it was in *B*. In the same way that by so much as the line *DC* is longer than *EC*, by so much the pulling hand at C had to be higher when the point of contact of the bit ring was in *E* than when it was in *D*. But *EC* differs more from *DC* than *FC* does from *BC*. And therefore, by so much as the difference of these two differences amounts to, by so much the hand rises higher with the displacement of the point of contact of the bit ring from *D* to *E* than with the displacement from *B* to *F*. But the displacement or the arc *BF* is equal to the displacement or the arc *DE* by the preliminary; therefore, if the displacements of *B* and *D* are equal, the hand at *C* rises higher with the displacement of *D* than with the displacement of *B*. And consequently, if the hand rose to the same height with both, the displacement from *B* to *F* would have to be greater than the displacement from *D* to *E*. But the greater displacement from *B* to *F* also causes greater displacement of the eye, and consequently of the curb chain, than the smaller displacement from *D* to *E*. Therefore, the hand having risen to the same height with both, the displacement of the curb chain caused by pulling from *B* to *F* will be greater than that of the curb chain caused by pulling from *D* to *E*. But the greater displacement or greater rise of the curb chain presses more tightly against the horse's chin, and consequently causes the bit to press more tightly against the gums than a smaller rise of the curb chain. Therefore, when the hand at *C* pulls to the same height, the rider hurts the horse more if the point of contact of the bit ring is in *B* of the cheek *AB* than in *D* of the cheek *AD*. And consequently, when the point of contact *B* of the bit ring is further away from the horse's breast, this causes greater severity than D.

1st NOTE

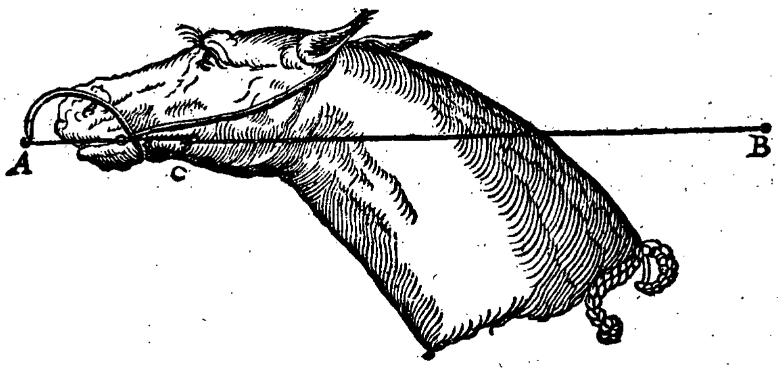
Since the angle *ADC* is nearer to a right angle than the angle *ABC*, which is much more acute, the power of the hand at *C* exerts greater force on the cheek *AD* than the same power of the hand at *C* does on the cheek *AB*, by the corollary of the 24th proposition of the 1st book of the Art of Weighing. But because someone might think this to be contrary to the foregoing proof, we give to this the same reply as in the note to the 2nd proposition, to wit that the question is not what force the hand at *C* exerts, but if the hand pulls so much more tightly on the angle *ABC* than on the angle *ADC* that it rises equally in both cases, it is asked which pulling then involves the greatest severity.

2nd NOTE

In addition to the foregoing cause of the severity, there is sometimes a second cause, as follows: The closer the point of contact of the bit ring comes to the horse's breast, the closer the intermediate chain also comes to the breast, if the common manner used in bridle-making is followed. But if that intermediate chain peerts borst an, sonder teghen kin of tantvlees meerder persing te maken: Maer een ander tueghelrincx raeckpunt verder vande borst sijnde, en de tusscheketen daerom oock verder, soo volght daer uyt datmen die stanghen verder achterwaert na de borst sal connen trecken als d'ander, eer de tusscheketen de borst gheraeckt, waer uyt oock openbaerlick meerder wreetheyt moet volgen. Doch en valt daer af niet te segghen als de tusscheketen na d'een en d'ander wijse de borst niet en raeckt.

3 MERCK.

T'ghebeurt ettelicke peerden datse hun selfs van t'gheprang des tooms verlossen, mette mont om hooch te steken, ghelijck de byghevoughde form anwijst: Sulcx dat hun alsdan den Ruyter niet dwingen en can, maer loopen daerse willen: Nochtans mocht ymant segghen, is dan des teughelrincx raeckpunt verder van des peerts borst, als in ander ghestalt, inder voughen dat daer me den toom wreeder behoort te wesen, t'welck teghen de regel deses voorstels schijnt



te strijden. Hier op wort gheseyt, dat wanneer de ghespannen teughelriem A B, evewijdich is mette verdochte rechte lini van des teughelrincx raeckpunt A, tot des montsticx aspunt C, ghelijck dese ghestalt mebrengt, alsdan en can stijver trecking ant bovedeel gheen roersel gheven, noch de kinketen doen opgaen, en vervolghens en isser gheen wreetheyt, want hoe wel het montstick stijver achterwaert ghetrocken wort, dat en veroirsaeckt het boveschreven wreet geprang niet. Maer soo de ghespannen teughelriem noch hoogher waer alsvooren gheseyt is, hoemen dan stijver treckt, hoe openbaerlick de kinketen slapper wort. Sulcx dat dit een uytneming is in bekende oirsaken bestaende.

5 VOORSTEL.

De cortste kinketens gheven de meeste vvreetheyt.

Tis daer voor te houden, dattet gheprang des montsticx eerst begint als de kinketen teghen de kin gheraeckt: Maer tot een langhe kinketen moet de hant verder opgaen eerse de kin gheraeckt dan tot een corte, en daerom doetmen met eveveel beweeghnis des hants, meer geprang met corte kinketens dan met langhe. TBESLVYT. De cortste kinketens dan gheven de meeste wreetheyt, t'welck wy bewijsen moesten.

MERCKT.

comes so close that in the pulling of the bridle it touches the breast, the severity is at an end there; for even if one then pulls much more tightly, all this pressure will be exerted on the horse's breast, without producing more pressure against the chin or gums. But if another point of contact of the bit ring is further away from the breast, and the intermediate chain therefore also further away, it follows therefrom that it will be possible to pull those cheeks further back towards the breast than the others before the intermediate chain touches the breast, from which there must also evidently follow greater severity. But nothing can be said thereof if the intermediate chain in any way does not touch the breast.

3rd NOTE

It happens with many horses that they relieve themselves of the pressure of the bridle by raising their mouths, as the accompanying figure shows, so that the rider cannot then force them, but they run as they like. Nevertheless, someone might say: the point of contact of the bit ring is then further away from the horse's breast than in the other position, in such a way that therewith the bridle ought to be more severe, which seems to be contrary to the rule of this proposition. To this it is said that when the tight rein *AB* is parallel to the imaginary straight line from the point of contact *A* of the bit ring to the axial point *C* of the bit, as this position involves, then a tighter pulling at the upper cheek cannot cause any displacement or cause the curb chain to rise, and consequently there is no severity, for though the bit is pulled more tightly backwards, that does not cause the severe pressure described above. But if the tight rein is even higher than has been said above, the more tightly one pulls, the gentler the curb chain manifestly becomes. So this is an exception which can be understood from known causes.

5th PROPOSITION

The shortest curb chains cause the greatest severity.

It is to be assumed that the pressure of the bit does not begin until the curb chain touches the chin. But with a long curb chain the hand has to rise further before it touches the chin than with a short one, and therefore with the same movement of the hand greater pressure is exerted with short than with long curb chains. CONCLUSION. The shortest curb chains therefore cause the greatest severity, which we had to prove.

MERCKT.

Wy hebben hier boven ghefeyt daer voor te houden te fijn, dattet gheprang des montfticx eerft begint als de kinketen teghen de kin gheraeckt: doch ghebeuret wel dat de peerden eenich gheprang ghevoelen voor fulck gheraeckfel, ja met een toom fonder kinketen, t'een peert eer als t'ander, na datfe teer of hart van monde fijn: Oock na dat d'een toom van ftijver of flapper ftof, loffer of fluytender mocht ghemaeckt fijn als d'ander: Doch foo cleyn onfeker en onghelijck gheprang, en fchijnt gheen dieper onderfoucking noch befchrijving der omftandighen te vereyffchen, als van gheender acht wefende.

6 VOORSTEL.

Een prouftoom te maken, en daer uyt een ghebruyckelicke toom.

Wat prouftoom is hebben wy verclaert inde 17 bepaling. Om hier van het maeckfel te fegghen, dat mach aldus ghefchien: De gheftalt is ghelijck de volghende form aenwijft, alwaer A B twee ftanghen beteyckenen, die verlangt en vercort connen worden deur de fchuyvende fticken als C B, welcke ter begeerde langde connen vaft ghehecht worden mette fchrouven als D. Defe ftanghen draeyen elck op een bout als E, makende mettet boveftick fulcken houck of cromte alfmen begheert, en worden alfoo vaft ghehecht mette fchrouven F. De bovedeelen G H fijn eenvaerdigher dickte, foo lanck als de langfte diemen behouft. De ooghen als I fijn daer aen fchuyvende ghemaeckt, en worden met fchrouven als K vaft ghehecht ter plaets daermenfe begheert. Inder voughen dat hier mede foo wel het bovedeel als onderdeel fulcken langde gegeven wort alfmen wil.

T

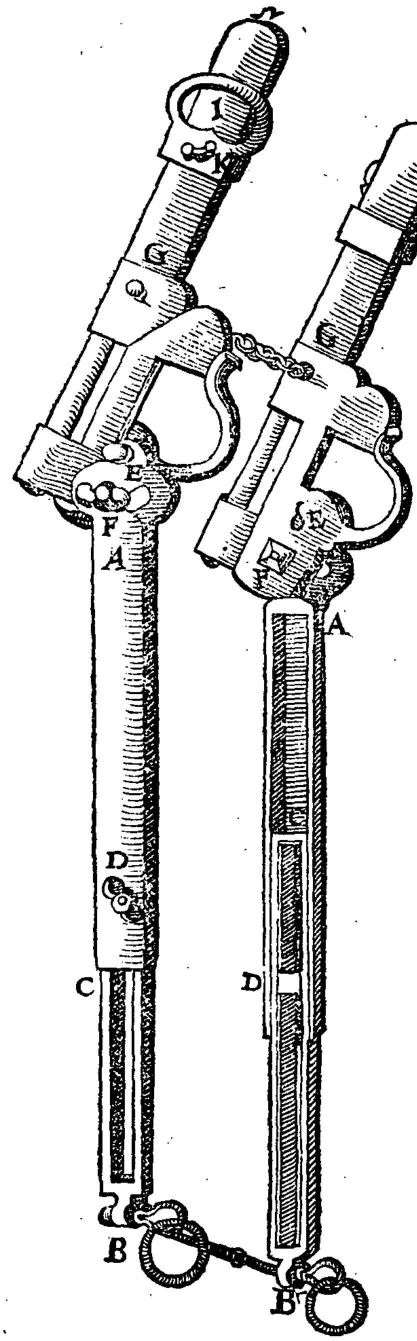
NOTE

We have said above that it is to be assumed that the pressure of the bit does not begin until the curb chain touches the chin. Yet it sometimes happens that horses feel some pressure before the curb chain touches the chin, nay, even with a bridle without a curb chain, according as a horse is tender or hard in the mouth. Also according as one bridle may be made of stiffer or softer material, more or less tightly fitting, than the other. But such a slight, uncertain, and dissimilar pressure does not seem to call for any deeper study or description of the circumstances, as being of no account.

6th PROPOSITION

To make a test bridle, and from that an actual bridle.

What a test bridle is, we have set forth in the 17th definition. As regards the construction, that may be effected as follows. The form is as shown in the following figure, where *AB* signifies two cheeks which can be lengthened and shortened by means of the sliding members *CB*, which can be fastened at the desired length with the screws *D*. Each of these cheeks pivots about a bolt *E*, including with the upper cheek such an angle or bend as is desired, and they are thus fastened with the screws *F*. The upper cheeks *GH* are of uniform thickness and as long as the longest that are required. The eyes *I* have been adapted to slide thereon, and are fastened with screws *K* in the place where they are desired, in such a way that thus both the upper and the lower cheek are given the desired length.

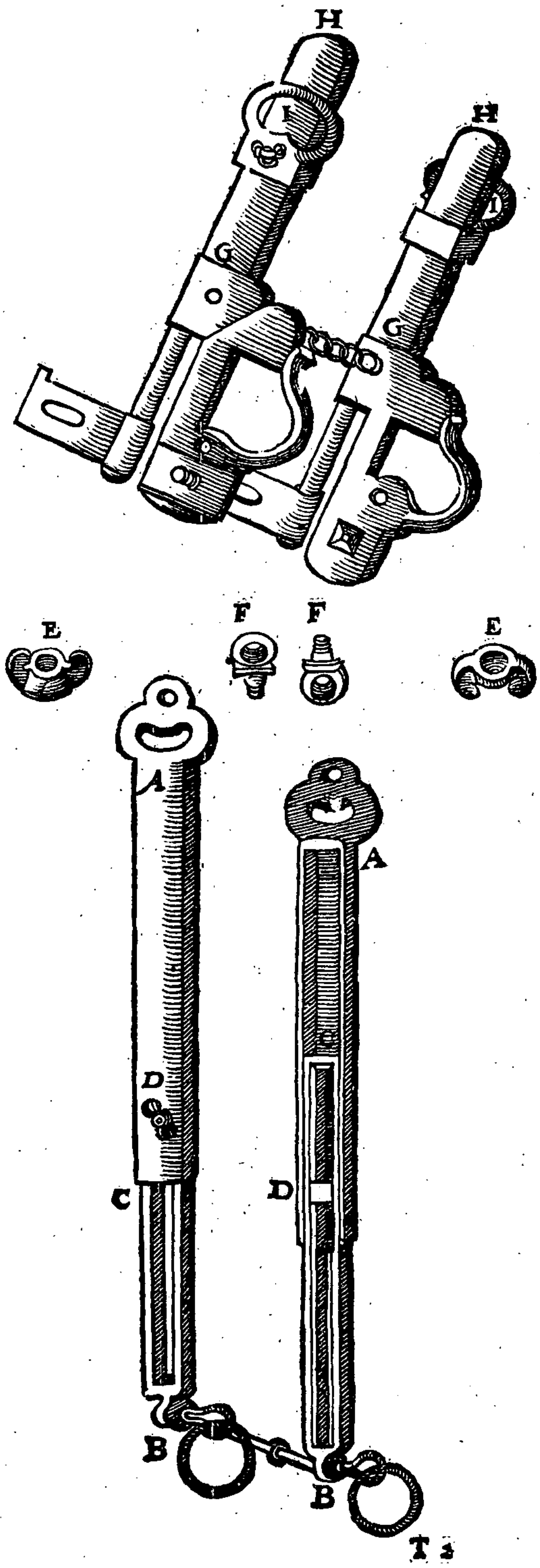


Tot hier toe is beſchreven de maniere des proufiooms int gheheel, de ſtic-ken by malcander vervoucht: Maer om noch breeder verclaring te doen vande form der ſtucken int beſonder, ſoo ſullen wy die hier nu verſcheyden ſtellen, alwaer de letteren andermael van beteyckening ſijn als vooren.

Dit is

So far the construction of the test bridle as a whole has been described, the pieces when assembled. But in order to give a more detailed exposition of the form of the pieces in particular, we shall now describe them here separately, the letters designating the same parts as before.

Dit is de form die sijn VORSTELICKE GENADE alsoo heeft doen maken, en metter daet bequaem bevint: doch alsmen daer in betering merckt, t'sal billich sijn die t'sijnen voordeele te nemen.



T 3

This is the form which his PRINCELY GRACE has had made in this manner, and which he finds suitable in practice; but if anyone sees a means of improving on it, he may take advantage thereof.

NV VANT MAKEN DES GHE- *bruyckelicken tooms deur t'behulp des prouftooms.*

An de prouftoom een montstick vervought sijnde na den eysch van t'peert, men sal deur t'behulp der schuyverkens, de langde der stanghen en bovedeelen, oock den raeckpunthouck, voor t'eerste stellen na t'ghene het voorghestelt peert schijnt te vereysschen: Maer t'selve an t'peert dadelick versocht sijnde, en bevonden wesende datter verandering moet gedaen sijn an een der vier saken, of an altemael, te weten verlanging of vercortingh der stanghen, verlanging of vercorting der bovedeelen, vermeerdering of vermindering des raeckpunthoucx, of verlanging of vercorting des kinketens, dat can van elck met luttel moeyte, groote sekerheyt, en seer haest gheschien; ia sonder den toom telcken af te moeten doen, oock sonder dat den Rijder behoeft af te stijghen. Nu de prouftoom soo ghestelt hebbende, datse voor dat peert past, men salse af doen, en een ghebruyckelicke toom doen maken, met sulcke keeren, form, en cyraet alsmen begheert, mits wel verstaende, dat de drie punten des raeckpunthoucx, even comen sulcken houck te maken als die des prouftooms, en de twee rechte verdochte linien dien houck begrijpende, oock vande selve langde als d'andere: Dat voort de tusschenketen, kinketen, en montstick, mede commen op dergelijcke ghestalt en form: T'welck soo sijnde, dees ghebruyckelicke toom moet het peert passen, en sal daer mede ter handt sijn, even als mette prouftoom, ghelijck sijn Vorstelicke Ghenade dat oock dadelick bevint.

Ettelicke van dese stof schrijvende, hebben gemaeckt toomen daermen verscheyden stanghen in mach steken met onghelijcke keeren, d'een crommer als d'ander: Maer het teughelrijncx raeckpunt op een selve plaets commende, soo en gheeft meerder noch minder cromheyt der keeren totte saeck niet, ghelijck int eerste Voorstel verclaert is: Of anders gheseyt, commende het teughelrijncx raeckpunt op een ander plaets, so en is meerder of minder cromheyt des stangs, de oirsaeck niet der veranderingh diemen inde regieringhe des peerts ghewaer wort, ghemerckt sulcx comt uyt verandering van plaets des teugelrijncx raeckpunt: Waer deur sulcke soucking sonder kennis der oirsaken soo moeylick en onseker valt, datter hun weynigh begheven tot deur soodanighe middel welpassende toomen te maken. T'besluyt. Wy hebben dan een prouftoom gemaeckt, en daer uyt een ghebruyckelicke toom na den eysch.

MERCKT.

Ymant overdenckende de ghemeene reghel der wichtighe ghedaenten van alle tuych daermen ghewelt mede doet, mocht segghen, dat wanneermen met even voorttreckingen des handts, de kinketen eveveel voortgancx geeft, t'mach mette langde der bovedeelen en stanghen sijn hoe't wil, daer volght een selve gheprang uyt. Om hier af by voorbeelt te spreken, gemaeckt sijnde twee toomen op even raeckpunthoucken, en de kinketen in d'een, met sulcken losheyt of verheyt vande kin als in d'ander, voort de stangen en bovedeelen *everedenich, doch van d'een kleender als van d'ander, de kinketen crijcht dan met eveveel voorttrecking des handts eveveel beweeghnis, en vervolghens een selve gheprang, t'welck ick deur een form breeder verclaren sal.

*Proportionales.

TGHE-

NOW AS TO THE CONSTRUCTION OF THE ACTUAL BRIDLE WITH THE AID OF THE TEST BRIDLE

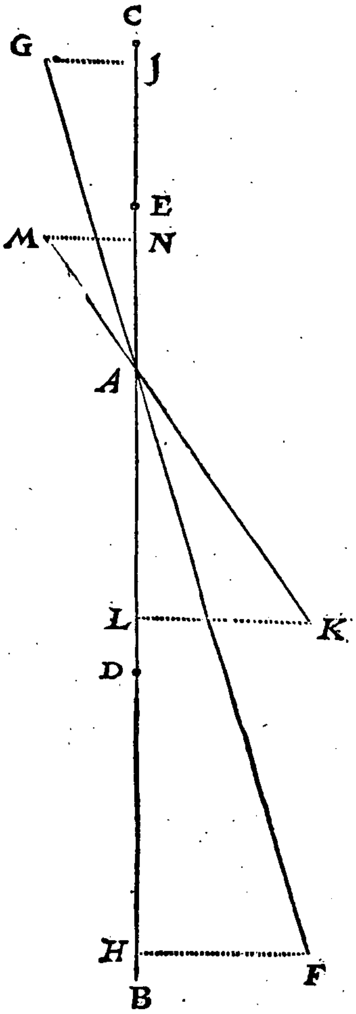
A bit having been attached to the test bridle according to the requirements of the horse, the length of the cheeks and the upper cheeks, and also the angle at the point of contact, shall first be adjusted by means of the sliding members in the way the horse in question seems to require. But when this has been tested in practice on the horse and it has been found that a change has to be made in one of the four members or all of them, to wit lengthening or shortening of the cheeks, lengthening or shortening of the upper cheeks, increasing or decreasing of the angle at the point of contact, or lengthening or shortening of the curb chain, this can be done with each of them with little trouble, great certainty, and very quickly, nay, even without having to take off the bridle every time, and also without the rider having to dismount. When the test bridle has been so adjusted that it fits the horse, it shall be taken off, and an actual bridle shall be caused to be made, with such twists, form, and adornments as may be desired, provided the three points of the angle at the point of contact make the same angle as that of the test bridle and the two straight imaginary lines comprehending that angle be also of the same length as the others, while further the intermediate chain, the curb chain, and the bit also have a similar position and form. This being so, this actual bridle is bound to fit the horse, and it will be held easily in hand, just as with the test bridle, as his PRINCELY GRACE indeed finds in practice.

Many writers dealing with this matter have made bridles into which may be mounted different cheeks with dissimilar twists, one more curved than the other. But if the point of contact of the bit ring comes in the same place, greater or lesser curvature is of no account in the matter, as has been set forth in the first Proposition. Or in other words: if the point of contact of the bit ring comes in another place, greater or lesser curvature of the cheek does not cause the change observed in the governing of the horse, seeing that this is due to a change of place of the point of contact of the bit ring. Owing to which such an examination without knowledge of the causes is so difficult and uncertain that few people try to make fitting bridles by such means. CONCLUSION. We have therefore made a test bridle, and from that an actual bridle, as required.

NOTE

Someone, reflecting on the common rule of the static properties of all devices with which force is exerted, might say that if with an equal amount of pulling of the hand the curb chain is equally advanced, no matter what the length of the upper cheeks and the cheeks, the same pressure will result therefrom. To give an example of this: two bridles being made with equal angles at the point of contact and the curb chain of one being just as loose or remote from the chin as the other, the cheeks and upper cheeks further being proportional, but those of the one smaller than those of the other, the curb chain is then displaced the same distance with an equal amount of pulling of the hand, and consequently it undergoes the same pressure, which I will explain more in detail by means of a figure.

TGHEGHEVEN. Laet A B een langhe ftang beteyckenen, A C heur lanck bovedeel inde voorghetrocken B A, daer na fy A D een corte ftang, A E heur cort bovedeel, in fulcken reden tot A D, als A C tot A B, voort fy ghetrocken A F even an A B, en A G inde voortgetrockẽ F A even an A C, en van F de lini F H rechthouckich op B C, oock G I rechthouckich op de felve B C, daer na A K even met A D, oock foo dat K L rechthouckich op B C even fy met F H, en A M inde voortghetrocken K A even met A E, en M N rechthouckich op B C. Dit fo wefende, laet ons nu nemen den teughelrinck B der langhe ftang, ghetrocken te fijn van B tot F, fulcx dat haer voortganck fy H F, en de corte ftang van D tot K, foo dat haer voortganck fy L K, en fal dan het oogh C des langften bovedeels ghecommen fijn an G, diens voortganck I G, en t'oogh E des cortften bovedeels an M, diens voortganck N M. Maer de voortganck H F en L K, is te houden voor des handts voorttreckingh an de teughelriem, om datfe daer me even fijn, en I G met M N voor de kinketens voortganck, als daer me oock even wefende. T'welc foo fijnde, wy moeten bewijfen dat N M even is met I G, waer uyt gelijck t'voornemen was te bewijfen een felve gheprang moet volghen.



T B E W Y S.

Den driehouck A K L, is ghelijck metten driehouck A M N, waer deur fy haer lijckftandighe fijden everedenich hebben, te weten *Holomoga latera.*

Ghelijck A K tot A M, alfoo K L tot M N.

Den driehouck A F H, is gelijck metten driehouck A I G, waer deur fy haer lijckftandighe fijden everedenich hebben, te weten

Ghelijck A F tot A G, alfoo F H tot G I.

Maer ghelijck A F tot A G, alfoo A K tot A M, daerom

Ghelijck A K tot A M, alfoo F H tot G I.

Maer F H is even met K L deur t'ghegheven, daerom

Ghelijck A K tot A M, alfoo K L tot G I.

Sulcx dat G I en M N, elck vierde everedenighe pael fijn der felve drie, te weten M N in d'eerfte everedenheyt, en G I in defe laetfte, waer deur fy even moeten wefen.

T 3 Nu dan

SUPPOSITION. Let *AB* signify a long cheek, *AC* its long upper part in *BA* produced. Further *AD* shall be a short cheek, *AE* its short upper part, having the same ratio to *AD* as *AC* to *AB*. Further there shall be drawn *AF* equal to *AB*, and *AG* in *FA* produced, equal to *AC*, and from *F* the line *FH* at right angles to *BC*, also *GI* at right angles to this same *BC*, further *AK* equal to *AD*, also so that *KL*, at right angles to *BC*, be equal to *FH*, and *AM*, in *KA* produced, equal to *AE*, and *MN* at right angles to *BC*. This being so, let us now assume the bit ring *B* of the long cheek to have been pulled from *B* to *F*, so that its advance be *HF*, and the short cheek from *D* to *K*, so that its advance be *LK*. Then the eye *C* of the longest upper cheek will have reached *G*, its advance being *IG*, and the eye *E* of the shortest upper cheek will have reached *M*, its advance being *NM*. But the advance *HF* and *LK* is to be considered the hand's pulling at the rein, because they are equal thereto, and *IG* with *MN* the advance of the curb chains, because they are equal to *IG*, from which, as was indended to be proved, there must result the same pressure.

PROOF

The triangle *AKL* is similar to the triangle *AMN*, in consequence of which their homologous sides are proportional, to wit

As *AK* is to *AM*, so is *KL* to *MN*.

The triangle *AFH* is similar to the triangle *AIG*, in consequence of which their homologous sides are proportional, to wit

As *AF* is to *AG*, so is *FH* to *GI*.

But as *AF* is to *AG*, so is *AK* to *AM*, therefore

As *AK* is to *AM*, so is *FH* to *GI*.

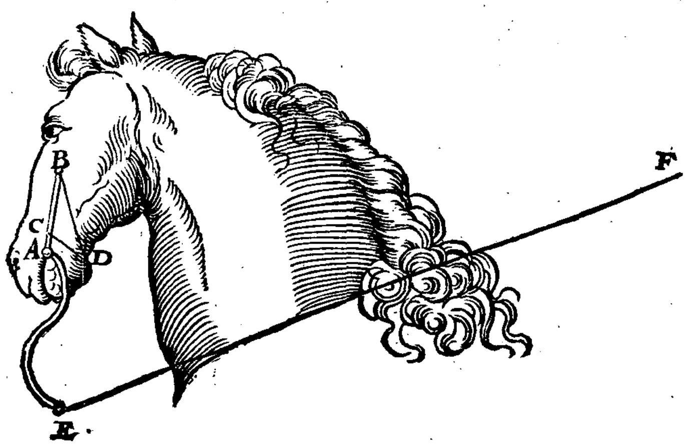
But *FH* is equal to *KL* by the supposition, therefore

As *AK* is to *AM*, so is *KL* to *GI*.

So that *GI* and *MN* are each the fourth proportional to the same three terms, to wit *MN* in the first proportion and *GI* in this last, in consequence of which they must be equal.

Nu dan de kinketen van d'een en d'ander toom aldus eveveel voortganck crijghende, waer uyt ymant dencken mocht sulcx een selve gheprang te geven, en datter nochtans groot verschil in valt, so sullen wy daer af wat breeder seggē.

D'ervaring leert, soo ettelicke oock schrijven, dat langer bovedeel en aen sommighe peerden het hooft hoogher doen verheffen als corter: Waer af sijn VORSTELICKE GENADE d'oirsaeck hout dusdanich te wesen: Laet A B een lanck bovedeel beteyckenen, A C een cort, B D de kinketen ant lang bovedeel, en C D de kinketen ant cort bovedeel. De langhe kinketen B D maeckt opt bovedeel een scherper houck dan de corter kinketen C D, want scherper is den



houck A B D, dan A C D. Hier me sietmen dat deur trecking des teughelriems E F, het bovedeel A B beweeghnis crijghende, soo perst de kinketen C D platter teghens t'peerts kin, dan de kinketen B D, welcke daer teghen meer opwaert druct: En t'peert om die opwaert persing te versachtē, verheft het hooft hooger.

Ymant soude hier op meughen segghen, dat by aldien sulcx de eyghenschap waer van langher bovedeelen, dat de * daet daer af niet alleen blijcken en soude an sommighe peerden, ghelijck boven ghesevt is, maer an allen, t'welck nochtans teghen d'ervaring te strijden by verscheyden betuycht wort, en onder anderen deur *le Sieur de la Broüe* int 3 bouck onder dit opschrift. *Effectus.*

Occasions pour lesquelles on doit faire l'œil de la branche plus haut ou plus bas que la mesure ordinaire.

Ick heb oock sijn VORSTELICKE GHENADE hooren bevestighen dadelick bevonden te hebben, dat verlanging van bovedeelen an sommige peerden het hooft dede dalen, an ettelicke verheffen: T'welck hy doen, ghelijck ander, met verwonderen ansach: Maer daer na hier op met kennis der Weeghconst lettende, heeft voor ghewis gehouden dit d'oirsaeck te wesen. Verlanging des bovedeels, t'welck meerder wreetheyt mebrengt op het tantvlees en teghen de kin deur het 3 voorstel, werckt twee verkeerde saken t'seffens, want deur de stijver perssing des montsticx teghen het tantvlees, is t'peert geneycht het hooft neerwaert te buyghen, om die weedom te versachten, maer deur de stijver opwaert perssing des kinketens teghen de kin, ist om die smerte te verminderen gheneycht het hooft opwaert te verheffen, gelijck wy boven verclaert hebben: Dese twee t'seffens aencommende, het souckt hem dadelick meest t'ontlasten

Now therefore the curb chain of one bridle as well as the other thus making the same advance, from which someone might conclude that this produces the same pressure, while nevertheless there is great difference between them, we will discuss this a little more in detail.

Experience teaches, as many writers affirm, that longer upper cheeks make some horses lift their heads higher than shorter ones, the cause of which is considered by his PRINCELY GRACE to be as follows: Let *AB* signify a long upper cheek, *AC* a short one, *BD* the curb chain on the long upper cheek and *CD* the curb chain on the short upper cheek. The long curb chain *BD* makes a more acute angle with the upper cheek than the shorter curb chain *CD*, for the angle *ABD* is more acute than *ACD*. Thus it is seen that when by the pulling of the rein *EF* the upper cheek *AB* is moved, the curb chain *CD* presses more flatly against the horse's chin than the curb chain *BD*, which presses against it in a more upward direction. And the horse, in order to relieve that upward pressure, will lift its head higher.

Someone might say to this that if this were the property of longer upper cheeks, the effect would be apparent not only with some horses, as has been said above, but with all, which is nevertheless stated by different people to be contrary to experience, among others by *le Sieur de la Brouë* [1]), in the 3rd book with the following heading:

> *Occasions pour lesquelles on doit faire l'oeil de la branche plus haut ou plus bas que la mesure ordinaire.*

I have also heard it affirmed by his PRINCELY GRACE that he has found in practice that the lengthening of the upper cheeks caused some horses to lower and many to lift their heads, which he watched, just like others, with surprise. But when he afterwards noted this with knowledge of the Art of Weighing, he held it for certain that this is the cause. Lengthening of the upper cheek, which involves greater severity to the gums and against the chin, by the 3rd proposition, simultaneously produces two contrary effects, for owing to the stiffer pressure of the bit against the gums the horse is inclined to bend its head downwards, in order to relieve that pain, but owing to the stiffer upward pressure of the curb chain against the chin it tends, in order to relieve that pain, to lift its head upwards, as we have set forth above. When these two tendencies act simultaneously, it tries in practice to relieve itself most of that which causes it the greatest pain.

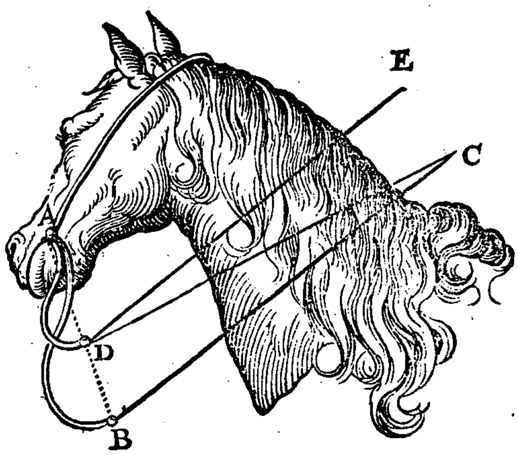
[1]) The work referred to is: *Le cavalerice françois composé par* Salomon de la Broue. Paris 1602. Livre III, ch. 25; p. 62.

van t'ghene hem de meeste weedom aendoet: Maer sommige peerden sijn teer van tantvlees en hart van kin, ander verkeert, hart van tantvlees en teer van kin, waer uyt volght dattet een peert deur verlanging des bovedeels het hooft leger buycht, het ander hoogher verheft: Maer om int ghemeen daer af te spreken, alle langher bovedeelen intansien der opwaert persing des kinketens alleen, veroirsaken eenige genegentheyt des peerts tot verheffiing des hoofts, hoe wel het nochtans om d'ander meerder smerte t'verkeerde wel mocht te werc stellē.

Uyt het voorgaende valt te besluyten, datmen tot peerden die uyter natuer het hooft hooch genouch dragen, en het tantvlees niet te teer en hebben, soude meughen ghebruycken corter bovedeelen met een sluytender kinketen, te meer dat langer bovedeelen en losser kinketens met een stercke snack ghetrocken wesende, het montstick en kinketen veel harder, als met een slach ancommende, de peerden den mont bederven, meer als corte bovedeelen, en sluytender kinketens, die sachter ancommen, en nochtans daer na eveveel persing gheven. Ten anderen dat al te langhe kinketens als B D, lichtelick over de kin slibberen, sonder dat den Ruyter het peert dan regieren can, welck ongheval de kinketens, als C D niet onderworpen en sijn.

Merckt noch dat alsmen niet ghedronghen en is langhe bovedeelen te nemen om t'peert sijn hooft te doen verheffen, (t'welck ghebeurt als de teerheyt des tantvlees niet en overtreft de teerheyt des kins) soo machmen een seer cort bovedeel ghebruycken, en de stanghen van langde soose best vougen: Daer na vermeerderen of verminderen de wreetheyt na sijn wille, met verlanging of vercorting der kinketen.

Maer want sijn VORSTELICKE GHENADE dese eygenschappen seer nauwe deurgront heeft, soo sal ick hier stellen noch wat ander onghelijckheyt, tusschen de boveschreven toomen met everedenighe stangen en bovedeelen:



Laet tot dien eynde A B een lange sijn, diens teugelriem B C, en A D een corte, diens teughelriem D E, en met haer bovedeelen neem ick everedenich. Alwaert nu dat dese twee stanghen om die everedenheyt een selve gheprang gaven, soo ist nochtans kennelick dat de treckende hant niet tot een selve plaets en soude moeten blijvē, maer sose op B treckende, is an C, sy sal op D treckende, moeten sijn by E, sulcx dat D E * evewijdeghe is met B C, want treckende de teughelriem van D tot C, sy maeckt op de rechte lini A B een ander houck dan D E, t'welck openbaerlick verandering moet mebrenghen, te weten minder wreetheyt an C, dan an E.

* Parallela.

DES TOOMPRANGS
EYNDE.

But some horses have tender gums and a hard chin, others on the contrary have hard gums and a tender chin, from which it follows that in consequence of lengthening of the upper cheek one horse will bend its head lower, and another will lift it higher. But to speak of this in a general way: all longer upper cheeks, as regards the upward pressure of the curb chain alone, cause some inclination in the horse to lift its head, though nevertheless, because of the other greater pain, it might do the contrary.

From the foregoing it may be concluded that for horses which by nature carry their heads high enough and do not have too tender gums one might use shorter upper cheeks with a more closely fitting curb chain, the more so because longer upper cheeks and looser curb chains, being pulled too tightly, the bit and the curb chain acting much harder, abruptly, spoil the horses' mouths more than short upper cheeks and more closely fitting curb chains, which act more gently and nevertheless cause further the same amount of pressure. Secondly, too long curb chains, such as *BD*, will hang too loosely on the chin, the rider then being unable to govern the horse, to which inconvenience curb chains such as *CD* are not subject.

It should also be noted that if it is not necessary to take long upper cheeks in order to make the horse lift its head (which happens when the tenderness of the gums does not exceed the tenderness of the chin), one may use a very short upper cheek and cheeks of the length that is most suitable, and then increase or decreasse the severity at will, by lengthening or shortening the curb chain.

But because his PRINCELY GRACE has studied these properties very closely, I will here describe another dissimilarity between the bridles described above with proportional cheeks and upper cheeks.

To this end let *AB* be a long cheek, whose rein is *BC*, and *AD* a short cheek, whose rein is *DE*, and I take them to be proportional to their upper cheeks. Even if these two cheeks, because of that proportionality, produced the same pressure, it is nevertheless obvious that the pulling hand would not have to remain in the same place; but if, pulling at *B*, it is in *C*, when pulling at *D*, it will have to be in *E*, so that *DE* is parallel to *BC*, for when the rein is pulled from *D* to *C*, it makes a different angle with the straight line *AB* than *DE*, which manifestly is bound to cause a change, to wit less severity at *C* than at *E*.

END OF THE PRESSURE OF THE BRIDLE.

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